Chapter 1, Problem 1

How many coulombs are represented by these amounts of electrons:

(a) 6.482×10^{17} (b) 1.24×10^{18}

(c) 2.46×10^{19} (d) 1.628×10^{20}

Chapter 1, Solution 1

(a) q = 6.482x10^{17} x [-1.602x10^{-19} C] = **-0.10384 C**

(b) q = 1. 24x10^{18} x [-1.602x10^{-19} C] = **-0.19865 C**

(c) q = 2.46x10^{19} x [-1.602x10^{-19} C] = **-3.941 C**

(d) q = 1.628x10^{20} x [-1.602x10^{-19} C] = **-26.08 C**

Chapter 1, Problem 2.

Determine the current flowing through an element if the charge flow is given by

(a) $q(t) = (3t + 8)$ mC

(b) $q(t) = (8t^2 + 4t\text{-}2)$ C

(c) $q(t) = (3e^{-t} - 5e^{-2t})$ nC

(d) $q(t) = 10\sin 120\pi$ t pC

(e) $q(t) = 20e^{-4t}\cos 50t$ μC

Chapter 1, Solution 2

(a) i = dq/dt = 3 mA

(b) i = dq/dt = (16t + 4) A

(c) i = dq/dt = (-3e^{-t} + 10e^{-2t}) nA

(d) i=dq/dt = $1200\pi \cos 120\pi t$ pA

(e) i =dq/dt = $-e^{-4t}(80\cos 50t + 1000\sin 50t)$ μ**A**

Chapter 1, Problem 3.

Find the charge $q(t)$ flowing through a device if the current is:

(a) $i(t) = 3\text{A}, q(0) = 1\text{C}$

(b) $i(t) = (2t + 5)\text{mA}, q(0) = 0$

(c) $i(t) = 20\cos(10t + \pi/6)\mu\text{A}, q(0) = 2\,\mu\,\text{C}$

(d) $i(t) = 10e^{-30t}\sin 40t\text{A}, q(0) = 0$

Chapter 1, Solution 3

(a) $\text{q(t)} = \int \text{i(t)dt} + \text{q(0)} = \underline{(3\text{t} + 1)\ \ \text{C}}$

(b) $\text{q(t)} = \int (2\text{t} + \text{s})\ \text{dt} + \text{q(v)} = \underline{(\text{t}^2 + 5\text{t})\ \ \text{mC}}$

(c) $\text{q(t)} = \int 20\ \cos\ (10\text{t} + \pi/6) + \text{q(0)} = \underline{(2\sin(10t + \pi/6) + 1)\mu\text{C}}$

(d) $$\text{q(t)} = \int 10\text{e}^{-30\text{t}}\sin 40\text{t} + \text{q(0)} = \frac{10\text{e}^{-30\text{t}}}{900 + 1600}(-30\sin 40\text{t} - 40\cos\text{t})$$
$$= \underline{-\ \text{e}^{-30\text{t}}(0.16\cos 40\ \text{t} + 0.12\ \sin 40\text{t})\ \text{C}}$$

Chapter 1, Problem 4.

A current of 3.2 A flows through a conductor. Calculate how much charge passes through any cross-section of the conductor in 20 seconds.

Chapter 1, Solution 4

$\text{q} = \text{it} = 3.2 \times 20 =$ **$\underline{64\ \text{C}}$**

Chapter 1, Problem 5.

Determine the total charge transferred over the time interval of $0 \le \text{t} \le 10\text{s}$ when $i(t) = \frac{1}{2}t$ A.

Chapter 1, Solution 5

$$q = \int idt = \int_0^{10} \frac{1}{2}tdt = \frac{t^2}{4}\Bigg|_0^{10} = \underline{25\ \text{C}}$$

Chapter 1, Problem 6.

The charge entering a certain element is shown in Fig. 1.23. Find the current at:
(a) $t = 1$ ms (b) $t = 6$ ms (c) $t = 10$ ms

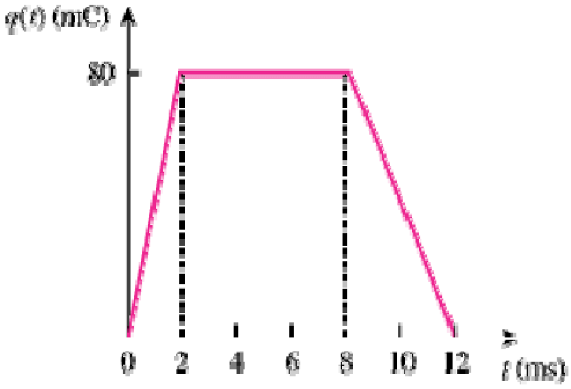


Figure 1.23

Chapter 1, Solution 6

(a) At t = 1ms, $i = \frac{dq}{dt} = \frac{80}{2} =$ **40 A**

(b) At t = 6ms, $i = \frac{dq}{dt} =$ **0 A**

(c) At t = 10ms, $i = \frac{dq}{dt} = \frac{80}{4} =$ **–20 A**

Chapter 1, Problem 7.

The charge flowing in a wire is plotted in Fig. 1.24. Sketch the corresponding current.

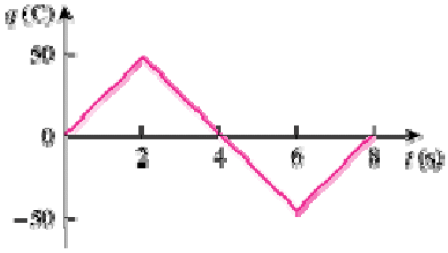
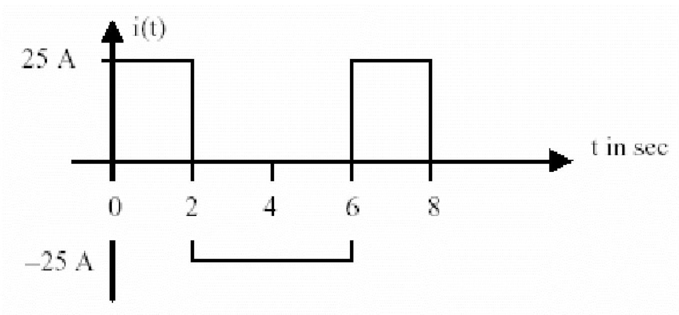


Figure 1.24

Chapter 1, Solution 7

$$i = \frac{dq}{dt} = \begin{cases} 25A, & 0 < t < 2 \\ -25A, & 2 < t < 6 \\ 25A, & 6 < t < 8 \end{cases}$$

which is sketched below:



Chapter 1, Problem 8.

The current flowing past a point in a device is shown in Fig. 1.25. Calculate the total charge through the point.

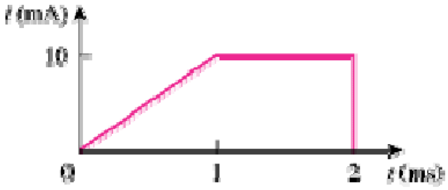


Figure 1.25

Chapter 1, Solution 8

$$q = \int i dt = \frac{10 \times 1}{2} + 10 \times 1 = \underline{15\ \mu C}$$

Chapter 1, Problem 9.

The current through an element is shown in Fig. 1.26. Determine the total charge that passed through the element at:
(a) $t = 1$ s (b) $t = 3$ s (c) $t = 5$ s

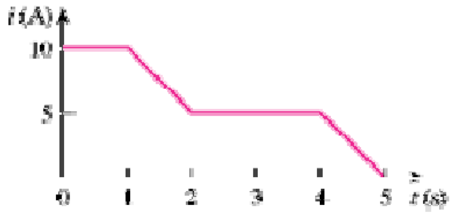


Figure 1.26

Chapter 1, Solution 9

(a) $q = \int i dt = \int_0^1 10\, dt = \underline{10\text{ C}}$

(b) $q = \int_0^3 i dt = 10 \times 1 + \left(10 - \frac{5 \times 1}{2}\right) + 5 \times 1$
$= 15 + 7.5 + 5 = \underline{22.5\text{C}}$

(c) $q = \int_0^5 i dt = 10 + 10 + 10 = \underline{30\text{ C}}$

Chapter 1, Problem 10.

A lightning bolt with 8 kA strikes an object for 15 μs. How much charge is deposited on the object?

Chapter 1, Solution 10

$q = it = 8x10^3x15x10^{-6} = \underline{\textbf{120 mC}}$

Chapter 1, Problem 11.

A rechargeable flashlight battery is capable of delivering 85 mA for about 12 h. How much charge can it release at that rate? If its terminals voltage is 1.2 V, how much energy can the battery deliver?

Chapter 1, Solution 11

q= it = 85 x10^{-3} x 12 x 60 x 60 = 3,672 C

E = pt = ivt = qv = 3672 x1.2 = 4406.4 J

Chapter 1, Problem 12.

If the current flowing through an element is given by

$$i(t) = \begin{cases} 3t\text{A}, & 0 < t < 6\text{s} \\ 18\text{A}, & 6 < t < 10\text{s} \\ -12\text{ A}, & 10 < t < 15\text{s} \\ 0, & t > 15\text{s} \end{cases}$$

Plot the charge stored in the element over 0 < t < 20s.

Chapter 1, Solution 12

For 0 < t < 6s, assuming q(0) = 0,

$$q(t) = \int_0^t idt + q(0) = \int_0^t 3tdt + 0 = 1.5t^2$$

At t=6, q(6) = $1.5(6)^2$ = 54

For 6 < t < 10s,

$$q(t) = \int_6^t idt + q(6) = \int_6^t 18dt + 54 = 18t - 54$$

At t=10, q(10) = 180 – 54 = 126

For 10<t<15s,

$$q(t) = \int_{10}^t idt + q(10) = \int_{10}^t (-12)dt + 126 = -12t + 246$$

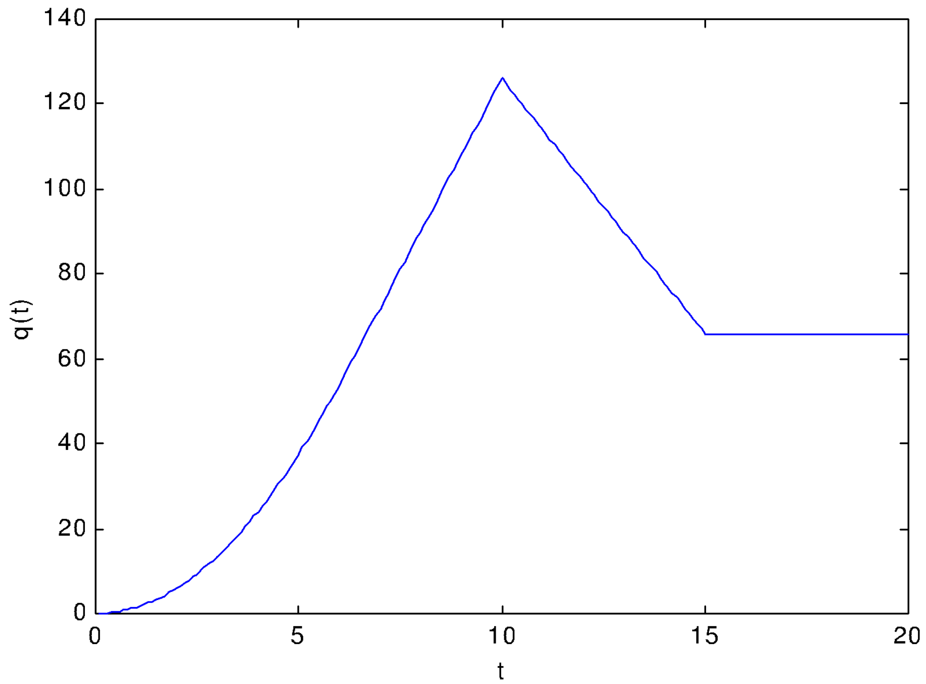
At t=15, q(15) = -12x15 + 246 = 66
For 15<t<20s,

$$q(t) = \int_{15}^{t} 0dt + q(15) = 66$$

Thus,

$$q(t) = \begin{cases} 1.5t^2 \ \textbf{C}, \ \ \textbf{0 < t < 6s} \\ 18t - 54 \ \textbf{C}, \ \ \textbf{6 < t < 10s} \\ -12t + 246 \ \textbf{C}, \ \ \textbf{10 < t < 15s} \\ 66 \ \textbf{C}, \ \ \textbf{15 < t < 20s} \end{cases}$$

The plot of the charge is shown below.



Chapter 1, Problem 13.

The charge entering the positive terminal of an element is

$$q = 10 \sin 4\pi t \text{ mC}$$

while the voltage across the element (plus to minus) is

$$v = 2 \cos 4\pi t \text{ V}$$

(a) Find the power delivered to the element at t = 0.3 s
(b) Calculate the energy delivered to the element between 0 and 0.6s.

Chapter 1, Solution 13

(a) $i = \frac{dq}{dt} = 40\pi \cos 4\pi t$ mA

$p = vi = 80\pi \cos^2 4\pi t$ mW

At t=0.3s,

$$p = 80\pi \cos^2(4\pi x 0.3) = \underline{164.5 \text{ mW}}$$

(b) $W = \int p dt = 80\pi \int_0^{0.6} \cos^2 4\pi t dt = 40\pi \int_0^{0.6} [1 + \cos 8\pi t] dt$ mJ

$$W = 40\pi \left[0.6 + \frac{1}{8\pi} \sin 8\pi t \Big|_0^{0.6} \right] = \underline{78.34 \text{ mJ}}$$

Chapter 1, Problem 14.

The voltage v across a device and the current I through it are

$$v(t) = 5\cos 2t \text{ V}, \quad i(t) = 10\left(1 - e^{-0.5t}\right) \text{A}$$

Calculate:

(a) the total charge in the device at t = 1 s

(b) the power consumed by the device at t = 1 s.

Chapter 1, Solution 14

(a)
$$q = \int i dt = \int_0^1 10\left(1 - e^{-0.5t}\right)dt = 10\left(t + 2e^{-0.5t}\right)\Big|_0^1$$
$$= 10\left(1 + 2e^{-0.5} - 2\right) = \underline{2.131 \text{ C}}$$

(b)
$$p(t) = v(t)i(t)$$
$$p(1) = 5\cos 2 \cdot 10(1 - e^{-0.5}) = (-2.081)(3.935)$$
$$= \underline{\mathbf{-8.188 \text{ W}}}$$

Chapter 1, Problem 15.

The current entering the positive terminal of a device is $i(t) = 3e^{-2t}$ A and the voltage across the device is $v(t) = 5\, di/dt$ V.

(a) Find the charge delivered to the device between t = 0 and t = 2 s.

(b) Calculate the power absorbed.

(c) Determine the energy absorbed in 3 s.

Chapter 1, Solution 15

(a)
$$q = \int i dt = \int_0^2 3e^{-2t} dt = \frac{-3}{2}e^{2t}\Big|_0^2$$
$$= -1.5\left(e^{-4} - 1\right) =$$
$$\underline{\mathbf{1.4725 \text{ C}}}$$

(b)
$$v = \frac{5di}{dt} = -6e^{2t}(5) = -30e^{-2t}$$
$$p = vi = \underline{-90\, e^{-4t} \text{W}}$$

(c)
$$w = \int p dt = -90\int_0^3 e^{-4t}\, dt = \frac{-90}{-4}e^{-4t}\Big|_0^3 = \underline{-22.5 \text{ J}}$$

Chapter 1, Problem 16.

Figure 1.27 shows the current through and the voltage across a device. (a) Sketch the power delivered to the device for t >0. (b) Find the total energy absorbed by the device for the period of 0< t < 4s.

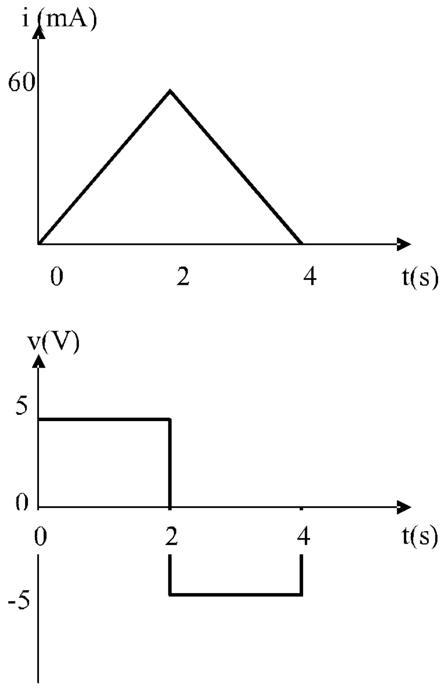


Figure 1.27 For Prob. 1.16.

Chapter 1, Solution 16

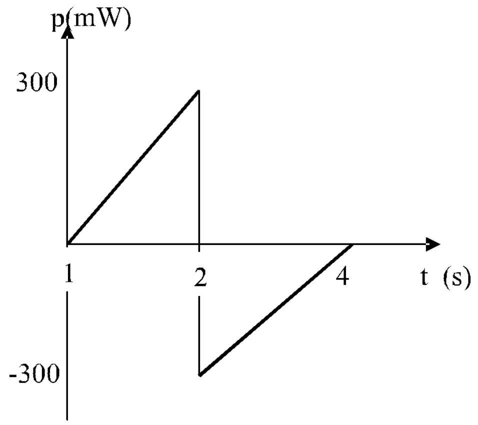
(a)

$$i(t) = \begin{cases} 30t \text{ mA}, & 0 < t < 2 \\ 120\text{-}30t \text{ mA}, & 2 < t < 4 \end{cases}$$

$$v(t) = \begin{cases} 5 \text{ V}, & 0 < t < 2 \\ -5 \text{ V}, & 2 < t < 4 \end{cases}$$

$$p(t) = \begin{cases} 150t \text{ mW}, & 0 < t < 2 \\ -600+150t \text{ mW}, & 2 < t < 4 \end{cases}$$

which is sketched below.



(b) From the graph of p,

$$W = \int_0^4 p\,dt = \underline{0 \text{ J}}$$

Chapter 1, Problem 17.

Figure 1.28 shows a circuit with five elements. If

$p_1 = -205$ W, $p_2 = 60$ W, $p_4 = 45$ W, $p_5 = 30$ W,

calculate the power p_3 received or delivered by element 3.

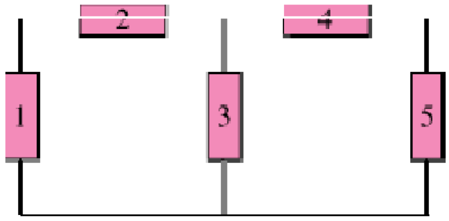


Figure 1.28

Chapter 1, Solution 17

Σ p = 0 $\rightarrow$ -205 + 60 + 45 + 30 + p_3 = 0

p_3 = 205 – 135 = 70 W

Thus element 3 receives **70 W.**

Chapter 1, Problem 18.

Find the power absorbed by each of the elements in Fig. 1.29.

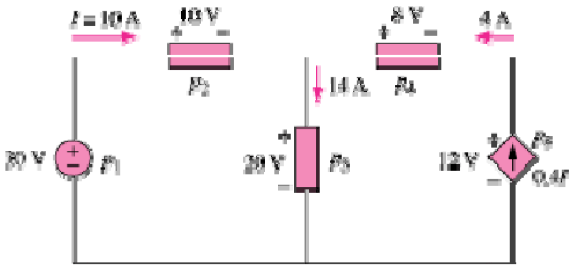


Figure 1.29

Chapter 1, Solution 18

$p_1 = 30(-10) =$ **-300 W**
$p_2 = 10(10) =$ **100 W**
$p_3 = 20(14) =$ **280 W**
$p_4 = 8(-4) =$ **-32 W**
$p_5 = 12(-4) =$ **-48 W**

Chapter 1, Problem 19.

Find I in the network of Fig. 1.30.

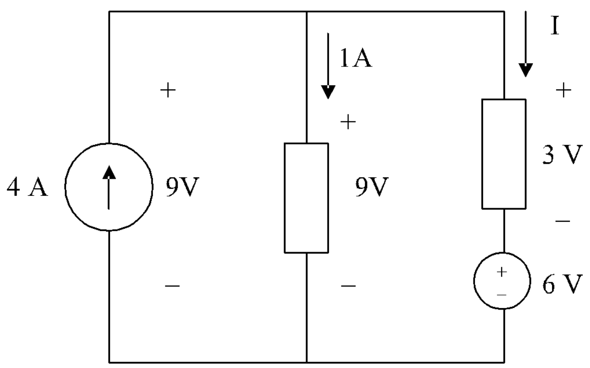


Figure 1.30 For Prob. 1.19.

Chapter 1, Solution 19

$$I = 4 - 1 = \underline{3\ \text{A}}$$

Or using power conservation,

$$9x4 = 1x9 + 3I + 6I = 9 + 9I$$

$$4 = 1 + I \text{ or } I = \underline{\mathbf{3\ A}}$$

Chapter 1, Problem 20.

Find V_0 in the circuit of Fig. 1.31.

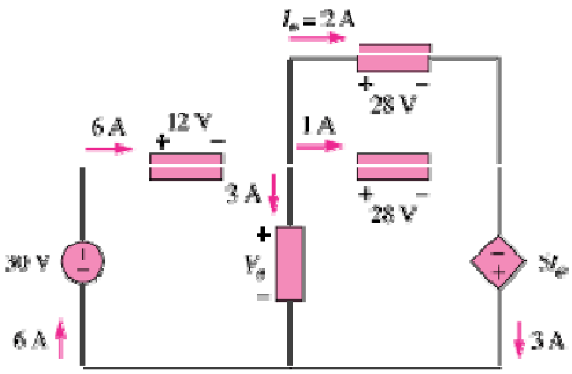


Figure 1.31

Chapter 1, Solution 20

Since Σ p = 0

$-30\times6 + 6\times12 + 3V_0 + 28 + 28\times2 - 3\times10 = 0$

$72 + 84 + 3V_0 = 210$ or $3V_0 = 54$

$V_0 =$ **<u>18 V</u>**

Chapter 1, Problem 21.

A 60-W, incandescent bulb operates at 120 V. How many electrons and coulombs flow through the bulb in one day?

Chapter 1, Solution 21

$p = vi \longrightarrow i = \frac{p}{v} = \frac{60}{120} = 0.5 \text{ A}$

q = it = 0.5x24x60x60 = 43200 <u>C</u>

$N_e = qx6.24x10^{18} = \underline{2.696x10^{23} \text{ electrons}}$

Chapter 1, Problem 22.

A lightning bolt strikes an airplane with 30 kA for 2 ms. How many coulombs of charge are deposited on the plane?

Chapter 1, Solution 22

$q = it = 30x10^3 x2x10^{-3} = \underline{60\ C}$

Chapter 1, Problem 23.

A 1.8-kW electric heater takes 15 min to boil a quantity of water. If this is done once a day and power costs 10 cents per kWh, what is the cost of its operation for 30 days?

Chapter 1, Solution 23

W = pt = 1.8x(15/60) x30 kWh = 13.5kWh
C = 10cents x13.5 = **$1.35**

Chapter 1, Problem 24.

A utility company charges 8.5 cents/kWh. If a consumer operates a 40-W light bulb continuously for one day, how much is the consumer charged?

Chapter 1, Solution 24

W = pt = 40 x24 Wh = 0.96 kWh
C = 8.5 cents x0.96 = **8.16 cents**

Chapter 1, Problem 25.

A 1.2-kW toaster takes roughly 4 minutes to heat four slices of bread. Find the cost of operating the toaster once per day for 1 month (30 days). Assume energy costs 9 cents/kWh.

Chapter 1, Solution 25

$$\text{Cost} = 1.2\ \text{kW} \times \frac{4}{60}\text{hr} \times 30 \times 9\ \text{cents/kWh} = \underline{21.6\ \text{cents}}$$

Chapter 1, Problem 26.

A flashlight battery has a rating of 0.8 ampere-hours (Ah) and a lifetime of 10 hours.

(a) How much current can it deliver?
(b) How much power can it give if its terminal voltage is 6 V?
(c) How much energy is stored in the battery in kWh?

Chapter 1, Solution 26

(a) $i = \dfrac{0.8\text{A}\cdot\text{h}}{10\text{h}} = \underline{80\text{ mA}}$

(b) $p = vi = 6 \times 0.08 = \underline{\mathbf{0.48\ W}}$

(c) $w = pt = 0.48 \times 10\text{ Wh} = \underline{\mathbf{0.0048\ kWh}}$

Chapter 1, Problem 27.

A constant current of 3 A for 4 hours is required to charge an automotive battery. If the terminal voltage is 10 + *t*/2 V, where *t* is in hours,

(a) how much charge is transported as a result of the charging?
(b) how much energy is expended?
(c) how much does the charging cost? Assume electricity costs 9 cents/kWh.

Chapter 1, Solution 27

(a) Let $T = 4\text{h} = 4 \times 3600$

$$q = \int i dt = \int_0^T 3dt = 3T = 3 \times 4 \times 3600 = \underline{43.2\text{ kC}}$$

(b) $W = \int p dt = \int_0^T vi dt = \int_0^T (3)\left(10 + \dfrac{0.5t}{3600}\right)dt$

$$= 3\left(10t + \frac{0.25t^2}{3600}\right)\Bigg|_0^{4\times3600} = 3[40 \times 3600 + 0.25 \times 16 \times 3600]$$

$$= \underline{475.2\text{ kJ}}$$

(c) $W = 475.2\text{ kWs}, \quad (\text{J} = \text{Ws})$

$$\text{Cost} = \frac{475.2}{3600}\text{kWh} \times 9\text{ cent} = \underline{1.188\text{ cents}}$$

Chapter 1, Problem 28.

A 30-W incandescent lamp is connected to a 120-V source and is left burning continuously in an otherwise dark staircase. Determine:

(a) the current through the lamp,
(b) the cost of operating the light for one non-leap year if electricity costs 12 cents per kWh.

Chapter 1, Solution 28

(a) $i = \frac{P}{V} = \frac{30}{120} = \underline{0.25\ \text{A}}$

(b) $\text{W} = \text{pt} = 30 \times 365 \times 24\ \text{Wh} = 262.8\ \text{kWh}$

$\text{Cost} = \$0.12 \times 262.8 = \underline{\$31.54}$

Chapter 1, Problem 29.

An electric stove with four burners and an oven is used in preparing a meal as follows.

Burner 1: 20 minutes
Burner 2: 40 minutes
Burner 3: 15 minutes
Burner 4: 45 minutes
Oven: 30 minutes

If each burner is rated at 1.2 kW and the oven at 1.8 kW, and electricity costs 12 cents per kWh, calculate the cost of electricity used in preparing the meal.

Chapter 1, Solution 29

$$w = pt = 1.2\text{kW}\frac{(20+40+15+45)}{60}\text{hr} + 1.8\,\text{kW}\left(\frac{30}{60}\right)\text{hr}$$
$$= 2.4 + 0.9 = 3.3\,\text{kWh}$$
$$\text{Cost} = 12\ \text{cents} \times 3.3 = \underline{39.6\ \text{cents}}$$

Chapter 1, Problem 30.

Reliant Energy (the electric company in Houston, Texas) charges customers as follows:

Monthly charge \$6
First 250 kWh @ \$0.02/kWh
All additional kWh @ \$0.07/kWh

If a customer uses 1,218 kWh in one month, how much will Reliant Energy charge?

Chapter 1, Solution 30

Monthly charge = \$6
First 250 kWh @ \$0.02/kWh = \$5
Remaining 968 kWh @ \$0.07/kWh= \$67.76
Total = **<u>\$78.76</u>**

Chapter 1, Problem 31.

In a household, a 120-W PC is run for 4 hours/day, while a 60-W bulb runs for 8 hours/day. If the utility company charges \$0.12/kWh, calculate how much the household pays per year on the PC and the bulb.

Chapter 1, Solution 31

Total energy consumed = 365(120x4 + 60x8) W
Cost = \$0.12x365x960/1000 = **<u>\$42.05</u>**

Chapter 1, Problem 32.

A telephone wire has a current of 20 μA flowing through it. How long does it take for a charge of 15 C to pass through the wire?

Chapter 1, Solution 32

$i = 20\ \mu A$

$q = 15\ C$

$t = q/i = 15/(20\times10^{-6}) = \mathbf{\underline{750\times10^3\ hrs}}$

Chapter 1, Problem 33.

A lightning bolt carried a current of 2 kA and lasted for 3 ms. How many coulombs of charge were contained in the lightning bolt?

Chapter 1, Solution 33

$$i = \frac{dq}{dt} \rightarrow q = \int i dt = 2000 \times 3 \times 10^{-3} = \underline{6\,C}$$

Chapter 1, Problem 34.

Figure 1.32 shows the power consumption of a certain household in one day. Calculate: (a) the total energy consumed in kWh, (b) the average power per hour.

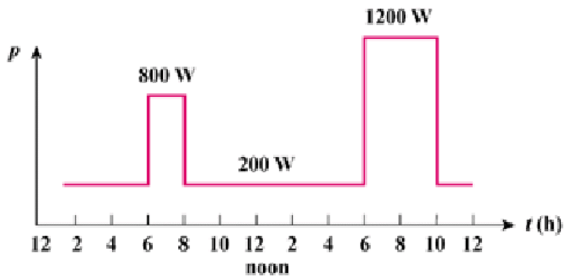


Figure 1.32

Chapter 1, Solution 34

(a) Energy = $\sum pt$ = 200 x 6 + 800 x 2 + 200 x 10 + 1200 x 4 + 200 x 2 = **10 kWh**

(b) Average power = 10,000/24 = **416.7 W**

Chapter 1, Problem 35.

The graph in Fig. 1.33 represents the power drawn by an industrial plant between 8:00 and 8:30 A.M. Calculate the total energy in MWh consumed by the plant.

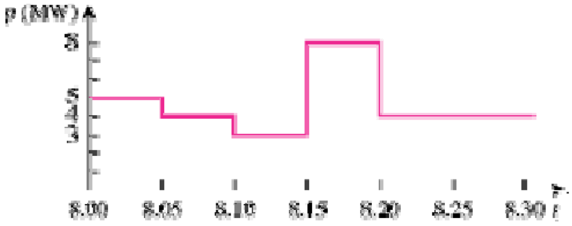


Figure 1.33

Chapter 1, Solution 35

energy = (5x5 + 4x5 + 3x5 + 8x5 + 4x10)/60 = **2.333 MWhr**

Chapter 1, Problem 36.

A battery may be rated in ampere-hours (Ah). A lead-acid battery is rated at 160 Ah.

(a) What is the maximum current it can supply for 40 h?
(b) How many days will it last if it is discharged at 1 mA?

Chapter 1, Solution 36

(a) $i = \frac{160\text{A}\cdot\text{h}}{40} = \underline{4\text{ A}}$

(b) $t = \frac{160\text{Ah}}{0.001\text{A}} = \frac{160{,}000\text{h}}{24\text{h / day}} = \underline{6{,}667\text{ days}}$

Chapter 1, Problem 37.

A 12-V battery requires a total charge of 40 ampere-hours during recharging. How many joules are supplied to the battery?

Chapter 1, Solution 37

W = pt = vit = 12x 40x 60x60 = **1.728 MJ**

Chapter 1, Problem 38.

How much energy does a 10-hp motor deliver in 30 minutes? Assume that 1 horsepower = 746 W.

Chapter 1, Solution 38

P = 10 hp = 7460 W

$W = pt = 7460 \times 30 \times 60 \text{ J} = \mathbf{13.43 \times 10^6 \text{ J}}$

Chapter 1, Problem 39.

A 600-W TV receiver is turned on for 4 hours with nobody watching it. If electricity costs 10 cents/kWh, how much money is wasted?

Chapter 1, Solution 39

W = pt = 600x4 = 2.4 kWh
C = 10cents x2.4 = **24 cents**

Chapter 2, Problem 1.

The voltage across a 5-kΩ resistor is 16 V. Find the current through the resistor.

Chapter 2, Solution 1

v = iR i = v/R = (16/5) mA = **3.2 mA**

Chapter 2, Problem 2.

Find the hot resistance of a lightbulb rated 60 W, 120 V.

Chapter 2, Solution 2

p = v^2/R → **R** = v^2/p = 14400/60 = **240 ohms**

Chapter 2, Problem 3.

A bar of silicon is 4 cm long with a circular cross section. If the resistance of the bar is 240 Ω at room temperature, what is the cross-sectional radius of the bar?

Chapter 2, Solution 3

For silicon, $\rho = 6.4x10^2$ Ω-m. $A = \pi r^2$. Hence,

$$R = \frac{\rho L}{A} = \frac{\rho L}{\pi r^2} \longrightarrow r^2 = \frac{\rho L}{\pi R} = \frac{6.4x10^2 x 4x10^{-2}}{\pi x 240} = 0.033953$$

r = **0.1843 m**

Chapter 2, Problem 4.

(a) Calculate current *i* in Fig. 2.68 when the switch is in position 1.
(b) Find the current when the switch is in position 2.

Chapter 2, Solution 4

(a) **i** = 3/100 = **30 mA**
(b) **i** = 3/150 = **20 mA**

Chapter 2, Problem 5.

For the network graph in Fig. 2.69, find the number of nodes, branches, and loops.

Chapter 2, Solution 5

n = 9; l = 7; **b** = n + l – 1 = **<u>15</u>**

Chapter 2, Problem 6.

In the network graph shown in Fig. 2.70, determine the number of branches and nodes.

Chapter 2, Solution 6

n = 12; l = 8; **b** = n + l –1 = **<u>19</u>**

Chapter 2, Problem 7.

Determine the number of branches and nodes in the circuit of Fig. 2.71.

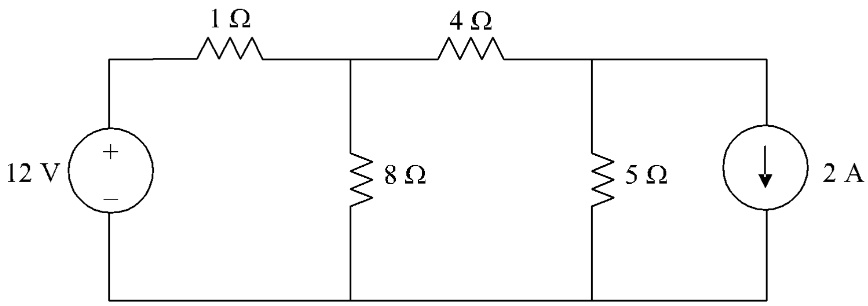


Figure 2.71 For Prob. 2.7.

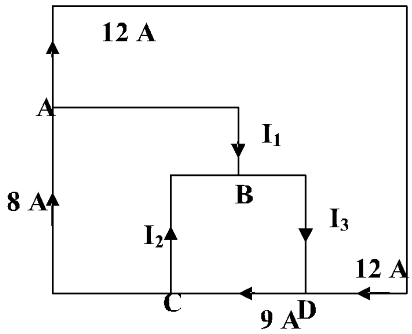
Chapter 2, Solution 7

<u>6 branches and 4 nodes.</u>

Chapter 2, Problem 8.

Use KCL to obtain currents i_1, i_2, and i_3 in the circuit shown in Fig. 2.72.

Chapter 2, Solution 8



At node a, $8 = 12 + i_1 \longrightarrow$ **$i_1 = -4A$**

At node c, $9 = 8 + i_2 \longrightarrow$ **$i_2 = 1A$**

At node d, $9 = 12 + i_3 \longrightarrow$ **$i_3 = -3A$**

Chapter 2, Problem 9.

Find i_1, i_2, and i_3 in Fig. 2.73.

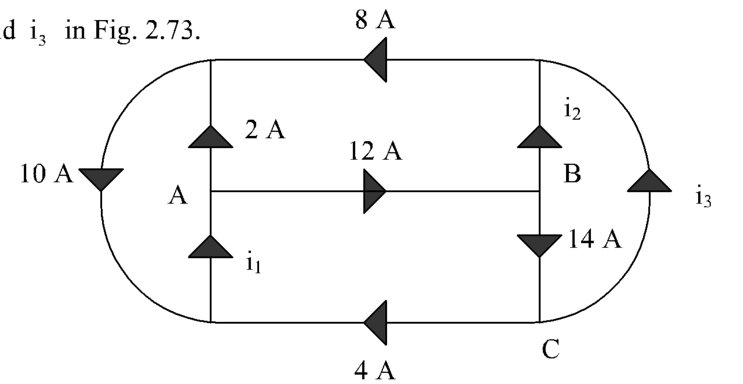


Figure 2.73 For Prob. 2.9.

Chapter 2, Solution 9

At A, $2+12 = i_1 \longrightarrow i_1 = \underline{14\text{ A}}$

At B, $12 = i_2 + 14 \longrightarrow i_2 = \underline{-2\text{ A}}$

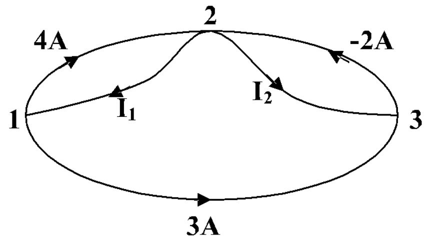
At C, $14 = 4 + i_3 \longrightarrow i_3 = \underline{10\text{ A}}$

Chapter 2, Problem 10.

In the circuit in Fig. 2.67 decrease in R_3 leads to a decrease of:

(a) current through R_3
(b) voltage through R_3
(c) voltage across R_1
(d) power dissipated in R_2
(e) none of the above

Chapter 2, Solution 10



At node 1, $4 + 3 = i_1 \longrightarrow$ **$i_1 =$ <u>7A</u>**
At node 3, $3 + i_2 = -2 \longrightarrow$ **$i_2 =$ <u>-5A</u>**

Chapter 2, Problem 11.

In the circuit of Fig. 2.75, calculate V_1 and V_2.

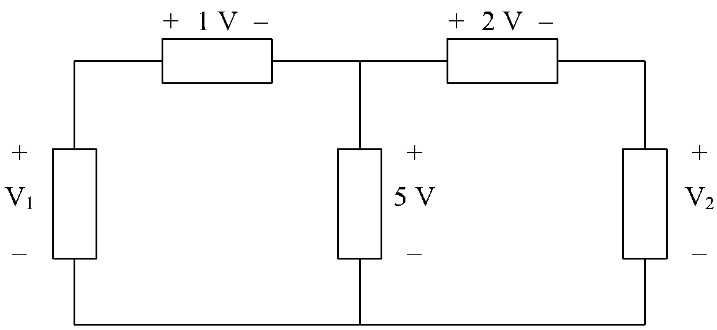


Figure 2.75 For Prob. 2.11.

Chapter 2, Solution 11

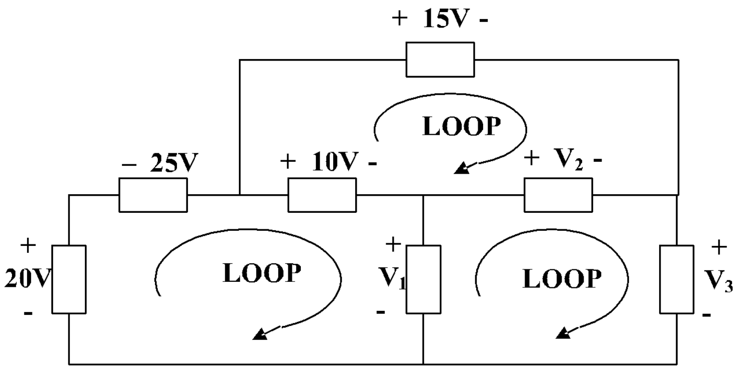
$-V_1 + 1 + 5 = 0 \quad \longrightarrow \quad V_1 = \underline{6\ V}$

$-5 + 2 + V_2 = 0 \quad \longrightarrow \quad V_2 = \underline{3\ V}$

Chapter 2, Problem 12.

In the circuit in Fig. 2.76, obtain v_1, v_2, and v_3.

Chapter 2, Solution 12



For loop 1, $-20 -25 +10 + v_1 = 0$ ⟶ $\underline{v_1 = 35v}$

For loop 2, $-10 +15 -v_2 = 0$ ⟶ $\underline{v_2 = 5v}$

For loop 3, $-V_1 + V_2 + V_3 = 0$ ⟶ $\underline{v_3 = 30v}$

Chapter 2, Problem 13.

For the circuit in Fig. 2.77, use KCL to find the branch currents I_1 to I_4.

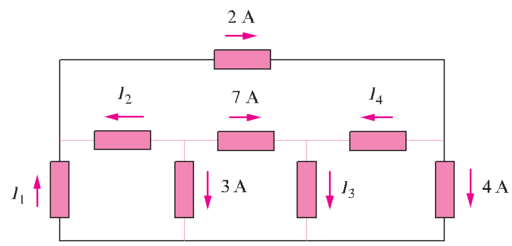
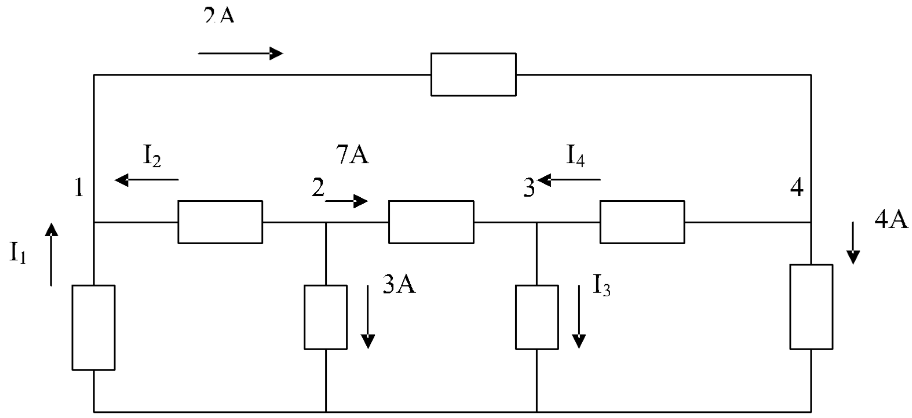


Figure 2.77

Chapter 2, Solution 13



At node 2,

$$3+7+I_2=0 \quad \longrightarrow \quad I_2=-10A$$

At node 1,

$$I_1+I_2=2 \quad \longrightarrow \quad I_1=2-I_2=12A$$

At node 4,

$$2=I_4+4 \quad \longrightarrow \quad I_4=2-4=-2A$$

At node 3,

$$7+I_4=I_3 \quad \longrightarrow \quad I_3=7-2=5A$$

Hence,

$$\underline{I_1=12A, \quad I_2=-10A, \quad I_3=5A, \quad I_4=-2A}$$

Chapter 2, Problem 14.

Given the circuit in Fig. 2.78, use KVL to find the branch voltages V_1 to V_4.

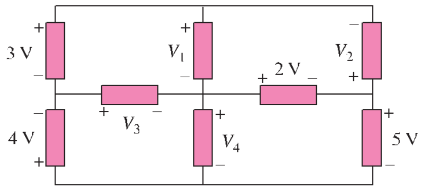
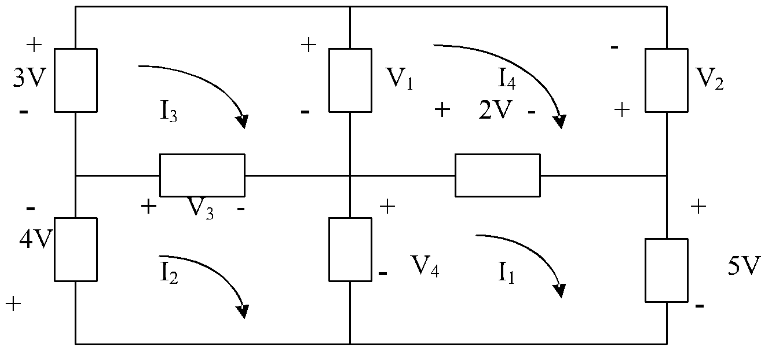


Figure 2.78

Chapter 2, Solution 14



For mesh 1,

$-V_4 + 2 + 5 = 0 \quad \longrightarrow \quad V_4 = 7V$

For mesh 2,

$+4 + V_3 + V_4 = 0 \quad \longrightarrow \quad V_3 = -4 - 7 = -11V$

For mesh 3,

$-3 + V_1 - V_3 = 0 \quad \longrightarrow \quad V_1 = V_3 + 3 = -8V$

For mesh 4,

$-V_1 - V_2 - 2 = 0 \quad \longrightarrow \quad V_2 = -V_1 - 2 = 6V$

Thus,

$\underline{V_1 = -8V, \quad V_2 = 6V, \quad V_3 = -11V, \quad V_4 = 7V}$

Chapter 2, Problem 15.

Calculate v and i_x in the circuit of Fig. 2.79.

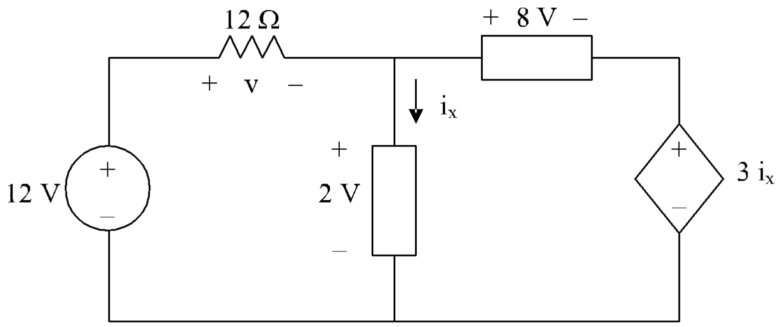


Figure 2.79 For Prob. 2.15.

Chapter 2, Solution 15

For loop 1, $-12 + v + 2 = 0$, $v =$ **<u>10 V</u>**

For loop 2, $-2 + 8 + 3i_x = 0$, $i_x =$ **<u>–2 A</u>**

Chapter 2, Problem 16.

Determine V_o in the circuit in Fig. 2.80.

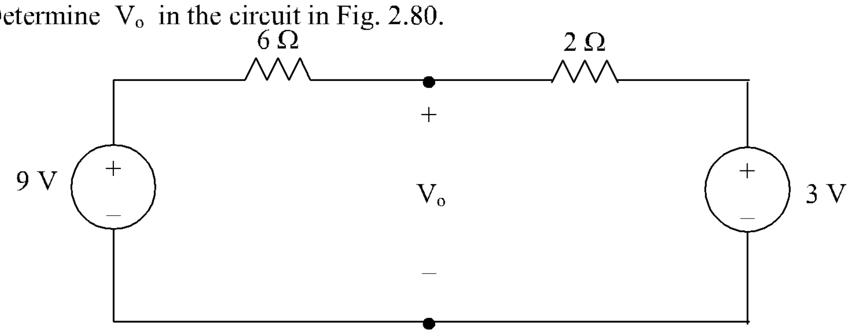


Figure 2.80 For Prob. 2.16.

Chapter 2, Solution 16

Apply KVL,

$-9 + (6+2)I + 3 = 0$, $8I = 9-3=6$, $I = 6/8$

Also,

$-9 + 6I + V_o = 0$

$V_o = 9- 6I =$ **<u>4.5 V</u>**

Chapter 2, Problem 17.

Obtain v_1 through v_3 in the circuit in Fig. 2.78.

Chapter 2, Solution 17

Applying KVL around the entire outside loop we get,

$$-24 + v_1 + 10 + 12 = 0 \text{ or } v_1 = \underline{\mathbf{2V}}$$

Applying KVL around the loop containing v_2, the 10-volt source, and the 12-volt source we get,

$$v_2 + 10 + 12 = 0 \text{ or } v_2 = \underline{\mathbf{-22V}}$$

Applying KVL around the loop containing v_3 and the 10-volt source we get,

$$-v_3 + 10 = 0 \text{ or } v_3 = \underline{\mathbf{10V}}$$

Chapter 2, Problem 18.

Find I and V_{ab} in the circuit of Fig. 2.79.

Chapter 2, Solution 18

APPLYING KVL,

$$-30 - 10 + 8 + I(3+5) = 0$$

$$8I = 32 \longrightarrow I = \underline{\mathbf{4A}}$$

$$-V_{ab} + 5I + 8 = 0 \longrightarrow V_{ab} = \underline{\mathbf{28V}}$$

Chapter 2, Problem 19.

From the circuit in Fig. 2.80, find *I*, the power dissipated by the resistor, and the power supplied by each source.

Chapter 2, Solution 19

APPLYING KVL AROUND THE LOOP, WE OBTAIN

$$-12 + 10 - (-8) + 3i = 0 \longrightarrow \underline{\mathbf{i = -2A}}$$

Power dissipated by the resistor:

$$p_{3\Omega} = i^2R = 4(3) = \underline{\mathbf{12W}}$$

Power supplied by the sources:

$$p_{12V} = 12\,((-2)) = \underline{\mathbf{-24W}}$$

$$p_{10V} = 10\,(-(-2)) = \underline{\mathbf{20W}}$$

$$p_{8V} = (-8)(-2) = \underline{\mathbf{16W}}$$

Chapter 2, Problem 20.

Determine i_o in the circuit of Fig. 2.81.

Chapter 2, Solution 20

APPLYING KVL AROUND THE LOOP,

$$-36 + 4i_0 + 5i_0 = 0 \longrightarrow \underline{\mathbf{i_0 = 4A}}$$

Chapter 2, Problem 21.

Find V_x in the circuit of Fig. 2.85.

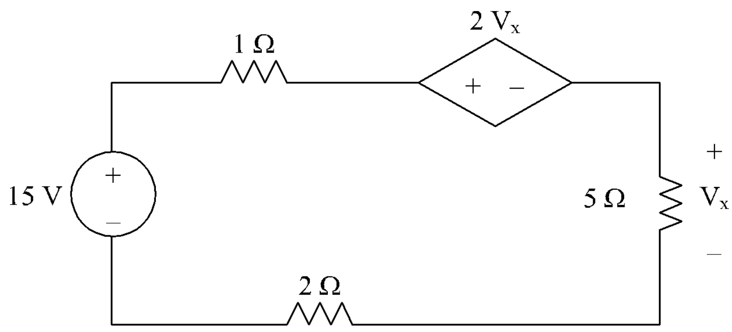


Figure 2.85 For Prob. 2.21.

Chapter 2, Solution 21

Applying KVL,

$$-15 + (1+5+2)I + 2V_x = 0$$

But $V_x = 5I$,

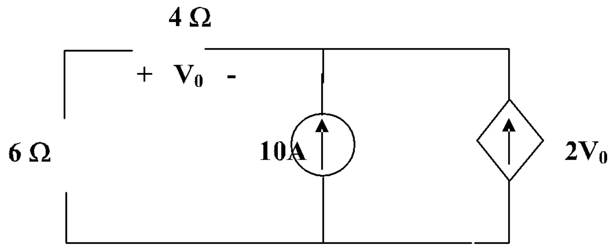
$$-15 + 8I + 10I = 0, \qquad I = 5/6$$

$$V_x = 5I = 25/6 = \underline{\textbf{4.167 V}}$$

Chapter 2, Problem 22.

Find V_o in the circuit in Fig. 2.85 and the power dissipated by the controlled source.

Chapter 2, Solution 22



At the node, KCL requires that

$$\frac{v_0}{4} + 10 + 2v_0 = 0 \longrightarrow \quad v_0 = \underline{-4.444V}$$

The current through the controlled source is

$i = 2V_0 = -8.888A$

and the voltage across it is

$$v = (6 + 4)\, i_0 \text{ (where } i_0 = v_0/4) = 10\frac{v_0}{4} = -11.111$$

Hence,

$p_2\, v_i = (-8.888)(-11.111) = \underline{\mathbf{98.75\ W}}$

Chapter 2, Problem 23.

In the circuit shown in Fig. 2.87, determine v_x and the power absorbed by the 12-Ω resistor.

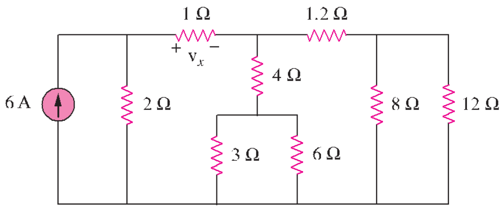
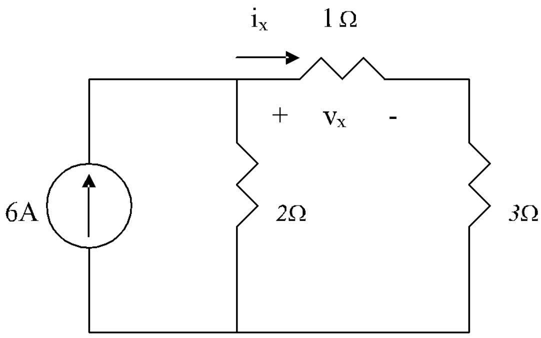


Figure 2.87

Chapter 2, Solution 23

8//12 = 4.8, 3//6 = 2, (4 + 2)//(1.2 + 4.8) = 6//6 = 3
The circuit is reduced to that shown below.



Applying current division,

$$i_x = \frac{2}{2+1+3}(6A) = 2A, \qquad v_x = 1i_x = \underline{2V}$$

The current through the 1.2-Ω resistor is $0.5i_x = 1A$. The voltage across the 12-Ω resistor is 1 x 4.8 = 4.8 V. Hence the power is

$$p = \frac{v^2}{R} = \frac{4.8^2}{12} = \underline{1.92W}$$

Chapter 2, Problem 24.

For the circuit in Fig. 2.86, find V_o / V_s in terms of α, R_1, R_2, R_3, and R_4. If $R_1 = R_2 = R_3 = R_4$, what value of α will produce $| V_o / V_s | = 10$?

Chapter 2, Solution 24

(a) $$I_0 = \frac{V_s}{R_1 + R_2}$$

$$V_0 = -\alpha I_0 (R_3 \| R_4) = -\frac{\alpha V_s}{R_1 + R_2} \cdot \frac{R_3 R_4}{R_3 + R_4}$$

$$\frac{V_0}{Vs} = \frac{-\alpha R_3 R_4}{(R_1 + R_2)(R_3 + R_4)}$$

(b) If $R_1 = R_2 = R_3 = R_4 = R$,

$$\left|\frac{V_0}{V_S}\right| = \frac{\alpha}{2R} \cdot \frac{R}{2} = \frac{\alpha}{4} = 10 \longrightarrow \alpha = \underline{\mathbf{40}}$$

Chapter 2, Problem 25.

For the network in Fig. 2.88, find the current, voltage, and power associated with the 20-kΩ resistor.

Chapter 2, Solution 25

$$V_0 = 5 \text{ x } 10^{-3} \text{ x } 10 \text{ x } 10^3 = 50\text{V}$$

Using current division,

$$I_{20} = \frac{5}{5+20}(0.01x50) = \underline{\mathbf{0.1\ A}}$$

$$V_{20} = 20 \text{ x } 0.1 \text{ kV} = \underline{\mathbf{2\ kV}}$$

$$p_{20} = I_{20} V_{20} = \underline{\mathbf{0.2\ kW}}$$

Chapter 2, Problem 26.
For the circuit in Fig. 2.90, i_o =2 A. Calculate i_x and the total power dissipated by the circuit.

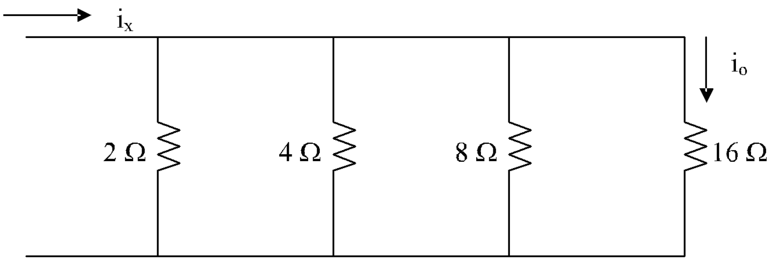


Figure 2.90 For Prob. 2.26.

Chapter 2, Solution 26

If i_{16}= i_o = 2A, then v = 16x2 = 32 V

$$i_8 = \frac{v}{8} = 4 \text{ A}, \quad i_4 = \frac{v}{4} = 8 \text{ A}, \qquad i_2 = \frac{v}{2} = 16$$

$$i_x = i_2 + i_4 + i_8 + i_{16} = 16 + 8 + 4 + 2 = \underline{30 \text{ A}}$$

$$P = \sum i^2 R = 16^2 x2 + 8^2 x4 + 4^2 x8 + 2^2 x16 = 960 \text{ W}$$

or

$$P = i_x v = 30x32 = \underline{960 \text{ W}}$$

Chapter 2, Problem 27.
Calculate V_o in the circuit of Fig. 2.91.

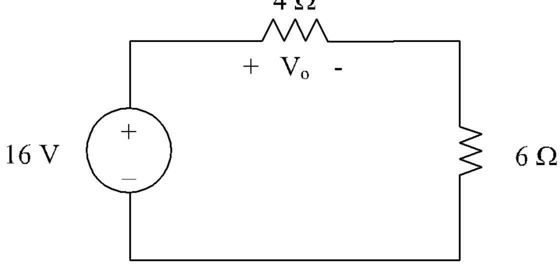


Figure 2.91 For Prob. 2.27.

Chapter 2, Solution 27
Using voltage division,

$$V_o = \frac{4}{4+16}(16\text{V}) = \underline{6.4 \text{ V}}$$

Chapter 2, Problem 28.

Find v_1, v_2, and v_3 in the circuit in Fig. 2.91.

Chapter 2, Solution 28

We first combine the two resistors in parallel

$$15 \| 10 = 6\ \Omega$$

We now apply voltage division,

$$v_1 = \frac{14}{14+6}(40) = \underline{\mathbf{28\ V}}$$

$$v_2 = v_3 = \frac{6}{14+6}(40) = 12\ V$$

Hence, $v_1 = \underline{\mathbf{28\ V}}$, $v_2 = \underline{\mathbf{12\ V}}$, $v_s = \underline{\mathbf{12\ V}}$

Chapter 2, Problem 29.

All resistors in Fig. 2.93 are 1 Ω each. Find R_{eq}.

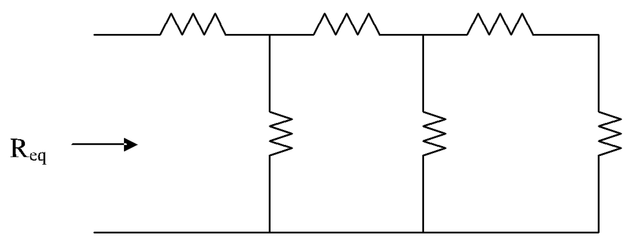


Figure 2.93 For Prob. 2.29.

Chapter 2, Solution 29

$$R_{eq} = 1 + 1//(1 + 1//2) = 1 + 1//(1+ 2/3) =1+ 1//5/3 = \underline{\mathbf{1.625\ \Omega}}$$

Chapter 2, Problem 30.

Find R_{eq} for the circuit in Fig. 2.94.

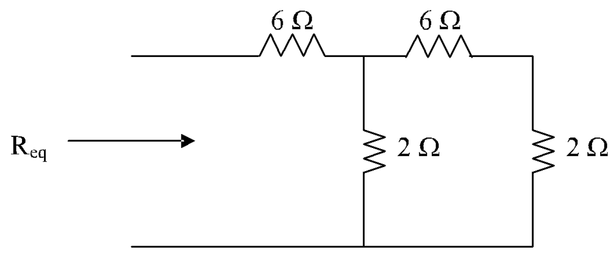


Figure 2.94 For Prob. 2.30.

Chapter 2, Solution 30

We start by combining the 6-ohm resistor with the 2-ohm one. We then end up with an 8-ohm resistor in parallel with a 2-ohm resistor.

$$(2x8)/(2+8) = 1.6\ \Omega$$

This is in series with the 6-ohm resistor which gives us,

$$R_{eq} = 6+1.6 = \underline{\mathbf{7.6\ \Omega}}.$$

Chapter 2, Problem 31.

For the circuit in Fig. 2.95, determine i_1 to i_5.

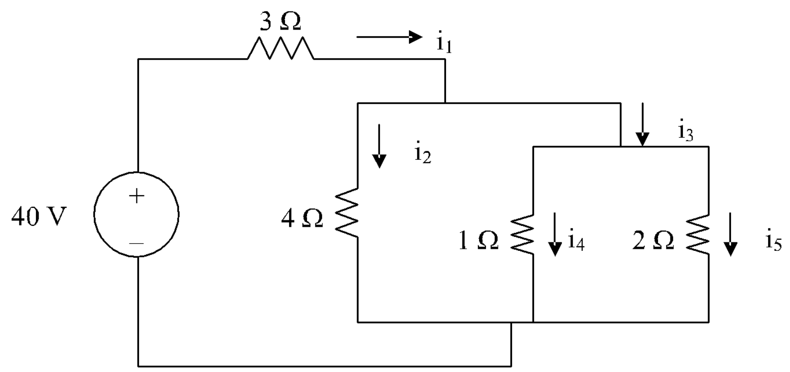


Figure 2.95 For Prob. 2.31.

Chapter 2, Solution 31

$$R_{eq} = 3 + 2 // 4 // 1 = 3 + \frac{1}{1/2 + 1/4 + 1} = 3.5714$$

$$i_1 = \frac{40}{3.5714} = \underline{11.2 \text{ A}}$$

$$v_1 = 0.5714 x i_1 = 6.4\text{V}, \quad i_2 = \frac{v_1}{4} = \underline{1.6 \text{ A}}$$

$$i_4 = \frac{v_1}{1} = \underline{6.4 \text{ A}}, \quad i_5 = \frac{v_1}{2} = \underline{3.2 \text{ A}}, \quad i_3 = i_4 + i_5 = \underline{9.6 \text{ A}}$$

Chapter 2, Problem 32.

Find i_1 through i_4 in the circuit in Fig. 2.96.

Chapter 2, Solution 32

We first combine resistors in parallel.

$$20\|30 = \frac{20x30}{50} = 12\ \Omega$$

$$10\|40 = \frac{10x40}{50} = 8\ \Omega$$

Using current division principle,

$$i_1 + i_2 = \frac{8}{8+12}(20) = 8A, i_3 + i_4 = \frac{12}{20}(20) = 12A$$

$$i_1 = \frac{20}{50}(8) = \underline{\mathbf{3.2\ A}}$$

$$i_2 = \frac{30}{50}(8) = \underline{\mathbf{4.8\ A}}$$

$$i_3 = \frac{10}{50}(12) = \underline{\mathbf{2.4A}}$$

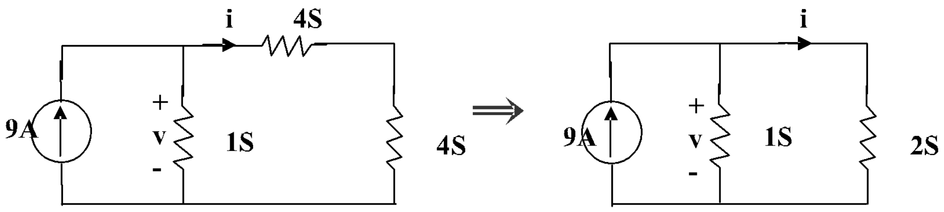
$$i_4 = \frac{40}{50}(12) = \underline{\mathbf{9.6\ A}}$$

Chapter 2, Problem 33.

Obtain v and i in the circuit in Fig. 2.97.

Chapter 2, Solution 33

Combining the conductance leads to the equivalent circuit below



$$6S\|3S = \frac{6x3}{9} = 2\text{S} \text{ and } 2\text{S} + 2\text{S} = 4\text{S}$$

Using current division,

$$i = \frac{1}{1+\frac{1}{2}}(9) = \underline{6\text{ A}},\ v = 3(1) = \underline{3\text{ V}}$$

Chapter 2, Problem 34.

Using series/parallel resistance combination, find the equivalent resistance seen by the source in the circuit of Fig. 2.98. Find the overall dissipated power.

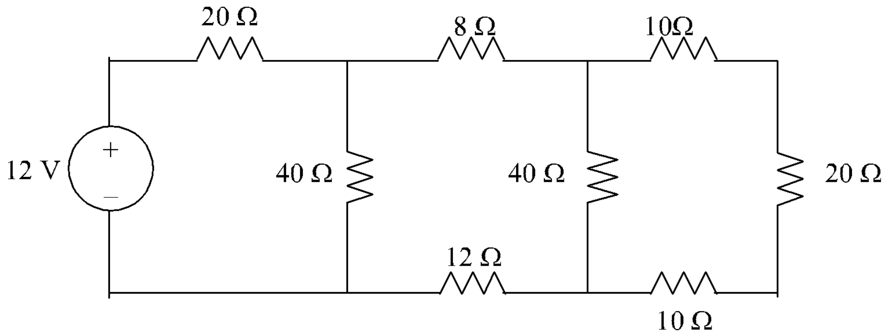


Figure 2.98 For Prob. 2.34.

Chapter 2, Solution 34

40//(10 + 20 + 10)= 20 Ω,
40//(8+12 + 20) = 20 Ω

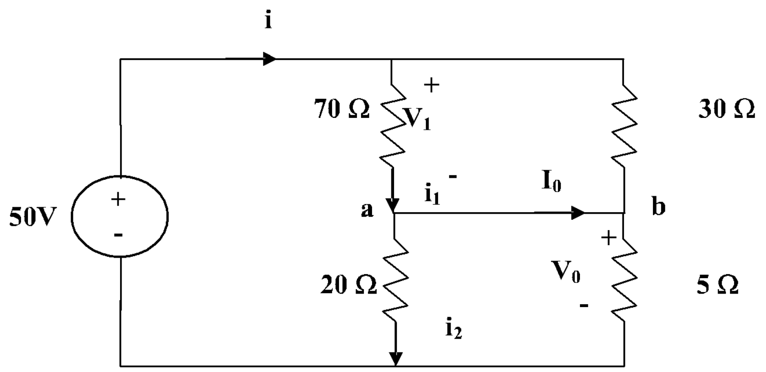
$$R_{eq} = 20 + 20 = \underline{40\ \Omega}$$

$$I = \frac{V}{R_{eq}} = 12/40, \qquad P = VI = \frac{12^2}{40} = \underline{3.6\ \text{W}}$$

Chapter 2, Problem 35.

Calculate V_o and I_o in the circuit of Fig. 2.99.

Chapter 2, Solution 35



Combining the versions in parallel,

$$70\|30 = \frac{70x30}{100} = 21\Omega\ , \qquad 20\|5 = \frac{20x5}{25} = 4\ \Omega$$

$$i = \frac{50}{21+4} = 2\ A$$

$$v_i = 21i = 42\ V,\ v_0 = 4i = 8\ V$$

$$i_1 = \frac{v_1}{70} = 0.6\ A,\ i_2 = \frac{v_2}{20} = 0.4\ A$$

At node a, KCL must be satisfied

$$i_1 = i_2 + I_0 \longrightarrow 0.6 = 0.4 + I_0 \longrightarrow I_0 = 0.2\ A$$

Hence $v_0 =$ **8 V** and $I_0 =$ **0.2A**

Chapter 2, Problem 36.

Find i and V_o in the circuit of Fig. 2.100.

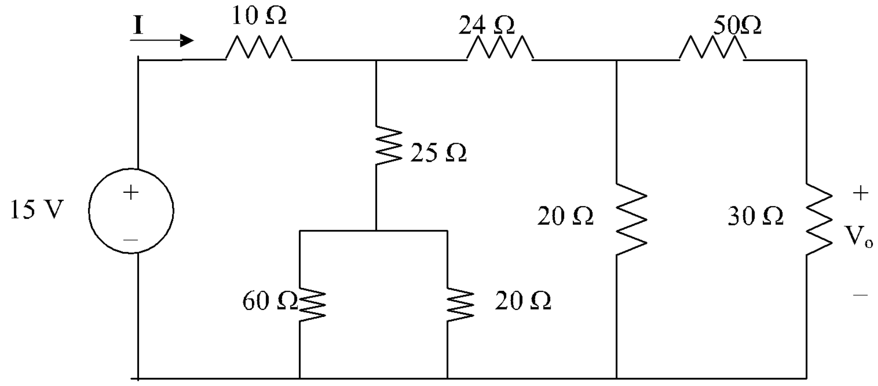


Figure 2.100 For Prob. 2.36.

Chapter 2, Solution 36

20//(30+50) = 16, 24 + 16 = 40, 60//20 = 15

$$R_{eq} = 10 + (15+25)//40 = 10 + 20 = 30$$

$$i = \frac{v_s}{R_{eq}} = \frac{15}{30} = \underline{0.5\text{ A}}$$

If i_1 is the current through the 24-Ω resistor and i_o is the current through the 50-Ω resistor, using current division gives

$$i_1 = \frac{40}{40+40} i = 0.25\text{ A}, \quad i_o = \frac{20}{20+80} i_1 = 0.05\text{ A}$$

$$v_o = 30 i_o = 30x0.05 = \underline{1.5\text{ V}}$$

Chapter 2, Problem 37.

Find R for the circuit in Fig. 2.101.

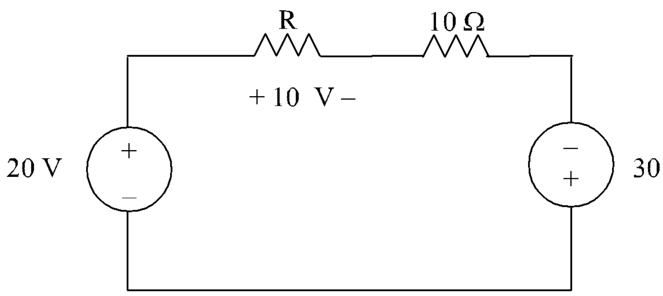


Figure 2.101 For Prob. 2.37.

Chapter 2, Solution 37

Applying KVL,

$$-20 + 10 + 10I - 30 = 0, \quad I = 4$$

$$10 = RI \quad \longrightarrow \quad R = \frac{10}{I} = \underline{2.5\ \Omega}$$

Chapter 2, Problem 38.

Find R_{eq} and i_o in the circuit of Fig. 2.102.

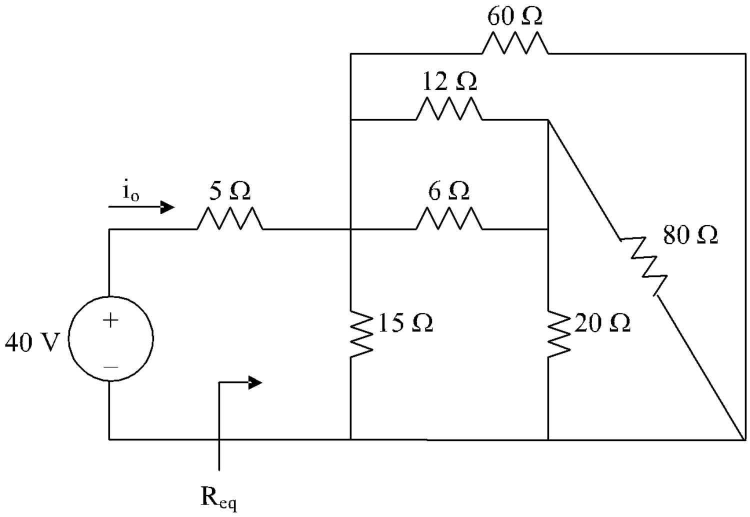
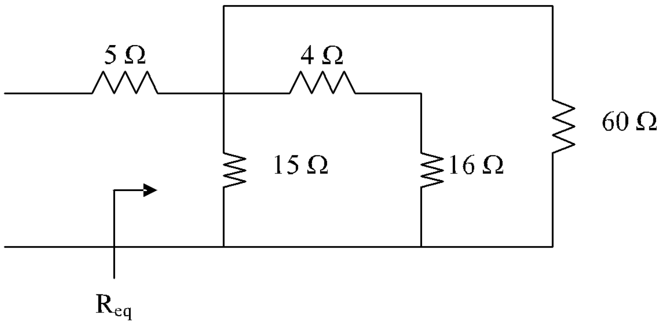


Figure 2.102 For Prob. 2.38

Chapter 2, Solution 38

20//80 = 80x20/100 = 16, 6//12 = 6x12/18 = 4
The circuit is reduced to that shown below.



(4 + 16)//60 = 20x60/80 = 15

$R_{eq} = 15//15 + 5 = \underline{12.5\ \Omega}$

$$i_o = \frac{40}{R_{eq}} = \underline{3.2\ \text{A}}$$

Chapter 2, Problem 39.

Evaluate R_{eq} for each of the circuits shown in Fig. 2.103.

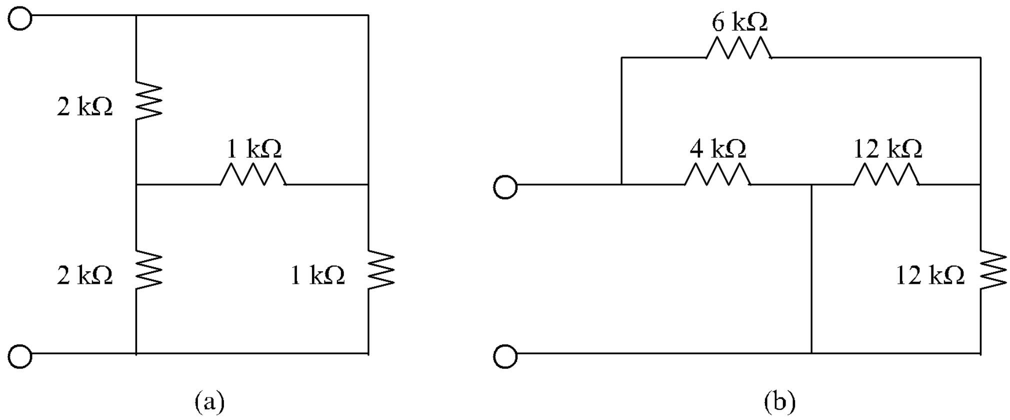


Figure 2.103 For Prob. 2.39.

Chapter 2, Solution 39

(a) We note that the top 2k-ohm resistor is actually in parallel with the first 1k-ohm resistor. This can be replaced (2/3)k-ohm resistor. This is now in series with the second 2k-ohm resistor which produces a 2.667k-ohm resistor which is now in parallel with the second 1k-ohm resistor. This now leads to,

$$R_{eq} = [(1x2.667)/3.667]k = \underline{\mathbf{727.3\ \Omega}}.$$

(b) We note that the two 12k-ohm resistors are in parallel producing a 6k-ohm resistor. This is in series with the 6k-ohm resistor which results in a 12k-ohm resistor which is in parallel with the 4k-ohm resistor producing,

$$R_{eq} = [(4x12)/16]k = \underline{\mathbf{3\ k\Omega}}.$$

Chapter 2, Problem 40.

For the ladder network in Fig. 2.104, find I and R_{eq}.

Chapter 2, Solution 40

$$REQ = 3 + 4 \| (2 + 6 \| 3) = 3 + 2 = \underline{\mathbf{5\Omega}}$$

$$I = \frac{10}{Req} = \frac{10}{5} = \underline{\mathbf{2\ A}}$$

Chapter 2, Problem 41.

If R_{eq} = 50 Ω in the circuit in Fig. 2.105, find *R*.

Chapter 2, Solution 41

Let R_0 = combination of three 12Ω resistors in parallel

$$\frac{1}{R_o} = \frac{1}{12} + \frac{1}{12} + \frac{1}{12} \longrightarrow R_o = 4$$

$$R_{eq} = 30 + 60 \| (10 + R_0 + R) = 30 + 60 \| (14 + R)$$

$$50 = 30 + \frac{60(14 + R)}{74 + R} \longrightarrow 74 + R = 42 + 3R$$

or R = **16 Ω**

Chapter 2, Problem 42.

Reduce each of the circuits in Fig. 2.106 to a single resistor at terminals *a-b*.

Chapter 2, Solution 42

(a) $R_{ab} = 5 \| (8 + 20 \| 30) = 5 \| (8 + 12) = \frac{5x20}{25} =$ **4 Ω**

(b) $R_{ab} = 2 + 4 \| (5 + 3) \| 8 + 5 \| 10 \| 4 = 2 + 4 \| 4 + 5 \| 2.857 = 2 + 2 + 1.8181 =$ **5.818 Ω**

Chapter 2, Problem 43

Calculate the equivalent resistance R_{ab} at terminals *a*-*b* for each of the circuits in Fig.2.107.

Chapter 2, Solution 43

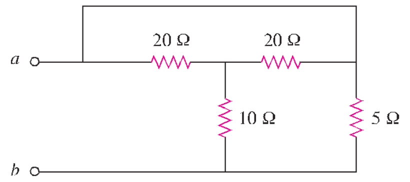
(a) $$R_{ab} = 5\|20 + 10\|40 = \frac{5x20}{25} + \frac{400}{50} = 4 + 8 = \underline{\mathbf{12\ \Omega}}$$

(b) $$60\|20\|30 = \left(\frac{1}{60} + \frac{1}{20} + \frac{1}{30}\right)^{-1} = \frac{60}{6} = 10\Omega$$

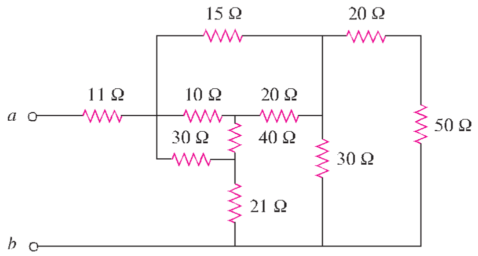
$$R_{ab} = 80\|(10+10) = \frac{80+20}{100} = \underline{\mathbf{16\ \Omega}}$$

Chapter 2, Problem 44.

For each of the circuits in Fig. 2.108, obtain the equivalent resistance at terminals *a-b*.



(a)



(b)

Figure 2.108

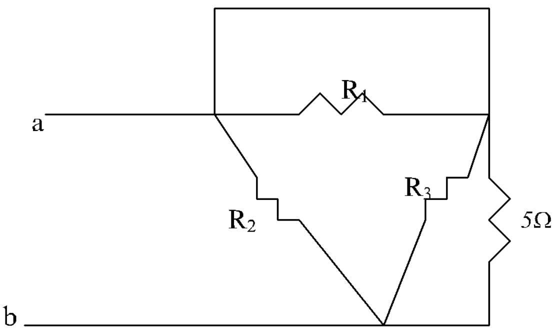
Chapter 2, Solution 44

(a) Convert T to Y and obtain

$$R_1 = \frac{20x20 + 20x10 + 10x20}{10} = \frac{800}{10} = 80\Omega$$

$$R_2 = \frac{800}{20} = 40\Omega = R_3$$

The circuit becomes that shown below.



$R_1//0 = 0$, $R_3//5 = 40//5 = 4.444\,\Omega$

$R_{ab} = R_2\,//(0 + 4.444) = 40\,//4.444 = \underline{4\Omega}$

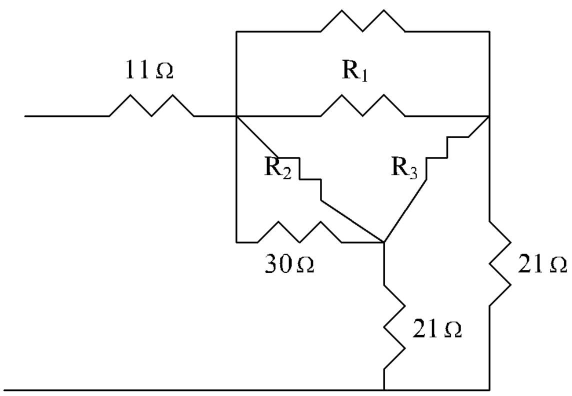
(b) $30//(20+50) = 30//70 = 21\ \Omega$

Convert the T to Y and obtain

$$R_1 = \frac{20x10 + 10x40 + 40x20}{40} = \frac{1400}{40} = 35\Omega$$

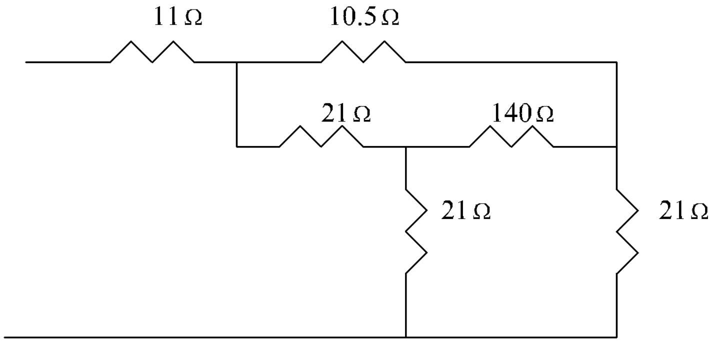
$$R_2 = \frac{1400}{20} = 70\Omega\,, \quad R_3 = \frac{1400}{10} = 140\Omega$$

The circuit is reduce 15Ω t shown below.

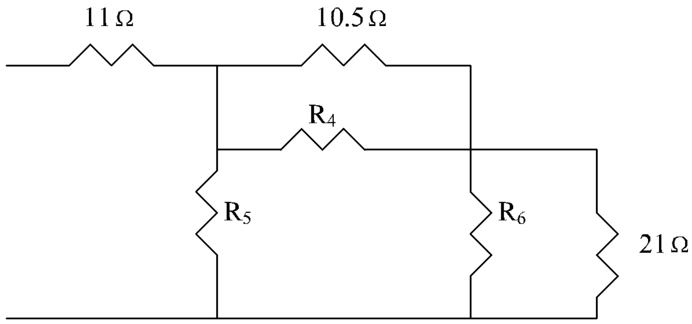


Combining the resistors in parallel

$R_1//15 = 35//15 = 10.5$, $30//R_2 = 30//70 = 21$
leads to the circuit below.



Coverting the T to Y leads to the circuit below.



$$R_4 = \frac{21x140 + 140x21 + 21x21}{21} = \frac{6321}{21} = 301\Omega = R_6$$

$$R_5 = \frac{6321}{140} = 45.15$$

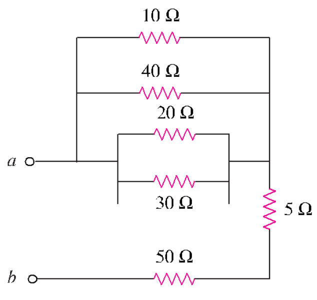
$10.5//301 = 10.15$, $301//21 = 19.63$

$R_5//(10.15 + 19.63) = 45.15//29.78 = 17.94$

$$R_{ab} = 11 + 17.94 = \underline{28.94\Omega}$$

Chapter 2, Problem 45.

Find the equivalent resistance at terminals *a-b* of each circuit in Fig. 2.109.



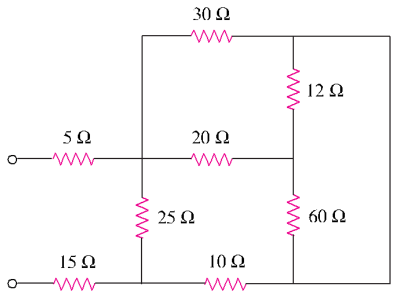


Figure 2.109

Chapter 2, Solution 45

(a) 10//40 = 8, 20//30 = 12, 8//12 = 4.8

$R_{ab} = 5 + 50 + 4.8 = \underline{59.8\Omega}$

(b) 12 and 60 ohm resistors are in parallel. Hence, 12//60 = 10 ohm. This 10 ohm and 20 ohm are in series to give 30 ohm. This is in parallel with 30 ohm to give 30//30 = 15 ohm. And 25//(15+10) = 12.5. Thus $R_{ab} = 5 + 12.8 + 15 = \underline{32.5\Omega}$

Chapter 2, Problem 46.
Find I in the circuit of Fig. 2.110.

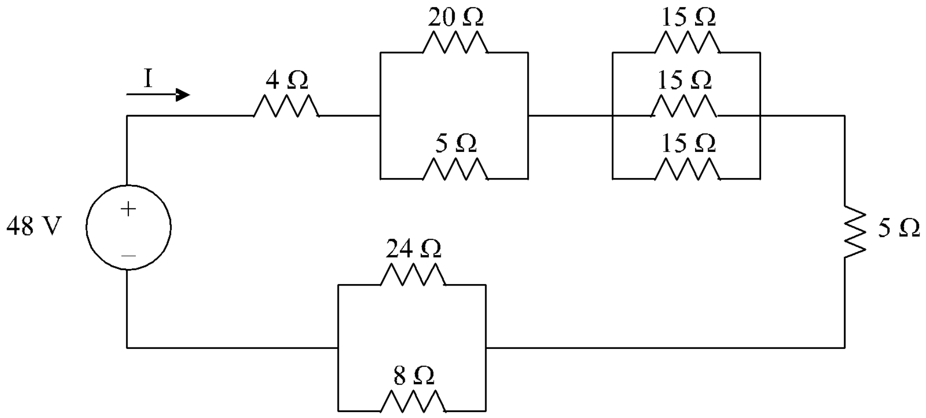


Figure 2.110 For Prob. 2.46.

Chapter 2, Solution 46

$$R_{eq} = 4 + 5 // 20 + \frac{1}{3} x 15 + 5 + 24 // 8 = 4 + 4 + 5 + 5 + 6 = 24$$

I = 48/24 = **<u>2 A</u>**

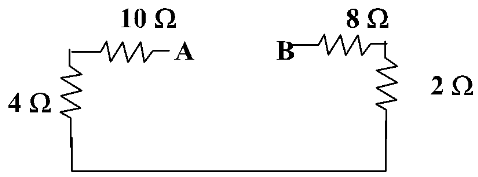
Chapter 2, Problem 47.

Find the equivalent resistance R_{ab} in the circuit of Fig. 2.111.

Chapter 2, Solution 47

$$5 \| 20 = \frac{5x20}{25} = 4\Omega$$

$$6 \| 3 = \frac{6x3}{9} = 2\Omega$$



$R_{ab} = 10 + 4 + 2 + 8 =$ **<u>24 Ω</u>**

Chapter 2, Problem 48.

Convert the circuits in Fig. 2.112 from Y to Δ.

Chapter 2, Solution 48

(A) $R_A = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} = \frac{100+100+100}{10} = 30$

$R_a = R_b = R_c = \underline{\mathbf{30\ \Omega}}$

(b) $R_a = \frac{30x20 + 30x50 + 20x50}{30} = \frac{3100}{30} = 103.3\Omega$

$R_b = \frac{3100}{20} = 155\Omega, \quad R_c = \frac{3100}{50} = 62\Omega$

$R_a = \underline{\mathbf{103.3\ \Omega}}$, $R_b = \underline{\mathbf{155\ \Omega}}$, $R_c = \underline{\mathbf{62\ \Omega}}$

Chapter 2, Problem 49.

Transform the circuits in Fig. 2.113 from Δ to Y.

Chapter 2, Solution 49

(A) $R_1 = \frac{R_aR_c}{R_a + R_b + R_c} = \frac{12*12}{36} = 4\Omega$

$R_1 = R_2 = R_3 = \underline{\mathbf{4\ \Omega}}$

(b) $R_1 = \frac{60x30}{60+30+10} = 18\Omega$

$R_2 = \frac{60x10}{100} = 6\Omega$

$R_3 = \frac{30x10}{100} = 3\Omega$

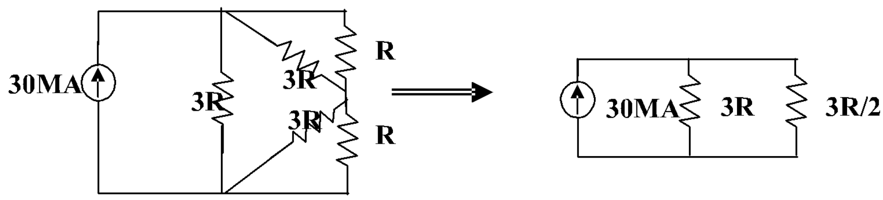
$\mathbf{R_1 = 18\Omega, R_2 = 6\Omega, R_3 = 3\Omega}$

Chapter 2, Problem 50.

What value of *R* in the circuit of Fig. 2.114 would cause the current source to deliver 800 mW to the resistors.

Chapter 2, Solution 50

Using $R_\Delta = 3R_Y = 3R$, we obtain the equivalent circuit shown below:



$$3R \| R = \frac{3RxR}{4R} = \frac{3}{4}R$$

$$3R \| (3RxR)/(4R) = 3/(4R)$$

$$3R \| \left(\frac{3}{4}R + \frac{3}{4}R\right) = 3R \| \frac{3}{2}R = \frac{3Rx\frac{3}{2}R}{3R + \frac{3}{2}R = R}$$

$P = I^2R \longrightarrow 800 \text{ x } 10^{-3} = (30 \text{ x } 10^{-3})^2 R$

R = **889 Ω**

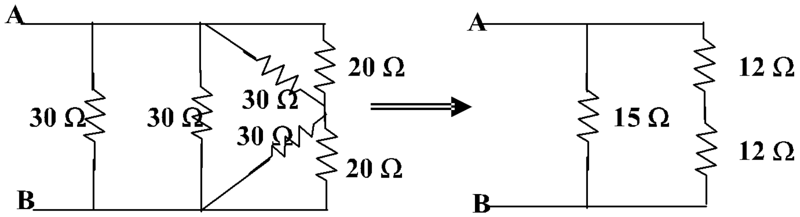
Chapter 2, Problem 51.

Obtain the equivalent resistance at the terminals *a-b* for each of the circuits in Fig. 2.115.

Chapter 2, Solution 51

(a) $30\|30 = 15\Omega$ and $30\|20 = 30x20/(50) = 12\Omega$

$R_{ab} = 15\|(12+12) = 15x24/(39) = \underline{\mathbf{9.231\ \Omega}}$



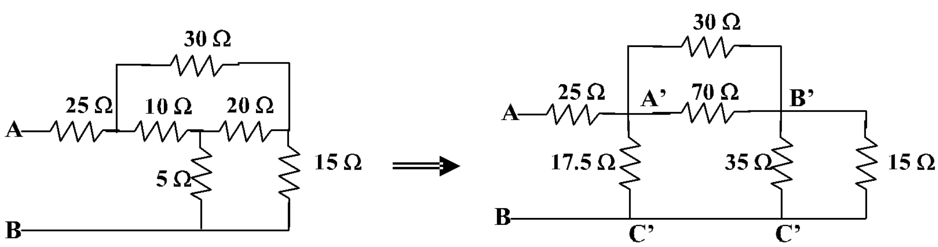
(b) Converting the T-subnetwork into its equivalent Δ network gives

$R_{a'b'}$ = 10x20 + 20x5 + 5x10/(5) = 350/(5) = 70 Ω
$R_{b'c'}$ = 350/(10) = 35Ω, Ra'c' = 350/(20) = 17.5 Ω

Also $30\|70 = 30x70/(100) = 21\Omega$ and 35/(15) = 35x15/(50) = 10.5

$R_{ab} = 25 + 17.5\|(21+10.5) = 25 + 17.5\|31.5$

$R_{ab} = \underline{\mathbf{36.25\ \Omega}}$



Chapter 2, Problem 52.

For the circuit shown in Fig. 2.116, find the equivalent resistance. All resistors are 1Ω.

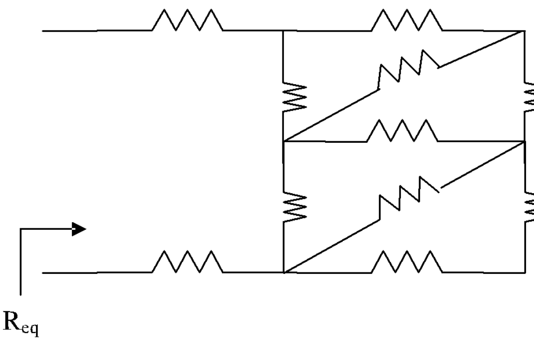
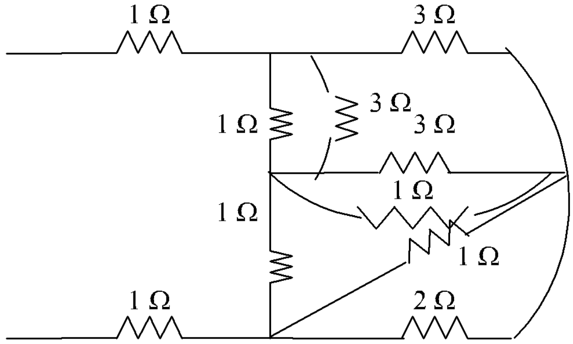


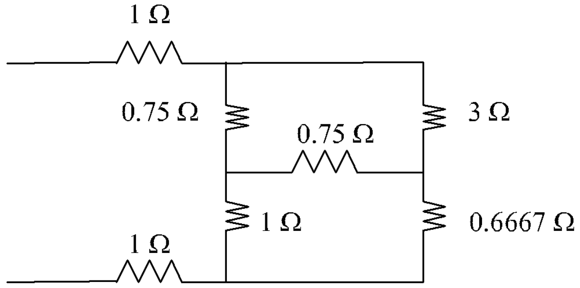
Figure 2.116 For Prob. 2.52.

Chapter 2, Solution 52

Converting the wye-subnetwork to delta-subnetwork, we obtain the circuit below.



3//1 = 3x1/4 = 0.75, 2//1 =2x1/3 = 0.6667. Combining these resistances leads to the circuit below.

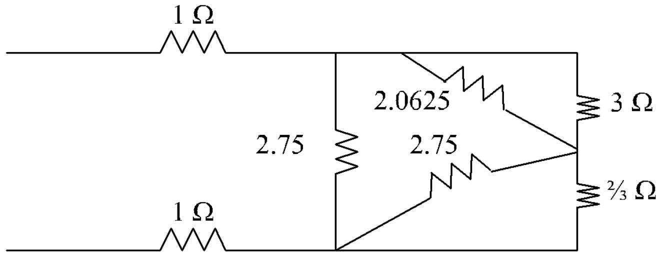


We now convert the wye-subnetwork to the delta-subnetwork.

$$R_a = \frac{0.75x1 + 0.75x1 + 0.75^2}{1} = 2.0625$$

$$R_b = R_c = \frac{2.0625}{0.75} = 2.75$$

This leads to the circuit below.



$$R = 3//2.0625 + 2.75//\frac{2}{3} = \frac{3x2.065}{5.0625} + \frac{2.75x2/3}{2/3+2.75} = 1.7607$$

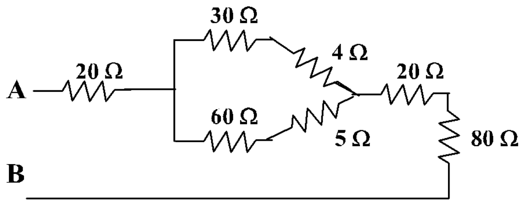
$$R_{eq} = 1 + 1 + 2.75//1.7607 = 2 + \frac{2.75x1.7607}{2.75+1.7607} = \underline{3.0734\ \Omega}$$

Chapter 2, Problem 53.

Obtain the equivalent resistance R_{ab} in each of the circuits of Fig. 2.117. In (b), all resistors have a value of 30 Ω.

Chapter 2, Solution 53

(a) Converting one Δ to T yields the equivalent circuit below:



$$R_{a'n} = \frac{40x10}{40+10+50} = 4\Omega,\ R_{b'n} = \frac{10x50}{100} = 5\Omega,\ R_{c'n} = \frac{40x50}{100} = 20\Omega$$

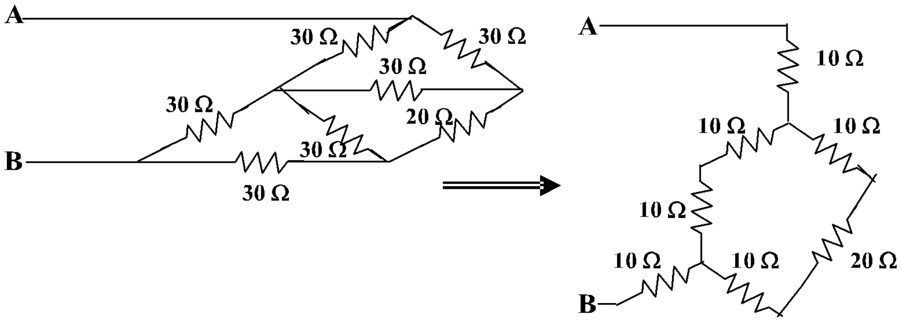
$R_{ab} = 20 + 80 + 20 + (30+4)||(60+5) = 120 + 34||65$

$R_{ab} =$ **142.32 Ω**

(c) We combine the resistor in series and in parallel.

$$30||(30+30) = \frac{30x60}{90} = 20\Omega$$

We convert the balanced Δ s to Ts as shown below:



$R_{ab} = 10 + (10+10)||(10+20+10) + 10 = 20 + 20||40$

R_{ab} = 33.33 Ω

Chapter 2, Problem 54.

Consider the circuit in Fig. 2.118. Find the equivalent resistance at terminals: (a) *a-b*, (b) *c-d*.

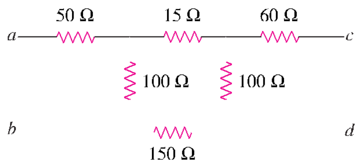


Figure 2.118

Chapter 2, Solution 54

(a) $R_{ab} = 50 + 100 / /(150 + 100 + 150) = 50 + 100 / /400 = \underline{130\Omega}$

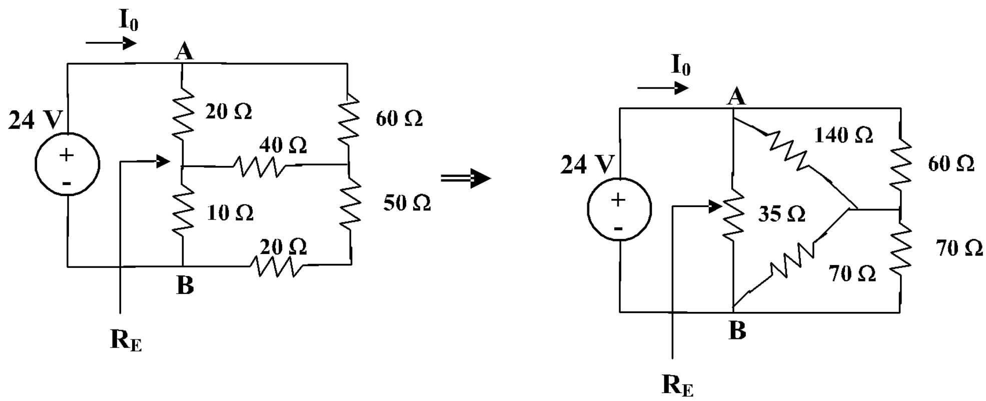
(b) $R_{ab} = 60 + 100 / /(150 + 100 + 150) = 60 + 100 / /400 = \underline{140\Omega}$

Chapter 2, Problem 55.

Calculate I_o in the circuit of Fig. 2.119.

Chapter 2, Solution 55

We convert the T to Δ.



$$R_{ab} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} = \frac{20x40 + 40x10 + 10x20}{40} = \frac{1400}{40} = 35\Omega$$

$R_{ac} = 1400/(10) = 140\Omega$, $R_{bc} = 1400/(20) = 70\Omega$

$70 \| 70 = 35$ and $140 \| 160 = 140x60/(200) = 42$

$R_{eq} = 35 \| (35 + 42) = 24.0625\Omega$

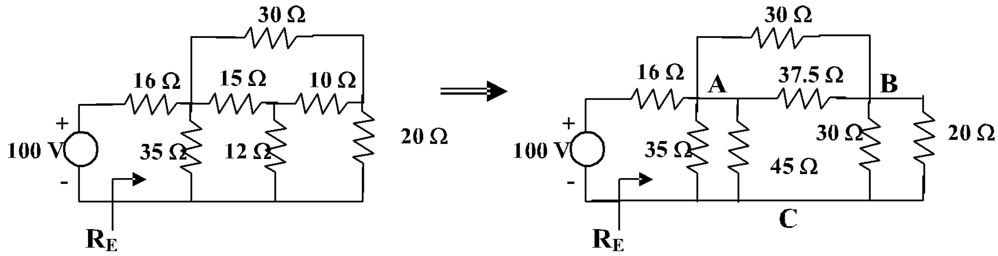
$I_0 = 24/(R_{ab}) =$ **997.4mA**

Chapter 2, Problem 56.

Determine *V* in the circuit of Fig. 1.120.

Chapter 2, Solution 56

We need to find R_{eq} and apply voltage division. We first tranform the Y network to Δ.



$$R_{ab} = \frac{15x10 + 10x12 + 12x15}{12} = \frac{450}{12} = 37.5\Omega$$

R_{ac} = 450/(10) = 45Ω, R_{bc} = 450/(15) = 30Ω

Combining the resistors in parallel,

$$30||20 = (600/50) = 12\ \Omega,$$

$$37.5||30 = (37.5x30/67.5) = 16.667\ \Omega$$

$$35||45 = (35x45/80) = 19.688\ \Omega$$

$$R_{eq} = 19.688||(12 + 16.667) = 11.672\Omega$$

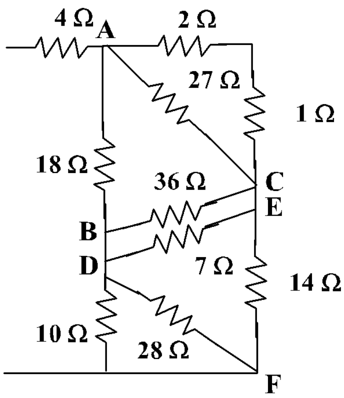
By voltage division,

$$v = \frac{11.672}{11.672 + 16}100 = \underline{\mathbf{42.18\ V}}$$

Chapter 2, Problem 57.

Find R_{eq} and I in the circuit of Fig. 2.121.

Chapter 2, Solution 57



$$R_{ab} = \frac{6x12 + 12x8 + 8x6}{12} = \frac{216}{12} = 18\ \Omega$$

$R_{ac} = 216/(8) = 27\Omega$, $R_{bc} = 36\ \Omega$

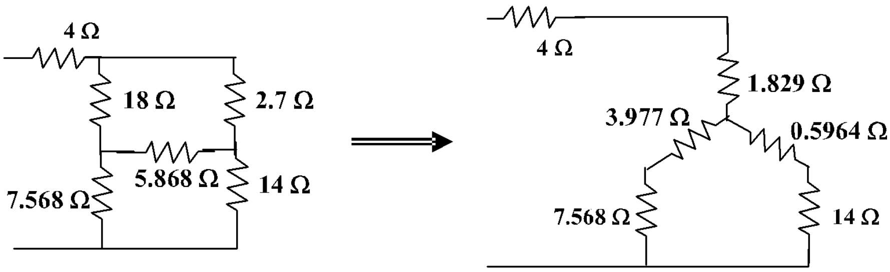
$$R_{de} = \frac{4x2 + 2x8 + 8x4}{8} = \frac{56}{8} 7\ \Omega$$

$R_{ef} = 56/(4) = 14\Omega$, $R_{df} = 56/(2) = 28\ \Omega$

Combining resistors in parallel,

$$10\|28 = \frac{280}{38} = 7.368\Omega,\ \ 36\|7 = \frac{36x7}{43} = 5.868\Omega$$

$$27\|3 = \frac{27x3}{30} = 2.7\Omega$$



$$R_{an} = \frac{18x2.7}{18+2.7+5.867} = \frac{18x2.7}{26.567} = 1.829\ \Omega$$

$$R_{bn} = \frac{18x5.868}{26.567} = 3.977\ \Omega$$

$$R_{cn} = \frac{5.868x2.7}{26.567} = 0.5904\ \Omega$$

$$R_{eq} = 4 + 1.829 + (3.977 + 7.368)\|(0.5964 + 14)$$

$$= 5.829 + 11.346\|14.5964 = \underline{\mathbf{12.21\ \Omega}}$$

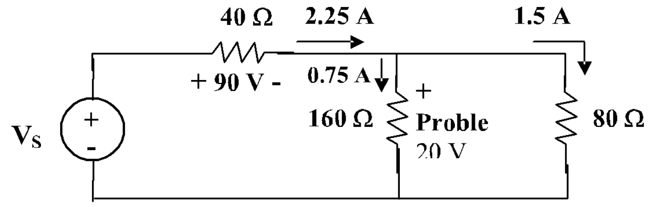
$i = 20/(R_{eq}) =$ **<u>1.64 A</u>**

Chapter 2, Problem 58.

The lightbulb in Fig. 2.122 is rated 120 V, 0.75 A. Calculate V_s to make the lightbulb operate at the rated conditions.

Chapter 2, Solution 58

The resistor of the bulb is 120/(0.75) = 160Ω



Once the 160Ω and 80Ω resistors are in parallel, they have the same voltage 120V. Hence the current through the 40Ω resistor is

$$40(0.75 + 1.5) = 2.25 \text{ x } 40 = 90$$

Thus

$$v_s = 90 + 120 = \underline{\textbf{210 V}}$$

Chapter 2, Problem 59.

Three lightbulbs are connected in series to a 100-V battery as shown in Fig. 2.123. Find the current *I* through the bulbs.

Chapter 2, Solution 59

TOTAL POWER P = 30 + 40 + 50 + 120 W = VI

OR I = P/(V) = 120/(100) = **1.2 A**

Chapter 2, Problem 60.
If the three bulbs of Prob. 2.59 are connected in parallel to the 100-V battery, calculate the current through each bulb.

Chapter 2, Solution 60

$p = iv \qquad\qquad i = p/(v)$

$i_{30W} = 30/(100) = \underline{\mathbf{0.3\ A}}$

$i_{40W} = 40/(100) = \underline{\mathbf{0.4\ A}}$

$i_{50W} = 50/(100) = \underline{\mathbf{0.5\ A}}$

Chapter 2, Problem 61.
As a design engineer, you are asked to design a lighting system consisting of a 70-W power supply and two lightbulbs as shown in Fig. 2.124. You must select the two bulbs from the following three available bulbs.

$R_1 = 80\Omega$, cost = \$0.60 (standard size)
$R_2 = 90\Omega$, cost = \$0.90 (standard size)
$R_3 = 100\ \Omega$, cost = \$0.75 (nonstandard size)
The system should be designed for minimum cost such that $I = 1.2$ A ± 5 percent.

Chapter 2, Solution 61

There are three possibilities, but they must also satisfy the current range of 1.2 + 0.06 = 1.26 and 1.2 – 0.06 = 1.14.

(a) Use R_1 and R_2:

$R = R_1 \| R_2 = 80 \| 90 = 42.35\Omega$

$p = i^2R = 70W$

$i^2 = 70/42.35 = 1.6529$ or $i = 1.2857$ (which is outside our range)

cost = \$0.60 + \$0.90 = \$1.50

(b) Use R_1 and R_3:

$R = R_1 \| R_3 = 80 \| 100 = 44.44\ \Omega$

$i^2 = 70/44.44 = 1.5752$ or $i = 1.2551$ (which is within our range), cost = \$1.35

(c) Use R_2 and R_3:

$R = R_2 \| R_3 = 90 \| 100 = 47.37\Omega$

$i^2 = 70/47.37 = 1.4777$ or $i = 1.2156$ (which is within our range), cost = \$1.65

Note that cases (b) and (c) satisfy the current range criteria and (b) is the cheaper of the two, hence the correct choice is:

$\underline{\mathbf{R_1\ and\ R_3}}$

Chapter 2, Problem 62.

A three-wire system supplies two loads *A* and *B* as shown in Fig. 2.125. Load *A* consists of a motor drawing a current of 8 A, while load *B* is a PC drawing 2 A. Assuming 10 h/day of use for 365 days and 6 cents/kWh, calculate the annual energy cost of the system.

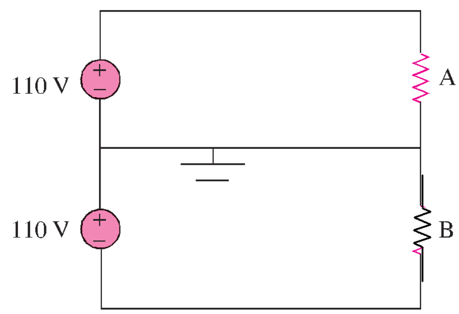


Figure 2.125

Chapter 2, Solution 62

$p_A = 110 \text{x} 8 = 880$ W, $p_B = 110 \text{x} 2 = 220$ W

Energy cost = \$0.06 x 365 x10 x (880 + 220)/1000 = <u>\$240.90</u>

Chapter 2, Problem 63.

If an ammeter with an internal resistance of 100 Ω and a current capacity of 2 mA is to measure 5 A, determine the value of the resistance needed. Calculate the power dissipated in the shunt resistor.

Chapter 2, Solution 63

Use eq. (2.61),

$$R_n = \frac{I_m}{I - I_m} R_m = \frac{2 \text{x} 10^{-3} \text{x} 100}{5 - 2 \text{x} 10^{-3}} = 0.04\Omega$$

$I_n = I - I_m = 4.998$ A

$p = I_n^2 R = (4.998)^2 (0.04) = 0.9992 \cong \underline{\mathbf{1\ W}}$

Chapter 2, Problem 64.

The potentiometer (adjustable resistor) R_x in Fig. 2.126 is to be designed to adjust current I_x from 1 A to 10 A. Calculate the values of R and R_x to achieve this.

Chapter 2, Solution 64

When $R_x = 0$, $i_x = 10A$ $\qquad R = \frac{110}{10} = 11\ \Omega$

When R_x is maximum, $i_x = 1A \longrightarrow R + R_x = \frac{110}{1} = 110\ \Omega$

i.e., $R_x = 110 - R = 99\ \Omega$

Thus, $R = \underline{11\ \Omega}$, $\quad R_x = \underline{99\ \Omega}$

Chapter 2, Problem 65.

A d'Arsonval meter with an internal resistance of 1 kΩ requires 10 mA to produce full-scale deflection. Calculate the value of a series resistance needed to measure 50 V of full scale.

Chapter 2, Solution 65

$$R_n = \frac{V_{fs}}{I_{fs}} - R_m = \frac{50}{10mA} - 1\ k\Omega = \underline{\mathbf{4\ K\Omega}}$$

Chapter 2, Problem 66.

A 20-kΩ/V voltmeter reads 10 V full scale,

(a) What series resistance is required to make the meter read 50 V full scale?
(b) What power will the series resistor dissipate when the meter reads full scale?

Chapter 2, Solution 66

$$20\ k\Omega/V = \text{sensitivity} = \frac{1}{I_{fs}}$$

$$\text{i.e., } I_{fs} = \frac{1}{20} k\Omega/V = 50\ \mu A$$

The intended resistance $R_m = \frac{V_{fs}}{I_{fs}} = 10(20k\Omega/V) = 200k\Omega$

(a) $\quad R_n = \frac{V_{fs}}{i_{fs}} - R_m = \frac{50\ V}{50\mu A} - 200\ k\Omega = \underline{\mathbf{800\ k\Omega}}$

(b) $\quad p = I_{fs}^2 R_n = (50\ \mu A)^2 (800\ k\Omega) = \underline{\mathbf{2\ mW}}$

Chapter 2, Problem 67.

(c) Obtain the voltage v_o in the circuit of Fig. 2.127.
(d) Determine the voltage v'_o measured when a voltmeter with 6-kΩ internal resistance is connected as shown in Fig. 2.127.
(e) The finite resistance of the meter introduces an error into the measurement. Calculate the percent error as

$$\left|\frac{v_o - v'_o}{v_o}\right| \times 100\%.$$

(f) Find the percent error if the internal resistance were 36 kΩ.

Chapter 2, Solution 67

(c) By current division,

$i_0 = 5/(5 + 5)$ (2 mA) = 1 mA
$V_0 = (4\ k\Omega)\ i_0 = 4 \times 10^3 \times 10^{-3} =$ **4 V**

(d) $4k\|6k = 2.4k\Omega$. By current division,

$$i'_0 = \frac{5}{1+2.4+5}(2mA) = 1.19\ mA$$

$$v'_0 = (2.4\ k\Omega)(1.19\ mA) = \mathbf{2.857\ V}$$

(e) $\% \text{ error} = \left|\frac{v_0 - v'_0}{v_0}\right| \text{ x } 100\% = \frac{1.143}{4}\text{x}100 = \mathbf{28.57\%}$

(f) $4k\|36\ k\Omega = 3.6\ k\Omega$. By current division,

$$i'_0 = \frac{5}{1+3.6+5}(2mA) = 1.042mA$$

$$v'_0(3.6k\Omega)(1.042\ mA) = 3.75V$$

$$\% \text{ error} = \left|\frac{v - v'_0}{v_0}\right|\text{x}100\% = \frac{0.25\text{x}100}{4} = \mathbf{6.25\%}$$

Chapter 2, Problem 68.

(f) Find the current *i* in the circuit of Fig. 2.128(a).

(g) An ammeter with an internal resistance of 1 Ω is inserted in the network to measure *i'* as shown in Fig. 2.128 (b). What is i''?

(h) Calculate the percent error introduced by the meter as

$$\left|\frac{i-i'}{i}\right|\times 100\%$$

Chapter 2, Solution 68

(F) $40 = 24 \| 60\Omega$

$$i = \frac{4}{16+24} = \underline{\mathbf{0.1\ A}}$$

(G) $$i' = \frac{4}{16+1+24} = \underline{\mathbf{0.09756\ A}}$$

(H) $$\%\ \text{error} = \frac{0.1-0.09756}{0.1} \text{x}100\% = \underline{\mathbf{2.44\%}}$$

Chapter 2, Problem 69.

A voltmeter is used to measure V_o in the circuit in Fig. 2.122. The voltmeter model consists of an ideal voltmeter in parallel with a 100-kΩ resistor. Let V_s = 40 V, R_s = 10 kΩ, and R_1 = 20 kΩ. Calculate V_o with and without the voltmeter when
(a) R_2 = 1 kΩ (b) R_2 = 10 kΩ
(c) R_2 = 100 kΩ

Chapter 2, Solution 69

With the voltmeter in place,

$$V_0 = \frac{R_2 \| R_m}{R_1 + R_S + R_2 \| R_m} V_S$$

where R_m = 100 kΩ without the voltmeter,

$$V_0 = \frac{R_2}{R_1 + R_2 + R_S} V_S$$

(a) When R_2 = 1 kΩ, $R_m \| R_2 = \frac{100}{101}$kΩ

$$V_0 = \frac{\frac{100}{101}}{\frac{100}{101} + 30}(40) = \underline{\textbf{1.278 V (with)}}$$

$$V_0 = \frac{1}{1+30}(40) = \underline{\textbf{1.29 V (without)}}$$

(b) When R_2 = 10 kΩ, $R_2 \| R_m = \frac{1000}{110} = 9.091\text{k}\Omega$

$$V_0 = \frac{9.091}{9.091+30}(40) = \underline{\textbf{9.30 V (with)}}$$

$$V_0 = \frac{10}{10+30}(40) = \underline{\textbf{10 V (without)}}$$

(c) When R_2 = 100 kΩ, $R_2 \| R_m = 50\text{k}\Omega$

$$V_0 = \frac{50}{50+30}(40) = \underline{\textbf{25 V (with)}}$$

$$V_0 = \frac{100}{100+30}(40) = \underline{\textbf{30.77 V (without)}}$$

Chapter 2, Problem 70.

(a) Consider the Wheatstone Bridge shown in Fig. 2.130. Calculate v_a, v_b, and
(b) Rework part (a) if the ground is placed at a instead of o.

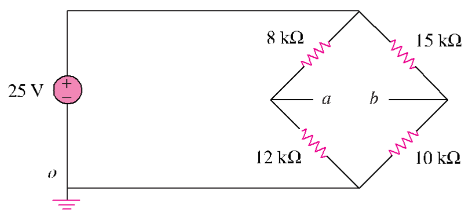


Figure 2.130

Chapter 2, Solution 70

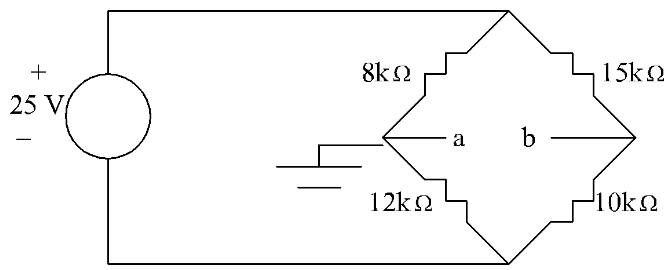
(a) Using voltage division,

$$v_a = \frac{12}{12+8}(25) = \underline{15V}$$

$$v_b = \frac{10}{10+15}(25) = \underline{10V}$$

$$v_{ab} = v_a - v_b = 15 - 10 = \underline{5V}$$

(b)



$v_a = \underline{\mathbf{0}}$; $v_{ac} = -(8/(8+12))25 = -10V$; $v_{cb} = (15/(15+10))25 = 15V$.

$v_{ab} = v_{ac} + v_{cb} = -10 + 15 = \underline{\mathbf{5V}}$.

$v_b = -v_{ab} = \underline{\mathbf{-5V}}$.

Chapter 2, Problem 71.

Figure 2.131 represents a model of a solar photovoltaic panel. Given that v_s = 30 **V**, R_1 = 20 Ω, I_L = 1 **A**, find R_L.

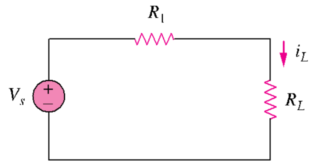
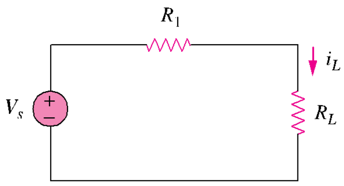


Figure 2.131

Chapter 2, Solution 71



Given that v_s = 30 **V**, R_1 = 20 Ω, I_L = 1 **A**, find R_L.

$$v_s = i_L(R_1 + R_L) \longrightarrow R_L = \frac{v_s}{i_L} - R_1 = \frac{30}{1} - 20 = \underline{10\Omega}$$

Chapter 2, Problem 72.
Find V_o in the two-way power divider circuit in Fig. 2.132.

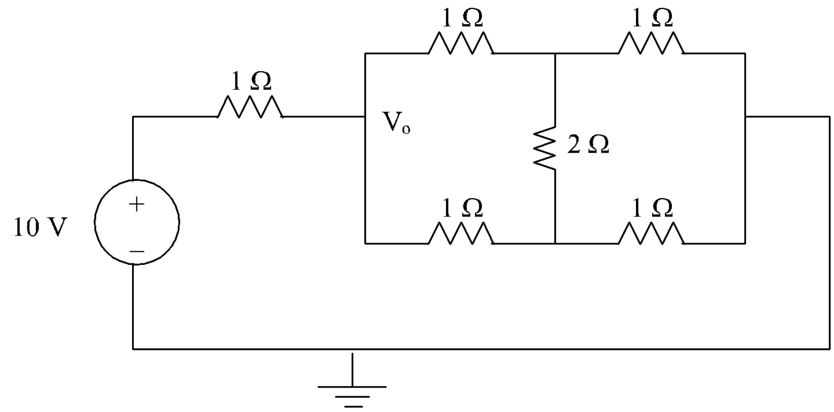
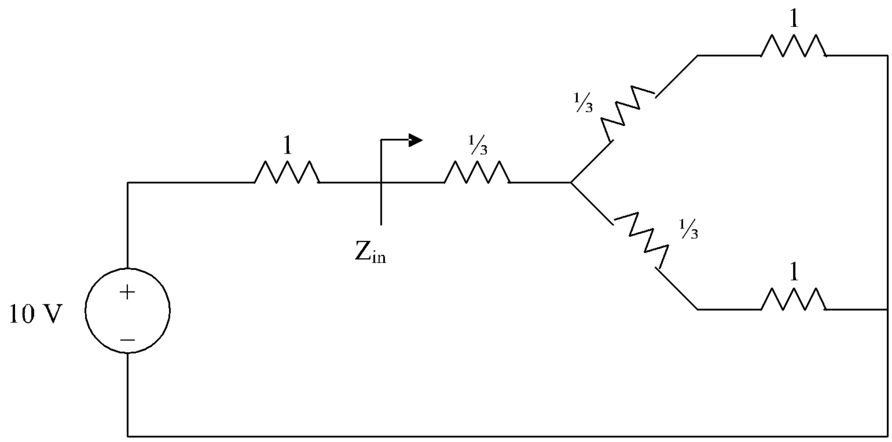


Figure 2.132 For Prob. 2.72.

Chapter 2, Solution 72
Converting the delta subnetwork into wye gives the circuit below.



$$Z_{in} = \frac{1}{3} + (1+\frac{1}{3})//(1+\frac{1}{3}) = \frac{1}{3} + \frac{1}{2}(\frac{4}{3}) = 1\ \Omega$$

$$V_o = \frac{Z_{in}}{1+Z_{in}}(10) = \frac{1}{1+1}(10) = \underline{5\text{ V}}$$

Chapter 2, Problem 73.

An ammeter model consists of an ideal ammeter in series with a 20-Ω resistor. It is connected with a current source and an unknown resistor R_x as shown in Fig. 2.133. The ammeter reading is noted. When a potentiometer R is added and adjusted until the ammeter reading drops to one half its previous reading, then R = 65 Ω. What is the value of R_x?

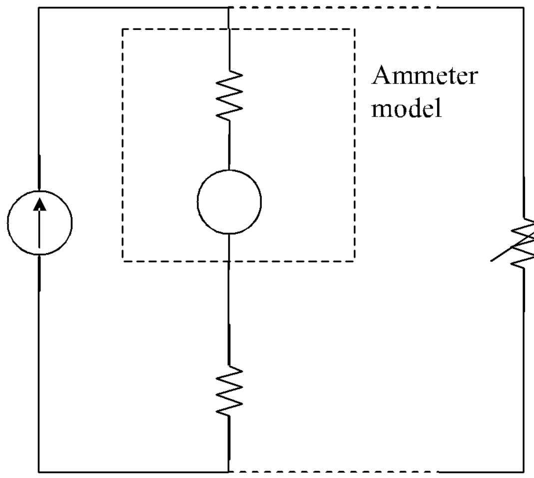


Figure 2.133

Chapter 2, Solution 73

By the current division principle, the current through the ammeter will be one-half its previous value when

$R = 20 + R_x$
$65 = 20 + R_x \longrightarrow R_x = \underline{\mathbf{45\ \Omega}}$

Chapter 2, Problem 74.

The circuit in Fig. 2.134 is to control the speed of a motor such that the motor draws currents 5 A, 3 A, and 1 A when the switch is at high, medium, and low positions, respectively. The motor can be modeled as a load resistance of 20 mΩ. Determine the series dropping resistances R_1, R_2, and R_3.

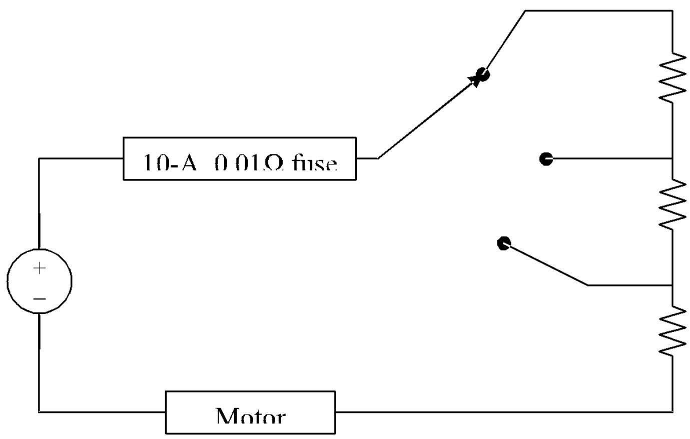


Figure 134

Chapter 2, Solution 74

With the switch in high position,

$$6 = (0.01 + R_3 + 0.02) \times 5 \longrightarrow R_3 = \underline{\mathbf{1.17\ \Omega}}$$

At the medium position,

$$6 = (0.01 + R_2 + R_3 + 0.02) \times 3 \longrightarrow R_2 + R_3 = 1.97$$

or $R_2 = 1.97 - 1.17 = \underline{\mathbf{0.8\ \Omega}}$

At the low position,

$$6 = (0.01 + R_1 + R_2 + R_3 + 0.02) \times 1 \longrightarrow R_1 + R_2 + R_3 = 5.97$$

$R_1 = 5.97 - 1.97 = \underline{\mathbf{4\ \Omega}}$

Chapter 2, Problem 75.

Find Z_{ab} in the four-way power divider circuit in Fig. 2.135. Assume each element is 1Ω.

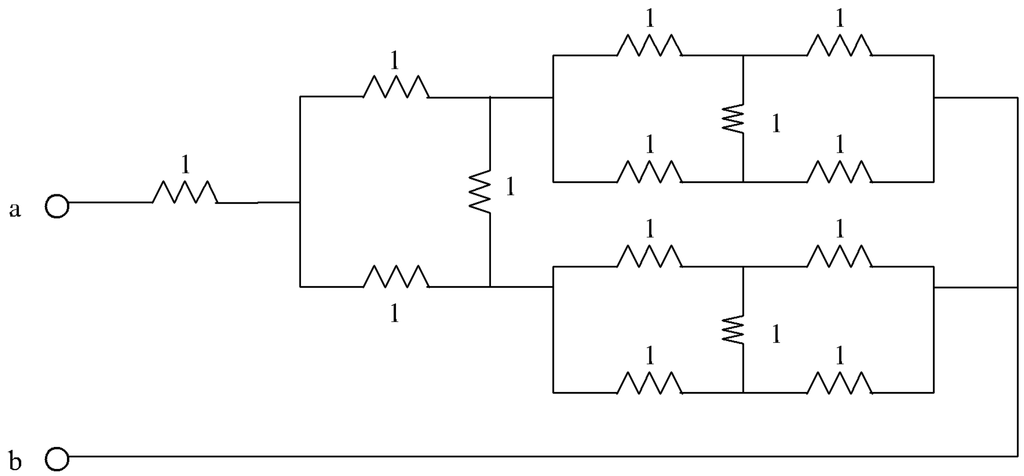
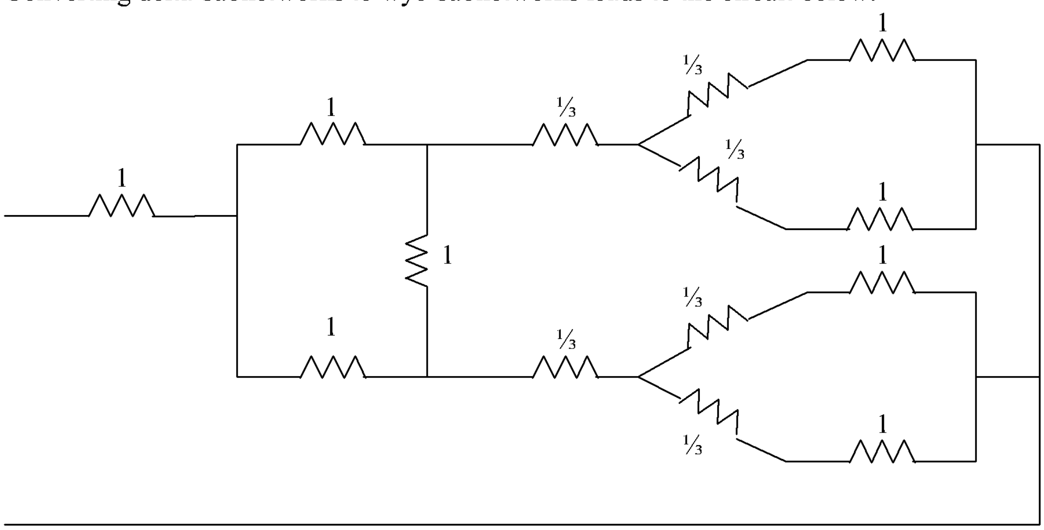


Figure 2.135 For Prob. 2.75.

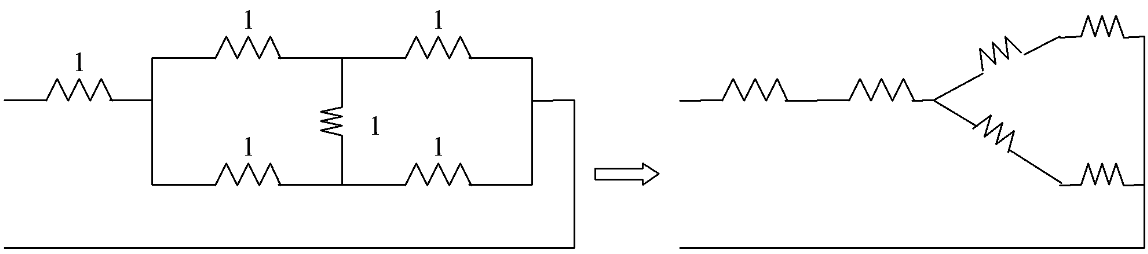
Chapter 2, Solution 75

Converting delta-subnetworks to wye-subnetworks leads to the circuit below.



$$\frac{1}{3} + (1 + \frac{1}{3}) // (1 + \frac{1}{3}) = \frac{1}{3} + \frac{1}{2}(\frac{4}{3}) = 1$$

With this combination, the circuit is further reduced to that shown below.



$$Z_{ab} = 1 + \frac{1}{3} + (1 + \frac{1}{3}) // (1 + \frac{1}{3}) = 1 + 1 = \underline{2\ \Omega}$$

Chapter 2, Problem 76.

Repeat Prob. 2.75 for the eight-way divider shown in Fig. 2.136.

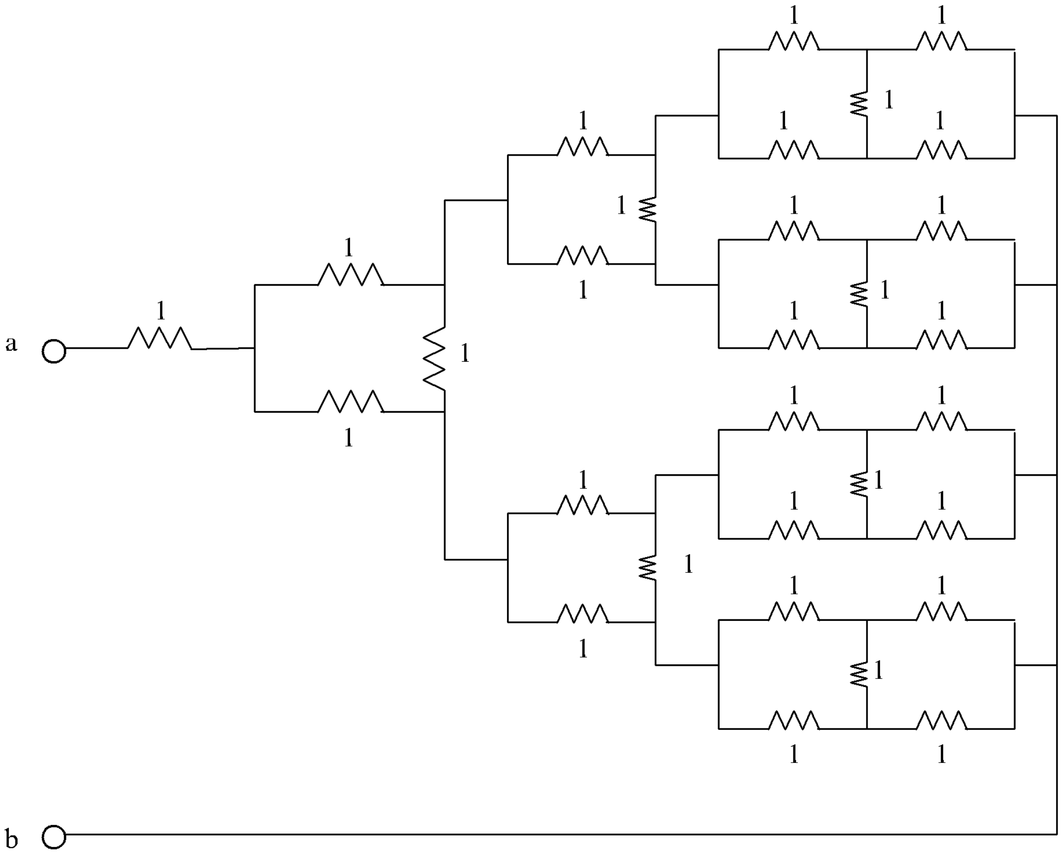


Figure 2.136 For Prob. 2.76.

Chapter 2, Solution 76

Z_{ab}= 1 + 1 = **2 Ω**

Chapter 2, Problem 77.

Suppose your circuit laboratory has the following standard commercially available resistors in large quantities:

1.8 Ω	20 Ω	300 Ω	24 kΩ	56 kΩ

Using series and parallel combinations and a minimum number of available resistors, how would you obtain the following resistances for an electronic circuit design?

(a) 5 Ω (b) 311.8 Ω
(c) 40 kΩ (d) 52.32 kΩ

Chapter 2, Solution 77

(a) $5\ \Omega = 10 \| 10 = 20 \| 20 \| 20 \| 20$

i.e., **four 20 Ω resistors in parallel.**

(b) $311.8 = 300 + 10 + 1.8 = 300 + 20 \| 20 + 1.8$

i.e., **one 300Ω resistor in series with 1.8Ω resistor and a parallel combination of two 20Ω resistors.**

(c) $40\text{k}\Omega = 12\text{k}\Omega + 28\text{k}\Omega = 24 \| 24\text{k} + 56\text{k} \| 56\text{k}$

i.e., **Two 24kΩ resistors in parallel connected in series with two 56kΩ resistors in parallel.**

(d) $42.32\text{k}\Omega = 42l + 320$

$= 24\text{k} + 28\text{k} = 320$

$= 24\text{k} = 56\text{k} \| 56\text{k} + 300 + 20$

i.e., **A series combination of a 20Ω resistor, 300Ω resistor, 24kΩ resistor, and a parallel combination of two 56kΩ resistors.**

Chapter 2, Problem 78.

In the circuit in Fig. 2.137, the wiper divides the potentiometer resistance between αR and $(1 - \alpha)R$, $0 \le \alpha \le 1$. Find v_o / v_s.

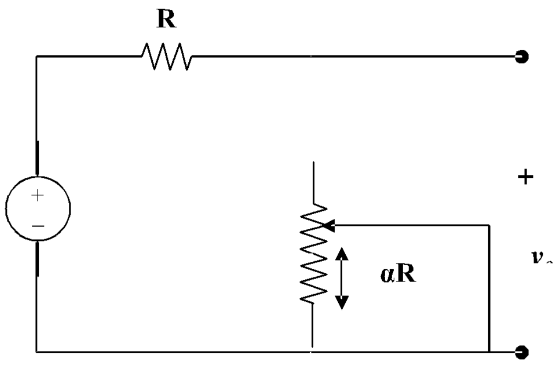
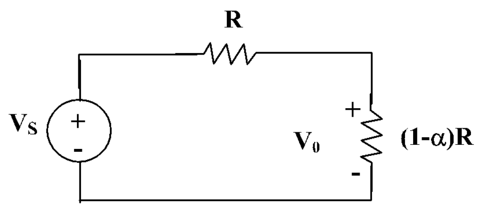


Figure 137

Chapter 2, Solution 78

The equivalent circuit is shown below:



$$V_0 = \frac{(1-\alpha)R}{R+(1-\alpha)R} V_S = \frac{1-\alpha}{2-\alpha} V_S$$

$$\underline{\frac{V_0}{V_S} = \frac{1-\alpha}{2-\alpha}}$$

Chapter 2, Problem 79.

An electric pencil sharpener rated 240 mW, 6 V is connected to a 9-V battery as shown in Fig. 2.138. Calculate the value of the series-dropping resistor R_x needed to power the sharpener.

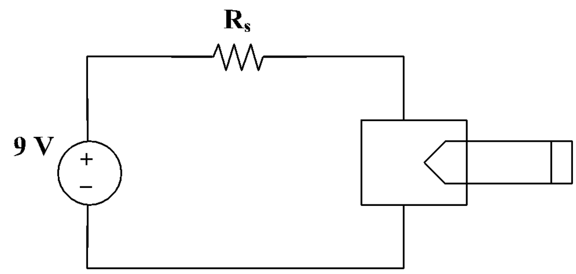


Figure 138

Chapter 2, Solution 79

Since $p = v^2/R$, the resistance of the sharpener is

$R = v^2/(p) = 6^2/(240 \times 10^{-3}) = 150\Omega$
$I = p/(v) = 240 \text{ mW}/(6V) = 40 \text{ mA}$

Since R and R_x are in series, I flows through both.

$IR_x = V_x = 9 - 6 = 3 \text{ V}$
$R_x = 3/(I) = 3/(40 \text{ mA}) = 3000/(40) = \underline{\mathbf{75\ \Omega}}$

Chapter 2, Problem 80.

A loudspeaker is connected to an amplifier as shown in Fig. 2.139. If a 10-Ω loudspeaker draws the maximum power of 12 W from the amplifier, determine the maximum power a 4-Ω loudspeaker will draw.

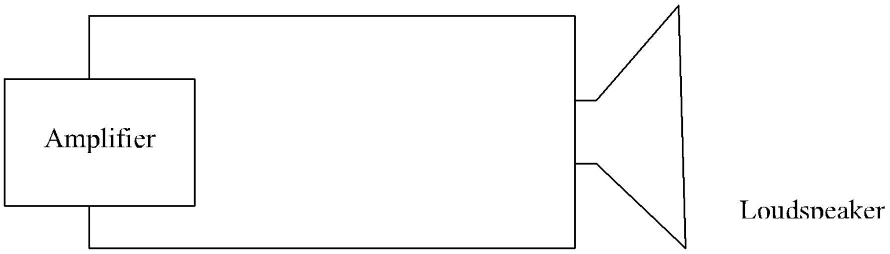
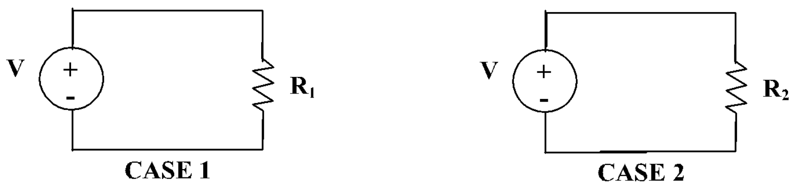


Figure 139

Chapter 2, Solution 80

The amplifier can be modeled as a voltage source and the loudspeaker as a resistor:



Hence $p = \frac{V^2}{R}, \quad \frac{p_2}{p_1} = \frac{R_1}{R_2} \longrightarrow p_2 = \frac{R_1}{R_2}p_1 = \frac{10}{4}(12) = \underline{\textbf{30 W}}$

Chapter 2, Problem 81.

In a certain application, the circuit in Figure 2.140 must be designed to meet these two criteria:

(a) $V_o / V_s = 0.05$ (b) $R_{eq} = 40$ kΩ

If the load resistor 5 kΩ is fixed, find R_1 and R_2 to meet the criteria.

Chapter 2, Solution 81

Let R_1 and R_2 be in kΩ.

$$R_{eq} = R_1 + R_2 \| 5 \qquad (1)$$

$$\frac{V_0}{V_S} = \frac{5\|R_2}{5\|R_2 + R_1} \qquad (2)$$

From (1) and (2), $0.05 = \frac{5\|R_1}{40} \longrightarrow 2 = 5\|R_2 = \frac{5R_2}{5+R_2}$ or $R_2 = 3.333$ kΩ

From (1), $40 = R_1 + 2 \longrightarrow R_1 = 38$ kΩ

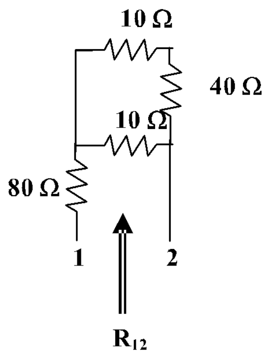
Thus **$R_1 = 38$ kΩ, $R_2 = 3.333$ kΩ**

Chapter 2, Problem 82.
The pin diagram of a resistance array is shown in Fig. 2.141. Find the equivalent resistance between the following:

(a) 1 and 2 (b) 1 and 3 (c) 1 and 4

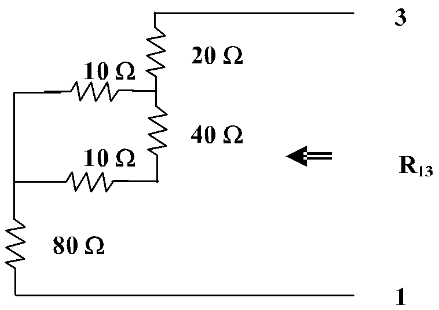
Chapter 2, Solution 82

(a)



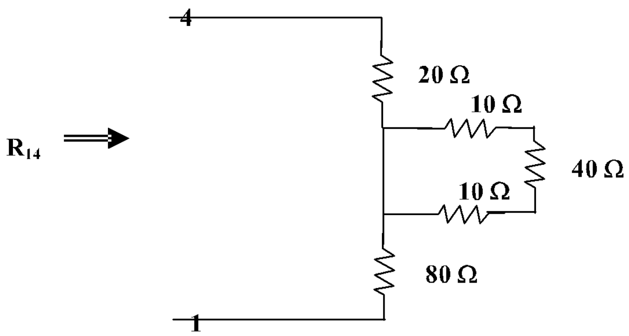
$R_{12} = 80 + 10 \| (10 + 40) = 80 + \frac{50}{6} = \underline{\mathbf{88.33\ \Omega}}$

(b)



$R_{13} = 80 + 10 \| (10 + 40) + 20 = 100 + 10 \| 50 = \underline{\mathbf{108.33\ \Omega}}$

(c)



$R_{14} = 80 + 0 \| (10 + 40 + 10) + 20 = 80 + 0 + 20 = \underline{\mathbf{100\ \Omega}}$

Chapter 2, Problem 83.

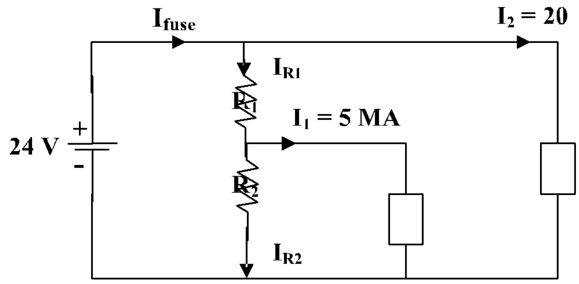
Two delicate devices are rated as shown in Fig. 2.142. Find the values of the resistors R_1 and R_2 needed to power the devices using a 24-V battery.

Chapter 2, Solution 83

The voltage across the fuse should be negligible when compared with 24 V (this can be checked later when we check to see if the fuse rating is exceeded in the final circuit). We can calculate the current through the devices.

$$I_1 = \frac{p_1}{V_1} = \frac{45\text{mW}}{9\text{V}} = 5\text{mA}$$

$$I_2 = \frac{p_2}{V_2} = \frac{480\text{mW}}{24} = 20\text{mA}$$



Let R_3 represent the resistance of the first device, we can solve for its value from knowing the voltage across it and the current through it.

$$R_3 = 9/0.005 = 1{,}800\ \Omega$$

This is an interesting problem in that it essentially has two unknowns, R_1 and R_2 but only one condition that need to be met and that the voltage across R_3 must equal 9 volts. Since the circuit is powered by a battery we could choose the value of R_2 which draws the least current, $R_2 = \infty$. Thus we can calculate the value of R_1 that give 9 volts across R_3.

$$9 = (24/(R_1 + 1800))1800 \text{ or } R_1 = (24/9)1800 - 1800 = \mathbf{\underline{3{,}000\Omega}}$$

This value of R_1 means that we only have a total of 25 mA flowing out of the battery through the fuse which means it will not open and produces a voltage drop across it of 0.05V. This is indeed negligible when compared with the 24-volt source.

Chapter 3, Problem 1.

Determine I_x in the circuit shown in Fig. 3.50 using nodal analysis.

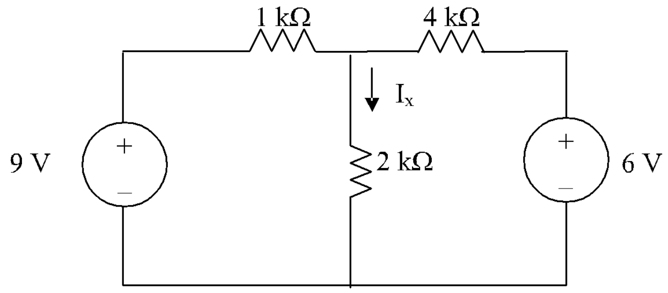


Figure 3.50 For Prob. 3.1.

Chapter 3, Solution 1

Let V_x be the voltage at the node between 1-kΩ and 4-kΩ resistors.

$$\frac{9-V_x}{1k}+\frac{6-V_x}{4k}=\frac{V_k}{2k} \longrightarrow V_x=6$$

$$I_x=\frac{V_x}{2k}=\underline{3\text{ mA}}$$

Chapter 3, Problem 2.

For the circuit in Fig. 3.51, obtain v_1 and v_2.

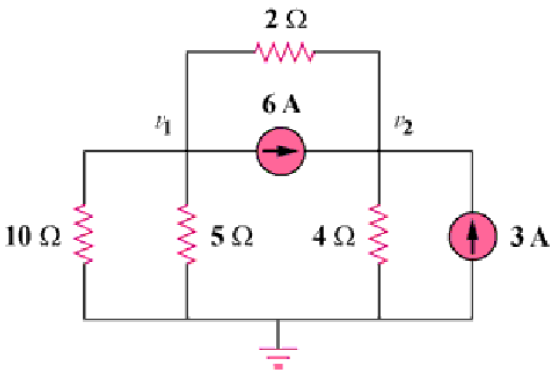


Figure 3.51

Chapter 3, Solution 2

At node 1,

$$\frac{-v_1}{10} - \frac{v_1}{5} = 6 + \frac{v_1 - v_2}{2} \longrightarrow 60 = -8v_1 + 5v_2 \qquad (1)$$

At node 2,

$$\frac{v_2}{4} = 3 + 6 + \frac{v_1 - v_2}{2} \longrightarrow 36 = -2v_1 + 3v_2 \qquad (2)$$

Solving (1) and (2),

v_1 = **<u>0 V</u>**, v_2 = **<u>12 V</u>**

Chapter 3, Problem 3.

Find the currents i_1 through i_4 and the voltage v_o in the circuit in Fig. 3.52.

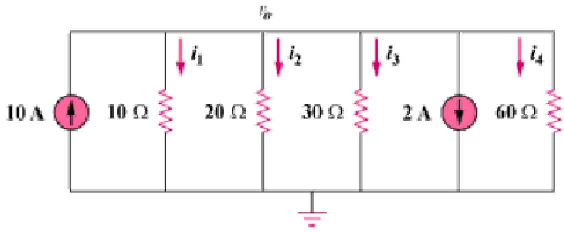


Figure 3.52

Chapter 3, Solution 3

Applying KCL to the upper node,

$$10 = \frac{v_0}{10} + \frac{v_o}{20} + \frac{v_o}{30} + 2 + \frac{v_0}{60} \longrightarrow v_0 = \underline{\mathbf{40\ V}}$$

$$i_1 = \frac{v_0}{10} = \underline{\mathbf{4\ A}},\ i_2 = \frac{v_0}{20} = \underline{\mathbf{2\ A}},\ i_3 = \frac{v_0}{30} = \underline{\mathbf{1.3333\ A}},\ i_4 = \frac{v_0}{60} = \underline{\mathbf{666.7\ mA}}$$

Chapter 3, Problem 4.

Given the circuit in Fig. 3.53, calculate the currents i_1 through i_4.

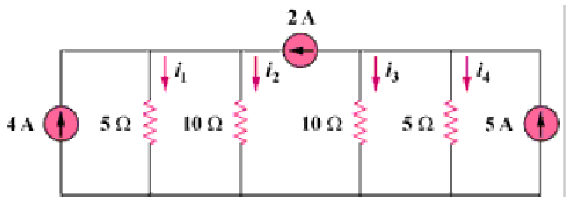
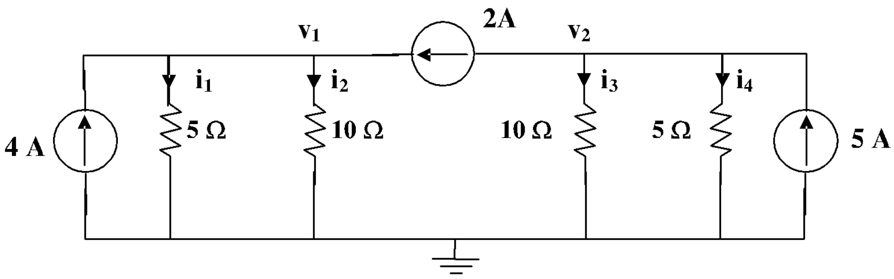


Figure 3.53

Chapter 3, Solution 4



At node 1,

$$4 + 2 = v_1/(5) + v_1/(10) \longrightarrow v_1 = 20$$

At node 2,

$$5 - 2 = v_2/(10) + v_2/(5) \longrightarrow v_2 = 10$$

$i_1 = v_1/(5) =$ **4 A**, $i_2 = v_1/(10) =$ **2 A**, $i_3 = v_2/(10) =$ **1 A**, $i_4 = v_2/(5) =$ **2 A**

Chapter 3, Problem 5.

Obtain v_0 in the circuit of Fig. 3.54.

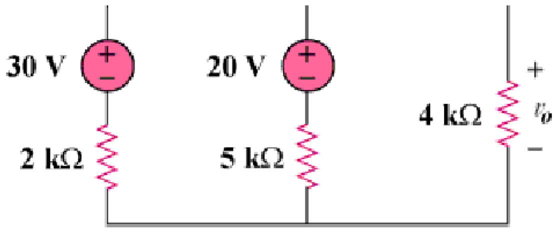


Figure 3.54

Chapter 3, Solution 5

Apply KCL to the top node.

$$\frac{30 - v_0}{2k} + \frac{20 - v_0}{5k} = \frac{v_0}{4k} \longrightarrow v_0 = \underline{\mathbf{20\ V}}$$

Chapter 3, Problem 6.

Use nodal analysis to obtain v_0 in the circuit in Fig. 3.55.

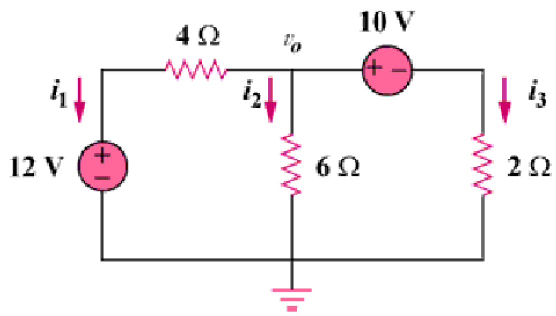


Figure 3.55

Chapter 3, Solution 6

$i_1 + i_2 + i_3 = 0$

$$\frac{v_2 - 12}{4} + \frac{v_0}{6} + \frac{v_0 - 10}{2} = 0$$

or $v_0 = \underline{\mathbf{8.727\ V}}$

Chapter 3, Problem 7.

Apply nodal analysis to solve for V_x in the circuit in Fig. 3.56.

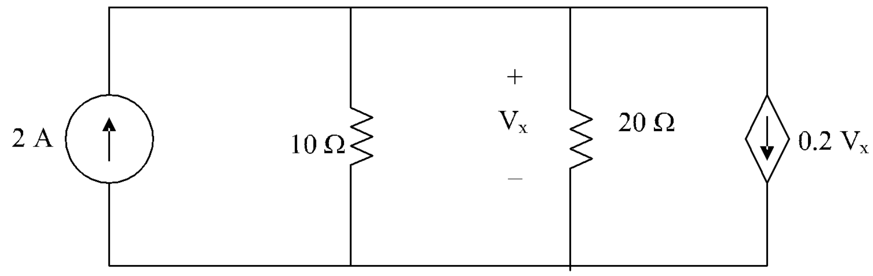


Figure 3.56 For Prob. 3.7.

Chapter 3, Solution 7

$$-2+\frac{V_X-0}{10}+\frac{V_X-0}{20}+0.2V_X=0$$

$0.35V_x = 2$ or $V_x =$ **<u>5.714 V</u>**.

Substituting into the original equation for a check we get,

$$0.5714 + 0.2857 + 1.1428 = 1.9999 \text{ checks!}$$

Chapter 3, Problem 8.

Using nodal analysis, find v_0 in the circuit in Fig. 3.57.

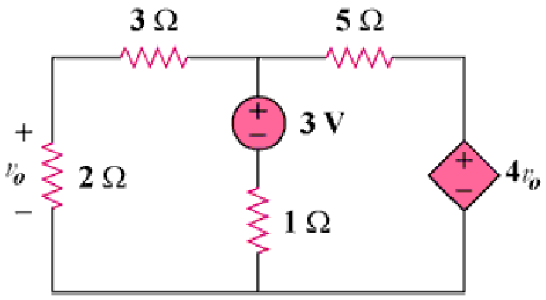
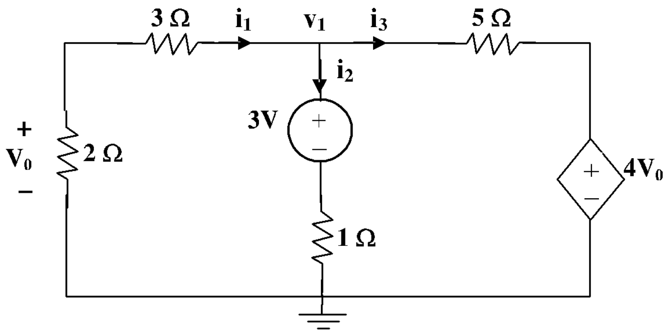


Figure 3.57

Chapter 3, Solution 8



$$i_1 + i_2 + i_3 = 0 \longrightarrow \frac{v_1}{5} + \frac{v_1 - 3}{1} + \frac{v_1 - 4v_0}{5} = 0$$

But $\quad v_0 = \frac{2}{5}v_1$ so that $v_1 + 5v_1 - 15 + v_1 - \frac{8}{5}v_1 = 0$

or $v_1 = 15x5/(27) = 2.778$ V, therefore $v_o = 2v_1/5 =$ **1.1111 V**

Chapter 3, Problem 9.

Determine I_b in the circuit in Fig. 3.58 using nodal analysis.

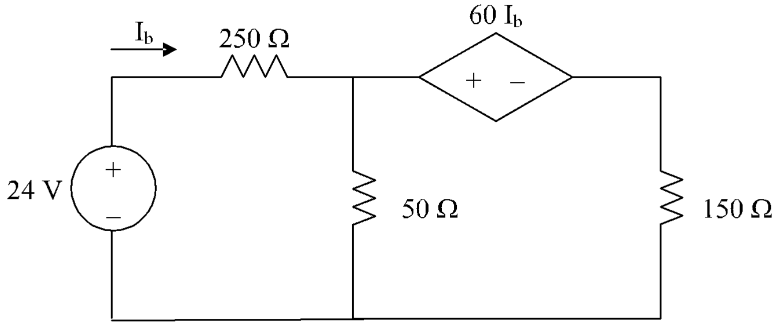


Figure 3.58 For Prob. 3.9.

Chapter 3, Solution 9

Let V_1 be the unknown node voltage to the right of the 250-Ω resistor. Let the ground reference be placed at the bottom of the 50-Ω resistor. This leads to the following nodal equation:

$$\frac{V_1 - 24}{250} + \frac{V_1 - 0}{50} + \frac{V_1 - 60I_b - 0}{150} = 0$$

simplifying we get

$$3V_1 - 72 + 15V_1 + 5V_1 - 300I_b = 0$$

But $I_b = \frac{24 - V_1}{250}$. Substituting this into the nodal equation leads to

$$24.2V_1 - 100.8 = 0 \quad \text{or } V_1 = 4.165 \text{ V}.$$

Thus, $I_b = (24 - 4.165)/250 =$ **79.34 mA**.

Chapter 3, Problem 10.

Find i_0 in the circuit in Fig. 3.59.

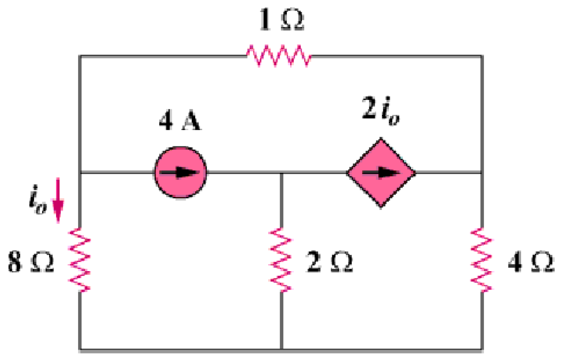
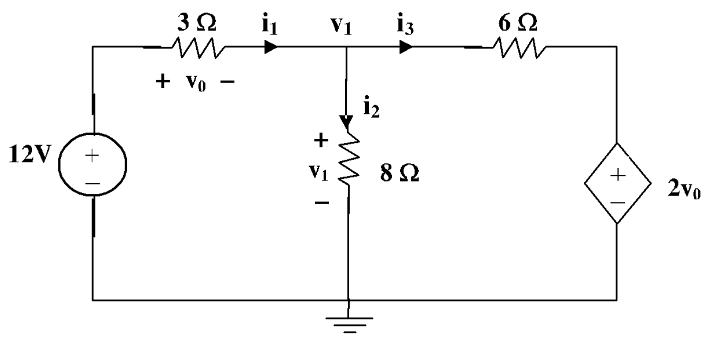


Figure 3.59

Chapter 3, Solution 10



At the non-reference node,

$$\frac{12 - v_1}{3} = \frac{v_1}{8} + \frac{v_1 - 2v_0}{6} \qquad (1)$$

But

$$-12 + v_0 + v_1 = 0 \longrightarrow v_0 = 12 - v_1 \qquad (2)$$

Substituting (2) into (1),

$$\frac{12 - v_1}{3} = \frac{v_1}{8} + \frac{3v_1 - 24}{6} \longrightarrow v_0 = \underline{\mathbf{3.652\ V}}$$

Chapter 3, Problem 11.

Find V_o and the power dissipated in all the resistors in the circuit of Fig. 3.60.

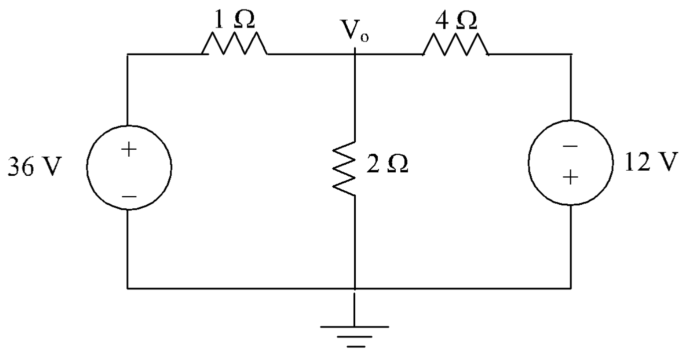


Figure 3.60 For Prob. 3.11.

Chapter 3, Solution 11

At the top node, KVL gives

$$\frac{V_o - 36}{1} + \frac{V_o - 0}{2} + \frac{V_o - (-12)}{4} = 0$$

$1.75V_o = 33$ or $V_o = 18.857V$

$P_{1\Omega} = (36–18.857)^2/1 =$ **293.9 W**

$P_{2\Omega} = (V_o)^2/2 = (18.857)^2/2 =$ **177.79 W**

$P_{4\Omega} = (18.857+12)^2/4 =$ **238 W**.

Chapter 3, Problem 12.

Using nodal analysis, determine V_o in the circuit in Fig. 3.61.

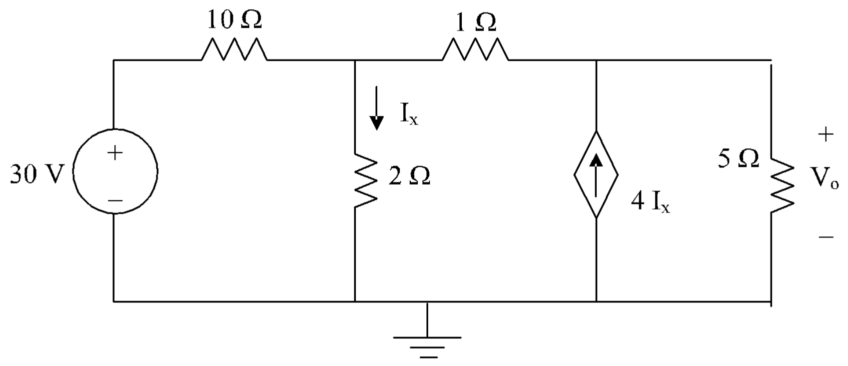
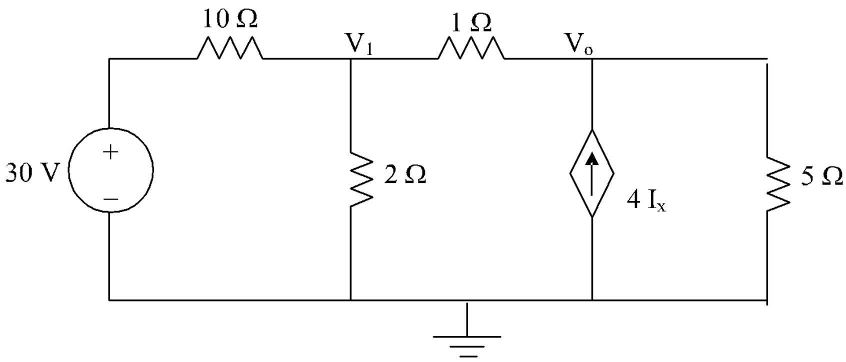


Figure 3.61 For Prob. 3.12.

Chapter 3, Solution 12

There are two unknown nodes, as shown in the circuit below.



At node 1,

$$\frac{V_1 - 30}{10} + \frac{V_1 - 0}{2} + \frac{V_1 - V_o}{1} = 0 \qquad (1)$$

$$16V_1 - 10V_o = 30$$

At node o,

$$\frac{V_o - V_1}{1} - 4I_x + \frac{V_o - 0}{5} = 0 \qquad (2)$$

$$-5V_1 + 6V_o - 20I_x = 0$$

But $I_x = V_1/2$. Substituting this in (2) leads to

$$-15V_1 + 6V_o = 0 \text{ or } V_1 = 0.4V_o \qquad (3)$$

Substituting (3) into 1,

$$16(0.4V_o) - 10V_o = 30 \text{ or } V_o = \underline{\mathbf{-8.333\ V}}.$$

Chapter 3, Problem 13.

Calculate v_1 and v_2 in the circuit of Fig. 3.62 using nodal analysis.

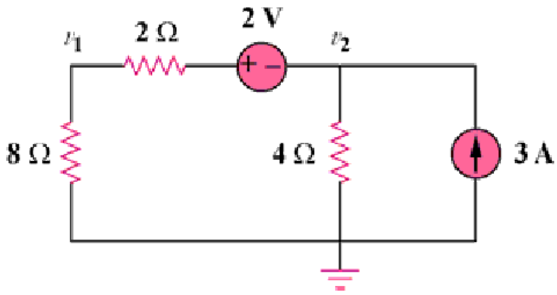


Figure 3.62

Chapter 3, Solution 13

At node number 2, $[(v_2 + 2) – 0]/10 + v_2/4 = 3$ or v_2 = <u>**8 volts**</u>

But, $I = [(v_2 + 2) – 0]/10 = (8 + 2)/10 = 1$ amp and $v_1 = 8x1$ = <u>**8volts**</u>

Chapter 3, Problem 14.

Using nodal analysis, find v_o in the circuit of Fig. 3.63.

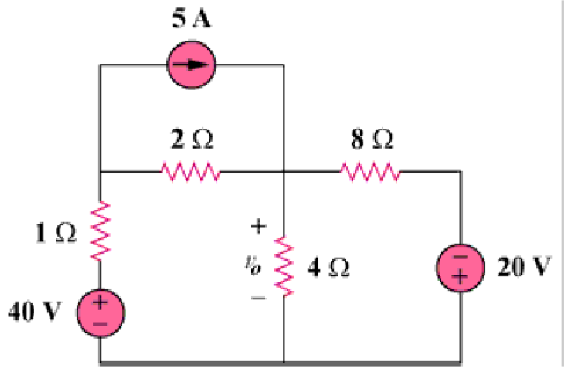
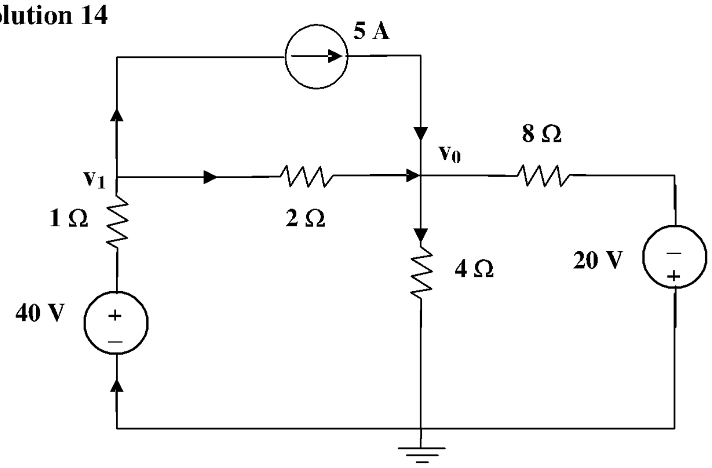


Figure 3.63

Chapter 3, Solution 14



At node 1, $\frac{v_1 - v_0}{2} + 5 = \frac{40 - v_0}{1} \longrightarrow v_1 + v_0 = 70$ (1)

At node 0, $\frac{v_1 - v_0}{2} + 5 = \frac{v_0}{4} + \frac{v_0 + 20}{8} \longrightarrow 4v_1 - 7v_0 = -20$ (2)

Solving (1) and (2), v_0 = **27.27 V**

Chapter 3, Problem 15.

Apply nodal analysis to find i_o and the power dissipated in each resistor in the circuit of Fig. 3.64.

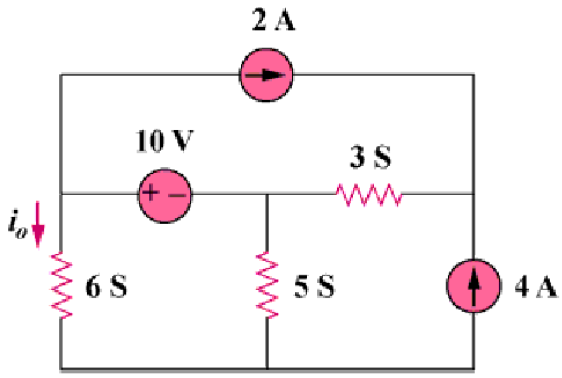
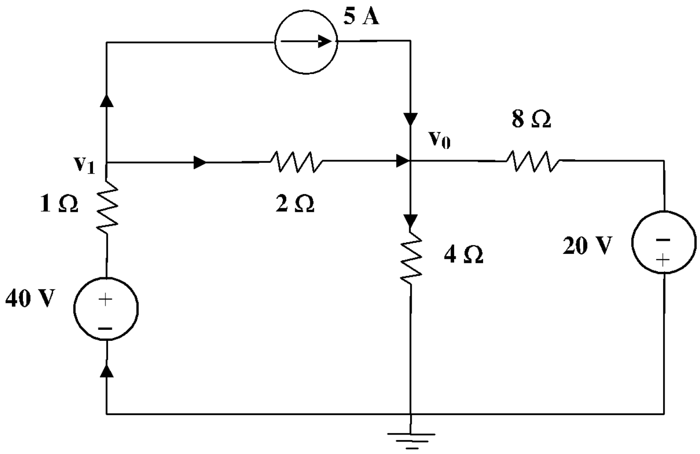


Figure 3.64

Chapter 3, Solution 15



Nodes 1 and 2 form a supernode so that $v_1 = v_2 + 10$ (1)

At the supernode, $2 + 6v_1 + 5v_2 = 3\ (v_3 - v_2) \longrightarrow 2 + 6v_1 + 8v_2 = 3v_3$ (2)

At node 3, $2 + 4 = 3\ (v_3 - v_2) \longrightarrow v_3 = v_2 + 2$ (3)

Substituting (1) and (3) into (2),

$$2 + 6v_2 + 60 + 8v_2 = 3v_2 + 6 \longrightarrow v_2 = \frac{-56}{11}$$

$$v_1 = v_2 + 10 = \frac{54}{11}$$

$i_0 = 6v_i = \underline{\mathbf{29.45\ A}}$

$$P_{65} = \frac{v_1^2}{R} = v_1^2 G = \left(\frac{54}{11}\right)^2 6 = \underline{\mathbf{144.6\ W}}$$

$$P_{55} = v_2^2 G = \left(\frac{-56}{11}\right)^2 5 = \underline{\mathbf{129.6\ W}}$$

$$P_{35} = (v_L - v_3)^2 G = (2)^2 3 = \underline{\mathbf{12\ W}}$$

Chapter 3, Problem 16.

Determine voltages v_1 through v_3 in the circuit of Fig. 3.65 using nodal analysis.

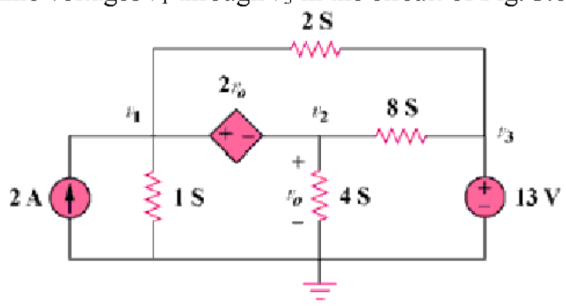
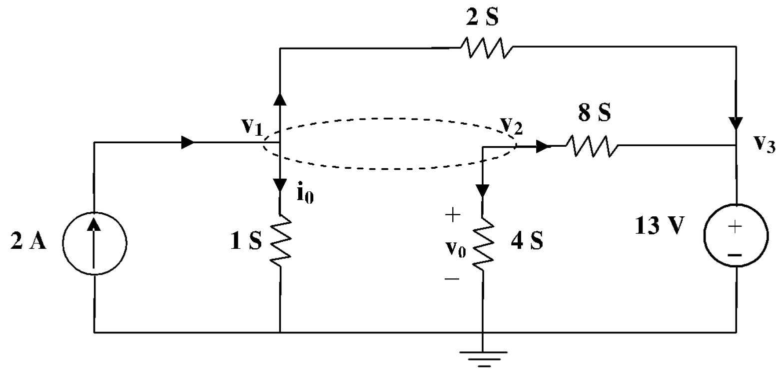


Figure 3.65

Chapter 3, Solution 16



At the supernode,

$2 = v_1 + 2(v_1 - v_3) + 8(v_2 - v_3) + 4v_2$, which leads to $2 = 3v_1 + 12v_2 - 10v_3$ (1)

But

$v_1 = v_2 + 2v_0$ and $v_0 = v_2$.

Hence

$$v_1 = 3v_2 \quad (2)$$
$$v_3 = 13V \quad (3)$$

Substituting (2) and (3) with (1) gives,

$v_1 =$ **18.858 V**, $v_2 =$ **6.286 V**, $v_3 =$ **13 V**

Chapter 3, Problem 17.

Using nodal analysis, find current i_o in the circuit of Fig. 3.66.

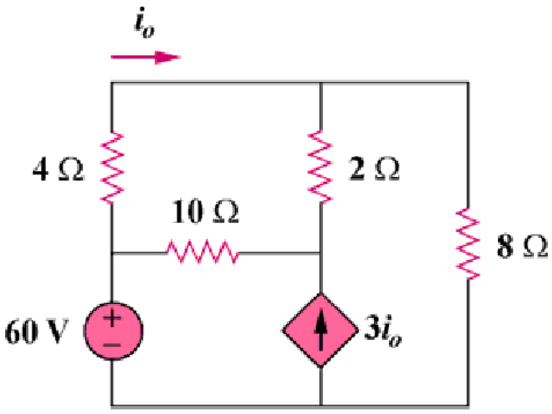
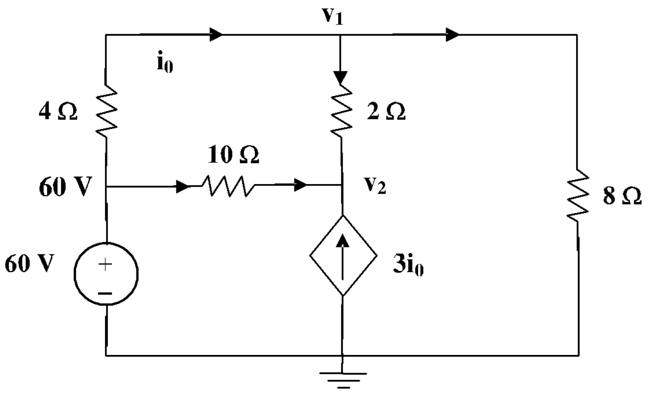


Figure 3.66

Chapter 3, Solution 17



At node 1, $\frac{60 - v_1}{4} = \frac{v_1}{8} + \frac{v_1 - v_2}{2}$ $\qquad$ $120 = 7v_1 - 4v_2$ $\qquad$ (1)

At node 2, $3i_0 + \frac{60 - v_2}{10} + \frac{v_1 - v_2}{2} = 0$

But $i_0 = \frac{60 - v_1}{4}$.

Hence

$$\frac{3(60 - v_1)}{4} + \frac{60 - v_2}{10} + \frac{v_1 - v_2}{2} = 0 \longrightarrow 1020 = 5v_1 + 12v_2 \qquad (2)$$

Solving (1) and (2) gives $v_1 = 53.08$ V. Hence $i_0 = \frac{60 - v_1}{4} =$ $\underline{\textbf{1.73 A}}$

Chapter 3, Problem 18.

Determine the node voltages in the circuit in Fig. 3.67 using nodal analysis.

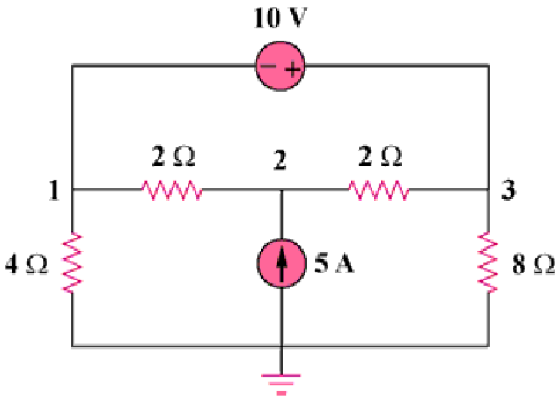
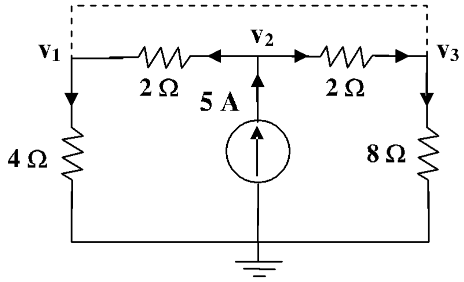
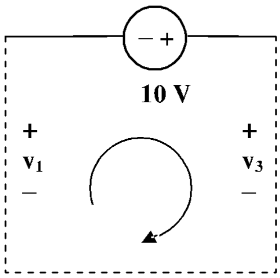


Figure 3.67

Chapter 3, Solution 18



(a)



(b)

At node 2, in Fig. (a), $5 = \frac{v_2 - v_1}{2} + \frac{v_2 - v_3}{2} \longrightarrow 10 = -v_1 + 2v_2 - v_3$ (1)

At the supernode, $\frac{v_2 - v_1}{2} + \frac{v_2 - v_3}{2} = \frac{v_1}{4} + \frac{v_3}{8} \longrightarrow 40 = 2v_1 + v_3$ (2)

From Fig. (b), $-v_1 - 10 + v_3 = 0 \longrightarrow v_3 = v_1 + 10$ (3)

Solving (1) to (3), we obtain $v_1 =$ **<u>10 V</u>**, $v_2 =$ **<u>20 V</u>** $= v_3$

Chapter 3, Problem 19.

Use nodal analysis to find v_1, v_2, and v_3 in the circuit in Fig. 3.68.

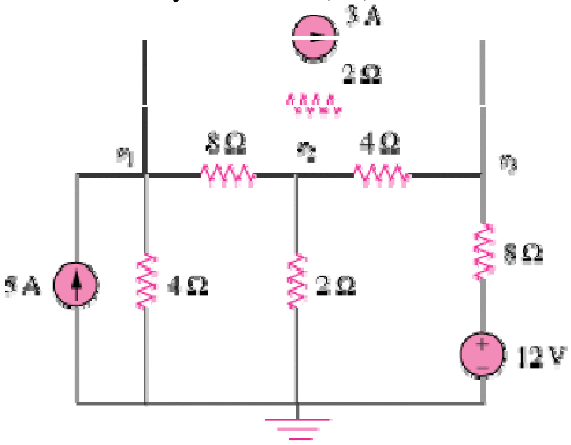


Figure 3.68

Chapter 3, Solution 19

At node 1,

$$5 = 3 + \frac{V_1 - V_3}{2} + \frac{V_1 - V_2}{8} + \frac{V_1}{4} \longrightarrow 16 = 7V_1 - V_2 - 4V_3 \qquad (1)$$

At node 2,

$$\frac{V_1 - V_2}{8} = \frac{V_2}{2} + \frac{V_2 - V_3}{4} \longrightarrow 0 = -V_1 + 7V_2 - 2V_3 \qquad (2)$$

At node 3,

$$3 + \frac{12 - V_3}{8} + \frac{V_1 - V_3}{2} + \frac{V_2 - V_3}{4} = 0 \longrightarrow -36 = 4V_1 + 2V_2 - 7V_3 \quad (3)$$

From (1) to (3),

$$\begin{pmatrix} 7 & -1 & -4 \\ -1 & 7 & -2 \\ 4 & 2 & -7 \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} 16 \\ 0 \\ -36 \end{pmatrix} \longrightarrow AV = B$$

Using MATLAB,

$$V = A^{-1}B = \begin{bmatrix} 10 \\ 4.933 \\ 12.267 \end{bmatrix} \longrightarrow \underline{V_1 = 10\text{ V},\ V_2 = 4.933\text{ V},\ V_3 = 12.267\text{ V}}$$

Chapter 3, Problem 20.

For the circuit in Fig. 3.69, find v_1, v_2, and v_3 using nodal analysis.

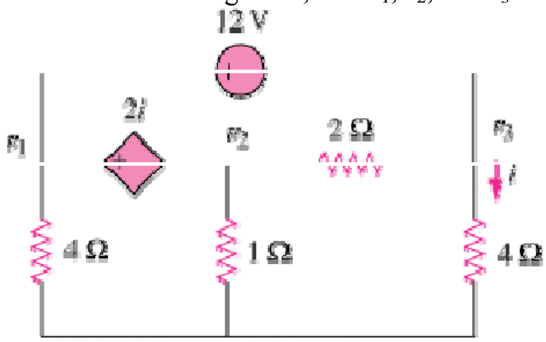
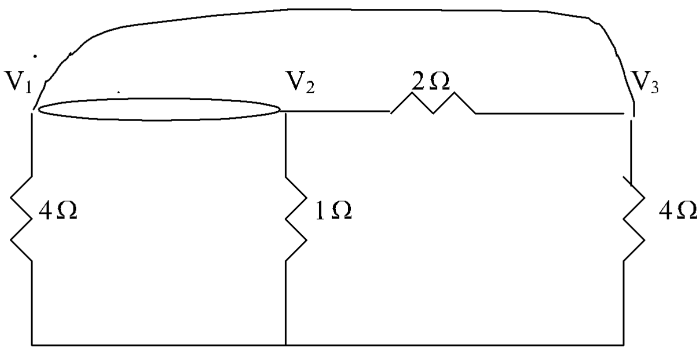


Figure 3.69

Chapter 3, Solution 20

Nodes 1 and 2 form a supernode; so do nodes 1 and 3. Hence

$$\frac{V_1}{4} + \frac{V_2}{1} + \frac{V_3}{4} = 0 \quad \longrightarrow \quad V_1 + 4V_2 + V_3 = 0 \qquad (1)$$



Between nodes 1 and 3,

$$-V_1 + 12 + V_3 = 0 \quad \longrightarrow \quad V_3 = V_1 - 12 \qquad (2)$$

Similarly, between nodes 1 and 2,

$$V_1 = V_2 + 2i \qquad (3)$$

But $i = V_3 / 4$. Combining this with (2) and (3) gives

$$V_2 = 6 + V_1 / 2 \qquad (4)$$

Solving (1), (2), and (4) leads to

$$\underline{V_1 = -3\text{V}, \quad V_2 = 4.5\text{V}, V_3 = -15\text{V}}$$

Chapter 3, Problem 21.

For the circuit in Fig. 3.70, find v_1 and v_2 using nodal analysis.

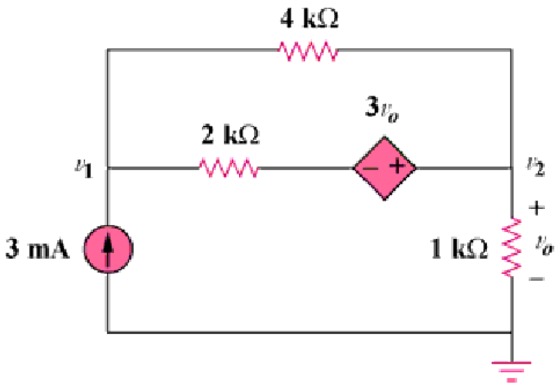
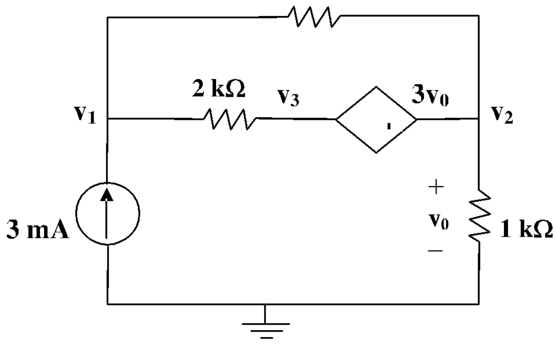
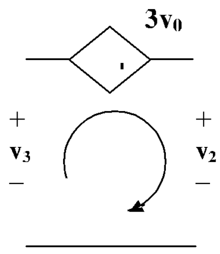


Figure 3.70

Chapter 3, Solution 21



(a)



(b)

Let v_3 be the voltage between the 2kΩ resistor and the voltage-controlled voltage source.
At node 1,

$$3x10^{-3} = \frac{v_1 - v_2}{4000} + \frac{v_1 - v_3}{2000} \longrightarrow 12 = 3v_1 - v_2 - 2v_3 \qquad (1)$$

At node 2,

$$\frac{v_1 - v_2}{4} + \frac{v_1 - v_3}{2} = \frac{v_2}{1} \longrightarrow 3v_1 - 5v_2 - 2v_3 = 0 \qquad (2)$$

Note that $v_0 = v_2$. We now apply KVL in Fig. (b)

$$- v_3 - 3v_2 + v_2 = 0 \longrightarrow v_3 = - 2v_2 \qquad (3)$$

From (1) to (3),

$v_1 = \underline{\mathbf{1\ V}}$, $v_2 = \underline{\mathbf{3\ V}}$

Chapter 3, Problem 22.

Determine $\boldsymbol{v_1}$ and $\boldsymbol{v_2}$ in the circuit in Fig. 3.71.

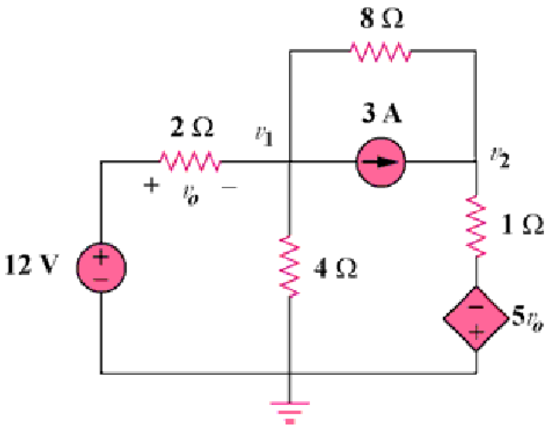


Figure 3.71

Chapter 3, Solution 22

At node 1, $\dfrac{12 - v_0}{2} = \dfrac{v_1}{4} + 3 + \dfrac{v_1 - v_0}{8} \longrightarrow 24 = 7v_1 - v_2$ (1)

At node 2, $3 + \dfrac{v_1 - v_2}{8} = \dfrac{v_2 + 5v_2}{1}$

But, $v_1 = 12 - v_1$

Hence, $24 + v_1 - v_2 = 8\ (v_2 + 60 + 5v_1) = 4$ V

$456 = 41v_1 - 9v_2$ (2)

Solving (1) and (2),

$v_1 =$ **<u>- 10.91 V</u>**, $v_2 =$ **<u>- 100.36 V</u>**

Chapter 3, Problem 23.

Use nodal analysis to find V_o in the circuit of Fig. 3.72.

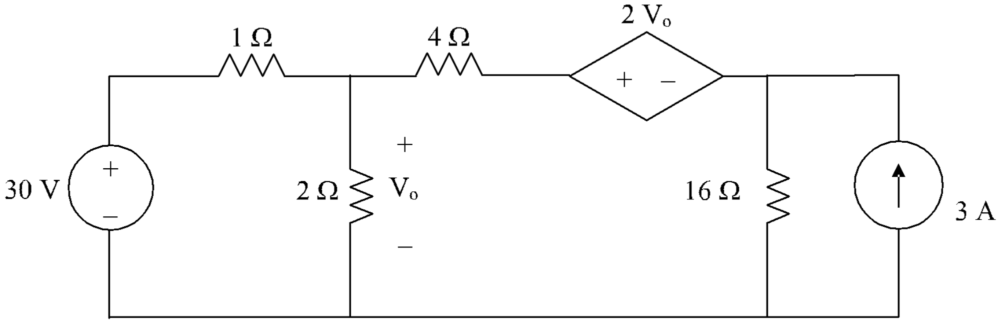
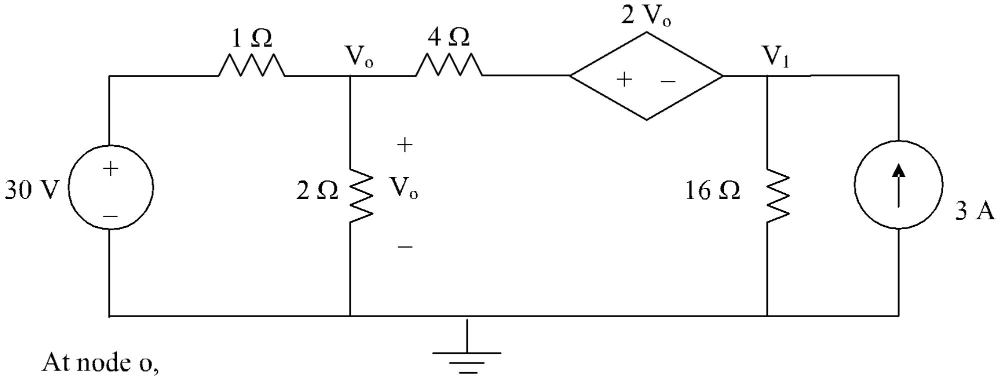


Figure 3.72 For Prob. 3.23.

Chapter 3, Solution 23

We apply nodal analysis to the circuit shown below.



At node o,

$$\frac{V_o - 30}{1} + \frac{V_o - 0}{2} + \frac{V_o - (2V_o + V_1)}{4} = 0 \rightarrow 1.25V_o - 0.25V_1 = 30$$

(1)

At node 1,

$$\frac{(2V_o + V_1) - V_o}{4} + \frac{V_1 - 0}{16} - 3 = 0 \rightarrow 5V_1 + 4V_o = 48 \qquad (2)$$

From (1), $V_1 = 5V_o - 120$. Substituting this into (2) yields
$29V_o = 648$ or V_o = **<u>22.34 V</u>**.

Chapter 3, Problem 24.

Use nodal analysis and MATLAB to find V_o in the circuit in Fig. 3.73.

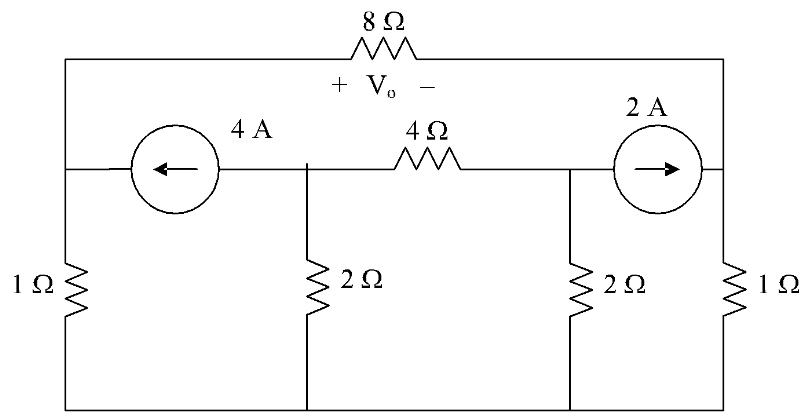
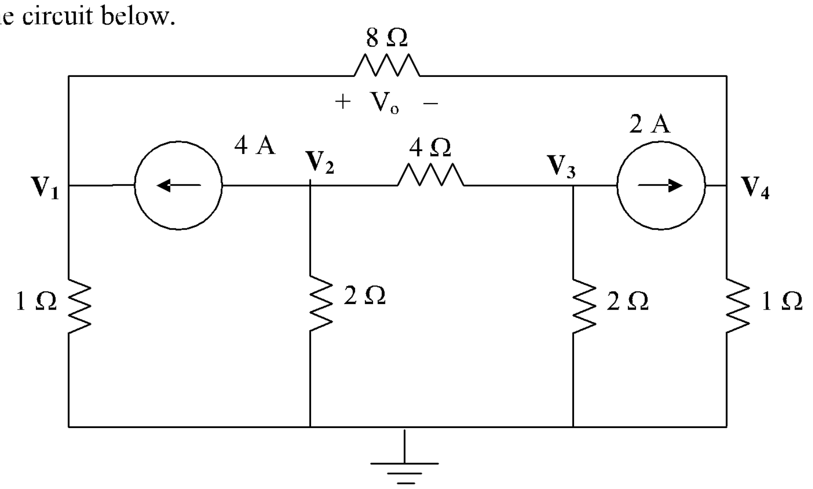


Figure 3.73 For Prob. 3.24.

Chapter 3, Solution 24

Consider the circuit below.



$$\frac{V_1 - 0}{1} - 4 + \frac{V_1 - V_4}{8} = 0 \rightarrow 1.125V_1 - 0.125V_4 = 4 \qquad (1)$$

$$+4 + \frac{V_2 - 0}{2} + \frac{V_2 - V_3}{4} = 0 \rightarrow 0.75V_2 - 0.25V_3 = -4 \qquad (2)$$

$$\frac{V_3 - V_2}{4} + \frac{V_3 - 0}{2} + 2 = 0 \rightarrow -0.25V_2 + 0.75V_3 = -2 \qquad (3)$$

$$-2 + \frac{V_4 - V_1}{8} + \frac{V_4 - 0}{1} = 0 \rightarrow -0.125V_1 + 1.125V_4 = 2 \qquad (4)$$

$$\begin{bmatrix} 1.125 & 0 & 0 & -0.125 \\ 0 & 0.75 & -0.25 & 0 \\ 0 & -0.25 & 0.75 & 0 \\ -0.125 & 0 & 0 & 1.125 \end{bmatrix} V = \begin{bmatrix} 4 \\ -4 \\ -2 \\ 2 \end{bmatrix}$$

Now we can use MATLAB to solve for the unknown node voltages.

```
>> Y=[1.125,0,0,-0.125;0,0.75,-0.25,0;0,-0.25,0.75,0;-0.125,0,0,1.125]

Y =
   1.1250        0        0  -0.1250
        0   0.7500  -0.2500        0
        0  -0.2500   0.7500        0
  -0.1250        0        0   1.1250

>> I=[4,-4,-2,2]'

I =
    4
   -4
   -2
    2

>> V=inv(Y)*I

V =
   3.8000
  -7.0000
  -5.0000
   2.2000
```

$V_o = V_1 - V_4 = 3.8 - 2.2 =$ **1.6 V**.

Chapter 3, Problem 25.

Use nodal analysis along with MATLAB to determine the node voltages in Fig. 3.74.

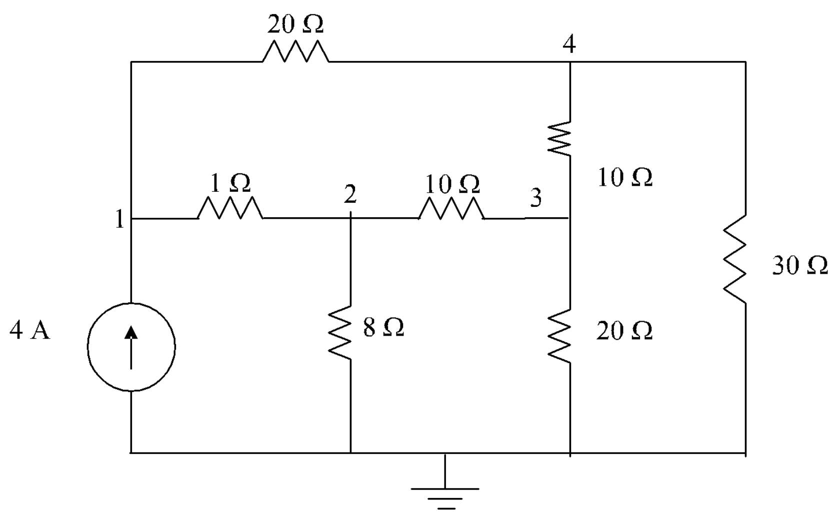
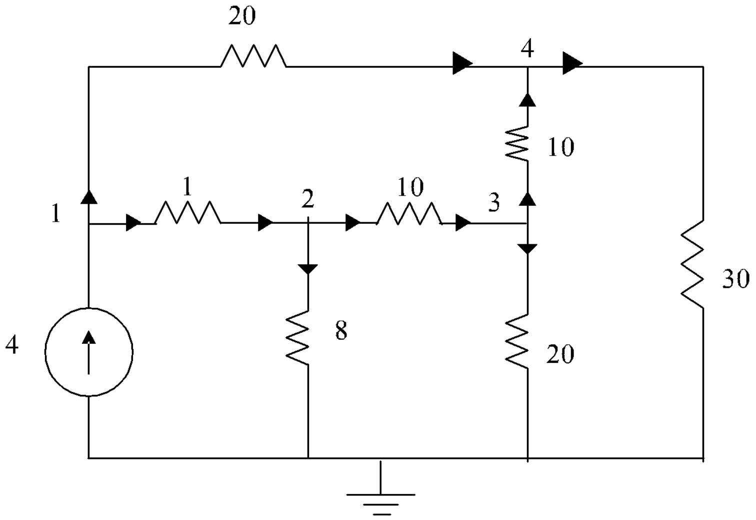


Figure 3.74 For Prob. 3.25.

Chapter 3, Solution 25

Consider the circuit shown below.



At node 1.

$$4 = \frac{V_1 - V_2}{1} + \frac{V_1 - V_4}{20} \longrightarrow 80 = 21V_1 - 20V_2 - V_4 \qquad (1)$$

At node 2,

$$\frac{V_1 - V_2}{1} = \frac{V_2}{8} + \frac{V_2 - V_3}{10} \longrightarrow 0 = -80V_1 + 98V_2 - 8V_3 \qquad (2)$$

At node 3,

$$\frac{V_2 - V_3}{10} = \frac{V_3}{20} + \frac{V_3 - V_4}{10} \longrightarrow 0 = -2V_2 + 5V_3 - 2V_4 \qquad (3)$$

At node 4,

$$\frac{V_1 - V_4}{20} + \frac{V_3 - V_4}{10} = \frac{V_4}{30} \longrightarrow 0 = 3V_1 + 6V_3 - 11V_4 \qquad (4)$$

Putting (1) to (4) in matrix form gives:

$$\begin{bmatrix} 80 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 21 & -20 & 0 & -1 \\ -80 & 98 & -8 & 0 \\ 0 & -2 & 5 & -2 \\ 3 & 0 & 6 & -11 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

$$B = A\ V \longrightarrow V = A^{-1} B$$

Using MATLAB leads to

V_1 = **25.52 V**, V_2 = **22.05 V**, V_3 = **14.842 V**, V_4 = **15.055 V**

Chapter 3, Problem 26.

Calculate the node voltages v_1, v_2, and v_3 in the circuit of Fig. 3.75.

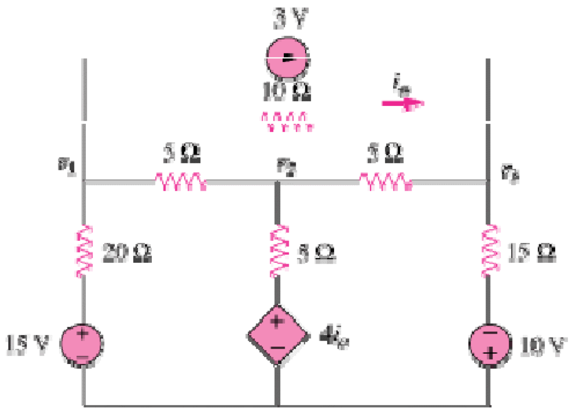


Figure 3.75

Chapter 3, Solution 26

At node 1,

$$\frac{15-V_1}{20}=3+\frac{V_1-V_3}{10}+\frac{V_1-V_2}{5} \quad\longrightarrow\quad -45=7V_1-4V_2-2V_3 \qquad (1)$$

At node 2,

$$\frac{V_1-V_2}{5}+\frac{4I_o-V_2}{5}=\frac{V_2-V_3}{5} \qquad (2)$$

But $I_o=\dfrac{V_1-V_3}{10}$. Hence, (2) becomes

$$0=7V_1-15V_2+3V_3 \qquad (3)$$

At node 3,

$$3+\frac{V_1-V_3}{10}+\frac{-10-V_3}{15}+\frac{V_2-V_3}{5}=0 \quad\longrightarrow\quad 70=-3V_1-6V_2+11V_3 \qquad (4)$$

Putting (1), (3), and (4) in matrix form produces

$$\begin{pmatrix} 7 & -4 & -2 \\ 7 & -15 & 3 \\ -3 & -6 & 11 \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix}=\begin{pmatrix} -45 \\ 0 \\ 70 \end{pmatrix} \quad\longrightarrow\quad AV=B$$

Using MATLAB leads to

$$V=A^{-1}B=\begin{pmatrix} -7.19 \\ -2.78 \\ 2.89 \end{pmatrix}$$

Thus,

V_1 = <u>**–7.19V**</u>; V_2 = <u>**–2.78V**</u>; V_3 = <u>**2.89V**</u>.

Chapter 3, Problem 27.

Use nodal analysis to determine voltages v_1, v_2, and v_3 in the circuit in Fig. 3.76.

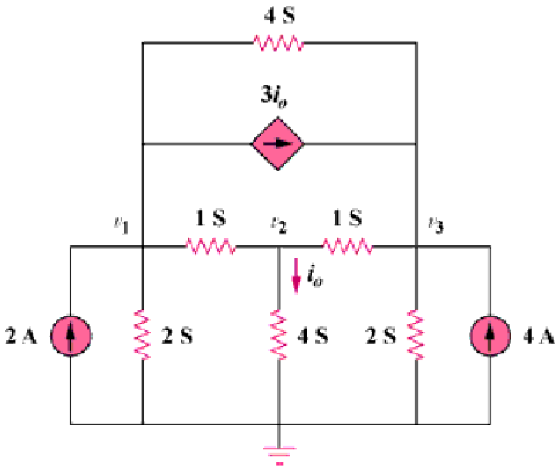


Figure 3.76

Chapter 3, Solution 27

At node 1,

$$2 = 2v_1 + v_1 - v_2 + (v_1 - v_3)4 + 3i_0, \quad i_0 = 4v_2. \text{ Hence,}$$

$$2 = 7v_1 + 11v_2 - 4v_3 \qquad (1)$$

At node 2,

$$v_1 - v_2 = 4v_2 + v_2 - v_3 \longrightarrow 0 = -v_1 + 6v_2 - v_3 \qquad (2)$$

At node 3,

$$2v_3 = 4 + v_2 - v_3 + 12v_2 + 4(v_1 - v_3)$$

or

$$-4 = 4v_1 + 13v_2 - 7v_3 \qquad (3)$$

In matrix form,

$$\begin{bmatrix} 7 & 11 & -4 \\ 1 & -6 & 1 \\ 4 & 13 & -7 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -4 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 7 & 11 & -4 \\ 1 & -6 & 1 \\ 4 & 13 & -7 \end{vmatrix} = 176, \quad \Delta_1 = \begin{vmatrix} 2 & 11 & -4 \\ 0 & -6 & 1 \\ -4 & 13 & -7 \end{vmatrix} = 110$$

$$\Delta_2 = \begin{vmatrix} 7 & 2 & -4 \\ 1 & 0 & 1 \\ 4 & -4 & -7 \end{vmatrix} = 66, \quad \Delta_3 = \begin{vmatrix} 7 & 11 & 2 \\ 1 & -6 & 0 \\ 4 & 13 & -4 \end{vmatrix} = 286$$

$$v_1 = \frac{\Delta_1}{\Delta} = \frac{110}{176} = 0.625\text{V}, \quad v_2 = \frac{\Delta_2}{\Delta} = \frac{66}{176} = 0.375\text{V}$$

$$v_3 = \frac{\Delta_3}{\Delta} = \frac{286}{176} = 1.625\text{V}.$$

v_1 = **625 mV**, v_2 = **375 mV**, v_3 = **1.625 V**.

Chapter 3, Problem 28.

Use *MATLAB* to find the voltages at nodes *a*, *b*, *c*, and *d* in the circuit of Fig. 3.77.

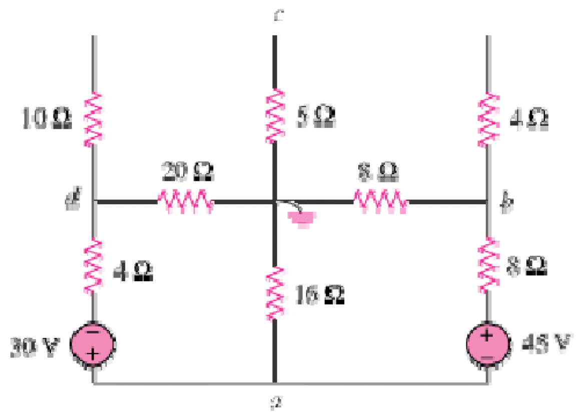


Figure 3.77

Chapter 3, Solution 28

At node c,

$$\frac{V_d - V_c}{10} = \frac{V_c - V_b}{4} + \frac{V_c}{5} \longrightarrow 0 = -5V_b + 11V_c - 2V_d \qquad (1)$$

At node b,

$$\frac{V_a + 45 - V_b}{8} + \frac{V_c - V_b}{4} = \frac{V_b}{8} \longrightarrow -45 = V_a - 4V_b + 2V_c \qquad (2)$$

At node a,

$$\frac{V_a - 30 - V_d}{4} + \frac{V_a}{16} + \frac{V_a + 45 - V_b}{8} = 0 \longrightarrow 30 = 7V_a - 2V_b - 4V_d \quad (3)$$

At node d,

$$\frac{V_a - 30 - V_d}{4} = \frac{V_d}{20} + \frac{V_d - V_c}{10} \longrightarrow 150 = 5V_a + 2V_c - 7V_d \qquad (4)$$

In matrix form, (1) to (4) become

$$\begin{pmatrix} 0 & -5 & 11 & -2 \\ 1 & -4 & 2 & 0 \\ 7 & -2 & 0 & -4 \\ 5 & 0 & 2 & -7 \end{pmatrix} \begin{pmatrix} V_a \\ V_b \\ V_c \\ V_d \end{pmatrix} = \begin{pmatrix} 0 \\ -45 \\ 30 \\ 150 \end{pmatrix} \longrightarrow AV = B$$

We use MATLAB to invert A and obtain

$$V = A^{-1}B = \begin{pmatrix} -10.14 \\ 7.847 \\ -1.736 \\ -29.17 \end{pmatrix}$$

Thus,

$$\underline{V_a = -10.14\,\text{V}, \; V_b = 7.847\,\text{V}, \; V_c = -1.736\,\text{V}, \; V_d = -29.17\,\text{V}}$$

Chapter 3, Problem 29.

Use *MATLAB* to solve for the node voltages in the circuit of Fig. 3.78.

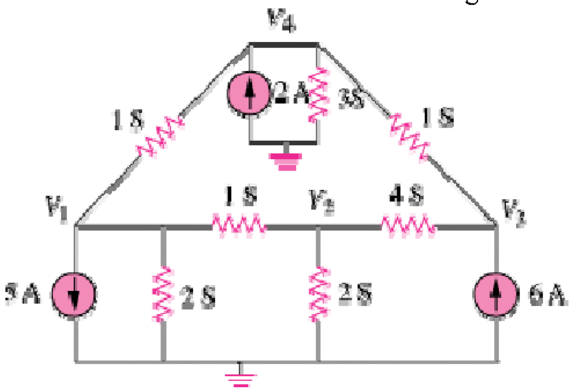


Figure 3.78

Chapter 3, Solution 29

At node 1,

$$5 + V_1 - V_4 + 2V_1 + V_1 - V_2 = 0 \quad \longrightarrow \quad -5 = 4V_1 - V_2 - V_4 \qquad (1)$$

At node 2,

$$V_1 - V_2 = 2V_2 + 4(V_2 - V_3) = 0 \quad \longrightarrow \quad 0 = -V_1 + 7V_2 - 4V_3 \qquad (2)$$

At node 3,

$$6 + 4(V_2 - V_3) = V_3 - V_4 \quad \longrightarrow \quad 6 = -4V_2 + 5V_3 - V_4 \qquad (3)$$

At node 4,

$$2 + V_3 - V_4 + V_1 - V_4 = 3V_4 \quad \longrightarrow \quad 2 = -V_1 - V_3 + 5V_4 \qquad (4)$$

In matrix form, (1) to (4) become

$$\begin{pmatrix} 4 & -1 & 0 & -1 \\ -1 & 7 & -4 & 0 \\ 0 & -4 & 5 & -1 \\ -1 & 0 & -1 & 5 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{pmatrix} = \begin{pmatrix} -5 \\ 0 \\ 6 \\ 2 \end{pmatrix} \quad \longrightarrow \quad AV = B$$

Using MATLAB,

$$V = A^{-1}B = \begin{pmatrix} -0.7708 \\ 1.209 \\ 2.309 \\ 0.7076 \end{pmatrix}$$

i.e.

$\underline{V_1 = -0.7708\,\text{V},\ V_2 = 1.209\,\text{V},\ V_3 = 2.309\,\text{V},\ V_4 = 0.7076\,\text{V}}$

Chapter 3, Problem 30.

Using nodal analysis, find $\boldsymbol{v_o}$ and $\boldsymbol{i_o}$ in the circuit of Fig. 3.79.

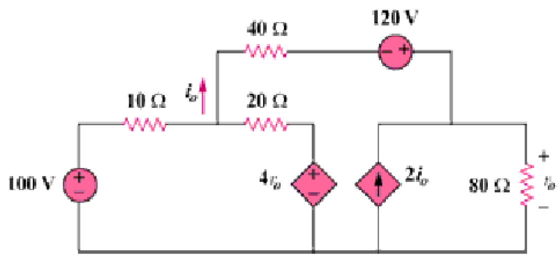
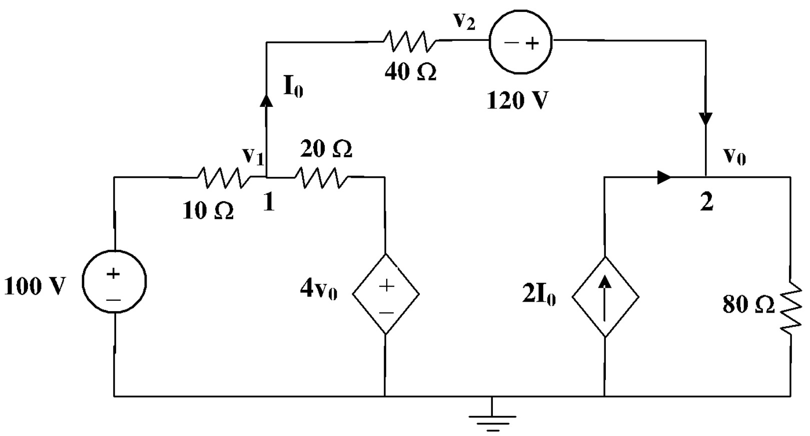


Figure 3.79

Chapter 3, Solution 30



At node 1,

$$\frac{v_1 - v_2}{40} = \frac{100 - v_1}{10} + \frac{4v_o - v_1}{20} \qquad (1)$$

But, $v_o = 120 + v_2 \longrightarrow v_2 = v_o - 120$. Hence (1) becomes

$$7v_1 - 9v_o = 280 \qquad (2)$$

At node 2,

$$I_o + 2I_o = \frac{v_o - 0}{80}$$

$$3\left(\frac{v_1 + 120 - v_o}{40}\right) = \frac{v_o}{80}$$

or

$$6v_1 - 7v_o = -720 \qquad (3)$$

from (2) and (3),

$$\begin{bmatrix} 7 & -9 \\ 6 & -7 \end{bmatrix}\begin{bmatrix} v_1 \\ v_o \end{bmatrix} = \begin{bmatrix} 280 \\ -720 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 7 & -9 \\ 6 & -7 \end{vmatrix} = -49 + 54 = 5$$

$$\Delta_1 = \begin{vmatrix} 280 & -9 \\ -720 & -7 \end{vmatrix} = -8440, \quad \Delta_2 = \begin{vmatrix} 7 & 280 \\ 6 & -720 \end{vmatrix} = -6720$$

$$v_1 = \frac{\Delta_1}{\Delta} = \frac{-8440}{5} = -1688, \quad v_o = \frac{\Delta_2}{\Delta} = \frac{-6720}{5} - 1344V$$

$$I_o = \underline{\mathbf{-5.6\ A}}$$

Chapter 3, Problem 31.

Find the node voltages for the circuit in Fig. 3.80.

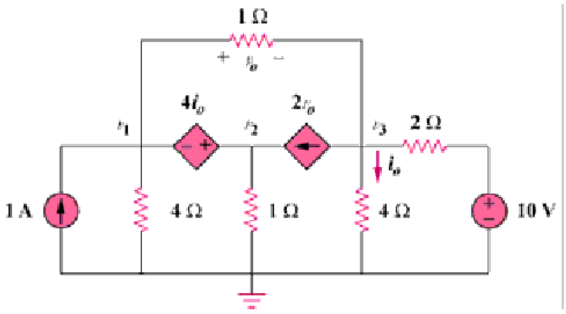
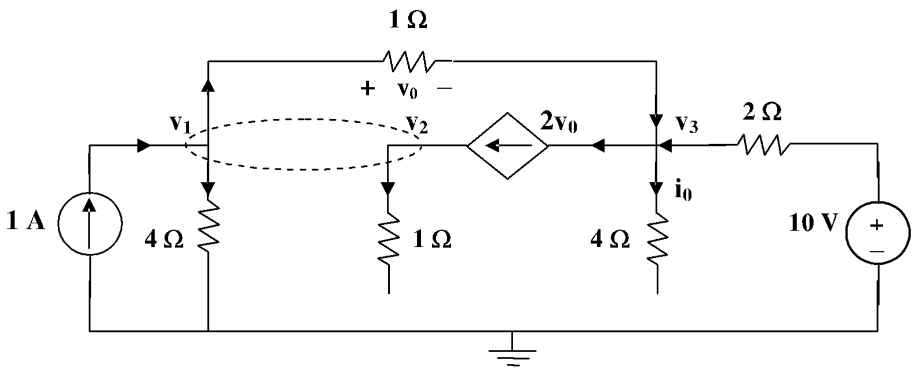


Figure 3.80

Chapter 3, Solution 31



At the supernode,

$$1 + 2v_0 = \frac{v_1}{4} + \frac{v_2}{1} + \frac{v_1 - v_3}{1} \qquad (1)$$

But $v_o = v_1 - v_3$. Hence (1) becomes,

$$4 = -3v_1 + 4v_2 + 4v_3 \qquad (2)$$

At node 3,

$$2v_o + \frac{v_3}{4} = v_1 - v_3 + \frac{10 - v_3}{2}$$

or

$$20 = 4v_1 + 0v_2 - v_3 \qquad (3)$$

At the supernode, $v_2 = v_1 + 4i_o$. But $i_o = \frac{v_3}{4}$. Hence,

$$v_2 = v_1 + v_3 \qquad (4)$$

Solving (2) to (4) leads to,

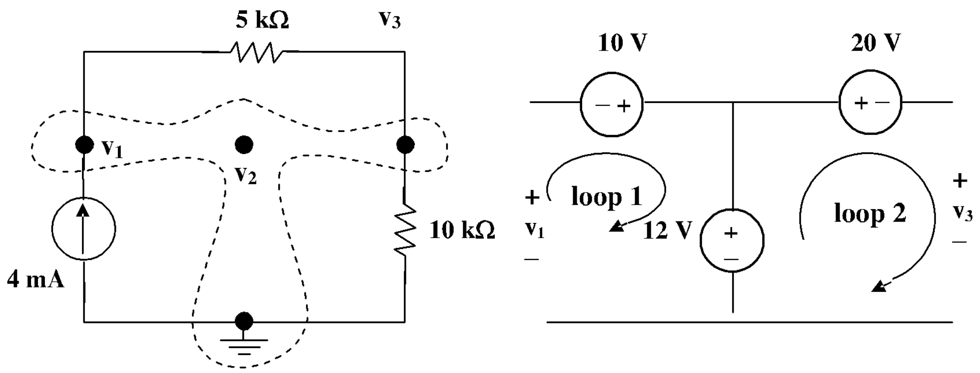
v_1 = **4.97V**, v_2 = **4.85V**, v_3 = **–0.12V**.

Chapter 3, Problem 32.

Obtain the node voltages v_1, v_2, and v_3 in the circuit of Fig. 3.81.

Figure 3.81

Chapter 3, Solution 32



(a) (b)

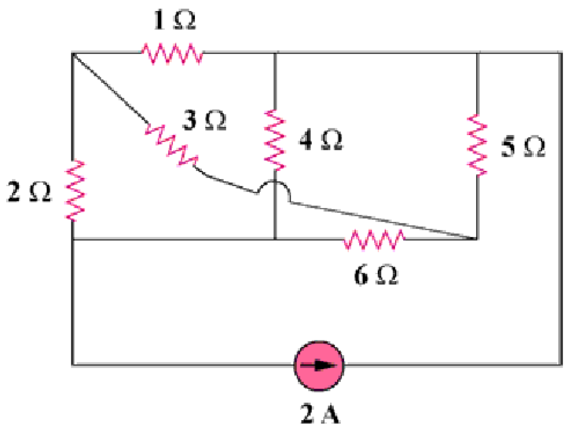
We have a supernode as shown in figure (a). It is evident that $v_2 = 12$ V, Applying KVL to loops 1and 2 in figure (b), we obtain,

$$-v_1 - 10 + 12 = 0 \text{ or } v_1 = 2 \text{ and } -12 + 20 + v_3 = 0 \text{ or } v_3 = -8 \text{ V}$$

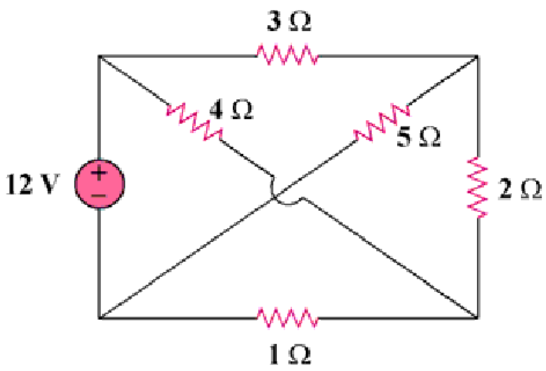
Thus, $v_1 = \underline{\mathbf{2\ V}}$, $v_2 = \underline{\mathbf{12\ V}}$, $v_3 = \underline{\mathbf{-8V}}$.

Chapter 3, Problem 33.

Which of the circuits in Fig. 3.82 is planar? For the planar circuit, redraw the circuits with no crossing branches.



(a)

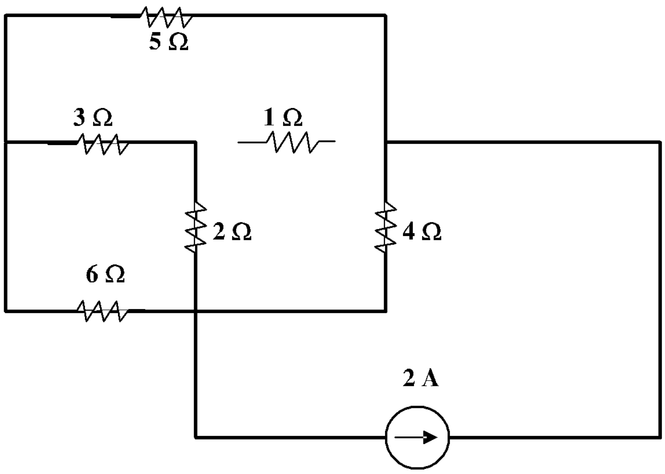


(b)

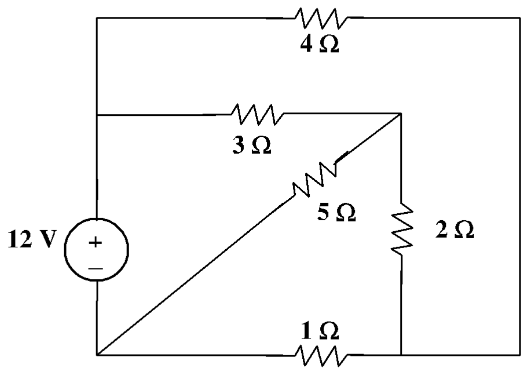
Figure 3.82

Chapter 3, Solution 33

(a) This is a **planar** circuit. It can be redrawn as shown below.



(b) This is a **planar** circuit. It can be redrawn as shown below.



Chapter 3, Problem 34.

Determine which of the circuits in Fig. 3.83 is planar and redraw it with no crossing branches.

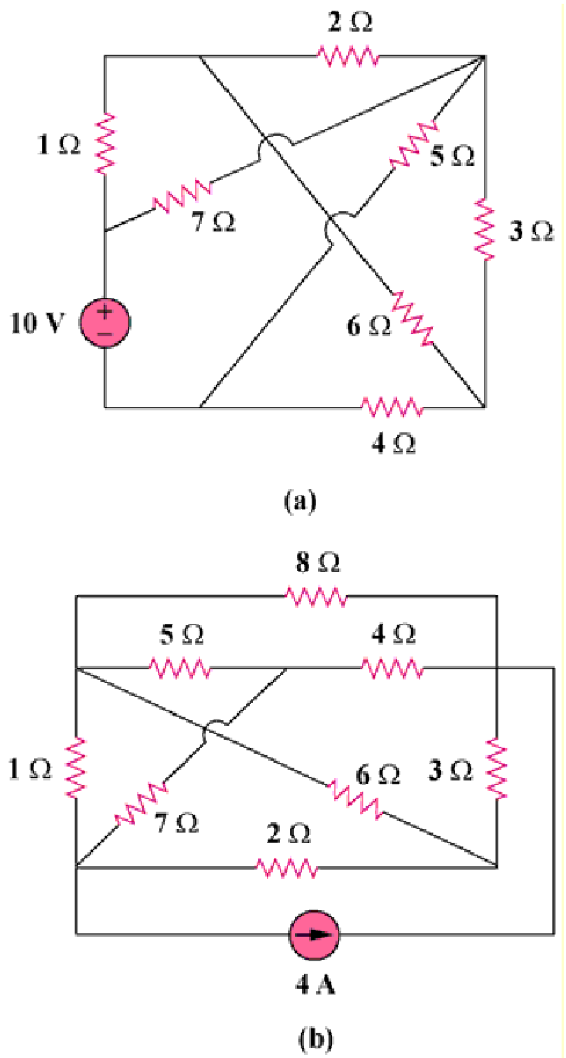
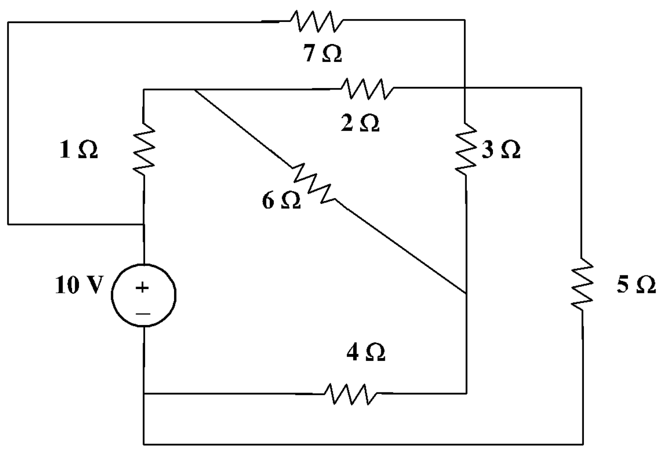


Figure 3.83

Chapter 3, Solution 34

(a) This is a **planar** circuit because it can be redrawn as shown below,



(b) This is a **non-planar** circuit.

Chapter 3, Problem 35.

Rework Prob. 3.5 using mesh analysis.

Chapter 3, Problem 5

Obtain v_0 in the circuit of Fig. 3.54.

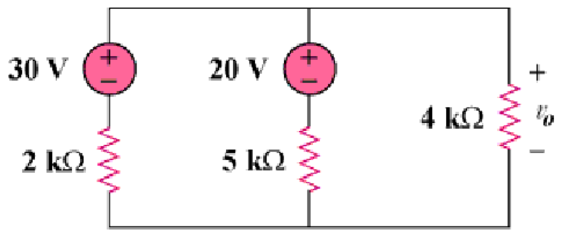
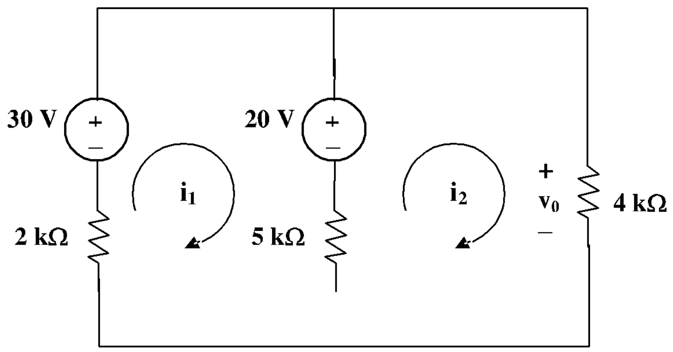


Figure 3.54

Chapter 3, Solution 35



Assume that i_1 and i_2 are in mA. We apply mesh analysis. For mesh 1,

$$-30 + 20 + 7i_1 - 5i_2 = 0 \quad \text{or} \quad 7i_1 - 5i_2 = 10 \qquad (1)$$

For mesh 2,

$$-20 + 9i_2 - 5i_1 = 0 \quad \text{or} \quad -5i_1 + 9i_2 = 20 \qquad (2)$$

Solving (1) and (2), we obtain, $i_2 = 5$.

$$v_0 = 4i_2 = \underline{\textbf{20 volts}}.$$

Chapter 3, Problem 36.

Rework Prob. 3.6 using mesh analysis.

Chapter 3, Problem 6

Use nodal analysis to obtain v_0 in the circuit in Fig. 3.55.

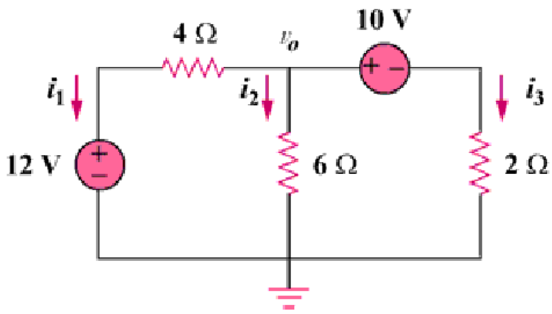
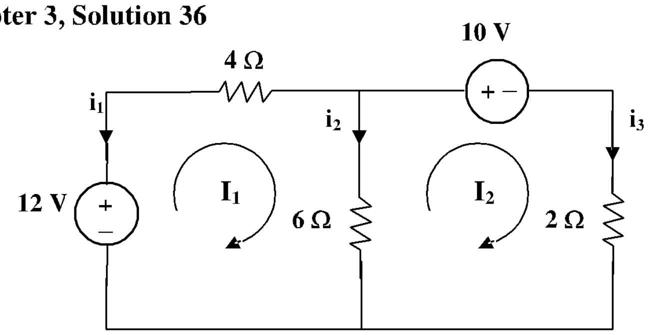


Figure 3.55

Chapter 3, Solution 36



Applying mesh analysis gives,

$$12 = 10I_1 - 6I_2$$

$$-10 = -6I_1 + 8I_2$$

or

$$\begin{bmatrix} 6 \\ -5 \end{bmatrix} = \begin{bmatrix} 5 & -3 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 5 & -3 \\ -3 & 4 \end{vmatrix} = 11, \quad \Delta_1 = \begin{vmatrix} 6 & -3 \\ -5 & 4 \end{vmatrix} = 9, \quad \Delta_2 = \begin{vmatrix} 5 & 6 \\ -3 & -5 \end{vmatrix} = -7$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{9}{11}, \quad I_2 = \frac{\Delta_2}{\Delta} = \frac{-7}{11}$$

$i_1 = -I_1 = -9/11 = -0.8181$ A, $i_2 = I_1 - I_2 = 10/11 = 1.4545$ A.

$v_o = 6i_2 = 6x1.4545 =$ **8.727 V**.

Chapter 3, Problem 37.

Rework Prob. 3.8 using mesh analysis.

Chapter 3, Problem 8

Using nodal analysis, find v_0 in the circuit in Fig. 3.57.

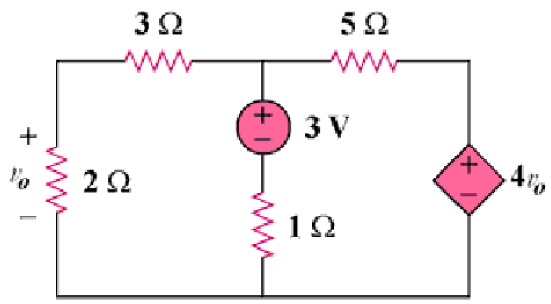
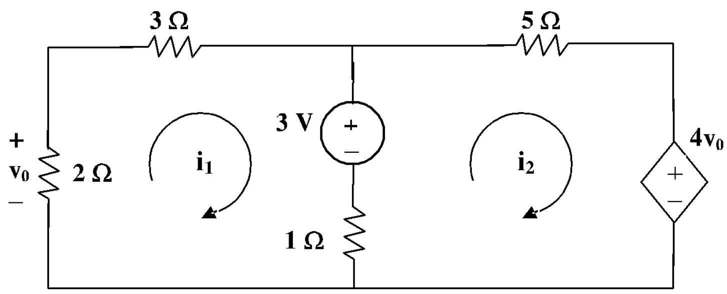


Figure 3.57

Chapter 3, Solution 37



Applying mesh analysis to loops 1 and 2, we get,

$6i_1 - 1i_2 + 3 = 0$ which leads to $i_2 = 6i_1 + 3$ (1)

$-1i_1 + 6i_2 - 3 + 4v_0 = 0$ (2)

But, $v_0 = -2i_1$ (3)

Using (1), (2), and (3) we get $i_1 = -5/9$.

Therefore, we get $v_0 = -2i_1 = -2(-5/9) =$ **1.1111 volts**

Chapter 3, Problem 38.
Apply mesh analysis to the circuit in Fig. 3.84 and obtain I_o.

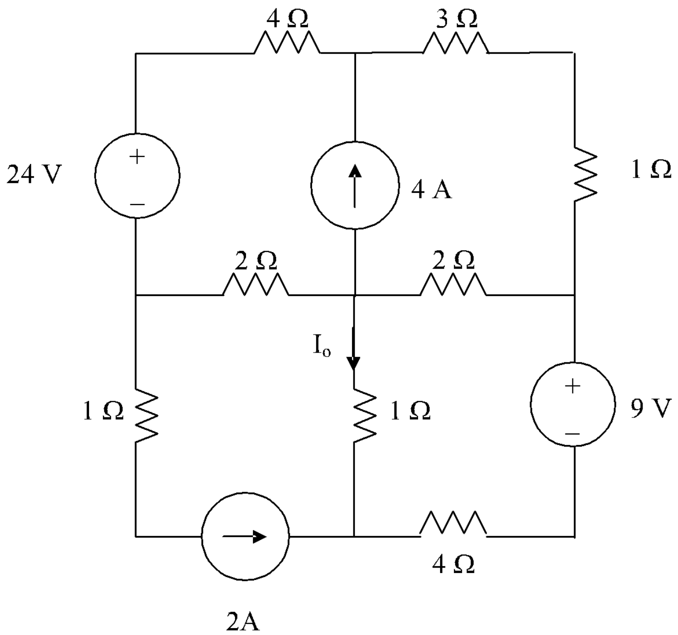
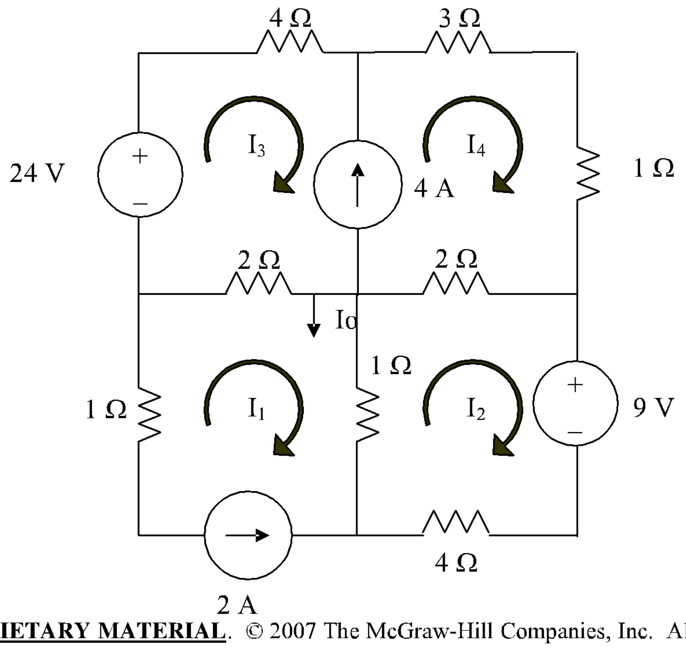


Figure 3.84 For Prob. 3.38.

Chapter 3, Solution 38
Consider the circuit below with the mesh currents.



I_1 =-2 A (1)

$1(I_2–I_1) + 2(I_2–I_4) + 9 + 4I_2 = 0$
$7I_2 – I_4 = –11$ (2)

$–24 + 4I_3 + 3I_4 + 1I_4 + 2(I_4–I_2) + 2(I_3 – I_1) = 0$ (super mesh)
$–2I_2 + 6 I_3 + 6I_4 = +24 – 4 = 20$ (3)

But, we need one more equation, so we use the constraint equation $–I_3 + I_4 = 4$. This now gives us three equations with three unknowns.

$$\begin{bmatrix} 7 & 0 & -1 \\ -2 & 6 & 6 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} -11 \\ 20 \\ 4 \end{bmatrix}$$

We can now use MATLAB to solve the problem.

```
>> Z=[7,0,-1;-2,6,6;0,-1,0]

Z =

   7   0  -1
  -2   6   6
   0  -1   0
>> V=[-11,20,4]'

V =

  -11
   20
    4
>> I=inv(Z)*V

I =

  -0.5500
  -4.0000
   7.1500
```

$I_o = I_1 – I_2 = –2 – 4 =$ **__–6 A__**.

Check using the super mesh (equation (3)): 1.1 – 24 + 42.9 = 20!

Chapter 3, Problem 39.

Determine the mesh currents i_1 and i_2 in the circuit shown in Fig. 3.85.

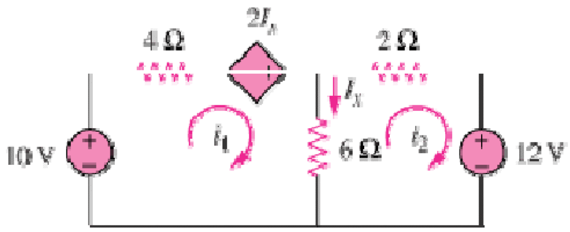


Figure 3.85

Chapter 3, Solution 39

For mesh 1,

$$-10 - 2I_x + 10I_1 - 6I_2 = 0$$

But $I_x = I_1 - I_2$. Hence,

$10 = -2I_1 + 2I_2 + 10I_1 - 6I_2 \quad \longrightarrow \quad 5 = 4I_1 - 2I_2 \qquad (1)$

For mesh 2,

$12 + 8I_2 - 6I_1 = 0 \quad \longrightarrow \quad 6 = 3I_1 - 4I_2 \qquad (2)$

Solving (1) and (2) leads to

$\underline{I_1 = 0.8\text{ A},\ \ I_2 = \text{-}0.9\text{A}}$

Chapter 3, Problem 40.

For the bridge network in Fig. 3.86, find I_o using mesh analysis.

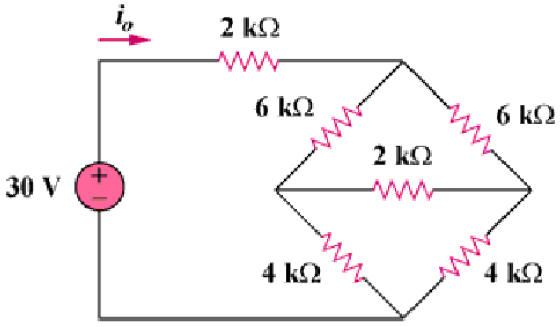
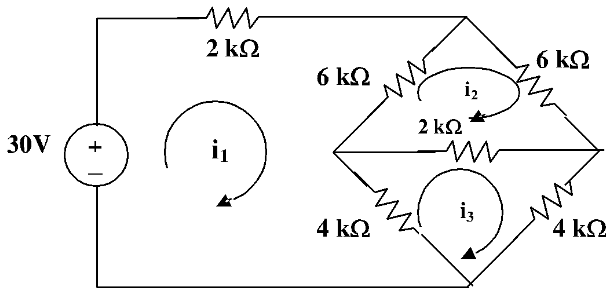


Figure 3.86

Chapter 3, Solution 40



Assume all currents are in mA and apply mesh analysis for mesh 1.

$$30 = 12i_1 - 6i_2 - 4i_3 \longrightarrow 15 = 6i_1 - 3i_2 - 2i_3 \qquad (1)$$

for mesh 2,

$$0 = -6i_1 + 14i_2 - 2i_3 \longrightarrow 0 = -3i_1 + 7i_2 - i_3 \qquad (2)$$

for mesh 2,

$$0 = -4i_1 - 2i_2 + 10i_3 \qquad 0 = -2i_1 - i_2 + 5i_3 \qquad (3)$$

Solving (1), (2), and (3), we obtain,

$$i_o = i_1 = \underline{\mathbf{4.286\ mA}}.$$

Chapter 3, Problem 41.

Apply mesh analysis to find i_o in Fig. 3.87.

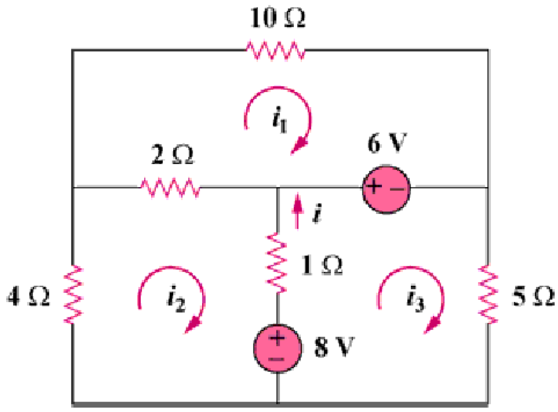
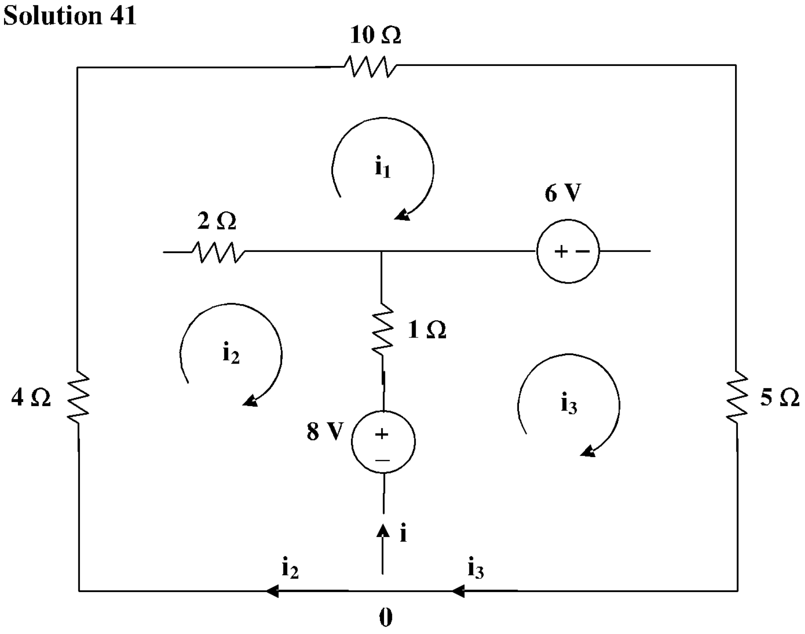


Figure 3.87

Chapter 3, Solution 41



For loop 1,

$$6 = 12i_1 - 2i_2 \longrightarrow 3 = 6i_1 - i_2 \qquad (1)$$

For loop 2,

$$-8 = -2i_1 + 7i_2 - i_3 \qquad (2)$$

For loop 3,

$$-8 + 6 + 6i_3 - i_2 = 0 \longrightarrow 2 = -i_2 + 6i_3 \qquad (3)$$

We put (1), (2), and (3) in matrix form,

$$\begin{bmatrix} 6 & -1 & 0 \\ 2 & -7 & 1 \\ 0 & -1 & 6 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 8 \\ 2 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 6 & -1 & 0 \\ 2 & -7 & 1 \\ 0 & -1 & 6 \end{vmatrix} = -234, \quad \Delta_2 = \begin{vmatrix} 6 & 3 & 0 \\ 2 & 8 & 1 \\ 0 & 2 & 6 \end{vmatrix} = 240$$

$$\Delta_3 = \begin{vmatrix} 6 & -1 & 3 \\ 2 & -7 & 8 \\ 0 & -1 & 2 \end{vmatrix} = -38$$

At node 0, $i + i_2 = i_3$ or $i = i_3 - i_2 = \dfrac{\Delta_3 - \Delta_2}{\Delta} = \dfrac{-38 - 240}{-234} =$ **1.188 A**

Chapter 3, Problem 42.

Determine the mesh currents in the circuit of Fig. 3.88.

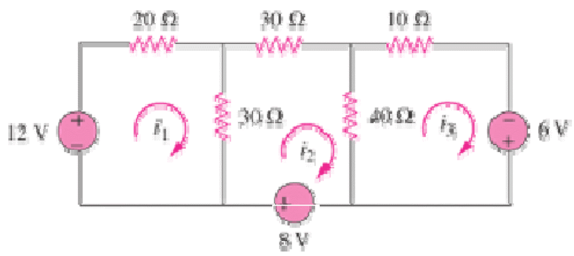


Figure 3.88

Chapter 3, Solution 42

For mesh 1,

$$-12+50I_1-30I_2=0 \quad\longrightarrow\quad 12=50I_1-30I_2 \qquad (1)$$

For mesh 2,

$$-8+100I_2-30I_1-40I_3=0 \quad\longrightarrow\quad 8=-30I_1+100I_2-40I_3 \qquad (2)$$

For mesh 3,

$$-6+50I_3-40I_2=0 \quad\longrightarrow\quad 6=-40I_2+50I_3 \qquad (3)$$

Putting eqs. (1) to (3) in matrix form, we get

$$\begin{pmatrix} 50 & -30 & 0 \\ -30 & 100 & -40 \\ 0 & -40 & 50 \end{pmatrix}\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix}=\begin{pmatrix} 12 \\ 8 \\ 6 \end{pmatrix} \quad\longrightarrow\quad AI=B$$

Using Matlab,

$$I=A^{-1}B=\begin{pmatrix} 0.48 \\ 0.40 \\ 0.44 \end{pmatrix}$$

i.e. $I_1 = 0.48$ A, $I_2 = 0.4$ A, $I_3 = 0.44$ A

Chapter 3, Problem 43.

Use mesh analysis to find v_{ab} and i_o in the circuit in Fig. 3.89.

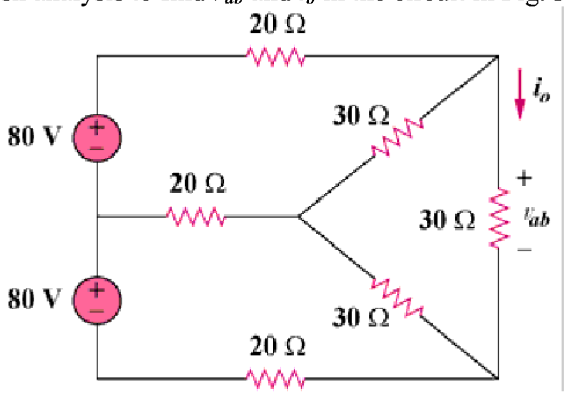
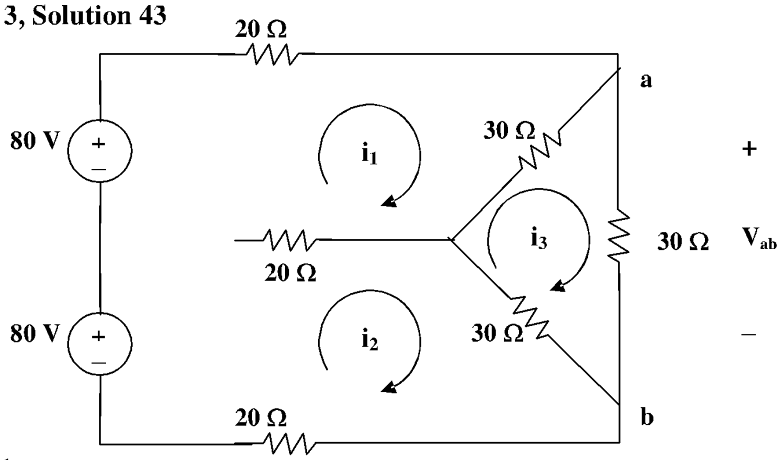


Figure 3.89

Chapter 3, Solution 43



For loop 1,

$$80 = 70i_1 - 20i_2 - 30i_3 \longrightarrow 8 = 7i_1 - 2i_2 - 3i_3 \quad (1)$$

For loop 2,

$$80 = 70i_2 - 20i_1 - 30i_3 \longrightarrow 8 = -2i_1 + 7i_2 - 3i_3 \quad (2)$$

For loop 3,

$$0 = -30i_1 - 30i_2 + 90i_3 \longrightarrow 0 = i_1 + i_2 - 3i_3 \quad (3)$$

Solving (1) to (3), we obtain $i_3 = 16/9$

$$I_o = i_3 = 16/9 = \underline{\mathbf{1.7778\ A}}$$

$$V_{ab} = 30i_3 = \underline{\mathbf{53.33\ V}}.$$

Chapter 3, Problem 44.

Use mesh analysis to obtain i_o in the circuit of Fig. 3.90.

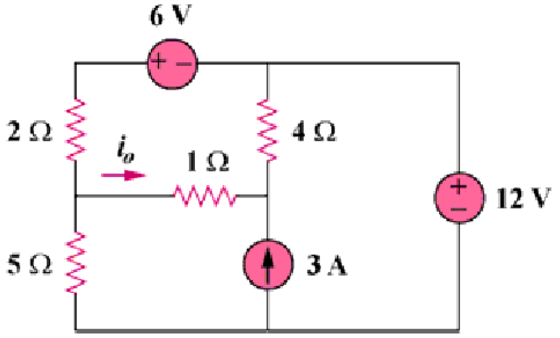
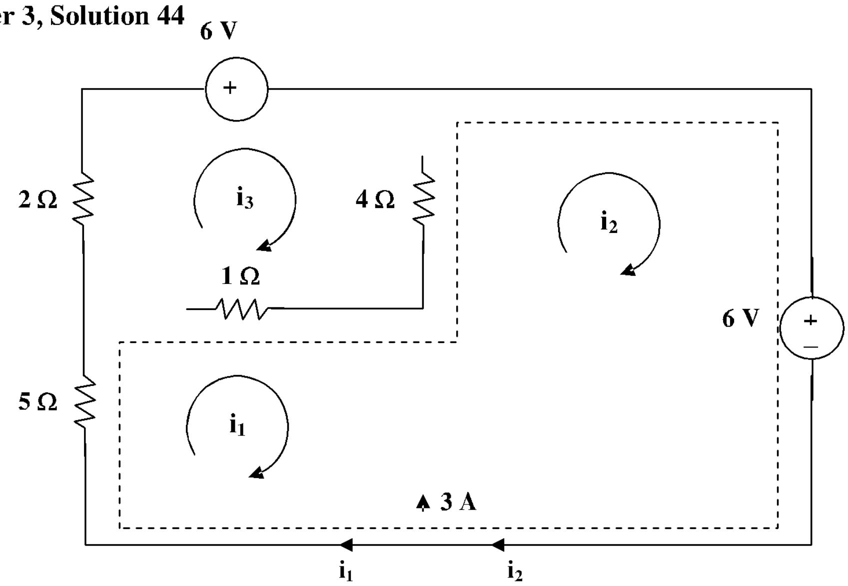


Figure 3.90

Chapter 3, Solution 44



Loop 1 and 2 form a supermesh. For the supermesh,

$$6i_1 + 4i_2 - 5i_3 + 12 = 0 \qquad (1)$$

For loop 3,

$$-i_1 - 4i_2 + 7i_3 + 6 = 0 \qquad (2)$$

Also,

$$i_2 = 3 + i_1 \qquad (3)$$

Solving (1) to (3), $i_1 = -3.067$, $i_3 = -1.3333$; $i_o = i_1 - i_3 =$ **-1.7333 A**

Chapter 3, Problem 45.

Find current ***i*** in the circuit in Fig. 3.91.

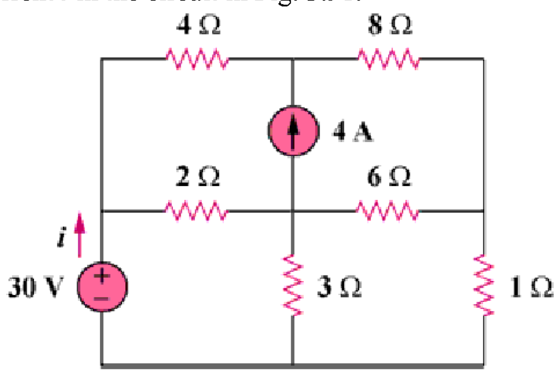
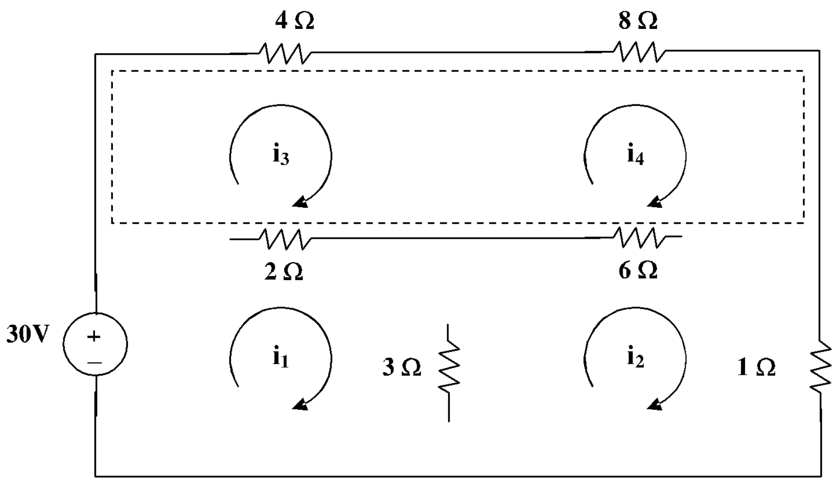


Figure 3.91

Chapter 3, Solution 45



For loop 1, $30 = 5i_1 - 3i_2 - 2i_3$ (1)

For loop 2, $10i_2 - 3i_1 - 6i_4 = 0$ (2)

For the supermesh, $6i_3 + 14i_4 - 2i_1 - 6i_2 = 0$ (3)

But $i_4 - i_3 = 4$ which leads to $i_4 = i_3 + 4$ (4)

Solving (1) to (4) by elimination gives $i = i_1 =$ **8.561 A**.

Chapter 3, Problem 46.

Calculate the mesh currents i_1 and i_2 in Fig. 3.92.

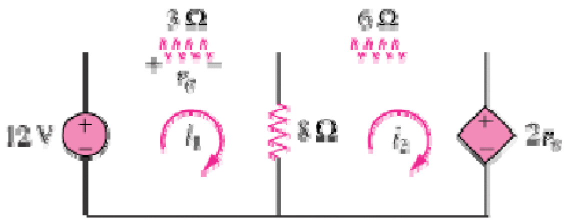


Figure 3.92

Chapter 3, Solution 46

For loop 1,

$-12 + 11i_1 - 8i_2 = 0 \quad \longrightarrow \quad 11i_1 - 8i_2 = 12 \qquad (1)$

For loop 2,

$-8i_1 + 14i_2 + 2v_o = 0$

But $v_o = 3i_1$,

$-8i_1 + 14i_2 + 6i_1 = 0 \quad \longrightarrow \quad i_1 = 7i_2 \qquad (2)$

Substituting (2) into (1),

$77i_2 - 8i_2 = 12 \quad \longrightarrow \quad \underline{i_2 = 0.1739}$ A and $\underline{i_1 = 7i_2 = 1.217}$ A

Chapter 3, Problem 47.

Rework Prob. 3.19 using mesh analysis.

Chapter 3, Problem 3.19

Use nodal analysis to find $\mathbf{V}_1$, $\mathbf{V}_2$, and $\mathbf{V}_3$ in the circuit in Fig. 3.68.

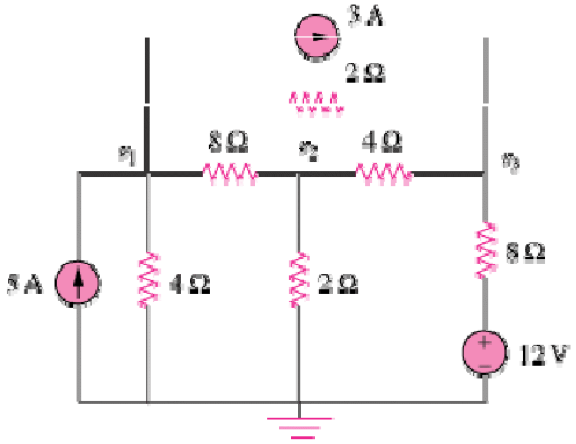
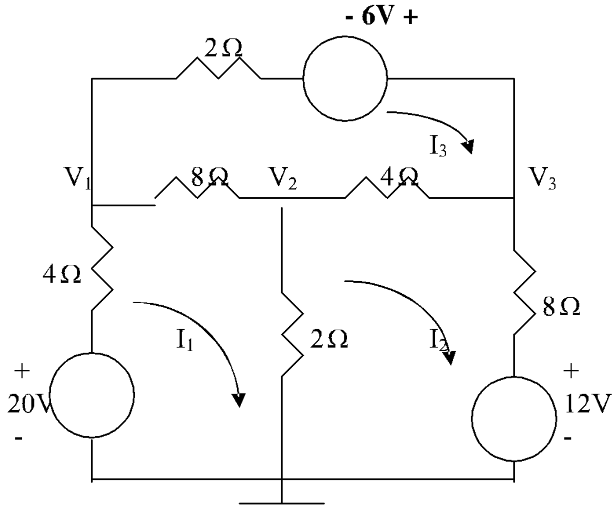


Figure 3.68

Chapter 3, Solution 47

First, transform the current sources as shown below.



For mesh 1,

$$-20+14I_1-2I_2-8I_3=0 \longrightarrow 10=7I_1-I_2-4I_3 \qquad (1)$$

For mesh 2,

$$12+14I_2-2I_1-4I_3=0 \longrightarrow -6=-I_1+7I_2-2I_3 \qquad (2)$$

For mesh 3,

$$-6+14I_3-4I_2-8I_1=0 \longrightarrow 3=-4I_1-2I_2+7I_3 \qquad (3)$$

Putting (1) to (3) in matrix form, we obtain

$$\begin{pmatrix} 7 & -1 & -4 \\ -1 & 7 & -2 \\ -4 & -2 & 7 \end{pmatrix}\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix}=\begin{pmatrix} 10 \\ -6 \\ 3 \end{pmatrix} \longrightarrow AI=B$$

Using MATLAB,

$$I=A^{-1}B=\begin{bmatrix} 2 \\ 0.0333 \\ 1.8667 \end{bmatrix} \longrightarrow I_1=2.5,\ I_2=0.0333, I_3=1.8667$$

But

$$I_1=\frac{20-V}{4} \longrightarrow V_1=20-4I_1=10\text{ V}$$

$$V_2=2(I_1-I_2)=4.933\text{ V}$$

Also,

$$I_2=\frac{V_3-12}{8} \longrightarrow V_3=12+8I_2=12.267\text{V}$$

Chapter 3, Problem 48.

Determine the current through the 10-kΩ resistor in the circuit in Fig. 3.93 using mesh analysis.

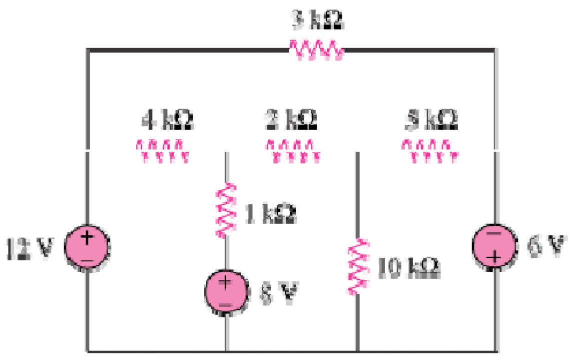
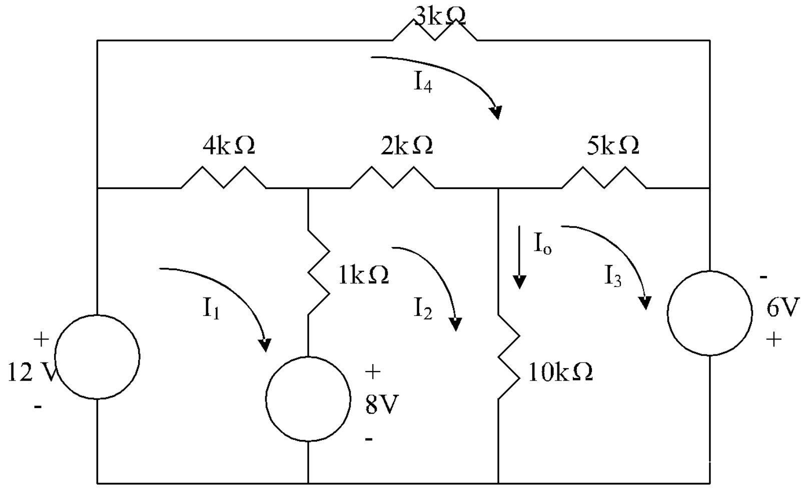


Figure 3.93

Chapter 3, Solution 48

We apply mesh analysis and let the mesh currents be in mA.



For mesh 1,

$$-12+8+5I_1-I_2-4I_4=0 \longrightarrow 4=5I_1-I_2-4I_4 \qquad (1)$$

For mesh 2,

$$-8+13I_2-I_1-10I_3-2I_4=0 \longrightarrow 8=-I_1+13I_2-10I_3-2I_4 \quad (2)$$

For mesh 3,

$$-6+15I_3-10I_2-5I_4=0 \longrightarrow 6=-10I_2+15I_3-5I_4 \qquad (3)$$

For mesh 4,

$$-4I_1-2I_2-5I_3+14I_4=0 \qquad (4)$$

Putting (1) to (4) in matrix form gives

$$\begin{pmatrix} 5 & -1 & 0 & -4 \\ -1 & 13 & -10 & -2 \\ 0 & -10 & 15 & -5 \\ -4 & -2 & -5 & 14 \end{pmatrix}\begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{pmatrix}=\begin{pmatrix} 4 \\ 8 \\ 6 \\ 0 \end{pmatrix} \longrightarrow AI=B$$

Using MATLAB,

$$I=A^{-1}B=\begin{pmatrix} 7.217 \\ 8.087 \\ 7.791 \\ 6 \end{pmatrix}$$

The current through the 10kΩ resistor is $I_o = I_2 - I_3$ = <u>0.2957 mA</u>

Chapter 3, Problem 49.

Find v_o and i_o in the circuit of Fig. 3.94.

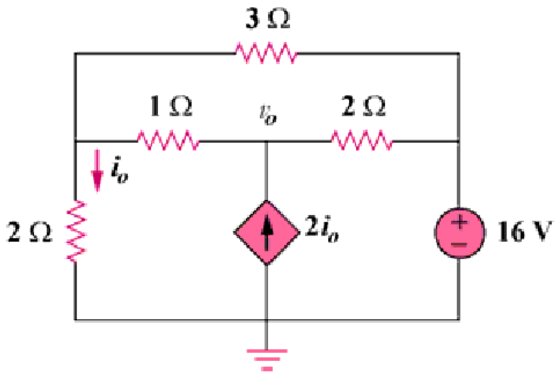
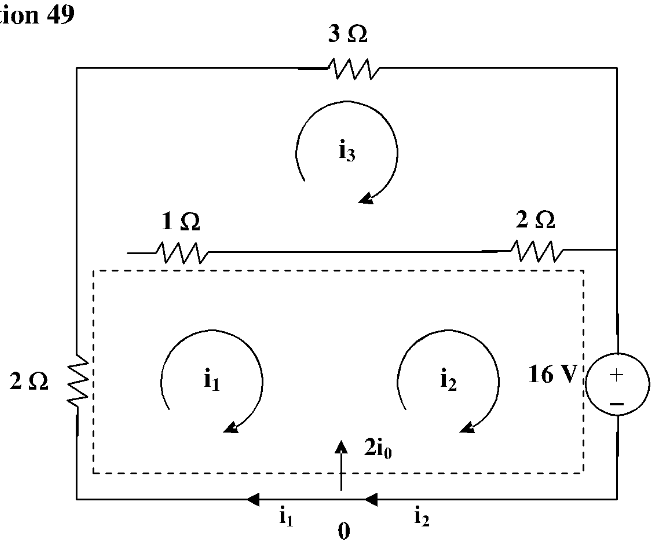
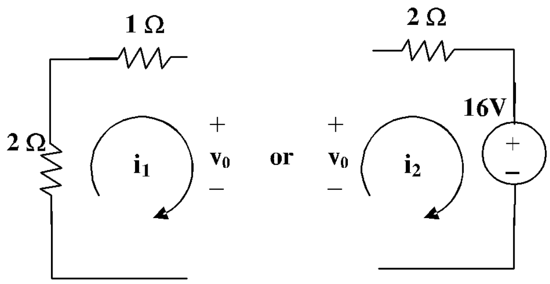


Figure 3.94

Chapter 3, Solution 49



(a)



(b)

For the supermesh in figure (a),

$$3i_1 + 2i_2 - 3i_3 + 16 = 0 \qquad (1)$$

At node 0, $i_2 - i_1 = 2i_0$ and $i_0 = -i_1$ which leads to $i_2 = -i_1$ (2)

For loop 3, $-i_1 - 2i_2 + 6i_3 = 0$ which leads to $6i_3 = -i_1$ (3)

Solving (1) to (3), $i_1 = (-32/3)$A, $i_2 = (32/3)$A, $i_3 = (16/9)$A

$i_0 = -i_1 =$ **<u>10.667 A</u>**, from fig. (b), $v_0 = i_3 - 3i_1 = (16/9) + 32 =$ **<u>33.78 V</u>**.

Chapter 3, Problem 50.

Use mesh analysis to find the current i_o in the circuit in Fig. 3.95.

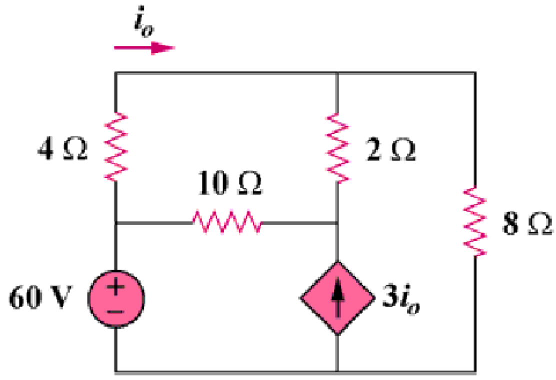
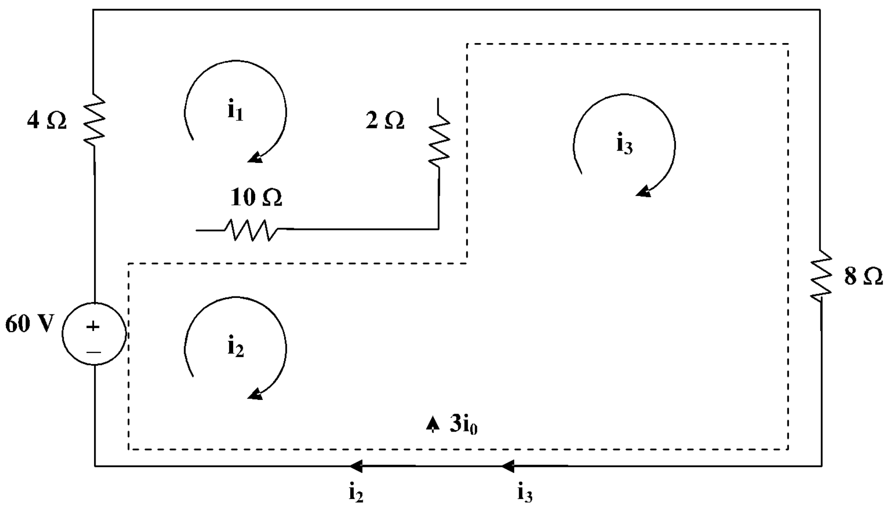


Figure 3.95

Chapter 3, Solution 50



For loop 1, $16i_1 - 10i_2 - 2i_3 = 0$ which leads to $8i_1 - 5i_2 - i_3 = 0$ (1)

For the supermesh, $-60 + 10i_2 - 10i_1 + 10i_3 - 2i_1 = 0$

or $-6i_1 + 5i_2 + 5i_3 = 30$ (2)

Also, $3i_0 = i_3 - i_2$ and $i_0 = i_1$ which leads to $3i_1 = i_3 - i_2$ (3)

Solving (1), (2), and (3), we obtain $i_1 = 1.731$ and $i_0 = i_1 =$ **1.731 A**

Chapter 3, Problem 51.

Apply mesh analysis to find v_o in the circuit in Fig. 3.96.

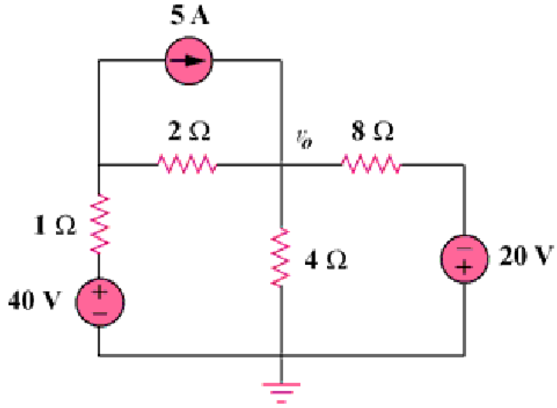
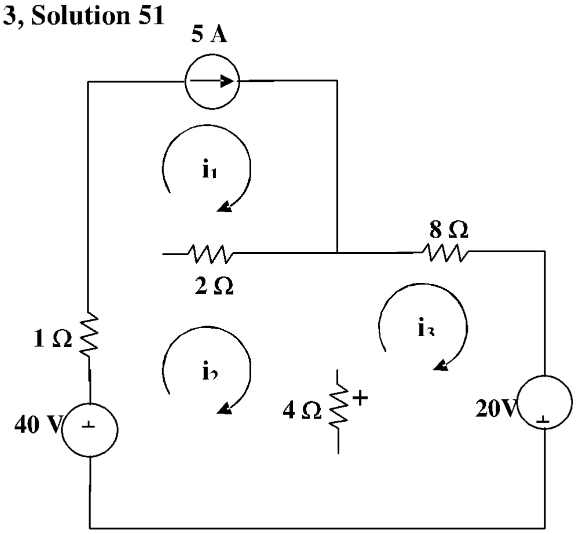


Figure 3.96

Chapter 3, Solution 51



For loop 1, $i_1 = 5A$ (1)

For loop 2, $-40 + 7i_2 - 2i_1 - 4i_3 = 0$ which leads to $50 = 7i_2 - 4i_3$ (2)

For loop 3, $-20 + 12i_3 - 4i_2 = 0$ which leads to $5 = -i_2 + 3i_3$ (3)

Solving with (2) and (3), $i_2 = 10$ A, $i_3 = 5$ A

And, $v_0 = 4(i_2 - i_3) = 4(10 - 5) =$ **20 V**.

Chapter 3, Problem 52.

Use mesh analysis to find i_1, i_2, and i_3 in the circuit of Fig. 3.97.

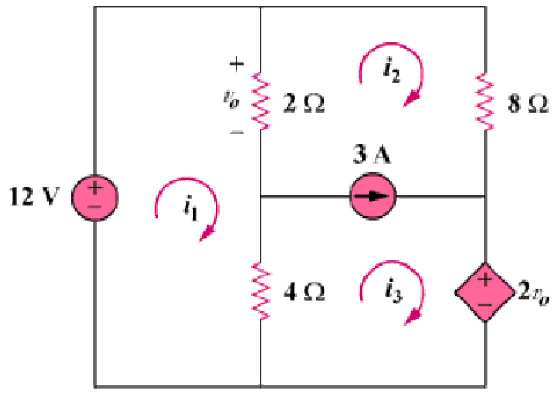
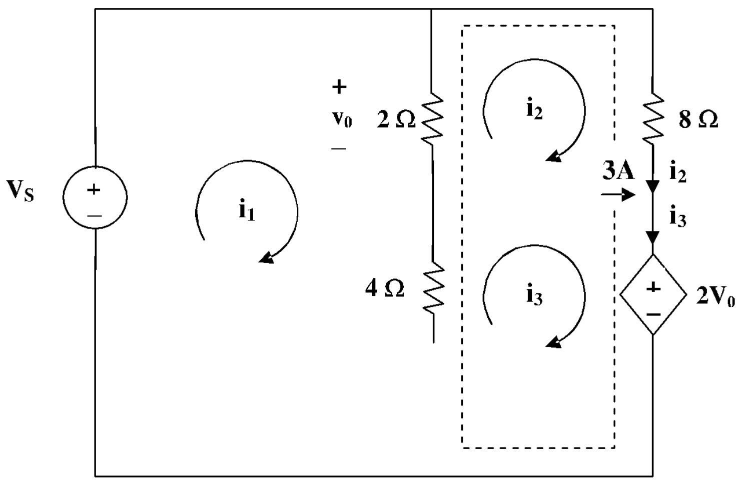


Figure 3.97

Chapter 3, Solution 52



For mesh 1,

$$2(i_1 - i_2) + 4(i_1 - i_3) - 12 = 0 \text{ which leads to } 3i_1 - i_2 - 2i_3 = 6 \qquad (1)$$

For the supermesh, $2(i_2 - i_1) + 8i_2 + 2v_0 + 4(i_3 - i_1) = 0$

But $v_0 = 2(i_1 - i_2)$ which leads to $-i_1 + 3i_2 + 2i_3 = 0$ (2)

For the independent current source, $i_3 = 3 + i_2$ (3)

Solving (1), (2), and (3), we obtain,

$i_1 =$ **<u>3.5 A</u>**, $i_2 =$ **<u>-0.5 A</u>**, $i_3 =$ **<u>2.5 A</u>**.

Chapter 3, Problem 53.

Find the mesh currents in the circuit of Fig. 3.98 using MATLAB.

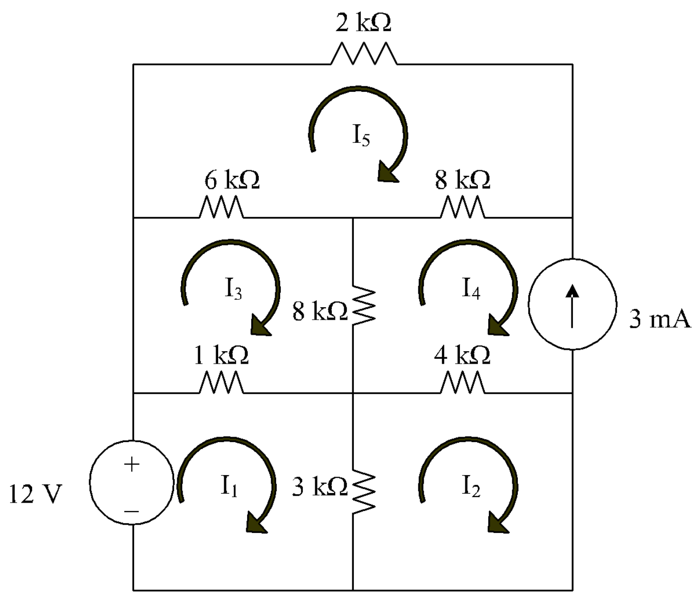


Figure 3.98 For Prob. 3.53.

Chapter 3, Solution 53

Applying mesh analysis leads to;

$-12 + 4kI_1 - 3kI_2 - 1kI_3 = 0$ (1)

$-3kI_1 + 7kI_2 - 4kI_4 = 0$

$-3kI_1 + 7kI_2 = -12$ (2)

$-1kI_1 + 15kI_3 - 8kI_4 - 6kI_5 = 0$

$-1kI_1 + 15kI_3 - 6k = -24$ (3)

$I_4 = -3mA$ (4)

$-6kI_3 - 8kI_4 + 16kI_5 = 0$

$-6kI_3 + 16kI_5 = -24$ (5)

Putting these in matrix form (having substituted I_4 = 3mA in the above),

$$\begin{bmatrix} 4 & -3 & -1 & 0 \\ -3 & 7 & 0 & 0 \\ -1 & 0 & 15 & -6 \\ 0 & 0 & -6 & 16 \end{bmatrix} k \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_5 \end{bmatrix} = \begin{bmatrix} 12 \\ -12 \\ -24 \\ -24 \end{bmatrix}$$

ZI = V

Using MATLAB,

```
>> Z = [4,-3,-1,0;-3,7,0,0;-1,0,15,-6;0,0,-6,16]

Z =

   4   -3   -1    0
  -3    7    0    0
  -1    0   15   -6
   0    0   -6   16

>> V = [12,-12,-24,-24]'

V =

  12
 -12
 -24
 -24
```

We obtain,

```
>> I = inv(Z)*V

      I =
```

<u>1.6196 mA</u>
<u>–1.0202 mA</u>
<u>–2.461 mA</u>
<u>3 mA</u>
<u>–2.423 mA</u>

Chapter 3, Problem 54.

Find the mesh currents i_1, i_2, and i_3 in the circuit in Fig. 3.99.

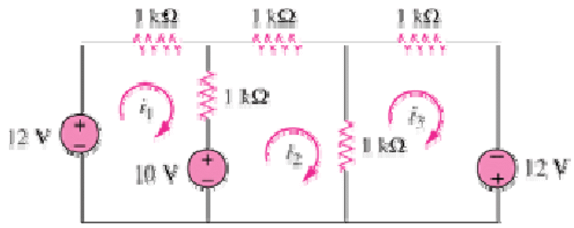


Figure 3.99

Chapter 3, Solution 54

Let the mesh currents be in mA. For mesh 1,

$$-12+10+2I_1-I_2=0 \longrightarrow 2=2I_1-I_2 \qquad (1)$$

For mesh 2,

$$-10+3I_2-I_1-I_3=0 \longrightarrow 10=-I_1+3I_2-I_3 \qquad (2)$$

For mesh 3,

$$-12+2I_3-I_2=0 \longrightarrow 12=-I_2+2I_3 \qquad (3)$$

Putting (1) to (3) in matrix form leads to

$$\begin{pmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{pmatrix}\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix}=\begin{pmatrix} 2 \\ 10 \\ 12 \end{pmatrix} \longrightarrow AI=B$$

Using MATLAB,

$$I=A^{-1}B=\begin{bmatrix} 5.25 \\ 8.5 \\ 10.25 \end{bmatrix} \longrightarrow \underline{I_1=5.25\,\text{mA},\ I_2=8.5\,\text{mA}, I_3=10.25\,\text{mA}}$$

Chapter 3, Problem 55.

In the circuit of Fig. 3.100, solve for i_1, i_2, and i_3.

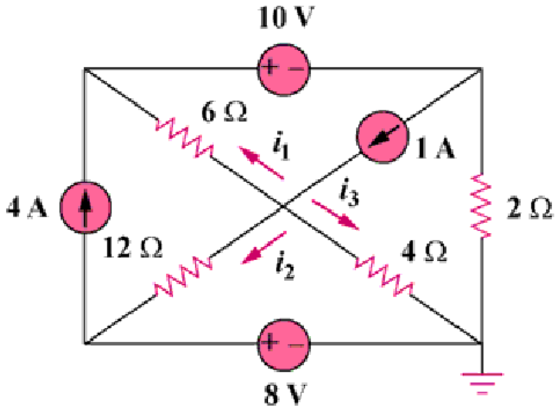
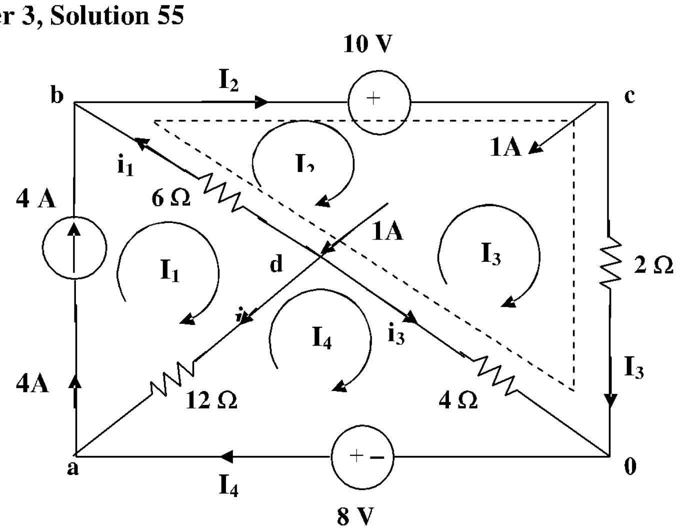


Figure 3.100

Chapter 3, Solution 55



It is evident that $I_1 = 4$ (1)

For mesh 4, $12(I_4 - I_1) + 4(I_4 - I_3) - 8 = 0$ (2)

For the supermesh $6(I_2 - I_1) + 10 + 2I_3 + 4(I_3 - I_4) = 0$

or $-3I_1 + 3I_2 + 3I_3 - 2I_4 = -5$ (3)

At node c, $I_2 = I_3 + 1$ (4)

Solving (1), (2), (3), and (4) yields, $I_1 = 4A$, $I_2 = 3A$, $I_3 = 2A$, and $I_4 = 4A$

At node b, $i_1 = I_2 - I_1 =$ **<u>-1A</u>**

At node a, $i_2 = 4 - I_4 =$ **<u>0A</u>**

At node 0, $i_3 = I_4 - I_3 =$ **<u>2A</u>**

Chapter 3, Problem 56.

Determine v_1 and v_2 in the circuit of Fig. 3.101.

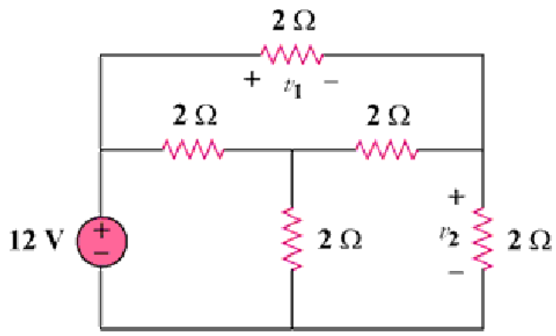
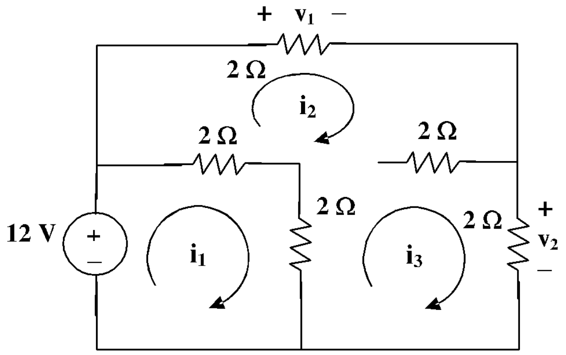


Figure 3.101

Chapter 3, Solution 56



For loop 1, $12 = 4i_1 - 2i_2 - 2i_3$ which leads to $6 = 2i_1 - i_2 - i_3$ (1)

For loop 2, $0 = 6i_2 - 2i_1 - 2\,i_3$ which leads to $0 = -i_1 + 3i_2 - i_3$ (2)

For loop 3, $0 = 6i_3 - 2i_1 - 2i_2$ which leads to $0 = -i_1 - i_2 + 3i_3$ (3)

In matrix form (1), (2), and (3) become,

$$\begin{bmatrix} 2 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 2 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{vmatrix} = 8, \quad \Delta_2 = \begin{vmatrix} 2 & 6 & -1 \\ -1 & 3 & -1 \\ -1 & 0 & 3 \end{vmatrix} = 24$$

$$\Delta_3 = \begin{vmatrix} 2 & -1 & 6 \\ -1 & 3 & 0 \\ -1 & -1 & 0 \end{vmatrix} = 24 \text{, therefore } i_2 = i_3 = 24/8 = 3\text{A},$$

$v_1 = 2i_2 =$ **6 volts**, $v = 2i_3 =$ **6 volts**

Chapter 3, Problem 57.

In the circuit in Fig. 3.102, find the values of R, V_1, and V_2 given that i_o = 18 mA.

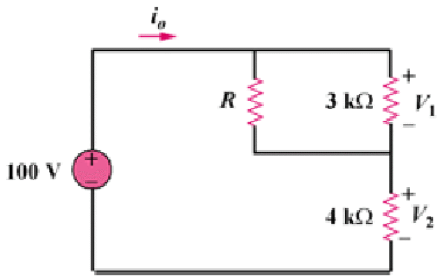


Figure 3.102

Chapter 3, Solution 57

Assume R is in kilo-ohms.

$V_2 = 4k\Omega x 18mA = \underline{72V}$, $\quad V_1 = 100 - V_2 = 100 - 72 = \underline{28V}$

Current through R is

$$i_R = \frac{3}{3+R} i_o, \qquad V_1 = i_R R \quad \longrightarrow \quad 28 = \frac{3}{3+R}(18)R$$

This leads to R = 84/26 = **3.23 kΩ**

Chapter 3, Problem 58.

Find $\mathbf{i}_1$, i_2, and i_3 the circuit in Fig. 3.103.

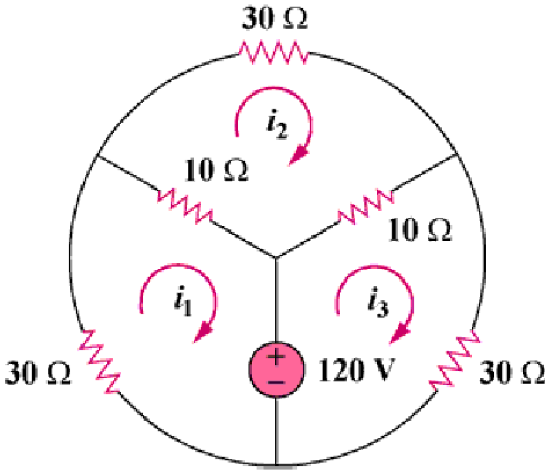
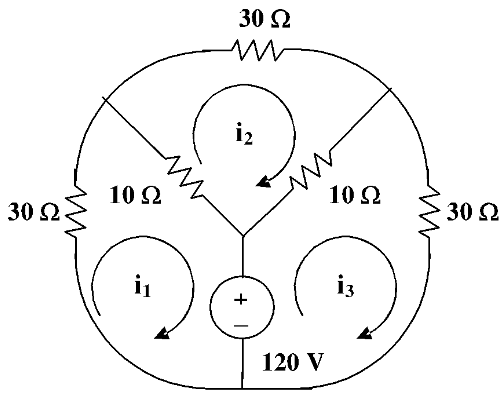


Figure 3.103

Chapter 3, Solution 58



For loop 1, $120 + 40i_1 - 10i_2 = 0$, which leads to $-12 = 4i_1 - i_2$ (1)

For loop 2, $50i_2 - 10i_1 - 10i_3 = 0$, which leads to $-i_1 + 5i_2 - i_3 = 0$ (2)

For loop 3, $-120 - 10i_2 + 40i_3 = 0$, which leads to $12 = -i_2 + 4i_3$ (3)

Solving (1), (2), and (3), we get, $i_1 =$ **-3A**, $i_2 =$ **0**, and $i_3 =$ **3A**

Chapter 3, Problem 59.

Rework Prob. 3.30 using mesh analysis.

Chapter 3, Problem 30.

Using nodal analysis, find $\boldsymbol{v_o}$ and $\boldsymbol{i_o}$ in the circuit of Fig. 3.79.

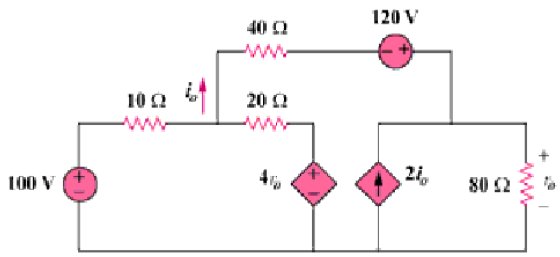
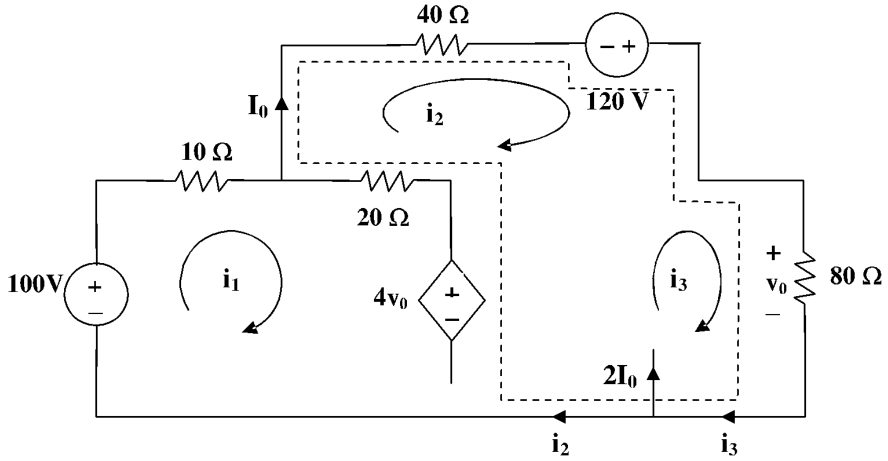


Figure 3.79

Chapter 3, Solution 59



For loop 1, $-100 + 30i_1 - 20i_2 + 4v_0 = 0$, where $v_0 = 80i_3$
or $5 = 1.5i_1 - i_2 + 16i_3$ (1)

For the supermesh, $60i_2 - 20i_1 - 120 + 80i_3 - 4\,v_0 = 0$, where $v_0 = 80i_3$
or $6 = -i_1 + 3i_2 - 12i_3$ (2)

Also, $2I_0 = i_3 - i_2$ and $I_0 = i_2$, hence, $3i_2 = i_3$ (3)

From (1), (2), and (3), $$\begin{bmatrix} 3 & -2 & 32 \\ -1 & 3 & -12 \\ 0 & 3 & -1 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 6 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 3 & -2 & 32 \\ -1 & 3 & -12 \\ 0 & 3 & -1 \end{vmatrix} = 5,\ \Delta_2 = \begin{vmatrix} 3 & 10 & 32 \\ -1 & 6 & -12 \\ 0 & 0 & -1 \end{vmatrix} = -28,\ \Delta_3 = \begin{vmatrix} 3 & -2 & 10 \\ -1 & 3 & 6 \\ 0 & 3 & 0 \end{vmatrix} = -84$$

$I_0 = i_2 = \Delta_2/\Delta = -28/5 =$ **-5.6 A**

$v_0 = 8i_3 = (-84/5)80 =$ **-1.344 kvolts**

Chapter 3, Problem 60.

Calculate the power dissipated in each resistor in the circuit in Fig. 3.104.

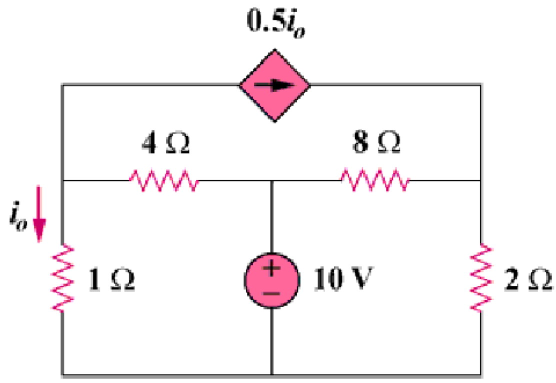
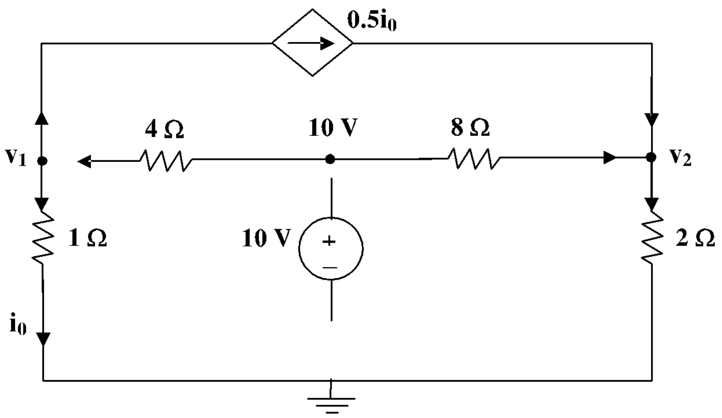


Figure 3.104

Chapter 3, Solution 60



At node 1, $(v_1/1) + (0.5v_1/1) = (10 - v_1)/4$, which leads to $v_1 = 10/7$

At node 2, $(0.5v_1/1) + ((10 - v_2)/8) = v_2/2$ which leads to $v_2 = 22/7$

$P_{1\Omega} = (v_1)^2/1 =$ **2.041 watts**, $P_{2\Omega} = (v_2)^2/2 =$ **4.939 watts**

$P_{4\Omega} = (10 - v_1)^2/4 =$ **18.38 watts**, $P_{8\Omega} = (10 - v_2)^2/8 =$ **5.88 watts**

Chapter 3, Problem 61.

Calculate the current gain i_o/i_s in the circuit of Fig. 3.105.

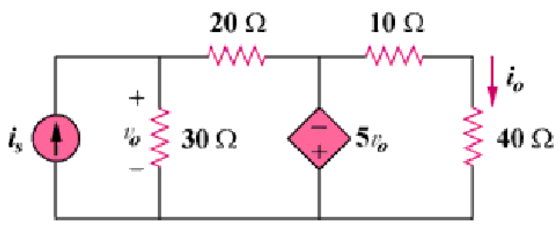
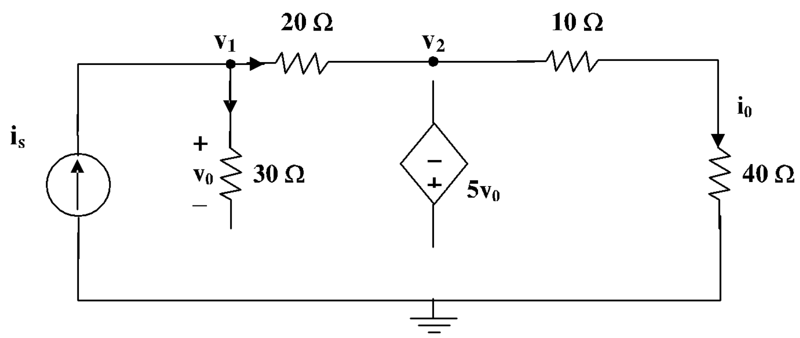


Figure 3.105

Chapter 3, Solution 61



At node 1, $i_s = (v_1/30) + ((v_1 - v_2)/20)$ which leads to $60i_s = 5v_1 - 3v_2$ (1)

But $v_2 = -5v_0$ and $v_0 = v_1$ which leads to $v_2 = -5v_1$

Hence, $60i_s = 5v_1 + 15v_1 = 20v_1$ which leads to $v_1 = 3i_s$, $v_2 = -15i_s$

$i_0 = v_2/50 = -15i_s/50$ which leads to $i_0/i_s = -15/50 =$ **<u>–0.3</u>**

Chapter 3, Problem 62.

Find the mesh currents i_1, i_2, and i_3 in the network of Fig. 3.106.

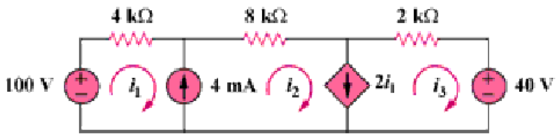
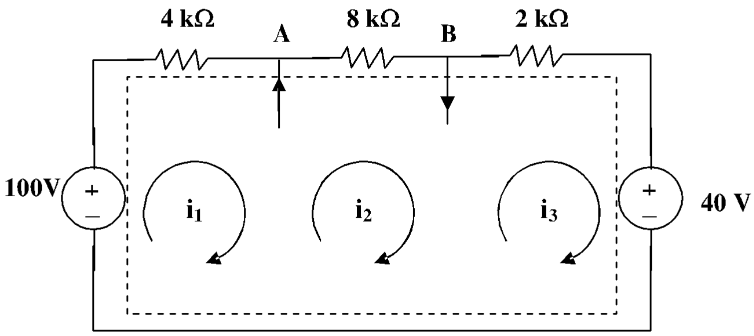


Figure 3.106

Chapter 3, Solution 62



We have a supermesh. Let all R be in kΩ, i in mA, and v in volts.

For the supermesh, $-100 + 4i_1 + 8i_2 + 2i_3 + 40 = 0$ or $30 = 2i_1 + 4i_2 + i_3$ (1)

At node A, $i_1 + 4 = i_2$ (2)

At node B, $i_2 = 2i_1 + i_3$ (3)

Solving (1), (2), and (3), we get i_1 = **<u>2 mA</u>**, i_2 = **<u>6 mA</u>**, and i_3 = **<u>2 mA</u>**.

Chapter 3, Problem 63.

Find v_x, and i_x in the circuit shown in Fig. 3.107.

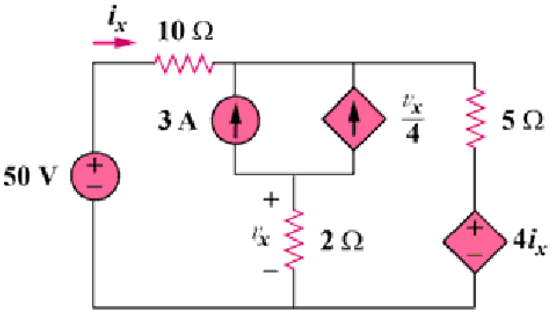
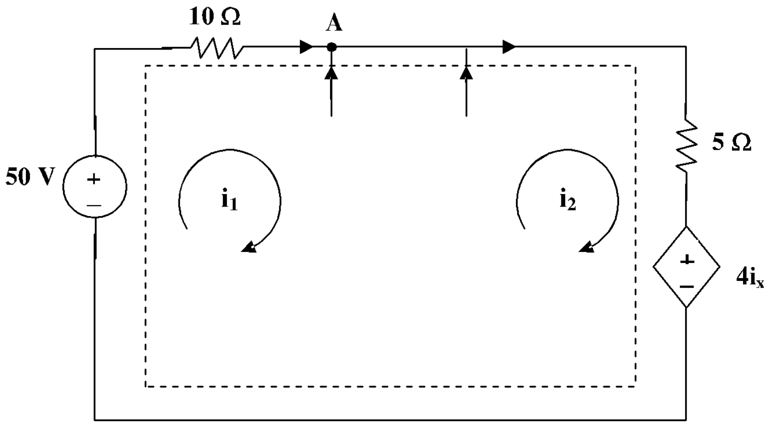


Figure 3.107

Chapter 3, Solution 63



For the supermesh, $-50 + 10i_1 + 5i_2 + 4i_x = 0$, but $i_x = i_1$. Hence,

$$50 = 14i_1 + 5i_2 \qquad (1)$$

At node A, $i_1 + 3 + (v_x/4) = i_2$, but $v_x = 2(i_1 - i_2)$, hence, $i_1 + 2 = i_2$ (2)

Solving (1) and (2) gives $i_1 = 2.105$ A and $i_2 = 4.105$ A

$v_x = 2(i_1 - i_2) =$ **–4 volts** and $i_x = i_2 - 2 =$ **2.105 amp**

Chapter 3, Problem 64.

Find v_o, and i_o in the circuit of Fig. 3.108.

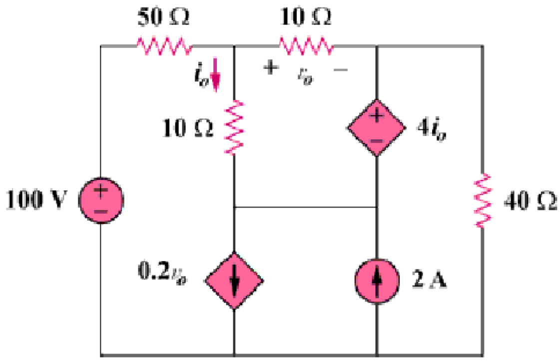
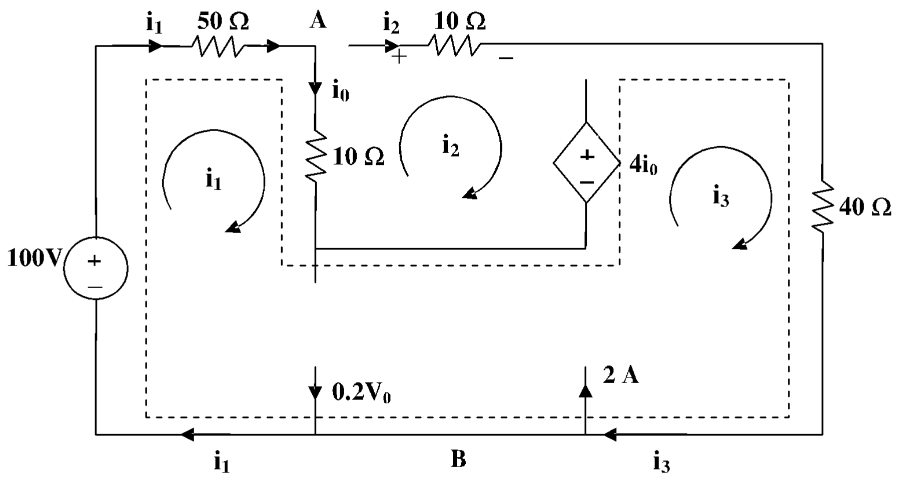


Figure 3.108

Chapter 3, Solution 64



For mesh 2, $20i_2 - 10i_1 + 4i_0 = 0$ (1)

But at node A, $i_o = i_1 - i_2$ so that (1) becomes $i_1 = (16/6)i_2$ (2)

For the supermesh, $-100 + 50i_1 + 10(i_1 - i_2) - 4i_0 + 40i_3 = 0$

or $50 = 28i_1 - 3i_2 + 20i_3$ (3)

At node B, $i_3 + 0.2v_0 = 2 + i_1$ (4)

But, $v_0 = 10i_2$ so that (4) becomes $i_3 = 2 + (2/3)i_2$ (5)

Solving (1) to (5), $i_2 = 0.11764$,

$v_0 = 10i_2 =$ **<u>1.1764 volts</u>**, $i_0 = i_1 - i_2 = (5/3)i_2 =$ **<u>196.07 mA</u>**

Chapter 3, Problem 65.

Use *MATLAB* to solve for the mesh currents in the circuit of Fig. 3.109.

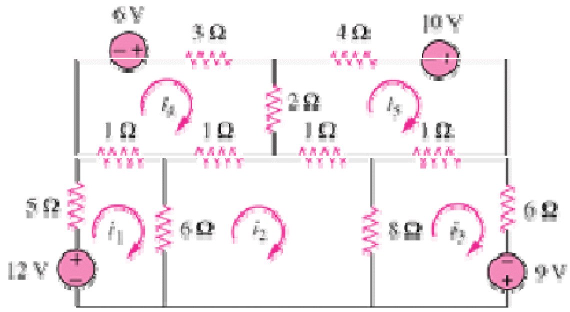


Figure 3.109

Chapter 3, Solution 65

For mesh 1,

$$-12 + 12I_1 - 6I_2 - I_4 = 0 \text{ or}$$

$$12 = 12I_1 - 6I_2 - I_4 \qquad (1)$$

For mesh 2,

$$-6I_1 + 16I_2 - 8I_3 - I_4 - I_5 = 0 \qquad (2)$$

For mesh 3,

$$-8I_2 + 15I_3 - I_5 - 9 = 0 \text{ or}$$

$$9 = -8I_2 + 15I_3 - I_5 \qquad (3)$$

For mesh 4,

$$-I_1 - I_2 + 7I_4 - 2I_5 - 6 = 0 \text{ or}$$

$$6 = -I_1 - I_2 + 7I_4 - 2I_5 \qquad (4)$$

For mesh 5,

$$-I_2 - I_3 - 2I_4 + 8I_5 - 10 = 0 \text{ or}$$

$$10 = -I_2 - I_3 - 2I_4 + 8I_5 \qquad (5)$$

Casting (1) to (5) in matrix form gives

$$\begin{pmatrix} 12 & -6 & 0 & 1 & 0 \\ -6 & 16 & -8 & -1 & -1 \\ 0 & -8 & 15 & 0 & -1 \\ -1 & -1 & 0 & 7 & -2 \\ 0 & -1 & -1 & -2 & 8 \end{pmatrix}\begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{pmatrix} = \begin{pmatrix} 12 \\ 0 \\ 9 \\ 6 \\ 10 \end{pmatrix} \longrightarrow \quad AI = B$$

Using MATLAB we input:
Z=[12,-6,0,-1,0;-6,16,-8,-1,-1;0,-8,15,0,-1;-1,-1,0,7,-2;0,-1,-1,-2,8]
and V=[12;0;9;6;10]

This leads to

```
>> Z=[12,-6,0,-1,0;-6,16,-8,-1,-1;0,-8,15,0,-1;-1,-1,0,7,-2;0,-1,-1,-2,8]

Z =
   12   -6    0   -1    0
   -6   16   -8   -1   -1
    0   -8   15    0   -1
   -1   -1    0    7   -2
    0   -1   -1   -2    8

>> V=[12;0;9;6;10]

V =
   12
    0
    9
    6
   10

>> I=inv(Z)*V

I =
   2.1701
   1.9912
   1.8119
   2.0942
   2.2489
```

Thus,

I = **[2.17, 1.9912, 1.8119, 2.094, 2.249] A**.

Chapter 3, Problem 66.

Write a set of mesh equations for the circuit in Fig. 3.110. Use MATLAB to determine the mesh currents.

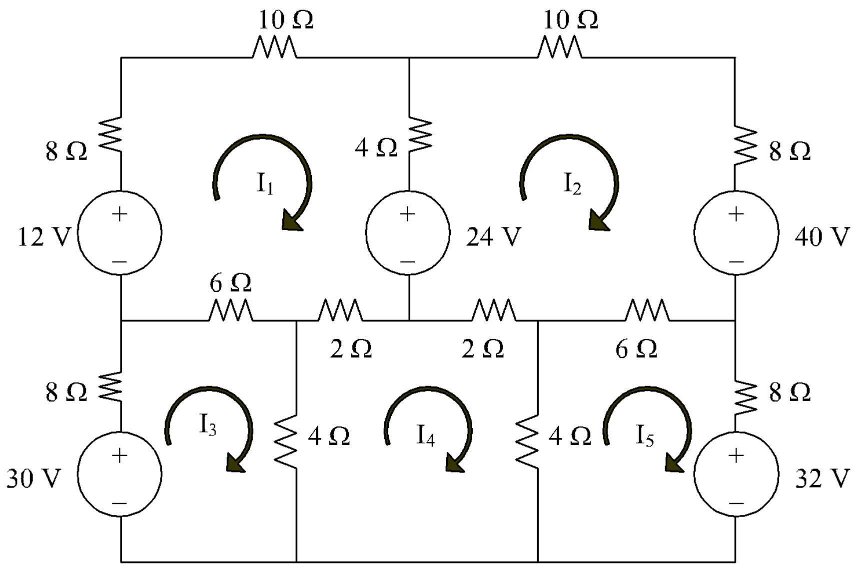


Figure 3.110 For Prob. 3.66.

Chapter 3, Solution 66

The mesh equations are obtained as follows.

$$-12 + 24 + 30I_1 - 4I_2 - 6I_3 - 2I_4 = 0$$

or

$$30I_1 - 4I_2 - 6I_3 - 2I_4 = -12 \qquad (1)$$

$$-24 + 40 - 4I_1 + 30I_2 - 2I_4 - 6I_5 = 0$$

or

$$-4I_1 + 30I_2 - 2I_4 - 6I_5 = -16 \qquad (2)$$

$$-6I_1 + 18I_3 - 4I_4 = 30 \qquad (3)$$

$$-2I_1 - 2I_2 - 4I_3 + 12I_4 - 4I_5 = 0 \qquad (4)$$

$$-6I_2 - 4I_4 + 18I_5 = -32 \qquad (5)$$

Putting (1) to (5) in matrix form

$$\begin{bmatrix} 30 & -4 & -6 & -2 & 0 \\ -4 & 30 & 0 & -2 & -6 \\ -6 & 0 & 18 & -4 & 0 \\ -2 & -2 & -4 & 12 & -4 \\ 0 & -6 & 0 & -4 & 18 \end{bmatrix} I = \begin{bmatrix} -12 \\ -16 \\ 30 \\ 0 \\ -32 \end{bmatrix}$$

ZI = V

Using MATLAB,

```
>> Z = [30,-4,-6,-2,0;
-4,30,0,-2,-6;
-6,0,18,-4,0;
-2,-2,-4,12,-4;
0,-6,0,-4,18]

Z =
   30   -4   -6   -2    0
   -4   30    0   -2   -6
   -6    0   18   -4    0
   -2   -2   -4   12   -4
    0   -6    0   -4   18

>> V = [-12,-16,30,0,-32]'

V =
  -12
  -16
   30
    0
  -32

>> I = inv(Z)*V

I =
```

-0.2779 A
-1.0488 A
1.4682 A
-0.4761 A
-2.2332 A

Chapter 3, Problem 67.

Obtain the node-voltage equations for the circuit in Fig. 3.111 by inspection. Then solve for V_o.

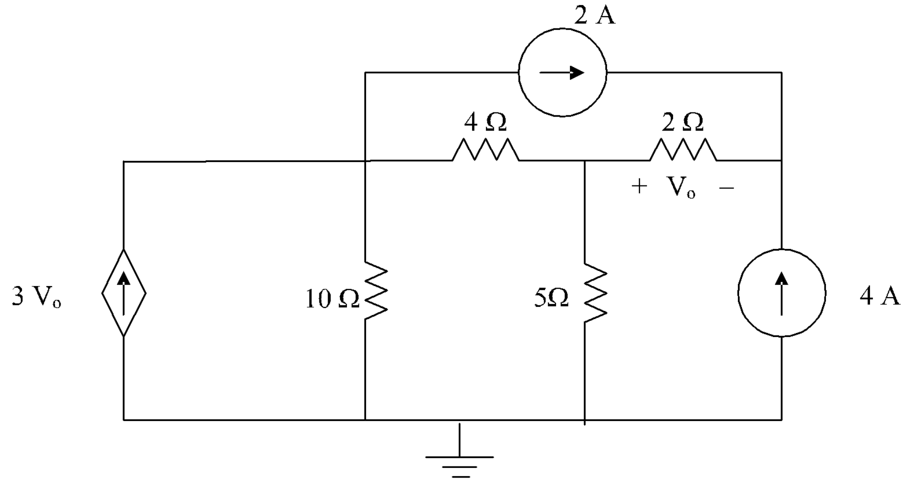
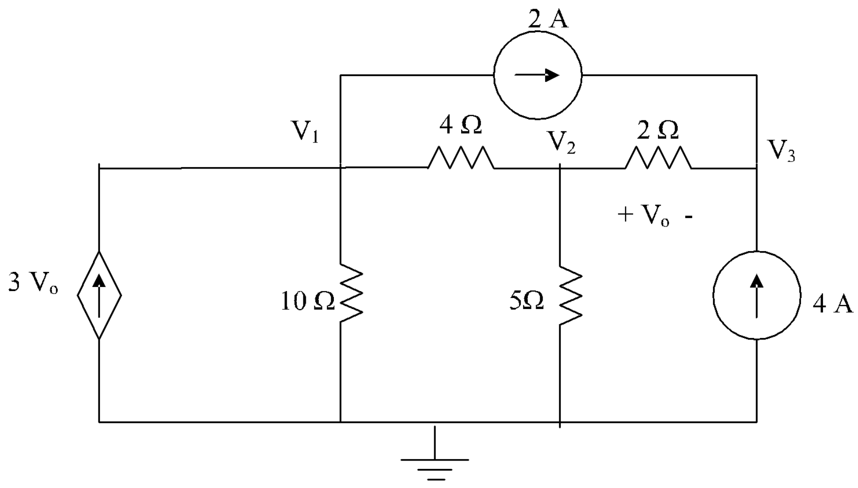


Figure 3.111 For Prob. 3.67.

Chapter 3, Solution 67

Consider the circuit below.



$$\begin{bmatrix} 0.35 & -0.25 & 0 \\ -0.25 & 0.95 & -0.5 \\ 0 & -0.5 & 0.5 \end{bmatrix} V = \begin{bmatrix} V_o \\ 0 \\ 6 \end{bmatrix}$$

Since we actually have four unknowns and only three equations, we need a constraint equation.

$$V_o = V_2 - V_3$$

Substituting this back into the matrix equation, the first equation becomes,

$$0.35V_1 - 3.25V_2 + 3V_3 = -2$$

This now results in the following matrix equation,

$$\begin{bmatrix} 0.35 & -3.25 & 3 \\ -0.25 & 0.95 & -0.5 \\ 0 & -0.5 & 0.5 \end{bmatrix} V = \begin{bmatrix} -2 \\ 0 \\ 6 \end{bmatrix}$$

Now we can use MATLAB to solve for V.

```
>> Y=[0.35,-3.25,3;-0.25,0.95,-0.5;0,-0.5,0.5]

Y =
    0.3500   -3.2500    3.0000
   -0.2500    0.9500   -0.5000
         0   -0.5000    0.5000

>> I=[-2,0,6]'

I =
    -2
     0
     6

>> V=inv(Y)*I

V =
 -164.2105
  -77.8947
  -65.8947
```

$V_o = V_2 - V_3 = -77.89 + 65.89 =$ **<u>–12 V</u>**.

Let us now do a quick check at node 1.

$$-3(-12) + 0.1(-164.21) + 0.25(-164.21+77.89) + 2 =$$
$+36 - 16.421 - 21.58 + 2 = -0.001$; answer checks!

Chapter 3, Problem 68.

Find the voltage V_o in the circuit of Fig. 3.112.

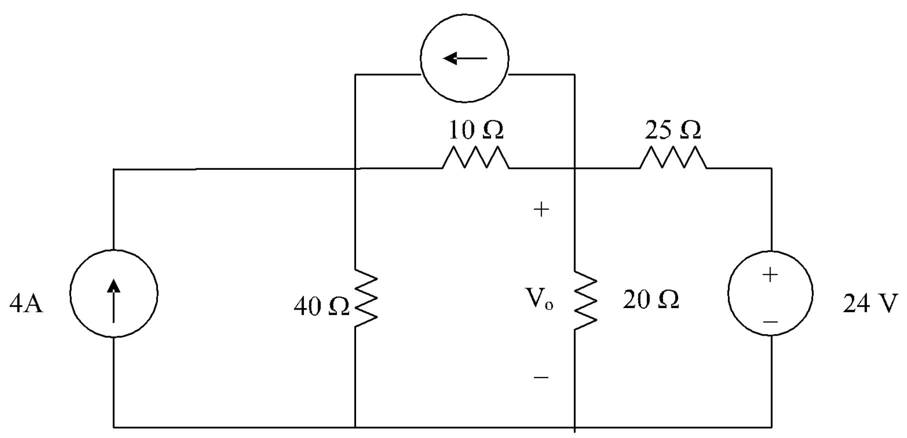
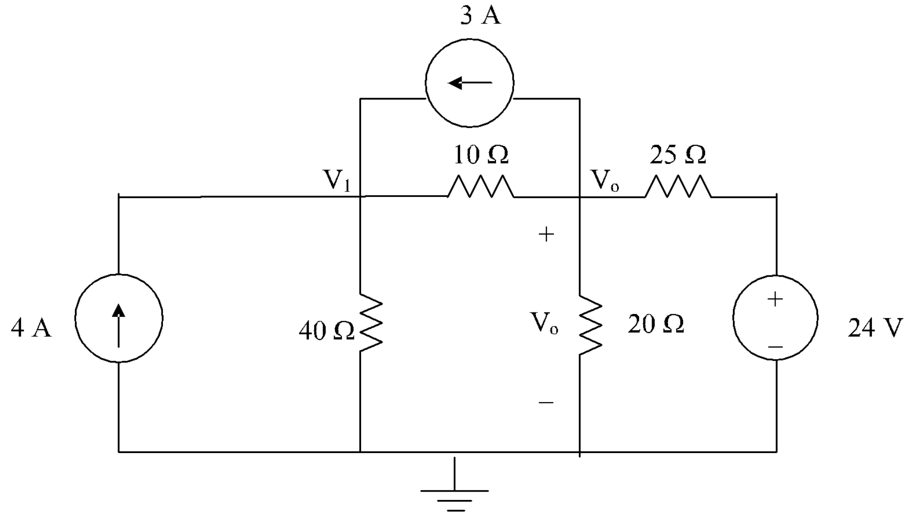


Figure 3.112 For Prob. 3.68.

Chapter 3, Solution 68

Consider the circuit below. There are two non-reference nodes.



$$\begin{bmatrix} 0.125 & -0.1 \\ -0.1 & 0.19 \end{bmatrix} V = \begin{bmatrix} +4+3 \\ -3+24/25 \end{bmatrix} = \begin{bmatrix} 7 \\ -2.04 \end{bmatrix}$$

Using MATLAB, we get,

```
>> Y=[0.125,-0.1;-0.1,0.19]

Y =

   0.1250   -0.1000
  -0.1000    0.1900

>> I=[7,-2.04]'

I =

   7.0000
  -2.0400

>> V=inv(Y)*I

V =

  81.8909
  32.3636
```

Thus, V_o = **32.36 V**.

We can perform a simple check at node V_o,

$3 + 0.1(32.36–81.89) + 0.05(32.36) + 0.04(32.36–24) =$
$3 – 4.953 + 1.618 + 0.3344 = – 0.0004$; answer checks!

Chapter 3, Problem 69.

For the circuit in Fig. 3.113, write the node voltage equations by inspection.

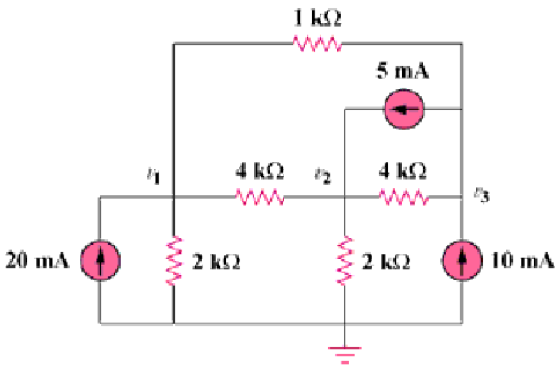


Figure 3.113

Chapter 3, Solution 69

Assume that all conductances are in mS, all currents are in mA, and all voltages are in volts.

G_{11} = (1/2) + (1/4) + (1/1) = 1.75, G_{22} = (1/4) + (1/4) + (1/2) = 1,
G_{33} = (1/1) + (1/4) = 1.25, G_{12} = -1/4 = -0.25, G_{13} = -1/1 = -1,
G_{21} = -0.25, G_{23} = -1/4 = -0.25, G_{31} = -1, G_{32} = -0.25

i_1 = 20, i_2 = 5, and i_3 = 10 – 5 = 5

The node-voltage equations are:

$$\begin{bmatrix} 1.75 & -0.25 & -1 \\ -0.25 & 1 & -0.25 \\ -1 & -0.25 & 1.25 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 20 \\ 5 \\ 5 \end{bmatrix}$$

Chapter 3, Problem 70.

Write the node-voltage equations by inspection and then determine values of V_1 and V_2 in the circuit in Fig. 3.114.

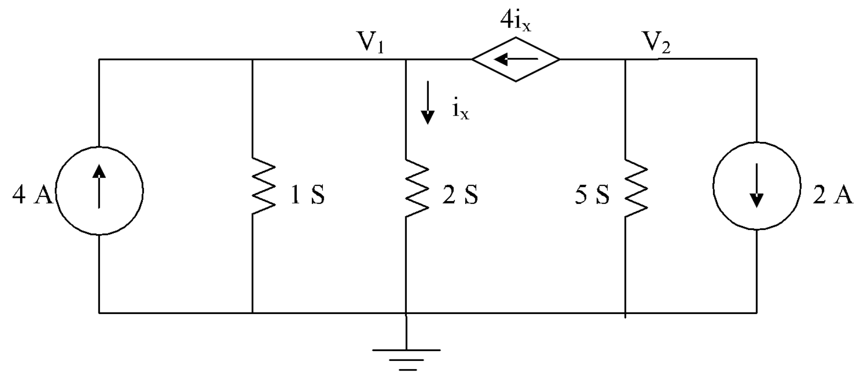


Figure 3.114 For Prob. 3.70.

Chapter 3, Solution 70

$$\begin{bmatrix} 3 & 0 \\ 0 & 5 \end{bmatrix} V = \begin{bmatrix} 4I_x + 4 \\ -4I_x - 2 \end{bmatrix}$$

With two equations and three unknowns, we need a constraint equation,

$I_x = 2V_1$, thus the matrix equation becomes,

$$\begin{bmatrix} -5 & 0 \\ 8 & 5 \end{bmatrix} V = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

This results in $V_1 = 4/(-5) =$ **<u>–0.8V</u>** and
$V_2 = [-8(-0.8) - 2]/5 = [6.4 - 2]/5 =$ **<u>0.88 V</u>**.

Chapter 3, Problem 71.

Write the mesh-current equations for the circuit in Fig. 3.115. Next, determine the values of I_1, I_2, and I_3.

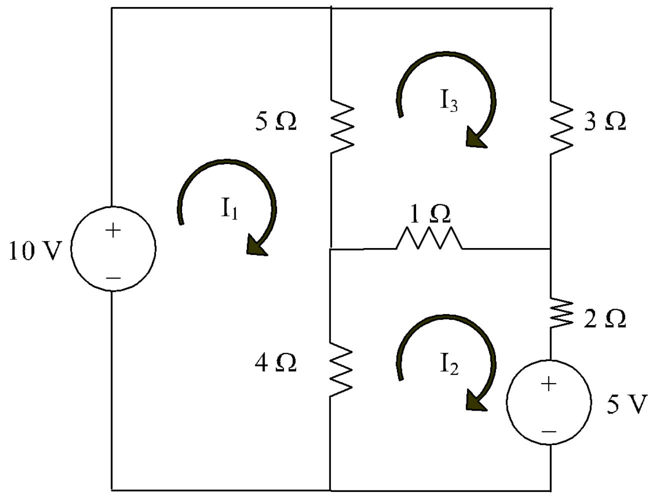


Figure 3.115 For Prob. 3.71.

Chapter 3, Solution 71

$$\begin{bmatrix} 9 & -4 & -5 \\ -4 & 7 & -1 \\ -5 & -1 & 9 \end{bmatrix} I = \begin{bmatrix} 10 \\ -5 \\ 0 \end{bmatrix}$$

We can now use MATLAB solve for our currents.

```
>> R=[9,-4,-5;-4,7,-1;-5,-1,9]

R =

     9    -4    -5
    -4     7    -1
    -5    -1     9

>> V=[10,-5,0]'

V =

    10
    -5
     0

>> I=inv(R)*V

I =
```

2.085 A
653.3 mA
1.2312 A

Chapter 3, Problem 72.

By inspection, write the mesh-current equations for the circuit in Fig. 3.116.

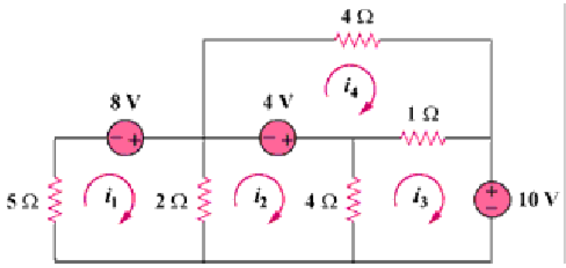


Figure 3.116

Chapter 3, Solution 72

$R_{11} = 5 + 2 = 7$, $R_{22} = 2 + 4 = 6$, $R_{33} = 1 + 4 = 5$, $R_{44} = 1 + 4 = 5$, $R_{12} = -2$, $R_{13} = 0 = R_{14}$, $R_{21} = -2$, $R_{23} = -4$, $R_{24} = 0$, $R_{31} = 0$, $R_{32} = -4$, $R_{34} = -1$, $R_{41} = 0 = R_{42}$, $R_{43} = -1$, we note that $R_{ij} = R_{ji}$ for all i not equal to j.

$v_1 = 8$, $v_2 = 4$, $v_3 = -10$, and $v_4 = -4$

Hence the mesh-current equations are:

$$\begin{bmatrix} 7 & -2 & 0 & 0 \\ -2 & 6 & -4 & 0 \\ 0 & -4 & 5 & -1 \\ 0 & 0 & -1 & 5 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \\ -10 \\ -4 \end{bmatrix}$$

Chapter 3, Problem 73.

Write the mesh-current equations for the circuit in Fig. 3.117.

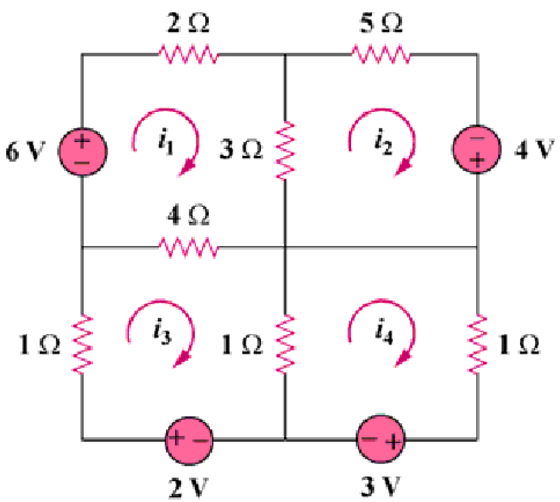


Figure 3.117

Chapter 3, Solution 73

$R_{11} = 2 + 3 + 4 = 9$, $R_{22} = 3 + 5 = 8$, $R_{33} = 1+1 + 4 = 6$, $R_{44} = 1 + 1 = 2$,
$R_{12} = -3$, $R_{13} = -4$, $R_{14} = 0$, $R_{23} = 0$, $R_{24} = 0$, $R_{34} = -1$

$v_1 = 6$, $v_2 = 4$, $v_3 = 2$, and $v_4 = -3$

Hence,

$$\begin{bmatrix} 9 & -3 & -4 & 0 \\ -3 & 8 & 0 & 0 \\ -4 & 0 & 6 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 2 \\ -3 \end{bmatrix}$$

Chapter 3, Problem 74.

By inspection, obtain the mesh-current equations for the circuit in Fig. 3.11.

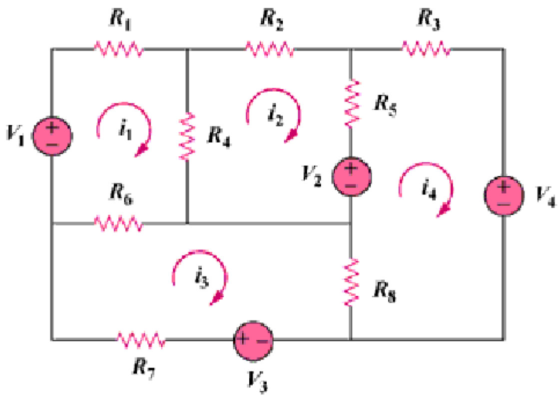


Figure 3.118

Chapter 3, Solution 74

$R_{11} = R_1 + R_4 + R_6$, $R_{22} = R_2 + R_4 + R_5$, $R_{33} = R_6 + R_7 + R_8$,
$R_{44} = R_3 + R_5 + R_8$, $R_{12} = -R_4$, $R_{13} = -R_6$, $R_{14} = 0$, $R_{23} = 0$,
$R_{24} = -R_5$, $R_{34} = -R_8$, again, we note that $R_{ij} = R_{ji}$ for all i not equal to j.

The input voltage vector is = $\begin{bmatrix} V_1 \\ -V_2 \\ V_3 \\ -V_4 \end{bmatrix}$

$$\begin{bmatrix} R_1 + R_4 + R_6 & -R_4 & -R_6 & 0 \\ -R_4 & R_2 + R_4 + R_5 & 0 & -R_5 \\ -R_6 & 0 & R_6 + R_7 + R_8 & -R_8 \\ 0 & -R_5 & -R_8 & R_3 + R_5 + R_8 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} V_1 \\ -V_2 \\ V_3 \\ -V_4 \end{bmatrix}$$

Chapter 3, Problem 75.

Use *PSpice* to solve Prob. 3.58.

Chapter 3, Problem 58

Find $\mathbf{i}_1$, i_2, and i_3 the circuit in Fig. 3.103.

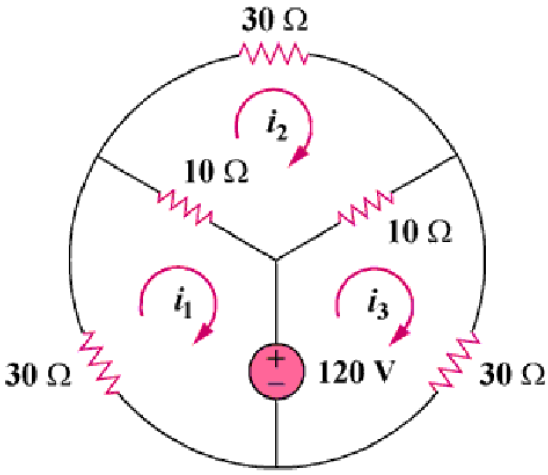
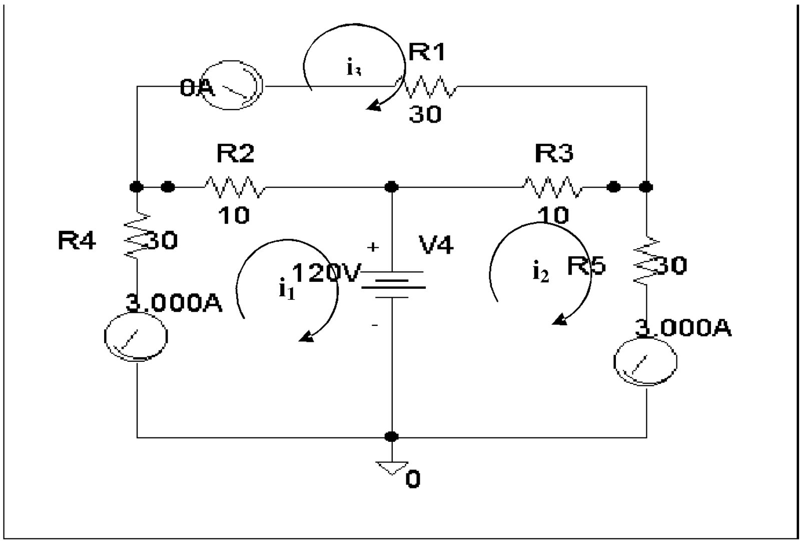


Figure 3.103

Chapter 3, Solution 75

```
* Schematics Netlist *

R_R4         $N_0002 $N_0001  30
R_R2         $N_0001 $N_0003  10
R_R1         $N_0005 $N_0004  30
R_R3         $N_0003 $N_0004  10
R_R5         $N_0006 $N_0004  30
V_V4         $N_0003 0 120V
v_V3         $N_0005 $N_0001 0
v_V2         0 $N_0006 0
v_V1         0 $N_0002 0
```



Clearly, i_1 = **<u>–3 amps</u>**, i_2 = **<u>0 amps</u>**, and i_3 = **<u>3 amps</u>**, which agrees with the answers in Problem 3.44.

Chapter 3, Problem 76.

Use *PSpice* to solve Prob. 3.27.

Chapter 3, Problem 27

Use nodal analysis to determine voltages v_1, v_2, and v_3 in the circuit in Fig. 3.76.

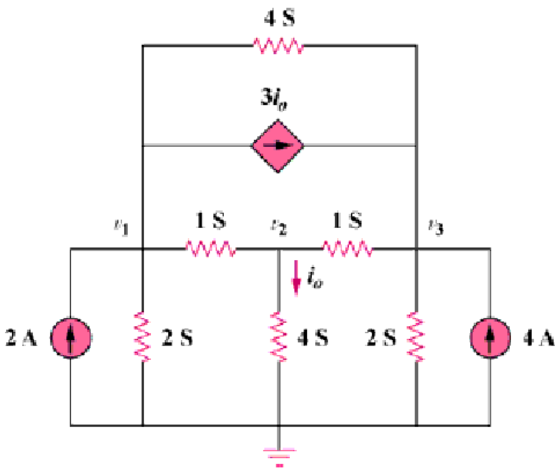
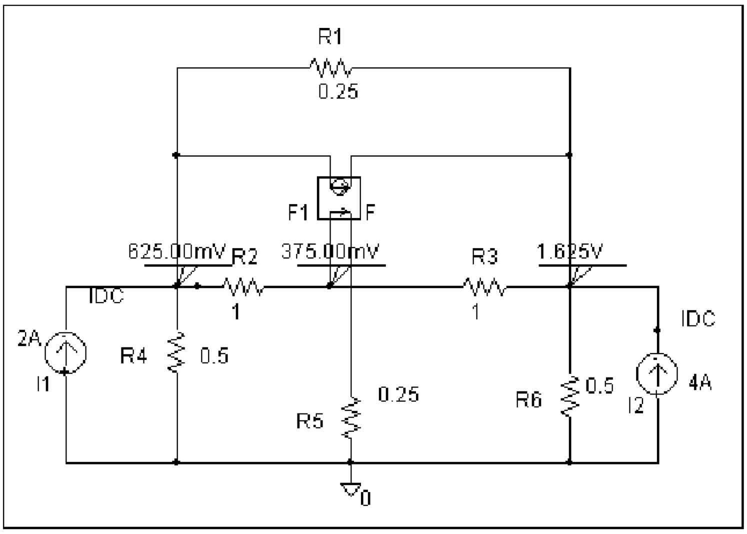


Figure 3.76

Chapter 3, Solution 76

```
* Schematics Netlist *

I_I2         0 $N_0001 DC 4A
R_R1         $N_0002 $N_0001  0.25
R_R3         $N_0003 $N_0001  1
R_R2         $N_0002 $N_0003  1
F_F1         $N_0002 $N_0001 VF_F1 3
VF_F1        $N_0003 $N_0004 0V
R_R4         0 $N_0002  0.5
R_R6         0 $N_0001  0.5
I_I1         0 $N_0002 DC 2A
R_R5         0 $N_0004  0.25
```



Clearly, v_1 = **625 mVolts**, v_2 = **375 mVolts**, and v_3 = **1.625 volts**, which agrees with the solution obtained in Problem 3.27.

Chapter 3, Problem 77.

Solve for V_1 and V_2 in the circuit of Fig. 3.119 using PSpice.

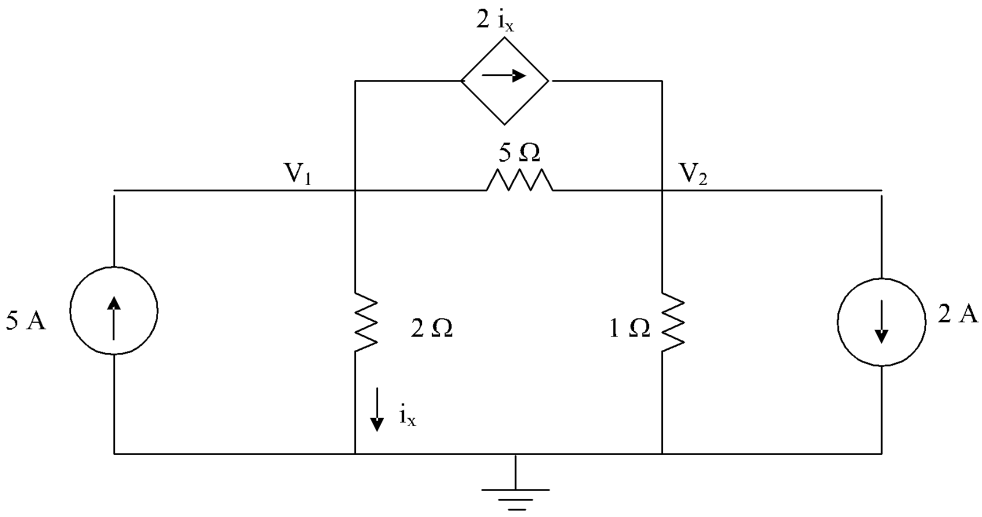
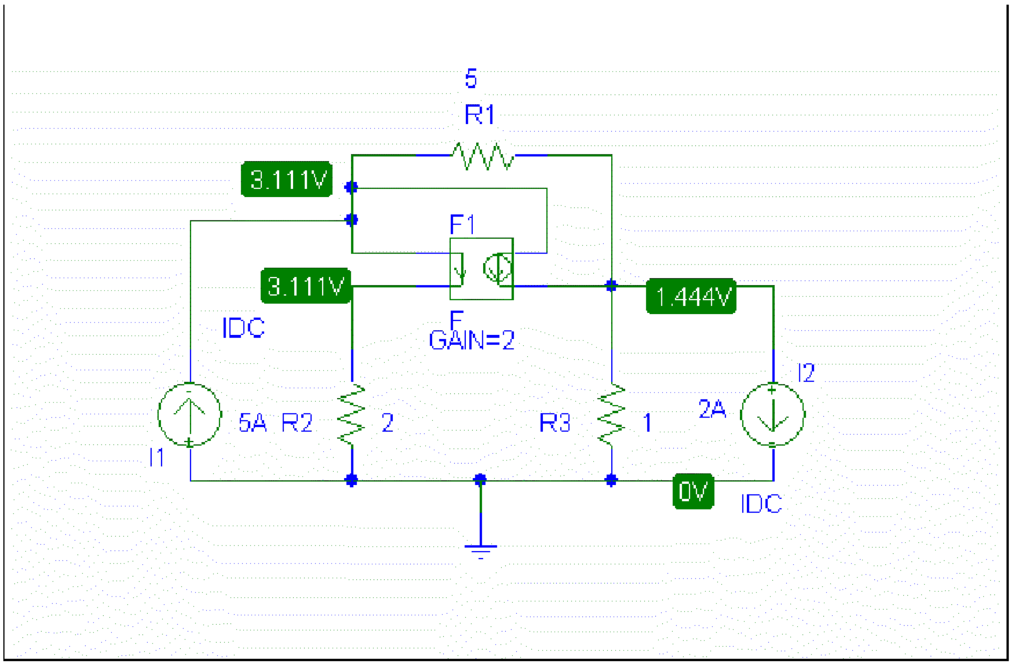


Figure 3.119 For Prob. 3.77.

Chapter 3, Solution 77



As a check we can write the nodal equations,

$$\begin{bmatrix} 1.7 & -0.2 \\ -1.2 & 1.2 \end{bmatrix} \mathbf{V} = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$$

Solving this leads to V_1 = **3.111 V** and V_2 = **1.4444 V**. The answer checks!

Chapter 3, Problem 78.

Solve Prob. 3.20 using *PSpice*.

Chapter 3, Problem 20

For the circuit in Fig. 3.69, find V_1, V_2, and V_3 using nodal analysis.

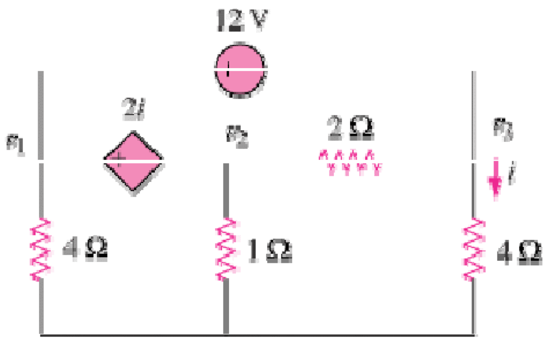
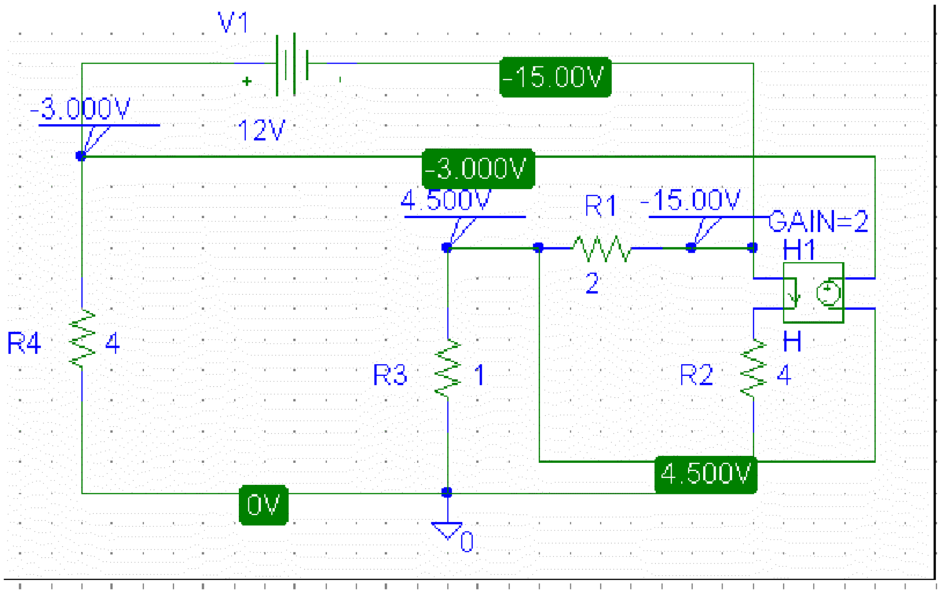


Figure 3.69

Chapter 3, Solution 78

The schematic is shown below. When the circuit is saved and simulated the node voltages are displaced on the pseudocomponents as shown. Thus,

$V_1 = -3\text{V}, \quad V_2 = 4.5\text{V}, V_3 = -15\text{V},$



Chapter 3, Problem 79.
Rework Prob. 3.28 using *PSpice*.

Chapter 3, Problem 28
Use MATLAB to find the voltages at nodes a, b, c, and d in the circuit of Fig. 3.77.

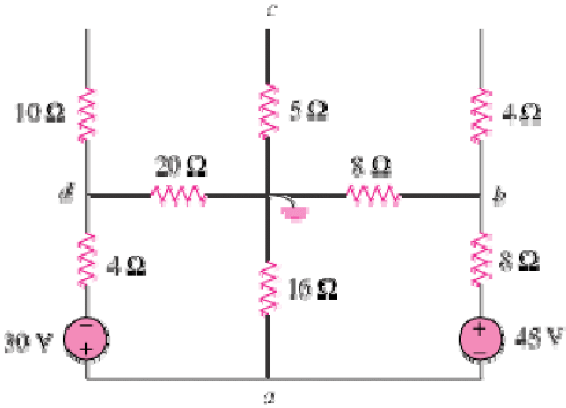
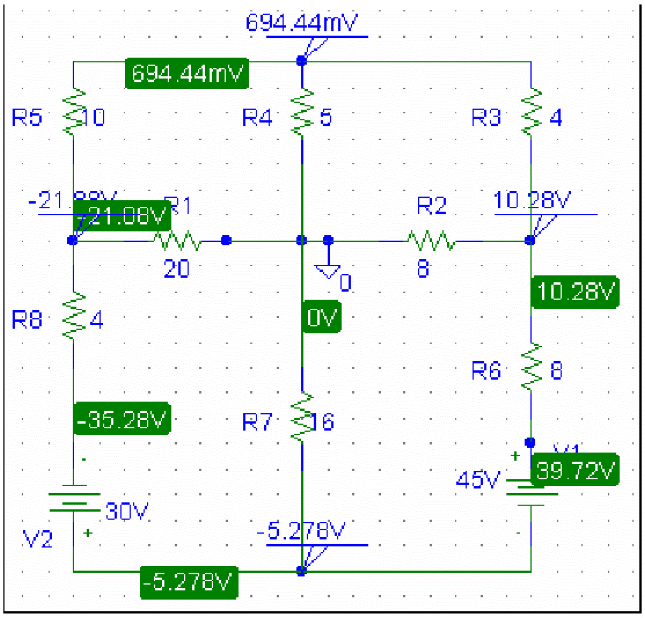


Figure 3.77

Chapter 3, Solution 79
The schematic is shown below. When the circuit is saved and simulated, we obtain the node voltages as displaced. Thus,

$\underline{V_a = -5.278\,\text{V}, \quad V_b = 10.28\,\text{V}, \quad V_c = 0.6944\,\text{V}, \quad V_d = -26.88\,\text{V}}$



Chapter 3, Problem 80.

Find the nodal voltage v_1 through v_4 in the circuit in Fig. 3.120 using *PSpice*.

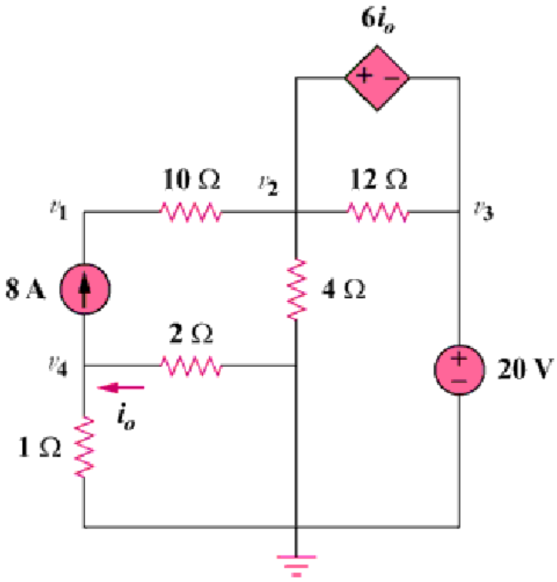
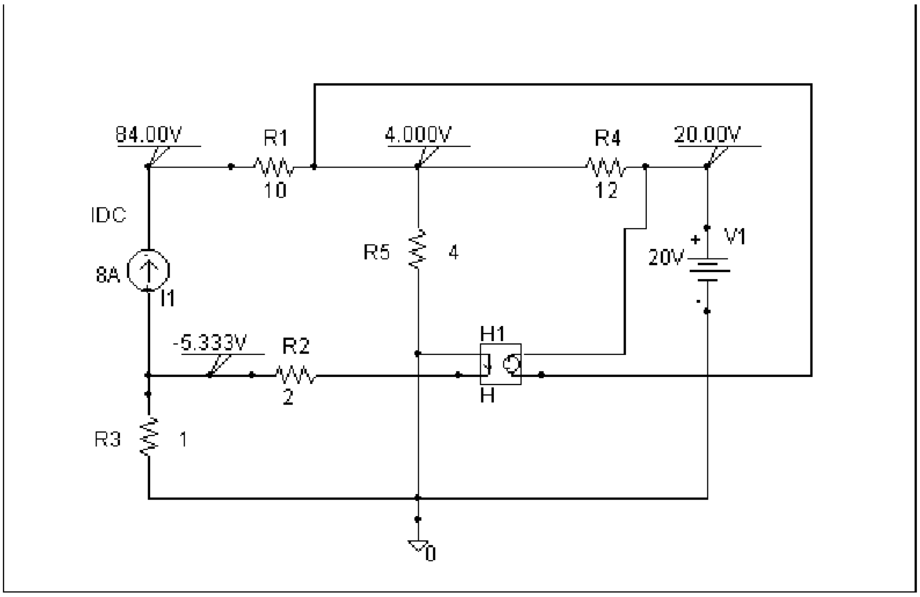


Figure 3.120

Chapter 3, Solution 80

```
* Schematics Netlist *

H_H1         $N_0002 $N_0003 VH_H1 6
VH_H1        0 $N_0001 0V
I_I1         $N_0004 $N_0005 DC 8A
V_V1         $N_0002 0 20V
R_R4         0 $N_0003  4
R_R1         $N_0005 $N_0003  10
R_R2         $N_0003 $N_0002  12
R_R5         0 $N_0004  1
R_R3         $N_0004 $N_0001  2
```



Clearly, v_1 = **84 volts**, v_2 = **4 volts**, v_3 = **20 volts**, and v_4 = **-5.333 volts**

Chapter 3, Problem 81.

Use *PSpice* to solve the problem in Example 3.4

Example 3.4

Find the node voltages in the circuit of Fig. 3.12.

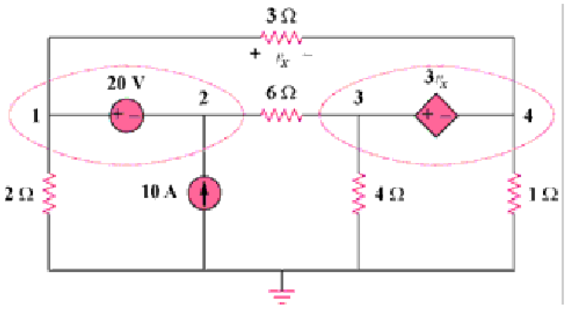
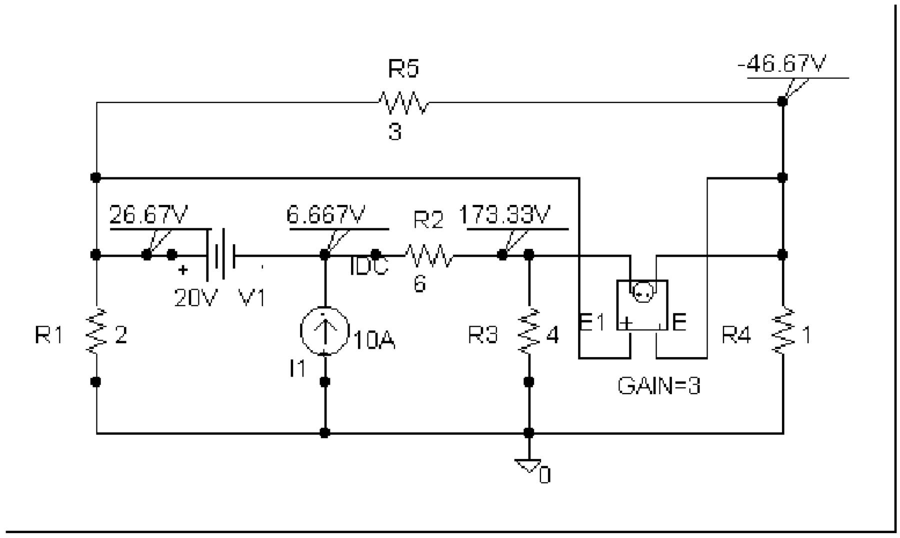


Figure 3.12

Chapter 3, Solution 81



Clearly, v_1 = **<u>26.67 volts</u>**, v_2 = **<u>6.667 volts</u>**, v_3 = **<u>173.33 volts</u>**, and v_4 = **<u>-46.67 volts</u>** which agrees with the results of Example 3.4.

This is the netlist for this circuit.

```
* Schematics Netlist *

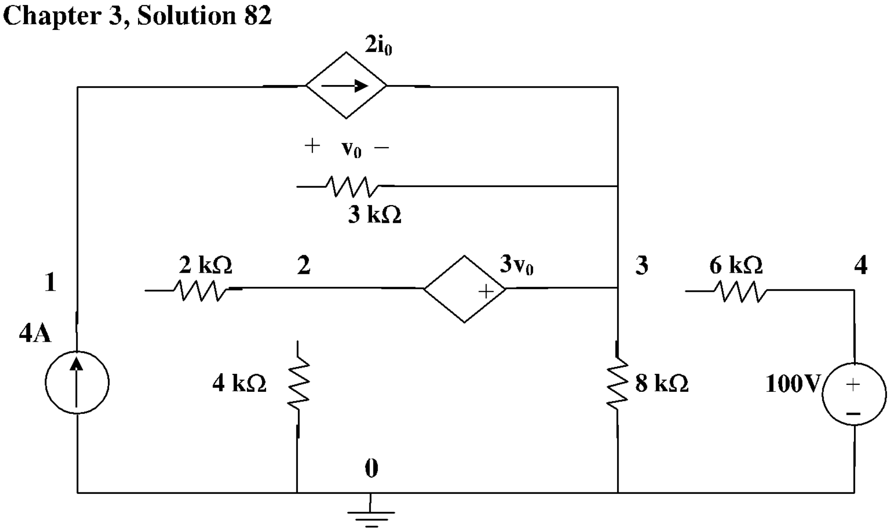
R_R1          0 $N_0001  2
R_R2          $N_0003 $N_0002  6
R_R3          0 $N_0002  4
R_R4          0 $N_0004  1
R_R5          $N_0001 $N_0004  3
I_I1          0 $N_0003 DC 10A
V_V1          $N_0001 $N_0003 20V
E_E1          $N_0002 $N_0004 $N_0001 $N_0004 3
```

Chapter 3, Problem 82.

If the Schematics Netlist for a network is as follows, draw the network.

R_R1	1	2	2K		
R_R2	2	0	4K		
R_R3	2	0	8K		
R_R4	3	4	6K		
R_R5	1	3	3K		
V_VS	4	0	DC	100	
I_IS	0	1	DC	4	
F_F1	1	3	VF_F1	2	
VF_F1	5	0	0V		
E_E1	3	2	1	3	3

Chapter 3, Solution 82



This network corresponds to the Netlist.

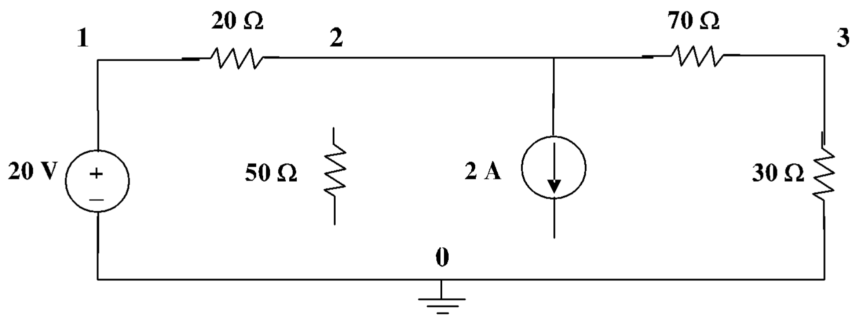
Chapter 3, Problem 83.

The following program is the Schematics Netlist of a particular circuit. Draw the circuit and determine the voltage at node 2.

```
R_R1   1  2  20
R_R2   2  0  50
R_R3   2  3  70
R_R4   3  0  30
V_VS   1  0  20V
I_IS   2  0  DC   2A
```

Chapter 3, Solution 83

The circuit is shown below.



When the circuit is saved and simulated, we obtain v_2 = **–12.5 volts**

Chapter 3, Problem 84.

Calculate v_o and i_o in the circuit of Fig. 3.121.

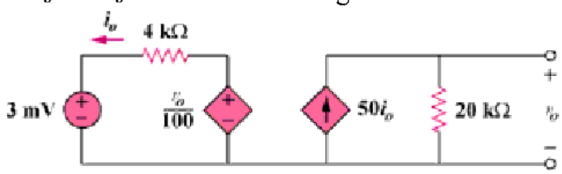


Figure 3.121

Chapter 3, Solution 84

From the output loop, $v_0 = 50i_0 x 20x10^3 = 10^6 i_0$ (1)

From the input loop, $3x10^{-3} + 4000i_0 - v_0/100 = 0$ (2)

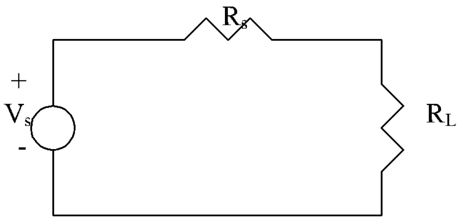
From (1) and (2) we get, i_0 = **0.5μA** and v_0 = **0.5 volt**.

Chapter 3, Problem 85.

An audio amplifier with resistance 9Ω supplies power to a speaker. In order that maximum power is delivered, what should be the resistance of the speaker?

Chapter 3, Solution 85

The amplifier acts as a source.



For maximum power transfer,

$$R_L = R_s = \underline{9\Omega}$$

Chapter 3, Problem 86.

For the simplified transistor circuit of Fig. 3.122, calculate the voltage $\boldsymbol{v_o}$.

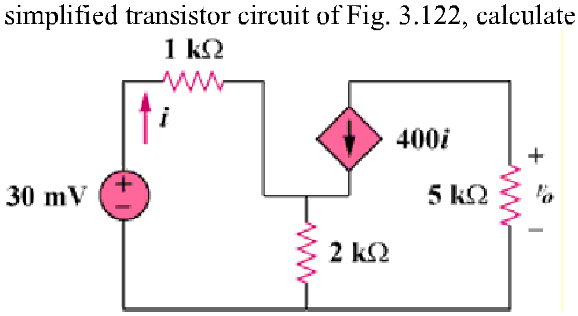


Figure 3.122

Chapter 3, Solution 86

Let v_1 be the potential across the 2 k-ohm resistor with plus being on top. Then,

$[(0.03 - v_1)/1k] + 400i = v_1/2k$ (1)

Assume that i is in mA. But, $i = (0.03 - v_1)/1$ (2)

Combining (1) and (2) yields,

v_1 = 29.963 mVolts and i = 37.4 nA, therefore,

$v_0 = -5000x400x37.4x10^{-9}$ = **<u>-74.8 mvolts</u>**

Chapter 3, Problem 87.

For the circuit in Fig. 3.123, find the gain $\boldsymbol{v_o/v_s}$.

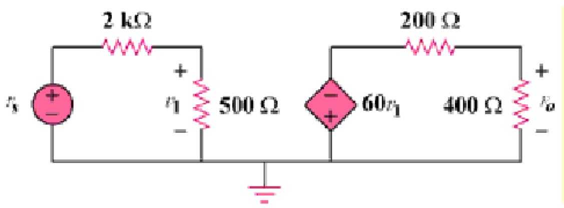


Figure 3.123

Chapter 3, Solution 87

$v_1 = 500(v_s)/(500 + 2000) = v_s/5$

$v_0 = -400(60v_1)/(400 + 2000) = -40v_1 = -40(v_s/5) = -8v_s,$

Therefore, $v_0/v_s = \underline{\mathbf{-8}}$

Chapter 3, Problem 88.

Determine the gain v_o/v_s of the transistor amplifier circuit in Fig. 3.124.

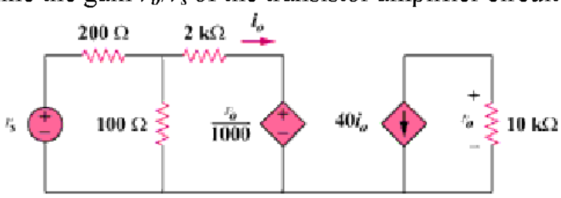


Figure 3.124

Chapter 3, Solution 88

Let v_1 be the potential at the top end of the 100-ohm resistor.

$$(v_s - v_1)/200 = v_1/100 + (v_1 - 10^{-3}v_0)/2000 \qquad (1)$$

For the right loop, $v_0 = -40i_0(10,000) = -40(v_1 - 10^{-3})10,000/2000,$

$$\text{or,}\ \ v_0 = -200v_1 + 0.2v_0 = -4x10^{-3}v_0 \qquad (2)$$

Substituting (2) into (1) gives, $(v_s + 0.004v_1)/2 = -0.004v_0 + (-0.004v_1 - 0.001v_0)/20$

This leads to $0.125v_0 = 10v_s$ or $(v_0/v_s) = 10/0.125 = \underline{\mathbf{-80}}$

Chapter 3, Problem 89.

For the transistor circuit shown in Fig. 3.125, find I_B and V_{CE}. Let $\beta = 100$ and V_{BE} = 0.7V.

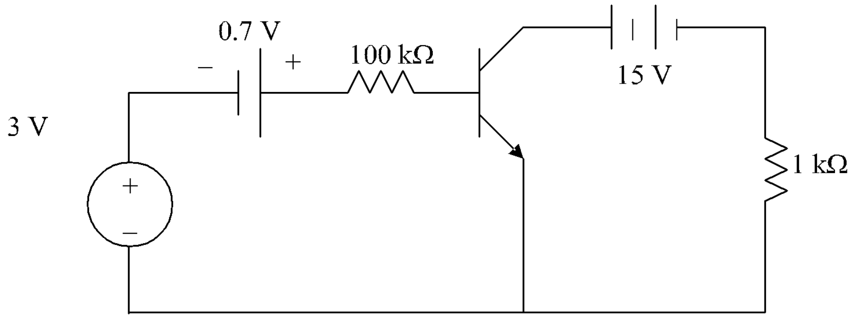
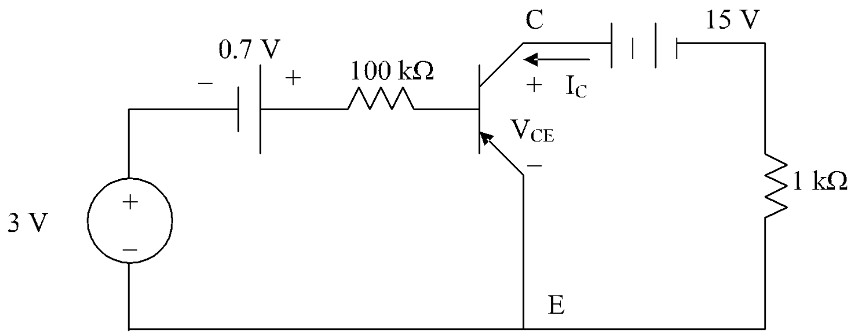


Figure 3.125 For Prob. 3.89.

Chapter 3, Solution 89

Consider the circuit below.



For the left loop, applying KVL gives

$$-3 - 0.7 + 100x10^3 I_B + V_{BE} = 0 \xrightarrow{V_{BE}=0.7} \underline{I_B = 30\ \mu A}$$

For the right loop,

$$-V_{CE} + 15 - I_c(1x10^3) = 0$$

But $I_C = \beta I_B = 100x30\ \mu A = 3\ mA$

$$V_{CE} = 15 - 3x10^{-3}x10^3 = \underline{12\ V}$$

Chapter 3, Problem 90.

Calculate v_s for the transistor in Fig. 3.126, given that $v_o = 4$ V, $\beta = 150$, $V_{BE} = 0.7$V.

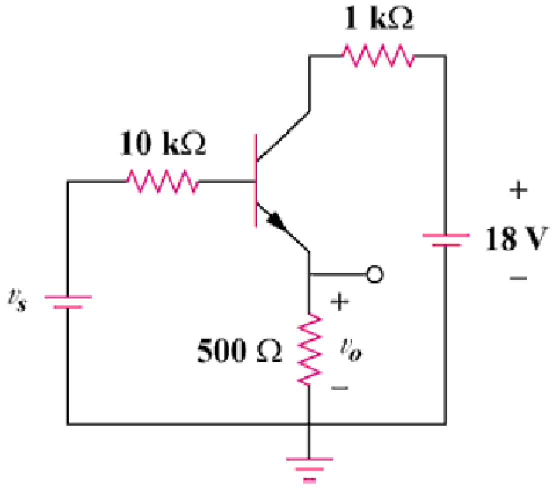
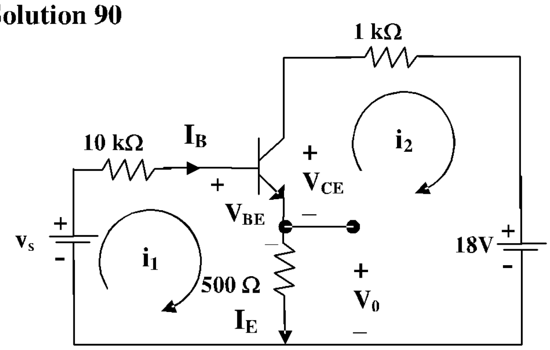


Figure 3.126

Chapter 3, Solution 90



For loop 1, $-v_s + 10k(I_B) + V_{BE} + I_E\,(500) = 0 = -v_s + 0.7 + 10{,}000I_B + 500(1 + \beta)I_B$

which leads to $v_s + 0.7 = 10{,}000I_B + 500(151)I_B = 85{,}500I_B$

But, $v_0 = 500I_E = 500x151I_B = 4$ which leads to $I_B = 5.298x10^{-5}$

Therefore, $v_s = 0.7 + 85{,}500I_B =$ **5.23 volts**

Chapter 3, Problem 91.

For the transistor circuit of Fig. 3.127, find I_B, V_{CE}, and v_o. Take $\beta = 200$, $V_{BE} = 0.7$V.

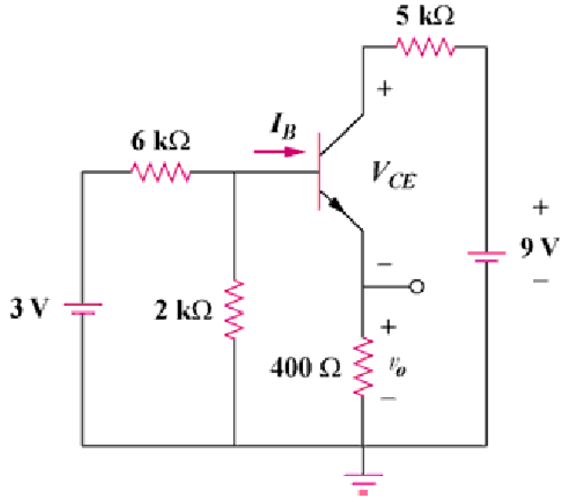
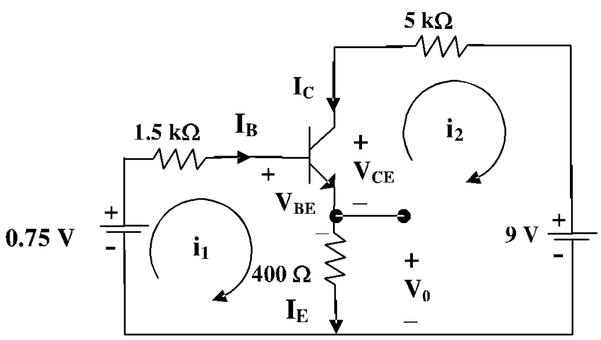


Figure 3.127

Chapter 3, Solution 91

We first determine the Thevenin equivalent for the input circuit.

$R_{Th} = 6||2 = 6x2/8 = 1.5$ kΩ and $V_{Th} = 2(3)/(2+6) = 0.75$ volts



For loop 1, $-0.75 + 1.5kI_B + V_{BE} + 400I_E = 0 = -0.75 + 0.7 + 1500I_B + 400(1 + \beta)I_B$

$I_B = 0.05/81{,}900 = \underline{\mathbf{0.61\ \mu A}}$

$v_0 = 400I_E = 400(1 + \beta)I_B = \underline{\mathbf{49\ mV}}$

For loop 2, $-400I_E - V_{CE} - 5kI_C + 9 = 0$, but, $I_C = \beta I_B$ and $I_E = (1 + \beta)I_B$

$V_{CE} = 9 - 5k\beta I_B - 400(1 + \beta)I_B = 9 - 0.659 = \underline{\mathbf{8.641\ volts}}$

Chapter 3, Problem 92.

Find I_B and V_C for the circuit in Fig. 3.128. Let $\beta = 100$, $V_{BE} = 0.7V$.

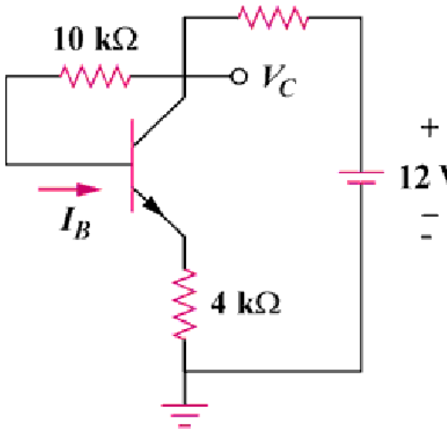
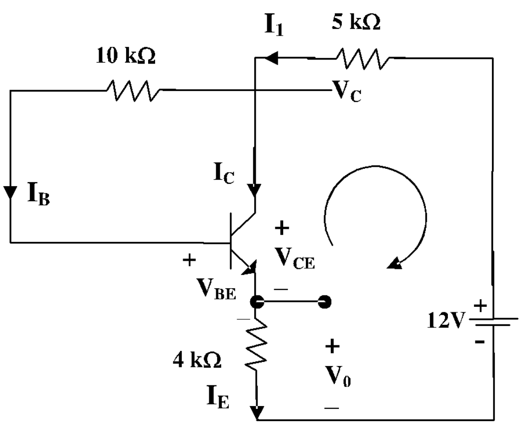


Figure 3.128

Chapter 3, Solution 92



$I_1 = I_B + I_C = (1 + \beta)I_B$ and $I_E = I_B + I_C = I_1$

Applying KVL around the outer loop,

$4kI_E + V_{BE} + 10kI_B + 5kI_1 = 12$

$12 - 0.7 = 5k(1 + \beta)I_B + 10kI_B + 4k(1 + \beta)I_B = 919kI_B$

$I_B = 11.3/919k = 12.296\ \mu A$

Also, $12 = 5kI_1 + V_C$ which leads to $V_C = 12 - 5k(101)I_B =$ **<u>5.791 volts</u>**

Chapter 3, Problem 93

Rework Example 3.11 with hand calculation.

In the circuit in Fig. 3.34, determine the currents i_1, i_2, and i_3.

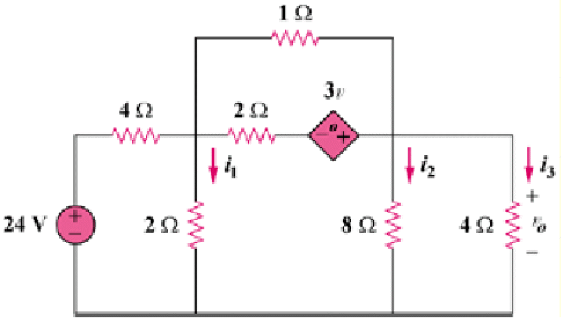
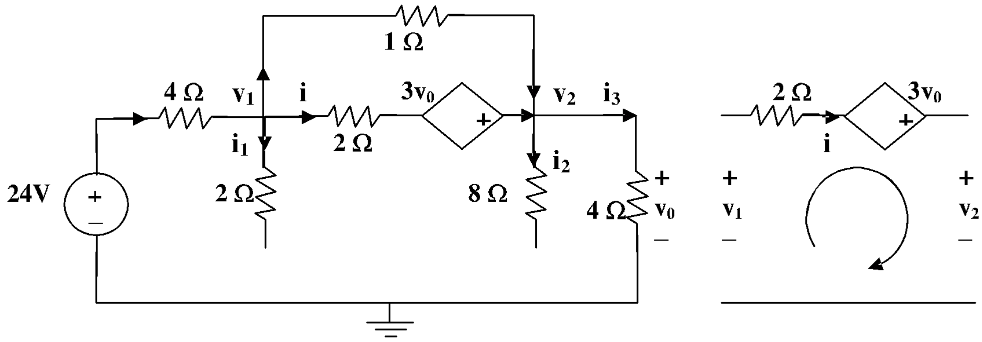


Figure 3.34

Chapter 3, Solution 93



From (b), $-v_1 + 2i - 3v_0 + v_2 = 0$ which leads to $i = (v_1 + 3v_0 - v_2)/2$

At node 1 in (a), $((24 - v_1)/4) = (v_1/2) + ((v_1 + 3v_0 - v_2)/2) + ((v_1 - v_2)/1)$, where $v_0 = v_2$

or $24 = 9v_1$ which leads to $v_1 =$ **2.667 volts**

At node 2, $((v_1 - v_2)/1) + ((v_1 + 3v_0 - v_2)/2) = (v_2/8) + v_2/4$, $v_0 = v_2$

$v_2 = 4v_1 =$ **10.66 volts**

Now we can solve for the currents, $i_1 = v_1/2 =$ **1.333 A**, $i_2 =$ **1.333 A**, and

$i_3 =$ **2.6667** A.

Chapter 4, Problem 1.

Calculate the current i_o in the circuit of Fig. 4.69. What does this current become when the input voltage is raised to 10 V?

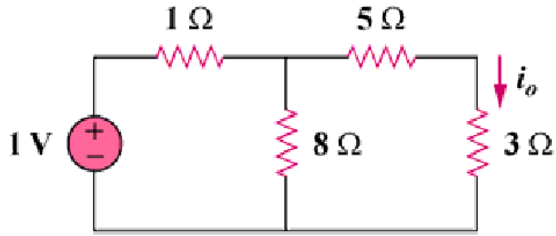
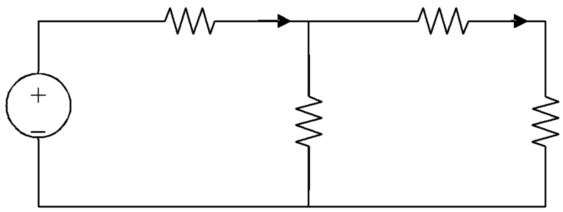


Figure 4.69

Chapter 4, Solution 1.



$$8\|(5+3) = 4\Omega, \quad i = \frac{1}{1+4} = \frac{1}{5}$$

$$i_o = \frac{1}{2}i = \frac{1}{10} = \underline{\mathbf{0.1A}}$$

Since the resistance remains the same we get i = 10/5 = 2A which leads to i_o = (1/2)i = (1/2)2 = **<u>1A</u>**.

Chapter 4, Problem 2.

Find v_o in the circuit of Fig. 4.70. If the source current is reduced to 1 μA, what is v_o?

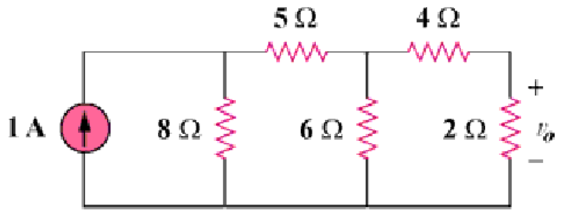
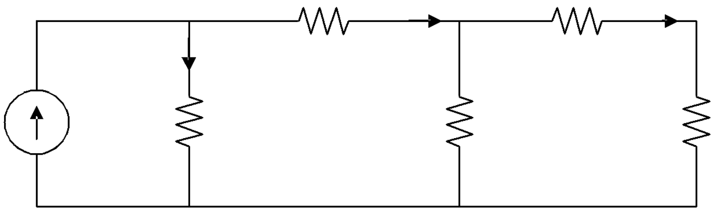


Figure 4.70

Chapter 4, Solution 2.

$$6 \| (4+2) = 3\Omega, \quad i_1 = i_2 = \frac{1}{2}A$$

$$i_o = \frac{1}{2} i_1 = \frac{1}{4}, \quad v_o = 2i_o = \underline{\mathbf{0.5V}}$$



If $i_s = 1\mu A$, then $v_o = \underline{\mathbf{0.5\mu V}}$

Chapter 4, Problem 3.

(a) In the circuit in Fig. 4.71, calculate v_o and I_o when $v_s = 1$ V.

(b) Find v_o and i_o when $v_s = 10$ V.

(c) What are v_o and I_o when each of the 1-Ω resistors is replaced by a 10-Ω resistor and $v_s = 10$ V?

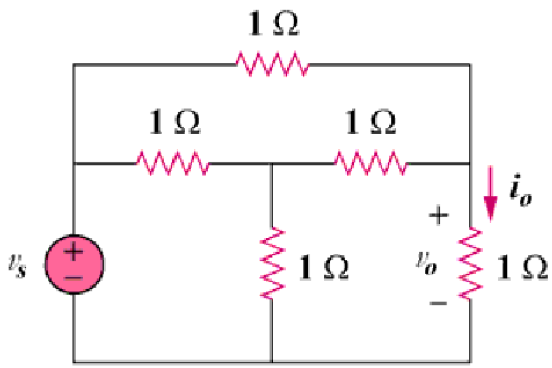
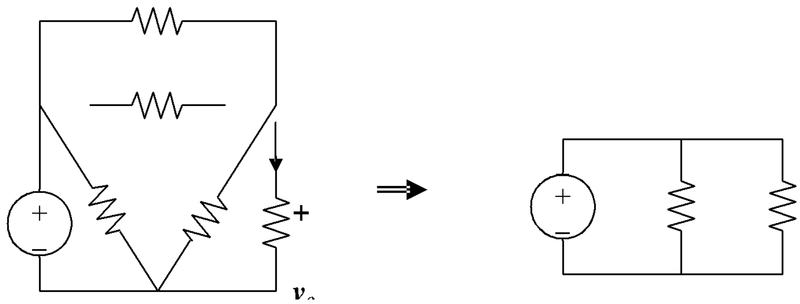


Figure 4.71

Chapter 4, Solution 3.



(a) We transform the Y sub-circuit to the equivalent Δ.

$$R \| 3R = \frac{3R^2}{4R} = \frac{3}{4}R, \quad \frac{3}{4}R + \frac{3}{4}R = \frac{3}{2}R$$

$$v_o = \frac{v_s}{2} \text{ independent of R}$$

$i_o = v_o/(R)$

When $v_s = 1$V, $v_o =$ __**0.5V**__, $i_o =$ __**0.5A**__

(b) When $v_s = 10$V, $v_o =$ __**5V**__, $i_o =$ __**5A**__

(c) When $v_s = 10$V and R = 10Ω,
$v_o =$ __**5V**__, $i_o =$ 10/(10) = __**500mA**__

Chapter 4, Problem 4.

Use linearity to determine i_o in the circuit in Fig. 4.72.

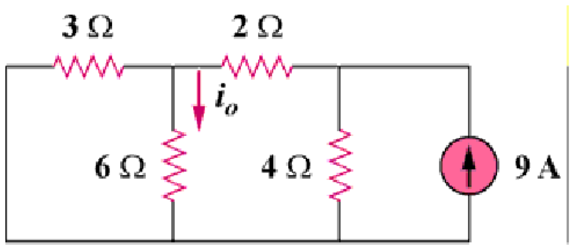
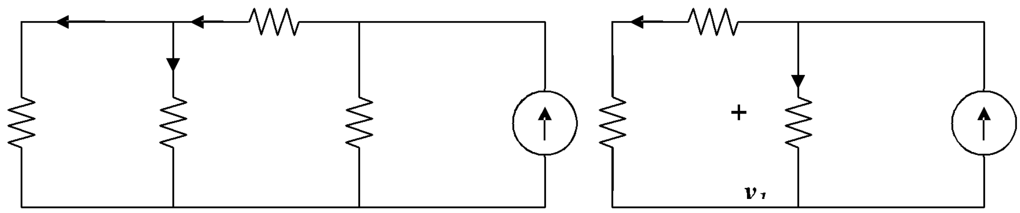


Figure 4.72

Chapter 4, Solution 4.

If $I_o = 1$, the voltage across the 6Ω resistor is 6V so that the current through the 3Ω resistor is 2A.



$$3\|6 = 2\Omega,\ v_o = 3(4) = 12V,\ i_1 = \frac{v_o}{4} = 3A.$$

Hence $I_s = 3 + 3 = 6A$

If $\quad I_s = 6A \longrightarrow I_o = 1$

$\quad\quad I_s = 9A \longrightarrow I_o = 9/6 =$ **<u>1.5A</u>**

Chapter 4, Problem 5.

For the circuit in Fig. 4.73, assume v_o = 1 V, and use linearity to find the actual value of v_o.

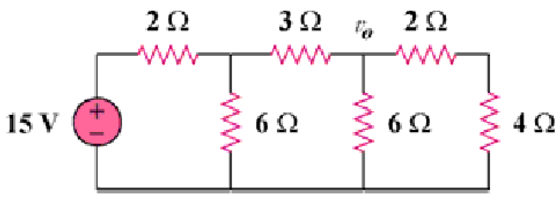
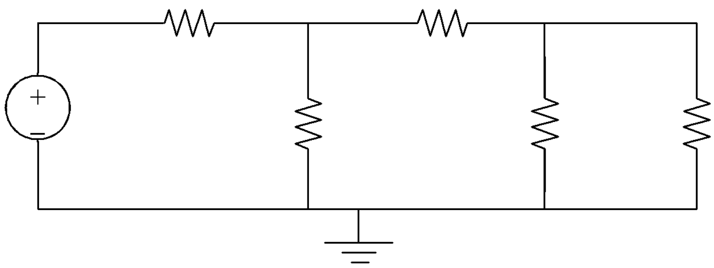


Figure 4.73

Chapter 4, Solution 5.



If $v_o = 1V$, $\quad V_1 = \left(\frac{1}{3}\right) + 1 = 2V$

$$V_s = 2\left(\frac{2}{3}\right) + v_1 = \frac{10}{3}$$

If $v_s = \frac{10}{3}$ ⟶ $v_o = 1$

Then $v_s = 15$ ⟶ $v_o = \frac{3}{10} x15 = \underline{\mathbf{4.5V}}$

Chapter 4, Problem 6.

For the linear circuit shown in Fig. 4.74, use linearity to complete the following table.

Experiment	V_s	V_o
1	12 V	4 V
2	--	16 V
3	1 V	--
4	--	-2V

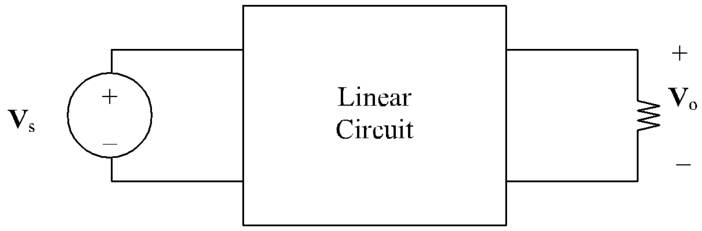


Figure 4.74 For Prob. 4.6.

Chapter 4, Solution 6.

Due to linearity, from the first experiment,

$$V_o = \frac{1}{3}V_s$$

Applying this to other experiments, we obtain:

Experiment	V_s	V_o
2	48	16 V
3	1 V	0.333 V
4	-6 V	-2V

Chapter 4, Problem 7.

Use linearity and the assumption that V_o = 1V to find the actual value of V_o in Fig. 4.75.

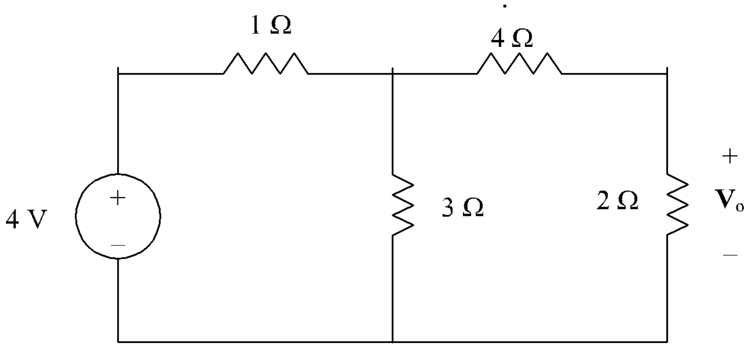


Figure 4.75 For Prob. 4.7.

Chapter 4, Solution 7.

If V_o = 1V, then the current through the 2-Ω and 4-Ω resistors is ½ = 0.5. The voltage across the 3-Ω resistor is ½ (4 + 2) = 3 V. The total current through the 1-Ω resistor is 0.5 +3/3 = 1.5 A. Hence the source voltage

$$v_s = 1x1.5 + 3 = 4.5 \text{ V}$$

If $v_s = 4.5 \longrightarrow 1V$

Then $v_s = 4 \longrightarrow \frac{1}{4.5}x4 = \underline{0.8889 \text{ V}}$ = **888.9 mV**.

Chapter 4, Problem 8.

Using superposition, find V_o in the circuit of Fig. 4.76.

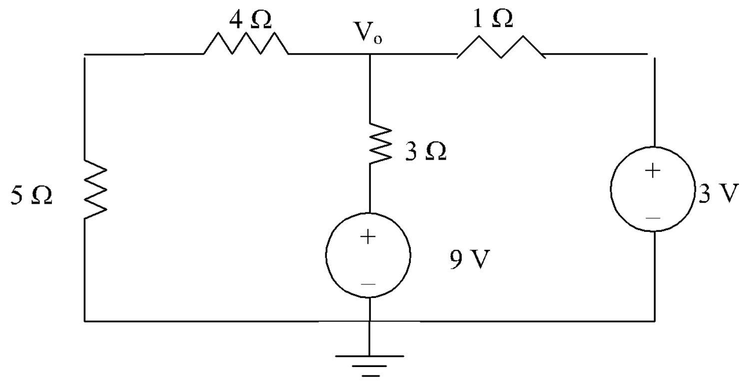
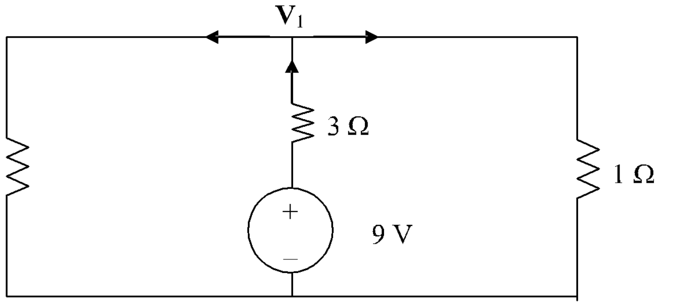


Figure 4.76 For Prob. 4.8.

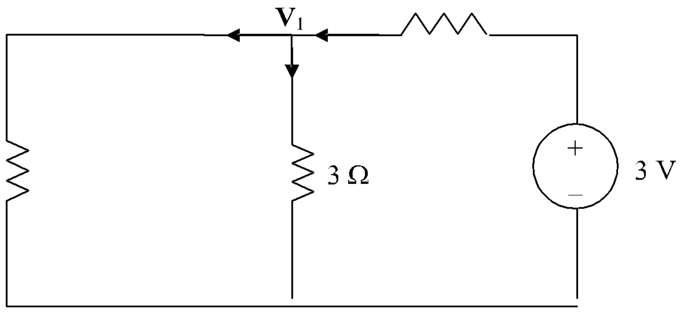
Chapter 4, Solution 8.

Let $V_o = V_1 + V_2$, where V_1 and V_2 are due to 9-V and 3-V sources respectively. To find V_1, consider the circuit below.



$$\frac{9-V_1}{3} = \frac{V_1}{9} + \frac{V_1}{1} \quad \longrightarrow \quad V_1 = 27/13 = 2.0769$$

To find V_2, consider the circuit below.



$$\frac{V_2}{9} + \frac{V_2}{3} = \frac{3-V_2}{1} \quad \longrightarrow \quad V_2 = 27/13 = 2.0769$$

$V_o = V_1 + V_2 =$ **<u>4.1538 V</u>**

Chapter 4, Problem 9.

Use superposition to find v_o in the circuit of Fig. 4.77.

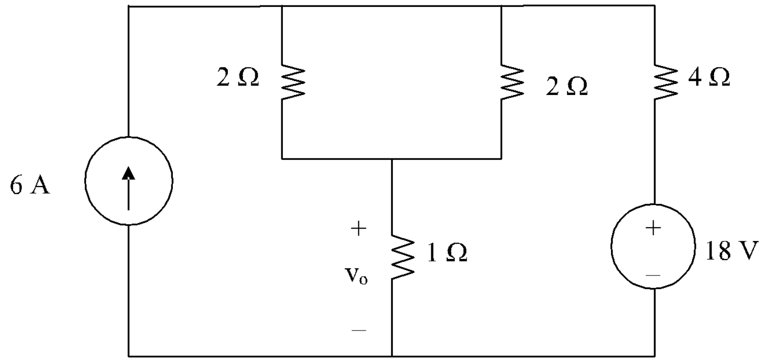
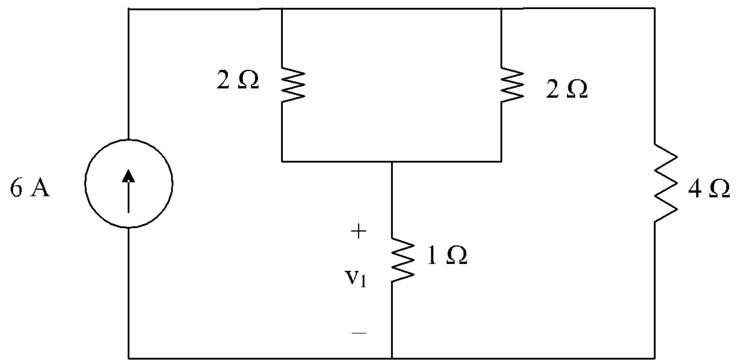


Figure 4.77 For Prob. 4.9.

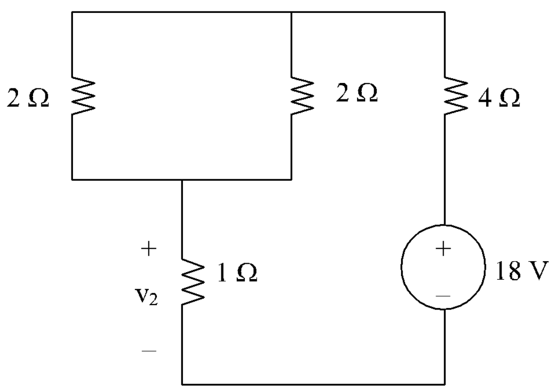
Chapter 4, Solution 9.

Let $v_o = v_1 + v_2$, where v_1 and v_2 are due to 6-A and 20-V sources respectively. We find v_1 using the circuit below.



2//2 = 1 Ω, $v_1 = 1x\frac{4}{4+2}(6A) = 4\ \text{V}$

We find v_2 using the circuit below.



$$v_2 = \frac{1}{1+1+4}(18) = 3\ \text{V}$$

$v_o = v_1 + v_2 = 4 + 3 =$ **7 V**

Chapter 4, Problem 10.

For the circuit in Fig. 4.78, find the terminal voltage V_{ab} using superposition.

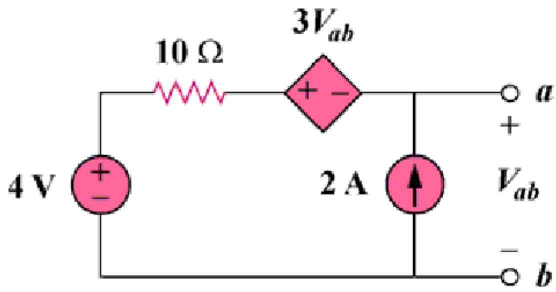
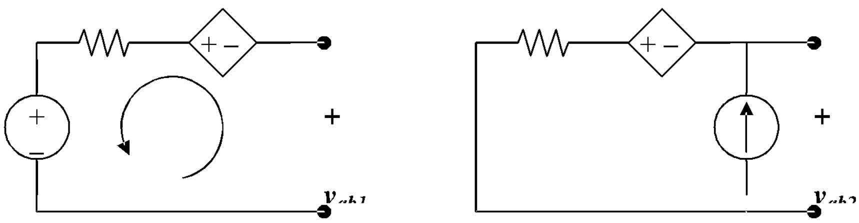


Figure 4.78

Chapter 4, Solution 10.

Let $v_{ab} = v_{ab1} + v_{ab2}$ where v_{ab1} and v_{ab2} are due to the 4-V and the 2-A sources respectively.



For v_{ab1}, consider Fig. (a). Applying KVL gives,

$$- v_{ab1} - 3\,v_{ab1} + 10x0 + 4 = 0, \text{ which leads to } v_{ab1} = 1 \text{ V}$$

For v_{ab2}, consider Fig. (b). Applying KVL gives,

$$- \quad v_{ab2} - 3v_{ab2} + 10x2 = 0, \text{ which leads to } v_{ab2} = 5$$

$$v_{ab} = 1 + 5 = \underline{\mathbf{6\ V}}$$

Chapter 4, Problem 11.

Use the superposition principle to find i_o and v_o in the circuit of Fig. 4.79.

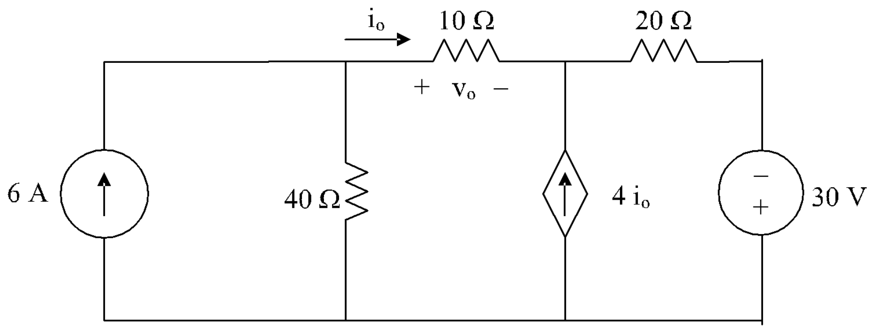
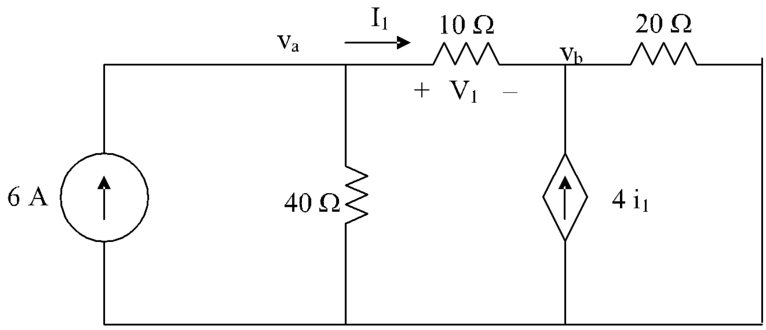


Figure 4.79 For Prob. 4.11.

Chapter 4, Solution 11.

Let $v_o = v_1 + v_2$, where v_1 and v_2 are due to the 6-A and 80-V sources respectively. To find v_1, consider the circuit below.



At node a,

$$6 = \frac{v_a}{40} + \frac{v_a - v_b}{10} \longrightarrow 240 = 5v_a - 4v_b \qquad (1)$$

At node b,

$$-I_1 - 4I_1 + (v_b - 0)/20 = 0 \text{ or } v_b = 100I_1$$

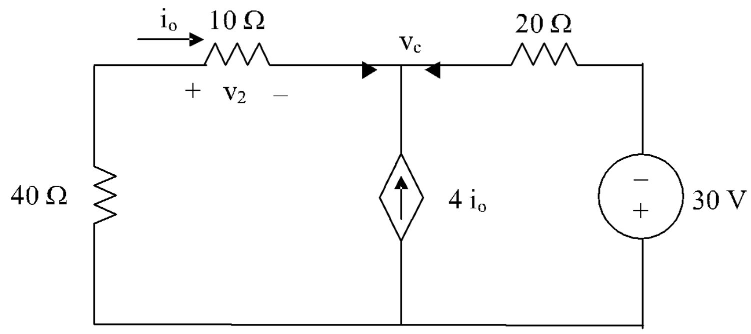
But $i_1 = \frac{v_a - v_b}{10}$ which leads to $100(v_a - v_b)10 = v_b$ or $v_b = 0.9091v_a$ (2)

Substituting (2) into (1),

$$5v_a - 3.636v_a = 240 \text{ or } v_a = 175.95 \text{ and } v_b = 159.96$$

However, $v_1 = v_a - v_b = 15.99$ V.

To find v_2, consider the circuit below.



$$\frac{0 - v_c}{50} + 4i_o + \frac{(-30 - v_c)}{20} = 0$$

But $i_o = \frac{(0 - v_c)}{50}$

$$-\frac{5v_c}{50} - \frac{(30 + v_c)}{20} = 0 \quad \longrightarrow \quad v_c = -10 \text{ V}$$

$$i_2 = \frac{0 - v_c}{50} = \frac{0 + 10}{50} = \frac{1}{5}$$

$$v_2 = 10i_2 = 2 \text{ V}$$

$v_o = v_1 + v_2 = 15.99 + 2 =$ **17.99 V** and $i_o = v_o/10 =$ **1.799 A**.

Chapter 4, Problem 12.

Determine v_o in the circuit in Fig. 4.80 using the superposition principle.

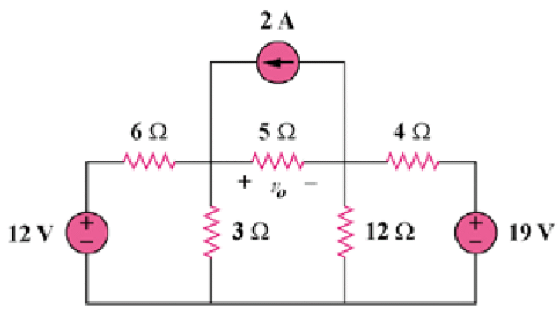
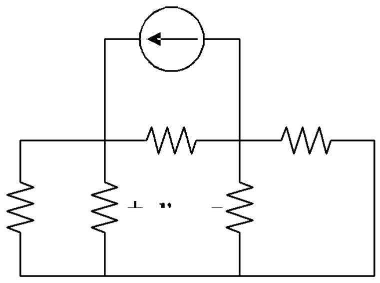
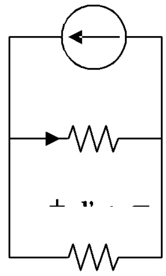


Figure 4.80

Chapter 4, Solution 12.

Let $v_o = v_{o1} + v_{o2} + v_{o3}$, where v_{o1}, v_{o2}, and v_{o3} are due to the 2-A, 12-V, and 19-V sources respectively. For v_{o1}, consider the circuit below.

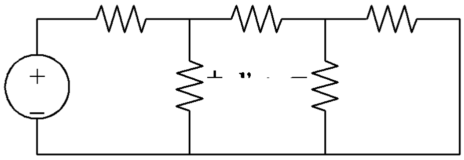


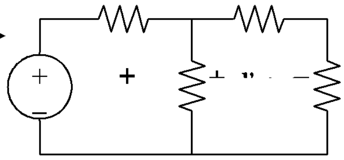


$6||3 = 2$ ohms, $4||12 = 3$ ohms. Hence,

$$i_o = 2/2 = 1, v_{o1} = 5io = 5 \text{ V}$$

For v_{o2}, consider the circuit below.

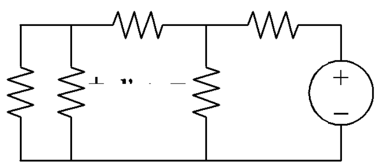


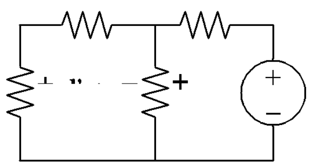


$$3||8 = 24/11, \ v_1 = [(24/11)/(6 + 24/11)]12 = 16/5$$

$$v_{o2} = (5/8)v_1 = (5/8)(16/5) = \underline{\mathbf{2\ V}}$$

For v_{o3}, consider the circuit shown below.





$$7||12 = (84/19) \text{ ohms}, \ v_2 = [(84/19)/(4 + 84/19)]19 = 9.975$$

$$v = (-5/7)v2 = -7.125$$

$v_o = 5 + 2 - 7.125 = \underline{\mathbf{-125\ mV}}$

Chapter 4, Problem 13.

Use superposition to find v_o in the circuit of Fig. 4.81.

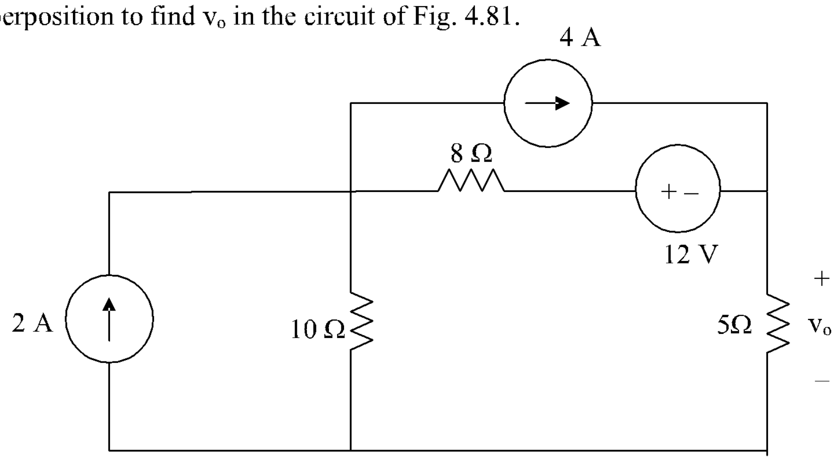
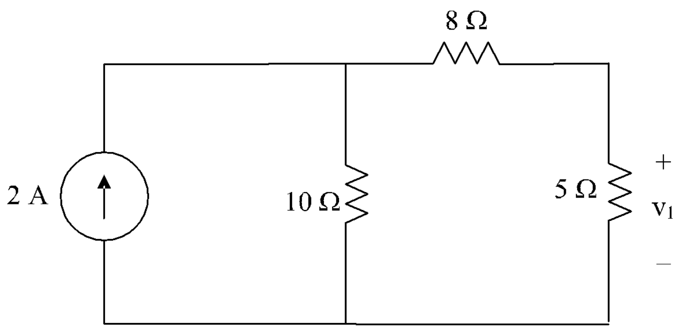


Figure 4.81 For Prob. 4.13.

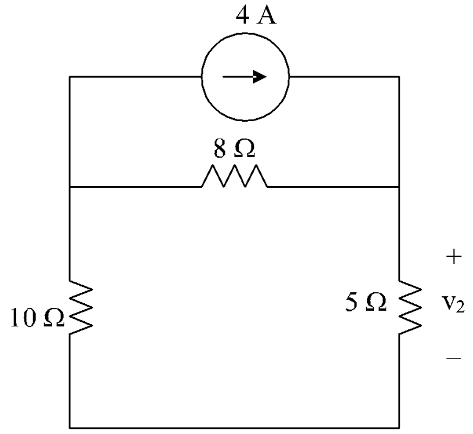
Chapter 4, Solution 13.

Let $v_o = v_1 + v_2 + v_3$, where v_1, v_2, and v_3 are due to the independent sources. To find v_1, consider the circuit below.



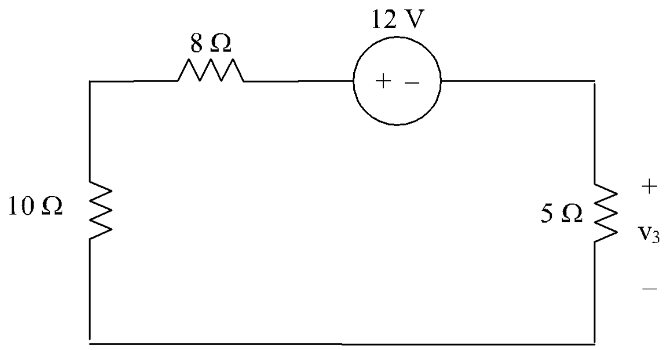
$$v_1 = 5x\frac{10}{10+8+5}x2 = 4.3478$$

To find v_2, consider the circuit below.



$$v_2 = 5x\frac{8}{8+10+5}x4 = 6.9565$$

To find v_3, consider the circuit below.



$$v_3 = -12\left(\frac{5}{5+10+8}\right) = -2.6087$$

$$v_o = v_1 + v_2 + v_3 = \underline{8.6956\ V} = \underline{\mathbf{8.696V}}.$$

Chapter 4, Problem 14.

Apply the superposition principle to find v_o in the circuit of Fig. 4.82.

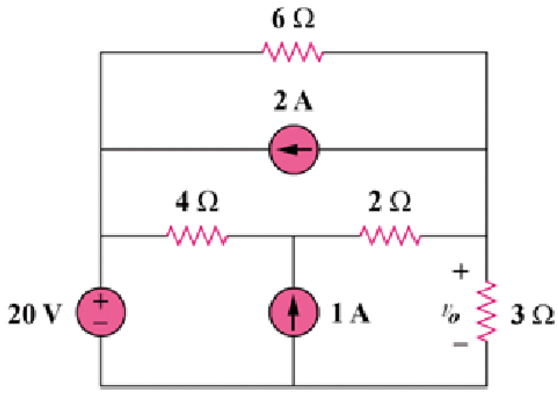
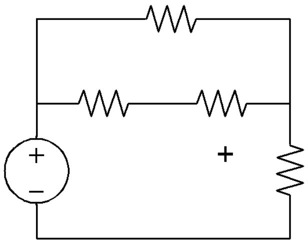


Figure 4.82

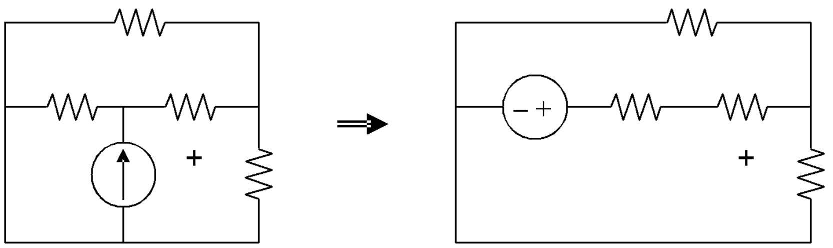
Chapter 4, Solution 14.

Let $v_o = v_{o1} + v_{o2} + v_{o3}$, where v_{o1}, v_{o2} , and v_{o3}, are due to the 20-V, 1-A, and 2-A sources respectively. For v_{o1}, consider the circuit below.



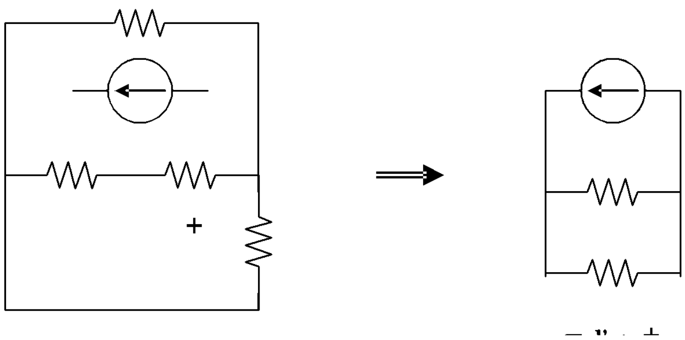
$$6||(4 + 2) = 3 \text{ ohms}, v_{o1} = (½)20 = 10 \text{ V}$$

For v_{o2}, consider the circuit below.



$$3||6 = 2 \text{ ohms}, v_{o2} = [2/(4 + 2 + 2)]4 = 1 \text{ V}$$

For v_{o3}, consider the circuit below.



$$6||(4 + 2) = 3, v_{o3} = (-1)3 = -3$$

$$v_o = 10 + 1 - 3 = \mathbf{8 \text{ V}}$$

Chapter 4, Problem 15.

For the circuit in Fig. 4.83, use superposition to find $\boldsymbol{i}$. Calculate the power delivered to the 3-Ω resistor.

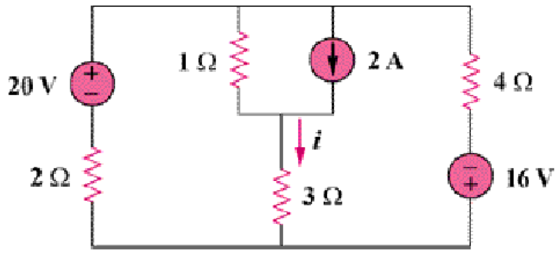
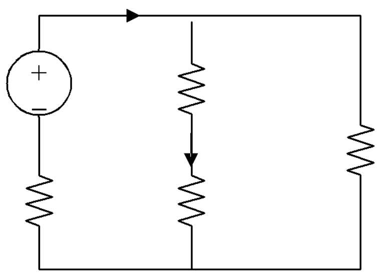


Figure 4.83

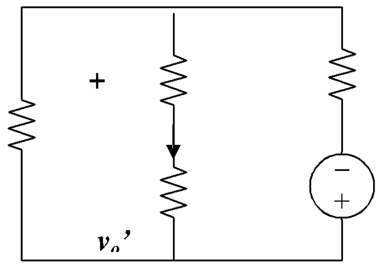
Chapter 4, Solution 15.

Let $i = i_1 + i_2 + i_3$, where i_1 , i_2 , and i_3 are due to the 20-V, 2-A, and 16-V sources. For i_1, consider the circuit below.



$$4||(3 + 1) = 2 \text{ ohms}, \text{ Then } i_o = [20/(2 + 2)] = 5 \text{ A}, \; i_1 = i_o/2 = 2.5 \text{ A}$$

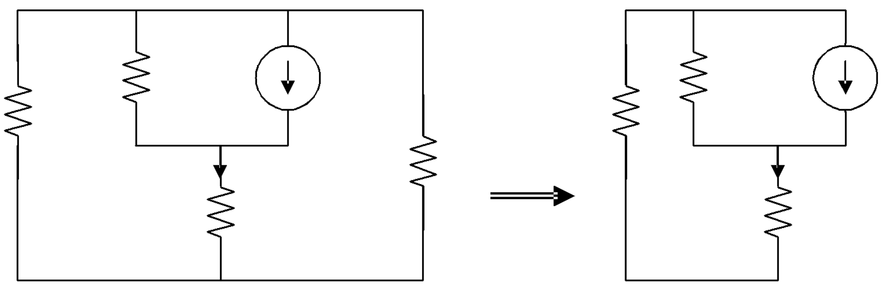
For i_3, consider the circuit below.



$$2||(1 + 3) = 4/3, \quad v_o' = [(4/3)/((4/3) + 4)](-16) = -4$$

$$i_3 = v_o'/4 = -1$$

For i_2, consider the circuit below.



$$2||4 = 4/3, \quad 3 + 4/3 = 13/3$$

Using the current division principle.

$$i_2 = [1/(1 + 13/2)]2 = 3/8 = 0.375$$

$$i = 2.5 + 0.375 - 1 = \mathbf{\underline{1.875\ A}}$$

$$p = i^2R = (1.875)^2 3 = \mathbf{\underline{10.55\ watts}}$$

Chapter 4, Problem 16.

Given the circuit in Fig. 4.84, use superposition to get i_o.

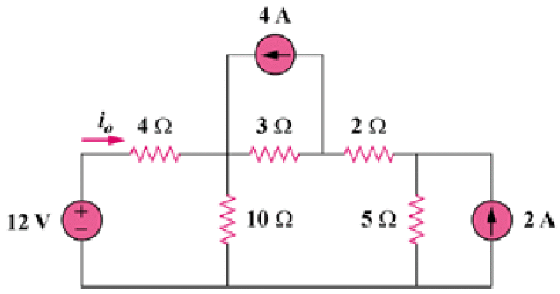
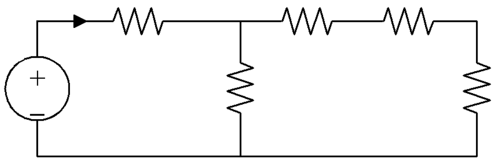


Figure 4.84

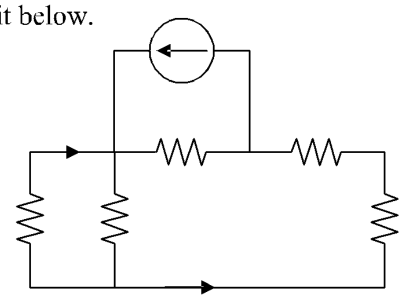
Chapter 4, Solution 16.

Let $i_o = i_{o1} + i_{o2} + i_{o3}$, where i_{o1}, i_{o2}, and i_{o3} are due to the 12-V, 4-A, and 2-A sources. For i_{o1}, consider the circuit below.



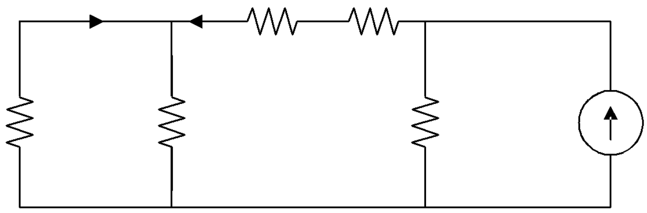
$$10||(3 + 2 + 5) = 5 \text{ ohms}, \ i_{o1} = 12/(5 + 4) = (12/9) \text{ A}$$

For i_{o2}, consider the circuit below.



$$2 + 5 + 4||10 = 7 + 40/14 = 69/7$$
$$i_1 = [3/(3 + 69/7)]4 = 84/90, \ i_{o2} =[-10/(4 + 10)]i_1 = -6/9$$

For i_{o3}, consider the circuit below.



$$3 + 2 + 4||10 = 5 + 20/7 = 55/7$$

$$i_2 = [5/(5 + 55/7)]2 = 7/9, \ i_{o3} = [-10/(10 + 4)]i_2 = -5/9$$

$$i_o = (12/9) - (6/9) - (5/9) = 1/9 = \mathbf{\underline{111.11\ mA}}$$

Chapter 4, Problem 17.

Use superposition to obtain v_x in the circuit of Fig. 4.85. Check your result using *PSpice*.

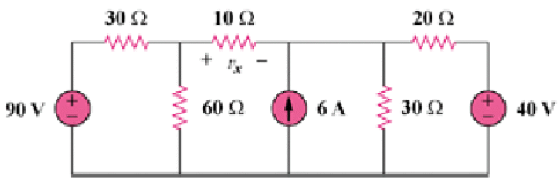
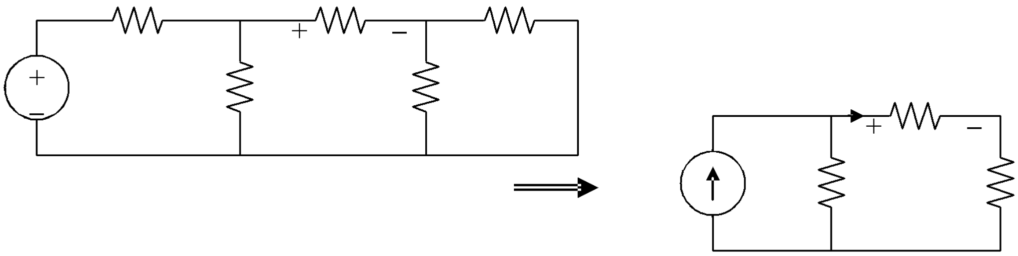


Figure 4.85

Chapter 4, Solution 17.

Let $v_x = v_{x1} + v_{x2} + v_{x3}$, where v_{x1}, v_{x2}, and v_{x3} are due to the 90-V, 6-A, and 40-V sources. For v_{x1}, consider the circuit below.

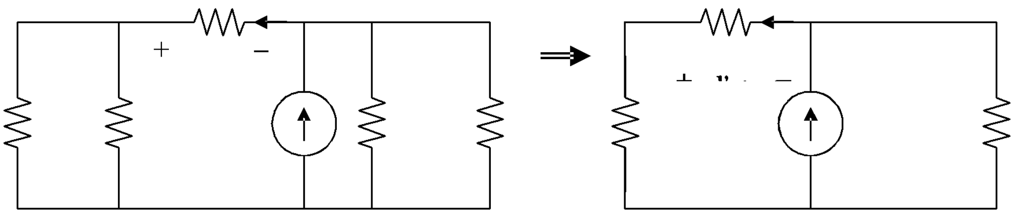


$$20||30 = 12 \text{ ohms}, \quad 60||30 = 20 \text{ ohms}$$

By using current division,

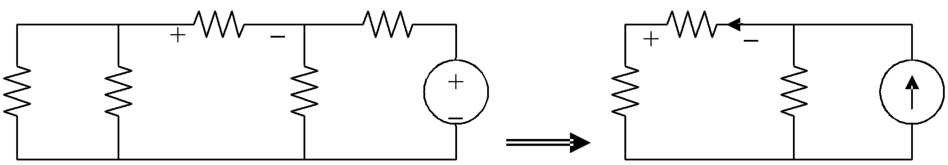
$$i_o = [20/(22 + 20)]3 = 60/42, \quad v_{x1} = 10i_o = 600/42 = 14.286 \text{ V}$$

For v_{x2}, consider the circuit below.



$$i_o' = [12/(12 + 30)]6 = 72/42, \quad v_{x2} = -10i_o' = -17.143 \text{ V}$$

For v_{x3}, consider the circuit below.



$i_o'' = [12/(12 + 30)]2 = 24/42, \ v_{x3} = -10i_o'' = -5.714 = [12/(12 + 30)]2 = 24/42, \ v_{x3}$
$= -10i_o'' = -5.714$

$$= [12/(12 + 30)]2 = 24/42, \quad v_{x3} = -10i_o'' = -5.714$$

$$v_x = 14.286 - 17.143 - 5.714 = \underline{\mathbf{-8.571\ V}}$$

Chapter 4, Problem 18.

Use superposition to find V_o in the circuit of Fig. 4.86.

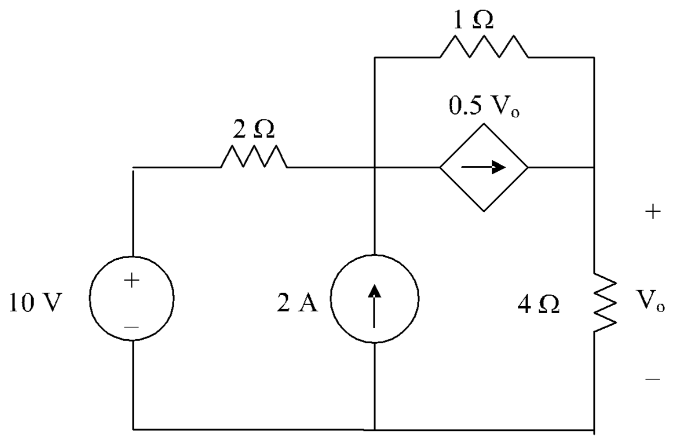
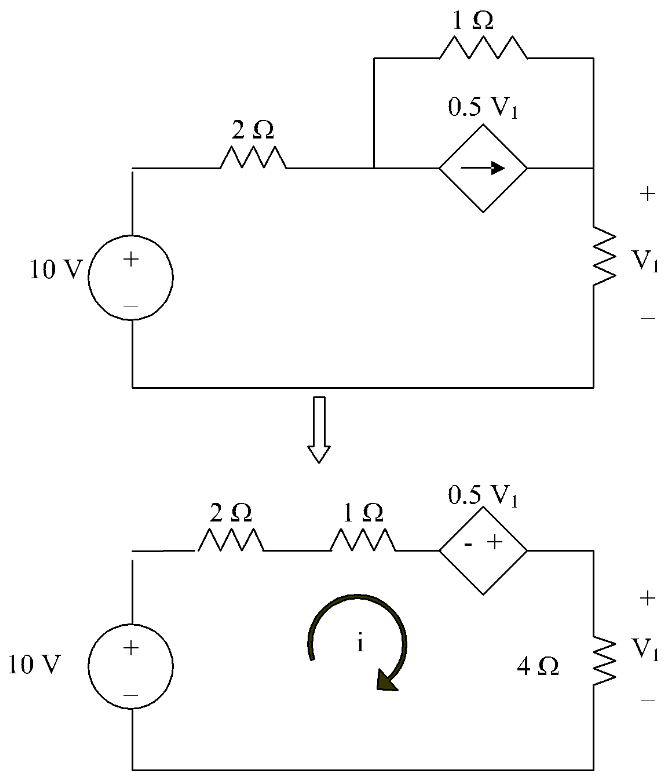


Figure 4.86 For Prob. 4.18.

Chapter 4, Solution 18.

Let $V_o = V_1 + V_2$, where V_1 and V_2 are due to 10-V and 2-A sources respectively. To find V_1, we use the circuit below.

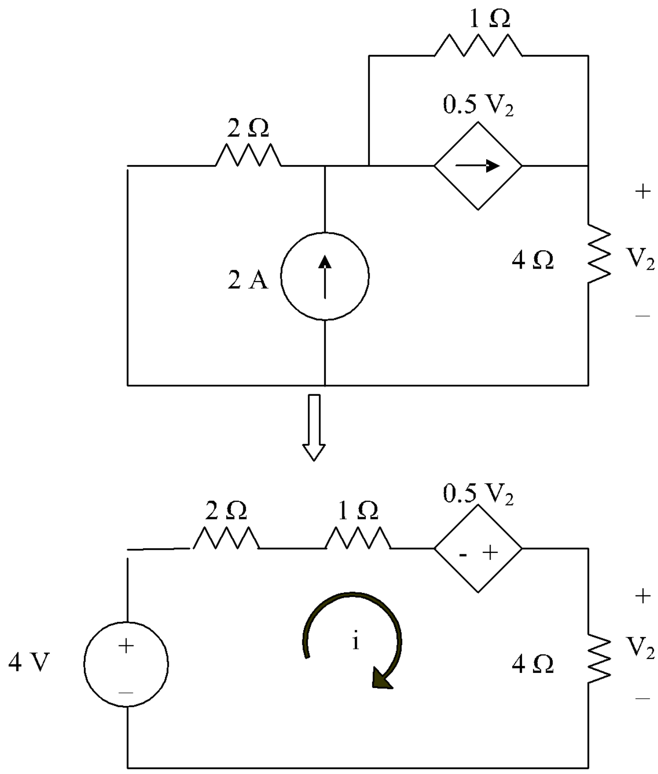


$$-10 + 7i - 0.5V_1 = 0$$

But $V_1 = 4i$

`$10 = 7i - 2i = 5i \longrightarrow i = 2, \quad V_1 = 8 \text{ V}$

To find V_2, we use the circuit below.



$- 4 + 7i – 0.5V_2 =0$

But $V_2 = 4i$

$$4 = 7i - 2i = 5i \quad \longrightarrow \quad i = 0.8, \quad V_2 = 4i = 3.2$$

$V_o = V_1 + V_2 = 8 +3.2$ =**11.2 V**

Chapter 4, Problem 19.

Use superposition to solve for v_x in the circuit of Fig. 4.87.

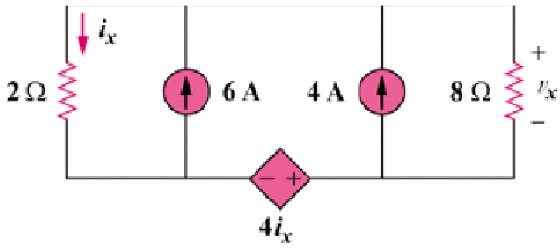
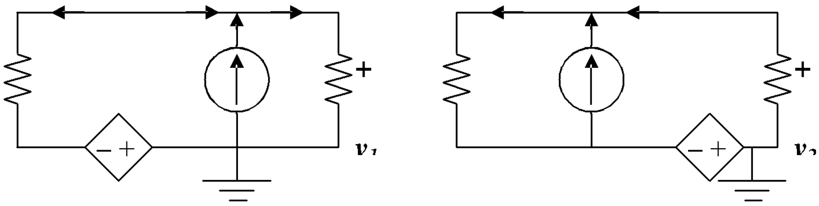


Figure 4.87

Chapter 4, Solution 19.

Let $v_x = v_1 + v_2$, where v_1 and v_2 are due to the 4-A and 6-A sources respectively.



To find v_1, consider the circuit in Fig. (a).

$$v_1/8 - 4 + (v_1 - (-4i_x))/2 = 0 \text{ or } (0.125+0.5)v_1 = 4 - 2i_x \text{ or } v_1 = 6.4 - 3.2i_x$$

But, $i_x = (v_1 - (-4i_x))/2$ or $i_x = -0.5v_1$. Thus,

$$v_1 = 6.4 + 3.2(0.5v_1), \text{ which leads to } v_1 = -6.4/0.6 = -10.667$$

To find v_2, consider the circuit shown in Fig. (b).

$$v_2/8 - 6 + (v_2 - (-4i_x))/2 = 0 \text{ or } v_2 + 3.2i_x = 9.6$$

But $i_x = -0.5v_2$. Therefore,

$$v_2 + 3.2(-0.5v_2) = 9.6 \text{ which leads to } v_2 = -16$$

Hence, $v_x = -10.667 - 16 = \underline{\mathbf{-26.67V}}$.

Checking,

$$i_x = -0.5v_x = 13.333\text{A}$$

Now all we need to do now is sum the currents flowing out of the top node.

$$13.333 - 6 - 4 + (-26.67)/8 = 3.333 - 3.333 = 0$$

Chapter 4, Problem 20.

Use source transformations to reduce the circuit in Fig. 4.88 to a single voltage source in series with a single resistor.

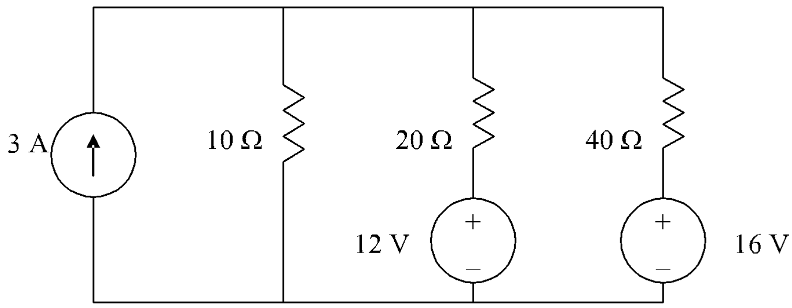
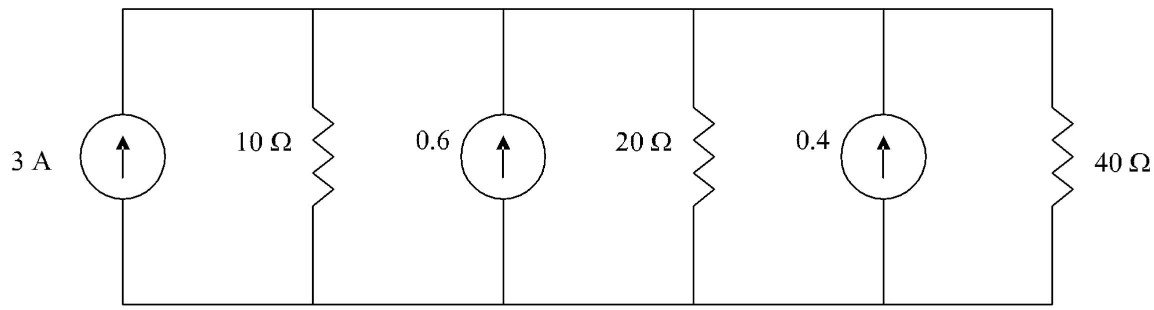


Figure 4.88 For Prob. 4.20.

Chapter 4, Solution 20.

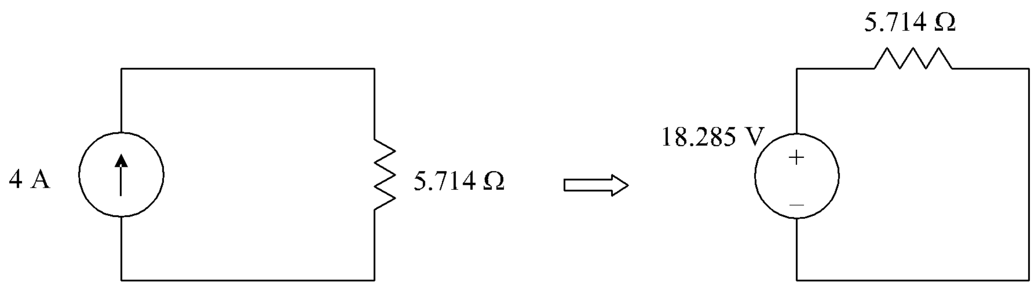
Convert the voltage sources to current sources and obtain the circuit shown below.



$$\frac{1}{R_{eq}} = \frac{1}{10} + \frac{1}{20} + \frac{1}{40} = 0.1 + 0.05 + 0.025 = 0.175 \longrightarrow R_{eq} = 5.714\ \Omega$$

$I_{eq} = 3 + 0.6 + 0.4 = 4$

Thus, the circuit is reduced as shown below. Please note, we that this is merely an exercise in combining sources and resistors. The circuit we have is an equivalent circuit which has no real purpose other than to demonstrate source transformation. In a practical situation, this would need some kind of reference and a use to an external circuit to be of real value.



Chapter 4, Problem 21.

Apply source transformation to determine v_o and i_o in the circuit in Fig. 4.89.

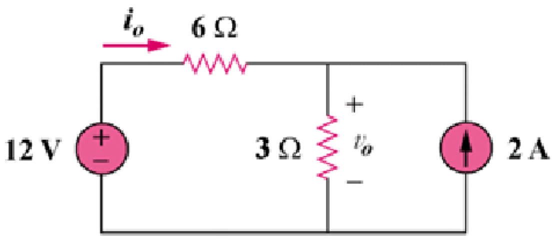
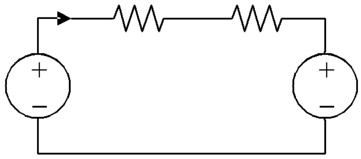
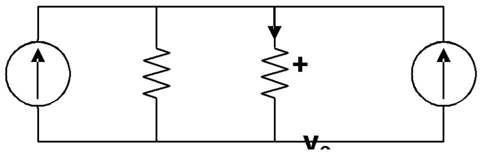


Figure 4.89

Chapter 4, Solution 21.

To get i_o, transform the current sources as shown in Fig. (a).





From Fig. (a), $-12 + 9i_o + 6 = 0$, therefore i_o = **666.7 mA**

To get v_o, transform the voltage sources as shown in Fig. (b).

$$i = [6/(3 + 6)](2 + 2) = 8/3$$

$$v_o = 3i = \underline{\mathbf{8\ V}}$$

Chapter 4, Problem 22.

Referring to Fig. 4.90, use source transformation to determine the current and power in the 8-Ω resistor.

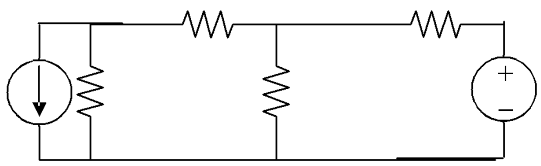
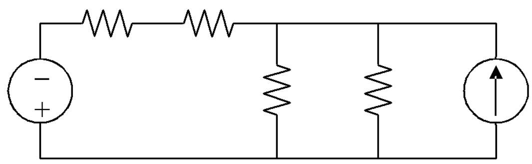
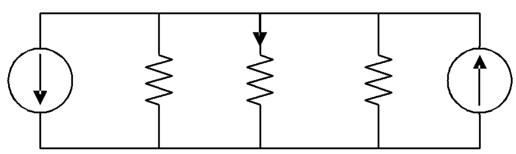


Figure 4.90

Chapter 4, Solution 22.

We transform the two sources to get the circuit shown in Fig. (a).





We now transform only the voltage source to obtain the circuit in Fig. (b).

10||10 = 5 ohms, i = [5/(5 + 4)](2 – 1) = 5/9 = **<u>555.5 mA</u>**

Chapter 4, Problem 23.

Referring to Fig. 4.91, use source transformation to determine the current and power in the 8-Ω resistor.

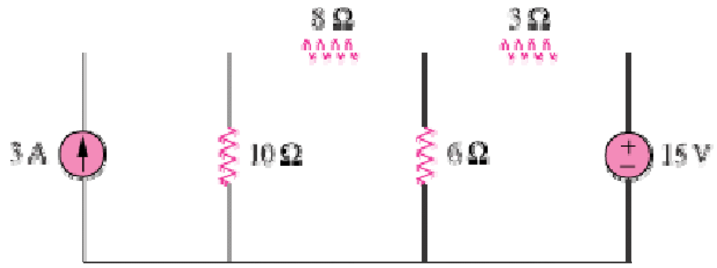
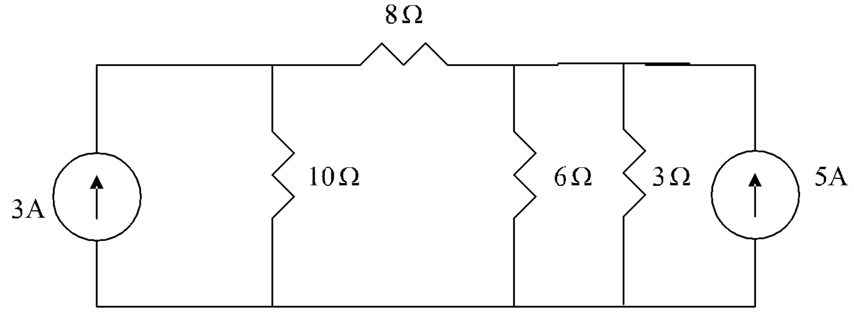


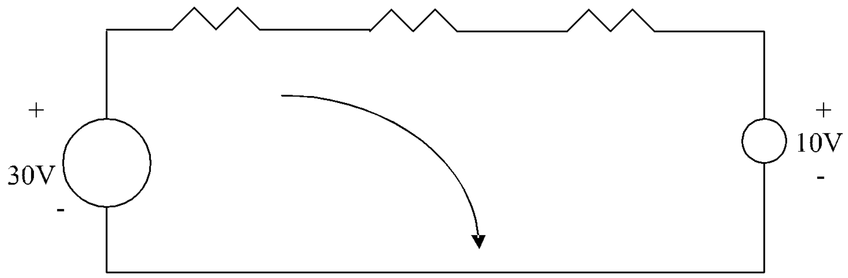
Figure 4.91

Chapter 4, Solution 23

If we transform the voltage source, we obtain the circuit below.



3//6 = 2-ohm. Convert the current sources to voltages sources as shown below.



Applying KVL to the loop gives

$$-30 + 10 + I(10 + 8 + 2) = 0 \quad \longrightarrow \quad \underline{I = 1\,\text{A}}$$

$$p = VI = I^2 R = \underline{8\,\text{W}}$$

Chapter 4, Problem 24.

Use source transformation to find the voltage V_x in the circuit of Fig. 4.92.

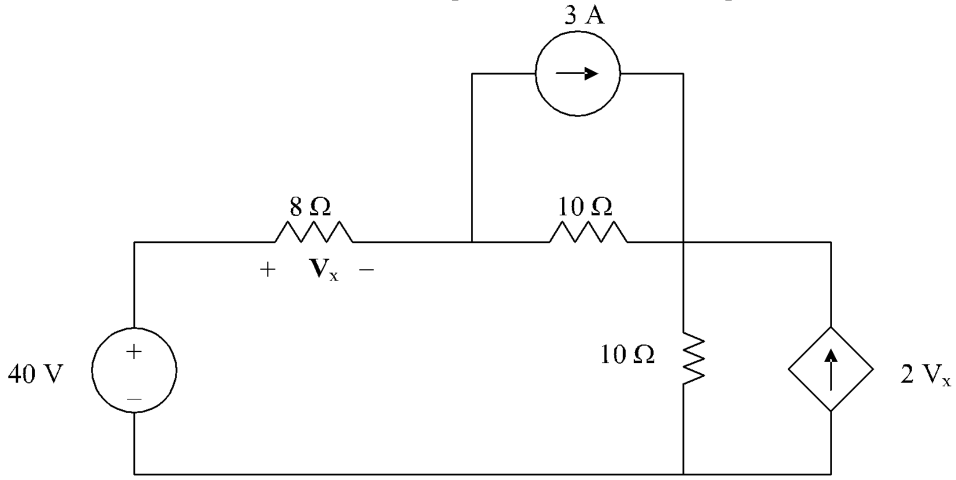


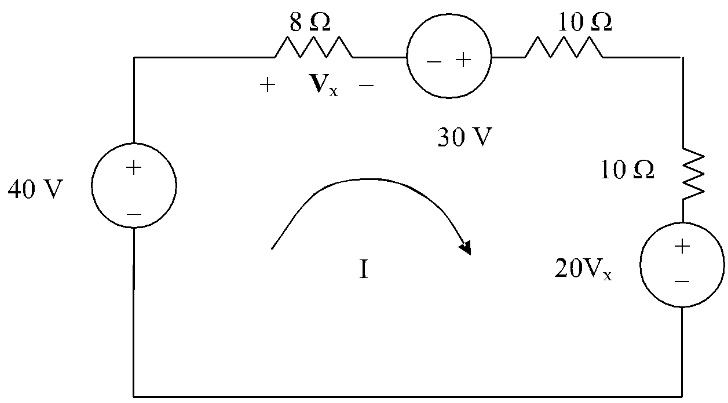
Figure 4.92 For Prob. 4.24.

Chapter 4, Solution 24.

Transform the two current sources in parallel with the resistors into their voltage source equivalents yield,

a 30-V source in series with a 10-Ω resistor and a $20V_x$-V sources in series with a 10-Ω resistor.

We now have the following circuit,



We now write the following mesh equation and constraint equation which will lead to a solution for V_x,

$28I - 70 + 20V_x = 0$ or $28I + 20V_x = 70$, but $V_x = 8I$ which leads to

$28I + 160I = 70$ or $I = 0.3723$ A or $V_x =$ **2.978 V**.

Chapter 4, Problem 25.

Obtain v_o in the circuit of Fig. 4.93 using source transformation. Check your result using *PSpice*.

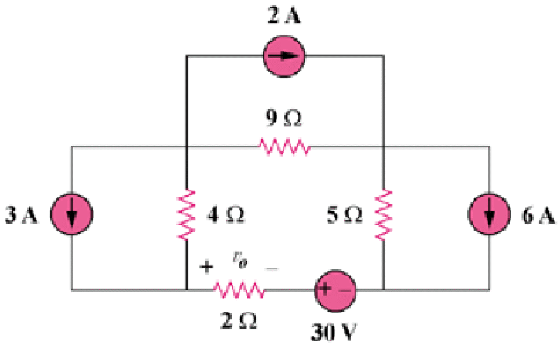
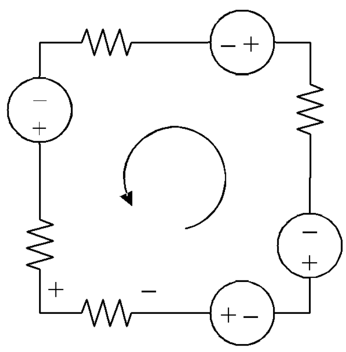


Figure 4.93

Chapter 4, Solution 25.

Transforming only the current source gives the circuit below.



Applying KVL to the loop gives,

$$–(4 + 9 + 5 + 2)i + 12 – 18 – 30 – 30 = 0$$

$$20i = –66 \text{ which leads to } i = –3.3$$

$v_o = 2i =$ **<u>–6.6 V</u>**

Chapter 4, Problem 26.

Use source transformation to find i_o in the circuit of Fig. 4.94.

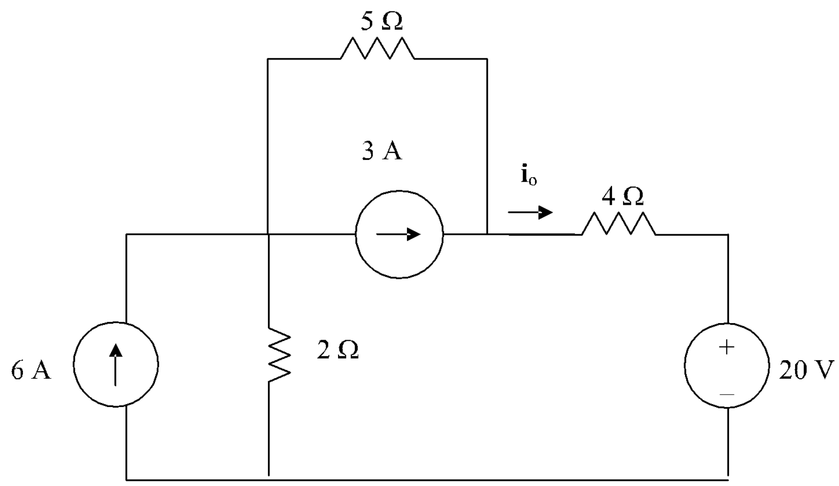
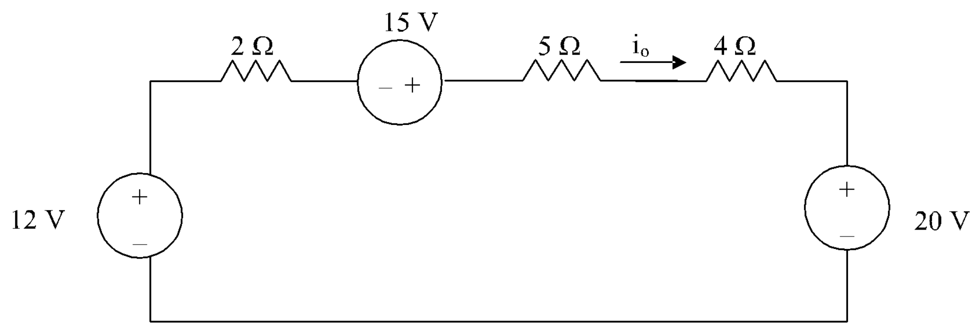


Figure 4.94 For Prob. 4.26.

Chapter 4, Solution 26.

Transforming the current sources gives the circuit below.



$-12 + 11i_o - 15 + 20 = 0$ or $11i_o = 7$ or $i_o =$ **636.4 mA**.

Chapter 4, Problem 27.

Apply source transformation to find v_x in the circuit of Fig. 4.95.

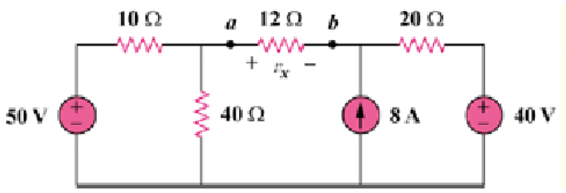


Figure 4.95

Chapter 4, Solution 27.

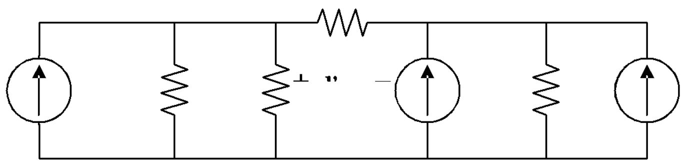
Transforming the voltage sources to current sources gives the circuit in Fig. (a).

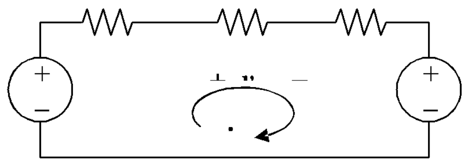
$$10||40 = 8 \text{ ohms}$$

Transforming the current sources to voltage sources yields the circuit in Fig. (b). Applying KVL to the loop,

$$-40 + (8 + 12 + 20)i + 200 = 0 \text{ leads to } i = -4$$

$$v_x \; 12i = \underline{\mathbf{-48\ V}}$$





Chapter 4, Problem 28.

Use source transformation to find I_o in Fig. 4.96.

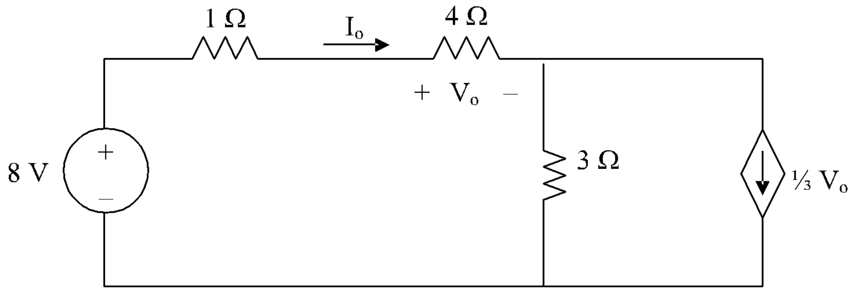
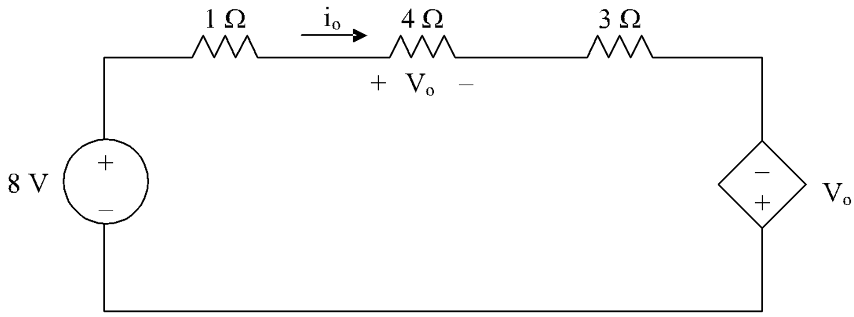


Figure 4.96 For Prob. 4.28.

Chapter 4, Solution 28.

Convert the dependent current source to a dependent voltage source as shown below.



Applying KVL,

$$-8 + i_o(1 + 4 + 3) - V_o = 0$$

But $V_o = 4i_o$

$$-8 + 8i_o - 4i_o = 0 \quad \longrightarrow \quad i_o = \underline{2\ \text{A}}$$

Chapter 4, Problem 29.

Use source transformation to find v_o in the circuit of Fig. 4.93.

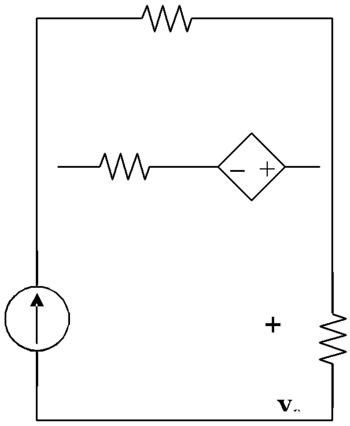
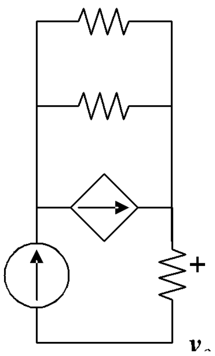
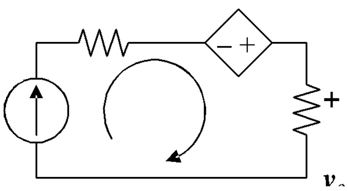


Figure 4.93

Chapter 4, Solution 29.

Transform the dependent voltage source to a current source as shown in Fig. (a). 2||4 = (4/3) k ohms





It is clear that i = 3 mA which leads to v_o = 1000i = **3 V**

If the use of source transformations was not required for this problem, the actual answer could have been determined by inspection right away since the only current that could have flowed through the 1 k ohm resistor is 3 mA.

Chapter 4, Problem 30.

Use source transformation on the circuit shown in Fig 4.98 to find i_x.

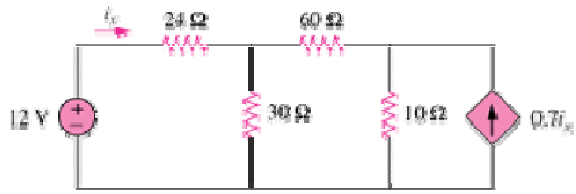
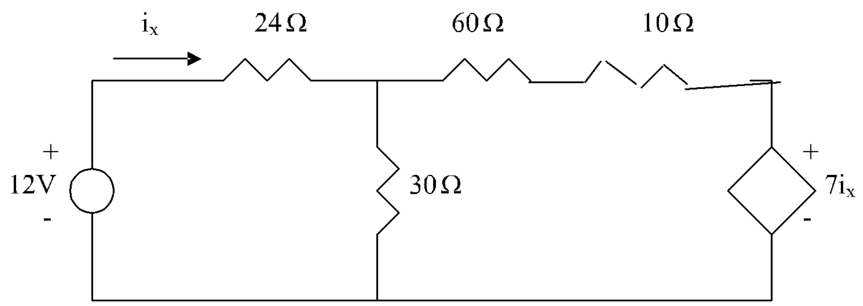


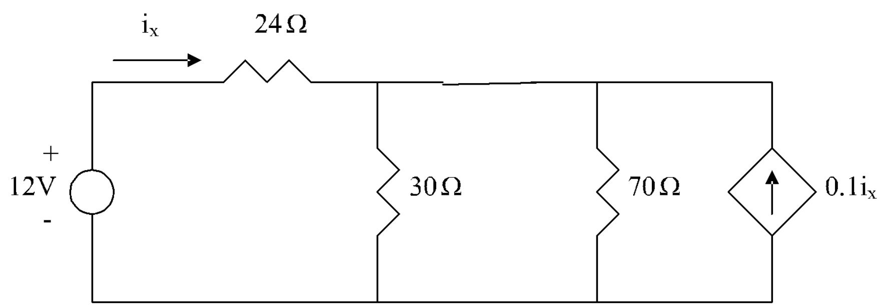
Figure 4.98

Chapter 4, Solution 30

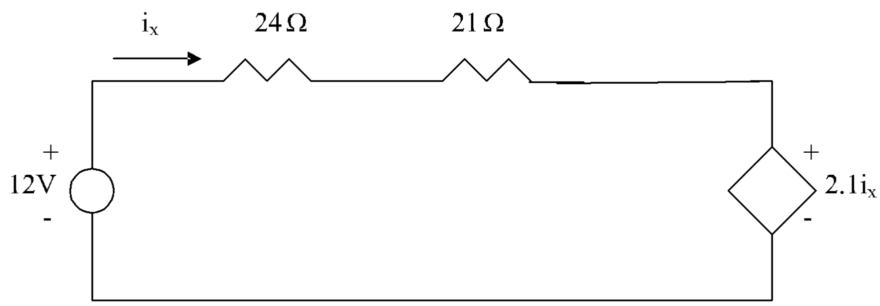
Transform the dependent current source as shown below.



Combine the 60-ohm with the 10-ohm and transform the dependent source as shown below.



Combining 30-ohm and 70-ohm gives 30//70 = 70x30/100 = 21-ohm. Transform the dependent current source as shown below.



Applying KVL to the loop gives

$$45i_x - 12 + 2.1i_x = 0 \quad \longrightarrow \quad i_x = \frac{12}{47.1} = \underline{254.8\text{ mA}}$$

Chapter 4, Problem 31.
Determine v_x in the circuit of Fig. 4.99 using source transformation.

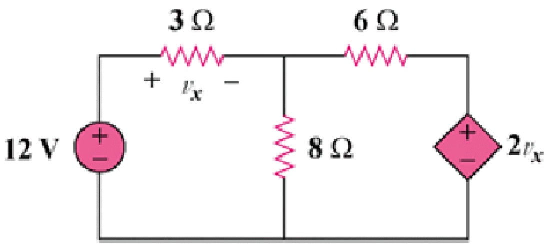
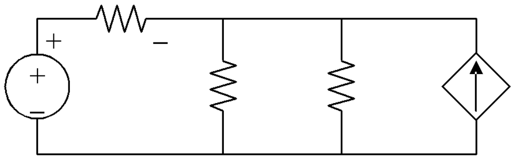
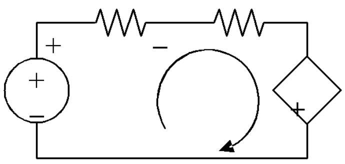


Figure 4.99

Chapter 4, Solution 31.
Transform the dependent source so that we have the circuit in Fig. (a). 6||8 = (24/7) ohms. Transform the dependent source again to get the circuit in Fig. (b).





From Fig. (b),

$$v_x = 3i, \text{ or } i = v_x/3.$$

Applying KVL,

$$-12 + (3 + 24/7)i + (24/21)v_x = 0$$

$$12 = [(21 + 24)/7]v_x/3 + (8/7)v_x, \text{ leads to } v_x = 84/23 = \underline{\mathbf{3.652\ V}}$$

Chapter 4, Problem 32.

Use source transformation to find i_x in the circuit of Fig. 4.100.

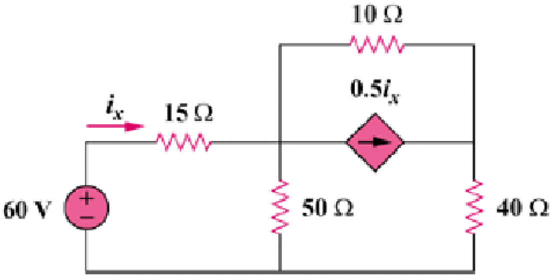
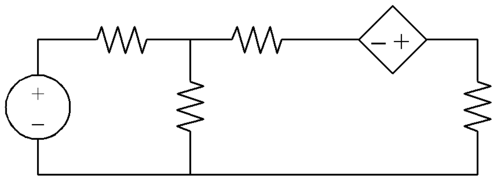
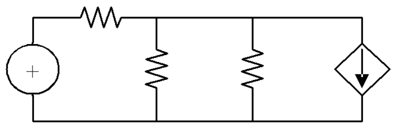


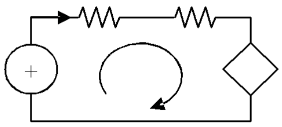
Figure 4.100

Chapter 4, Solution 32.

As shown in Fig. (a), we transform the dependent current source to a voltage source,







In Fig. (b), 50||50 = 25 ohms. Applying KVL in Fig. (c),

$-60 + 40i_x - 2.5i_x = 0$, or i_x = **<u>1.6 A</u>**

Chapter 4, Problem 33.

Determine R_{Th} and V_{Th} at terminals 1-2 of each of the circuits of Fig. 4.101.

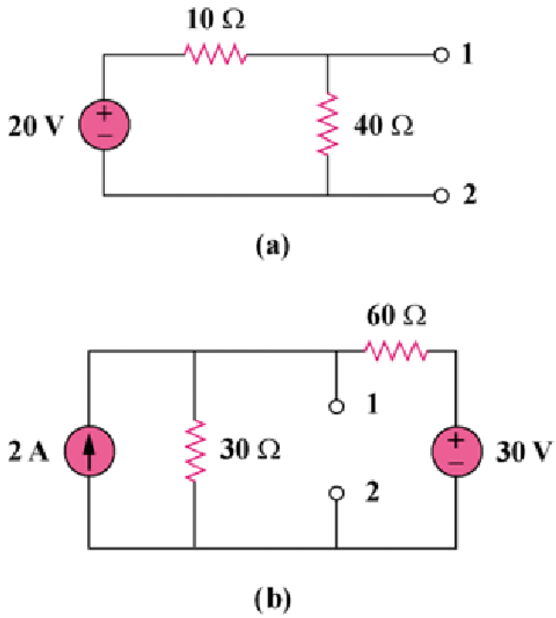


Figure 4.101

Chapter 4, Solution 33.

(a) $R_{Th} = 10||40 = 400/50 = \underline{\textbf{8 ohms}}$

$$V_{Th} = (40/(40 + 10))20 = \underline{\textbf{16 V}}$$

(b) $R_{Th} = 30||60 = 1800/90 = \underline{\textbf{20 ohms}}$

$$2 + (30 - v1)/60 = v_1/30, \text{ and } v_1 = V_{Th}$$

$$120 + 30 - v_1 = 2v_1, \text{ or } v_1 = 50 \text{ V}$$

$$V_{Th} = \underline{\textbf{50 V}}$$

Chapter 4, Problem 34.

Find the Thevenin equivalent at terminals ***a-b*** of the circuit in Fig. 4.102.

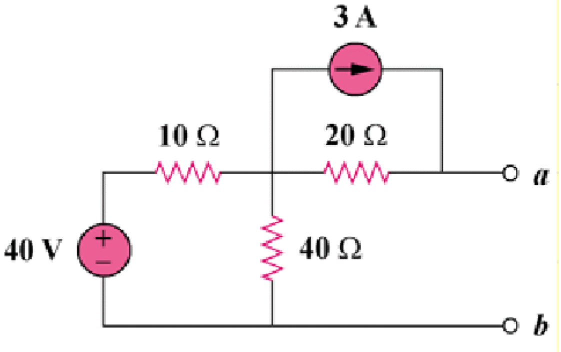
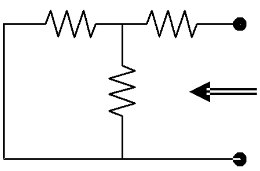
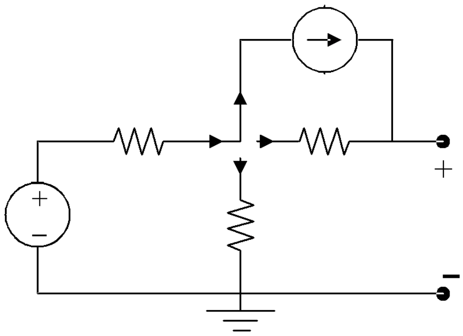


Figure 4.102

Chapter 4, Solution 34.

To find R_{Th}, consider the circuit in Fig. (a).





$$R_{Th} = 20 + 10||40 = 20 + 400/50 = \underline{\textbf{28 ohms}}$$

To find V_{Th}, consider the circuit in Fig. (b).

At node 1, $(40 - v_1)/10 = 3 + [(v_1 - v_2)/20] + v_1/40, \; 40 = 7v_1 - 2v_2$ (1)

At node 2, $3 + (v1 - v2)/20 = 0$, or $v1 = v2 - 60$ (2)

Solving (1) and (2), $v_1 = 32$ V, $v_2 = 92$ V, and $V_{Th} = v_2 = \underline{\textbf{92 V}}$

Chapter 4, Problem 35.

Use Thevenin's theorem to find v_o in Prob. 4.12.

Chapter 4, Problem 12**.**
Determine v_o in the circuit in Fig. 4.80 using the superposition principle.

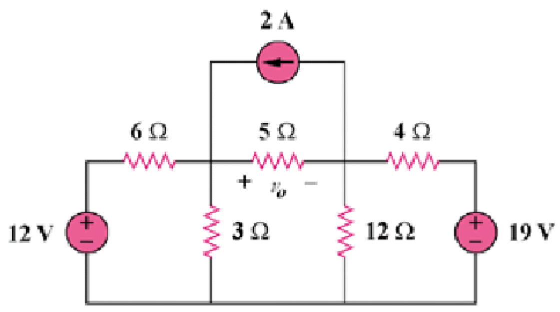


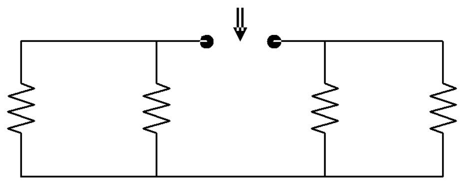
Figure 4.80

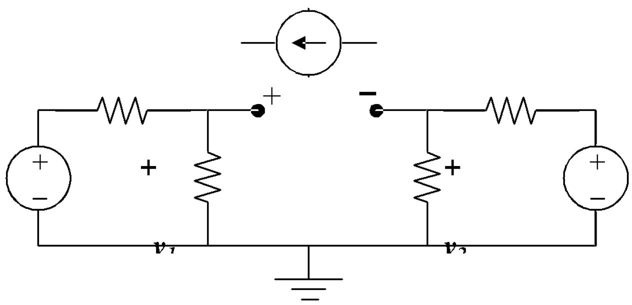
Chapter 4, Solution 35.

To find R_{Th}, consider the circuit in Fig. (a).

$$R_{Th} = R_{ab} = 6||3 + 12||4 = 2 + 3 = 5 \text{ ohms}$$

To find V_{Th}, consider the circuit shown in Fig. (b).

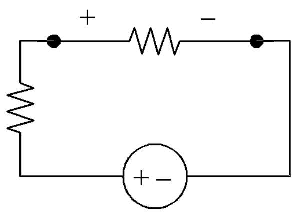




At node 1, $2 + (12 - v_1)/6 = v_1/3$, or $v_1 = 8$

At node 2, $(19 - v_2)/4 = 2 + v_2/12$, or $v_2 = 33/4$

But, $-v_1 + V_{Th} + v_2 = 0$, or $V_{Th} = v_1 - v_2 = 8 - 33/4 = -0.25$



$$v_o = V_{Th}/2 = -0.25/2 = \underline{\mathbf{-125\ mV}}$$

Chapter 4, Problem 36.

Solve for the current ***i*** in the circuit of Fig. 4.103 using Thevenin's theorem. (*Hint:* Find the Thevenin equivalent as seen by the 12-Ω resistor.)

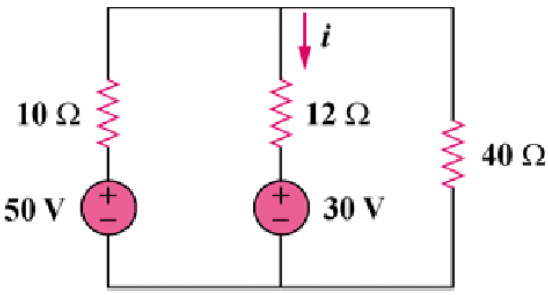
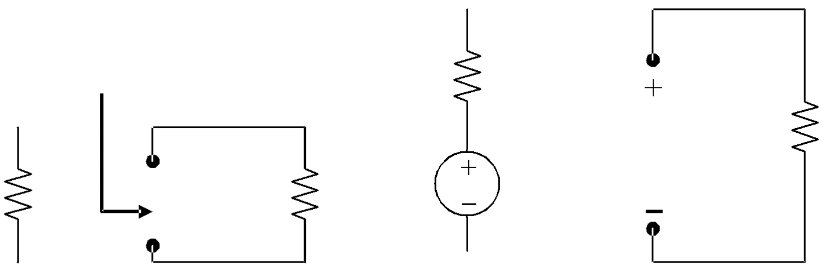


Figure 4.103

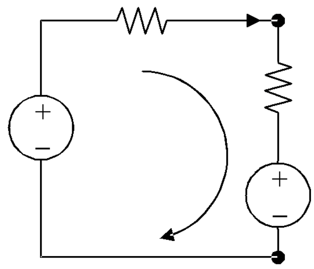
Chapter 4, Solution 36.

Remove the 30-V voltage source and the 20-ohm resistor.



From Fig. (a), $R_{Th} = 10||40 = 8$ ohms

From Fig. (b), $V_{Th} = (40/(10 + 40))50 = 40V$



The equivalent circuit of the original circuit is shown in Fig. (c). Applying KVL,

$$30 - 40 + (8 + 12)i = 0, \text{ which leads to } i = \underline{\mathbf{500mA}}$$

Chapter 4, Problem 37.

Find the Norton equivalent with respect to terminals *a*-*b* in the circuit shown in Fig. 4.100.

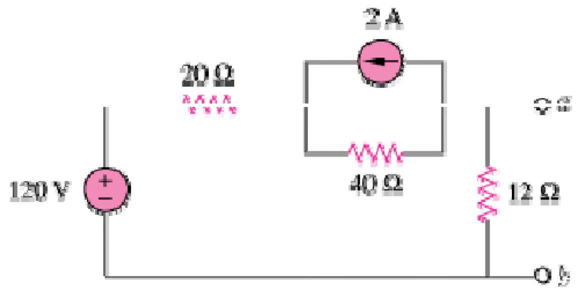
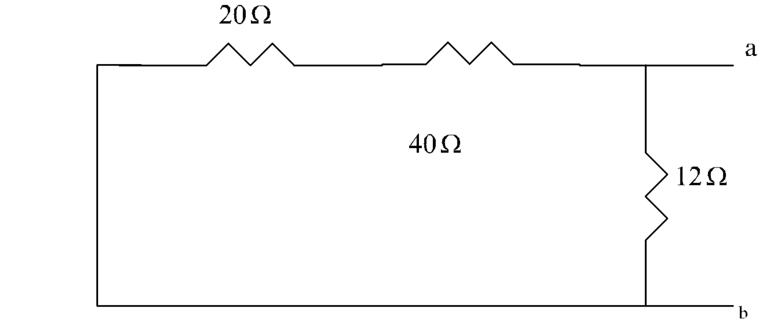


Figure 4.100

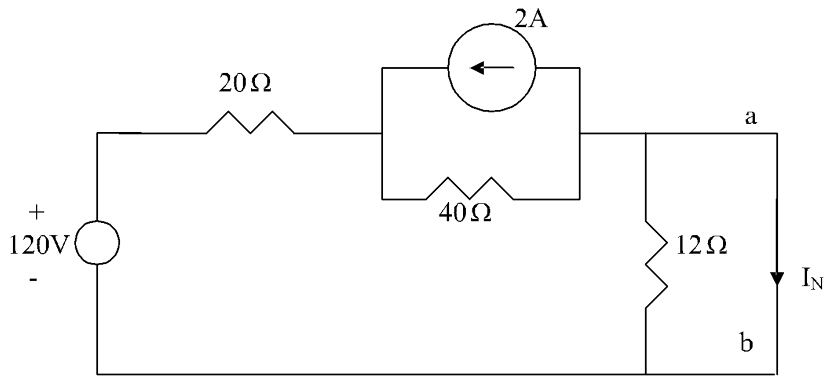
Chapter 4, Solution 37

R_N is found from the circuit below.

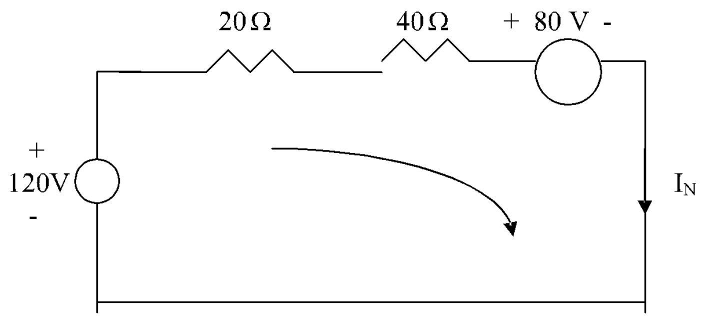


$R_N = 12//(20+40) = \underline{10\Omega}$

I_N is found from the circuit below.



Applying source transformation to the current source yields the circuit below.



Applying KVL to the loop yields

$-120 + 80 + 60I_N = 0 \quad \longrightarrow \quad I_N = 40/60 = \underline{\mathbf{666.7\ mA}}.$

Chapter 4, Problem 38.

Apply Thèvenin's theorem to find V_o in the circuit of Fig. 4.105.

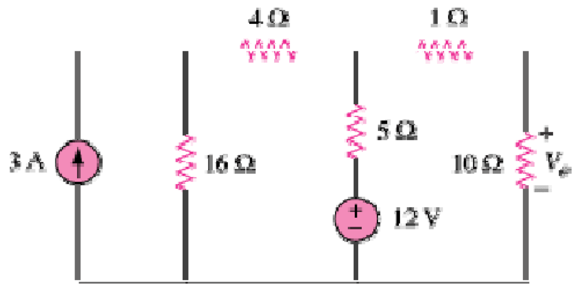
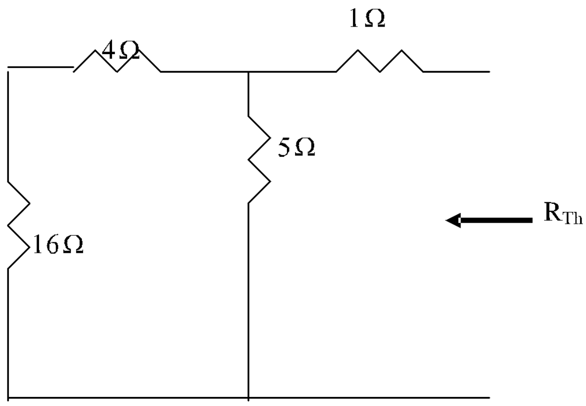


Figure 4.105

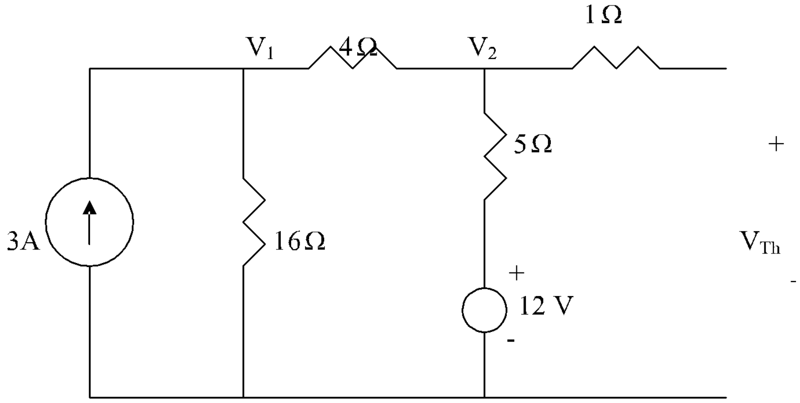
Chapter 4, Solution 38

We find Thevenin equivalent at the terminals of the 10-ohm resistor. For R_{Th}, consider the circuit below.



$$R_{Th} = 1 + 5//(4 + 16) = 1 + 4 = 5\Omega$$

For V_{Th}, consider the circuit below.



At node 1,

$$3 = \frac{V_1}{16} + \frac{V_1 - V_2}{4} \longrightarrow 48 = 5V_1 - 4V_2 \qquad (1)$$

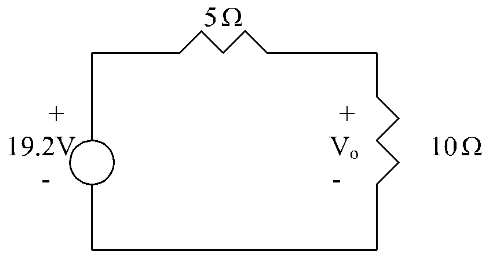
At node 2,

$$\frac{V_1 - V_2}{4} + \frac{12 - V_2}{5} = 0 \longrightarrow 48 = -5V_1 + 9V_2 \qquad (2)$$

Solving (1) and (2) leads to

$V_{Th} = V_2 = 19.2$

Thus, the given circuit can be replaced as shown below.



Using voltage division,

$$V_o = \frac{10}{10+5}(19.2) = 12.8 \text{ V}$$

Chapter 4, Problem 39.

Obtain the Thevenin equivalent at terminals a-b of the circuit in Fig. 4.106.

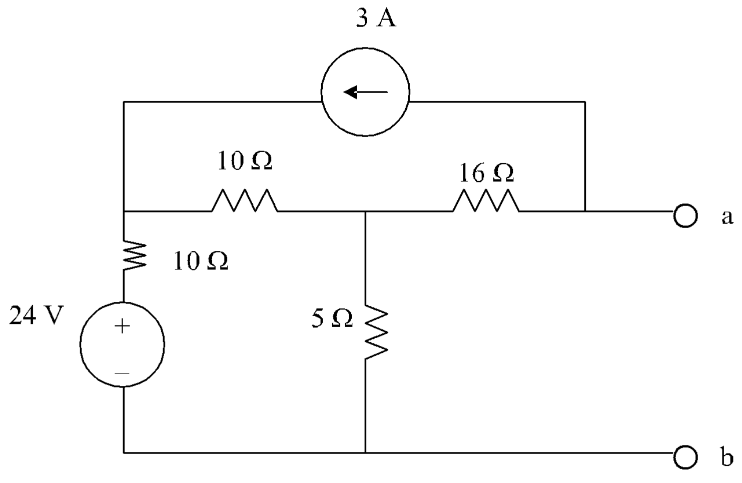
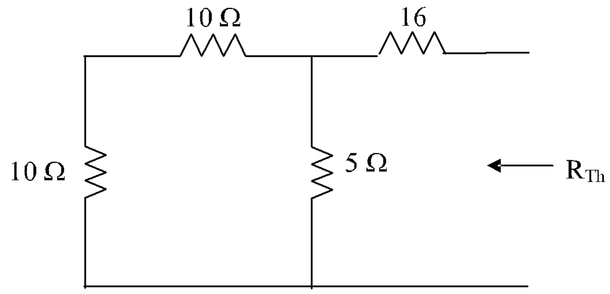


Figure 4.106 For Prob. 4.39.

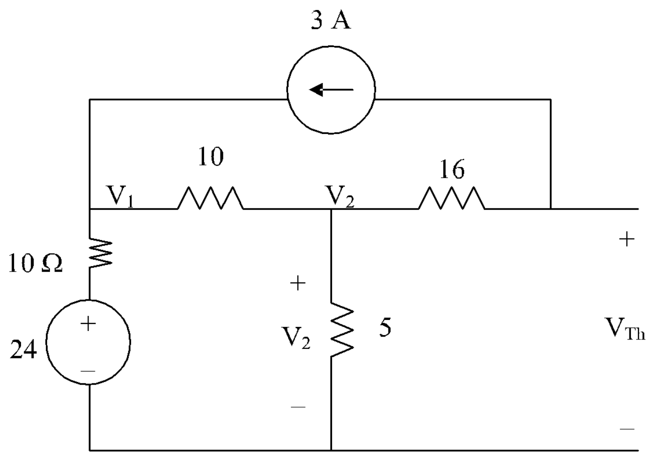
Chapter 4, Solution 39.

We obtain R_{Th} using the circuit below.



$$R_{Th} = 16 + 20 // 5 = 16 + \frac{20x5}{25} = \underline{20\ \Omega}$$

To find V_{Th}, we use the circuit below.



At node 1,

$$\frac{24 - V_1}{10} + 3 = \frac{V_1 - V_2}{10} \quad \longrightarrow \quad 54 = 2V_1 - V_2 \qquad (1)$$

At node 2,

$$\frac{V_1 - V_2}{10} = 3 + \frac{V_2}{5} \quad \longrightarrow \quad 60 = 2V_1 - 6V_2 \qquad (2)$$

Substracting (1) from (2) gives

$$6 = -5V_1 \quad \longrightarrow \quad V_2 = 1.2 \text{ V}$$

But

$$-V_2 + 16x3 + V_{Th} = 0 \quad \longrightarrow \quad V_{Th} = \underline{-49.2 \text{ V}}$$

Chapter 4, Problem 40.

Find the Thevenin equivalent at terminals a-b of the circuit in Fig. 4.107.

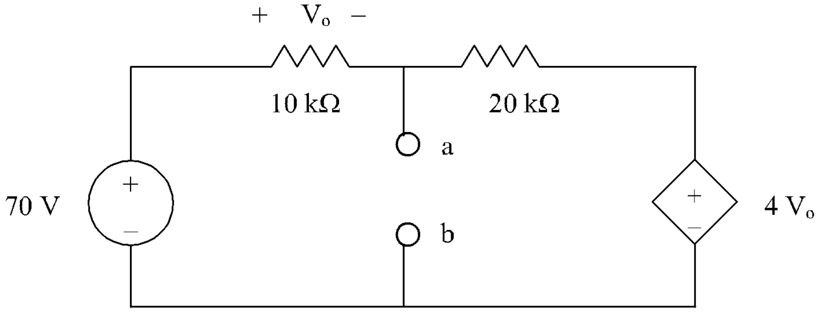


Figure 4.107 For Prob. 4.40.

Chapter 4, Solution 40.

To obtain V_{Th}, we apply KVL to the loop.

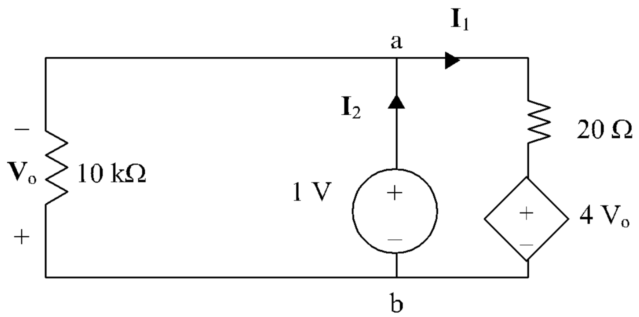
$$-70 + (10 + 20)kI + 4V_o = 0$$

But $V_o = 10kI$

$$70 = 70kI \longrightarrow I = 1mA$$

$$-70 + 10kI + V_{Th} = 0 \longrightarrow V_{Th} = \underline{60\ V}$$

To find R_{Th}, we remove the 70-V source and apply a 1-V source at terminals a-b, as shown in the circuit below.



We notice that V_o = -1 V.

$$-1 + 20kI_1 + 4V_o = 0 \longrightarrow I_1 = 0.25\ mA$$

$$I_2 = I_1 + \frac{1V}{10k} = 0.35\ mA$$

$$R_{Th} = \frac{1V}{I_2} = \frac{1}{0.35} k\Omega = \underline{2.857\ k\Omega}$$

Chapter 4, Problem 41.

Find the Thèvenin and Norton equivalents at terminals *a*-*b* of the circuit shown in Fig. 4.108.

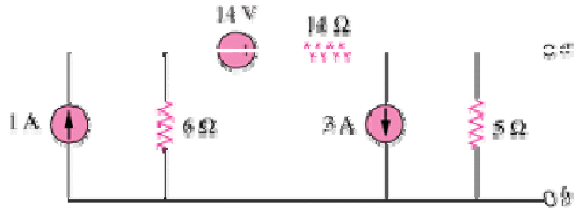
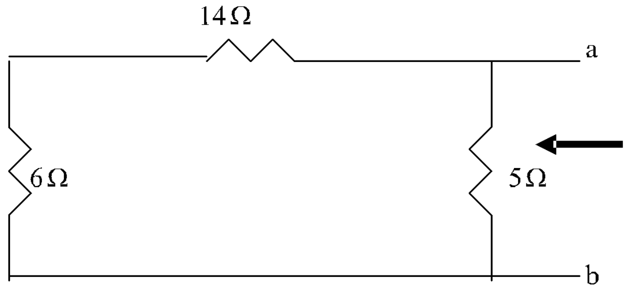


Figure 4.108

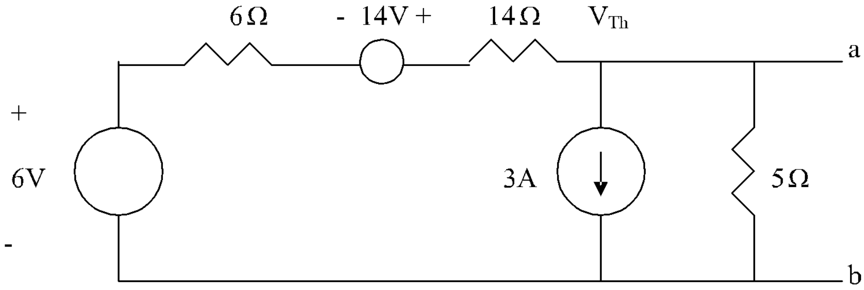
Chapter 4, Solution 41

To find R_{Th}, consider the circuit below



$$R_{Th} = 5 // (14 + 6) = 4\Omega = R_N$$

Applying source transformation to the 1-A current source, we obtain the circuit below.



At node a,

$$\frac{14 + 6 - V_{Th}}{6 + 14} = 3 + \frac{V_{Th}}{5} \quad \longrightarrow \quad V_{Th} = -8\text{ V}$$

$$I_N = \frac{V_{Th}}{R_{Th}} = (-8)/4 = -2\text{ A}$$

Thus,

$$\underline{R_{Th} = R_N = 4\Omega, \quad V_{Th} = -8\text{V}, \quad I_N = -2\text{ A}}$$

Chapter 4, Problem 42.

For the circuit in Fig. 4.109, find Thevenin equivalent between terminals ***a*** and ***b***.

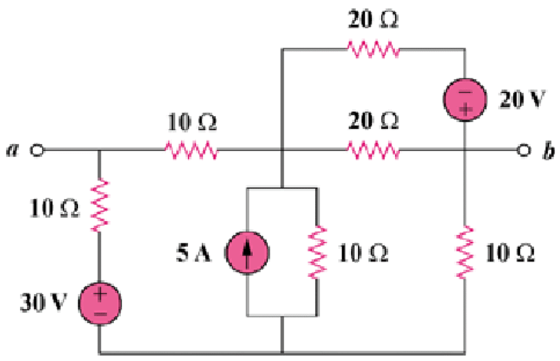
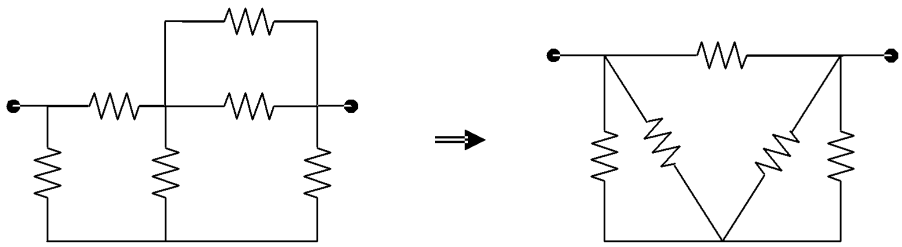


Figure 4.109

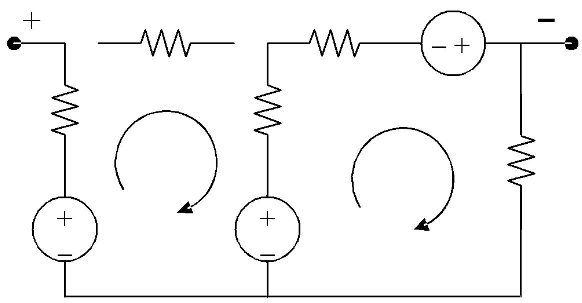
Chapter 4, Solution 42.

To find R_{Th}, consider the circuit in Fig. (a).



$20||20 = 10$ ohms. Transform the wye sub-network to a delta as shown in Fig. (b).
$10||30 = 7.5$ ohms. $R_{Th} = R_{ab} = 30||(7.5 + 7.5) =$ **<u>10 ohms</u>**.

To find V_{Th}, we transform the 20-V and the 5-V sources. We obtain the circuit shown in Fig. (c).



For loop 1, $-30 + 50 + 30i_1 - 10i_2 = 0$, or $-2 = 3i_1 - i_2$ (1)

For loop 2, $-50 - 10 + 30i_2 - 10i_1 = 0$, or $6 = -i_1 + 3i_2$ (2)

Solving (1) and (2), $i_1 = 0$, $i_2 = 2$ A

Applying KVL to the output loop, $-v_{ab} - 10i_1 + 30 - 10i_2 = 0$, $v_{ab} = 10$ V

$V_{Th} = v_{ab} =$ **<u>10 volts</u>**

Chapter 4, Problem 43.

Find the Thevenin equivalent looking into terminals ***a-b*** of the circuit in Fig. 4.110 and solve for i_x.

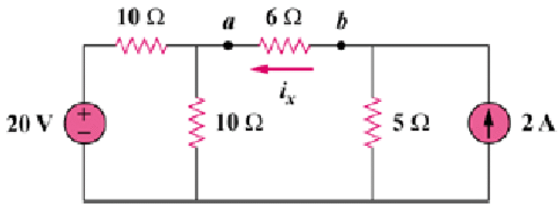
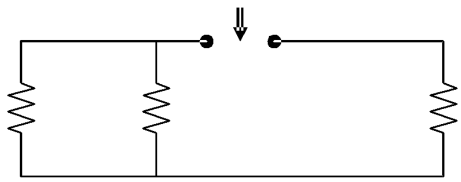
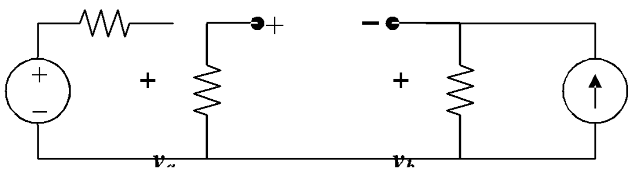


Figure 4.110

Chapter 4, Solution 43.

To find R_{Th}, consider the circuit in Fig. (a).





$$R_{Th} = 10||10 + 5 = \underline{\textbf{10 ohms}}$$

To find V_{Th}, consider the circuit in Fig. (b).

$$v_b = 2x5 = 10 \text{ V}, \ v_a = 20/2 = 10 \text{ V}$$

But, $-v_a + V_{Th} + v_b = 0$, or $V_{Th} = v_a - v_b = \underline{\textbf{0 volts}}$

Chapter 4, Problem 44.

For the circuit in Fig. 4.111, obtain the Thevenin equivalent as seen from terminals
(a) ***a-b*** (b) ***b-c***

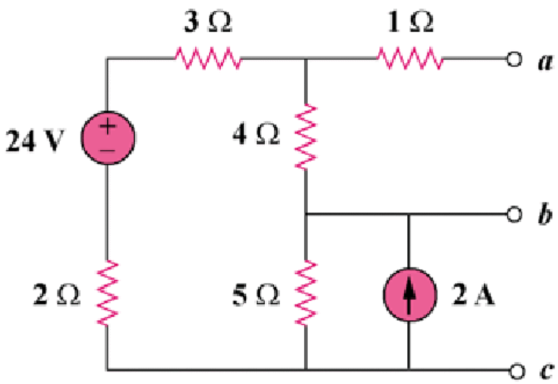


Figure 4.111

Chapter 4, Solution 44.

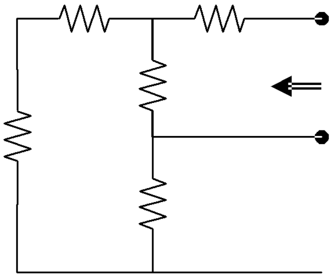
(a) For R_{Th}, consider the circuit in Fig. (a).

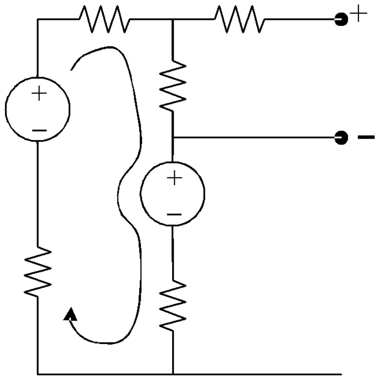
$$R_{Th} = 1 + 4||(3 + 2 + 5) = \underline{\textbf{3.857 ohms}}$$

For V_{Th}, consider the circuit in Fig. (b). Applying KVL gives,

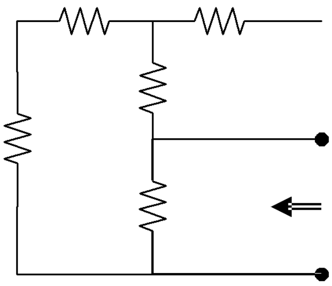
$$10 – 24 + i(3 + 4 + 5 + 2), \text{ or } i = 1$$

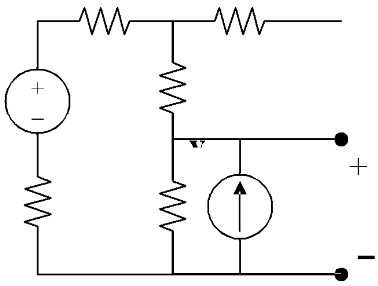
$$V_{Th} = 4i = \underline{\textbf{4 V}}$$





(b) For R_{Th}, consider the circuit in Fig. (c).





$$R_{Th} = 5||(2 + 3 + 4) = \underline{\textbf{3.214 ohms}}$$

To get V_{Th}, consider the circuit in Fig. (d). At the node, KCL gives,

$$[(24 – vo)/9] + 2 = vo/5, \text{ or } vo = 15$$

$$V_{Th} = vo = \underline{\mathbf{15\ V}}$$

Chapter 4, Problem 45.

Find the Thevenin equivalent of the circuit in Fig. 4.112.

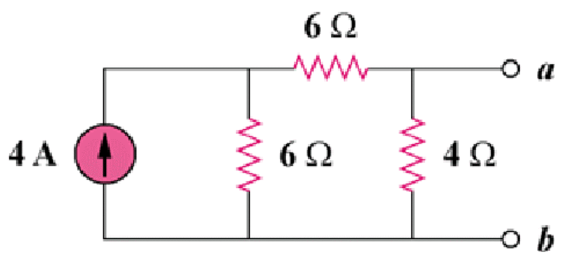
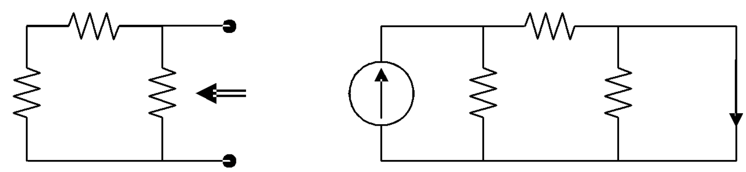


Figure 4.112

Chapter 4, Solution 45.

For R_N, consider the circuit in Fig. (a).



$$R_N = (6 + 6)||4 = \underline{\mathbf{3\ ohms}}$$

For I_N, consider the circuit in Fig. (b). The 4-ohm resistor is shorted so that 4-A current is equally divided between the two 6-ohm resistors. Hence,

$$I_N = 4/2 = \underline{\mathbf{2\ A}}$$

Chapter 4, Problem 46.
Find the Norton equivalent at terminals a-b of the circuit in Fig. 4.113.

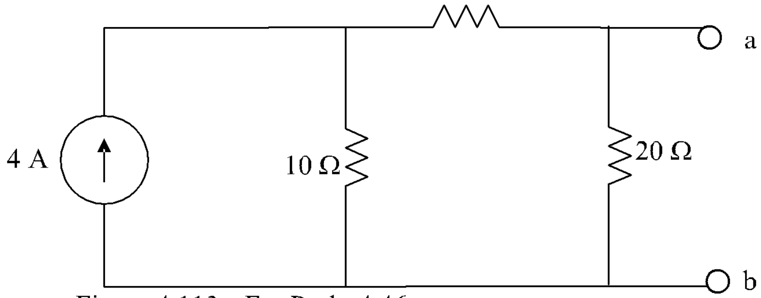
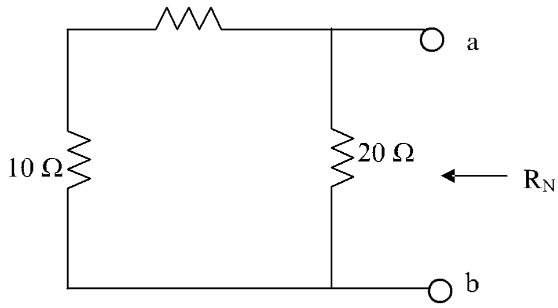
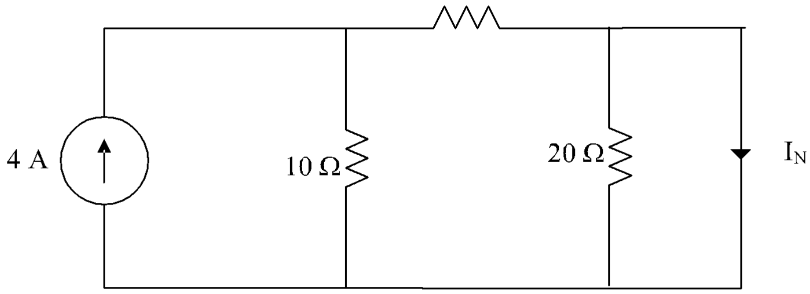


Figure 4.113 For Prob. 4.46.

Chapter 4, Solution 46.
R_N is found using the circuit below.



$R_N = 20//(10+10) = 10\ \Omega$
To find I_N, consider the circuit below.



The 20-Ω resistor is short-circuited and can be ignored.

$$I_N = \frac{1}{2} \times 4 = \underline{\mathbf{2\ A}}$$

Chapter 4, Problem 47.

Obtain the Thèvenin and Norton equivalent circuits of the circuit in Fig. 4.114 with respect to terminals *a* and *b*.

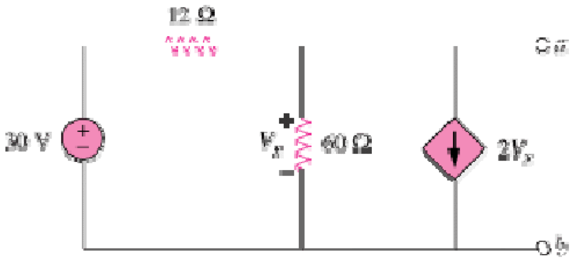


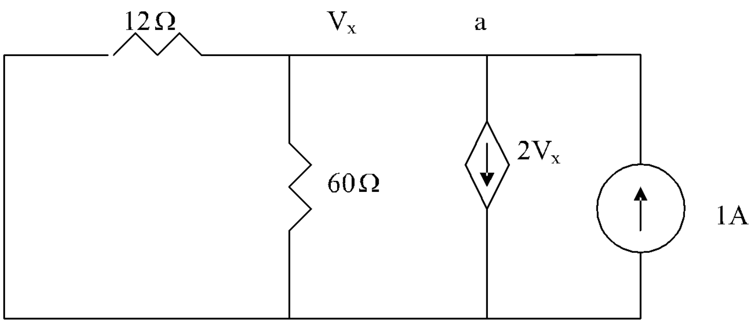
Figure 4.114

Chapter 4, Solution 47

Since $V_{Th} = V_{ab} = V_x$, we apply KCL at the node a and obtain

$$\frac{30 - V_{Th}}{12} = \frac{V_{Th}}{60} + 2V_{Th} \quad \longrightarrow \quad V_{Th} = 150/126 = 1.19\text{ V}$$

To find R_{Th}, consider the circuit below.



At node a, KCL gives

$$1 = 2V_x + \frac{V_x}{60} + \frac{V_x}{12} \quad \longrightarrow \quad V_x = 60/126 = 0.4762$$

$$R_{Th} = \frac{V_x}{1} = 0.4762\Omega, \qquad I_N = \frac{V_{Th}}{R_{Th}} = 1.19/0.4762 = 2.5$$

Thus,

$$\underline{V_{Th} = 1.19V, \qquad R_{Th} = R_N = 0.4762\Omega, \qquad I_N = 2.5\text{ A}}$$

Chapter 4, Problem 48.

Determine the Norton equivalent at terminals ***a-b*** for the circuit in Fig. 4.115.

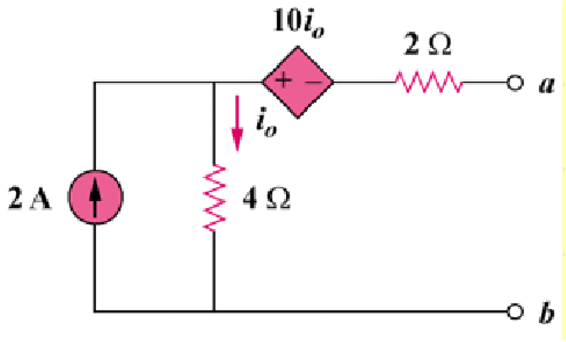
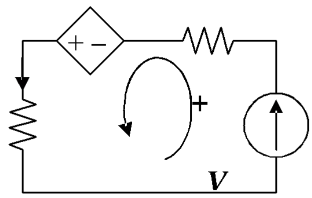
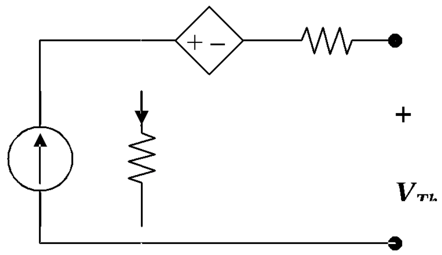


Figure 4.115

Chapter 4, Solution 48.

To get R_{Th}, consider the circuit in Fig. (a).





From Fig. (a), $\quad I_o = 1, \quad 6 - 10 - V = 0, \text{ or } V = -4$

$$R_N = R_{Th} = V/1 = \underline{\mathbf{-4\ ohms}}$$

To get V_{Th}, consider the circuit in Fig. (b),

$$I_o = 2, \; V_{Th} = -10I_o + 4I_o = -12\text{ V}$$

$$I_N = V_{Th}/R_{Th} = \underline{\mathbf{3A}}$$

Chapter 4, Problem 49.

Find the Norton equivalent looking into terminals ***a-b*** of the circuit in Fig. 4.102.

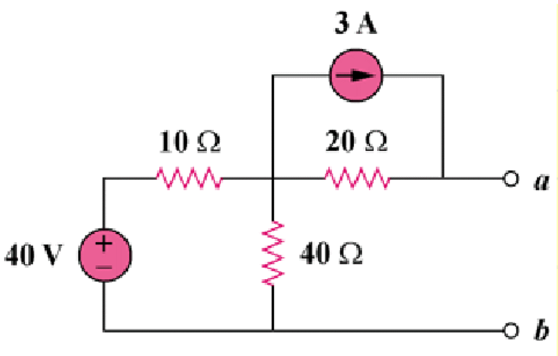
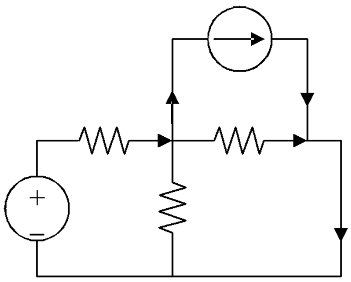


Figure 4.102

Chapter 4, Solution 49.

$R_N = R_{Th} =$ **28 ohms**

To find I_N, consider the circuit below,



At the node, $(40 - v_o)/10 = 3 + (v_o/40) + (v_o/20)$, or $v_o = 40/7$

$i_o = v_o/20 = 2/7$, but $I_N = I_{sc} = i_o + 3 =$ **3.286 A**

Chapter 4, Problem 50.

Obtain the Norton equivalent of the circuit in Fig. 4.116 to the left of terminals ***a***-***b***. Use the result to find current ***i***

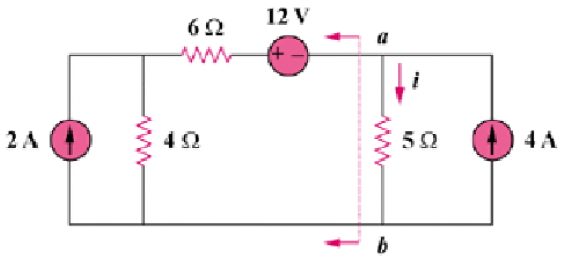
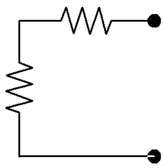
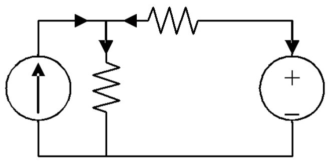


Figure 4.116

Chapter 4, Solution 50.

From Fig. (a), R_N = 6 + 4 = **<u>10 ohms</u>**

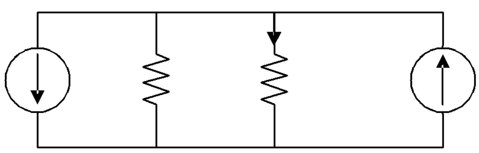




From Fig. (b), $2 + (12 – v)/6 = v/4$, or $v = 9.6$ V

$-I_N = (12 – v)/6 = 0.4$, which leads to I_N = **<u>-0.4 A</u>**

Combining the Norton equivalent with the right-hand side of the original circuit produces the circuit in Fig. (c).



$i = [10/(10 + 5)] (4 – 0.4)$ = **<u>2.4 A</u>**

Chapter 4, Problem 51.

Given the circuit in Fig. 4.117, obtain the Norton equivalent as viewed from terminals
(a) ***a-b*** (b) ***c-d***

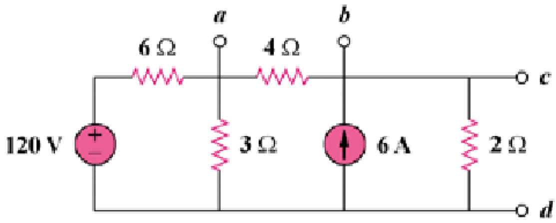
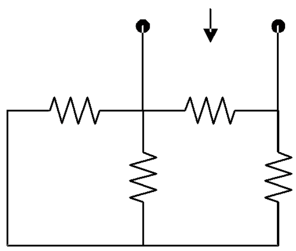


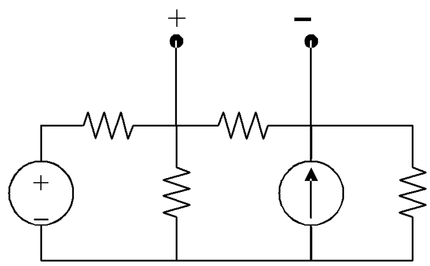
Figure 4.117

Chapter 4, Solution 51.

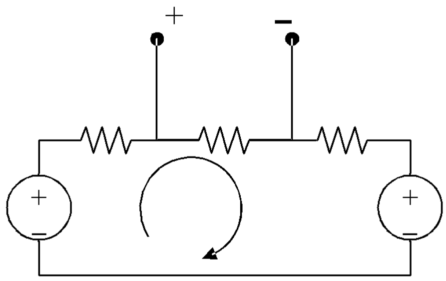
(a) From the circuit in Fig. (a),

$$R_N = 4||(2 + 6||3) = 4||4 = \underline{\textbf{2 ohms}}$$





For I_N or V_{Th}, consider the circuit in Fig. (b). After some source transformations, the circuit becomes that shown in Fig. (c).



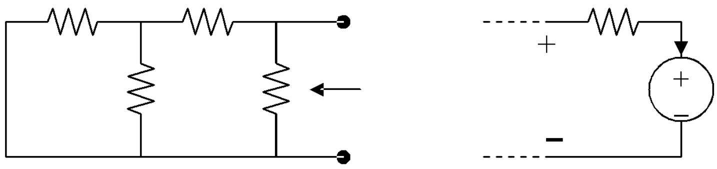
Applying KVL to the circuit in Fig. (c),

$$-40 + 8i + 12 = 0 \text{ which gives } i = 7/2$$

$$V_{Th} = 4i = 14 \text{ therefore } I_N = V_{Th}/R_N = 14/2 = \underline{\mathbf{7\ A}}$$

(b) To get R_N, consider the circuit in Fig. (d).

$$R_N = 2||(4 + 6||3) = 2||6 = \underline{\mathbf{1.5\ ohms}}$$



To get I_N, the circuit in Fig. (c) applies except that it needs slight modification as in Fig. (e).

$$i = 7/2,\ V_{Th} = 12 + 2i = 19,\ I_N = V_{Th}/R_N = 19/1.5 = \underline{\mathbf{12.667\ A}}$$

Chapter 4, Problem 52.

For the transistor model in Fig. 4.118, obtain the Thevenin equivalent at terminals ***a-b***.

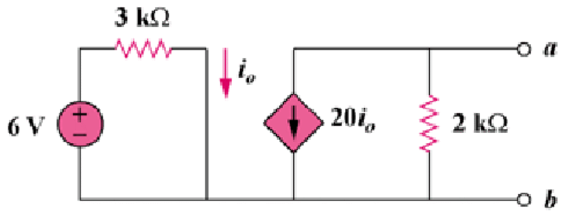
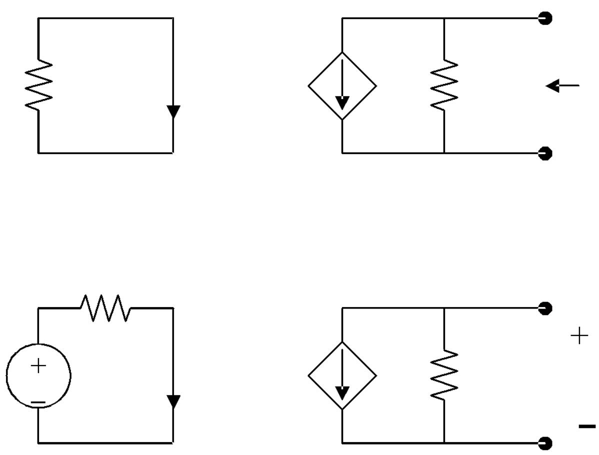


Figure 4.118

Chapter 4, Solution 52.

For R_{Th}, consider the circuit in Fig. (a).



For Fig. (a), $I_o = 0$, hence the current source is inactive and

$$R_{Th} = \underline{\mathbf{2\ k\ ohms}}$$

For V_{Th}, consider the circuit in Fig. (b).

$$I_o = 6/3k = 2 \text{ mA}$$

$$V_{Th} = (-20I_o)(2k) = -20x2x10^{-3}x2x10^{3} = \underline{\mathbf{-80\ V}}$$

Chapter 4, Problem 53.

Find the Norton equivalent at terminals ***a-b*** of the circuit in Fig. 4.119.

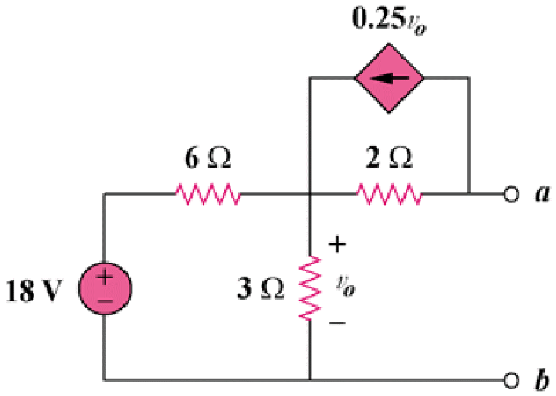
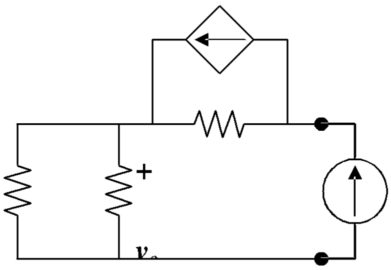
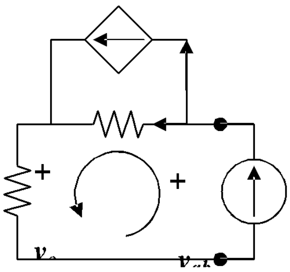


Figure 4.119

Chapter 4, Solution 53.

To get R_{Th}, consider the circuit in Fig. (a).





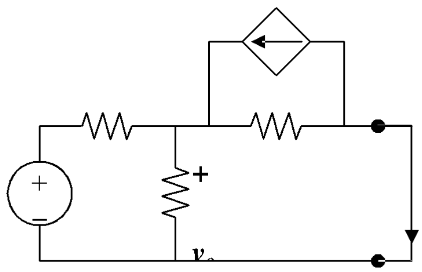
From Fig. (b),

$$v_o = 2x1 = 2V, \quad -v_{ab} + 2x(1/2) + v_o = 0$$

$$v_{ab} = 3V$$

$$R_N = v_{ab}/1 = \underline{\mathbf{3\ ohms}}$$

To get I_N, consider the circuit in Fig. (c).



$$[(18 - v_o)/6] + 0.25v_o = (v_o/2) + (v_o/3) \quad \text{or} \quad v_o = 4V$$

But, $\quad (v_o/2) = 0.25v_o + I_N$, which leads to $I_N = \underline{\mathbf{1\ A}}$

Chapter 4, Problem 54.

Find the Thèvenin equivalent between terminals *a*-*b* of the circuit in Fig. 4.120.

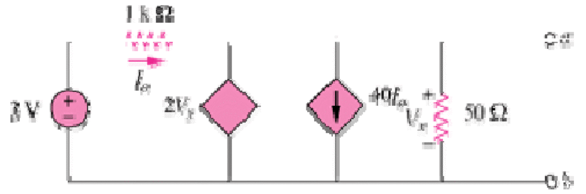


Figure 4.120

Chapter 4, Solution 54

To find $V_{Th}=V_x$, consider the left loop.

$$-3+1000i_o+2V_x=0 \longrightarrow 3=1000i_o+2V_x \qquad (1)$$

For the right loop,

$$V_x=-50x40i_o=-2000i_o \qquad (2)$$

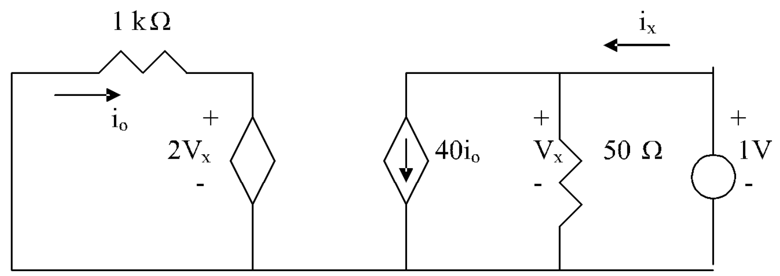
Combining (1) and (2),

$$3=1000i_o-4000i_o=-3000i_o \longrightarrow i_o=-1\text{mA}$$

$$V_x=-2000i_o=2 \longrightarrow \underline{V_{Th}=2}$$

To find R_{Th}, insert a 1-V source at terminals a-b and remove the 3-V independent source, as shown below.

.



$$V_x=1, \qquad i_o=-\frac{2V_x}{1000}=-2\text{mA}$$

$$i_x=40i_o+\frac{V_x}{50}=-80\text{mA}+\frac{1}{50}\text{A}=\text{-60mA}$$

$$R_{Th} = \frac{1}{i_x} = -1/0.060 = \underline{-16.67\Omega}$$

Chapter 4, Problem 55.

Obtain the Norton equivalent at terminals ***a***-***b*** of the circuit in Fig. 4.121.

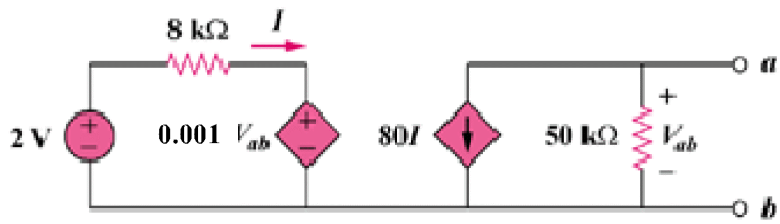
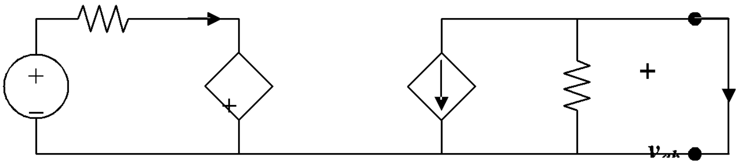


Figure 4.121

Chapter 4, Solution 55.

To get R_N, apply a 1 mA source at the terminals a and b as shown in Fig. (a).



We assume all resistances are in k ohms, all currents in mA, and all voltages in volts. At node a,

$$(v_{ab}/50) + 80I = 1 \quad (1)$$

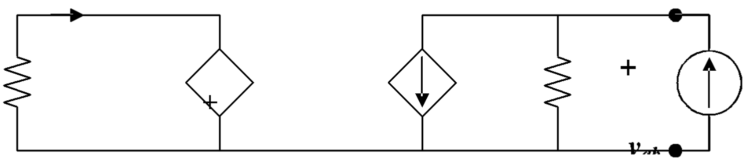
Also,

$$-8I = (v_{ab}/1000), \text{ or } I = -v_{ab}/8000 \quad (2)$$

From (1) and (2), $(v_{ab}/50) - (80v_{ab}/8000) = 1$, or $v_{ab} = 100$

$R_N = v_{ab}/1 =$ **<u>100 k ohms</u>**

To get I_N, consider the circuit in Fig. (b).



Since the 50-k ohm resistor is shorted,

$$I_N = -80I, \; v_{ab} = 0$$

Hence, $8i = 2$ which leads to $I = (1/4)$ mA

$I_N =$ **<u>-20 mA</u>**

Chapter 4, Problem 56.

Use Norton's theorem to find V_o in the circuit of Fig. 4.122.

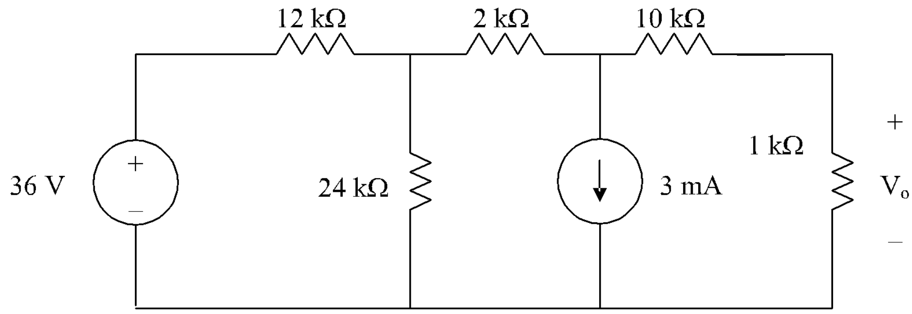
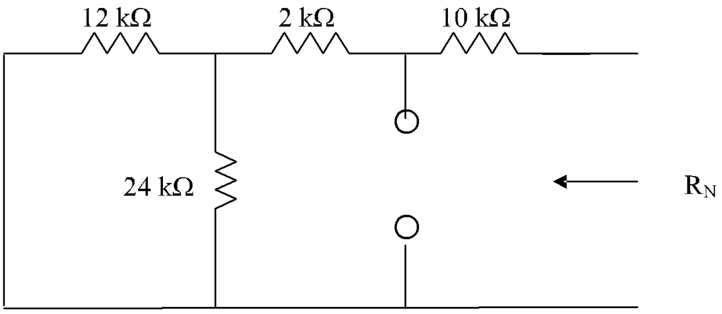


Figure 4.122 For Prob. 4.56.

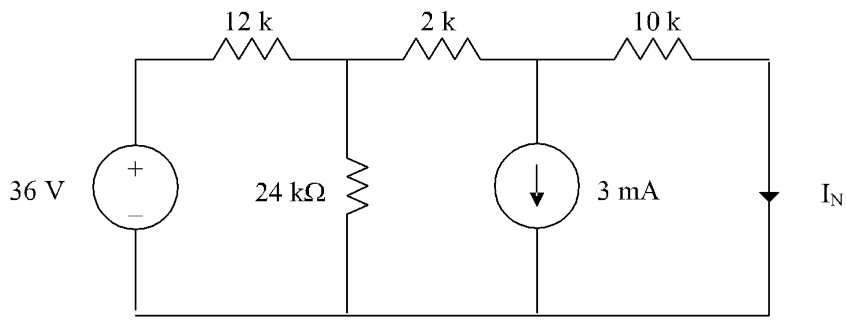
Chapter 4, Solution 56.

We remove the 1-kΩ resistor temporarily and find Norton equivalent across its terminals. R_N is obtained from the circuit below.

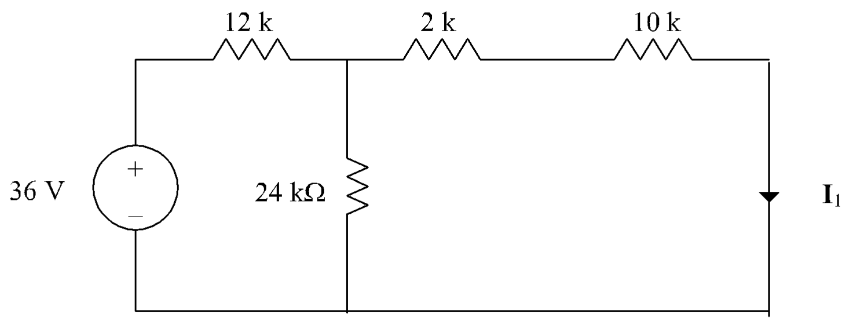


$R_N = 10 + 2 + 12//24 = 12+8 = 20$ kΩ

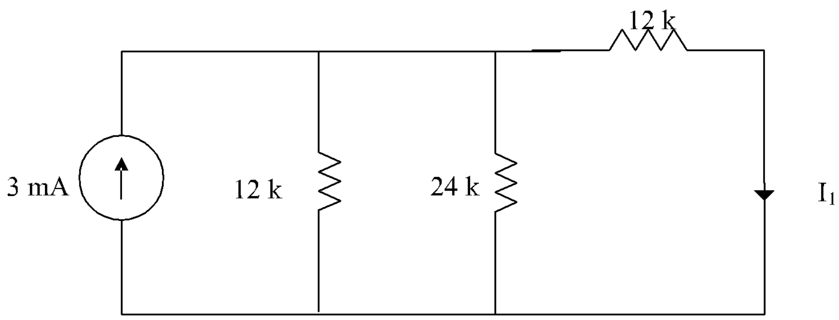
I_N is obtained from the circuit below.



We can use superposition theorem to find I_N. Let $I_N = I_1 + I_2$, where I_1 and I_2 are due to 16-V and 3-mA sources respectively. We find I_1 using the circuit below.



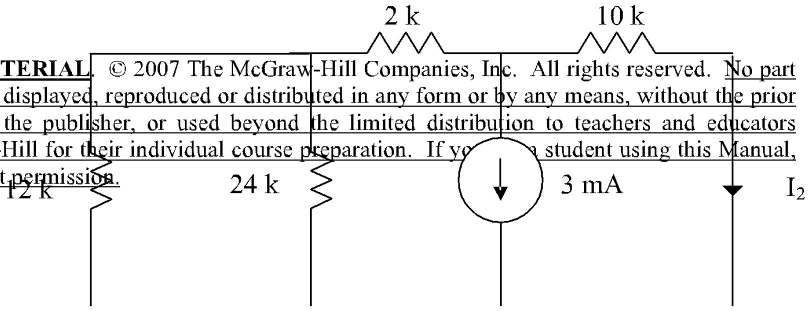
Using source transformation, we obtain the circuit below.



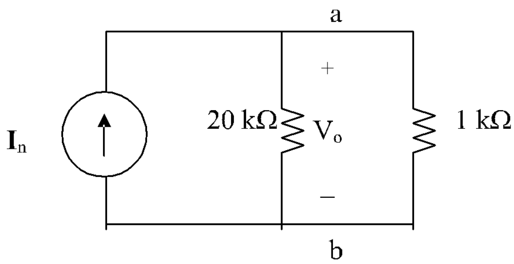
12//24 = 8 kΩ

$$I_1 = \frac{8}{8+12}(3mA) = 1.2 \text{ mA}$$

To find I_2, consider the circuit below.



2k + 12k//24 k = 10 kΩ
I_2=0.5(-3mA) = -1.5 mA
I_N = 1.2 –1.5 = -0.3 mA
The Norton equivalent with the 1-kΩ resistor is shown below



$$V_o = 1k\left(\frac{20}{20+1}\right)(-0.3\ \text{mA}) = \underline{-0.2857\ \text{V}}$$

Chapter 4, Problem 57.

Obtain the Thevenin and Norton equivalent circuits at the terminals ***a-b*** for the circuit in Fig. 4.123.

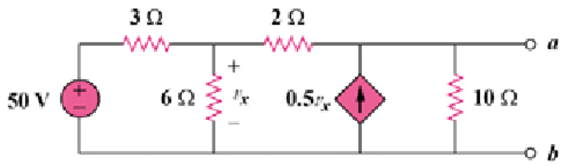
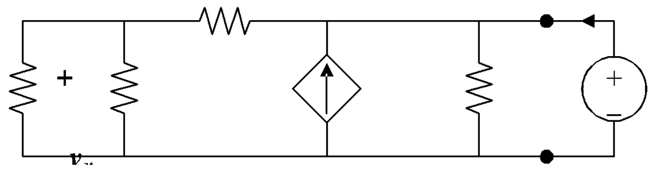


Figure 4.123

Chapter 4, Solution 57.

To find R_{Th}, remove the 50V source and insert a 1-V source at a – b, as shown in Fig. (a).



We apply nodal analysis. At node A,

$$i + 0.5v_x = (1/10) + (1 - v_x)/2, \text{ or } i + v_x = 0.6 \qquad (1)$$

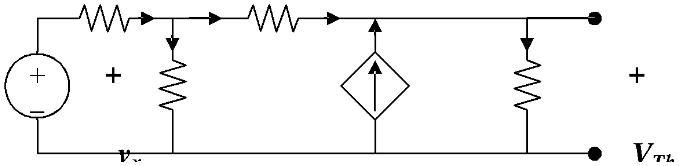
At node B,

$$(1 - v_o)/2 = (v_x/3) + (v_x/6), \text{ and } v_x = 0.5 \qquad (2)$$

From (1) and (2), $i = 0.1$ and

$$R_{Th} = 1/i = \underline{\textbf{10 ohms}}$$

To get V_{Th}, consider the circuit in Fig. (b).



At node 1, $(50 - v_1)/3 = (v_1/6) + (v_1 - v_2)/2$, or $100 = 6v_1 - 3v_2$ (3)

At node 2, $0.5v_x + (v_1 - v_2)/2 = v_2/10$, $v_x = v_1$, and $v_1 = 0.6v_2$ (4)

From (3) and (4),

$$v_2 = V_{Th} = \underline{\textbf{166.67 V}}$$

$$I_N = V_{Th}/R_{Th} = \underline{\textbf{16.667 A}}$$

$$R_N = R_{Th} = \underline{\textbf{10 ohms}}$$

Chapter 4, Problem 58.

The network in Fig. 4.124 models a bipolar transistor common-emitter amplifier connected to a load. Find the Thevenin resistance seen by the load.

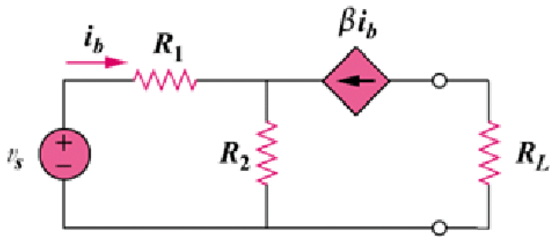
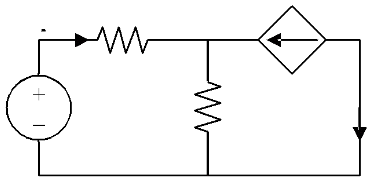


Figure 4.124

Chapter 4, Solution 58.

This problem does not have a solution as it was originally stated. The reason for this is that the load resistor is in series with a current source which means that the only equivalent circuit that will work will be a Norton circuit where the value of R_N = **infinity**. I_N can be found by solving for I_{sc}.



Writing the node equation at node v_o,

$$i_b + \beta i_b = v_o/R_2 = (1 + \beta)i_b$$

But $$i_b = (V_s - v_o)/R_1$$

$$v_o = V_s - i_b R_1$$

$$V_s - i_b R_1 = (1 + \beta)R_2 i_b, \text{ or } i_b = V_s/(R_1 + (1 + \beta)R_2)$$

$$I_{sc} = I_N = -\beta i_b = \underline{-\beta V_s/(R_1 + (1 + \beta)R_2)}$$

Chapter 4, Problem 59.

Determine the Thevenin and Norton equivalents at terminals ***a-b*** of the circuit in Fig. 4.125.

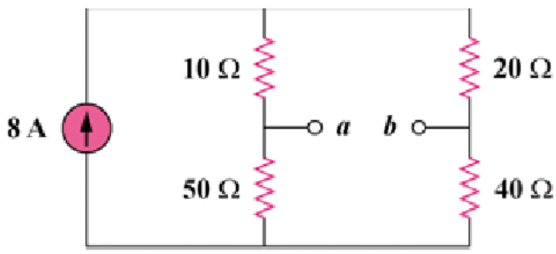
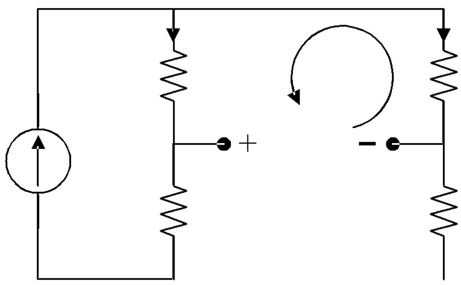


Figure 4.125

Chapter 4, Solution 59.

$R_{Th} = (10 + 20)||(50 + 40)\ 30||90 = \underline{\mathbf{22.5\ ohms}}$

To find V_{Th}, consider the circuit below.



$i_1 = i_2 = 8/2 = 4,\ 10i_1 + V_{Th} - 20i_2 = 0,\ \text{or}\ V_{Th} = 20i_2 - 10i_1 = 10i_1 = 10x4$

$$V_{Th} = \underline{\mathbf{40V}},\ \text{and}\ I_N = V_{Th}/R_{Th} = 40/22.5 = \underline{\mathbf{1.7778\ A}}$$

Chapter 4, Problem 60.

For the circuit in Fig. 4.126, find the Thevenin and Norton equivalent circuits at terminals ***a-b***.

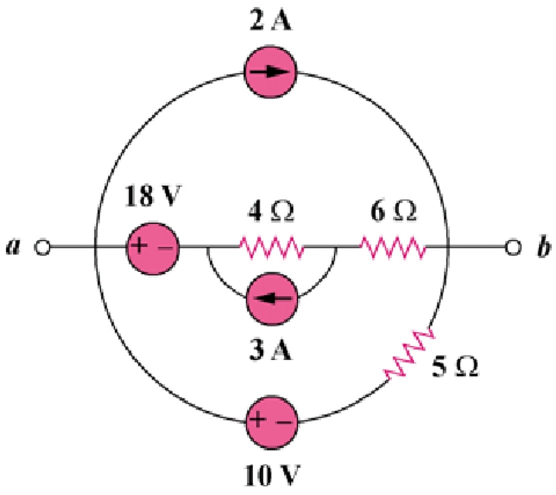
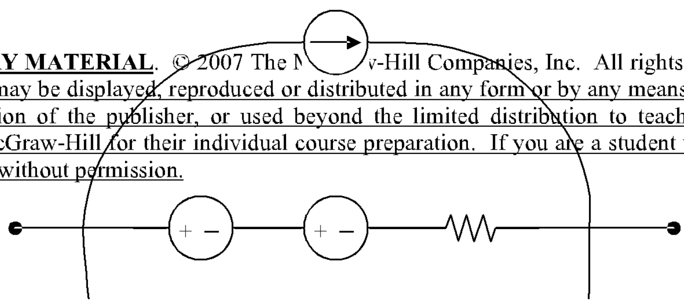
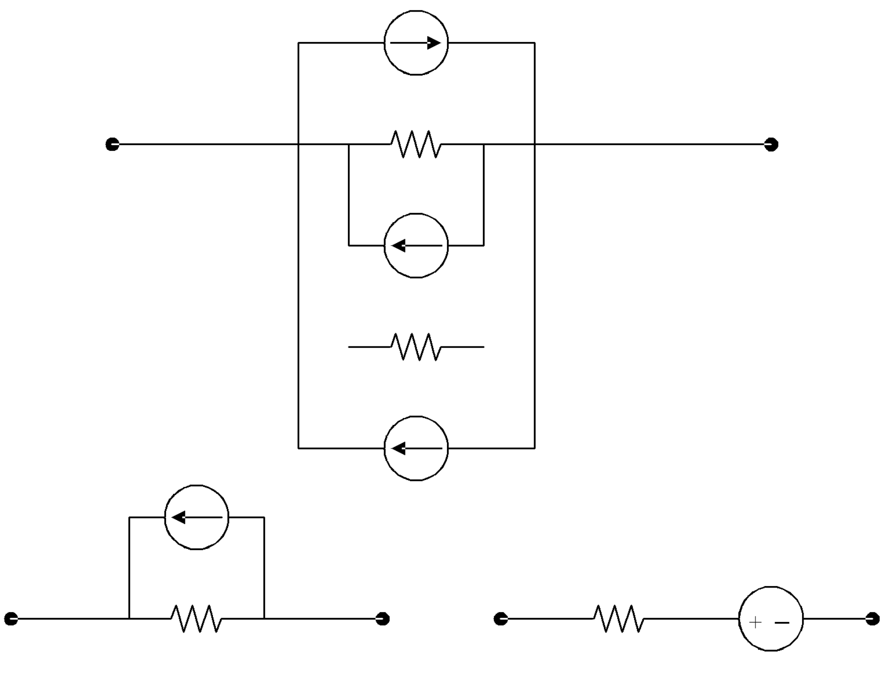


Figure 4.126

Chapter 4, Solution 60.

The circuit can be reduced by source transformations.





Chapter 4, Problem 61.

Obtain the Thevenin and Norton equivalent circuits at terminals ***a-b*** of the circuit in Fig. 4.127.

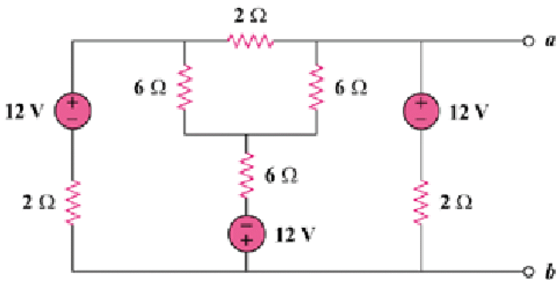


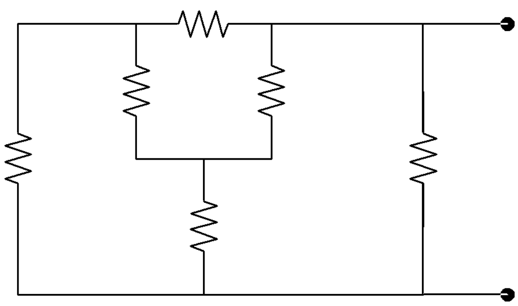
Figure 4.127

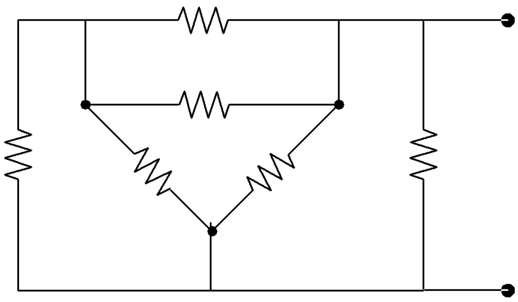
Chapter 4, Solution 61.

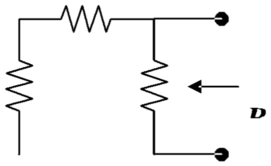
To find R_{Th}, consider the circuit in Fig. (a).

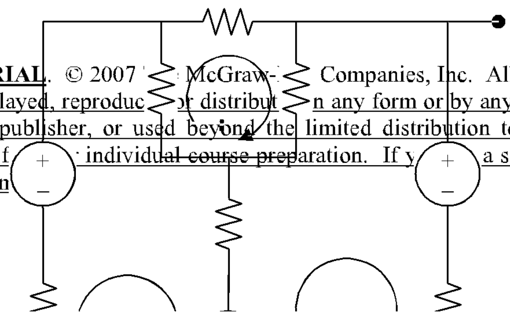
Let $\quad R = 2||18 = $ **1.8 ohms**, $\quad R_{Th} = 2R||R = (2/3)R = $ **1.2 ohms**.

To get V_{Th}, we apply mesh analysis to the circuit in Fig. (d).









$-12 - 12 + 14i_1 - 6i_2 - 6i_3 = 0$, and $7i_1 - 3i_2 - 3i_3 = 12$ (1)

$12 + 12 + 14i_2 - 6i_1 - 6i_3 = 0$, and $-3i_1 + 7i_2 - 3i_3 = -12$ (2)

$14i_3 - 6i_1 - 6i_2 = 0$, and $-3i_1 - 3i_2 + 7i_3 = 0$ (3)

This leads to the following matrix form for (1), (2) and (3),

$$\begin{bmatrix} 7 & -3 & -3 \\ -3 & 7 & -3 \\ -3 & -3 & 7 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ -12 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 7 & -3 & -3 \\ -3 & 7 & -3 \\ -3 & -3 & 7 \end{vmatrix} = 100, \quad \Delta_2 = \begin{vmatrix} 7 & 12 & -3 \\ -3 & -12 & -3 \\ -3 & 0 & 7 \end{vmatrix} = -120$$

$$i_2 = \Delta/\Delta_2 = -120/100 = -1.2 \text{ A}$$

$$V_{Th} = 12 + 2i_2 = \underline{\mathbf{9.6\ V}}, \text{ and } I_N = V_{Th}/R_{Th} = \underline{\mathbf{8\ A}}$$

Chapter 4, Problem 62.

Find the Thevenin equivalent of the circuit in Fig. 4.128.

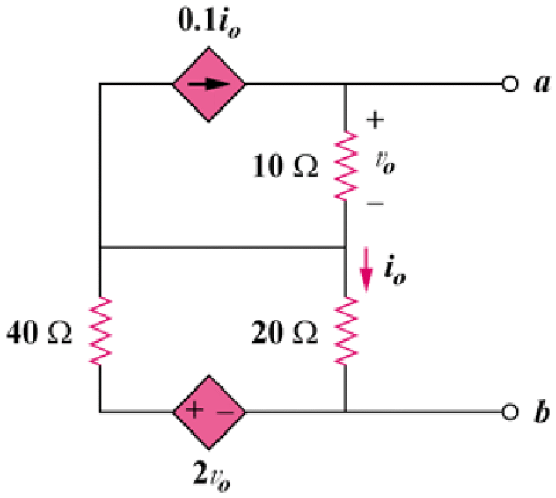
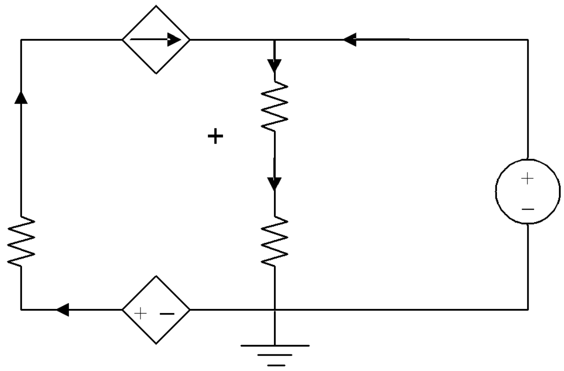


Figure 4.128

Chapter 4, Solution 62.

Since there are no independent sources, V_{Th} = 0 V

To obtain R_{Th}, consider the circuit below.



At node 2,

$$i_x + 0.1i_o = (1 - v_1)/10, \text{ or } 10i_x + i_o = 1 - v_1 \qquad (1)$$

At node 1,

$$(v_1/20) + 0.1i_o = [(2v_o - v_1)/40] + [(1 - v_1)/10] \qquad (2)$$

But $i_o = (v_1/20)$ and $v_o = 1 - v_1$, then (2) becomes,

$$1.1v_1/20 = [(2 - 3v_1)/40] + [(1 - v_1)/10]$$

$$2.2v_1 = 2 - 3v_1 + 4 - 4v_1 = 6 - 7v_1$$

or

$$v_1 = 6/9.2 \qquad (3)$$

From (1) and (3),

$$10i_x + v_1/20 = 1 - v_1$$

$$10i_x = 1 - v_1 - v_1/20 = 1 - (21/20)v_1 = 1 - (21/20)(6/9.2)$$

$$i_x = 31.52 \text{ mA}, \quad R_{Th} = 1/i_x = \underline{\textbf{31.73 ohms.}}$$

Chapter 4, Problem 63.

Find the Norton equivalent for the circuit in Fig. 4.129.

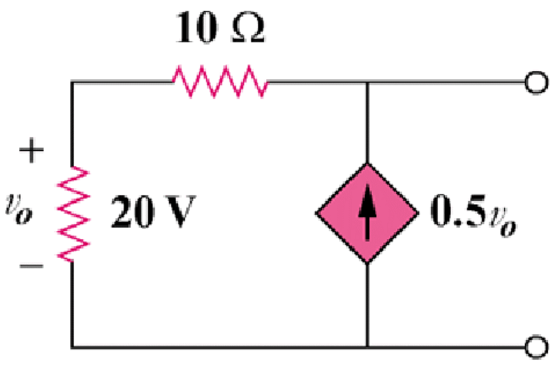
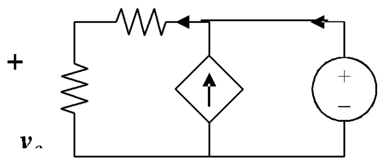


Figure 4.129

Chapter 4, Solution 63.

Because there are no independent sources, $I_N = I_{sc} =$ **<u>0 A</u>**

R_N can be found using the circuit below.



Applying KCL at node 1, $v_1 = 1$, and $v_o = (20/30)v_1 = 2/3$

$i_o = (v_1/30) - 0.5v_o = (1/30) - 0.5x2/3 = 0.03333 - 0.33333 = - 0.3$ A.

Hence,

$R_N = 1/(-0.3) =$ **<u>−3.333 ohms</u>**

Chapter 4, Problem 64.

Obtain the Thevenin equivalent seen at terminals ***a-b*** of the circuit in Fig. 4.130.

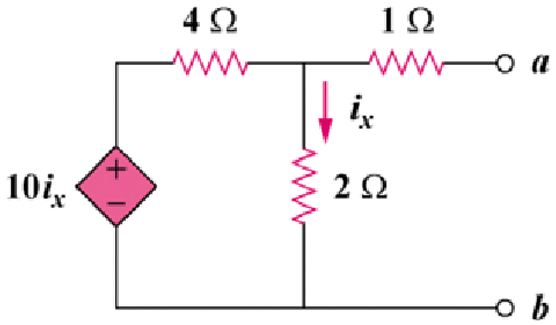
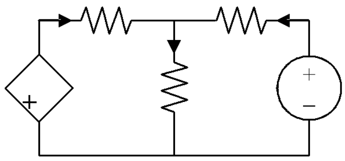


Figure 4.130

Chapter 4, Solution 64.

With no independent sources, V_{Th} = **0 V**. To obtain R_{Th}, consider the circuit shown below.



$$i_x = [(1 - v_o)/1] + [(10i_x - v_o)/4], \text{ or } 5v_o = 4 + 6i_x \qquad (1)$$

But $i_x = v_o/2$. Hence,

$$5v_o = 4 + 3v_o, \text{ or } v_o = 2,\ i_o = (1 - v_o)/1 = -1$$

$$\text{Thus, } R_{Th} = 1/i_o = \underline{\mathbf{-1\ ohm}}$$

Chapter 4, Problem 65.

For the circuit shown in Fig. 4.131, determine the relationship between V_o and I_o.

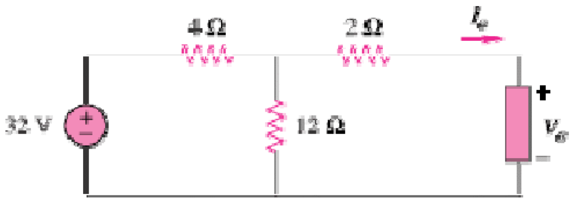


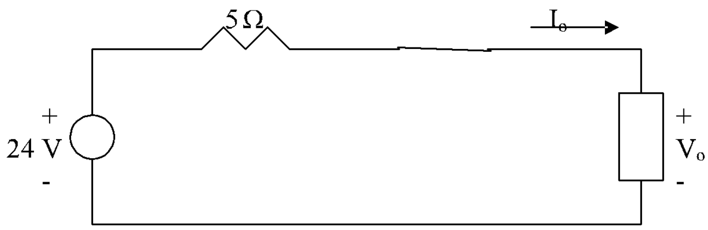
Figure 4.131

Chapter 4, Solution 65

At the terminals of the unknown resistance, we replace the circuit by its Thevenin equivalent.

$R_{Th} = 2 + 4//12 = 2 + 3 = 5\Omega, \qquad V_{Th} = \frac{12}{12+4}(32) = 24\text{ V}$

Thus, the circuit can be replaced by that shown below.



Applying KVL to the loop,

$$-24 + 5I_o + V_o = 0 \qquad \longrightarrow \qquad \underline{V_o = 24 - 5I_o}$$

Chapter 4, Problem 66.

Find the maximum power that can be delivered to the resistor ***R*** in the circuit in Fig. 4.132.

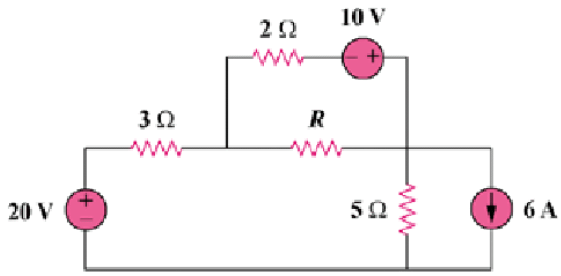
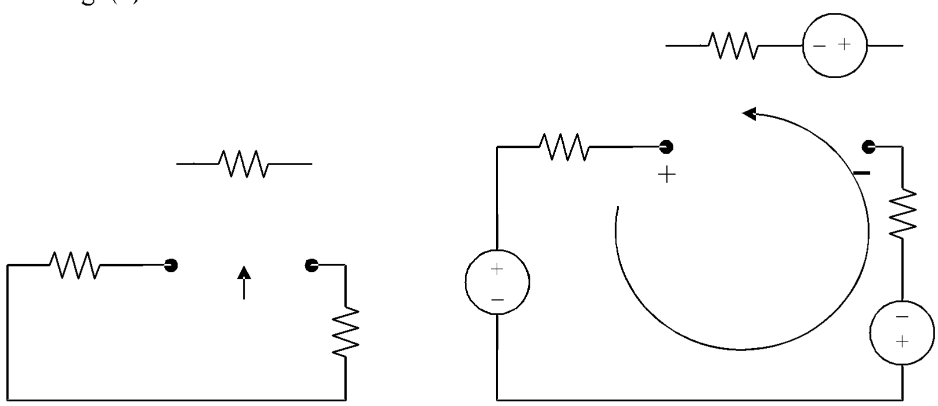


Figure 4.132

Chapter 4, Solution 66.

We first find the Thevenin equivalent at terminals a and b. We find R_{Th} using the circuit in Fig. (a).



$$R_{Th} = 2||(3 + 5) = 2||8 = \underline{\textbf{1.6 ohms}}$$

By performing source transformation on the given circuit, we obatin the circuit in (b). We now use this to find V_{Th}.

$$10i + 30 + 20 + 10 = 0, \text{ or } i = -6$$

$$V_{Th} + 10 + 2i = 0, \text{ or } V_{Th} = 2 \text{ V}$$

$$p = V_{Th}^2/(4R_{Th}) = (2)^2/[4(1.6)] = \underline{\textbf{625 m watts}}$$

Chapter 4, Problem 67.

The variable resistor R in Fig. 4.133 is adjusted until it absorbs the maximum power from the circuit. (a) Calculate the value of R for maximum power. (b) Determine the maximum power absorbed by R.

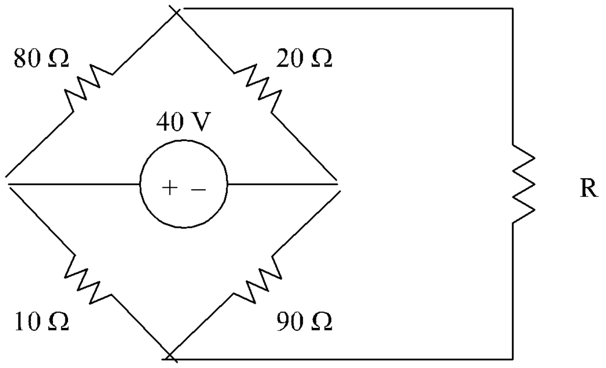
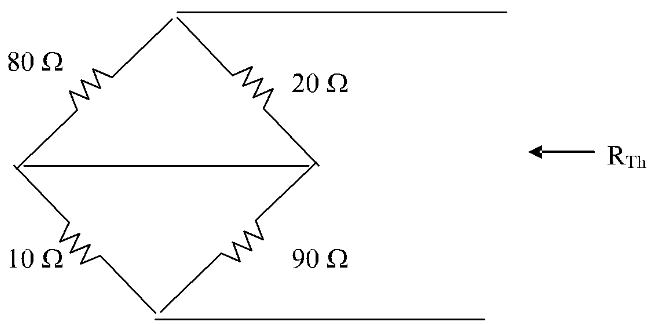


Figure 4.133 For Prob. 4.67.

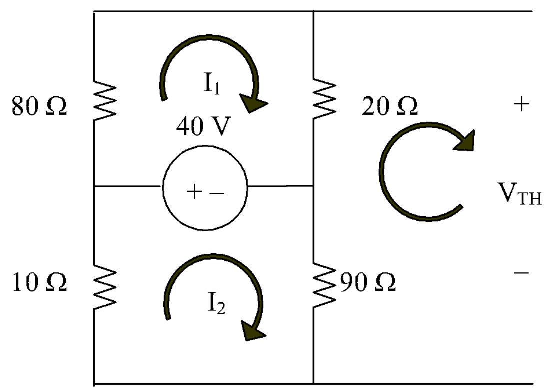
Chapter 4, Solution 67.

We first find the Thevenin equivalent. We find R_{Th} using the circuit below.



$$R_{Th} = 20//80 + 90//10 = 16 + 9 = 25\ \Omega$$

We find V_{Th} using the circuit below. We apply mesh analysis.



$$(80+20)i_1 - 40 = 0 \longrightarrow i_1 = 0.4$$
$$(10+90)i_2 + 40 = 0 \longrightarrow i_2 = -0.4$$
$$-90i_2 - 20i_1 + V_{Th} = 0 \longrightarrow V_{Th} = -28\text{ V}$$

(a) $R = R_{Th} = \underline{25\ \Omega}$

(b) $P_{max} = \dfrac{V_{Th}^2}{4R_{Th}} = \dfrac{(28)^2}{100} = \underline{7.84\text{ W}}$

Chapter 4, Problem 68.

Compute the value of ***R*** that results in maximum power transfer to the 10-Ω resistor in Fig. 4.134. Find the maximum power.

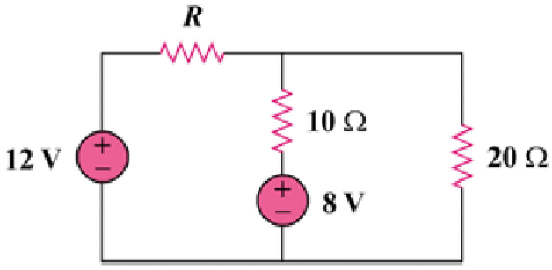
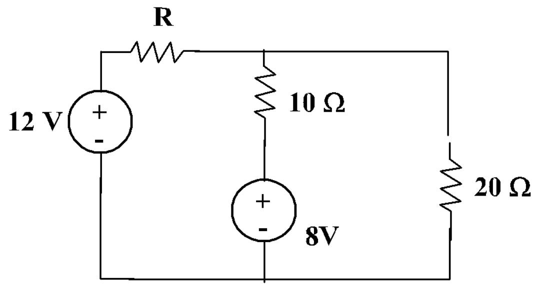


Figure 4.134

Chapter 4, Solution 68.
This is a challenging problem in that the load is already specified. This now becomes a "minimize losses" style problem. When a load is specified and internal losses can be adjusted, then the objective becomes, reduce R_{Thev} as much as possible, which will result in maximum power transfer to the load.



Removing the 10 ohm resistor and solving for the Thevenin Circuit results in:

$$R_{Th} = (Rx20/(R+20)) \text{ and a } V_{oc} = V_{Th} = 12x(20/(R+20)) + (-8)$$

As R goes to zero, R_{Th} goes to zero and V_{Th} goes to 4 volts, which produces the maximum power delivered to the 10-ohm resistor.

$$P = vi = v^2/R = 4x4/10 = 1.6 \text{ watts}$$

Notice that if R = 20 ohms which gives an R_{Th} = 10 ohms, then V_{Th} becomes -2 volts and the power delivered to the load becomes 0.1 watts, much less that the 1.6 watts.

It is also interesting to note that the internal losses for the first case are $12^2/20 = 7.2$ watts and for the second case are = to 12 watts. This is a significant difference.

Chapter 4, Problem 69.

Find the maximum power transferred to resistor ***R*** in the circuit of Fig. 4.135.

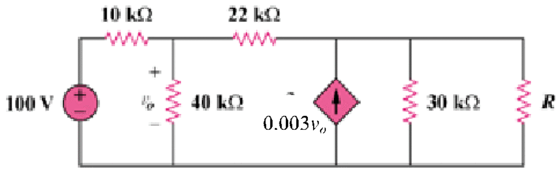
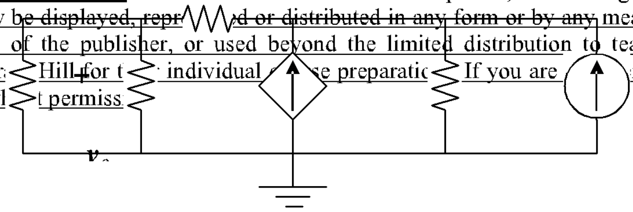


Figure 4.135

Chapter 4, Solution 69.



We need the Thevenin equivalent across the resistor R. To find R_{Th}, consider the circuit below.

Assume that all resistances are in k ohms and all currents are in mA.

$$10||40 \;=\; 8,\;\text{ and }\; 8 + 22 \;=\; 30$$

$$1 + 3v_o \;=\; (v_I/30) + (v_I/30) \;=\; (v_I/15)$$

$$15 + 45v_o \;=\; v_I$$

But $v_o \;=\; (8/30)v_I$, hence,

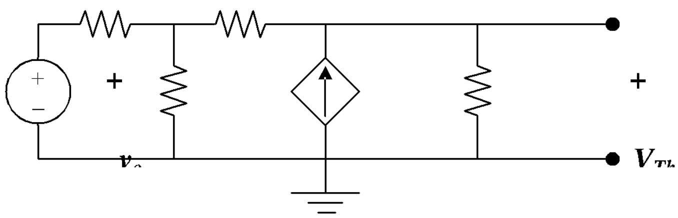
$$15 + 45x(8v_I/30)\;\; v_I,\;\text{ which leads to }\; v_I \;=\; 1.3636$$

$$R_{Th} \;=\; v_I/1 \;=\; -1.3636\text{ k ohms}$$

R_{Th} being negative indicates an active circuit and if you now make R equal to 1.3636 k ohms, then the active circuit will actually try to supply infinite power to the resistor. The correct answer is therefore:

$$p_R = \left(\frac{V_{Th}}{-1363.6+1363.6}\right)^2 1363.6 = \left(\frac{V_{Th}}{0}\right)^2 1363.6 \;=\; \underline{\infty}$$

It may still be instructive to find V_{Th}. Consider the circuit below.



$$(100 - v_o)/10 \;=\; (v_o/40) + (v_o - v_I)/22 \qquad (1)$$

$$[(v_o - v_I)/22] + 3v_o \;=\; (v_I/30) \qquad (2)$$

Solving (1) and (2),

$$v_I \;=\; V_{Th} \;=\; \text{-243.6 volts}$$

Chapter 4, Problem 70.

Determine the maximum power delivered to the variable resistor *R* shown in the circuit of Fig. 4.136.

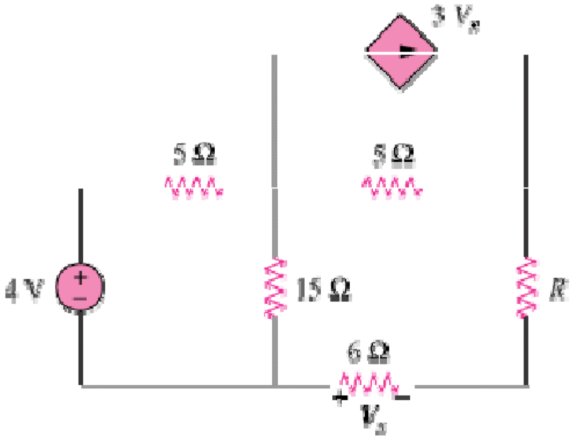
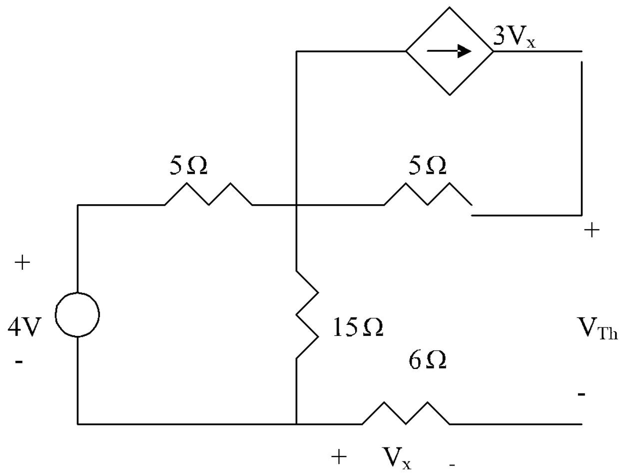


Figure 4.136

Chapter 4, Solution 70

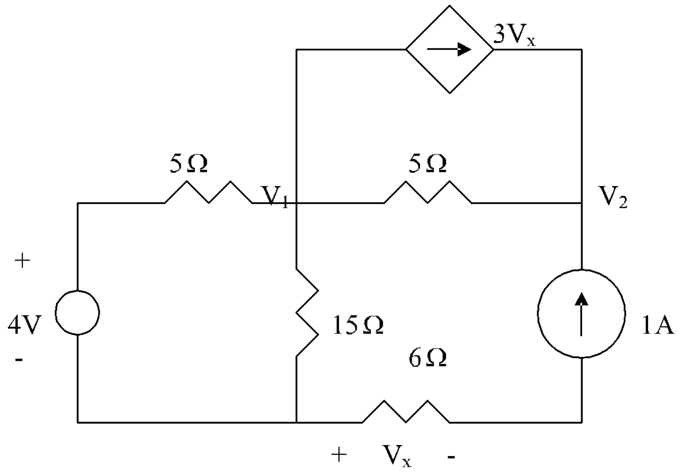
We find the Thevenin equivalent across the 10-ohm resistor. To find V_{Th}, consider the circuit below.



From the figure,

$$V_x = 0, \qquad V_{Th} = \frac{15}{15+5}(4) = 3\text{V}$$

To find R_{Th}, consider the circuit below:



At node 1,

$$\frac{4-V_1}{5} = 3V_x + \frac{V_1}{15} + \frac{V_1 - V_2}{5}, \qquad V_x = 6x1 = 6 \longrightarrow \qquad 258 = 3V_2 - 7V_1 \qquad (1)$$

At node 2,

$$1 + 3V_x + \frac{V_1 - V_2}{5} = 0 \longrightarrow \qquad V_1 = V_2 - 95 \qquad (2)$$

Solving (1) and (2) leads to V_2 = 101.75 V

$$R_{Th} = \frac{V_2}{1} = 101.75\Omega, \qquad p_{\max} = \frac{V_{Th}^{\ 2}}{4R_{Th}} = \frac{9}{4x101.75} = \underline{22.11\,\text{mW}}$$

Chapter 4, Problem 71.

For the circuit in Fig. 4.137, what resistor connected across terminals ***a-b*** will absorb maximum power from the circuit? What is that power?

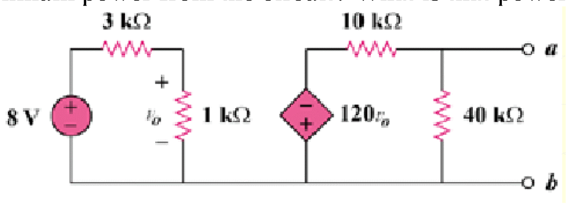
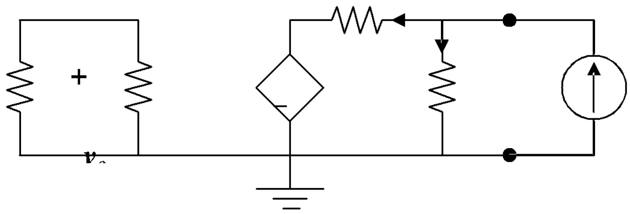


Figure 4.137

Chapter 4, Solution 71.

We need R_{Th} and V_{Th} at terminals a and b. To find R_{Th}, we insert a 1-mA source at the terminals a and b as shown below.



Assume that all resistances are in k ohms, all currents are in mA, and all voltages are in volts. At node a,

$$1 = (v_a/40) + [(v_a + 120v_o)/10], \text{ or } 40 = 5v_a + 480v_o \qquad (1)$$

The loop on the left side has no voltage source. Hence, $v_o = 0$. From (1), $v_a = 8$ V.

$$R_{Th} = v_a/1 \text{ mA} = 8 \text{ kohms}$$

To get V_{Th}, consider the original circuit. For the left loop,

$$v_o = (1/4)8 = 2 \text{ V}$$

For the right loop, $v_R = V_{Th} = (40/50)(-120v_o) = -192$

The resistance at the required resistor is

$$R = R_{Th} = \underline{\textbf{8 kohms}}$$

$$p = V_{Th}^2/(4R_{Th}) = (-192)^2/(4x8x10^3) = \underline{\textbf{1.152 watts}}$$

Chapter 4, Problem 72.

(a) For the circuit in Fig. 4.138, obtain the Thevenin equivalent at terminals ***a-b***.
(b) Calculate the current in $R_L = 8\Omega$.
(c) Find R_L for maximum power deliverable to R_L.
(d) Determine that maximum power.

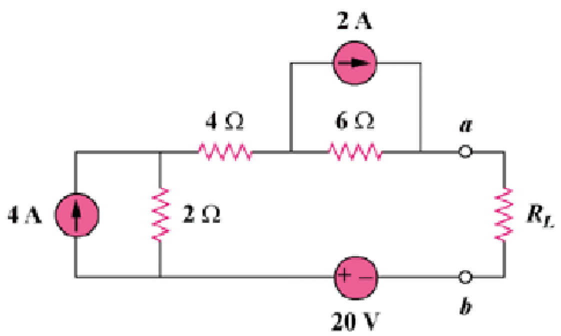


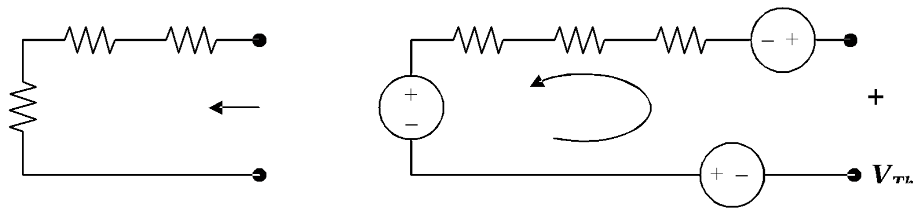
Figure 4.138

Chapter 4, Solution 72.

(a) R_{Th} and V_{Th} are calculated using the circuits shown in Fig. (a) and (b) respectively.

From Fig. (a), $R_{Th} = 2 + 4 + 6 = \underline{\textbf{12 ohms}}$

From Fig. (b), $-V_{Th} + 12 + 8 + 20 = 0$, or $V_{Th} = \underline{\textbf{40 V}}$



(b) $i = V_{Th}/(R_{Th} + R) = 40/(12 + 8) = \underline{\textbf{2A}}$

(c) For maximum power transfer, $R_L = R_{Th} = \underline{\textbf{12 ohms}}$

(d) $p = V_{Th}^2/(4R_{Th}) = (40)^2/(4x12) = \underline{\textbf{33.33 watts}}$.

Chapter 4, Problem 73.

Determine the maximum power that can be delivered to the variable resistor *R* in the circuit of Fig. 4.139.

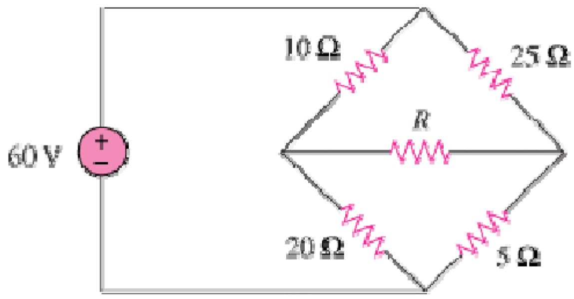
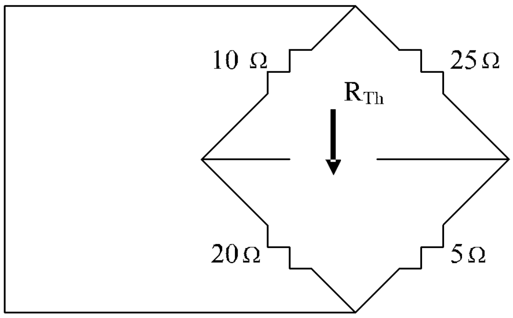


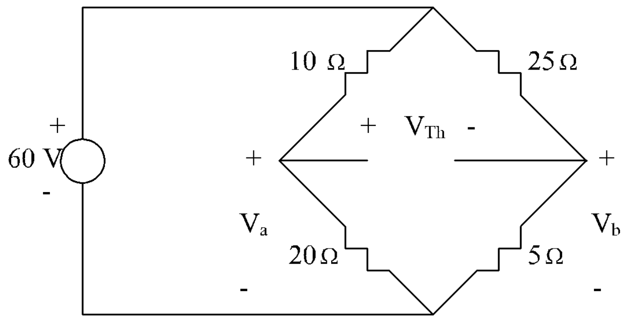
Figure 4.139

Chapter 4, Solution 73

Find the Thevenin's equivalent circuit across the terminals of R.



$R_{Th} = 10//20 + 25//5 = 325/30 = 10.833\Omega$



$V_a = \frac{20}{30}(60) = 40, \qquad V_b = \frac{5}{30}(60) = 10$

$-V_a + V_{Th} + V_b = 0 \longrightarrow V_{Th} = V_a - V_b = 40 - 10 = 30\text{ V}$

$$p_{max} = \frac{V_{Th}^{\ 2}}{4R_{Th}} = \frac{30^2}{4x10.833} = \underline{20.77\text{ W}}$$

Chapter 4, Problem 74.

For the bridge circuit shown in Fig. 4.140, find the load $\boldsymbol{R_L}$ for maximum power transfer and the maximum power absorbed by the load.

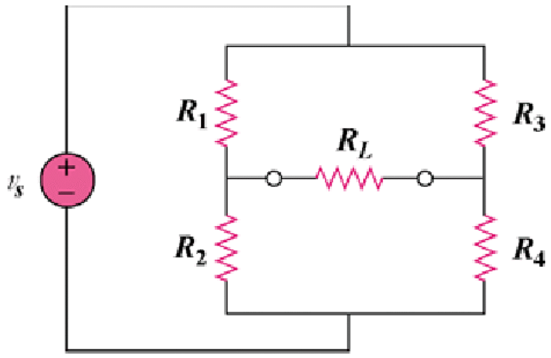


Figure 4.140

Chapter 4, Solution 74.

When R_L is removed and V_s is short-circuited,

$$R_{Th} = R_1||R_2 + R_3||R_4 = [R_1 R_2/(R_1 + R_2)] + [R_3 R_4/(R_3 + R_4)]$$

$$R_L = R_{Th} = \underline{\mathbf{(R_1 R_2 R_3 + R_1 R_2 R_4 + R_1 R_3 R_4 + R_2 R_3 R_4)/[(R_1 + R_2)(R_3 + R_4)]}}$$

When R_L is removed and we apply the voltage division principle,

$$V_{oc} = V_{Th} = v_{R2} - v_{R4}$$

$$= ([R_2/(R_1 + R_2)] - [R_4/(R_3 + R_4)])V_s = \{[(R_2R_3) - (R_1R_4)]/[(R_1 + R_2)(R_3 + R_4)]\}V_s$$

$$p_{max} = V_{Th}^2/(4R_{Th})$$

$$= \{[(R_2R_3) - (R_1R_4)]^2/[(R_1 + R_2)(R_3 + R_4)]^2\}V_s^2[(R_1 + R_2)(R_3 + R_4)]/[4(a)]$$

$$\text{where } a = (R_1 R_2 R_3 + R_1 R_2 R_4 + R_1 R_3 R_4 + R_2 R_3 R_4)$$

$$p_{max} =$$

$$\underline{\mathbf{[(R_2R_3) - (R_1R_4)]^2V_s^2/[4(R_1 + R_2)(R_3 + R_4)\ (R_1 R_2 R_3 + R_1 R_2 R_4 + R_1 R_3 R_4 + R_2 R_3 R_4)]}}$$

Chapter 4, Problem 75.

For the circuit in Fig. 4.141, determine the value of ***R*** such that the maximum power delivered to the load is 3 mW.

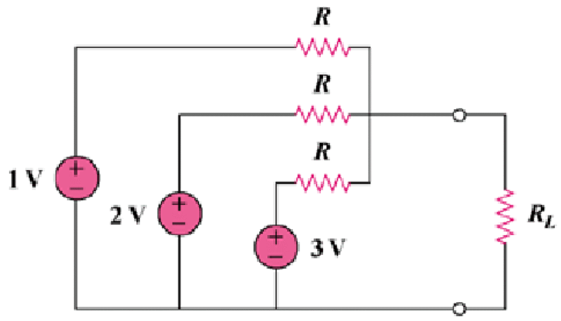
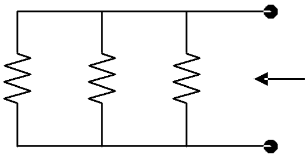
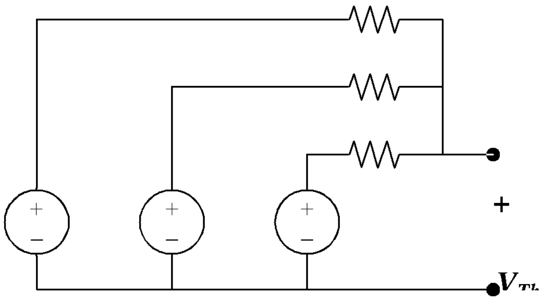


Figure 4.141

Chapter 4, Solution 75.

We need to first find R_{Th} and V_{Th}.





Consider the circuit in Fig. (a).

$$(1/R_{Th}) = (1/R) + (1/R) + (1/R) = 3/R$$

$$R_{Th} = R/3$$

From the circuit in Fig. (b),

$$((1 - v_o)/R) + ((2 - v_o)/R) + ((3 - v_o)/R) = 0$$

$$v_o = 2 = V_{Th}$$

For maximum power transfer,

$$R_L = R_{Th} = R/3$$

$$P_{max} = [(V_{Th})^2/(4R_{Th})] = 3 \text{ mW}$$

$$R_{Th} = [(V_{Th})^2/(4P_{max})] = 4/(4xP_{max}) = 1/P_{max} = R/3$$

$$R = 3/(3x10^{-3}) = \underline{\mathbf{1\ k\ ohms}}$$

Chapter 4, Problem 76.

Solve Prob. 4.34 using *PSpice.*

Chapter 4, Problem 34.

Find the Thevenin equivalent at terminals ***a-b*** of the circuit in Fig. 4.98.

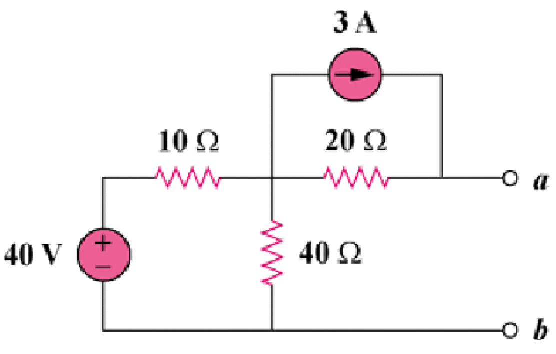


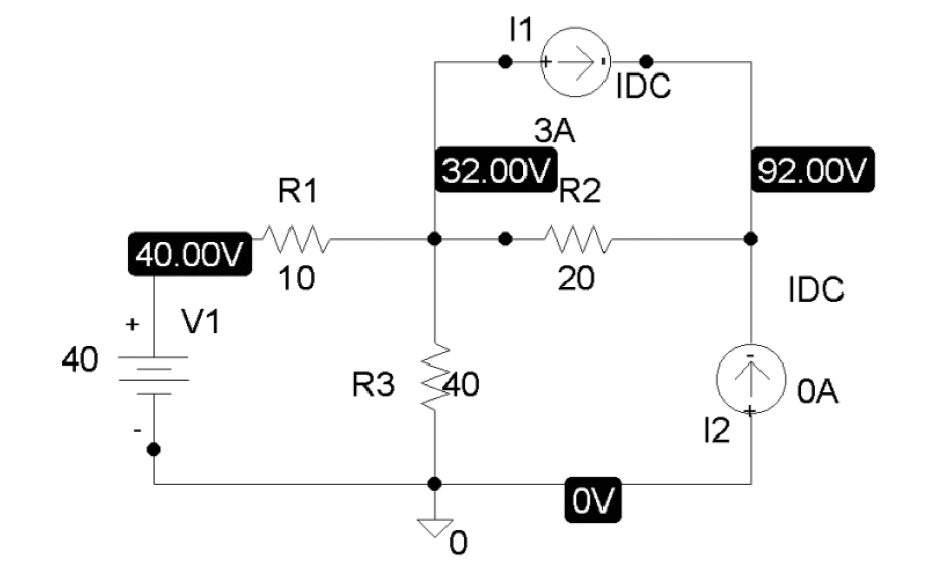
Figure 4.98

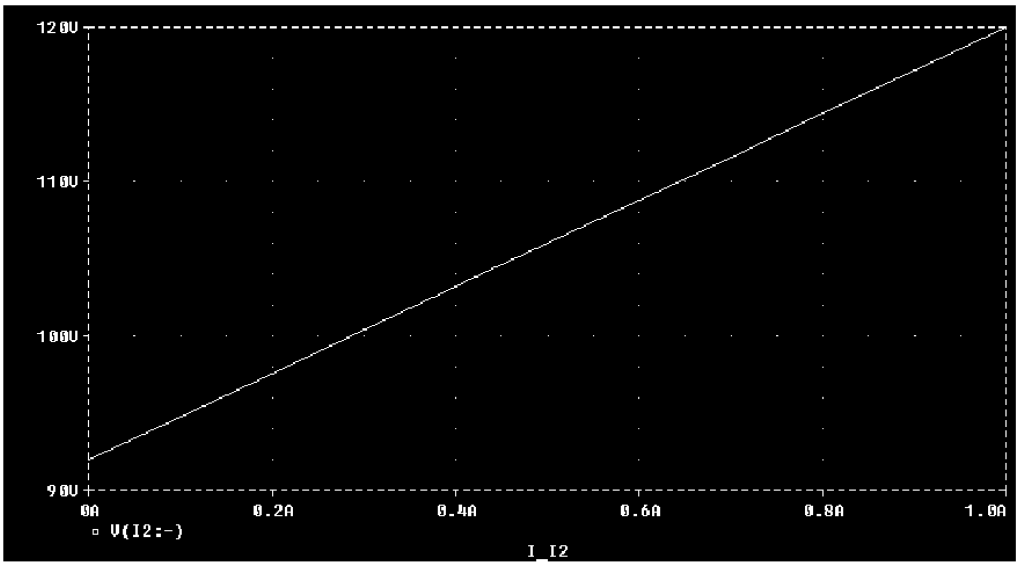
Chapter 4, Solution 76.

Follow the steps in Example 4.14. The schematic and the output plots are shown below. From the plot, we obtain,

$$V = \underline{\mathbf{92\ V}} \text{ [i = 0, voltage axis intercept]}$$

$$R = \text{Slope} = (120 - 92)/1 = \underline{\mathbf{28\ ohms}}$$





Chapter 4, Problem 77.

Solve Prob. 4.44 using *PSpice.*

Chapter 4, Problem 44.
For the circuit in Fig. 4.111, obtain the Thevenin equivalent as seen from terminals
(b) ***a-b*** (b) ***b-c***

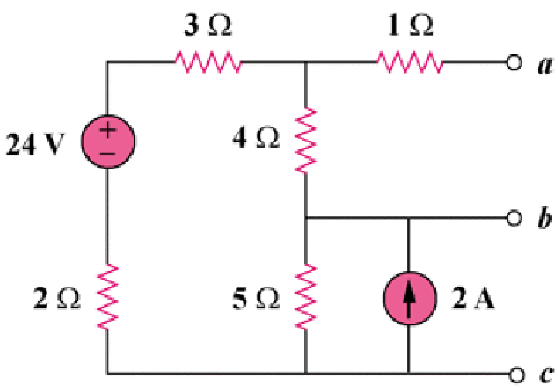
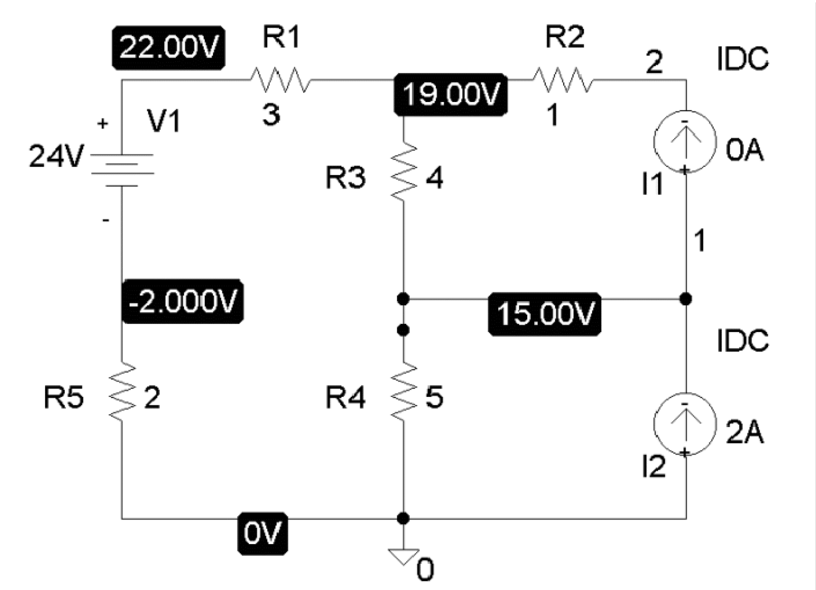


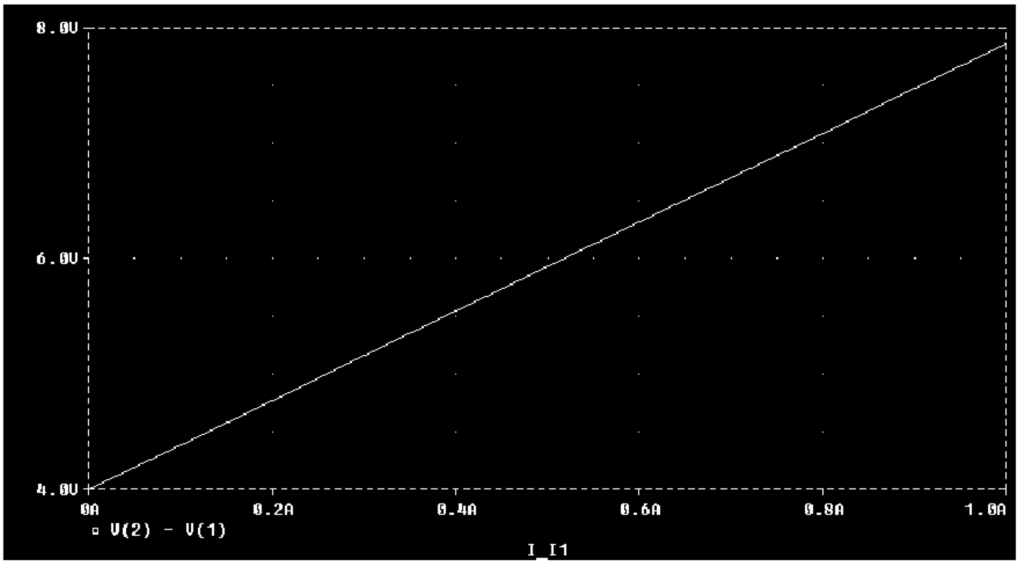
Figure 4.111

Chapter 4, Solution 77.

(a) The schematic is shown below. We perform a dc sweep on a current source, I1, connected between terminals a and b. We label the top and bottom of source I1 as 2 and 1 respectively. We plot V(2) – V(1) as shown.

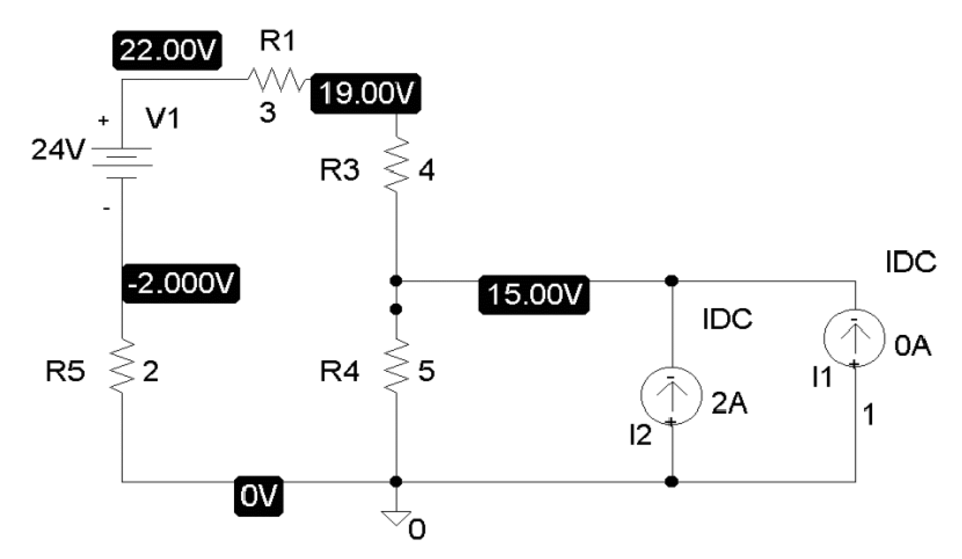
V_{Th} = **4 V** [zero intercept]
R_{Th} = (7.8 – 4)/1 = **3.8 ohms**

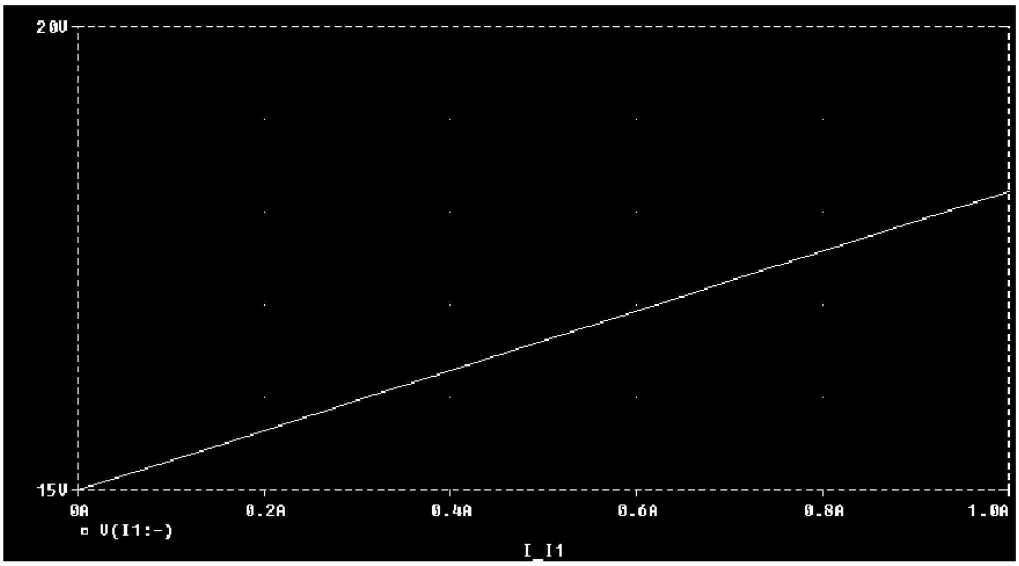




(b) Everything remains the same as in part (a) except that the current source, I1, is connected between terminals b and c as shown below. We perform a dc sweep on I1 and obtain the plot shown below. From the plot, we obtain,

$$V = \underline{\mathbf{15\ V}}\ \text{[zero intercept]}$$
$$R = (18.2 - 15)/1 = \underline{\mathbf{3.2\ ohms}}$$





Chapter 4, Problem 78.

Use *PSpice* to solve Prob. 4.52.

Chapter 4, Problem 52**.**

For the transistor model in Fig. 4.111, obtain the Thevenin equivalent at terminals ***a***-***b***.

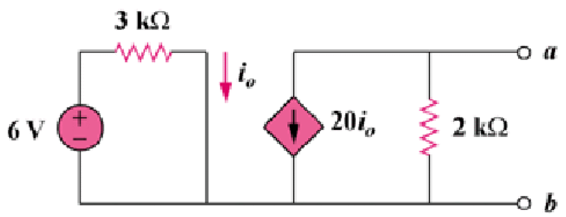


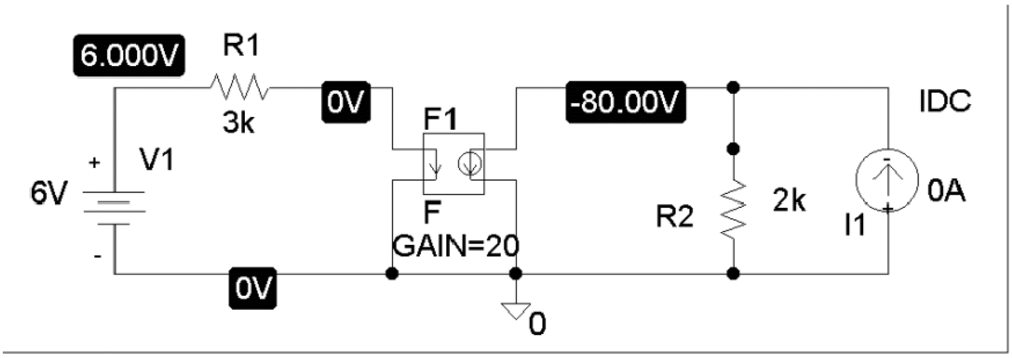
Figure 4.111

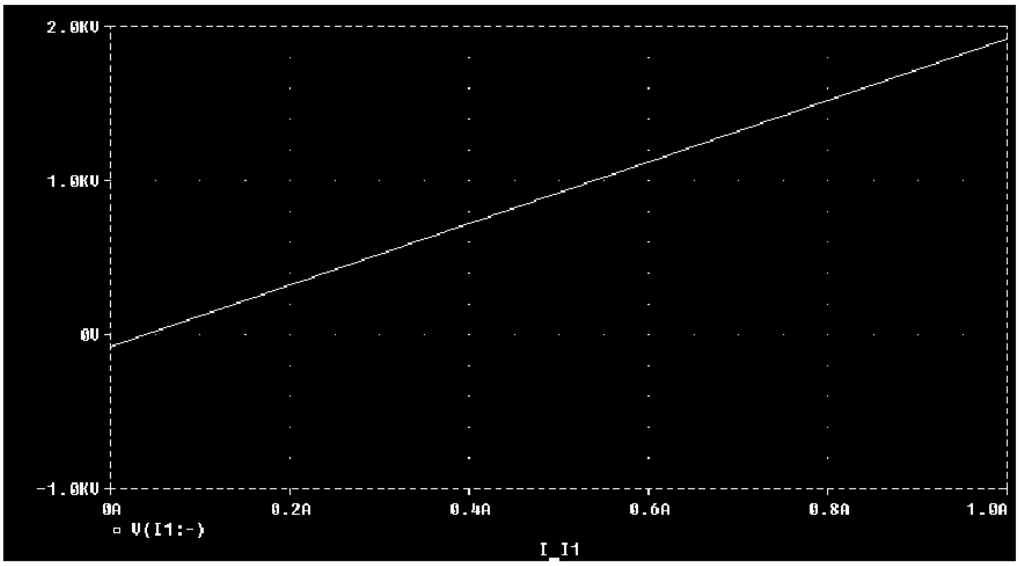
Chapter 4, Solution 78.

The schematic is shown below. We perform a dc sweep on the current source, I1, connected between terminals a and b. The plot is shown. From the plot we obtain,

$$V_{Th} = \underline{\mathbf{-80\ V}} \text{ [zero intercept]}$$

$$R_{Th} = (1920 - (-80))/1 = \underline{\mathbf{2\ k\ ohms}}$$





Chapter 4, Problem 79.

Obtain the Thevenin equivalent of the circuit in Fig. 4.123 using *PSpice*.

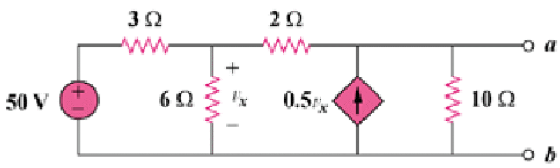


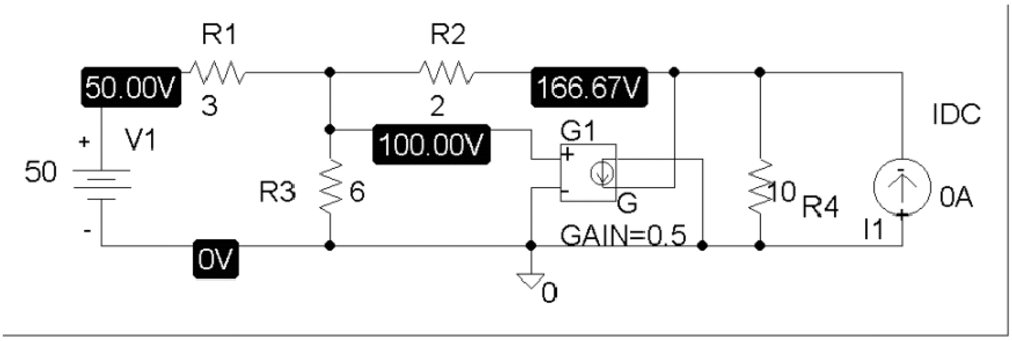
Figure 4.123

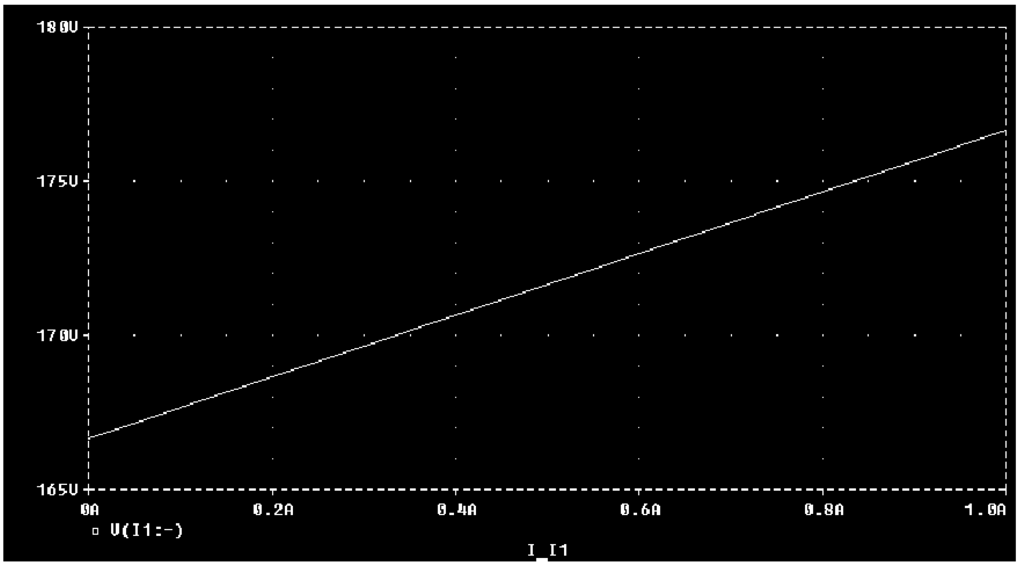
Chapter 4, Solution 79.

After drawing and saving the schematic as shown below, we perform a dc sweep on I1 connected across a and b. The plot is shown. From the plot, we get,

$$V = \underline{\mathbf{167\ V}} \quad \text{[zero intercept]}$$

$$R = (177 - 167)/1 = \underline{\mathbf{10\ ohms}}$$





Chapter 4, Problem 80.

Use *PSpice* to find the Thevenin equivalent circuit at terminals ***a*-*b*** of the circuit in Fig. 4.125.

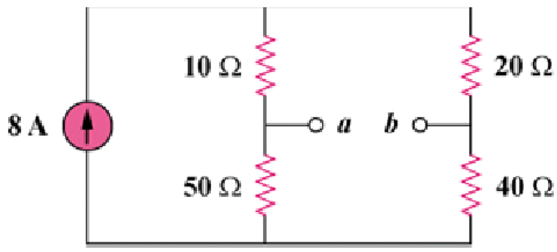


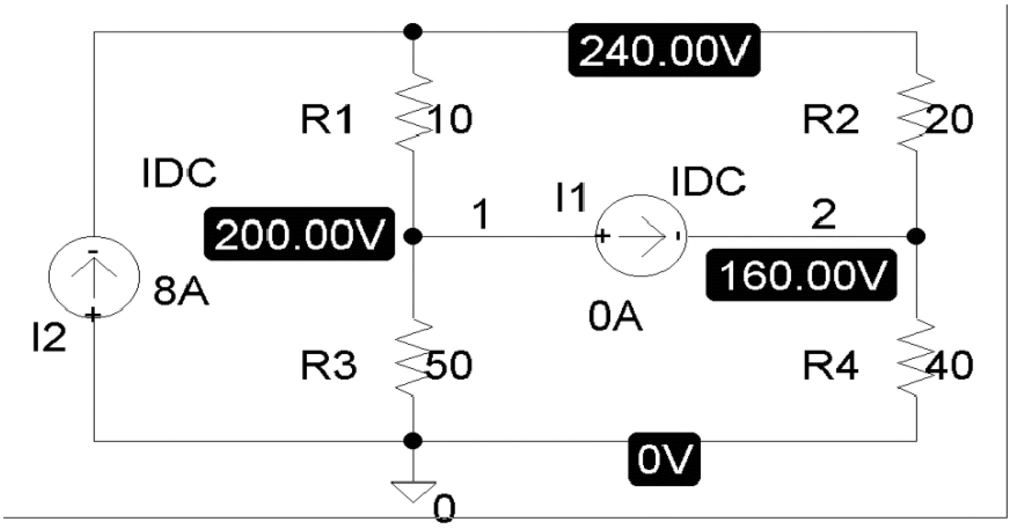
Figure 4.125

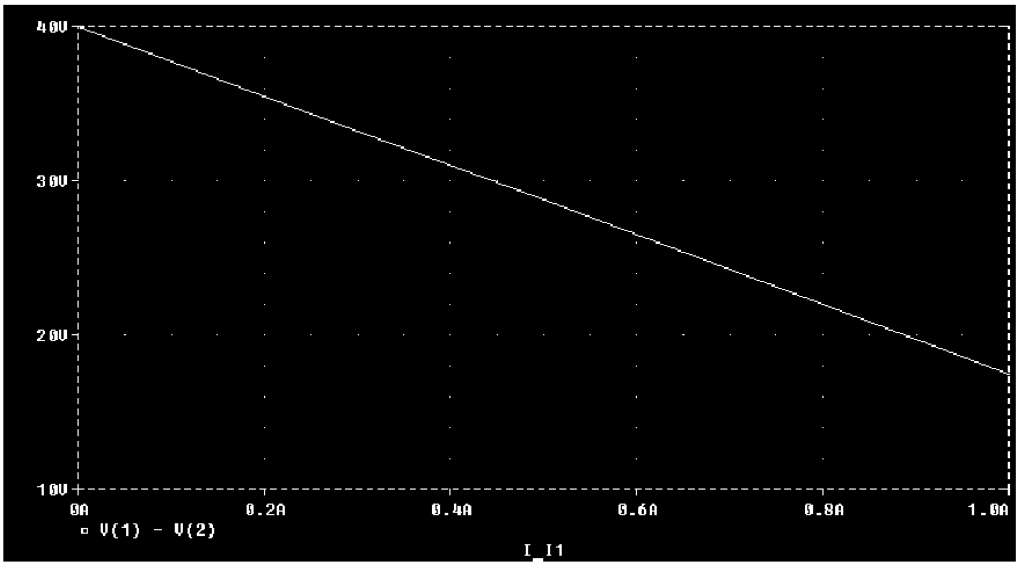
Chapter 4, Solution 80.

The schematic in shown below. We label nodes a and b as 1 and 2 respectively. We perform dc sweep on I1. In the Trace/Add menu, type v(1) – v(2) which will result in the plot below. From the plot,

$$V_{Th} = \underline{\mathbf{40\ V}} \text{ [zero intercept]}$$

$$R_{Th} = (40 - 17.5)/1 = \underline{\mathbf{22.5\ ohms}} \text{ [slope]}$$





Chapter 4, Problem 81.

For the circuit in Fig. 4.126, use *PSpice* to find the Thevenin equivalent at terminals ***a***-***b***.

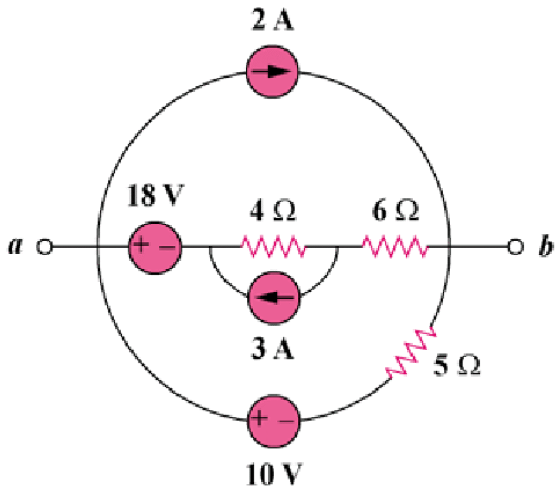


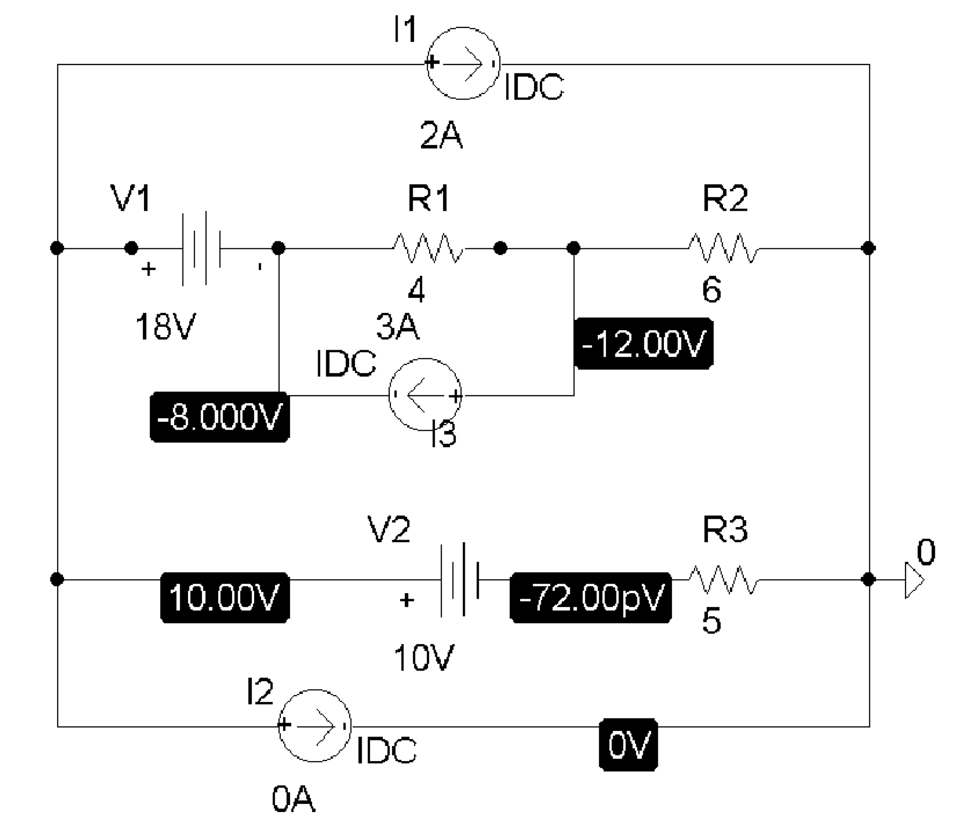
Figure 4.126

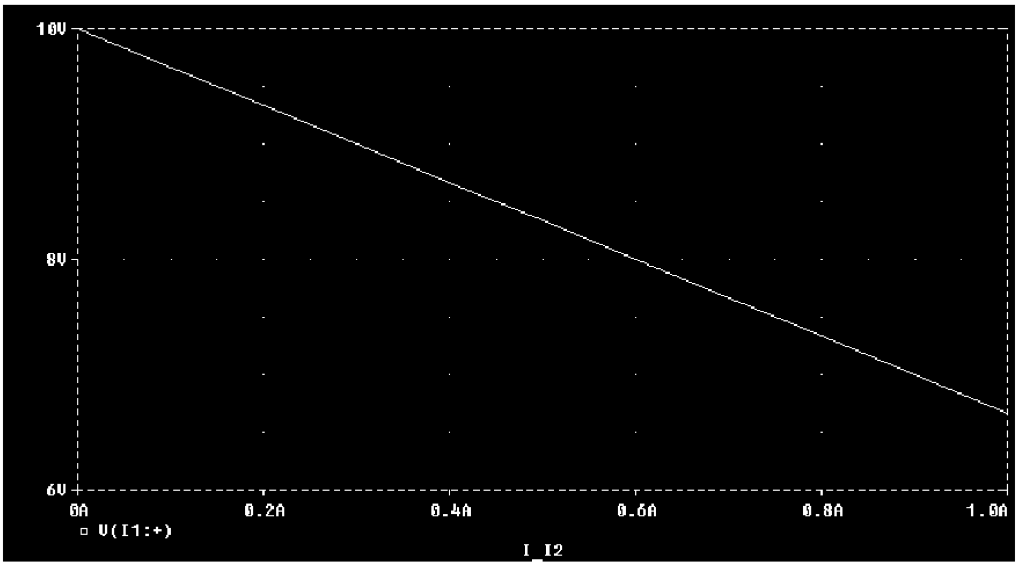
Chapter 4, Solution 81.

The schematic is shown below. We perform a dc sweep on the current source, I2, connected between terminals a and b. The plot of the voltage across I2 is shown below. From the plot,

$$V_{Th} = \underline{\mathbf{10\ V}}\ \text{[zero intercept]}$$

R_{Th} = (10 – 6.7)/1 = **3.3 ohms**. Note that this is in good agreement with the exact value of 3.333 ohms.




10V
8V
6V
0A
0.2A
0.4A
0.6A
0.8A
1.0A
V(I1:+)
I_I2

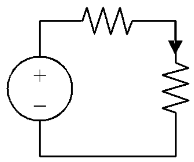
Chapter 4, Problem 82.

A battery has a short-circuit current of 20 A and an open-circuit voltage of 12 V. If the battery is connected to an electric bulb of resistance 2 Ω, calculate the power dissipated by the bulb.

Chapter 4, Solution 82.

$V_{Th} = V_{oc} = 12$ V, $I_{sc} = 20$ A

$$R_{Th} = V_{oc}/I_{sc} = 12/20 = 0.6 \text{ ohm.}$$



$$i = 12/2.6\,, \quad p = i^2R = (12/2.6)^2(2) = \underline{\textbf{42.6 watts}}$$

Chapter 4, Problem 83.

The following results were obtained from measurements taken between the two terminals of a resistive network.

Terminal Voltage	12 V	0 V
Terminal Current	0 V	1.5A

Find the Thevenin equivalent of the network.

Chapter 4, Solution 83.

$V_{Th} = V_{oc} = 12$ V, $I_{sc} = I_N = 1.5$ A

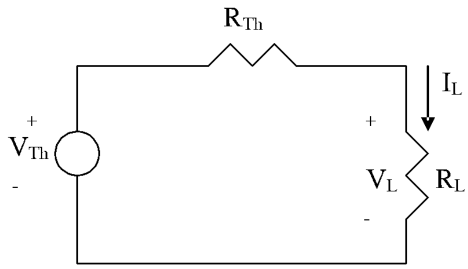
$$R_{Th} = V_{Th}/I_N = 8 \text{ ohms}, \quad V_{Th} = \underline{\textbf{12 V}}, \quad R_{Th} = \underline{\textbf{8 ohms}}$$

Chapter 4, Problem 84.

When connected to a 4-Ω resistor, a battery has a terminal voltage of 10.8 V but produces 12 V on open circuit. Determine the Thèvenin equivalent circuit for the battery.

Chapter 4, Solution 84

Let the equivalent circuit of the battery terminated by a load be as shown below.



For open circuit,

$$R_L = \infty, \quad \longrightarrow \quad V_{Th} = V_{oc} = V_L = \underline{10.8\text{ V}}$$

When R_L = 4 ohm, V_L=10.5,

$$I_L = \frac{V_L}{R_L} = 10.8/4 = 2.7$$

But

$$V_{Th} = V_L + I_L R_{Th} \quad \longrightarrow \quad R_{Th} = \frac{V_{Th} - V_L}{I_L} = \frac{12 - 10.8}{2.7} = \underline{0.4444\Omega}$$

Chapter 4, Problem 85.

The Thèvenin equivalent at terminals *a-b* of the linear network shown in Fig. 4.142 is to be determined by measurement. When a 10-kΩ resistor is connected to terminals *a-b*, the voltage V_{ab} is measured as 6 V. When a 30-kΩ resistor is connected to the terminals, V_{ab} is measured as 12 V. Determine: (a) the Thèvenin equivalent at terminals *a-b*, (b) V_{ab} when a 20-kΩ resistor is connected to terminals *a-b*.

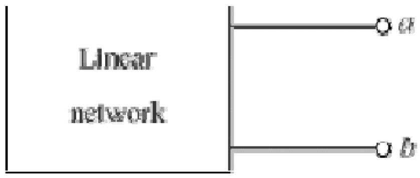
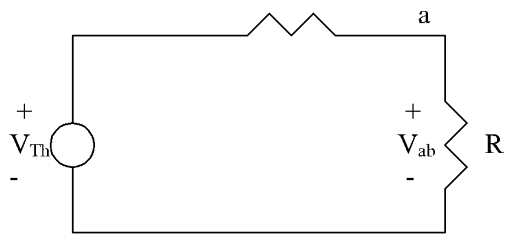


Figure 4.142

Chapter 4, Solution 85

(a) Consider the equivalent circuit terminated with R as shown below.



$$V_{ab} = \frac{R}{R+R_{Th}}V_{Th} \longrightarrow 6 = \frac{10}{10+R_{Th}}V_{Th}$$

or

$$60 + 6R_{Th} = 10V_{Th} \qquad (1)$$

where R_{Th} is in k-ohm.

Similarly,

$$12 = \frac{30}{30+R_{Th}}V_{Th} \longrightarrow 360 + 12R_{Th} = 30V_{Th} \qquad (2)$$

Solving (1) and (2) leads to

$\underline{V_{Th} = 24\text{ V},\ R_{Th} = 30k\Omega}$

(b) $V_{ab} = \frac{20}{20+30}(24) = \underline{9.6\text{ V}}$

Chapter 4, Problem 86.

A black box with a circuit in it is connected to a variable resistor. An ideal ammeter (with zero resistance) and an ideal voltmeter (with infinite resistance) are used to measure current and voltage as shown in Fig. 4.143. The results are shown in the table below.

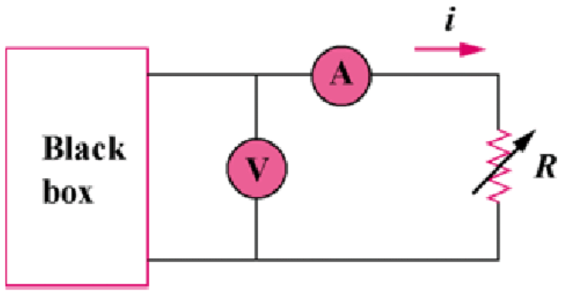


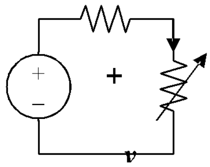
Figure 4.143

(a) Find i when $R = 4\ \Omega$.
(b) Determine the maximum power from the box.

$R(\Omega)$	V(V)	i(A)
2	3	1.5
8	8	1.0
14	10.5	0.75

Chapter 4, Solution 86.

We replace the box with the Thevenin equivalent.



$$V_{Th} = v + iR_{Th}$$

When i = 1.5, v = 3, which implies that $V_{Th} = 3 + 1.5R_{Th}$ (1)

When i = 1, v = 8, which implies that $V_{Th} = 8 + 1xR_{Th}$ (2)

From (1) and (2), R_{Th} = 10 ohms and V_{Th} = 18 V.

(a) When R = 4, i = $V_{Th}/(R + R_{Th})$ = 18/(4 + 10) = **1.2857 A**

(b) For maximum power, R = R_{TH}

$$P_{max} = (V_{Th})^2/4R_{Th} = 18^2/(4x10) = \textbf{8.1 watts}$$

Chapter 4, Problem 87.

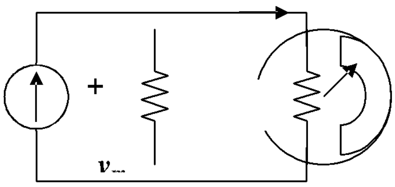
A transducer is modeled with a current source I_s and a parallel resistance R_s. The current at the terminals of the source is measured to be 9.975 mA when an ammeter with an internal resistance of 20 Ω is used.

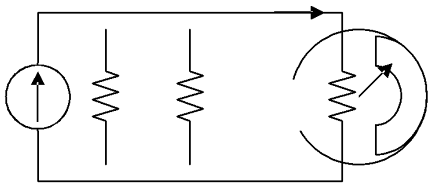
(a) If adding a 2-kΩ resistor across the source terminals causes the ammeter reading to fall to 9.876 mA, calculate I_s and R_s.

(b) What will the ammeter reading be if the resistance between the source terminals is changed to 4 kΩ?

Chapter 4, Solution 87.

(a)





From Fig. (a),

$$v_m = R_m i_m = 9.975 \text{ mA x } 20 = 0.1995 \text{ V}$$

$$I_s = 9.975 \text{ mA} + (0.1995/R_s) \qquad (1)$$

From Fig. (b),

$$v_m = R_m i_m = 20\text{x}9.876 = 0.19752 \text{ V}$$

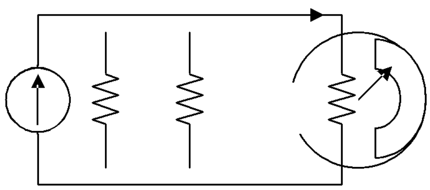
$$I_s = 9.876 \text{ mA} + (0.19752/2\text{k}) + (0.19752/R_s)$$

$$= 9.975 \text{ mA} + (0.19752/R_s) \qquad (2)$$

Solving (1) and (2) gives,

R_s = **<u>8 k ohms</u>**, I_s = **<u>10 mA</u>**

(b)



8k||4k = 2.667 k ohms

i_m' = [2667/(2667 + 20)](10 mA) = **<u>9.926 mA</u>**

Chapter 4, Problem 88.

Consider the circuit in Fig. 4.144. An ammeter with internal resistance R_i is inserted between *A* and *B* to measure I_o. Determine the reading of the ammeter if: (a) $R_i = 500\ \Omega$, (b) $R_i = 0\ \Omega$. (*Hint*: Find the Thèvenin equivalent circuit at terminals *A*-*B*.)

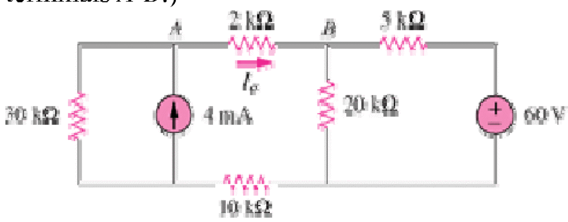
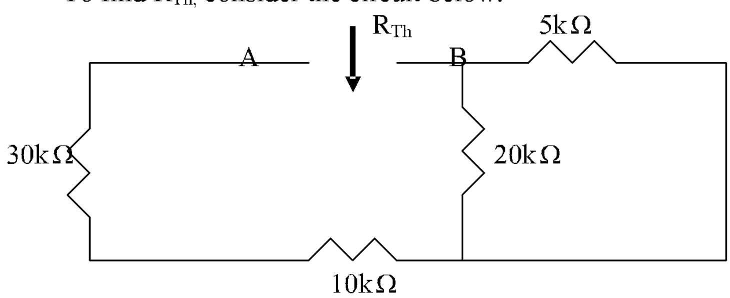


Figure 4.144

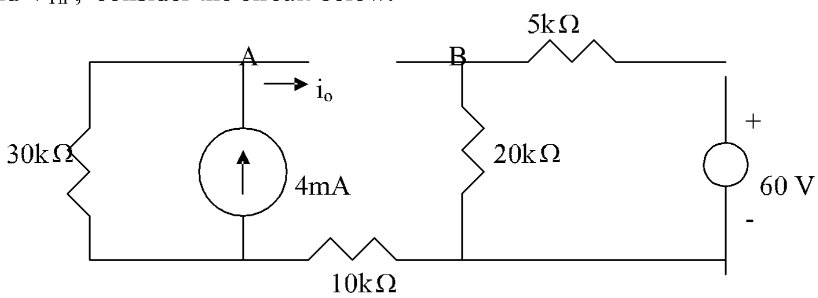
Chapter 4, Solution 88

To find R_{Th}, consider the circuit below.



$$R_{Th} = 30 + 10 + 20 // 5 = 44k\Omega$$

To find V_{Th}, consider the circuit below.



$$V_A = 30x4 = 120, \quad V_B = \frac{20}{25}(60) = 48, \quad V_{Th} = V_A - V_B = 72\text{ V}$$

Chapter 4, Problem 89.

Consider the circuit in Fig. 4.145. (a) Replace the resistor R_L by a zero resistance ammeter and determine the ammeter reading. (b) To verify the reciprocity theorem, interchange the ammeter and the 12-V source and determine the ammeter reading again.

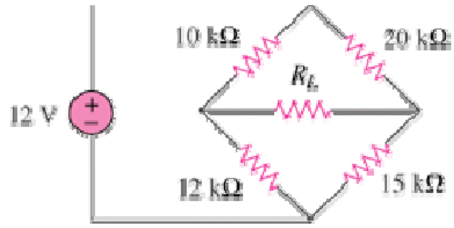
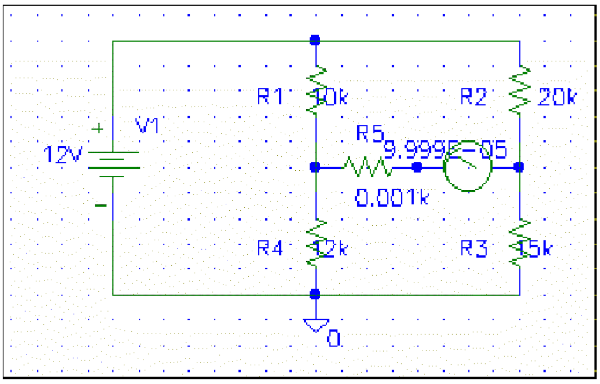


Figure 4.145

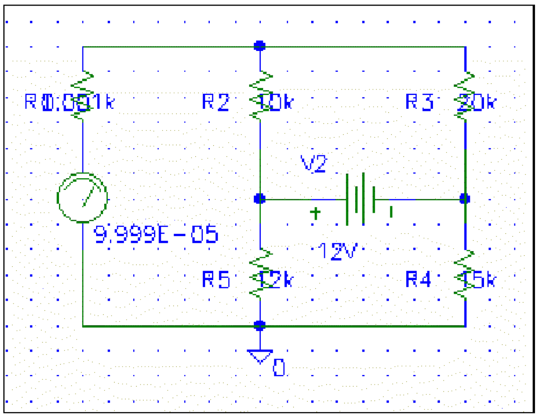
Chapter 4, Solution 89

It is easy to solve this problem using Pspice.

(a) The schematic is shown below. We insert IPROBE to measure the desired ammeter reading. We insert a very small resistance in series IPROBE to avoid problem. After the circuit is saved and simulated, the current is displaced on IPROBE as $99.99\mu\text{A}$.



(b) By interchanging the ammeter and the 12-V voltage source, the schematic is shown below. We obtain exactly the same result as in part (a).



Chapter 4, Problem 90.

The Wheatstone bridge circuit shown in Fig. 4.146 is used to measure the resistance of a strain gauge. The adjustable resistor has a linear taper with a maximum value of 100 Ω. If the resistance of the strain gauge is found to be 42.6 Ω, what fraction of the full slider travel is the slider when the bridge is balanced?

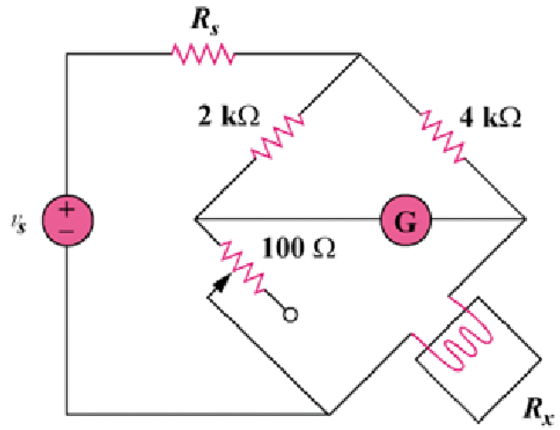


Figure 4.146

Chapter 4, Solution 90.

$R_x = (R_3/R_1)R_2 = (4/2)R_2 = 42.6, \; R_2 = 21.3$

which is (21.3ohms/100ohms)% = **21.3%**

Chapter 4, Problem 91.

(a) In the Wheatstone bridge circuit of Fig. 4.147 select the values of R_1 and R_3 such that the bridge can measure R_x in the reange of 0-10 Ω.

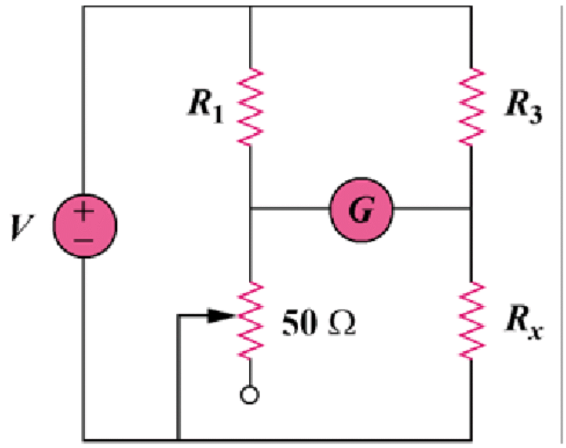


Figure 4.147

(b) Repeat for the range of 0-100 Ω.

Chapter 4, Solution 91.

$R_x = (R_3/R_1)R_2$

(a) Since $0 < R_2 < 50$ ohms, to make $0 < R_x < 10$ ohms requires that when $R_2 = 50$ ohms, $R_x = 10$ ohms.

$$10 = (R_3/R_1)50 \quad \text{or} \quad R_3 = R_1/5$$

so we select R_1 = **<u>100 ohms</u>** and R_3 = **<u>20 ohms</u>**

(b) For $0 < R_x < 100$ ohms

$$100 = (R_3/R_1)50, \quad \text{or} \quad R_3 = 2R_1$$

So we can select R_1 = **<u>100 ohms</u>** and R_3 = **<u>200 ohms</u>**

Chapter 4, Problem 92.
Consider the bridge circuit of Fig. 4.148. Is the bridge balanced? If the 10 Ω resistor is replaced by an 18-kΩ resistor, what resistor connected between terminals ***a-b*** absorbs the maximum power? What is this power?.

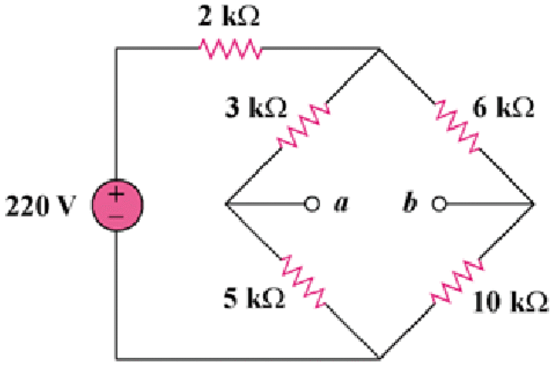
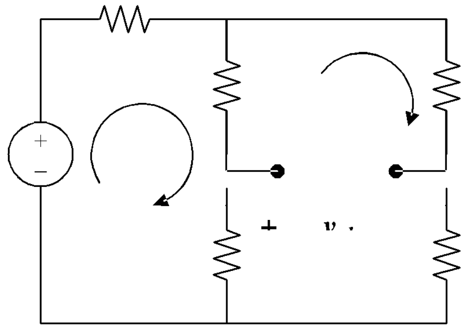


Figure 4.148

Chapter 4, Solution 92.
For a balanced bridge, $v_{ab} = 0$. We can use mesh analysis to find v_{ab}. Consider the circuit in Fig. (a), where i_1 and i_2 are assumed to be in mA.



$$220 = 2i_1 + 8(i_1 - i_2) \text{ or } 220 = 10i_1 - 8i_2 \quad (1)$$

$$0 = 24i_2 - 8i_1 \text{ or } i_2 = (1/3)i_1 \quad (2)$$

From (1) and (2),

$$i_1 = 30 \text{ mA and } i_2 = 10 \text{ mA}$$

Applying KVL to loop 0ab0 gives

$$5(i_2 - i_1) + v_{ab} + 10i_2 = 0 \text{ V}$$

Since $v_{ab} = 0$, the bridge is balanced.

When the 10 k ohm resistor is replaced by the 18 k ohm resistor, the gridge becomes unbalanced. (1) remains the same but (2) becomes

$$0 = 32i_2 - 8i_1, \text{ or } i_2 = (1/4)i_1 \qquad (3)$$

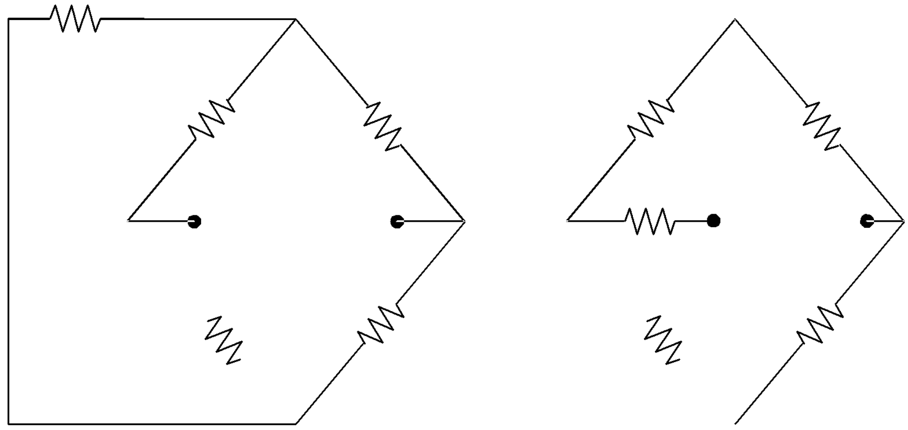
Solving (1) and (3),

$$i_1 = 27.5 \text{ mA}, \ i_2 = 6.875 \text{ mA}$$

$$v_{ab} = 5(i_1 - i_2) - 18i_2 = -20.625 \text{ V}$$

$$V_{Th} = v_{ab} = -20.625 \text{ V}$$

To obtain R_{Th}, we convert the delta connection in Fig. (b) to a wye connection shown in Fig. (c).



$$R_1 = 3x5/(2 + 3 + 5) = 1.5 \text{ k ohms}, \ R_2 = 2x3/10 = 600 \text{ ohms},$$

$$R_3 = 2x5/10 = 1 \text{ k ohm}.$$

$$R_{Th} = R_1 + (R_2 + 6)||(R_3 + 18) = 1.5 + 6.6||9 = 6.398 \text{ k ohms}$$

$$R_L = R_{Th} = \underline{\mathbf{6.398 \text{ k ohms}}}$$

$$P_{max} = (V_{Th})^2/(4R_{Th}) = (20.625)^2/(4x6.398) = \underline{\mathbf{16.622 \text{ mWatts}}}$$

Chapter 4, Problem 93.

The circuit in Fig. 4.149 models a common-emitter transistor amplifier. Find $\boldsymbol{i_x}$ using source transformation.

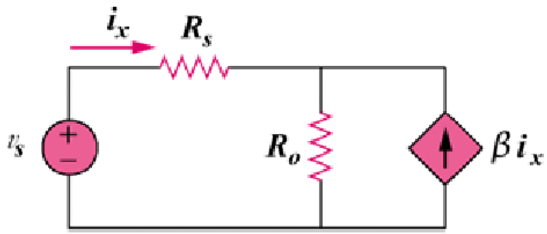
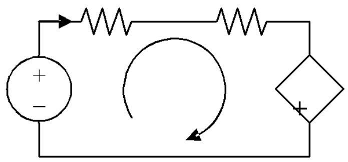


Figure 4.149

Chapter 4, Solution 93.



$$-V_s + (R_s + R_o)i_x + \beta R_o i_x = 0$$

$$i_x = \underline{\mathbf{V_s/(R_s + (1 + \beta)R_o)}}$$

Chapter 4, Problem 94.

An attenuator is an interface circuit that reduces the voltage level without changing the output resistance.

(a) By specifying $\boldsymbol{R_s}$ and $\boldsymbol{R_p}$ of the interface circuit in Fig. 4.150, design an attenuator that will meet the following requirements:

$$\frac{V_o}{V_g} = 0.125, \qquad R_{eq} = R_{\text{Th}} = R_g = 100\Omega$$

(b) Using the interface designed in part (a), calculate the current through a load of $\boldsymbol{R_L}$ = 50 Ω when $\boldsymbol{V_g}$ = 12 V.

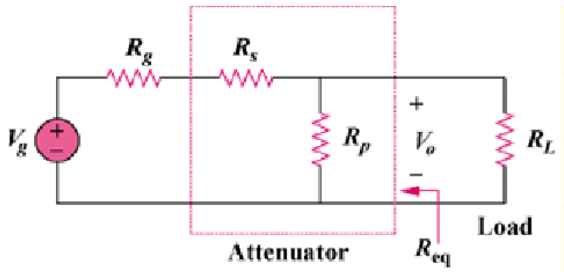


Figure 4.150

Chapter 4, Solution 94.

(a) $$V_o/V_g = R_p/(R_g + R_s + R_p) \quad (1)$$

$$R_{eq} = R_p||(R_g + R_s) = R_g$$

$$R_g = R_p(R_g + R_s)/(R_p + R_g + R_s)$$

$$R_gR_p + R_g^2 + R_gR_s = R_pR_g + R_pR_s$$

$$R_pR_s = R_g(R_g + R_s) \quad (2)$$

From (1), $$R_p/\alpha = R_g + R_s + R_p$$

$$R_g + R_s = R_p((1/\alpha) – 1) = R_p(1 - \alpha)/\alpha \quad (1a)$$

Combining (2) and (1a) gives,

$$R_s = [(1 - \alpha)/\alpha]R_{eq} \quad (3)$$

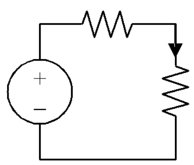
$$= (1 – 0.125)(100)/0.125 = \underline{\mathbf{700\ ohms}}$$

From (3) and (1a),

$$R_p(1 - \alpha)/\alpha = R_g + [(1 - \alpha)/\alpha]R_g = R_g/\alpha$$

$$R_p = R_g/(1 - \alpha) = 100/(1 – 0.125) = \underline{\mathbf{114.29\ ohms}}$$

(b)



$$V_{Th} = V_s = 0.125V_g = 1.5\ V$$

$$R_{Th} = R_g = 100\ ohms$$

$$I = V_{Th}/(R_{Th} + R_L) = 1.5/150 = \underline{\mathbf{10\ mA}}$$

Chapter 4, Problem 95.

A dc voltmeter with a sensitivity of 20 kΩ/V is used to find the Thevenin equivalent of a linear network. Readings on two scales are as follows:

(c) 0-10 V scale: 4 V

(d) 0-50 V scale: 5 V

Obtain the Thevenin voltage and the Thevenin resistance of the network.

Chapter 4, Solution 95.

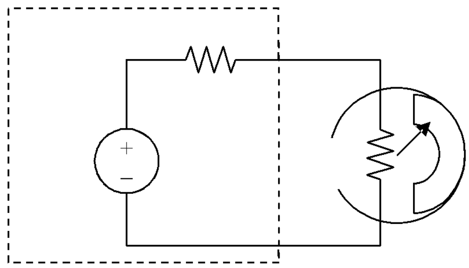
Let 1/sensitivity = 1/(20 k ohms/volt) = 50 μA

For the 0 – 10 V scale,

$$R_m = V_{fs}/I_{fs} = 10/50\ \mu A = 200\ k\ ohms$$

For the 0 – 50 V scale,

$$R_m = 50(20\ k\ ohms/V) = 1\ M\ ohm$$



$$V_{Th} = I(R_{Th} + R_m)$$

(a) A 4V reading corresponds to

$$I = (4/10)I_{fs} = 0.4x50\ \mu A = 20\ \mu A$$

$$V_{Th} = 20\ \mu A\ R_{Th} + 20\ \mu A\ 250\ k\ ohms$$

$$= 4 + 20\ \mu A\ R_{Th} \qquad (1)$$

(b) A 5V reading corresponds to

$$I = (5/50)I_{fs} = 0.1\ x\ 50\ \mu A = 5\ \mu A$$

$$V_{Th} = 5\ \mu A\ x\ R_{Th} + 5\ \mu A\ x\ 1\ M\ ohm$$

$$V_{Th} = 5 + 5\ \mu A\ R_{Th} \qquad (2)$$

From (1) and (2)

$$0 = -1 + 15\ \mu A\ R_{Th}$$ which leads to R_{Th} = **66.67 k ohms**

From (1),

$$V_{Th} = 4 + 20x10^{-6}x(1/(15x10^{-6})) = \mathbf{5.333\ V}$$

Chapter 4, Problem 96.

A resistance array is connected to a load resistor ***R*** and a 9-V battery as shown in Fig. 4.151.

(e) Find the value of ***R*** such that V_o = 1.8 V.

(f) Calculate the value of ***R*** that will draw the maximum current. What is the maximum current?

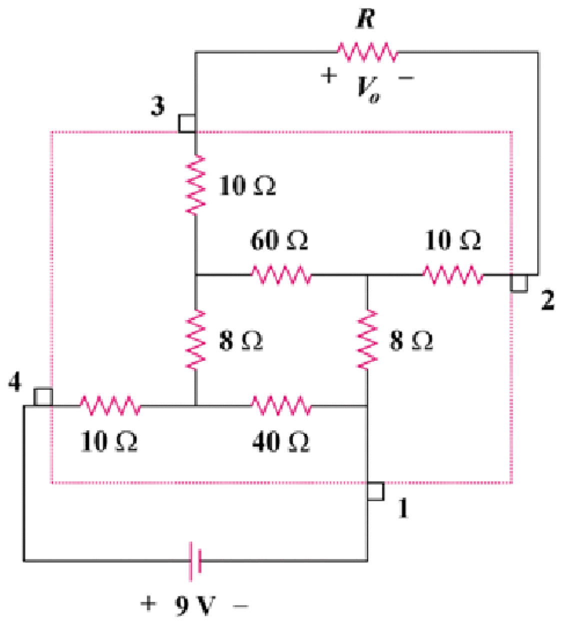
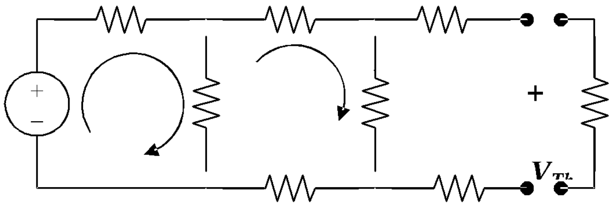
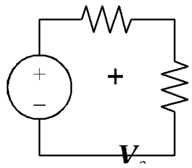


Figure 4.151

Chapter 4, Solution 96.

(a) The resistance network can be redrawn as shown in Fig. (a),





$$R_{Th} = 10 + 10 + 60||(8 + 8 + 10||40) = 20 + 60||24 = 37.14 \text{ ohms}$$

Using mesh analysis,

$$-9 + 50i_1 - 40i_2 = 0 \quad (1)$$
$$116i_2 - 40i_1 = 0 \text{ or } i_1 = 2.9i_2 \quad (2)$$

From (1) and (2), $i_2 = 9/105$

$$V_{Th} = 60i_2 = 5.143 \text{ V}$$

From Fig. (b),

$$V_o = [R/(R + R_{Th})]V_{Th} = 1.8$$

$$R/(R + 37.14) = 1.8/5.143 \text{ which leads to } R = \underline{\textbf{20 ohms}}$$

(b) $R = R_{Th} =$ **<u>37.14 ohms</u>**

$$I_{max} = V_{Th}/(2R_{Th}) = 5.143/(2x37.14) = \underline{\textbf{69.23 mA}}$$

Chapter 4, Problem 97.

A common-emitter amplifier circuit is shown in Fig. 4.152. Obtain the Thevenin equivalent to the left of points ***B*** and ***E***.

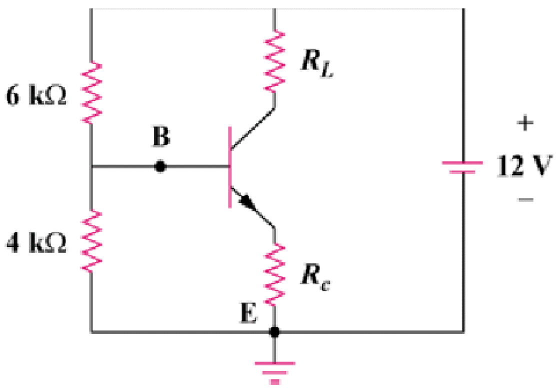
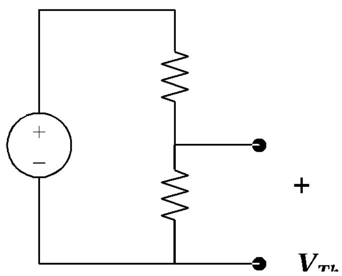


Figure 4.152

Chapter 4, Solution 97.



$$R_{Th} = R_1 || R_2 = 6||4 = \underline{\textbf{2.4 k ohms}}$$

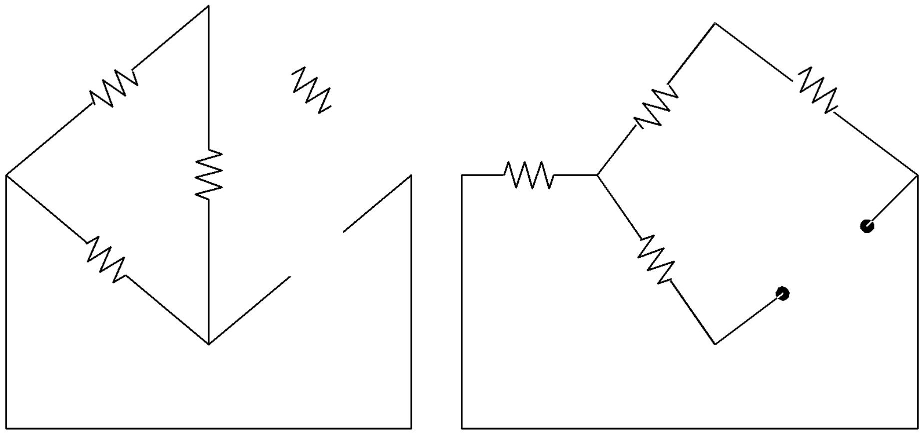
$$V_{Th} = [R_2/(R_1 + R_2)]v_s = [4/(6+4)](12) = \underline{\textbf{4.8 V}}$$

Chapter 4, Problem 98.

For Practice Prob. 4.18, determine the current through the 40-Ω resistor and the power dissipated by the resistor.

Chapter 4, Solution 98.

The 20-ohm, 60-ohm, and 14-ohm resistors form a delta connection which needs to be connected to the wye connection as shown in Fig. (b),



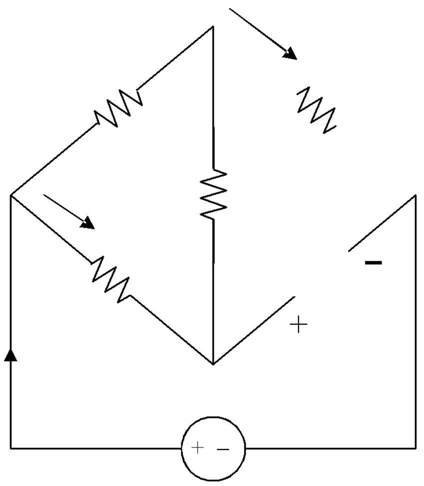
$$R_1 = 20x60/(20 + 60 + 14) = 1200/94 = 12.766 \text{ ohms}$$

$$R_2 = 20x14/94 = 2.979 \text{ ohms}$$

$$R_3 = 60x14/94 = 8.936 \text{ ohms}$$

$$R_{Th} = R_3 + R_1||(R_2 + 30) = 8.936 + 12.766||32.98 = 18.139 \text{ ohms}$$

To find V_{Th}, consider the circuit in Fig. (c).



$$I_T = 16/(30 + 15.745) = 349.8 \text{ mA}$$

$$I_1 = [20/(20 + 60 + 14)]I_T = 74.43 \text{ mA}$$

$$V_{Th} = 14I_1 + 30I_T = 11.536 \text{ V}$$

$$I_{40} = V_{Th}/(R_{Th} + 40) = 11.536/(18.139 + 40) = 198.42 \text{ mA}$$

$$P_{40} = I_{40}^2R = \mathbf{\underline{1.5748 \text{ watts}}}$$

Chapter 5, Problem 1.

The equivalent model of a certain op amp is shown in Fig. 5.43. Determine:

(a) the input resistance.
(b) the output resistance.
(c) the voltage gain in dB.

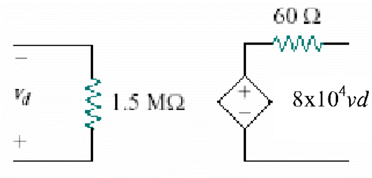


Figure 5.42 For Prob. 5.1.

Chapter 5, Solution 1.

(a) R_{in} = **1.5 MΩ**
(b) R_{out} = **60 Ω**
(c) $A = 8x10^4$
Therefore $A_{dB} = 20 \log 8x10^4$ = **98.0 dB**

Chapter 5, Problem 2

The open-loop gain of an op amp is 100,000. Calculate the output voltage when there are inputs of +10 μV on the inverting terminal and + 20 μV on the noninverting terminal.

Chapter 5, Solution 2.

$v_0 = Av_d = A(v_2 - v_1)$

$= 10^5 (20-10) \times 10^{-6}$ = **1V**

Chapter 5, Problem 3

Determine the output voltage when .20 μV is applied to the inverting terminal of an op amp and +30 μV to its noninverting terminal. Assume that the op amp has an open-loop gain of 200,000.

Chapter 5, Solution 3.

$$v_0 = Av_d = A(v_2 - v_1)$$
$$= 2 \text{ x } 10^5 \, (30 + 20) \text{ x } 10^{-6} = \underline{\mathbf{10V}}$$

Chapter 5, Problem 4

The output voltage of an op amp is .4 V when the noninverting input is 1 mV. If the open-loop gain of the op amp is 2×10^6, what is the inverting input?

Chapter 5, Solution 4.

$$v_0 = Av_d = A(v_2 - v_1)$$
$$v_2 - v_1 = \frac{v_0}{A} = \frac{-4}{2x10^6} = -2\mu V$$

$$v_2 - v_1 = -2 \ \mu V = -0.002 \text{ mV}$$
$$1 \text{ mV} - v_1 = -0.002 \text{ mV}$$
$$v_1 = \underline{\mathbf{1.002 \ mV}}$$

Chapter 5, Problem 5.

For the op amp circuit of Fig. 5.44, the op amp has an open-loop gain of 100,000, an input resistance of 10 kΩ, and an output resistance of 100 Ω. Find the voltage gain v_o/v_i using the nonideal model of the op amp.

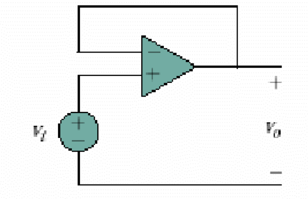
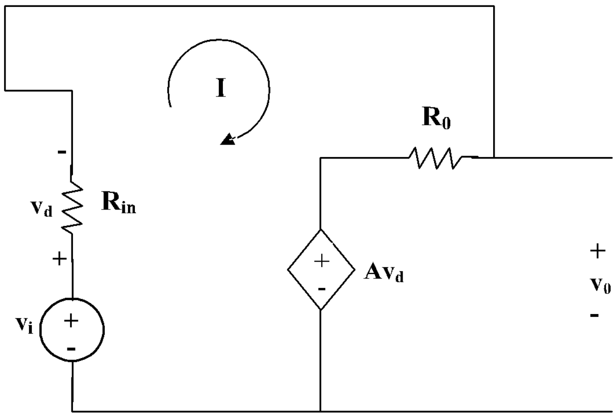


Figure 5.44 for Prob. 5.5

Chapter 5, Solution 5.



$$-v_i + Av_d + (R_i + R_0)\, I = 0 \qquad (1)$$

But $\quad v_d = R_i I,$

$$-v_i + (R_i + R_0 + R_i A)\, I = 0$$

$$I = \frac{v_i}{R_0 + (1 + A)R_i} \qquad (2)$$

$$-Av_d - R_0 I + v_0 = 0$$

$$v_0 = Av_d + R_0 I = (R_0 + R_i A)I \;=\; \frac{(R_0 + R_i A)v_i}{R_0 + (1 + A)R_i}$$

$$\frac{v_0}{v_i} = \frac{R_0 + R_i A}{R_0 + (1 + A)R_i} = \frac{100 + 10^4 x 10^5}{100 + (1 + 10^5)} \cdot 10^4$$

$$\cong \frac{10^9}{(1 + 10^5)} \cdot 10^4 = \frac{100{,}000}{100{,}001} = \underline{\mathbf{0.9999990}}$$

Chapter 5, Problem 6

Using the same parameters for the 741 op amp in Example 5.1, find v_o in the op amp circuit of Fig. 5.45.

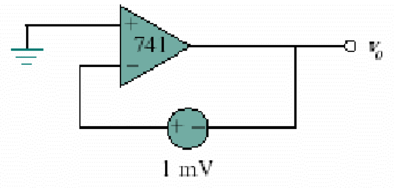
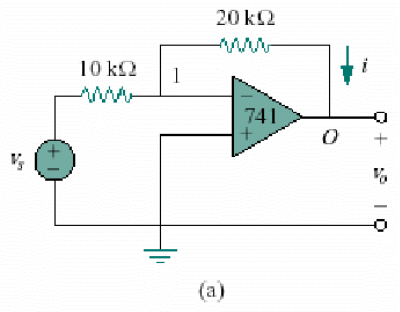


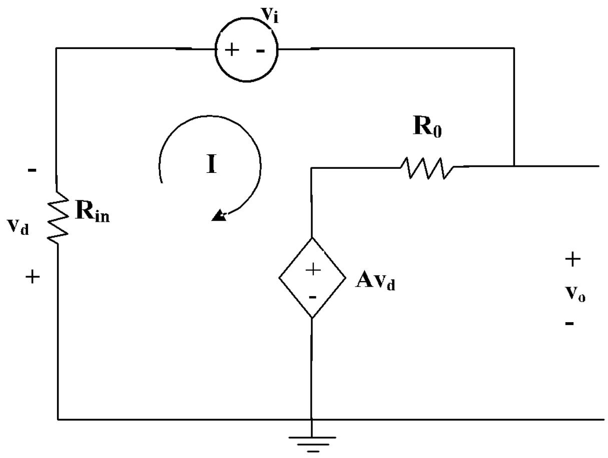
Figure 5.45 for Prob. 5.6

Example 5.1

A 741 op amp has an open-loop voltage gain of 2×10^5, input resistance of 2 MΩ, and output resistance of 50Ω. The op amp is used in the circuit of Fig. 5.6(a). Find the closed-loop gain v_o/v_s . Determine current i when $v_s = 2$ V.



Chapter 5, Solution 6.



$(R_0 + R_i)R + v_i + Av_d = 0$

But $\quad v_d = R_iI,$

$v_i + (R_0 + R_i + R_iA)I = 0$

$$I = \frac{-v_i}{R_0 + (1 + A)R_i} \qquad (1)$$

$-Av_d - R_0I + v_o = 0$

$v_o = Av_d + R_0I = (R_0 + R_iA)I$

Substituting for I in (1),

$$v_0 = -\left(\frac{R_0 + R_iA}{R_0 + (1 + A)R_i}\right)v_i$$

$$= -\frac{\left(50 + 2x10^6 x2x10^5\right)\cdot 10^{-3}}{50 + \left(1 + 2x10^5\right)x2x10^6}$$

$$\cong \frac{-200{,}000x2x10^6}{200{,}001x2x10^6}mV$$

$v_0 =$ **<u>-0.999995 mV</u>**

Chapter 5, Problem 7

The op amp in Fig. 5.46 has R_i = 100 kΩ, R_o = 100 Ω, A = 100,000. Find the differential voltage v_d and the output voltage v_o.

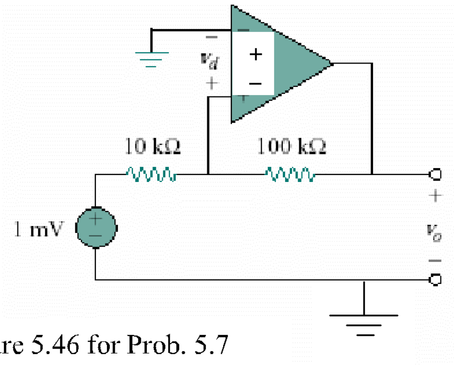
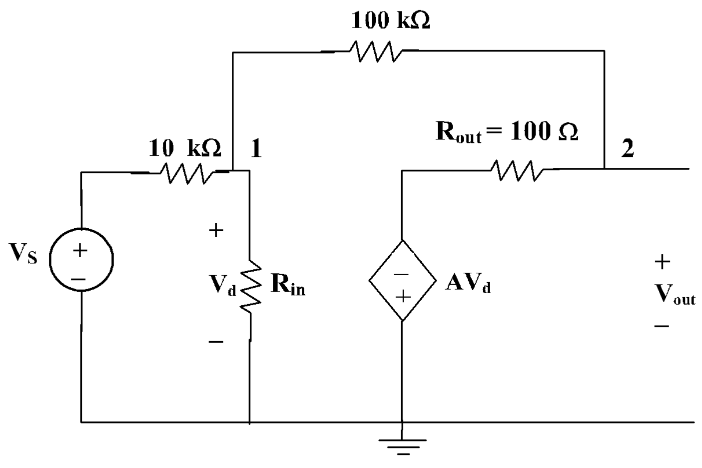


Figure 5.46 for Prob. 5.7

Chapter 5, Solution 7.



At node 1, $(V_S - V_1)/10\text{ k} = [V_1/100\text{ k}] + [(V_1 - V_0)/100\text{ k}]$

$10\,V_S - 10\,V_1 = V_1 + V_1 - V_0$

which leads to $V_1 = (10V_S + V_0)/12$

At node 2, $(V_1 - V_0)/100\text{ k} = (V_0 - (-AV_d))/100$

But $V_d = V_1$ and $A = 100{,}000$,

$V_1 - V_0 = 1000\,(V_0 + 100{,}000V_1)$

$0 = 1001V_0 + 99{,}999{,}999[(10V_S + V_0)/12]$

$0 = 83{,}333{,}332.5\,V_S + 8{,}334{,}334.25\,V_0$

which gives us $(V_0/V_S) = -10$ (for all practical purposes)

If $V_S = 1$ mV, then V_0 = **<u>–10 mV</u>**

Since $V_0 = A\,V_d = 100{,}000\,V_d$, then $V_d = (V_0/10^5)\text{ V}$ = **<u>–100 nV</u>**

Chapter 5, Problem 8

Obtain v_o for each of the op amp circuits in Fig. 5.47.

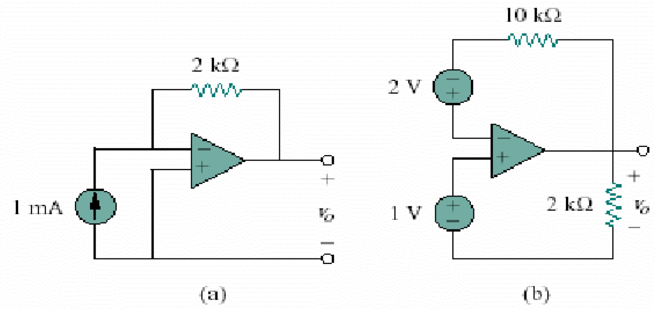


Figure 5.47 for Prob. 5.8

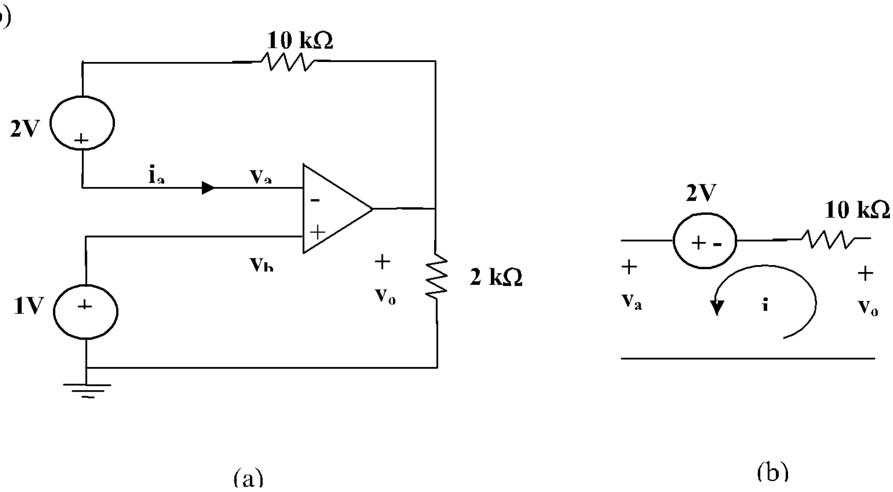
Chapter 5, Solution 8.

(a) If v_a and v_b are the voltages at the inverting and noninverting terminals of the op amp.

$v_a = v_b = 0$

$1\text{mA} = \dfrac{0 - v_0}{2k} \longrightarrow v_0 = \underline{-2V}$

(b)



Since $v_a = v_b = 1V$ and $i_a = 0$, no current flows through the 10 kΩ resistor. From Fig. (b),

$-v_a + 2 + v_0 = 0 \longrightarrow v_0 = v_a - 2 = 1 - 2 = \underline{-1V}$

Chapter 5, Problem 9

Determine v_o for each of the op amp circuits in Fig. 5.48.

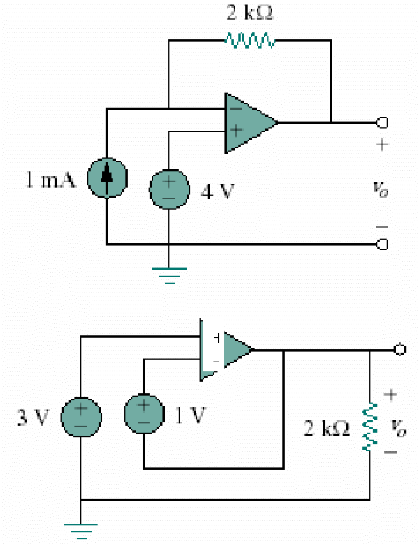


Figure 5.48 for Prob. 5.9

Chapter 5, Solution 9.

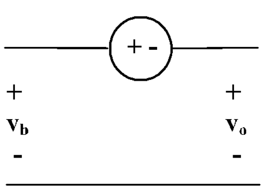
(a) Let v_a and v_b be respectively the voltages at the inverting and noninverting terminals of the op amp

$$v_a = v_b = 4V$$

At the inverting terminal,

$$1mA = \frac{4 - v_0}{2k} \longrightarrow v_0 = \underline{\mathbf{2V}}$$

(b)



Since $v_a = v_b = 3V$,

$$-v_b + 1 + v_o = 0 \longrightarrow v_o = v_b - 1 = \underline{\mathbf{2V}}$$

Chapter 5, Problem 10

Find the gain v_o/v_s of the circuit in Fig. 5.49.

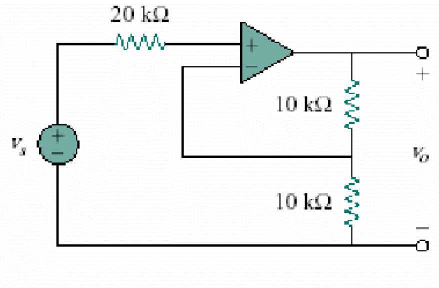


Figure 5.49 for Prob. 5.10

Chapter 5, Solution 10.

Since no current enters the op amp, the voltage at the input of the op amp is v_s. Hence

$$v_s = v_o\left(\frac{10}{10+10}\right) = \frac{v_o}{2} \longrightarrow \frac{v_o}{v_s} = \underline{2}$$

Chapter 5, Problem 11

Find v_o and i_o in the circuit in Fig. 5.50.

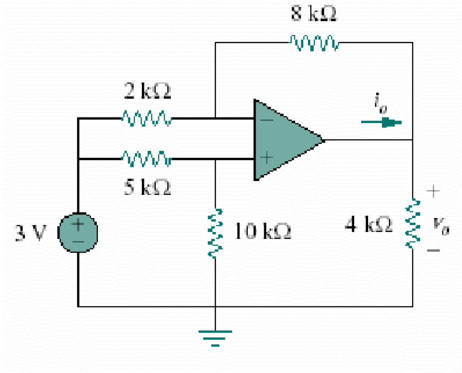
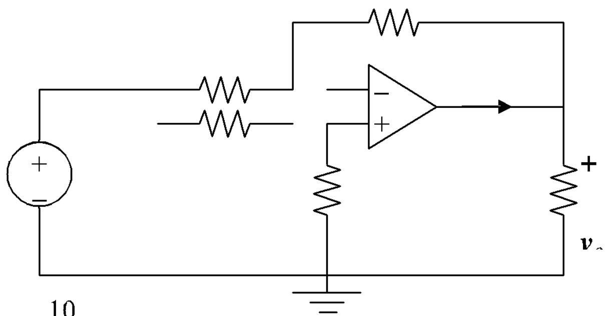


Figure 5.50 for Prob. 5.11

Chapter 5, Solution 11.



$$v_b = \frac{10}{10+5}(3) = 2V$$

At node a,

$$\frac{3 - v_a}{2} = \frac{v_a - v_o}{8} \longrightarrow 12 = 5v_a - v_o$$

But $v_a = v_b = 2V$,

$$12 = 10 - v_o \longrightarrow v_o = \underline{-2V}$$

$$-i_o = \frac{v_a - v_o}{8} + \frac{0 - v_o}{4} = \frac{2+2}{8} + \frac{2}{4} = 1mA$$

$$i_o = \underline{-1mA}$$

Chapter 5, Problem 12.

Calculate the voltage ratio v_o/v_s for the op amp circuit of Fig. 5.51. Assume that the op amp is ideal.

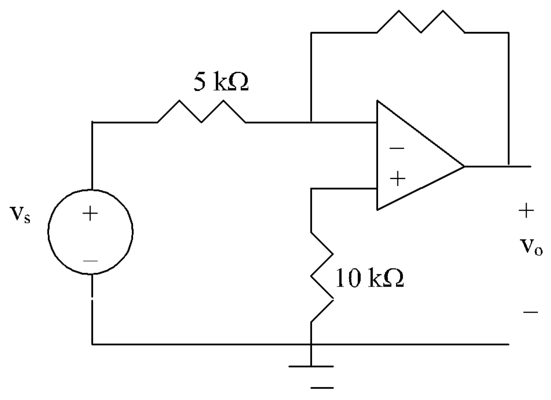


Figure 5.51 For Prob. 5.12.

Chapter 5, Solution 12.

This is an inverting amplifier.

$$v_o = -\frac{25}{5} v_s \longrightarrow \frac{v_o}{v_s} = \underline{-5}$$

Chapter 5, Problem 13

Find v_o and i_o in the circuit of Fig. 5.52.

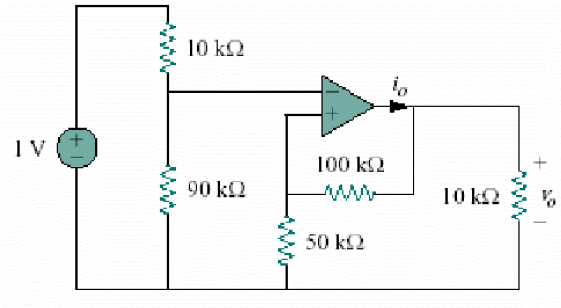
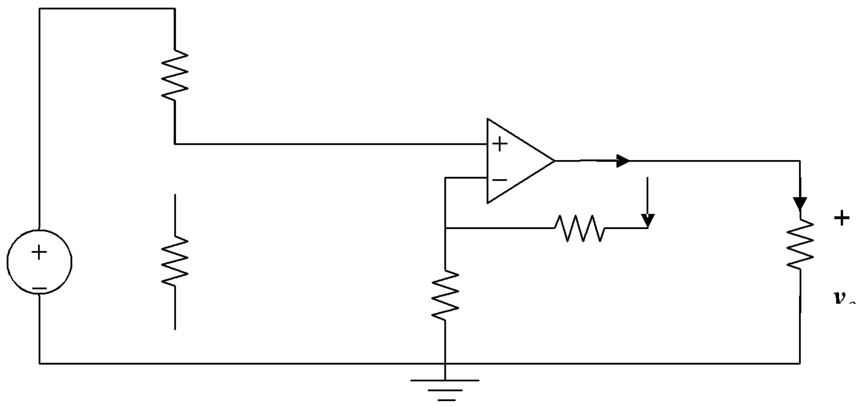


Figure 5.52 for Prob. 5.13

Chapter 5, Solution 13.



By voltage division,

$$v_a = \frac{90}{100}(1) = 0.9\text{V}$$

$$v_b = \frac{50}{150} v_o = \frac{v_o}{3}$$

But $v_a = v_b \longrightarrow \frac{v_0}{3} = 0.9 \longrightarrow v_o = \underline{\mathbf{2.7V}}$

$$i_o = i_1 + i_2 = \frac{v_o}{10k} + \frac{v_o}{150k} = 0.27\text{mA} + 0.018\text{mA} = \underline{\mathbf{288\ \mu A}}$$

Chapter 5, Problem 14

Determine the output voltage v_o in the circuit of Fig. 5.53.

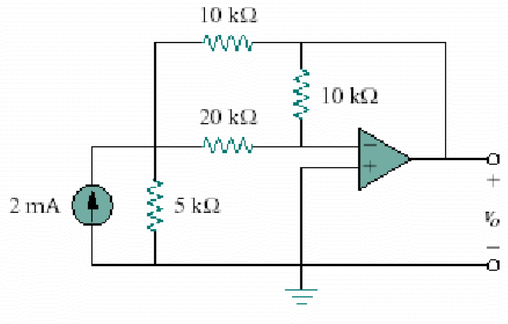
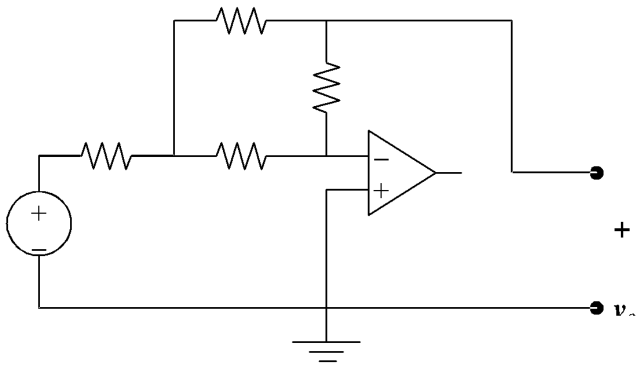


Figure 5.53 for Prob. 5.14

Chapter 5, Solution 14.

Transform the current source as shown below. At node 1,

$$\frac{10 - v_1}{5} = \frac{v_1 - v_2}{20} + \frac{v_1 - v_o}{10}$$



But $v_2 = 0$. Hence $40 - 4v_1 = v_1 + 2v_1 - 2v_o \longrightarrow 40 = 7v_1 - 2v_o$ (1)

At node 2, $\frac{v_1 - v_2}{20} = \frac{v_2 - v_o}{10}, \quad v_2 = 0$ or $v_1 = -2v_o$ (2)

From (1) and (2), $40 = -14v_o - 2v_o \longrightarrow v_o =$ **-2.5V**

Chapter 5, Problem 15

(a). Determine the ratio v_o/i_s in the op amp circuit of Fig. 5.54.
(b). Evaluate the ratio for R_1 = 20 kΩ, R_2 = 25 kΩ, R_3 = 40 2kOmega$.

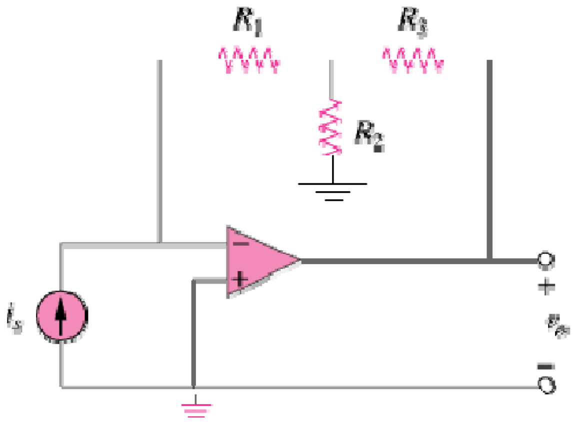


Figure 5.54

Chapter 5, Solution 15

(a) Let v_1 be the voltage at the node where the three resistors meet. Applying KCL at this node gives

$$i_s = \frac{v_1}{R_2} + \frac{v_1 - v_o}{R_3} = v_1\left(\frac{1}{R_2} + \frac{1}{R_3}\right) - \frac{v_o}{R_3} \qquad (1)$$

At the inverting terminal,

$$i_s = \frac{0 - v_1}{R_1} \longrightarrow v_1 = -i_s R_1 \qquad (2)$$

Combining (1) and (2) leads to

$$i_s\left(1 + \frac{R_1}{R_2} + \frac{R_1}{R_3}\right) = -\frac{v_o}{R_3} \longrightarrow \underline{\frac{v_o}{i_s} = -\left(R_1 + R_3 + \frac{R_1 R_3}{R_2}\right)}$$

(b) For this case,

$$\frac{v_o}{i_s} = -\left(20 + 40 + \frac{20x40}{25}\right)\text{k}\Omega = \underline{\text{- 92 k}\Omega}$$

Chapter 5, Problem 16

Obtain i_x and i_y in the op amp circuit in Fig. 5.55.

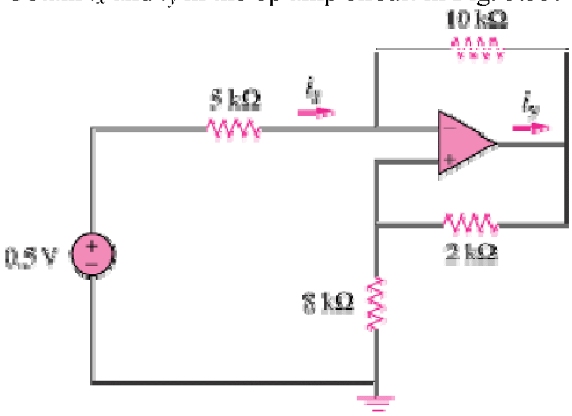
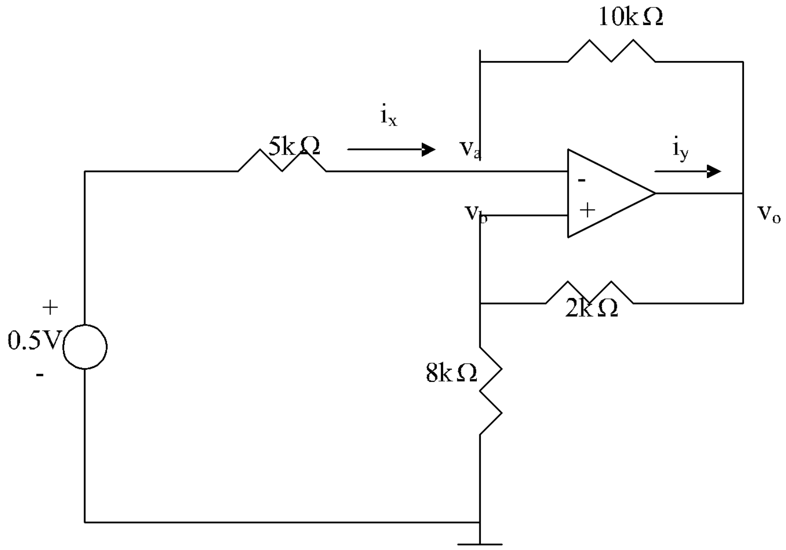


Figure 5.55

Chapter 5, Solution 16



Let currents be in mA and resistances be in kΩ. At node a,

$$\frac{0.5 - v_a}{5} = \frac{v_a - v_o}{10} \quad \longrightarrow \quad 1 = 3v_a - v_o \qquad (1)$$

But

$$v_a = v_b = \frac{8}{8+2} v_o \quad \longrightarrow v_o = \frac{10}{8} v_a \qquad (2)$$

Substituting (2) into (1) gives

$$1 = 3v_a - \frac{10}{8} v_a \quad \longrightarrow \quad v_a = \frac{8}{14}$$

Thus,

$$i_x = \frac{0.5 - v_a}{5} = -1/70 \text{ mA} = \underline{-14.28\,\mu\text{A}}$$

$$i_y = \frac{v_o - v_b}{2} + \frac{v_o - v_a}{10} = 0.6(v_o - v_a) = 0.6(\frac{10}{8} v_a - v_a) = \frac{0.6}{4} x \frac{8}{14} \text{ mA} = \underline{85.71\,\mu\text{A}}$$

Chapter 5, Problem 17

Calculate the gain v_o/v_i when the switch in Fig. 5.56 is in:
(a) position 1 (b) position 2 (c) position 3

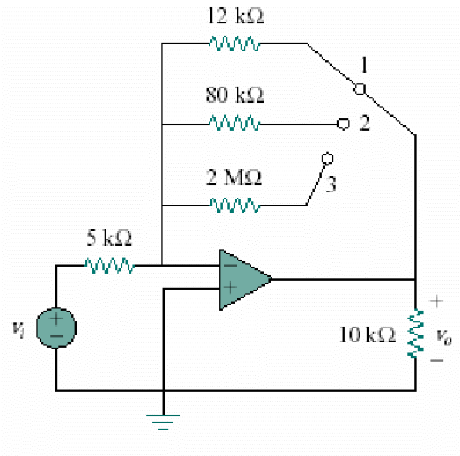


Figure 5.56

Chapter 5, Solution 17.

(a) $$G = \frac{v_o}{v_i} = -\frac{R_2}{R_1} = -\frac{12}{5} = \underline{\mathbf{-2.4}}$$

(b) $$\frac{v_o}{v_i} = -\frac{80}{5} = \underline{\mathbf{-16}}$$

(c) $$\frac{v_o}{v_i} = -\frac{2000}{5} = \underline{\mathbf{-400}}$$

*** Chapter 5, Problem 18.**

For the circuit in Fig. 5.57, find the Thevenin equivalent to the left of terminals a-b. Then calculate the power absorbed by the 20-kΩ resistor. Assume that the op amp is ideal.

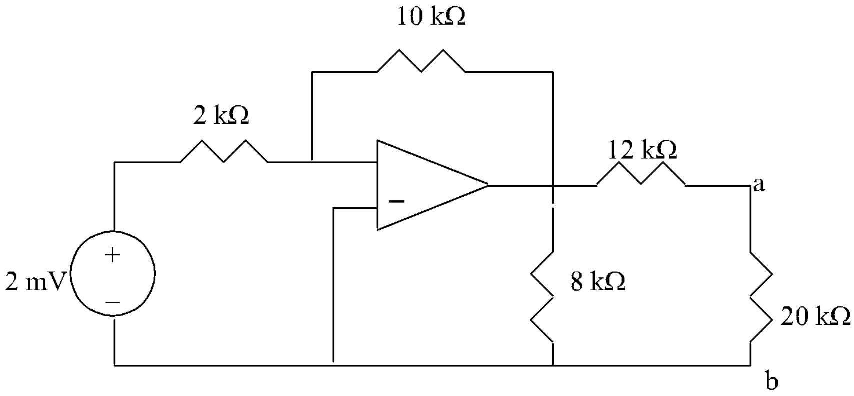
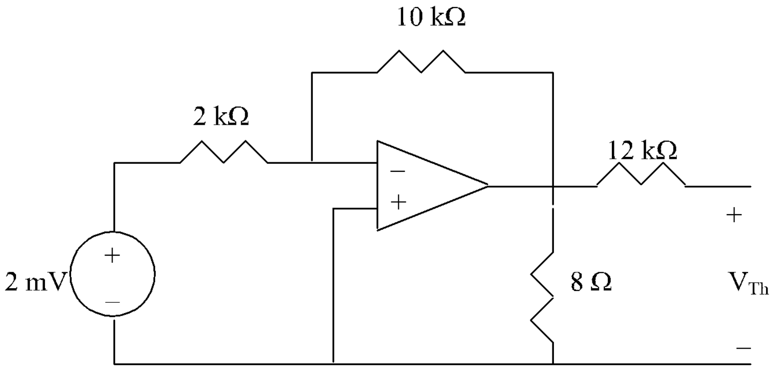


Figure 5.57 For Prob. 5.18.

Chapter 5, Solution 18.

We temporarily remove the 20-kΩ resistor. To find V_{Th}, we consider the circuit below.

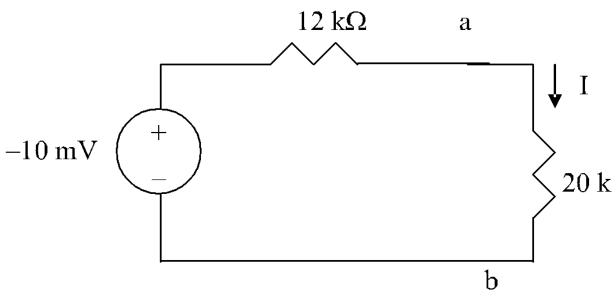


This is an inverting amplifier.

$$V_{Th} = -\frac{10k}{2k}(2mV) = \underline{-10mV}$$

To find R_{Th}, we note that the 8-kΩ resistor is across the output of the op amp which is acting like a voltage source so the only resistance seen looking in is the 12-kΩ resistor.

The Thevenin equivalent with the 20-kΩ resistor is shown below.



$I = -10m/(12k + 20k) = 0.3125x10^{-6}$ A

$p = I^2R = (0.3125x10^{-6})^2x20x10^3 =$ **1.9531 nW**

Chapter 5, Problem 19

Determine i_o in the circuit of Fig. 5.58.

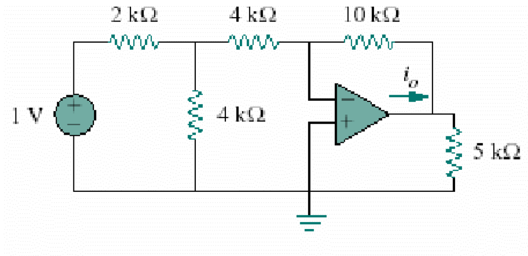
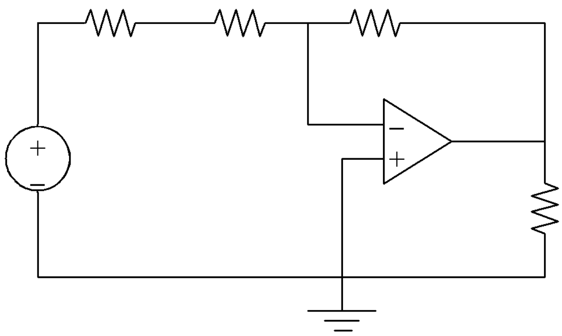


Figure 5.58

Chapter 5, Solution 19.

We convert the current source and back to a voltage source.

$$2\|4 = \frac{4}{3}$$



$$v_o = -\frac{10k}{\left(4 + \frac{4}{3}\right)k}\left(\frac{2}{3}\right) = -1.25V$$

$$i_o = \frac{v_o}{5k} + \frac{v_o - 0}{10k} = \underline{\mathbf{-0.375mA}}$$

Chapter 5, Problem 20

In the circuit in Fig. 5.59, calculate v_o if $v_s = 0$.

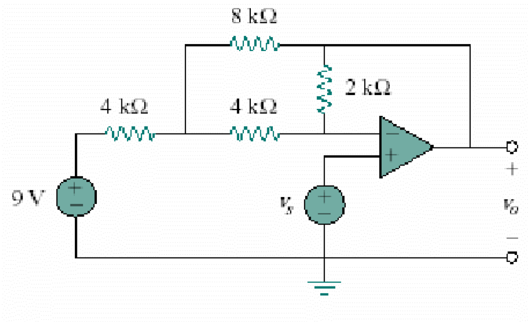
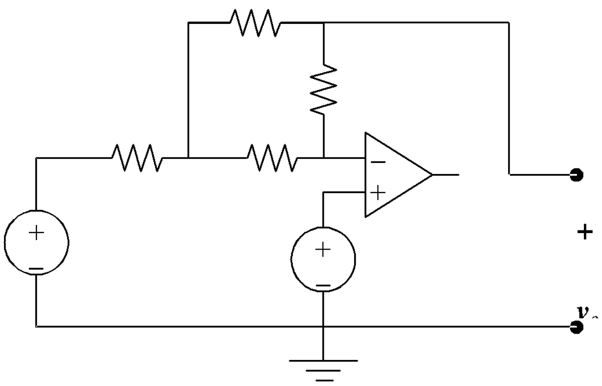


Figure 5.59

Chapter 5, Solution 20.



At node a,

$$\frac{9 - v_a}{4} = \frac{v_a - v_o}{8} + \frac{v_a - v_b}{4} \longrightarrow 18 = 5v_a - v_o - 2v_b \qquad (1)$$

At node b,

$$\frac{v_a - v_b}{4} = \frac{v_b - v_o}{2} \longrightarrow v_a = 3v_b - 2v_o \qquad (2)$$

But $v_b = v_s = 0$; (2) becomes $v_a = -2v_o$ and (1) becomes

$$-18 = -10v_o - v_o \longrightarrow v_o = -18/(11) = \underline{\mathbf{-1.6364V}}$$

Chapter 5, Problem 21.

Calculate v_o in the op amp circuit of Fig. 5.60.

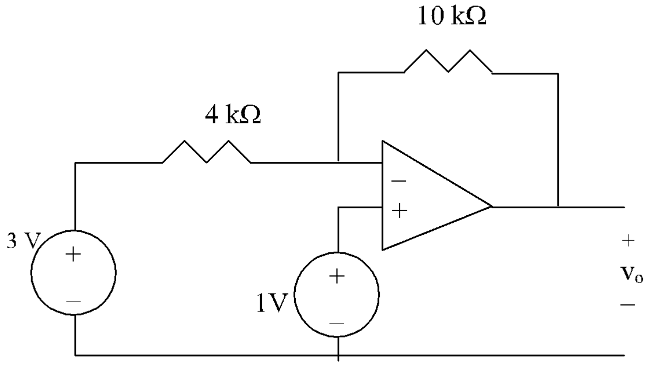


Figure 5.60 For Prob. 5.21.

Chapter 5, Solution 21.

Let the voltage at the input of the op amp be v_a.

$$v_a = 1\,\text{V}, \quad \frac{3\text{-}v_a}{4k} = \frac{v_a - v_o}{10k} \quad \longrightarrow \quad \frac{3\text{-}1}{4} = \frac{1 - v_o}{10}$$

v_o = **−4 V**.

Chapter 5, Problem 22

Design an inverting amplifier with a gain of -15.

Chapter 5, Solution 22.

$A_v = -R_f/R_i = -15.$

If $R_i = 10\text{k}\Omega$, then R_f = **150 kΩ.**

Chapter 5, Problem 23

For the op amp circuit in Fig. 5.61, find the voltage gain v_o/v_s.

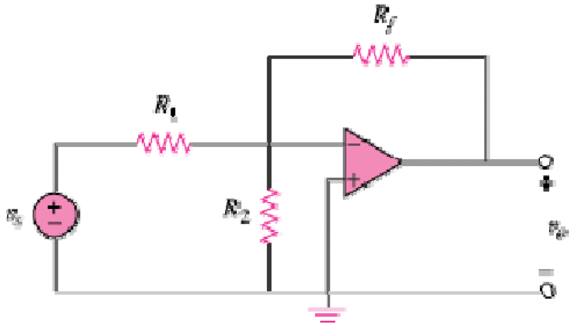


Figure 5.61

Chapter 5, Solution 23

At the inverting terminal, v=0 so that KCL gives

$$\frac{v_s - 0}{R_1} = \frac{0}{R_2} + \frac{0 - v_o}{R_f} \longrightarrow \underline{\frac{v_o}{v_s} = -\frac{R_f}{R_1}}$$

Chapter 5, Problem 24

In the circuit shown in Fig. 5.62, find k in the voltage transfer function $v_o = kv_s$.

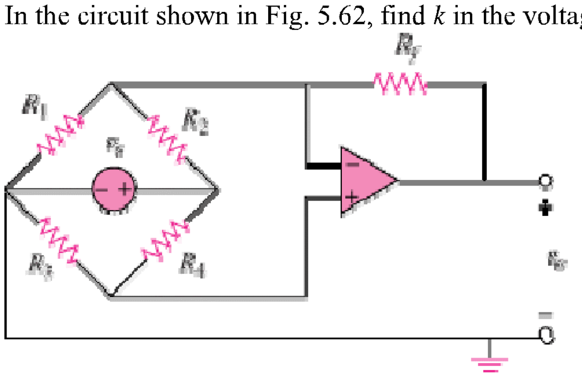
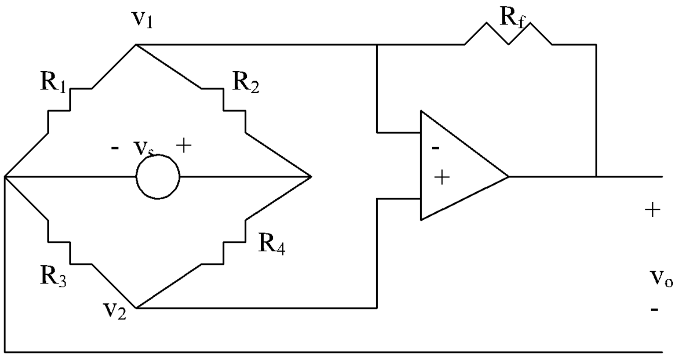


Figure 5.62

Chapter 5, Solution 24



We notice that $v_1 = v_2$. Applying KCL at node 1 gives

$$\frac{v_1}{R_1} + \frac{(v_1 - v_s)}{R_2} + \frac{v_1 - v_o}{R_f} = 0 \longrightarrow \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_f}\right)v_1 - \frac{v_s}{R_2} = \frac{v_o}{R_f} \quad (1)$$

Applying KCL at node 2 gives

$$\frac{v_1}{R_3} + \frac{v_1 - v_s}{R_4} = 0 \longrightarrow v_1 = \frac{R_3}{R_3 + R_4}v_s \quad (2)$$

Substituting (2) into (1) yields

$$v_o = R_f\left[\left(\frac{R_3}{R_1} + \frac{R_3}{R_f} - \frac{R_4}{R_2}\right)\left(\frac{R_3}{R_3 + R_4}\right) - \frac{1}{R_2}\right]v_s$$

i.e.

$$k = R_f\left[\left(\frac{R_3}{R_1} + \frac{R_3}{R_f} - \frac{R_4}{R_2}\right)\left(\frac{R_3}{R_3 + R_4}\right) - \frac{1}{R_2}\right]$$

Chapter 5, Problem 25.

Calculate v_o in the op amp circuit of Fig. 5.63.

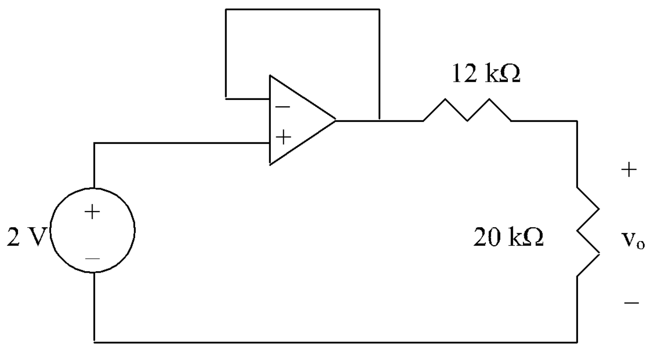


Figure 5.63 For Prob. 5.25.

Chapter 5, Solution 25.

This is a voltage follower. If v_1 is the output of the op amp,

$$v_1 = 2V$$

$$v_o = \frac{20k}{20k+12k} v_1 = \frac{20}{32}(12) = \underline{1.25\ V}$$

Chapter 5, Problem 26

Determine i_o in the circuit of Fig. 5.64.

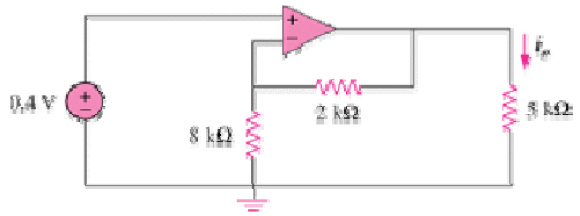
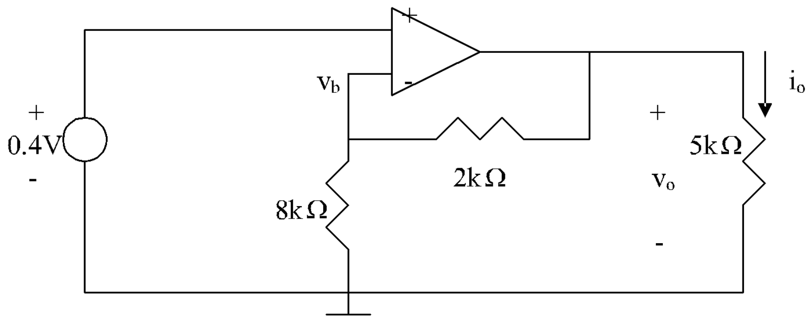


Figure 5.64

Chapter 5, Solution 26



$$v_b = 0.4 = \frac{8}{8+2} v_o = 0.8 v_o \qquad \longrightarrow \qquad v_o = 0.4/0.8 = 0.5\text{ V}$$

Hence,

$$i_o = \frac{v_o}{5k} = \frac{0.5}{5k} = \underline{0.1\text{ mA}}$$

Chapter 5, Problem 27.

Find v_o in the op amp circuit in Fig. 5.65.

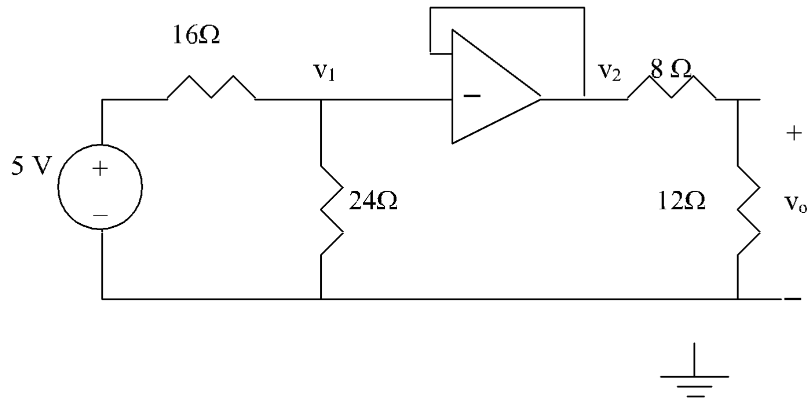


Figure 5.65 For Prob. 5.27.

Chapter 5, Solution 27.

This is a voltage follower.

$$v_1 = \frac{24}{24+16}(5) = 3V, \qquad v_2 = v_1 = 3V$$

$$v_o = \frac{12}{12+8}(3V) = \underline{1.8\ \text{V}}$$

Chapter 5, Problem 28

Find i_o in the op amp circuit of Fig. 5.66.

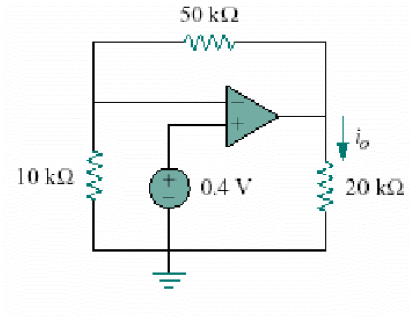
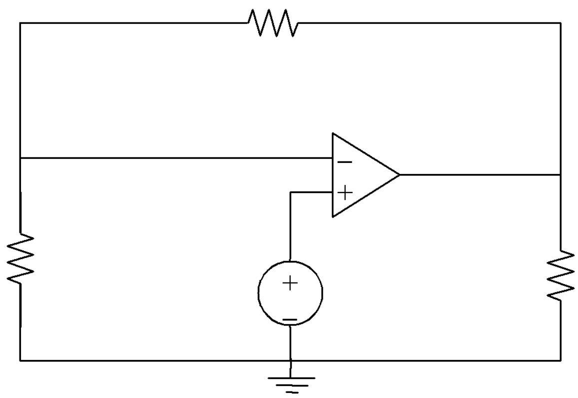


Figure 5.66

Chapter 5, Solution 28.



At node 1, $\frac{0 - v_1}{10k} = \frac{v_1 - v_o}{50k}$

But $v_1 = 0.4V$,

$-5v_1 = v_1 - v_o$, leads to $v_o = 6v_1 = \underline{\textbf{2.4V}}$

Alternatively, viewed as a noninverting amplifier,

$v_o = (1 + (50/10))\ (0.4V) = \underline{\textbf{2.4V}}$

$i_o = v_o/(20k) = 2.4/(20k) = \underline{\textbf{120 µA}}$

Chapter 5, Problem 29

Determine the voltage gain v_o/v_i of the op amp circuit in Fig. 5.67.

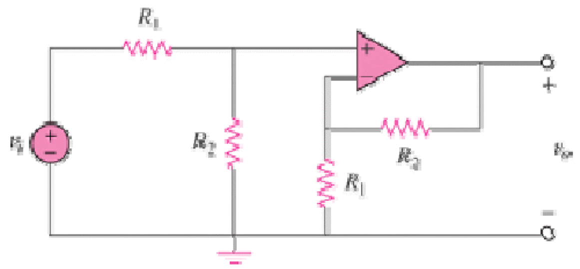
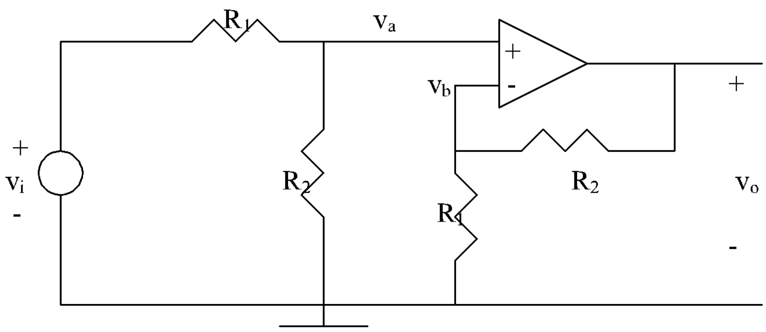


Figure 5.67

Chapter 5, Solution 29



$$v_a = \frac{R_2}{R_1 + R_2} v_i, \qquad v_b = \frac{R_1}{R_1 + R_2} v_o$$

But $v_a = v_b \qquad \longrightarrow \qquad \frac{R_2}{R_1 + R_2} v_i = \frac{R_1}{R_1 + R_2} v_o$

Or

$$\frac{v_o}{v_i} = \frac{R_2}{R_1}$$

Chapter 5, Problem 30

In the circuit shown in Fig. 5.68, find i_x and the power absorbed by the 20-Ω resistor.

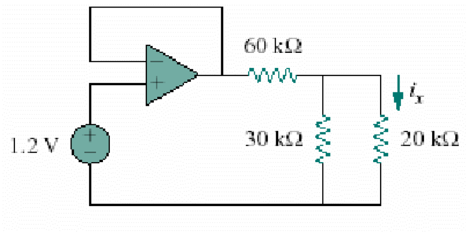


Figure 5.68

Chapter 5, Solution 30.

The output of the voltage becomes

$$v_o = v_i = 12$$
$$30\|20 = 12\text{k}\Omega$$

By voltage division,

$$v_x = \frac{12}{12+60}(1.2) = 0.2\text{V}$$

$$i_x = \frac{v_x}{20\text{k}} = \frac{0.2}{20\text{k}} = \underline{\mathbf{10\mu A}}$$

$$p = \frac{v_x^2}{R} = \frac{0.04}{20\text{k}} = \underline{\mathbf{2\mu W}}$$

Chapter 5, Problem 31

For the circuit in Fig. 5.69, find $\boldsymbol{i_x}$.

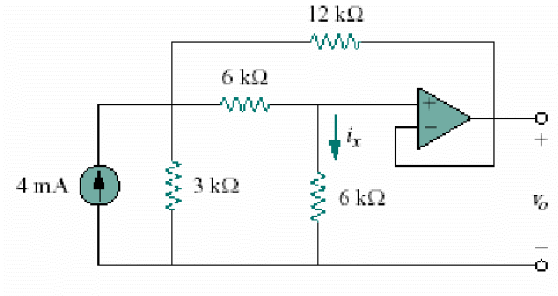
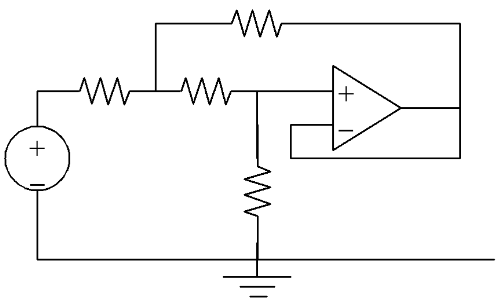


Figure 5.69

Chapter 5, Solution 31.

After converting the current source to a voltage source, the circuit is as shown below:



At node 1,

$$\frac{12 - v_1}{3} = \frac{v_1 - v_o}{6} + \frac{v_1 - v_o}{12} \longrightarrow 48 = 7v_1 - 3v_o \qquad (1)$$

At node 2,

$$\frac{v_1 - v_o}{6} = \frac{v_o - 0}{6} = i_x \longrightarrow v_1 = 2v_o \qquad (2)$$

From (1) and (2),

$$v_o = \frac{48}{11}$$

$$i_x = \frac{v_o}{6k} = \underline{\mathbf{727.2\mu A}}$$

Chapter 5, Problem 32

Calculate i_x and v_o in the circuit of Fig. 5.70. Find the power dissipated by the 60-kΩ resistor.

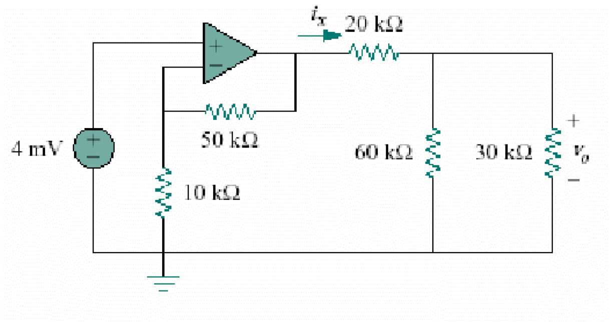


Figure 5.70

Chapter 5, Solution 32.

Let v_x = the voltage at the output of the op amp. The given circuit is a non-inverting amplifier.

$$v_x = \left(1 + \frac{50}{10}\right)(4 \text{ mV}) = 24 \text{ mV}$$

$$60 \| 30 = 20\text{k}\Omega$$

By voltage division,

$$v_o = \frac{20}{20+20} v_x = \frac{v_x}{2} = 12\text{mV}$$

$$i_x = \frac{v_x}{(20+20)\text{k}} = \frac{24\text{mV}}{40\text{k}} = \underline{\mathbf{600nA}}$$

$$p = \frac{v_o^2}{R} = \frac{144\text{x}10^{-6}}{60\text{x}10^3} = \underline{\mathbf{204nW}}$$

Chapter 5, Problem 33

Refer to the op amp circuit in Fig. 5.71. Calculate i_x and the power dissipated by the 3-kΩ resistor.

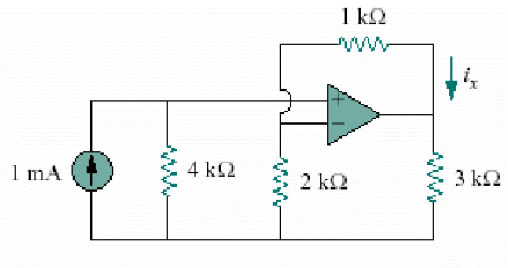
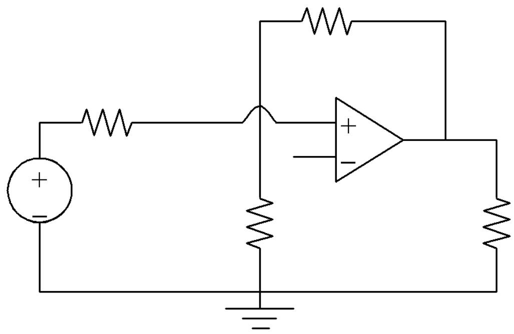


Figure 5.71

Chapter 5, Solution 33.

After transforming the current source, the current is as shown below:



This is a noninverting amplifier.

$$v_o = \left(1 + \frac{1}{2}\right)v_i = \frac{3}{2}v_i$$

Since the current entering the op amp is 0, the source resistor has a OV potential drop. Hence v_i = 4V.

$$v_o = \frac{3}{2}(4) = 6V$$

Power dissipated by the 3kΩ resistor is

$$\frac{v_o^2}{R} = \frac{36}{3k} = \underline{\textbf{12mW}}$$

$$i_x = \frac{v_a - v_o}{R} = \frac{4-6}{1k} = \underline{\textbf{-2mA}}$$

Chapter 5, Problem 34.

Given the op amp circuit shown in Fig. 5.72, express v_o in terms of v_1 and v_2.

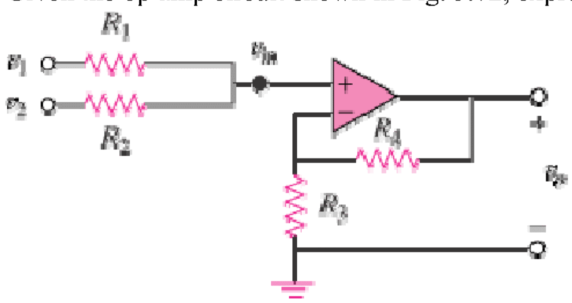


Figure 5.72

Chapter 5, Solution 34

$$\frac{v_1 - v_{in}}{R_1} + \frac{v_1 - v_{in}}{R_2} = 0 \qquad (1)$$

but

$$v_a = \frac{R_3}{R_3 + R_4} v_o \qquad (2)$$

Combining (1) and (2),

$$v_1 - v_a + \frac{R_1}{R_2} v_2 - \frac{R_1}{R_2} v_a = 0$$

$$v_a\left(1 + \frac{R_1}{R_2}\right) = v_1 + \frac{R_1}{R_2} v_2$$

$$\frac{R_3 v_o}{R_3 + R_4}\left(1 + \frac{R_1}{R_2}\right) = v_1 + \frac{R_1}{R_2} v_2$$

$$v_o = \frac{R_3 + R_4}{R_3\left(1 + \frac{R_1}{R_2}\right)}\left(v_1 + \frac{R_1}{R_2} v_2\right)$$

$$v_O = \frac{R_3 + R_4}{R_3(R_1 + R_2)}(v_1 R_2 + v_2)$$

Chapter 5, Problem 35

Design a non-inverting amplifier with a gain of 10.

Chapter 5, Solution 35.

$$A_v = \frac{v_o}{v_i} = 1 + \frac{R_f}{R_i} = 10 \longrightarrow R_f = 9R_i$$

If R_i = **10kΩ**, R_f = **90kΩ**

Chapter 5, Problem 36

For the circuit shown in Fig. 5.73, find the Thèvenin equivalent at terminals *a*-*b*. (*Hint*: To find R_{Th}, apply a current source i_o and calculate v_o.)

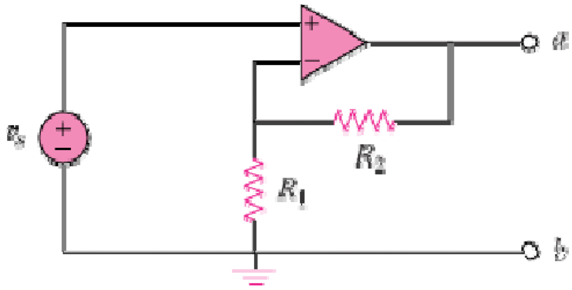


Figure 5.73

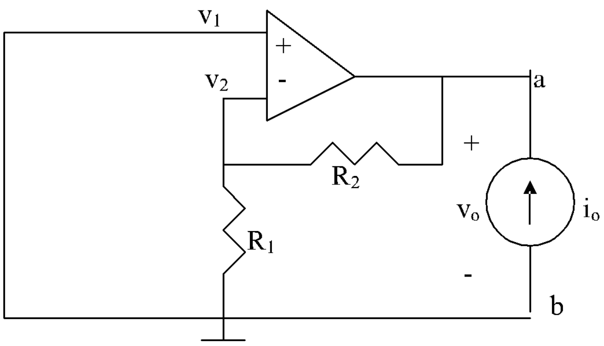
Chapter 5, Solution 36

$$V_{Th} = V_{ab}$$

But $\quad v_s = \frac{R_1}{R_1 + R_2} V_{ab}$. Thus,

$$V_{Th} = V_{ab} = \frac{R_1 + R_2}{R_1} v_s = (1 + \frac{R_2}{R_1}) v_s$$

To get R_{Th}, apply a current source I_o at terminals a-b as shown below.



Since the noninverting terminal is connected to ground, $v_1 = v_2 = 0$, i.e. no current passes through R_1 and consequently R_2. Thus, v_o=0 and

$$R_{Th} = \frac{v_o}{i_o} = 0$$

Chapter 5, Problem 37

Determine the output of the summing amplifier in Fig. 5.74.

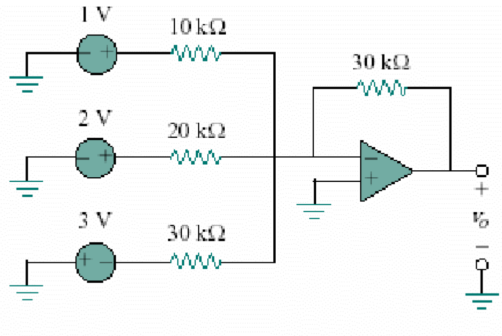


Figure 5.74

Chapter 5, Solution 37.

$$v_o = -\left[\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3\right]$$

$$= -\left[\frac{30}{10}(1) + \frac{30}{20}(2) + \frac{30}{30}(-3)\right]$$

v_o = <u>**–3V**</u>

Chapter 5, Problem 38

Calculate the output voltage due to the summing amplifier shown in Fig. 5.75.

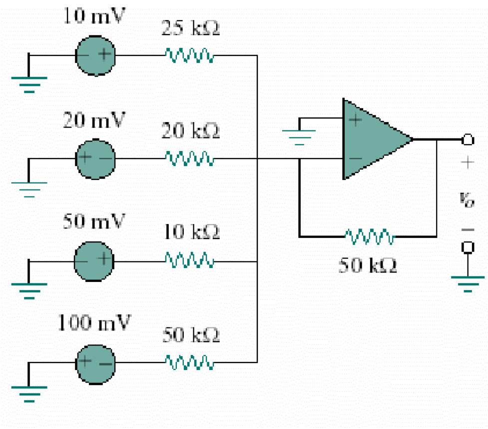


Figure 5.75

Chapter 5, Solution 38.

$$v_o = -\left[\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3 + \frac{R_f}{R_4}v_4\right]$$

$$= -\left[\frac{50}{25}(10) + \frac{50}{20}(-20) + \frac{50}{10}(50) + \frac{50}{50}(-100)\right]$$

= **<u>-120mV</u>**

Chapter 5, Problem 39

For the op amp circuit in Fig. 5.76, determine the value of v_2 in order to make v_o = -16.5 V.

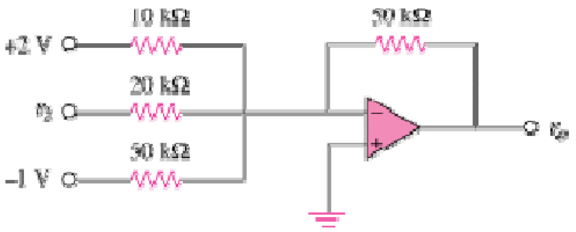


Figure 5.76

Chapter 5, Solution 39

This is a summing amplifier.

$$v_o = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3\right) = -\left(\frac{50}{10}(2) + \frac{50}{20}v_2 + \frac{50}{50}(-1)\right) = -9 - 2.5v_2$$

Thus,

$$v_o = -16.5 = -9 - 2.5v_2 \quad \longrightarrow \quad \underline{v_2 = 3\text{ V}}$$

Chapter 5, Problem 40.

Find v_o in terms of v_1, v_2, and v_3, in the circuit of Fig. 5.77.

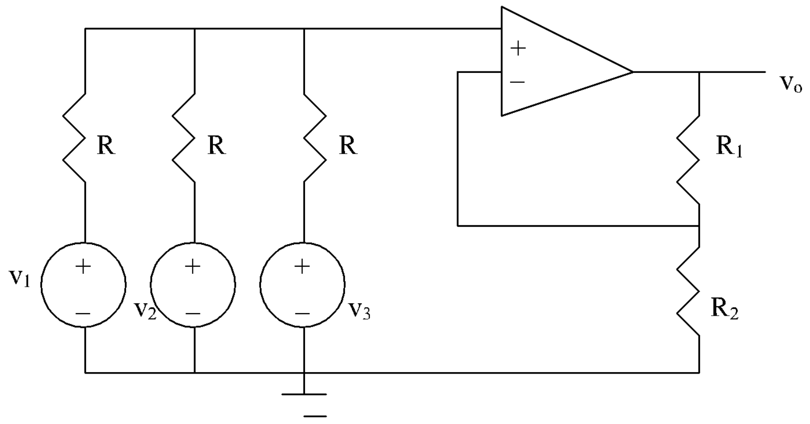


Figure 5.77 For Prob. 5.40.

Chapter 5, Solution 40.

Applying KCL at node a, where node a is the input to the op amp.

$$\frac{v_1 - v_a}{R} + \frac{v_2 - v_a}{R} + \frac{v_3 - v_a}{R} = 0 \quad \text{or} \quad v_a = (v_1 + v_2 + v_3)/3$$

$$v_o = (1 + R_1/R_2)v_a = \underline{\mathbf{(1 + R_1/R_2)(v_1 + v_2 + v_3)/3}}.$$

Chapter 5, Problem 41

An *averaging amplifier* is a summer that provides an output equal to the average of the inputs. By using proper input and feedback resistor values, one can get

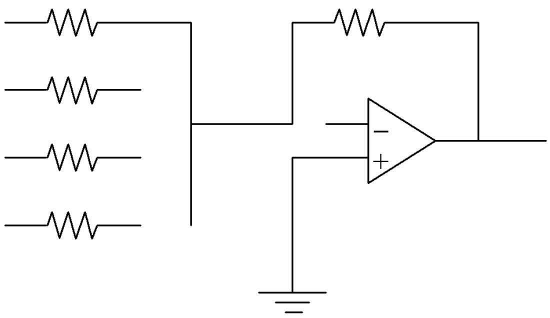
$$-v_{out} = \frac{1}{4}(v_1 + v_2 + v_3 + v_4)$$

Using a feedback resistor of 10 kΩ, design an averaging amplifier with four inputs.

Chapter 5, Solution 41.

$$R_f/R_i = 1/(4) \longrightarrow R_i = 4R_f = 40\text{k}\Omega$$

The averaging amplifier is as shown below:



Chapter 5, Problem 42

A three-input summing amplifier has input resistors with $R_1 = R_2 = R_3 = 30$ kΩ. To produce an averaging amplifier, what value of feedback resistor is needed?

Chapter 5, Solution 42

$$R_f = \frac{1}{3}R_1 = \underline{10\ \text{k}\Omega}$$

Chapter 5, Problem 43

A four-input summing amplifier has $R_1 = R_2 = R_3 = R_4 = 12$ kΩ. What value of feedback resistor is needed to make it an averaging amplifier?

Chapter 5, Solution 43.

In order for

$$v_o = \left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3 + \frac{R_f}{R_4}v_4\right)$$

to become

$$v_o = -\frac{1}{4}(v_1 + v_2 + v_3 + v_4)$$

$$\frac{R_f}{R_i} = \frac{1}{4} \longrightarrow R_f = \frac{R_i}{4} = \frac{12}{4} = \underline{\mathbf{3k\Omega}}$$

Chapter 5, Problem 44

Show that the output voltage v_o of the circuit in Fig. 5.78 is

$$v_o = \frac{(R_3 + R_4)}{R_3(R_1 + R_2)}(R_2 v_1 + R_1 v_2)$$

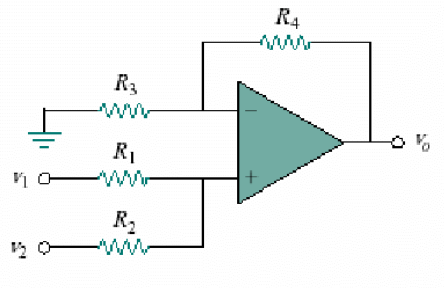
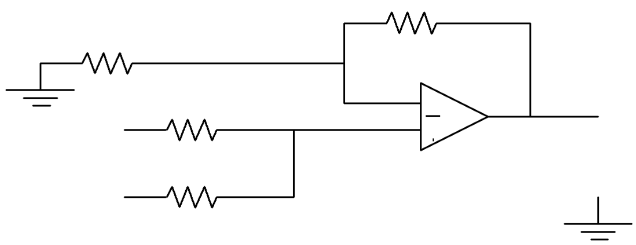


Figure 5.78

Chapter 5, Solution 44.



At node b, $\frac{v_b - v_1}{R_1} + \frac{v_b - v_2}{R_2} = 0 \longrightarrow v_b = \frac{\frac{v_1}{R_1} + \frac{v_2}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}}$ (1)

At node a, $\frac{0 - v_a}{R_3} = \frac{v_a - v_o}{R_4} \longrightarrow v_a = \frac{v_o}{1 + R_4 / R_3}$ (2)

But $v_a = v_b$. We set (1) and (2) equal.

$$\frac{v_o}{1 + R_4 / R_3} = \frac{R_2 v_1 + R_1 v_2}{R_1 + R_2}$$

or

$$v_o = \frac{(R_3 + R_4)}{R_3(R_1 + R_2)}(R_2 v_1 + R_1 v_2)$$

Chapter 5, Problem 45

Design an op amp circuit to perform the following operation:

$$v_o = 3v_1 - 2v_2$$

All resistances must be ≤ 100 kΩ.

Chapter 5, Solution 45.

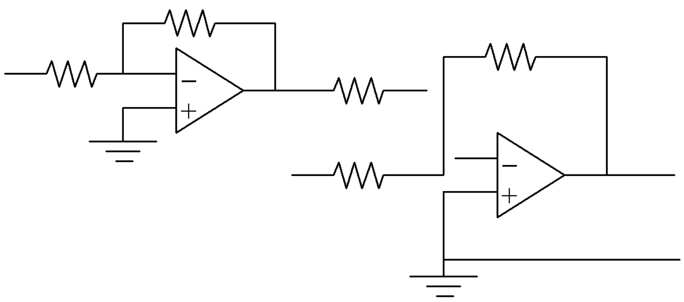
This can be achieved as follows:

$$v_o = -\left[\frac{R}{R/3}(-v_1) + \frac{R}{R/2}v_2\right]$$

$$= -\left[\frac{R_f}{R_1}(-v_1) + \frac{R_f}{R_2}v_2\right]$$

i.e. $R_f = R$, $R_1 = R/3$, and $R_2 = R/2$

Thus we need an inverter to invert v_1, and a summer, as shown below ($R<100k\Omega$).



Chapter 5, Problem 46

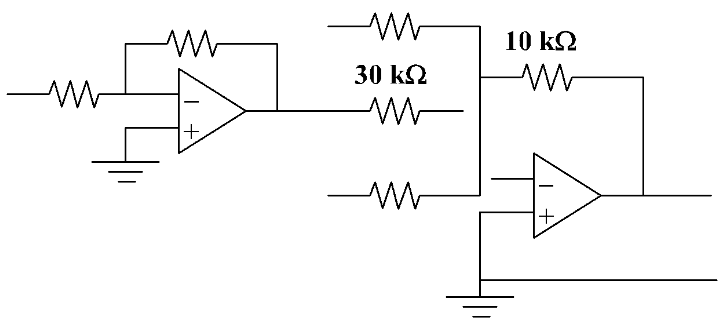
Using only two op amps, design a circuit to solve

$$-v_{out} = \frac{v_1 - v_2}{3} + \frac{v_3}{2}$$

Chapter 5, Solution 46.

$$-v_o = \frac{v_1}{3} + \frac{1}{3}(-v_2) + \frac{1}{2}v_3 = \frac{R_f}{R_1}v_1 + \frac{R_x}{R_2}(-v_2) + \frac{R_f}{R_3}v_3$$

i.e. $R_3 = 2R_f$, $R_1 = R_2 = 3R_f$. To get $-v_2$, we need an inverter with $R_f = R_i$. If $R_f = 10\text{k}\Omega$, a solution is given below.



Chapter 5, Problem 47.

The circuit in Fig. 5.79 is for a difference amplifier. Find v_o given that v_1 =1V and v_2 = 2V.

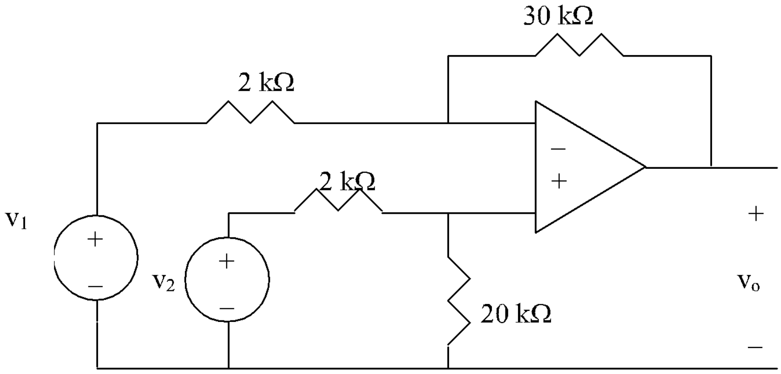


Figure 5.79 For Prob. 5.47.

Chapter 5, Solution 47.

Using eq. (5.18), $R_1 = 2k\Omega,\ R_2 = 30k\Omega,\ R_3 = 2k\Omega,\ R_4 = 20k\Omega$

$$v_o = \frac{30(1+2/30)}{2(1+2/20)}V_2 - \frac{30}{2}V_1 = \frac{32}{2.2}(2) - 15(1) = \underline{14.09\ \text{V}}$$

Chapter 5, Problem 48

The circuit in Fig. 5.80 is a differential amplifier driven by a bridge. Find v_o.

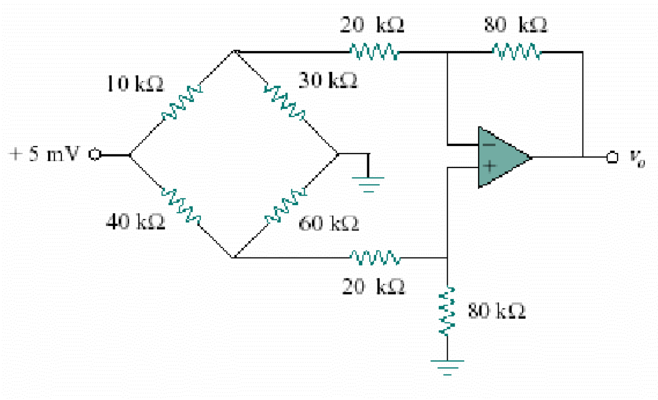
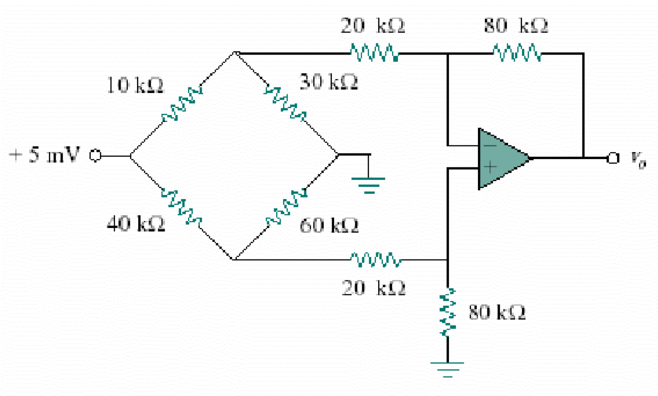


Figure 5.80

Chapter 5, Solution 48.
We can break this problem up into parts. The 5 mV source separates the lower circuit from the upper. In addition, there is no current flowing into the input of the op amp which means we now have the 40-kohm resistor in series with a parallel combination of the 60-kohm resistor and the equivalent 100-kohm resistor.



Thus, $40k + (60x100k)/(160) = 77.5k$

which leads to the current flowing through this part of the circuit,

$$i = 5m/77.5k = 6.452x10^{-8}$$

The voltage across the 60k and equivalent 100k is equal to,

$$v = ix37.5k = 2.419mV$$

We can now calculate the voltage across the 80-kohm resistor.

$$v_{80} = 0.8x2.419m = 1.9352mV$$

which is also the voltage at both inputs of the op amp and the voltage between the 20-kohm and 80-kohm resistors in the upper circuit. Let v_1 be the voltage to the left of the 20-kohm resistor of the upper circuit and we can write a node equation at that node.

$$(v_1\text{–}5m)/(10k) + v_1/30k + (v_1\text{–}1.9352m)/20k = 0$$

or $$6v_1 - 30m + 2v_1 + 3v_1 - 5.806m = 0$$

or $$v_1 = 35.806m/11 = 3.255mV$$

The current through the 20k-ohm resistor, left to right, is,

$$i_{20} = (3.255m\text{–}1.9352m)/20k = 6.599x10^{-8}\text{ A}$$

thus, $$v_o = 1.9352m - 6.599x10^{-8}x80k$$
$$= 1.9352m - 5.2792m = \underline{\mathbf{-3.344\ mV}}.$$

Chapter 5, Problem 49

Design a difference amplifier to have a gain of 2 and a common mode input resistance of 10 kΩ at each input.

Chapter 5, Solution 49.

$$R_1 = R_3 = 10k\Omega,\ R_2/(R_1) = 2$$

i.e. $$R_2 = 2R_1 = 20k\Omega = R_4$$

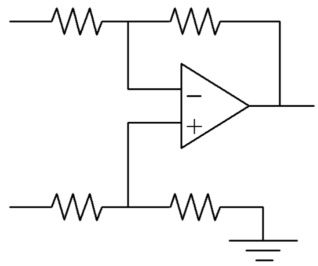
Verify: $$v_o = \frac{R_2}{R_1}\frac{1+R_1/R_2}{1+R_3/R_4}v_2 - \frac{R_2}{R_1}v_1$$

$$= 2\frac{(1+0.5)}{1+0.5}v_2 - 2v_1 = 2(v_2 - v_1)$$

Thus, $R_1 = R_3 =$ **10kΩ**, $R_2 = R_4 =$ **20kΩ**

Chapter 5, Problem 50
Design a circuit to amplify the difference between two inputs by 2.
(a) Use only one op amp.
(b) Use two op amps.

Chapter 5, Solution 50.
(a) We use a difference amplifier, as shown below:



$$v_o = \frac{R_2}{R_1}(v_2 - v_1) = 2(v_2 - v_1), \text{ i.e. } R_2/R_1 = 2$$

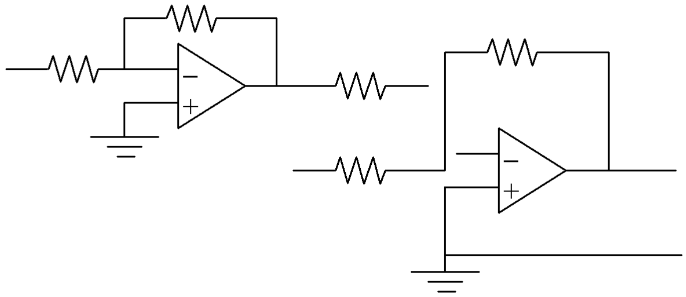
If R_1 = **<u>10 kΩ</u>** then R_2 = **<u>20kΩ</u>**

(b) We may apply the idea in Prob. 5.35.

$$v_0 = 2v_1 - 2v_2$$
$$= -\left[\frac{R}{R/2}(-v_1) + \frac{R}{R/2}v_2\right]$$
$$= -\left[\frac{R_f}{R_1}(-v_1) + \frac{R_f}{R_2}v_2\right]$$

i.e. $R_f = R$, $R_1 = R/2 = R_2$

We need an inverter to invert v_1 and a summer, as shown below. We may let R = 10kΩ.

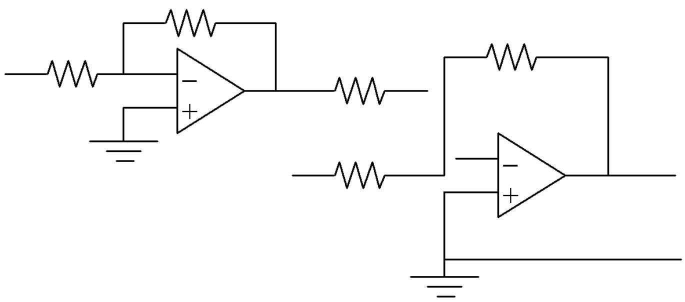


Chapter 5, Problem 51

Using two op amps, design a subtractor.

Chapter 5, Solution 51.

We achieve this by cascading an inverting amplifier and two-input inverting summer as shown below:



Verify:

$v_o = -v_a - v_2$

But $v_a = -v_1$. Hence

$v_o = v_1 - v_2$.

Chapter 5, Problem 52

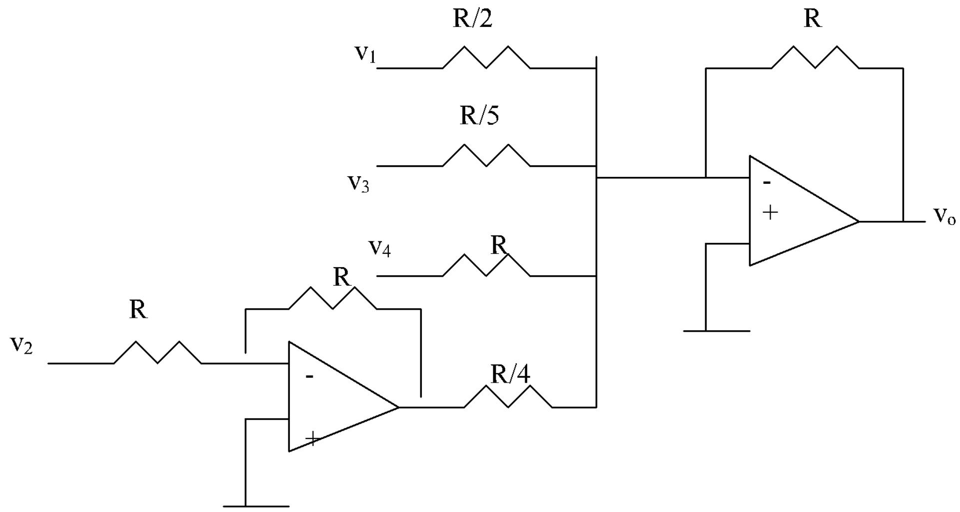
Design an op amp circuit such that

$v_o = - 2v_1 + 4v_2 - 5v_3 - v_4$

Let all the resistors be in the range of 5 to 100 kΩ.

Chapter 5, Solution 52

A summing amplifier shown below will achieve the objective. An inverter is inserted to invert v_2. Let $R = 10\ k\Omega$.



Chapter 5, Problem 53

The ordinary difference amplifier for fixed-gain operation is shown in Fig. 5.81(a). It is simple and reliable unless gain is made variable. One way of providing gain adjustment without losing simplicity and accuracy is to use the circuit in Fig. 5.81(b). Another way is to use the circuit in Fig. 5.81(c). Show that:

(a) for the circuit in Fig. 5.81(a),

$$\frac{v_o}{v_i} = \frac{R_2}{R_1}$$

(b) for the circuit in Fig. 5.81(b),

$$\frac{v_o}{v_i} = \frac{R_2}{R_1} \frac{1}{1 + \dfrac{R_1}{2R_G}}$$

(c) for the circuit in Fig. 5.81(c),

$$\frac{v_o}{v_i} = \frac{R_2}{R_1}\left(1 + \frac{R_2}{2R_G}\right)$$

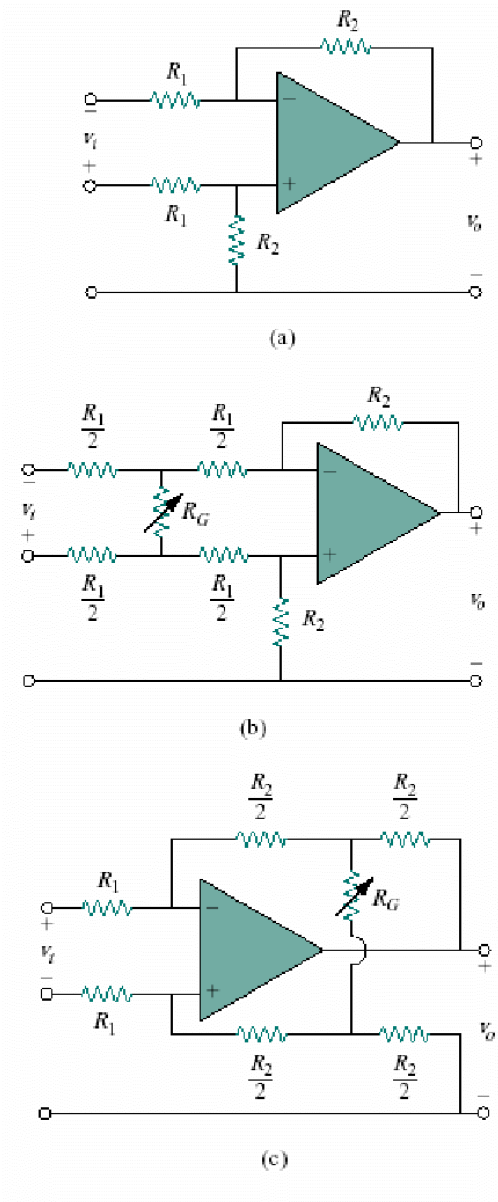
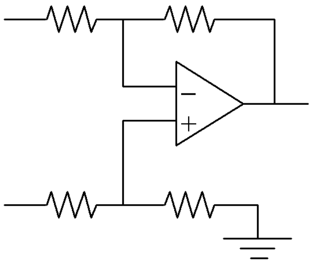

R2
R1
vi
R1
R2
vo
(a)
R1/2
R1/2
R2
RG
vi
R1/2
R1/2
R2
vo
(b)
R2/2
R2/2
R1
RG
vi
R1
vo
R2/2
R2/2
(c)

Figure 5.81

Chapter 5, Solution 53.

(a)



At node a,

$$\frac{v_1 - v_a}{R_1} = \frac{v_a - v_o}{R_2} \longrightarrow v_a = \frac{R_2 v_1 + R_1 v_o}{R_1 + R_2} \qquad (1)$$

At node b, $$v_b = \frac{R_2}{R_1 + R_2} v_2 \qquad (2)$$

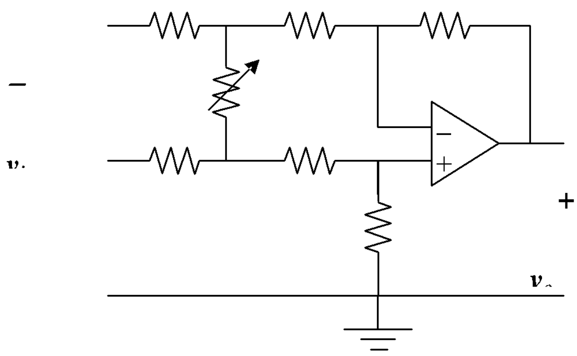
But $v_a = v_b$. Setting (1) and (2) equal gives

$$\frac{R_2}{R_1 + R_2} v_2 = \frac{R_2 v_1 + R_1 v_o}{R_1 + R_2}$$

$$v_2 - v_1 = \frac{R_1}{R_2} v_o = v_i$$

$$\frac{v_o}{v_i} = \frac{R_2}{R_1}$$

(b)



At node A, $\dfrac{v_1 - v_A}{R_1/2} + \dfrac{v_B - v_A}{R_g} = \dfrac{v_A - v_a}{R_1/2}$

or $$v_1 - v_A + \frac{R_1}{2R_g}(v_B - v_A) = v_A - v_a \qquad (1)$$

At node B, $\dfrac{v_2 - v_B}{R_1/2} = \dfrac{v_B - v_A}{R_1/2} + \dfrac{v_B - v_b}{R_g}$

or $$v_2 - v_B - \frac{R_1}{2R_g}(v_B - v_A) = v_B - v_b \qquad (2)$$

Subtracting (1) from (2),

$$v_2 - v_1 - v_B + v_A - \frac{2R_1}{2R_g}(v_B - v_A) = v_B - v_A - v_b + v_a$$

Since, $v_a = v_b$,

$$\frac{v_2 - v_1}{2} = \left(1 + \frac{R_1}{2R_g}\right)(v_B - v_A) = \frac{v_i}{2}$$

or $$v_B - v_A = \frac{v_i}{2} \cdot \frac{1}{1 + \dfrac{R_1}{2R_g}} \qquad (3)$$

But for the difference amplifier,

$$v_o = \frac{R_2}{R_1/2}(v_B - v_A)$$

or

$$v_B - v_A = \frac{R_1}{2R_2} v_o \qquad (4)$$

Equating (3) and (4), $\frac{R_1}{2R_2} v_o = \frac{v_i}{2} \cdot \frac{1}{1+\frac{R_1}{2R_g}}$

$$\frac{v_o}{v_i} = \frac{R_2}{R_1} \cdot \frac{1}{1+\frac{R_1}{2R_g}}$$

(c) At node a, $\frac{v_1 - v_a}{R_1} = \frac{v_a - v_A}{R_2/2}$

$$v_1 - v_a = \frac{2R_1}{R_2}v_a - \frac{2R_1}{R_2}v_A \qquad (1)$$

At node b, $v_2 - v_b = \frac{2R_1}{R_2}v_b - \frac{2R_1}{R_2}v_B \qquad (2)$

Since $v_a = v_b$, we subtract (1) from (2),

$$v_2 - v_1 = \frac{-2R_1}{R_2}(v_B - v_A) = \frac{v_i}{2}$$

or $v_B - v_A = \frac{-R_2}{2R_1}v_i \qquad (3)$

At node A,

$$\frac{v_a - v_A}{R_2/2} + \frac{v_B - v_A}{R_g} = \frac{v_A - v_o}{R/2}$$

$$v_a - v_A + \frac{R_2}{2R_g}(v_B - v_A) = v_A - v_o \qquad (4)$$

At node B, $\frac{v_b - v_B}{R/2} - \frac{v_B - v_A}{R_g} = \frac{v_B - 0}{R/2}$

$$v_b - v_B - \frac{R_2}{2R_g}(v_B - v_A) = v_B \qquad (5)$$

Subtracting (5) from (4),

$$v_B - v_A + \frac{R_2}{R_g}(v_B - v_A) = v_A - v_B - v_o$$

$$2(v_B - v_A)\left(1 + \frac{R_2}{2R_g}\right) = -v_o \qquad (6)$$

Combining (3) and (6),

$$\frac{-R_2}{R_1}v_i\left(1 + \frac{R_2}{2R_g}\right) = -v_o$$

$$\underline{\frac{v_o}{v_i} = \frac{R_2}{R_1}\left(1 + \frac{R_2}{2R_g}\right)}$$

Chapter 5, Problem 54.

Determine the voltage transfer ratio v_o/v_s in the op amp circuit of Fig. 5.82, where R =10 kΩ.

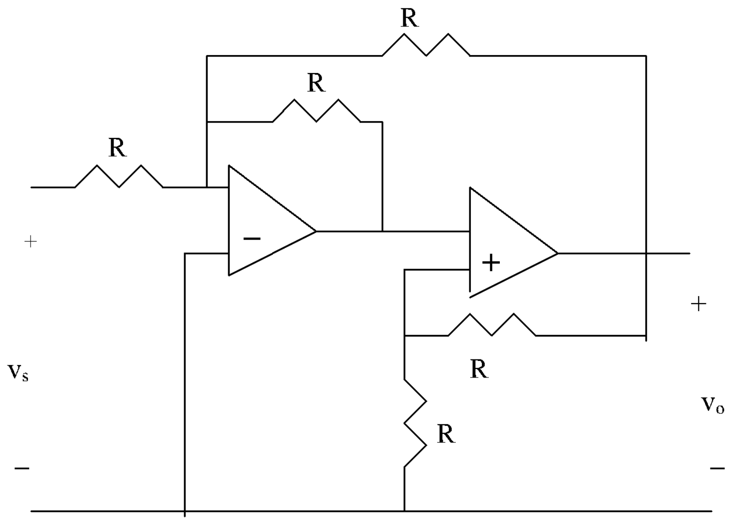


Figure 5.82 For Prob. 5.54.

Chapter 5, Solution 54.

The first stage is a summer (please note that we let the output of the first stage be v_1).

$$v_1 = -\left(\frac{R}{R}v_s + \frac{R}{R}v_o\right) = -v_s - v_o$$

The second stage is a noninverting amplifier

$$v_o = (1 + R/R)v_1 = 2v_1 = 2(-v_s - v_o) \quad \text{or } 3v_o = -2v_s$$

$$v_o/v_s = \underline{\mathbf{-0.6667}}.$$

Chapter 5, Problem 55

In a certain electronic device, a three-stage amplifier is desired, whose overall voltage gain is 42 dB. The individual voltage gains of the first two stages are to be equal, while the gain of the third is to be one-fourth of each of the first two. Calculate the voltage gain of each.

Chapter 5, Solution 55.

Let $A_1 = k$, $A_2 = k$, and $A_3 = k/(4)$

$A = A_1A_2A_3 = k^3/(4)$

$20\text{Log}_{10}A = 42$

$\text{Log}_{10}A = 2.1 \longrightarrow A = 10^{2.1} = 125.89$

$k^3 = 4A = 503.57$

$k = \sqrt[3]{503.57} = 7.956$

Thus $A_1 = A_2 =$ **7.956**, $A_3 =$ **1.989**

Chapter 5, Problem 56.

Calculate the gain of the op amp circuit shown in Fig. 5.83.

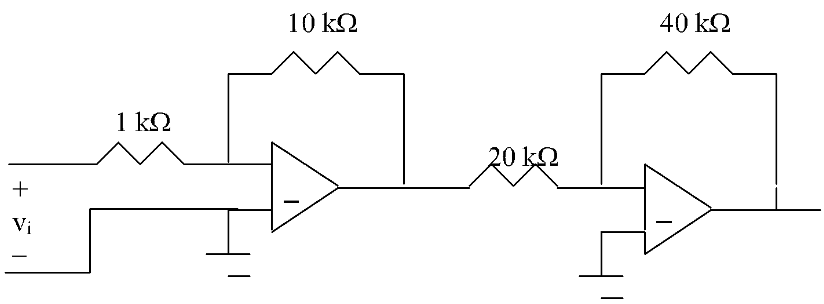


Figure 5.83 For Prob. 5.56.

Chapter 5, Solution 56.

Each stage is an inverting amplifier. Hence.

$$\frac{V_o}{V_s} = (-\frac{10}{1})(-\frac{40}{20}) = \underline{20}$$

Chapter 5, Problem 57.

Find v_o in the op amp circuit of Fig. 5.84.

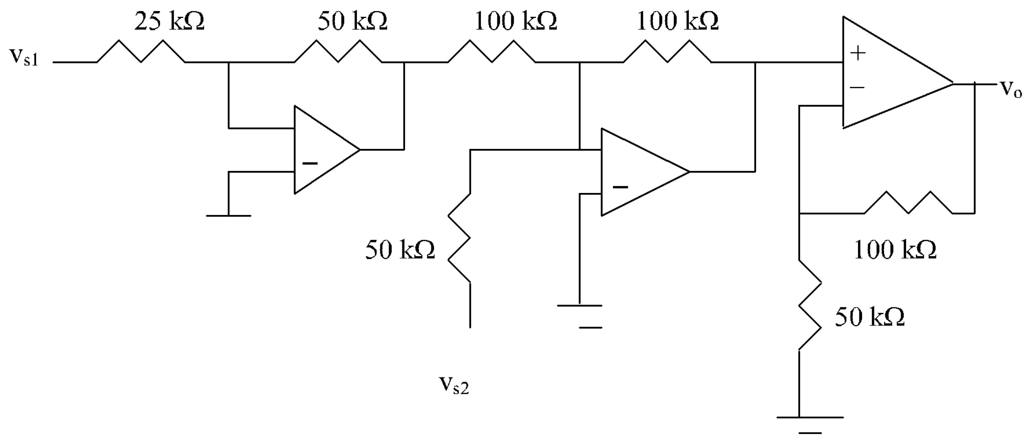


Figure 5.84 For Prob. 5.57.

Chapter 5, Solution 57.

Let v_1 be the output of the first op amp and v_2 be the output of the second op amp.

The first stage is an inverting amplifier.

$$v_1 = -\frac{50}{25}v_{s1} = -2v_{s1}$$

The second state is a summer.

$$v_2 = -(100/50)v_{s2} - (100/100)v_1 = -2v_{s2} + 2v_{s1}$$

The third state is a noninverting amplifier

$$v_o = (1+\frac{100}{50})v_2 = 3v_2 = \underline{6v_{s1} - 6v_{s2}}$$

Chapter 5, Problem 58

Calculate i_o in the op amp circuit of Fig. 5.85.

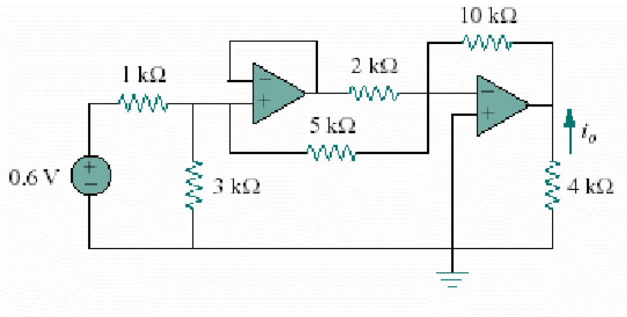


Figure 5.85

Chapter 5, Solution 58.

Looking at the circuit, the voltage at the right side of the 5-kΩ resistor must be at 0V if the op amps are working correctly. Thus the 1-kΩ is in series with the parallel combination of the 3-kΩ and the 5-kΩ. By voltage division, the input to the voltage follower is:

$$v_1 = \frac{3\|5}{1+3\|5}(0.6) = 0.3913\text{V} = \text{to the output of the first op amp.}$$

Thus

$$v_o = -10((0.3913/5)+(0.3913/2)) = -2.739 \text{ V}.$$

$$i_o = \frac{0 - v_o}{4k} = \underline{\textbf{0.6848 mA}}$$

Chapter 5, Problem 59.

In the op amp circuit of Fig. 5.86, determine the voltage gain v_o/v_s. Take R = 10 kΩ.

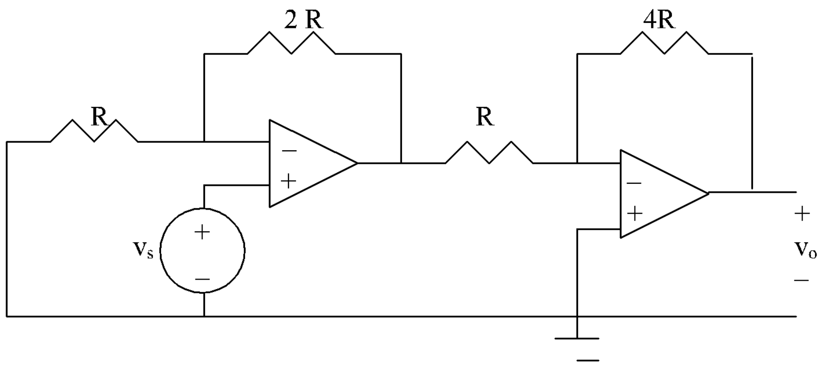


Figure 5.86 For Prob. 5.59.

Chapter 5, Solution 59.

The first stage is a noninverting amplifier. If v_1 is the output of the first op amp,

$$v_1 = (1 + 2R/R)v_s = 3v_s$$

The second stage is an inverting amplifier

$$v_o = -(4R/R)v_1 = -4v_1 = -4(3v_s) = -12v_s$$

$$v_o/v_s = \underline{\mathbf{-12}}.$$

Chapter 5, Problem 60.

Calculate v_o/v_i in the op amp circuit in Fig. 5.87.

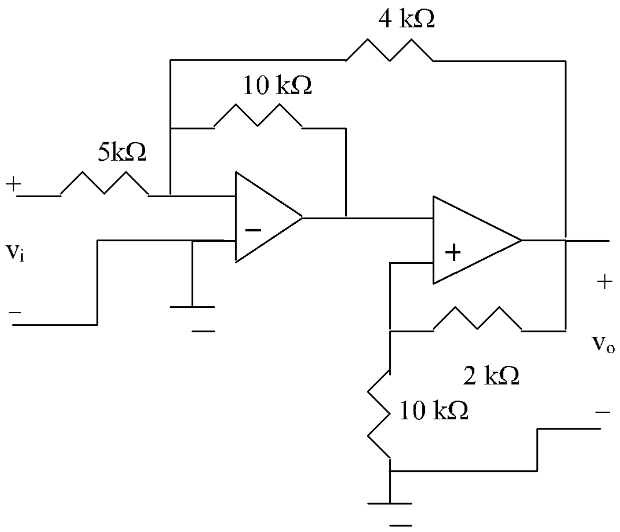


Figure 5.87 For Prob. 5.60.

Chapter 5, Solution 60.

The first stage is a summer. Let V_1 be the output of the first stage.

$$v_1 = -\frac{10}{5}v_i - \frac{10}{4}v_o \longrightarrow v_1 = -2v_i - 2.5v_o \qquad (1)$$

By voltage division,

$$v_1 = \frac{10}{10+2}v_o = \frac{5}{6}v_o \qquad (2)$$

Combining (1) and (2),

$$\frac{5}{6}v_o = -2v_1 - 2.5v_0 \longrightarrow \frac{10}{3}v_0 = -2v_i$$

$$\frac{v_o}{v_i} = -6/10 = \underline{-0.6}$$

Chapter 5, Problem 61.

Determine v_o in the circuit of Fig. 5.88.

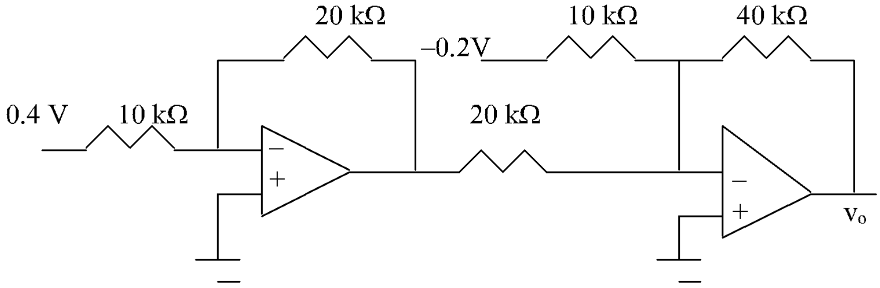


Figure 5.88 For Prob. 5.61.

Chapter 5, Solution 61.

The first op amp is an inverter. If v_1 is the output of the first op amp,

$$v_1 = -\frac{200}{100}(0.4) = -0.8V$$

The second op amp is a summer

$$V_o = \frac{-40}{10}(0.2) - \frac{40}{20}(0.8) = 0.8 + 1.6 = \underline{2.4\ \text{V}}$$

Chapter 5, Problem 62

Obtain the closed-loop voltage gain v_o/v_i of the circuit in Fig. 5.89.

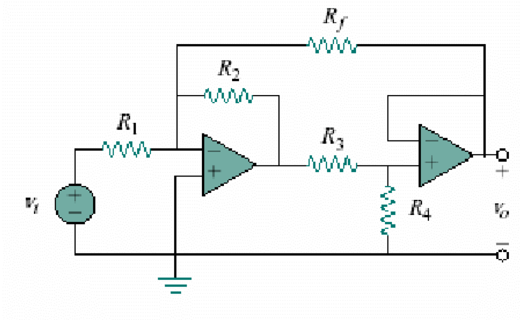


Figure 5.89

Chapter 5, Solution 62.

Let v_1 = output of the first op amp
v_2 = output of the second op amp

The first stage is a summer

$$v_1 = -\frac{R_2}{R_1}v_i - \frac{R_2}{R_f}v_o \qquad (1)$$

The second stage is a follower. By voltage division

$$v_o = v_2 = \frac{R_4}{R_3 + R_4}v_1 \longrightarrow v_1 = \frac{R_3 + R_4}{R_4}v_o \qquad (2)$$

From (1) and (2),

$$\left(1 + \frac{R_3}{R_4}\right)v_o = -\frac{R_2}{R_1}v_i - \frac{R_2}{R_f}v_o$$

$$\left(1 + \frac{R_3}{R_4} + \frac{R_2}{R_f}\right)v_o = -\frac{R_2}{R_1}v_i$$

$$\frac{v_o}{v_i} = -\frac{R_2}{R_1}\cdot\frac{1}{1 + \frac{R_3}{R_4} + \frac{R_2}{R_f}} = \frac{-R_2R_4R_f}{R_1(R_2R_4 + R_3R_f + R_4R_f)}$$

Chapter 5, Problem 63

Determine the gain v_o/v_i of the circuit in Fig. 5.90.

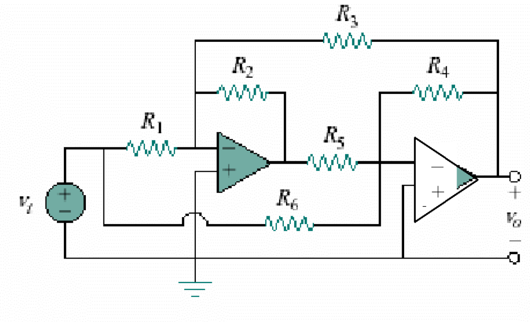


Figure 5.90

Chapter 5, Solution 63.

The two op amps are summers. Let v_1 be the output of the first op amp. For the first stage,

$$v_1 = -\frac{R_2}{R_1}v_i - \frac{R_2}{R_3}v_o \qquad (1)$$

For the second stage,

$$v_o = -\frac{R_4}{R_5}v_1 - \frac{R_4}{R_6}v_i \qquad (2)$$

Combining (1) and (2),

$$v_o = \frac{R_4}{R_5}\left(\frac{R_2}{R_1}\right)v_i + \frac{R_4}{R_5}\left(\frac{R_2}{R_3}\right)v_o - \frac{R_4}{R_6}v_i$$

$$v_o\left(1 - \frac{R_2R_4}{R_3R_5}\right) = \left(\frac{R_2R_4}{R_1R_5} - \frac{R_4}{R_6}\right)v_i$$

$$\frac{v_o}{v_i} = \frac{\dfrac{R_2R_4}{R_1R_5} - \dfrac{R_4}{R_6}}{1 - \dfrac{R_2R_4}{R_3R_5}}$$

Chapter 5, Problem 64

For the op amp circuit shown in Fig. 5.91, find v_o/v_s.

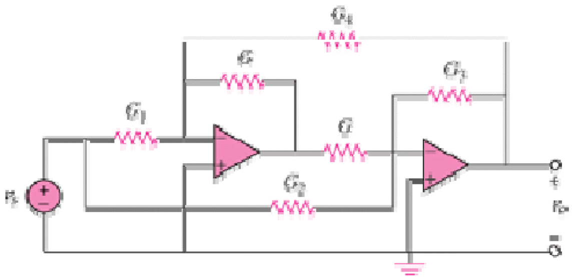
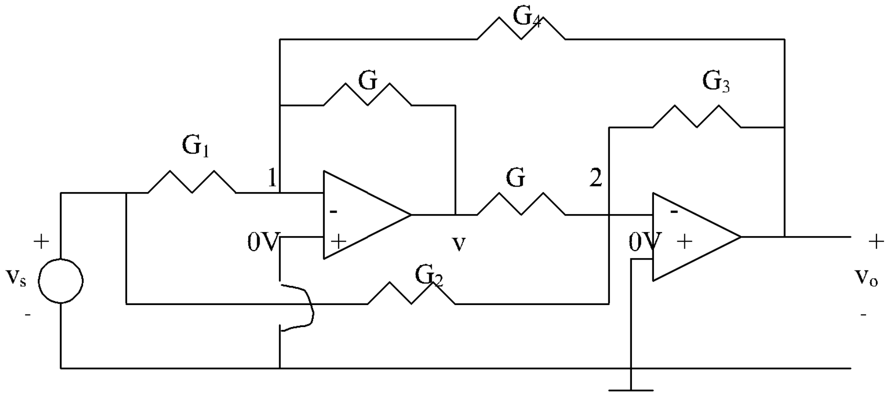


Figure 5.91

Chapter 5, Solution 64



At node 1, v_1=0 so that KCL gives

$$G_1 v_s + G_4 v_o = -Gv \qquad (1)$$

At node 2,

$$G_2 v_s + G_3 v_o = -Gv \qquad (2)$$

From (1) and (2),

$$G_1 v_s + G_4 v_o = G_2 v_s + G_3 v_o \longrightarrow (G_1 - G_2)v_s = (G_3 - G_4)v_o$$

or

$$\frac{v_o}{v_s} = \frac{G_1 - G_2}{G_3 - G_4}$$

Chapter 5, Problem 65

Find v_o in the op amp circuit of Fig. 5.92.

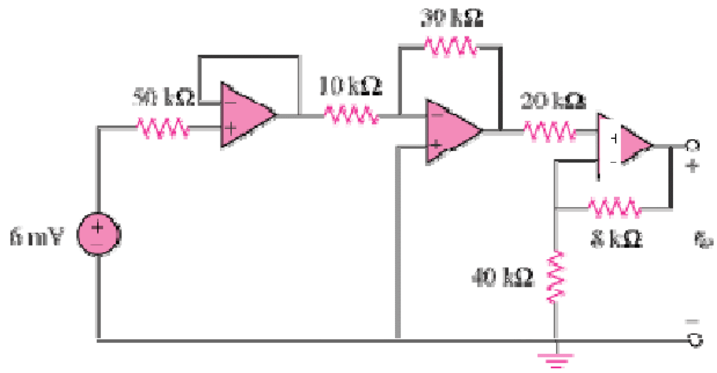


Figure 5.92

Chapter 5, Solution 65

The output of the first op amp (to the left) is 6 mV. The second op amp is an inverter so that its output is

$$v_o' = -\frac{30}{10}(6\text{mV}) = \text{-}18\,\text{mV}$$

The third op amp is a noninverter so that

$$v_o' = \frac{40}{40+8}v_o \quad \longrightarrow \quad v_o = \frac{48}{40}v_o' = \underline{-21.6\,\text{mV}}$$

Chapter 5, Problem 66

For the circuit in Fig. 5.93, find v_o.

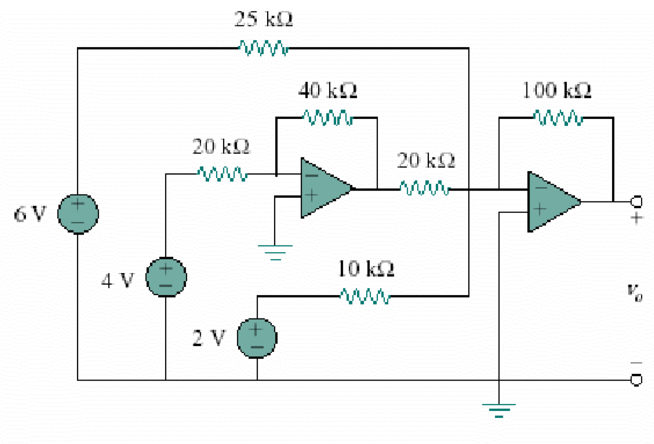


Figure 5.93

Chapter 5, Solution 66.

$$v_o = \frac{-100}{25}(6) - \frac{100}{20}\left(-\frac{40}{20}\right)(4) - \frac{100}{10}(2)$$
$$= -24 + 40 - 20 = \underline{\textbf{-4V}}$$

Chapter 5, Problem 67

Obtain the output v_o in the circuit of Fig. 5.94.

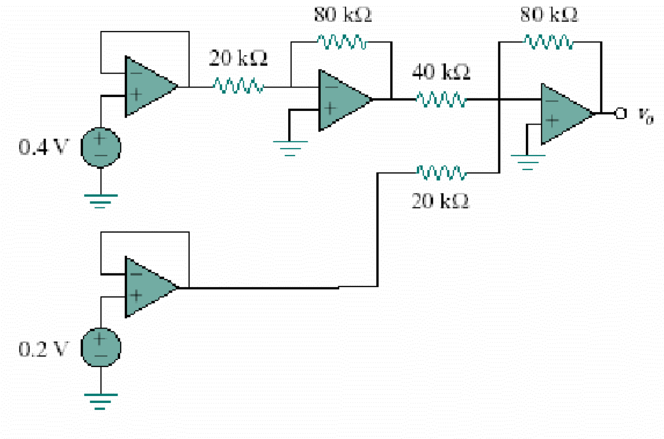


Figure 5.94

Chapter 5, Solution 67.

$$v_o = -\frac{80}{40}\left(-\frac{80}{20}\right)(0.2) - \frac{80}{20}(0.2)$$

$$= 3.2 - 0.8 = \underline{\textbf{2.4V}}$$

Chapter 5, Problem 68.

Find v_o in the circuit in Fig. 5.95, assuming that $R_f = \infty$ (open circuit).

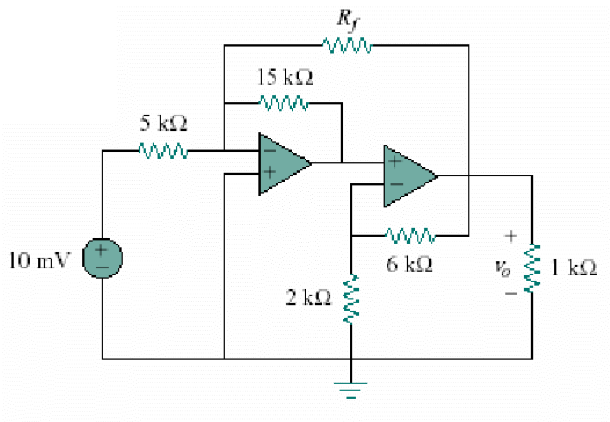


Figure 5.95

Chapter 5, Solution 68.

If $R_q = \infty$, the first stage is an inverter.

$$V_a = -\frac{15}{5}(10) = -30\text{mV}$$

when V_a is the output of the first op amp.

The second stage is a noninverting amplifier.

$$v_o = \left(1 + \frac{6}{2}\right)v_a = (1+3)(-30) = \underline{\mathbf{-120mV}}$$

Chapter 5, Problem 69

Repeat the previous problem if $R_f = 10\ \text{k}\Omega$.

5.68 Find v_o in the circuit in Fig. 5.93, assuming that $R_f = \infty$ (open circuit).

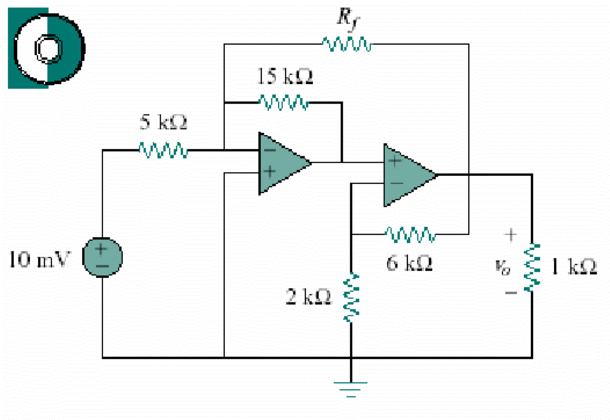


Figure 5.93

Chapter 5, Solution 69.

In this case, the first stage is a summer

$$v_a = -\frac{15}{5}(10) - \frac{15}{10}v_o = -30 - 1.5v_o$$

For the second stage,

$$v_o = \left(1 + \frac{6}{2}\right)v_a = 4v_a = 4(-30 - 1.5v_o)$$

$$7v_o = -120 \longrightarrow v_o = -\frac{120}{7} = \underline{\textbf{-17.143mV}}$$

Chapter 5, Problem 70

Determine v_o in the op amp circuit of Fig. 5.96.

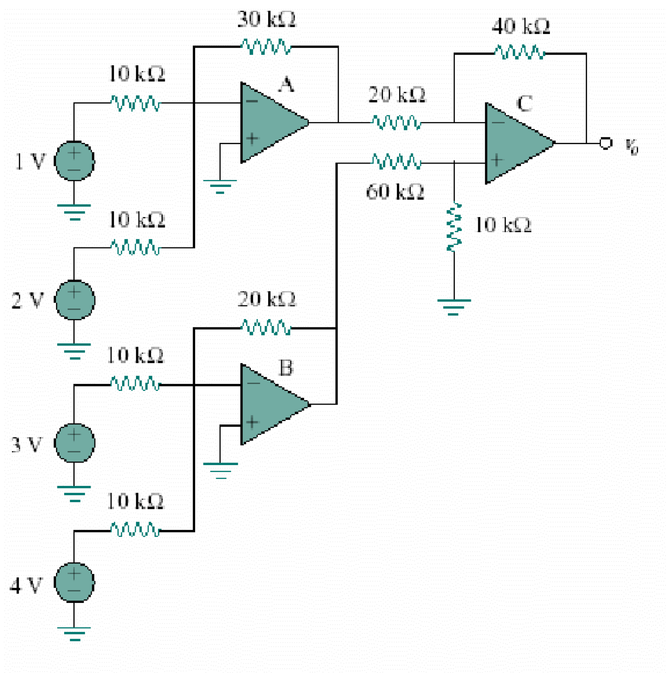


Figure 5.96

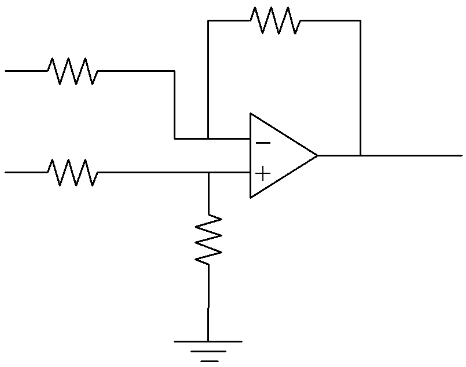
Chapter 5, Solution 70.

The output of amplifier A is

$$v_A = -\frac{30}{10}(1) - \frac{30}{10}(2) = -9$$

The output of amplifier B is

$$v_B = -\frac{20}{10}(3) - \frac{20}{10}(4) = -14$$



$$v_b = \frac{10}{60+10}(-14) = -2V$$

At node a, $\frac{v_A - v_a}{20} = \frac{v_a - v_o}{40}$

But $v_a = v_b$ = -2V, 2(-9+2) = $-2-v_o$

Therefore, v_o = **12V**

Chapter 5, Problem 71

Determine v_o in the op amp circuit in Fig. 5.97.

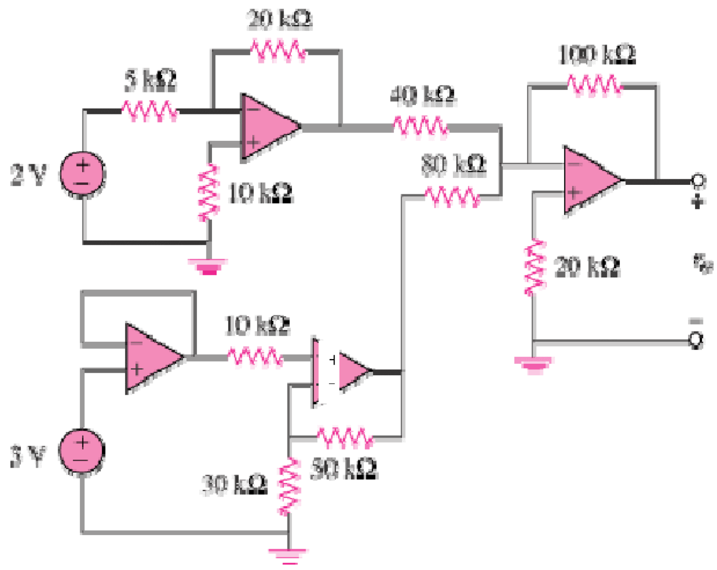
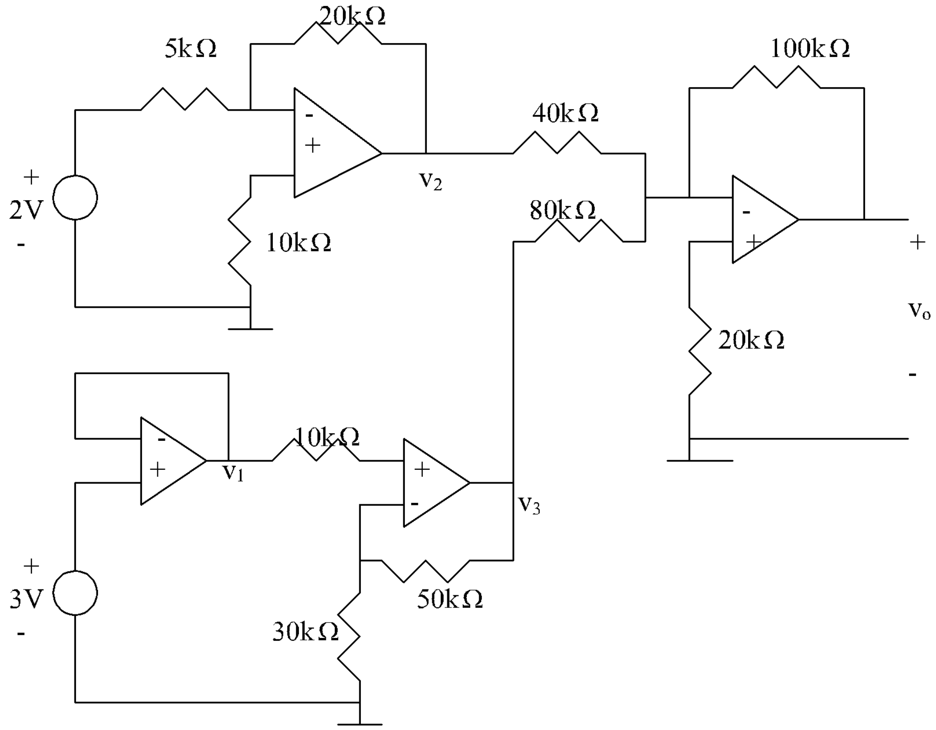


Figure 5.97

Chapter 5, Solution 71



$$v_1 = 3, \qquad v_2 = -\frac{20}{5}(2) = -8, \qquad v_3 = (1 + \frac{50}{30})v_1 = 8$$

$$v_o = -\left(\frac{100}{40}v_2 + \frac{100}{80}v_3\right) = -(-20 + 10) = \underline{10\ \text{V}}$$

Chapter 5, Problem 72

Find the load voltage v_L in the circuit of Fig. 5.98.

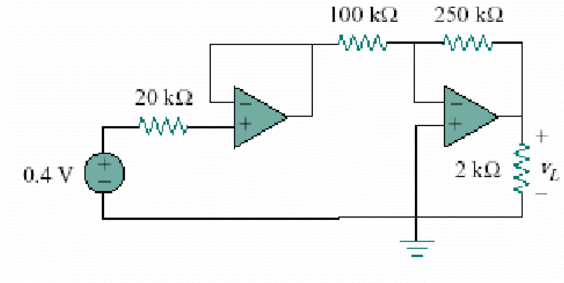


Figure 5.98

Chapter 5, Solution 72.

Since no current flows into the input terminals of ideal op amp, there is no voltage drop across the 20 kΩ resistor. As a voltage summer, the output of the first op amp is

$$v_{01} = 0.4$$

The second stage is an inverter

$$v_2 = -\frac{250}{100} v_{01}$$
$$= -2.5(0.4) = \underline{\mathbf{-1V}}$$

Chapter 5, Problem 73

Determine the load voltage v_L in the circuit of Fig. 5.99.

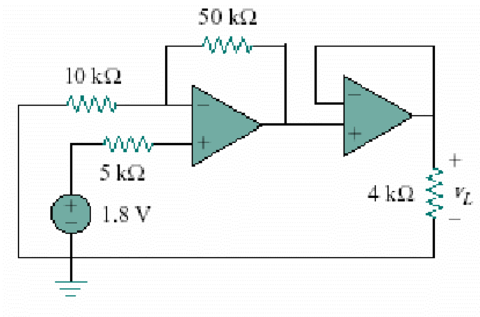


Figure 5.99

Chapter 5, Solution 73.

The first stage is an inverter. The output is

$$v_{01} = -\frac{50}{10}(-1.8) + 1.8 = 10.8\text{V}$$

The second stage is

$$v_2 = v_{01} = \underline{\mathbf{10.8V}}$$

Chapter 5, Problem 74

Find i_o in the op amp circuit of Fig. 5.100.

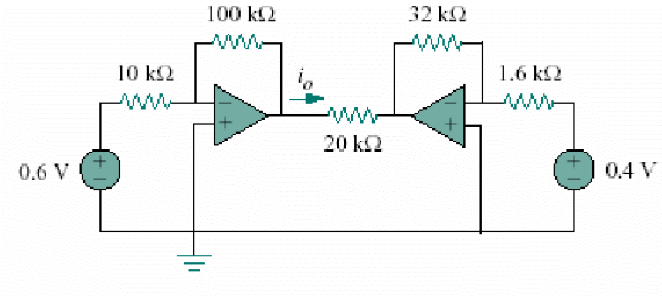


Figure 5.100

Chapter 5, Solution 74.

Let v_1 = output of the first op amp
v_2 = input of the second op amp.

The two sub-circuits are inverting amplifiers

$$v_1 = -\frac{100}{10}(0.6) = -6V$$

$$v_2 = -\frac{32}{1.6}(0.4) = -8V$$

$$i_o = \frac{v_1 - v_2}{20k} = -\frac{-6+8}{20k} = \underline{\mathbf{100\ \mu A}}$$

Chapter 5, Problem 75

Rework Example 5.11 using the nonideal op amp LM324 instead of uA741.

Example 5.11 - Use *PSpice* to solve the op amp circuit for Example 5.1.

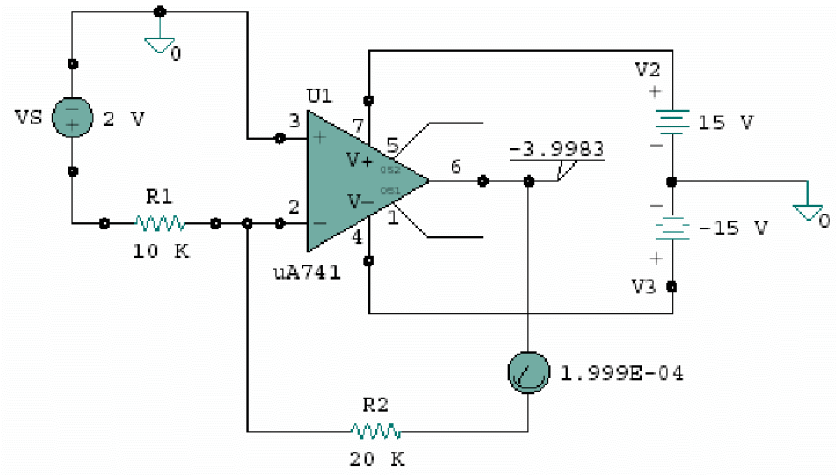


Figure 5.34 Schematic for Example 5.11.

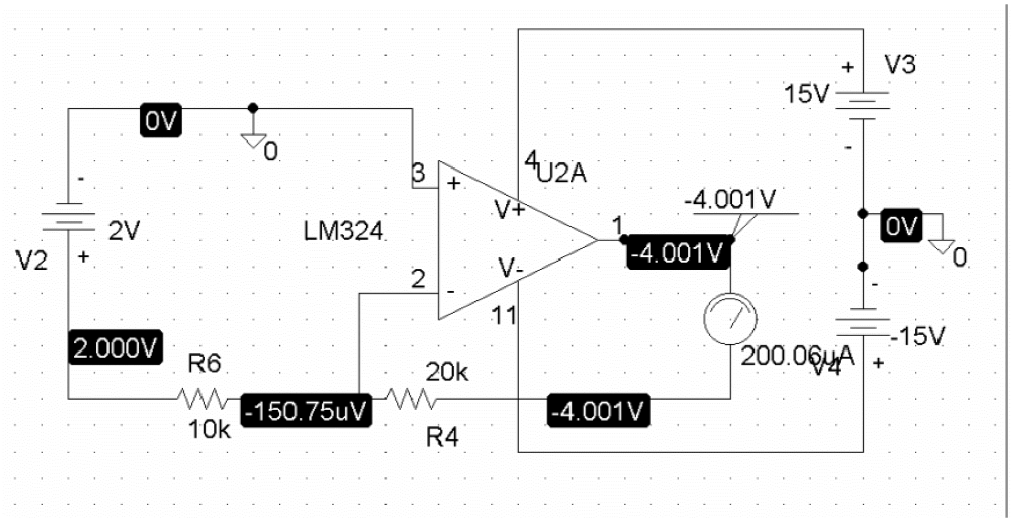
Chapter 5, Solution 75.

The schematic is shown below. Pseudo-components VIEWPOINT and IPROBE are involved as shown to measure v_o and i respectively. Once the circuit is saved, we click Analysis | Simulate. The values of v and i are displayed on the pseudo-components as:

$$i = \underline{\mathbf{200\ \mu A}}$$

$$(v_o/v_s) = -4/2 = \underline{-2}$$

The results are slightly different than those obtained in Example 5.11.



Chapter 5, Problem 76

Solve Prob. 5.19 using *PSpice* and op amp uA741.

5.19 Determine i_o in the circuit of Fig. 5.57.

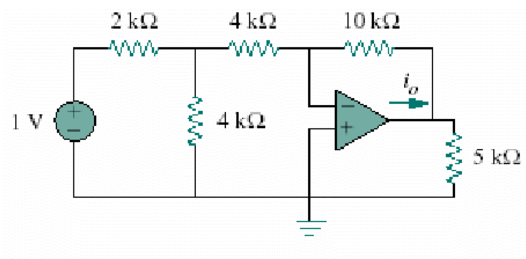
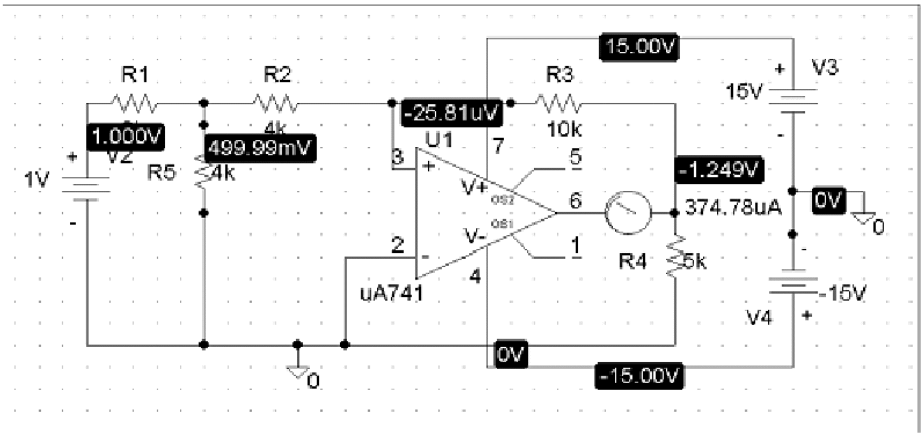


Figure 5.57

Chapter 5, Solution 76.

The schematic is shown below. IPROBE is inserted to measure i_o. Upon simulation, the value of i_o is displayed on IPROBE as

$$i_o = \underline{-374.78\ \mu A}$$



Chapter 5, Problem 77

Solve Prob. 5.48 using *PSpice* and op amp LM324.

5.48 The circuit in Fig. 5.78 is a differential amplifier driven by a bridge. Find v_o.

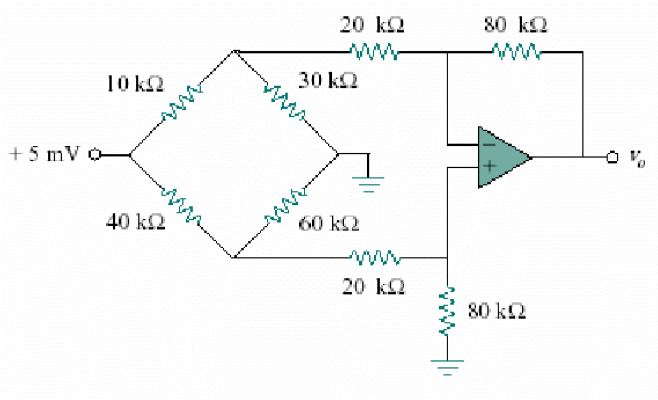
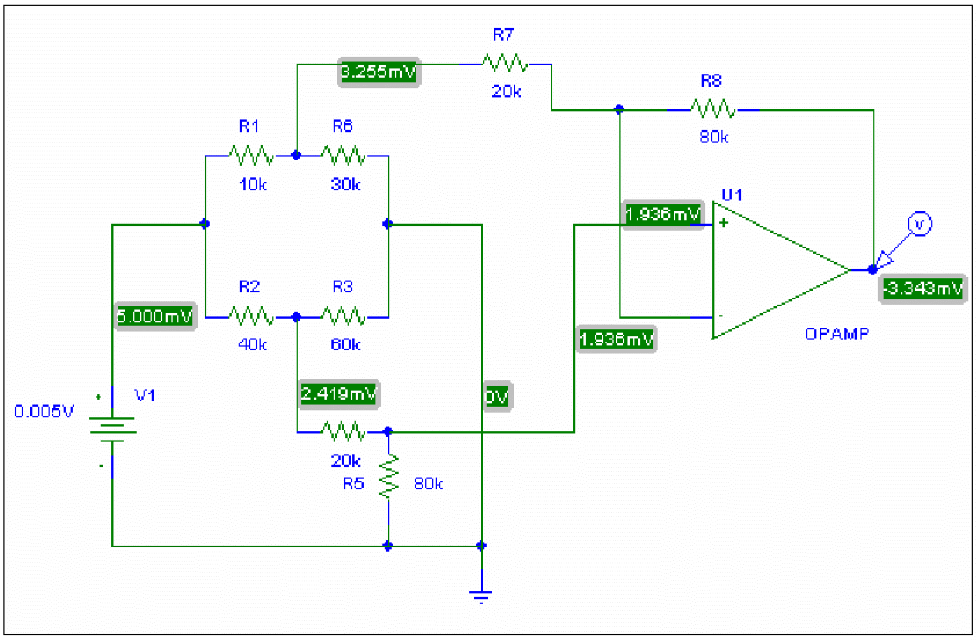


Figure 5.78

Chapter 5, Solution 77.

The schematic for the PSpice solution is shown below.

Note that the output voltage, <u>**–3.343 mV**</u>, agrees with the answer to problem, 5.48.



Chapter 5, Problem 78

Use *PSpice* to obtain v_o in the circuit of Fig. 5.101.

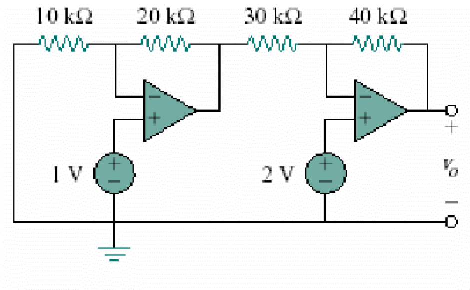
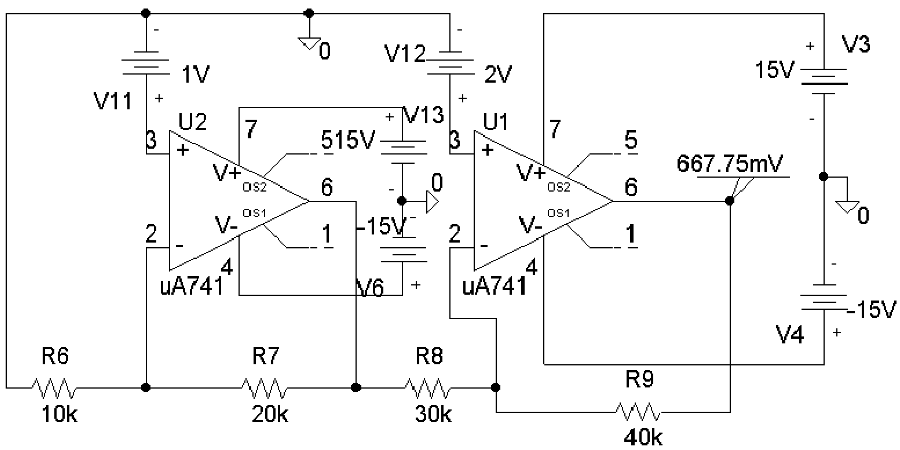


Figure 5.101

Chapter 5, Solution 78.

The circuit is constructed as shown below. We insert a VIEWPOINT to display v_o. Upon simulating the circuit, we obtain,

$$v_o = \underline{\mathbf{667.75\ mV}}$$



Chapter 5, Problem 79

Determine v_o in the op amp circuit of Fig. 5.102 using *PSpice*.

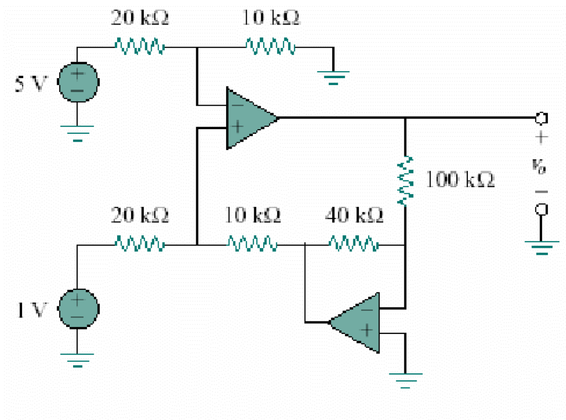
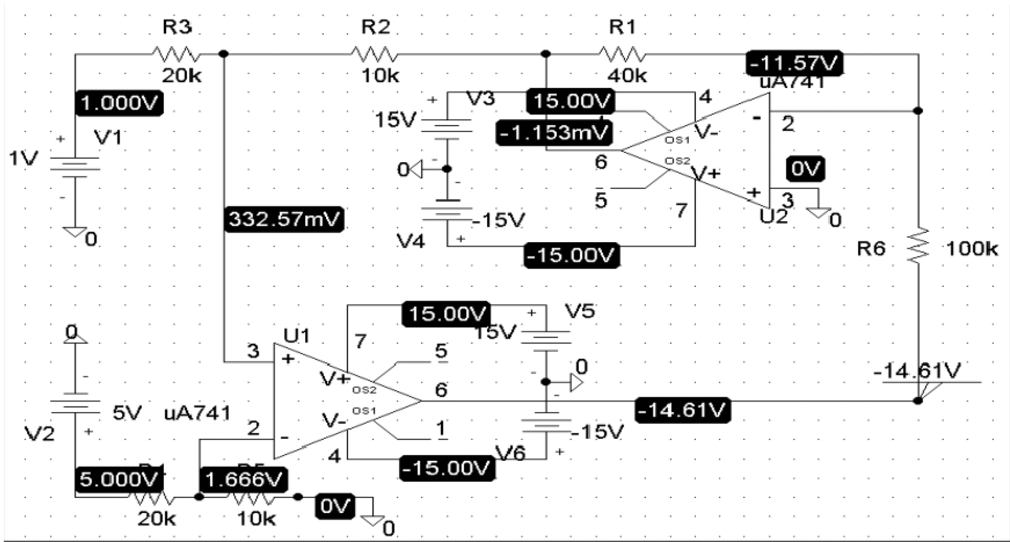


Figure 5.102

Chapter 5, Solution 79.

The schematic is shown below. A pseudo-component VIEWPOINT is inserted to display v_o. After saving and simulating the circuit, we obtain,

$$v_o = \underline{\mathbf{-14.61\ V}}$$



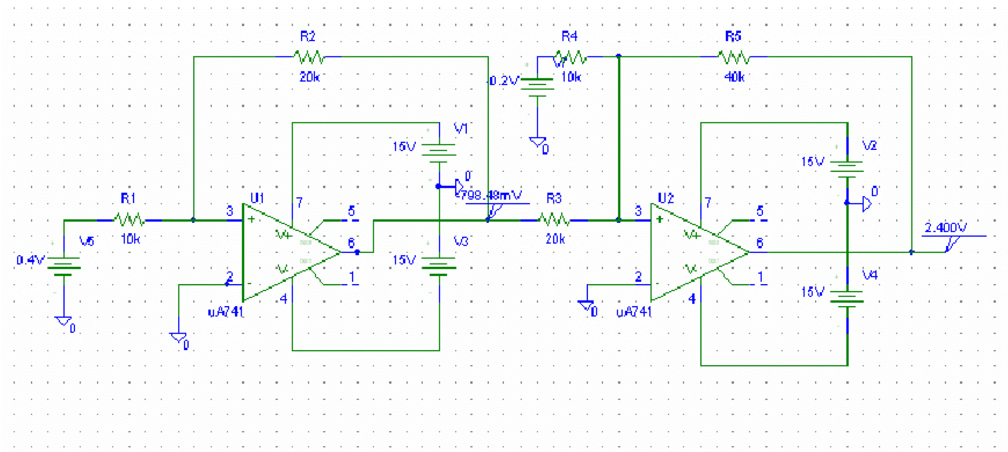
Chapter 5, Problem 80.

Use PSpice to solve Prob. 5.61.

Chapter 5, Solution 80.

The schematic is as shown below. After it is saved and simulated, we obtain

$$v_o = \underline{\mathbf{2.4\ V}}.$$



Chapter 5, Problem 81

Use *PSpice* to verify the results in Example 5.9. Assume nonideal op amps LM324.

Example 5.9 - Determine *vo* and *io* in the op amp circuit in Fig. 5.30.
Answer: 10 V, 1 mA.

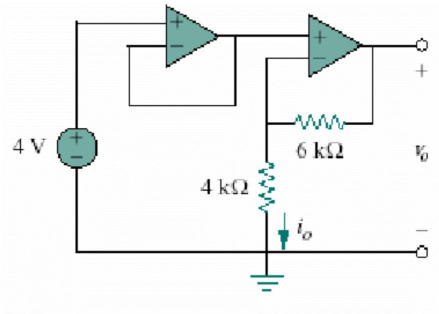


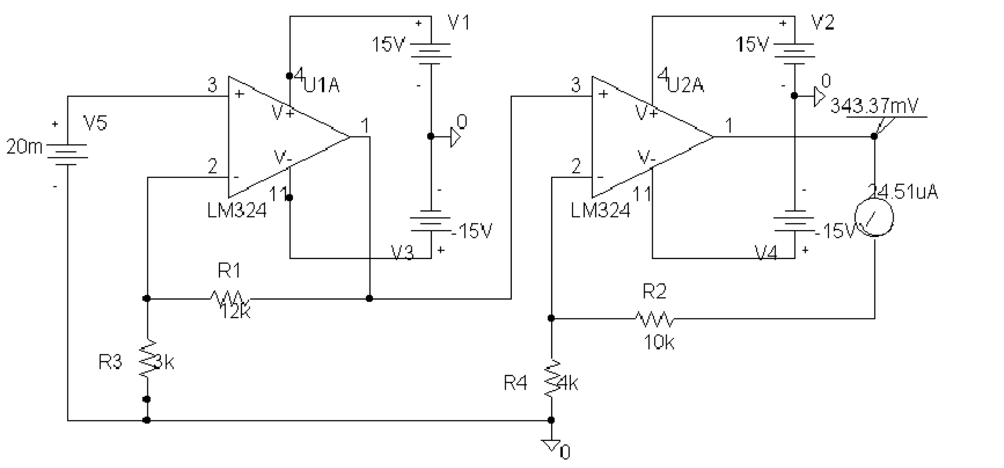
Figure 5.30 For Practice Prob. 5.9.

Chapter 5, Solution 81.

The schematic is shown below. We insert one VIEWPOINT and one IPROBE to measure v_o and i_o respectively. Upon saving and simulating the circuit, we obtain,

$$v_o = \underline{\mathbf{343.4\ mV}}$$

$$i_o = \underline{\mathbf{24.51\ \mu A}}$$



Chapter 5, Problem 82

A five-bit DAC covers a voltage range of 0 to 7.75 V. Calculate how much voltage each bit is worth.

Chapter 5, Solution 82.

The maximum voltage level corresponds to

$$11111 = 2^5 - 1 = 31$$

Hence, each bit is worth $(7.75/31) = \underline{\mathbf{250\ mV}}$

Chapter 5, Problem 83

Design a six-bit digital-to-analog converter.

(a) If $|V_o| = 1.1875$ V is desired, what should $[V_1V_2V_3V_4V_5V_6]$ be?

(b) Calculate $|V_o|$ if $[V_1V_2V_3V_4V_5V_6] = [011011]$.

(c) What is the maximum value $|V_o|$ can assume?

Chapter 5, Solution 83.

The result depends on your design. Hence, let R_G = 10 k ohms, R_1 = 10 k ohms, R_2 = 20 k ohms, R_3 = 40 k ohms, R_4 = 80 k ohms, R_5 = 160 k ohms, R_6 = 320 k ohms, then,

$$-v_o = (R_f/R_1)v_1 + \text{---------} + (R_f/R_6)v_6$$

$$= v_1 + 0.5v_2 + 0.25v_3 + 0.125v_4 + 0.0625v_5 + 0.03125v_6$$

(a) $|v_o| = 1.1875 = 1 + 0.125 + 0.0625 = 1 + (1/8) + (1/16)$ which implies,

$$[v_1\ v_2\ v_3\ v_4\ v_5\ v_6] = \underline{\mathbf{[100110]}}$$

(b) $|v_o| = 0 + (1/2) + (1/4) + 0 + (1/16) + (1/32) = (27/32) = \underline{\mathbf{843.75\ mV}}$

(c) This corresponds to [1 1 1 1 1 1].

$|v_o| = 1 + (1/2) + (1/4) + (1/8) + (1/16) + (1/32) = 63/32 = \underline{\mathbf{1.96875\ V}}$

Chapter 5, Problem 84

A four-bit *R-2R ladder* DAC is presented in Fig. 5.103.

(a) Show that the output voltage is given by

$$-V_o = R_f\left(\frac{V_1}{2R}+\frac{V_2}{4R}+\frac{V_3}{8R}+\frac{V_4}{16R}\right)$$

(b) If $R_f = 12$ kΩ and $R = 10$ kΩ, find $|V_o|$ for $[V_1V_2V_3V_4] = [1011]$ and $[V_1V_2V_3V_4] = [0101]$.

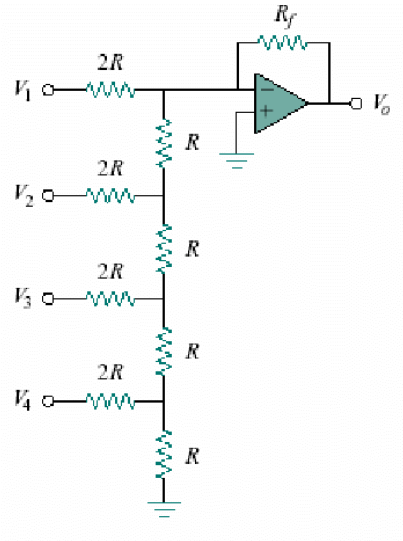
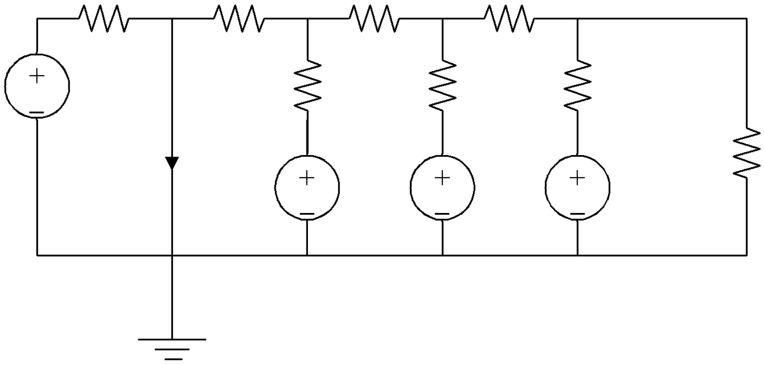


Figure 5.103

Chapter 5, Solution 84.

For (a), the process of the proof is time consuming and the results are only approximate, but close enough for the applications where this device is used.

(a) The easiest way to solve this problem is to use superposition and to solve for each term letting all of the corresponding voltages be equal to zero. Also, starting with each current contribution (i_k) equal to one amp and working backwards is easiest.



For the first case, let $v_2 = v_3 = v_4 = 0$, and $i_1 = 1A$.

Therefore, $v_1 = 2R$ volts or $i_1 = v_1/(2R)$.

Second case, let $v_1 = v_3 = v_4 = 0$, and $i_2 = 1A$.

Therefore, $v_2 = 85R/21$ volts or $i_2 = 21v_2/(85R)$. Clearly this is not ($1/4^{th}$), so where is the difference? $(21/85) = 0.247$ which is a really good approximation for 0.25. Since this is a practical electronic circuit, the result is good enough for all practical purposes.

Now for the third case, let $v_1 = v_2 = v_4 = 0$, and $i_3 = 1A$.

Therefore, $v_3 = 8.5R$ volts or $i_3 = v_3/(8.5R)$. Clearly this is not $(1/8^{th})$, so where is the difference? $(1/8.5) = 0.11765$ which is a really good approximation for 0.125. Since this is a practical electronic circuit, the result is good enough for all practical purposes.

Finally, for the fourth case, let $v_1 = v_2 = v_4 = 0$, and $i_3 = 1A$.

Therefore, $v_4 = 16.25R$ volts or $i_4 = v_4/(16.25R)$. Clearly this is not $(1/16^{th})$, so where is the difference? $(1/16.25) = 0.06154$ which is a really good approximation for 0.0625. Since this is a practical electronic circuit, the result is good enough for all practical purposes.

Please note that a goal of a lot of electronic design is to come up with practical circuits that are economical to design and build yet give the desired results.

(b) If $R_f = 12$ k ohms and $R = 10$ k ohms,

$$-v_o = (12/20)[v_1 + (v_2/2) + (v_3/4) + (v_4/8)]$$

$$= 0.6[v_1 + 0.5v_2 + 0.25v_3 + 0.125v_4]$$

For $[v_1\ v_2\ v_3\ v_4] = [1\ 0\ 1 1]$,

$$|v_o| = 0.6[1 + 0.25 + 0.125] = \underline{\mathbf{825\ mV}}$$

For $[v_1\ v_2\ v_3\ v_4] = [0\ 1\ 0\ 1]$,

$$|v_o| = 0.6[0.5 + 0.125] = \underline{\mathbf{375\ mV}}$$

Chapter 5, Problem 85.

In the op amp circuit of Fig. 5.104, find the value of R so that the power absorbed by the 10-kΩ resistor is 10 mW. Take $v_s = 2V$.

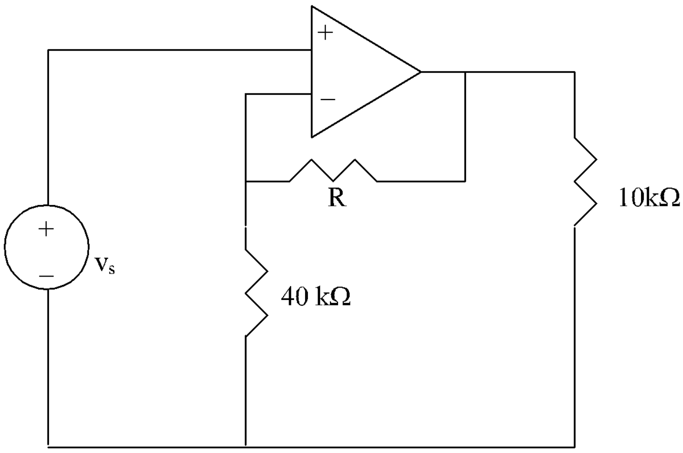


Figure 5.104 For Prob. 5.85.

Chapter 5, Solution 85.

This is a noninverting amplifier.

$$v_o = (1 + R/40k)v_s = (1 + R/40k)2$$

The power being delivered to the 10-kΩ give us

$$P = 10\text{ mW} = (v_o)^2/10k \quad \text{or} \quad v_o = \sqrt{10^{-2} x 10^4} = 10V$$

Returning to our first equation we get

$$10 = (1 + R/40k)2 \quad \text{or} \quad R/40k = 5 - 1 = 4$$

Thus, R = **<u>160 kΩ</u>**.

Chapter 5, Problem 86

Assuming a gain of 200 for an IA, find its output voltage for:
(a) $v_1 = 0.402$ V and $v_2 = 0.386$ V
(b) $v_1 = 1.002$ V and $v_2 = 1.011$ V.

Chapter 5, Solution 86.

$$v_o = A(v_2 - v_1) = 200(v_2 - v_1)$$

(a) $$v_o = 200(0.386 - 0.402) = \underline{\mathbf{-3.2\ V}}$$

$$v_o = 200(1.011 - 1.002) = \underline{\mathbf{1.8\ V}}$$

Chapter 5, Problem 87

Figure 5.105 displays a two-op-amp instrumentation amplifier. Derive an expression for v_o in terms of v_1 and v_2. How can this amplifier be used as a subtractor?

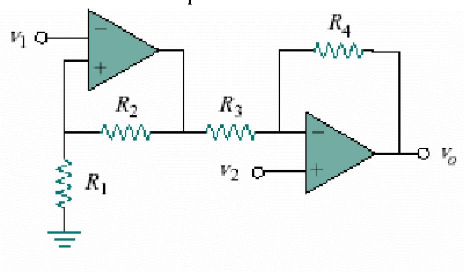


Figure 5.105

Chapter 5, Solution 87.

The output, v_a, of the first op amp is,

$$v_a = (1 + (R_2/R_1))v_1 \qquad (1)$$

Also, $$v_o = (-R_4/R_3)v_a + (1 + (R_4/R_3))v_2 \qquad (2)$$

Substituting (1) into (2),

$$v_o = (-R_4/R_3)(1 + (R_2/R_1))v_1 + (1 + (R_4/R_3))v_2$$

Or, $$v_o = \underline{\mathbf{(1 + (R_4/R_3))v_2 - (R_4/R_3 + (R_2R_4/R_1R_3))v_1}}$$

If $R_4 = R_1$ and $R_3 = R_2$, then,

$$v_o = (1 + (R_4/R_3))(v_2 - v_1)$$

which is a subtractor with a gain of $\underline{\mathbf{(1 + (R_4/R_3))}}$.

Chapter 5, Problem 88

Figure 5.106 shows an instrumentation amplifier driven by a bridge. Obtain the gain v_o/v_i of the amplifier.

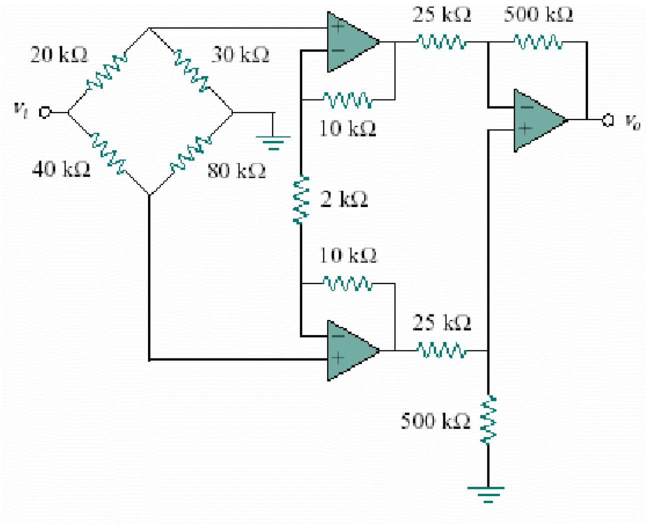


Figure 5.106

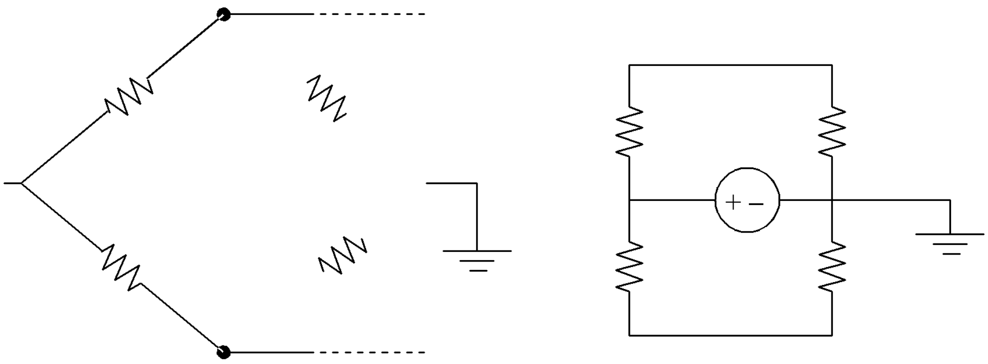
Chapter 5, Solution 88.

We need to find V_{Th} at terminals a – b, from this,

$$v_o = (R_2/R_1)(1 + 2(R_3/R_4))V_{Th} = (500/25)(1 + 2(10/2))V_{Th}$$

$$= 220V_{Th}$$

Now we use Fig. (b) to find V_{Th} in terms of v_i.



$$v_a = (3/5)v_i, \quad v_b = (2/3)v_i$$

$$V_{Th} = v_b - v_a \quad (1/15)v_i$$

$$(v_o/v_i) = A_v = -220/15 = \underline{\mathbf{-14.667}}$$

Chapter 5, Problem 89.

Design a circuit that provides a relationship between output voltage v_o and input voltage v_s such that $v_o = 12v_s - 10$. Two op amps, a 6-V battery and several resistors are available.

Chapter 5, Solution 89.

A **summer** with $v_o = -v_1 - (5/3)v_2$ where v_2 = **6-V battery** and an **inverting amplifier** with $v_1 = -12v_s$.

Chapter 5, Problem 90

The op amp circuit in Fig. 5.107 is a *current amplifier.* Find the current gain i_o/i_s of the amplifier.

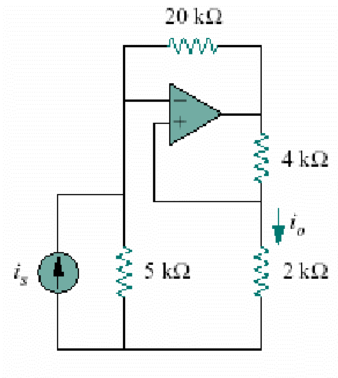
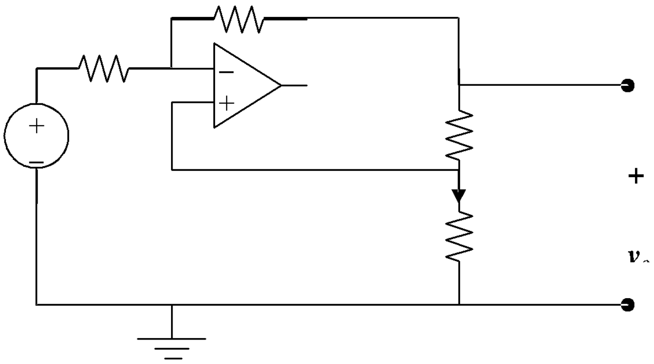


Figure 5.107

Chapter 5, Solution 90.

Transforming the current source to a voltage source produces the circuit below,

At node b, $v_b = (2/(2+4))v_o = v_o/3$



At node a, $(5i_s - v_a)/5 = (v_a - v_o)/20$

But $v_a = v_b = v_o/3$. $20i_s - (4/3)v_o = (1/3)v_o - v_o$, or $i_s = v_o/30$

$i_o = [(2/(2+4))/2]v_o = v_o/6$

$i_o/i_s = (v_o/6)/(v_o/30) = \underline{5}$

Chapter 5, Problem 91

A noninverting current amplifier is portrayed in Fig. 5.108. Calculate the gain i_o/i_s. Take $R_1 = 8$ kΩ and $R_2 = 1$ kΩ.

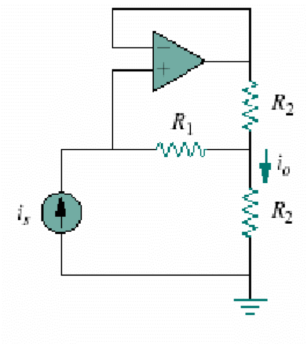
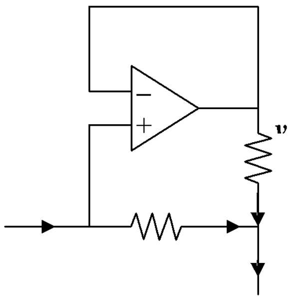


Figure 5.108

Chapter 5, Solution 91.



$$i_o = i_1 + i_2 \quad (1)$$

But $$i_1 = i_s \quad (2)$$

R_1 and R_2 have the same voltage, v_o, across them.

$$R_1 i_1 = R_2 i_2, \text{ which leads to } i_2 = (R_1/R_2)i_1 \quad (3)$$

Substituting (2) and (3) into (1) gives,

$$i_o = i_s(1 + R_1/R_2)$$

$$i_o/i_s = 1 + (R_1/R_2) = 1 + 8/1 = \underline{\mathbf{9}}$$

Chapter 5, Problem 92

Refer to the *bridge amplifier* shown in Fig. 5.109. Determine the voltage gain v_o/v_i .

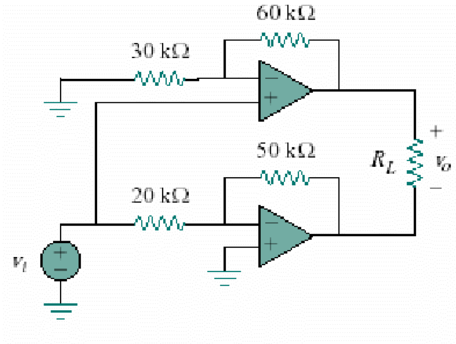


Figure 5.109

Chapter 5, Solution 92

The top op amp circuit is a non-inverter, while the lower one is an inverter. The output at the top op amp is

$$v_1 = (1 + 60/30)v_i = 3v_i$$

while the output of the lower op amp is

$$v_2 = -(50/20)v_i = -2.5v_i$$

Hence, $$v_o = v_1 - v_2 = 3v_i + 2.5v_i = 5.5v_i$$

$$v_o/v_i = \underline{\mathbf{5.5}}$$

Chapter 5, Problem 93

A voltage-to-current converter is shown in Fig. 5.110, which means that $i_L = Av_i$ if $R_1R_2 = R_3R_4$. Find the constant term A.

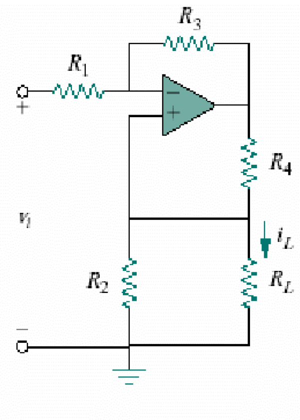
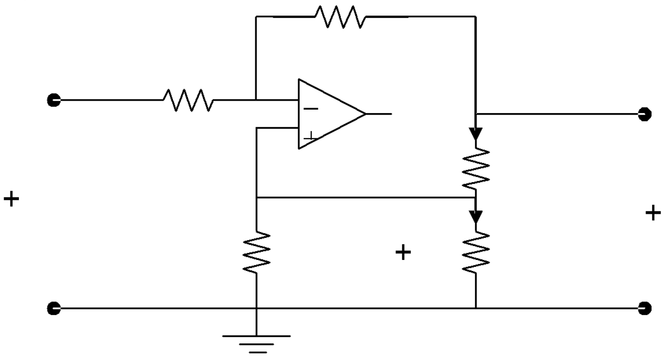


Figure 5.110

Chapter 5, Solution 93.



At node a, $(v_i - v_a)/R_1 = (v_a - v_o)/R_3$

$v_i - v_a = (R_1/R_2)(v_a - v_o)$

$v_i + (R_1/R_3)v_o = (1 + R_1/R_3)v_a$ (1)

But $v_a = v_b = v_L$. Hence, (1) becomes

$v_i = (1 + R_1/R_3)v_L - (R_1/R_3)v_o$ (2)

$i_o = v_o/(R_4 + R_2||R_L)$, $i_L = (R_2/(R_2 + R_L))i_o = (R_2/(R_2 + R_L))(v_o/(R_4 + R_2||R_L))$

Or, $v_o = i_L[(R_2 + R_L)(R_4 + R_2||R_L)/R_2$ (3)

But, $v_L = i_LR_L$ (4)

Substituting (3) and (4) into (2),

$v_i = (1 + R_1/R_3)\, i_LR_L - R_1[(R_2 + R_L)/(R_2R_3)](R_4 + R_2||R_L)i_L$

$= [((R_3 + R_1)/R_3)R_L - R_1((R_2 + R_L)/(R_2R_3)(R_4 + (R_2R_L/(R_2 + R_L))]i_L$

$= (1/A)i_L$

Thus,

$$A = \frac{1}{\left(1+\frac{R_1}{R_3}\right)R_L - R_1\left(\frac{R_2+R_L}{R_2R_3}\right)\left(R_4+\frac{R_2R_L}{R_2+R_L}\right)}$$

Please note that A has the units of mhos. An easy check is to let every resistor equal 1-ohm and v_i equal to one amp. Going through the circuit produces i_L = 1A. Plugging into the above equation produces the same answer so the answer does check.

Chapter 6, Problem 1.

If the voltage across a 5-F capacitor is $2te^{-3t}$ V, find the current and the power.

Chapter 6, Solution 1.

$$i = C\frac{dv}{dt} = 5\left(2e^{-3t} - 6te^{-3t}\right) = \underline{10(1 - 3t)e^{-3t}\text{ A}}$$

$$p = vi = 10(1\text{-}3t)e^{-3t} \cdot 2t\, e^{-3t} = \underline{20t(1 - 3t)e^{-6t}\text{ W}}$$

Chapter 6, Problem 2.

A 20-μF capacitor has energy $w(t) = 10\cos^2 377t$ J. Determine the current through the capacitor.

Chapter 6, Solution 2.

$$w = \frac{1}{2}Cv^2 \quad\longrightarrow\quad v^2 = \frac{2W}{C} = \frac{20\cos^2 377t}{20x10^{-6}} = 10^6\cos^2 377t$$

v = $\pm 10^3$cos(377t) V, let us assume the v = +cos(377t) mV, this then leads to,

$$i = C(dv/dt) = 20x10^{-6}(-377\sin(377t)10^{-3}) = \underline{\mathbf{-7.54\sin(377t)\ A}}.$$

Please note that if we had chosen the negative value for v, then i would have been positive.

Chapter 6, Problem 3.

In 5 s, the voltage across a 40-mF capacitor changes from 160 V to 220 V. Calculate the average current through the capacitor.

Chapter 6, Solution 3.

$$i = C\frac{dv}{dt} = 40x10^{-3}\frac{220 - 160}{5} = \underline{\mathbf{480\ mA}}$$

Chapter 6, Problem 4.

A current of 6 sin 4*t* A flows through a 2-F capacitor. Find the voltage *v(t)* across the capacitor given that *v*(0) = 1 V.

Chapter 6, Solution 4.

$$v = \frac{1}{C}\int_0^t i dt + v(0)$$

$$= \frac{1}{2}\int_0^t 6\sin 4t dt + 1 = \left(-\frac{3}{4}\cos 4t\right)\Big|_0^t + 1 = -0.75\cos 4t + 0.75 + 1$$

= **<u>1.75 – 0.75 cos 4t V</u>**

Chapter 6, Problem 5.

The voltage across a 4-μF capacitor is shown in Fig. 6.45. Find the current waveform.

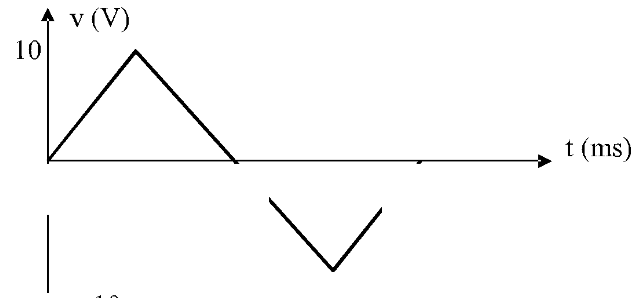


Figure 6.45 For Prob. 6.5.

Chapter 6, Solution 5.

$$v = \begin{cases} 5000t, & 0 < t < 2ms \\ 20 - 5000t, & 2 < t < 6ms \\ -40 + 5000t, & 6 < t < 8ms \end{cases}$$

$$i = C\frac{dv}{dt} = \frac{4x10^{-6}}{10^{-3}}\begin{cases} 5, & 0 < t < 2ms \\ -5, & 2 < t < 6ms \\ 5, & 6 < t < 8ms \end{cases} = \begin{cases} 20 \text{ mA}, & 0 < t < 2ms \\ -20 \text{ mA}, & 2 < t < 6ms \\ 20 \text{ mA}, & 6 < t < 8ms \end{cases}$$

Chapter 6, Problem 6.

The voltage waveform in Fig. 6.46 is applied across a 30-μF capacitor. Draw the current waveform through it.

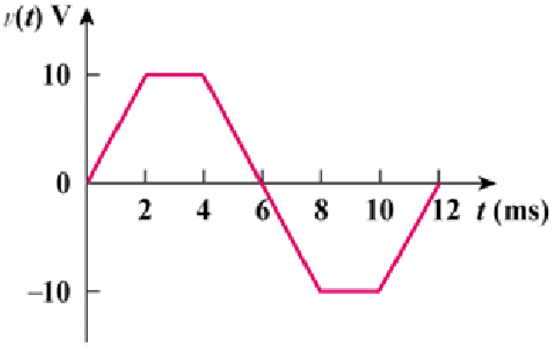


Figure 6.46

Chapter 6, Solution 6.

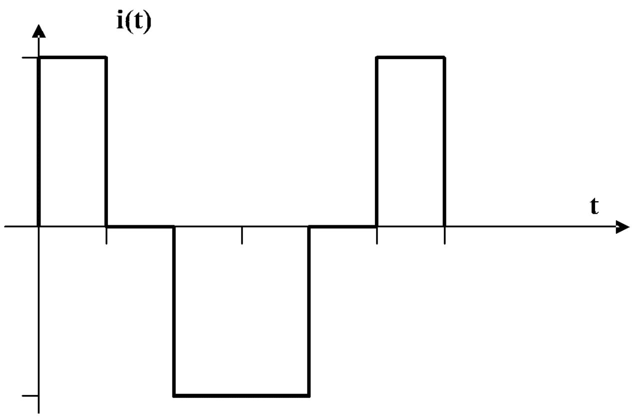
$i = C\frac{dv}{dt} = 30x10^{-6}$ x slope of the waveform.

For example, for 0 < t < 2,

$$\frac{dv}{dt} = \frac{10}{2x10^{-3}}$$

$$i = C\frac{dv}{dt} = 30x10^{-6} x \frac{10}{2x10^{-3}} = 150mA$$

Thus the current i is sketched below.



Chapter 6, Problem 7.

At t=0, the voltage across a 50-mF capacitor is 10 V. Calculate the voltage across the capacitor for $t > 0$ when current 4t mA flows through it.

Chapter 6, Solution 7.

$$v = \frac{1}{C}\int idt + v(t_o) = \frac{1}{50x10^{-3}}\int_o^t 4tx10^{-3}\,dt + 10$$

$$= \frac{2t^2}{50} + 10 = \underline{\mathbf{0.04t^2 + 10\ V}}$$

Chapter 6, Problem 8.

A 4-mF capacitor has the terminal voltage

$$v = \begin{cases} 50\text{ V}, & t \le 0 \\ Ae^{-100t} + Be^{-600t}\text{ V}, & t \ge 0 \end{cases}$$

If the capacitor has initial current of 2A, find:

(a) the constants A and B,
(b) the energy stored in the capacitor at $t = 0$,
(c) the capacitor current for $t > 0$.

Chapter 6, Solution 8.

(a) $i = C\frac{dv}{dt} = -100ACe^{-100t} - 600BCe^{-600t}$ (1)

$i(0) = 2 = -100AC - 600BC \longrightarrow 5 = -A - 6B$ (2)

$v(0^+) = v(0^-) \longrightarrow 50 = A + B$ (3)

Solving (2) and (3) leads to

A=61, B=-11

(b) $\text{Energy} = \frac{1}{2}Cv^2(0) = \frac{1}{2}x4x10^{-3}x2500 = \underline{5\text{ J}}$

(c) From (1),

$$i = -100x61x4x10^{-3}e^{-100t} - 600x11x4x10^{-3}e^{-600t} = \underline{-24.4e^{-100t} - 26.4e^{-600t}\ \text{A}}$$

Chapter 6, Problem 9.

The current through a 0.5-F capacitor is $6(1\text{-}e^{-t})$A.
Determine the voltage and power at t=2 s. Assume v(0) = 0.

Chapter 6, Solution 9.

$$v(t) = \frac{1}{1/2}\int_0^t 6\left(1 - e^{-t}\right)dt + 0 = 12\left(t + e^{-t}\right)\Big|_0^t \text{ V} = 12(t + e^{-t}) - 12$$

$v(2) = 12(2 + e^{-2}) - 12 =$ **<u>13.624 V</u>**

$p = iv = [12\,(t + e^{-t}) - 12]6(1\text{-}e^{-t})$

$p(2) = [12\,(2 + e^{-2}) - 12]6(1\text{-}e^{-2}) =$ **<u>70.66 W</u>**

Chapter 6, Problem 10.

The voltage across a 2-mF capacitor is shown in Fig. 6.47. Determine the current through the capacitor.

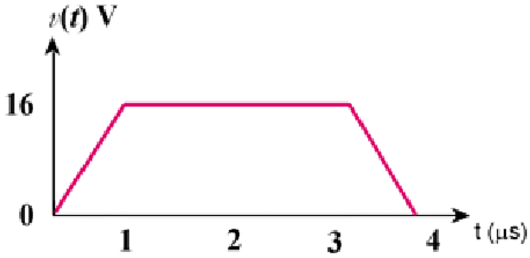


Figure 6.47

Chapter 6, Solution 10

$$i = C\frac{dv}{dt} = 2x10^{-3}\frac{dv}{dt}$$

$$v = \begin{cases} 16t, & 0 < t < 1\mu s \\ 16, & 1 < t < 3\ \mu s \\ 64 - 16t, & 3 < t < 4\mu s \end{cases}$$

$$\frac{dv}{dt} = \begin{cases} 16x10^6, & 0 < t < 1\mu s \\ 0, & 1 < t < 3\ \mu s \\ -16x10^6, & 3 < t < 4\mu s \end{cases}$$

$$i(t) = \begin{cases} 32\text{ kA}, & 0 < t < 1\mu s \\ 0, & 1 < t < 3\ \mu s \\ -32\text{ kA}, & 3 < t < 4\mu s \end{cases}$$

Chapter 6, Problem 11.

3. A 4-mF capacitor has the current waveform shown in Fig. 6.48. Assuming that v(0)=10V, sketch the voltage waveform v(t).

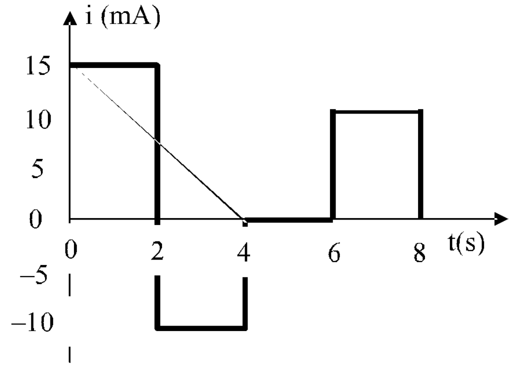


Figure 6.48 For Prob. 6.11.

Chapter 6, Solution 11.

$$v = \frac{1}{C}\int_0^t i dt + v(0) = 10 + \frac{1}{4x10^{-3}}\int_0^t i(t) dt$$

For 0<t <2, i(t)=15mA, V(t)= 10+ $v = 10 + \frac{10^3}{4x10^{-3}}\int_0^t 15 dt = 10 + 3.76t$

v(2) = 10+7.5 =17.5

For 2 < t <4, i(t) = –10 mA

$$v(t) = \frac{1}{4x10^{-3}}\int_2^t i(t) dt + v(2) = -\frac{10x10^{-3}}{4x10^{-3}}\int_2^t dt + 17.5 = 22.5 + 2.5t$$

v(4)=22.5-2.5x4 =12.5

For 4<t<6, i(t) = 0, $v(t) = \frac{1}{4x10^{-3}}\int_2^t 0 dt + v(4) = 12.5$

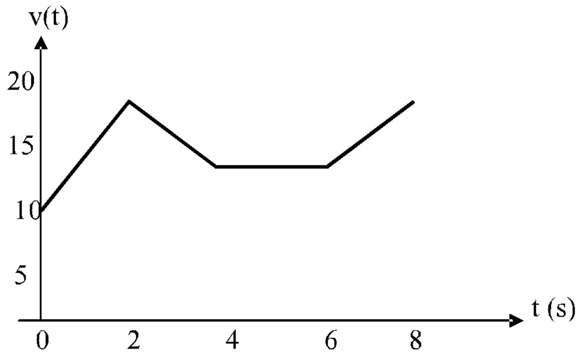
For 6<t<8, i(t) = 10 mA

$$v(t) = \frac{10x10^3}{4x10^{-3}}\int_4^t dt + v(6) = 2.5(t-6) + 12.5 = 2.5t - 2.5$$

Hence,

$$v(t) = \begin{cases} 10 + 3.75t \text{ V}, & 0 < t < 2s \\ 22.5 - 2.5t \text{ V}, & 2 < t < 4s \\ 12.5 \text{ V}, & 4 < t < 6s \\ 2.5t - 2.5 \text{ V}, & 6 < t < 8s \end{cases}$$

which is sketched below.



Chapter 6, Problem 12.

A voltage of $6e^{-2000t}$ V appears across a parallel combination of a 100-mF capacitor and a 12-Ω resistor. Calculate the power absorbed by the parallel combination.

Chapter 6, Solution 12.

$$i_R = \frac{V}{R} = \frac{6}{12} e^{-2000t} = 0.5e^{-2000t}$$

$$i_c = C\frac{dv}{dt} = 100x10^{-3} x6(-2000)e^{-2000t} = -1200e^{-2000t}$$

$$i = i_R + i_C = -1199.5e^{-2000t}$$

$$p = vi = \underline{-7197e^{-4000t} \text{ W}}$$

Chapter 6, Problem 13.

Find the voltage across the capacitors in the circuit of Fig. 6.49 under dc conditions.

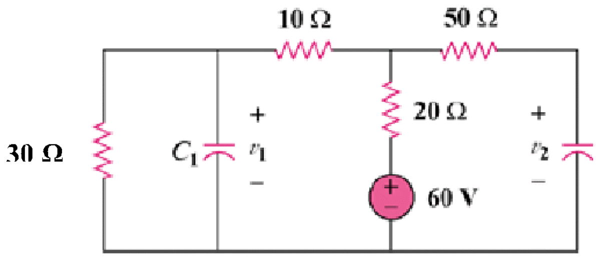
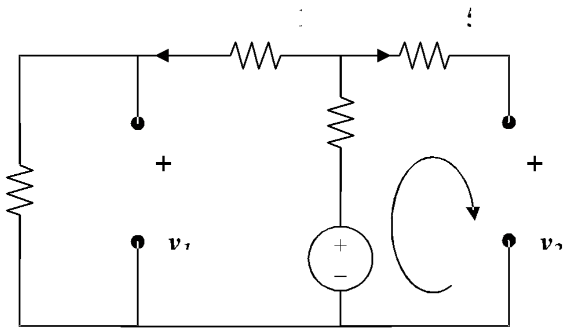


Figure 6.49

Chapter 6, Solution 13.

Under dc conditions, the circuit becomes that shown below:



$i_2 = 0$, $i_1 = 60/(30+10+20) = 1A$

$v_1 = 30i_1 = 30V$, $v_2 = 60–20i_1 = 40V$

Thus, $\underline{v_1 = 30V, v_2 = 40V}$

Chapter 6, Problem 14.

Series-connected 20-pF and 60-pF capacitors are placed in parallel with series-connected 30-pF and 70-pF capacitors. Determine the equivalent capacitance.

Chapter 6, Solution 14.

20 pF is in series with 60pF = 20*60/80=15 pF
30-pF is in series with 70pF = 30x70/100=21pF
15pF is in parallel with 21pF = 15+21 = **36 pF**

Chapter 6, Problem 15.

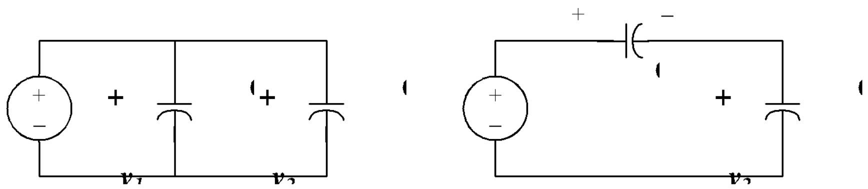
Two capacitors (20 μF and 30 μF) are connected to a 100-V source. Find the energy stored in each capacitor if they are connected in:

(a) parallel (b) series

Chapter 6, Solution 15.

In parallel, as in Fig. (a),

$v_1 = v_2 = 100$



$$w_{20} = \frac{1}{2}Cv^2 = \frac{1}{2}x20x10^{-6}x100^2 = \underline{\mathbf{100\ mJ}}$$

$$w_{30} = \frac{1}{2}x30x10^{-6}x100^2 = \underline{\mathbf{150\ mJ}}$$

(b) When they are connected in series as in Fig. (b):

$$v_1 = \frac{C_2}{C_1 + C_2}V = \frac{30}{50}x100 = 60, \ v_2 = 40$$

$$w_{20} = \frac{1}{2}x30x10^{-6}x60^2 = \underline{\mathbf{36\ mJ}}$$

$$w_{30} = \frac{1}{2}x30x10^{-6}x40^2 = \underline{\mathbf{24\ mJ}}$$

Chapter 6, Problem 16.

The equivalent capacitance at terminals *a*-*b* in the circuit in Fig. 6.50 is 30 μF. Calculate the value of *C*.

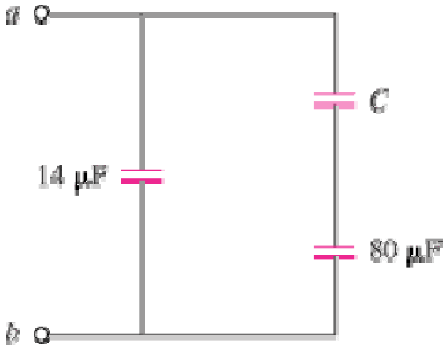


Figure 6.50

Chapter 6, Solution 16

$$C_{eq} = 14 + \frac{Cx80}{C+80} = 30 \longrightarrow \underline{C = 20\ \mu\text{F}}$$

Chapter 6, Problem 17.

Determine the equivalent capacitance for each of the circuits in Fig. 6.51.

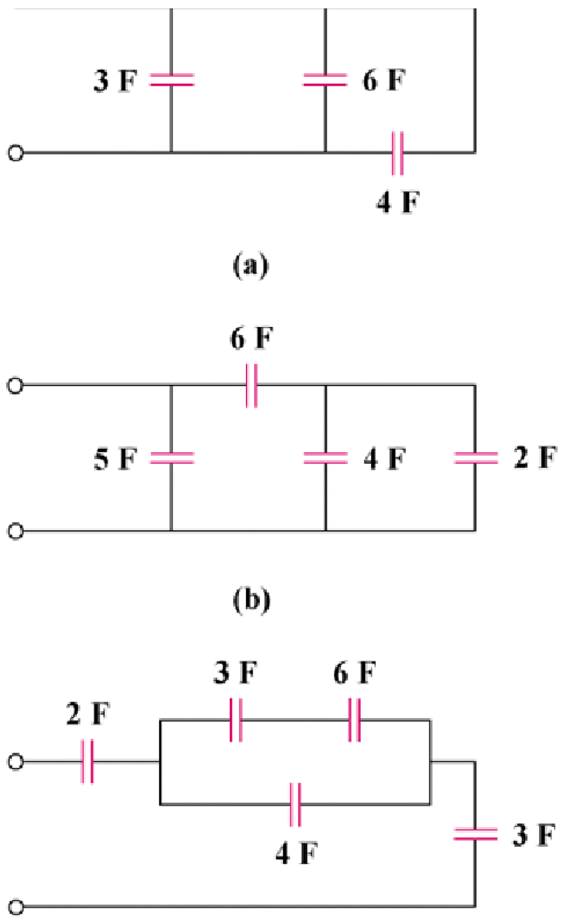


Figure 6.51

Chapter 6, Solution 17.

(a) 4F in series with 12F = 4 x 12/(16) = 3F
3F in parallel with 6F and 3F = 3+6+3 = 12F
4F in series with 12F = 3F
i.e. C_{eq} = **<u>3F</u>**

(b) C_{eq} = 5 + [6x(4 + 2)/(6+4+2)] = 5 + (36/12) = 5 + 3 = **<u>8F</u>**

(c) 3F in series with 6F = (3 x 6)/9 = 2F

$$\frac{1}{C_{eq}} = \frac{1}{2} + \frac{1}{6} + \frac{1}{3} = 1$$

C_{eq} = **<u>1F</u>**

Chapter 6, Problem 18.

Find C_{eq} in the circuit of Fig. 6.52 if all capacitors are 4 μF

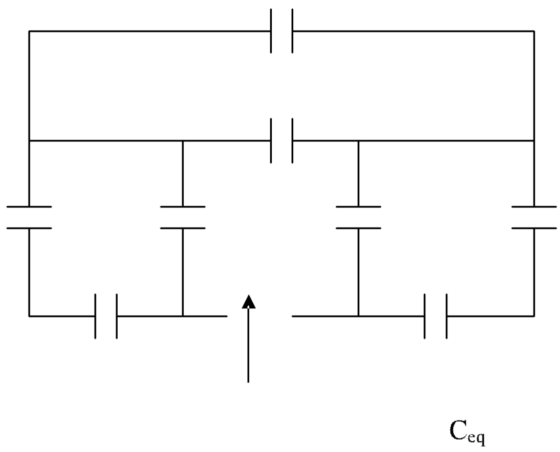
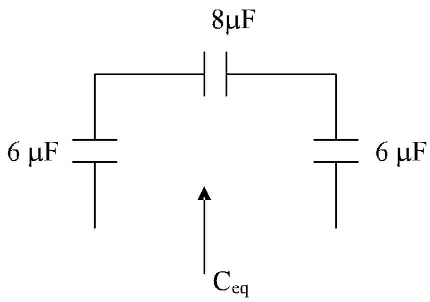


Figure 6.52 For Prob. 6.18.

Chapter 6, Solution 18.

4 μF in parallel with 4 μF = 8μF
4 μF in series with 4 μF = 2 μF
2 μF in parallel with 4 μF = 6 μF
Hence, the circuit is reduced to that shown below.



$$\frac{1}{C_{eq}} = \frac{1}{6} + \frac{1}{6} + \frac{1}{8} = 0.4583 \longrightarrow C_{eq} = \underline{2.1818\ \mu F}$$

Chapter 6, Problem 19.

Find the equivalent capacitance between terminals *a* and *b* in the circuit of Fig. 6.53. All capacitances are in μF.

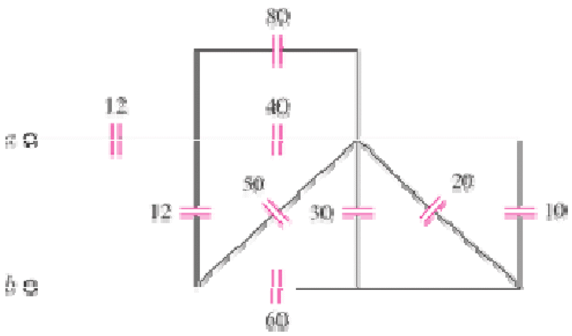


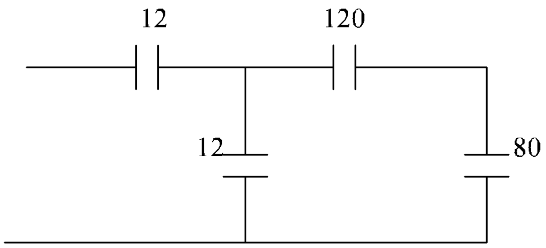
Figure 6.53

Chapter 6, Solution 19.

We combine 10-, 20-, and 30- μ F capacitors in parallel to get 60 μ F. The 60 - μ F capacitor in series with another 60- μ F capacitor gives 30 μ F.

30 + 50 = 80 μ F, 80 + 40 = 120 μ F

The circuit is reduced to that shown below.



120- μ F capacitor in series with 80 μ F gives (80x120)/200 = 48

48 + 12 = 60

60- μ F capacitor in series with 12 μ F gives (60x12)/72 = **<u>10 μ F</u>**

Chapter 6, Problem 20.

Find the equivalent capacitance at terminals a-b of the circuit in Fig. 6.54.

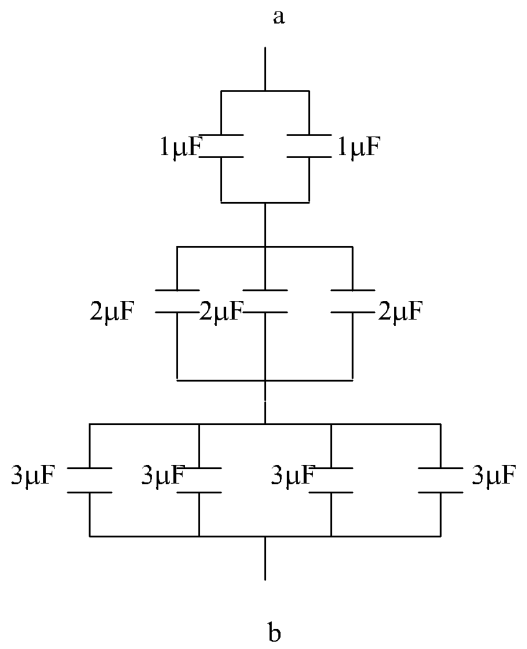
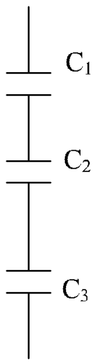


Figure 6.54 For Prob. 6.20.

Chapter 6, Solution 20.

Consider the circuit shown below.



$$C_1 = 1 + 1 = 2\mu F$$
$$C_2 = 2 + 2 + 2 = 6\mu F$$

$$C_3 = 4x3 = 12\mu F$$

$1/C_{eq} = (1/C_1) + (1/C_2) + (1/C_3) = 0.5 + 0.16667 + 0.08333 = 0.75x10^6$

C_{eq} = **1.3333 µF**.

Chapter 6, Problem 21.

Determine the equivalent capacitance at terminals ***a*** - ***b*** of the circuit in Fig. 6.55.

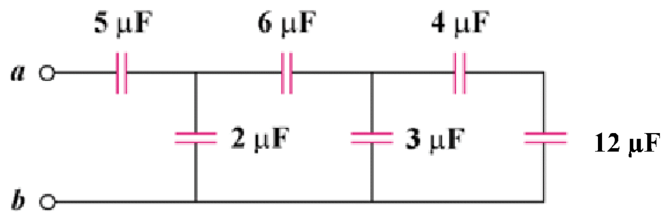


Figure 6.55

Chapter 6, Solution 21.

4μF in series with 12μF = (4x12)/16 = 3μF
3μF in parallel with 3μF = 6μF
6μF in series with 6μF = 3μF
3μF in parallel with 2μF = 5μF
5μF in series with 5μF = 2.5μF

Hence C_{eq} = **2.5μF**

Chapter 6, Problem 22.

Obtain the equivalent capacitance of the circuit in Fig. 6.56.

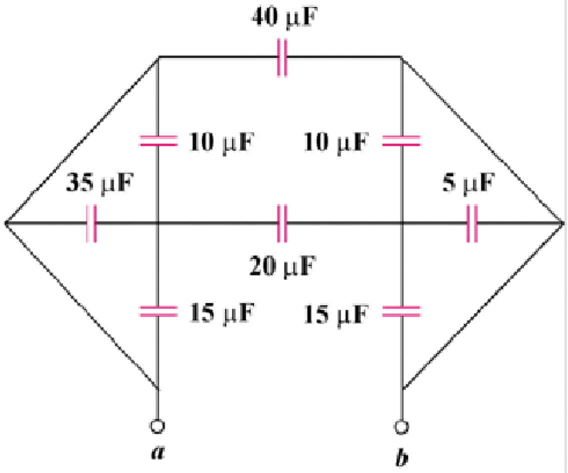
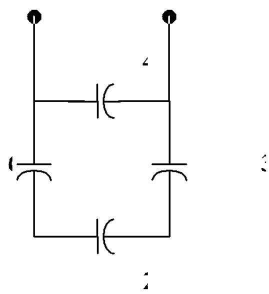


Figure 6.56

Chapter 6, Solution 22.

Combining the capacitors in parallel, we obtain the equivalent circuit shown below:



Combining the capacitors in series gives C^1_{eq}, where

$$\frac{1}{C^1_{eq}} = \frac{1}{60} + \frac{1}{20} + \frac{1}{30} = \frac{1}{10} \longrightarrow C^1_{eq} = 10\mu F$$

Thus

$$C_{eq} = 10 + 40 = \mathbf{\underline{50\ \mu F}}$$

Chapter 6, Problem 23.

For the circuit in Fig. 6.57, determine:

(a) the voltage across each capacitor,
(b) the energy stored in each capacitor.

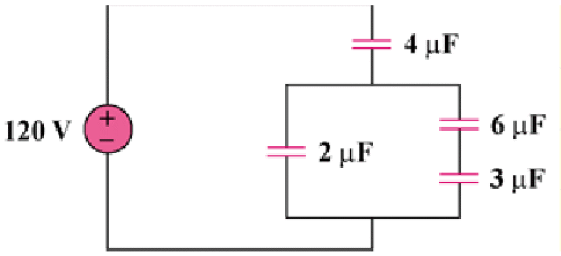


Figure 6.57

Chapter 6, Solution 23.

(a) 3μF is in series with 6μF   3x6/(9) = 2μF

$v_{4\mu F}$ = 1/2 x 120 = **60V**

$v_{2\mu F}$ = **60V**

$v_{6\mu F} = \frac{3}{6+3}(60) = \mathbf{20V}$

$v_{3\mu F}$ = 60 - 20 = **40V**

(b) Hence w = 1/2 Cv^2

$w_{4\mu F}$ = 1/2 x 4 x 10^{-6} x 3600 = **7.2mJ**

$w_{2\mu F}$ = 1/2 x 2 x 10^{-6} x 3600 = **3.6mJ**

$w_{6\mu F}$ = 1/2 x 6 x 10^{-6} x 400 = **1.2mJ**

$w_{3\mu F}$ = 1/2 x 3 x 10^{-6} x 1600 = **2.4mJ**

Chapter 6, Problem 24.

Repeat Prob. 6.23 for the circuit in Fig. 6.58.

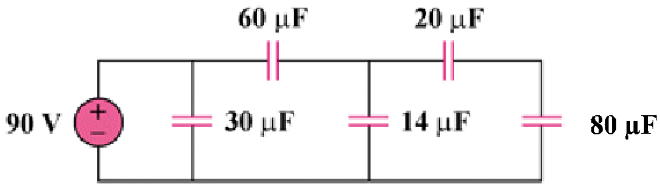


Figure 6.58

Chapter 6, Solution 24.

20μF is series with 80μF = 20x80/(100) = 16μF

14μF is parallel with 16μF = 30μF

(a) $v_{30\mu F} = \underline{\textbf{90V}}$

$v_{60\mu F} = \underline{\textbf{30V}}$

$v_{14\mu F} = \underline{\textbf{60V}}$

$v_{20\mu F} = \frac{80}{20+80} x60 = \underline{\textbf{48V}}$

$v_{80\mu F} = 60 - 48 = \underline{\textbf{12V}}$

(b) Since $w = \frac{1}{2} Cv^2$

$w_{30\mu F} = 1/2 \text{ x } 30 \text{ x } 10^{-6} \text{ x } 8100 = \underline{\textbf{121.5mJ}}$

$w_{60\mu F} = 1/2 \text{ x } 60 \text{ x } 10^{-6} \text{ x } 900 = \underline{\textbf{27mJ}}$

$w_{14\mu F} = 1/2 \text{ x } 14 \text{ x } 10^{-6} \text{ x } 3600 = \underline{\textbf{25.2mJ}}$

$w_{20\mu F} = 1/2 \text{ x } 20 \text{ x } 10^{-6} \text{ x } (48)^2 = \underline{\textbf{23.04mJ}}$

$w_{80\mu F} = 1/2 \text{ x } 80 \text{ x } 10^{-6} \text{ x } 144 = \underline{\textbf{5.76mJ}}$

Chapter 6, Problem 25.

(a) Show that the voltage-division rule for two capacitors in series as in Fig. 6.59(a) is

$$v_1 = \frac{C_2}{C_1 + C_2} v_s , \qquad v_2 = \frac{C_1}{C_1 + C_2} v_s$$

assuming that the initial conditions are zero.

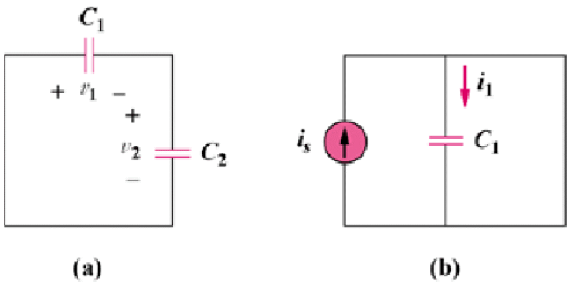


Figure 6.59

(b) For two capacitors in parallel as in Fig. 6.59(b), show that the current-division rule is

$$i_1 = \frac{C_1}{C_1 + C_2} i_s , \qquad i_2 = \frac{C_2}{C_1 + C_2} i_s$$

assuming that the initial conditions are zero.

Chapter 6, Solution 25.

(a) For the capacitors in series,

$$Q_1 = Q_2 \longrightarrow C_1v_1 = C_2v_2 \longrightarrow \frac{v_1}{v_2} = \frac{C_2}{C_1}$$

$$v_s = v_1 + v_2 = \frac{C_2}{C_1}v_2 + v_2 = \frac{C_1 + C_2}{C_1}v_2 \longrightarrow \underline{v_2 = \frac{C_1}{C_1 + C_2}v_s}$$

Similarly, $\underline{v_1 = \frac{C_2}{C_1 + C_2}v_s}$

(b) For capacitors in parallel

$$v_1 = v_2 = \frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

$$Q_s = Q_1 + Q_2 = \frac{C_1}{C_2}Q_2 + Q_2 = \frac{C_1 + C_2}{C_2}Q_2$$

or

$$Q_2 = \frac{C_2}{C_1 + C_2}$$

$$Q_1 = \frac{C_1}{C_1 + C_2}Q_s$$

$$i = \frac{dQ}{dt} \longrightarrow \underline{i_1 = \frac{C_1}{C_1 + C_2}i_s}, \quad \underline{i_2 = \frac{C_2}{C_1 + C_2}i_s}$$

Chapter 6, Problem 26.

Three capacitors, $C_1 = 5\ \mu F$, $C_2 = 10\ \mu F$, and $C_3 = 20\ \mu F$, are connected in parallel across a 150-V source. Determine:

(a) the total capacitance,
(b) the charge on each capacitor,
(c) the total energy stored in the parallel combination.

Chapter 6, Solution 26.

(a) $C_{eq} = C_1 + C_2 + C_3 = \underline{\mathbf{35\mu F}}$

(b) $Q_1 = C_1 v = 5 \text{ x } 150\mu C = \underline{\mathbf{0.75mC}}$
$Q_2 = C_2 v = 10 \text{ x } 150\mu C = \underline{\mathbf{1.5mC}}$
$Q_3 = C_3 v = 20 \text{ x } 150 = \underline{\mathbf{3mC}}$

(c) $w = \frac{1}{2} C_{eq} v^2 = \frac{1}{2} x35x150^2 \mu J = \underline{\mathbf{393.8mJ}}$

Chapter 6, Problem 27.

Given that four 4-µF capacitors can be connected in series and in parallel, find the minimum and maximum values that can be obtained by such series/parallel combinations.

Chapter 6, Solution 27.

If they are all connected in parallel, we get $C_T = 4x4\mu F = 16\mu F$

If they are all connected in series, we get

$$\frac{1}{C_T} = \frac{4}{4\mu F} \longrightarrow C_T = 1\mu F$$

All other combinations fall within these two extreme cases. Hence,

$\underline{C_{min} = 1\mu F, \quad C_{max} = 16\mu F}$

Chapter 6, Problem 28.

Obtain the equivalent capacitance of the network shown in Fig. 6.58.

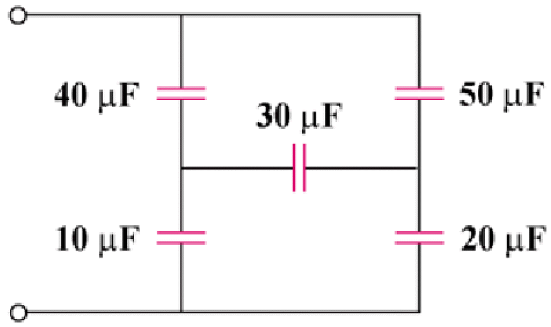
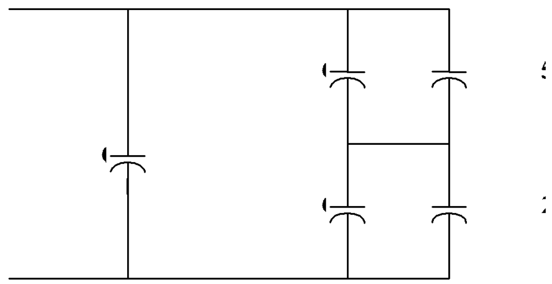


Figure 6.58

Chapter 6, Solution 28.

We may treat this like a resistive circuit and apply delta-wye transformation, except that R is replaced by 1/C.



$$\frac{1}{C_a} = \frac{\left(\frac{1}{10}\right)\left(\frac{1}{40}\right)+\left(\frac{1}{10}\right)\left(\frac{1}{30}\right)+\left(\frac{1}{30}\right)\left(\frac{1}{40}\right)}{\frac{1}{30}}$$

$$= \frac{3}{40}+\frac{1}{10}+\frac{1}{40}=\frac{2}{10}$$

$C_a = 5\mu F$

$$\frac{1}{C_b} = \frac{\frac{1}{400}+\frac{1}{300}+\frac{1}{1200}}{\frac{1}{10}} = \frac{2}{30}$$

$C_b = 15\mu F$

$$\frac{1}{C_c} = \frac{\frac{1}{400}+\frac{1}{300}+\frac{1}{1200}}{\frac{1}{40}} = \frac{4}{15}$$

$C_c = 3.75\mu F$

C_b in parallel with $50\mu F = 50 + 15 = 65\mu F$

C_c in series with $20\mu F = 23.75\mu F$

$65\mu F$ in series with $23.75\mu F = \dfrac{65x23.75}{88.75} = 17.39\mu F$

$17.39\mu F$ in parallel with $C_a = 17.39 + 5 = 22.39\mu F$

Hence C_{eq} = **$\underline{22.39\mu F}$**

Chapter 6, Problem 29.

Determine C_{eq} for each circuit in Fig. 6.61.

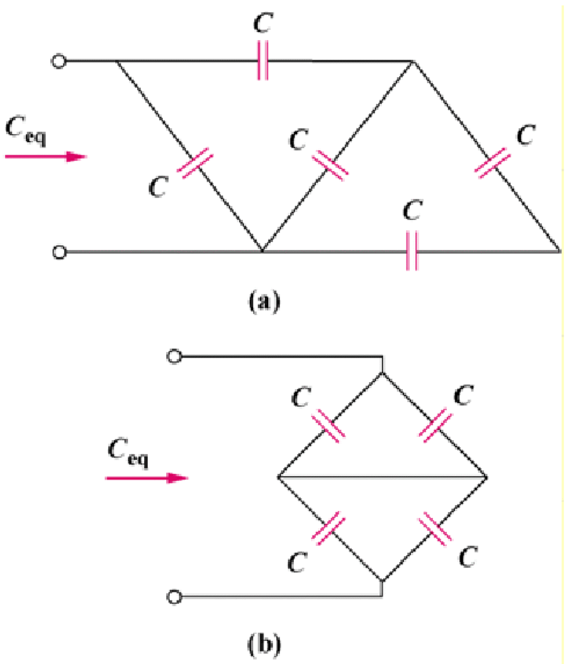


Figure 6.61

Chapter 6, Solution 29.

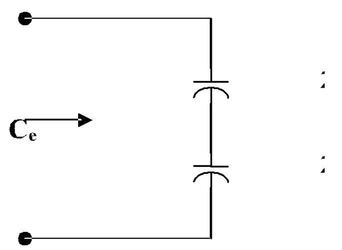
(a) C in series with C = C/(2)

C/2 in parallel with C = 3C/2

$$\frac{3C}{2} \text{ in series with } C = \frac{Cx\frac{3C}{2}}{5\frac{C}{2}} = \frac{3C}{5}$$

$$3\frac{C}{5} \text{ in parallel with } C = C + 3\frac{C}{5} = \underline{\mathbf{1.6\ C}}$$

(b)



$$\frac{1}{C_{eq}} = \frac{1}{2C} + \frac{1}{2C} = \frac{1}{C}$$

C_{eq} = **1 C**

Chapter 6, Problem 30.

Assuming that the capacitors are initially uncharged, find $v_o(t)$ in the circuit in Fig. 6.62.

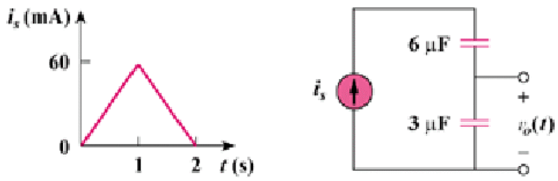


Figure 6.62

Chapter 6, Solution 30.

$$v_o = \frac{1}{C}\int_0^t i\,dt + i(0)$$

For $0 < t < 1$, $i = 60t$ mA,

$$v_o = \frac{10^{-3}}{3x10^{-6}}\int_0^t 60t\,dt + 0 = 10t^2 kV$$

$v_o(1) = 10kV$

For $1 < t < 2$, $i = 120 - 60t$ mA,

$$v_o = \frac{10^{-3}}{3x10^{-6}}\int_1^t (120 - 60t)dt + v_o(1)$$

$$= [40t - 10t^2]\Big|_1^t + 10kV$$

$$= 40t - 10t^2 - 20$$

$$v_o(t) = \begin{cases} 10t^2 kV, & 0 < t < 1 \\ 40t - 10t^2 - 20kV, & 1 < t < 2 \end{cases}$$

Chapter 6, Problem 31.

If $v(0)=0$, find $v(t)$, $i_1(t)$, and $i_2(t)$ in the circuit in Fig. 6.63.

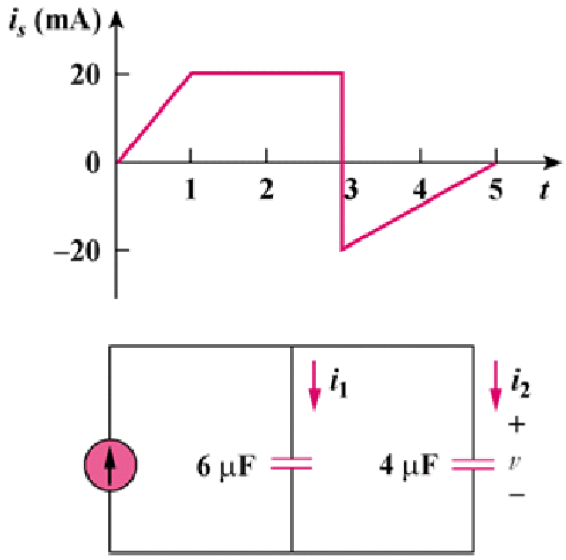


Figure 6.63

Chapter 6, Solution 31.

$$i_s(t) = \begin{cases} 20t\text{mA}, & 0 < t < 1 \\ 20\text{mA}, & 1 < t < 3 \\ -50 + 10t, & 3 < t < 5 \end{cases}$$

$$C_{eq} = 4 + 6 = 10\mu F$$

$$v = \frac{1}{C_{eq}} \int_0^t i dt + v(0)$$

For $0 < t < 1$,

$$v = \frac{10^{-3}}{10x10^{-6}} \int_0^t 20t\, dt + 0 = t^2 \text{ kV}$$

For $1 < t < 3$,

$$v = \frac{10^3}{10} \int_1^t 20dt + v(1) = 2(t-1) + 1\text{kV}$$

$$= 2t - 1\text{kV}$$

For $3 < t < 5$,

$$v = \frac{10^3}{10} \int_3^t 10(t-5)dt + v(3)$$

$$= \frac{t^2}{2} - 5t \Big|_3^t + 5\text{kV} = \frac{t^2}{2} - 5t + 15.5\text{kV}$$

$$v(t) = \begin{cases} t^2\text{kV}, & 0 < t < 1\text{s} \\ 2t - 1\text{kV}, & 1 < t < 3\text{s} \\ \frac{t^2}{2} - 5t + 15.5\text{kV}, & 3 < t < 5\text{s} \end{cases}$$

$$i_1 = C_1 \frac{dv}{dt} = 6x10^{-6} \frac{dv}{dt}$$

$$= \begin{cases} 12t\text{mA}, & 0 < t < 1\text{s} \\ 12\text{mA}, & 1 < t < 3\text{s} \\ 6t - 30\text{mA}, & 3 < t < 5\text{s} \end{cases}$$

$$i_2 = C_2 \frac{dv}{dt} = 4x10^{-6} \frac{dv}{dt}$$

$$= \begin{cases} 8t\text{mA}, & 0 < t < 1\text{s} \\ 8\text{mA}, & 1 < t < 3\text{s} \\ 4t - 20\text{mA}, & 3 < t < 5\text{s} \end{cases}$$

Chapter 6, Problem 32.

In the circuit in Fig. 6.64, let $i_s = 30e^{-2t}$ mA and $v_1(0) = 50$ V, $v_2(0) = 20$ V. Determine: (a) $v_1(t)$ and $v_2(t)$, (b) the energy in each capacitor at $t = 0.5$ s.

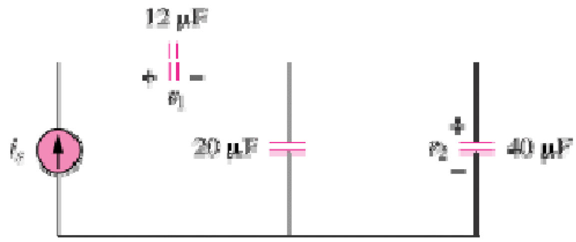


Figure 6.64

Chapter 6, Solution 32.

(a) $C_{eq} = (12x60)/72 = 10\ \mu F$

$$v_1 = \frac{10^{-3}}{12x10^{-6}}\int_0^t 30e^{-2t}dt + v_1(0) = \underline{-1250e^{-2t}\ |_0^t} + 50 = \underline{-1250e^{-2t} + 1300}V$$

$$v_2 = \frac{10^{-3}}{60x10^{-6}}\int_0^t 30e^{-2t}dt + v_2(0) = \underline{250e^{-2t}\ |_0^t} + 20 = \underline{-250e^{-2t} + 270V}$$

(b) At t=0.5s,

$$v_1 = -1250e^{-1} + 1300 = 840.2, \qquad v_2 = -250e^{-1} + 270 = 178.03$$

$$w_{12\mu F} = \frac{1}{2}x12x10^{-6}x(840.15)^2 = \underline{4.235\ \ J}$$

$$w_{20\mu F} = \frac{1}{2}x20x10^{-6}x(178.03)^2 = \underline{0.3169\ \ J}$$

$$w_{40\mu F} = \frac{1}{2}x40x10^{-6}x(178.03)^2 = \underline{0.6339\ J}$$

Chapter 6, Problem 33.

Obtain the Thèvenin equivalent at the terminals, *a-b*, of the circuit shown in Fig. 6.65. Please note that Thèvenin equivalent circuits do not generally exist for circuits involving capacitors and resistors. This is a special case where the Thèvenin equivalent circuit does exist.

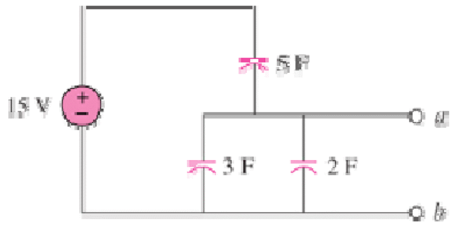


Figure 6.65

Chapter 6, Solution 33

Because this is a totally capacitive circuit, we can combine all the capacitors using the property that capacitors in parallel can be combined by just adding their values and we combine capacitors in series by adding their reciprocals. However, for this circuit we only have the three capacitors in parallel.

3 F + 2 F = 5 F (we need this to be able to calculate the voltage)

$C_{Th} = C_{eq} = 5+5 = 10$ F

The voltage will divide equally across the two 5 F capacitors. Therefore, we get:

$V_{Th} =$ **7.5 V**, $C_{Th} =$ **10 F**

Chapter 6, Problem 34.

The current through a 10-mH inductor is $6e^{-t/2}$ A. Find the voltage and the power at $t = 3$ s.

Chapter 6, Solution 34.

$i = 6e^{-t/2}$

$$v = L\frac{di}{dt} = 10x10^{-3}(6)\left(\frac{1}{2}\right)e^{-t/2}$$

$= -30e^{-t/2}$ mV

$v(3) = -30e^{-3/2}$ mV = **–6.694 mV**

$p = vi = -180e^{-t}$ mW

$p(3) = -180e^{-3}$ mW = **–8.962 mW**

Chapter 6, Problem 35.

An inductor has a linear change in current from 50 mA to 100 mA in 2 ms and induces a voltage of 160 mV. Calculate the value of the inductor.

Chapter 6, Solution 35.

$$v = L\frac{di}{dt} \longrightarrow L = \frac{v}{di/dt} = \frac{160x10^{-3}}{\frac{(100-50)x10^{-3}}{2x10^{-3}}} = \underline{6.4\ \text{mH}}$$

Chapter 6, Problem 36.

The current through a 12-mH inductor is $i(t) = 30te^{-2t}$ A, $t \geq 0$. Determine: (a) the voltage across the inductor, (b) the power being delivered to the inductor at t = 1 s, (c) the energy stored in the inductor at t = 1 s.

Chapter 6, Solution 36.

(a) $v = L\frac{di}{dt} = 12x10^{-3}(30e^{-2t} - 60te^{-2t}) = \underline{(0.36 - 0.72t)e^{-2t}\ \text{V}}$

(b) $p = vi = (0.36 - 0.72x1)e^{-2}x30x1e^{-2} = 0.36x30e^{-4} = \underline{-0.1978\ \text{W}}$

(c) $w = \frac{1}{2}Li^2 = 0.5x12x10^{-3}(30x1xe^{-2})^2 = \underline{\mathbf{98.9\ mJ}}$.

Chapter 6, Problem 37.

The current through a 12-mH inductor is 4 sin 100*t* A. Find the voltage, and also the energy stored in the inductor for $0 < t < \pi/200$ s.

Chapter 6, Solution 37.

$$v = L\frac{di}{dt} = 12x10^{-3}x4(100)\cos 100t$$

$$= \underline{\mathbf{4.8\ cos\ 100t\ V}}$$

p = vi = 4.8 x 4 sin 100t cos 100t = 9.6 sin 200t

$$w = \int_o^t pdt = \int_o^{11/200} 9.6\sin 200t$$

$$= -\frac{9.6}{200}\cos 200t\Big|_o^{11/200} \text{J}$$

$$= -48(\cos\pi - 1)\text{mJ} = \underline{\mathbf{96\ mJ}}$$

Please note that this problem could have also been done by using (½)Li².

Chapter 6, Problem 38.

The current through a 40-mH inductor is

$$i(t) = \begin{cases} 0, & t < 0 \\ te^{-2t}\ \text{A}, & t > 0 \end{cases}$$

Find the voltage ***v*(*t*)**.

Chapter 6, Solution 38.

$$v = L\frac{di}{dt} = 40x10^{-3}\left(e^{-2t} - 2te^{-2t}\right)dt$$

$$= \underline{40(1-2t)e^{-2t}\text{mV}, t > 0}$$

Chapter 6, Problem 39.

The voltage across a 200-mH inductor is given by

$v(t) = 3t^2 + 2t + 4$ V for $t > 0$.

Determine the current $i(t)$ through the inductor. Assume that $i(0) = 1$ A.

Chapter 6, Solution 39

$$v = L\frac{di}{dt} \longrightarrow i = \frac{1}{L}\int_0^t idt + i(0)$$

$$i = \frac{1}{200x10^{-3}}\int_0^t (3t^2 + 2t + 4)dt + 1$$

$$= 5(t^3 + t^2 + 4t)\Big|_0^t + 1$$

i(t) = $\underline{\mathbf{5t^3 + 5t^2 + 20t + 1\ A}}$

Chapter 6, Problem 40.

The current through a 5-mH inductor is shown in Fig. 6.66. Determine the voltage across the inductor at t=1,3, and 5ms.

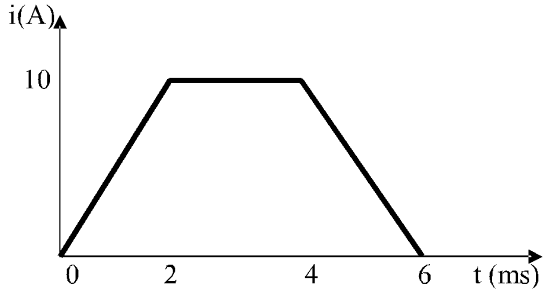


Figure 6.66 For Prob. 6.40.

Chapter 6, Solution 40.

$$i = \begin{cases} 5t, & 0 < t < 2ms \\ 10, & 2 < t < 4ms \\ 30 - 5t, & 4 < t < 6ms \end{cases}$$

$$v = L\frac{di}{dt} = \frac{5x10^{-3}}{10^{-3}} \begin{cases} 5, & 0 < t < 2ms \\ 0, & 2 < t < 4ms \\ -5, & 4 < t < 6ms \end{cases} = \begin{cases} 25, & 0 < t < 2ms \\ 0, & 2 < t < 4ms \\ -25, & 4 < t < 6ms \end{cases}$$

At t=1ms, v=25 V
At t=3ms, v=0 V
At t=5ms, v=-25 V

Chapter 6, Problem 41.

The voltage across a 2-H inductor is $20(1 - e^{-2t})$ V. If the initial current through the inductor is 0.3 A, find the current and the energy stored in the inductor at t = 1 s.

Chapter 6, Solution 41.

$$i = \frac{1}{L}\int_0^t v\,dt + C = \left(\frac{1}{2}\right)\int_0^t 20\left(1 - e^{-2t}\right)dt + C$$

$$= 10\left(t + \frac{1}{2}e^{-2t}\right)\Big|_0^t + C = 10t + 5e^{-2t} - 4.7\text{A}$$

Note, we get C = –4.7 from the initial condition for i needing to be 0.3 A.

We can check our results be solving for v = Ldi/dt.

$v = 2(10 – 10e^{-2t})$V which is what we started with.

At t = 1 s, $i = 10 + 5e^{-2} – 4.7 = 10 + 0.6767 – 4.7 =$ **5.977 A**

$$w = \frac{1}{2}L\,i^2 = \mathbf{35.72J}$$

Chapter 6, Problem 42.

If the voltage waveform in Fig. 6.67 is applied across the terminals of a 5-H inductor, calculate the current through the inductor. Assume ***i*(0)** = -1 A.

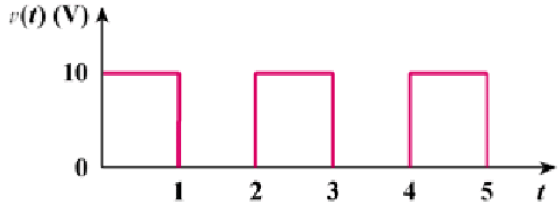


Figure 6.67

Chapter 6, Solution 42.

$$i = \frac{1}{L}\int_0^t v dt + i(0) = \frac{1}{5}\int_0^t v(t) dt - 1$$

For $0 < t < 1$, $i = \frac{10}{5}\int_0^t dt - 1 = 2t - 1$ A

For $1 < t < 2$, $i = 0 + i(1) = 1A$

For $2 < t < 3$, $i = \frac{1}{5}\int 10 dt + i(2) = 2t\Big|_t^2 + 1$

$= 2t - 3$ A

For $3 < t < 4$, $i = 0 + i(3) = 3$ A

For $4 < t < 5$, $i = \frac{1}{5}\int_4^t 10 dt + i(4) = 2t\Big|_4^t + 3$

$= 2t - 5$ A

$$\text{Thus,} \quad i(t) = \begin{bmatrix} 2t - 1A, & 0 < t < 1 \\ 1A, & 1 < t < 2 \\ 2t - 3A, & 2 < t < 3 \\ 3A, & 3 < t < 4 \\ 2t - 5, & 4 < t < 5 \end{bmatrix}$$

Chapter 6, Problem 43.

The current in an 80-mH inductor increases from 0 to 60 mA.
How much energy is stored in the inductor?

Chapter 6, Solution 43.

$$w = L\int_{-\infty}^{t} idt = \frac{1}{2}Li^2(t) - \frac{1}{2}Li^2(-\infty)$$
$$= \frac{1}{2}x80x10^{-3}x\left(60x10^{-3}\right)^2 - 0$$
$$= \underline{\mathbf{144\ \mu J}}$$

***Chapter 6, Problem 44.**

A 100-mH inductor is connected in parallel with a 2-kΩ resistor. The current through the inductor is $i(t) = 50e^{-400t}$ mA. (a) Find the voltage v_L across the inductor. (b) Find the voltage v_R across the resistor. (c) Is $v_R(t) + v_L(t) = 0$? (d) Calculate the energy in the inductor at t=0.

Chapter 6, Solution 44.

(a) $v_L = L\frac{di}{dt} = 100x10^{-3}(-400)x50x10^{-3}e^{-400t} = \underline{-2e^{-400t}\ V}$

(b) Since R and L are in parallel, $v_R = v_L = \underline{-2e^{-400t}\ V}$

(c) **No**

(d) $w = \frac{1}{2}Li^2 = 0.5x100x10^{-3}(0.05)^2 = \underline{\mathbf{125\ \mu J}}$.

Chapter 6, Problem 45.

If the voltage waveform in Fig. 6.68 is applied to a 10-mH inductor, find the inductor current ***i*(*t*)**. Assume ***i*(0)** = 0.

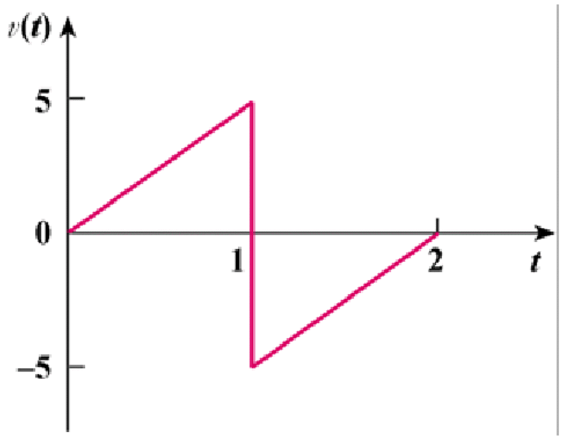


Figure 6.68

Chapter 6, Solution 45.

$$i(t) = \frac{1}{L}\int_0^t v(t) + i(0)$$

For $0 < t < 1$, $v = 5t$

$$i = \frac{1}{10x10^{-3}}\int_0^t 5t\,dt + 0$$

$$= 0.25t^2 \text{ kA}$$

For $1 < t < 2$, $v = -10 + 5t$

$$i = \frac{1}{10x10^{-3}}\int_1^t (-10 + 5t)dt + i(1)$$

$$= \int_1^t (0.5t - 1)dt + 0.25\text{kA}$$

$$= 1 - t + 0.25t^2 \text{ kA}$$

$$i(t) = \begin{cases} 0.25t^2\text{kA}, & 0 < t < 1\text{s} \\ 1 - t + 0.25t^2\text{kA}, & 1 < t < 2\text{s} \end{cases}$$

Chapter 6, Problem 46.

Find v_C, i_L, and the energy stored in the capacitor and inductor in the circuit of Fig. 6.69 under dc conditions.

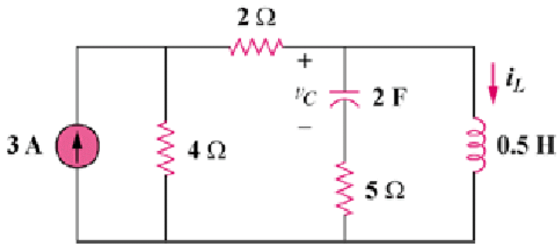
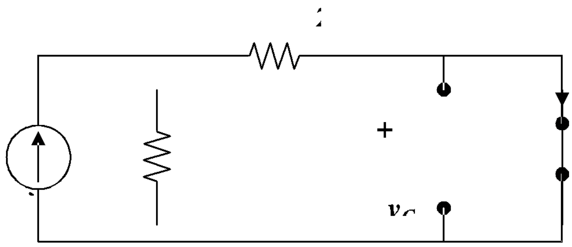


Figure 6.69

Chapter 6, Solution 46.

Under dc conditions, the circuit is as shown below:



By current division,

$$i_L = \frac{4}{4+2}(3) = \underline{\mathbf{2A}}, \quad v_c = \underline{\mathbf{0V}}$$

$$w_L = \frac{1}{2} L\, i_L^2 = \frac{1}{2}\left(\frac{1}{2}\right)(2)^2 = \underline{\mathbf{1J}}$$

$$w_c = \frac{1}{2} C\, v_c^2 = \frac{1}{2}(2)(v) = \underline{\mathbf{0J}}$$

Chapter 6, Problem 47.

For the circuit in Fig. 6.70, calculate the value of ***R*** that will make the energy stored in the capacitor the same as that stored in the inductor under dc conditions.

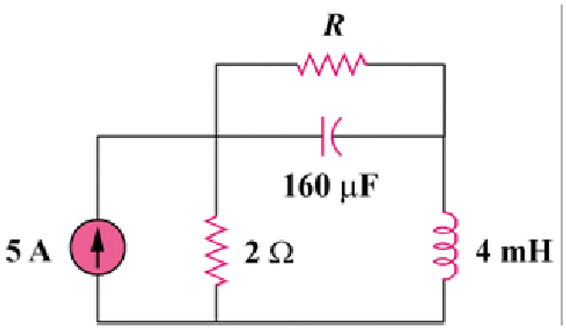
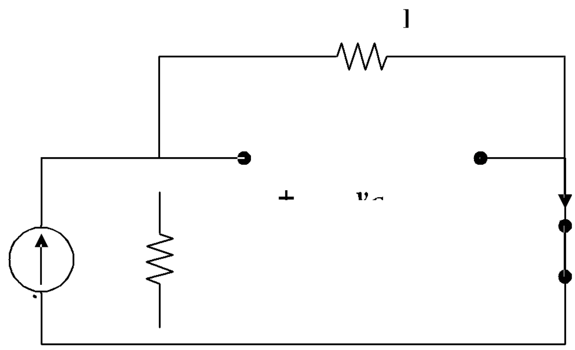


Figure 6.70

Chapter 6, Solution 47.

Under dc conditions, the circuit is equivalent to that shown below:



$$i_L = \frac{2}{R+2}(5) = \frac{10}{R+2}, \quad v_c = Ri_L = \frac{10R}{R+2}$$

$$w_c = \frac{1}{2}Cv_c^2 = 80x10^{-6}x\frac{100R^2}{(R+2)^2}$$

$$w_L = \frac{1}{2}Li_l^2 = 2x10^{-3}x\frac{100}{(R+2)^2}$$

If $w_c = w_L$,

$$80x10^{-6}x\frac{100R^2}{(R+2)^2} = \frac{2x10^{-3}x100}{(R+2)^2} \longrightarrow 80 \text{ x } 10^{-3}R^2 = 2$$

R = <u>5Ω</u>

Chapter 6, Problem 48.

Under steady-state dc conditions, find i and v in the circuit in Fig. 6.71.

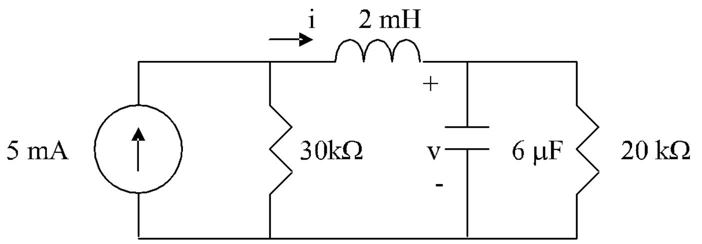
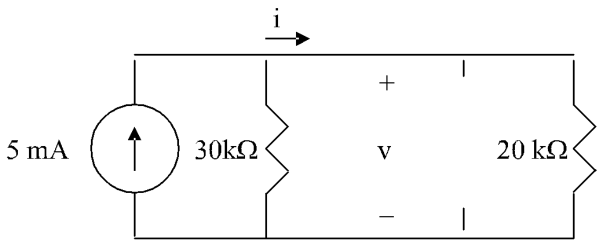


Figure 6.71 For Prob. 6.48.

Chapter 6, Solution 48.

Under steady-state, the inductor acts like a short-circuit, while the capacitor acts like an open circuit as shown below.



Using current division,

$$i = \frac{30k}{30k+20k}(5mA) = \underline{3\ mA}$$

v = 20ki = **60 V**

Chapter 6, Problem 49.

Find the equivalent inductance of the circuit in Fig. 6.72. Assume all inductors are 10 mH.

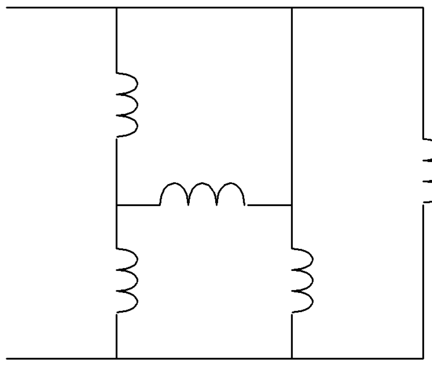
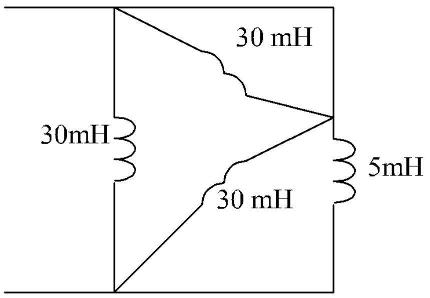


Figure 6.72 For Prob. 6.49.

Chapter 6, Solution 49.

Converting the wye-subnetwork to its equivalent delta gives the circuit below.



30//0 = 0, 30//5 = 30x5/35=4.286

$$L_{eq} = 30//4.286 = \frac{30x4.286}{34.286} = \underline{3.75 \text{ mH}}$$

Chapter 6, Problem 50.

An energy-storage network consists of series-connected 16-mH and 14-mH inductors in parallel with a series connected 24-mH and 36-mH inductors. Calculate the equivalent inductance.

Chapter 6, Solution 50.

16mH in series with 14 mH = 16+14=30 mH
24 mH in series with 36 mH = 24+36=60 mH
30mH in parallel with 60 mH = 30x60/90 = **20 mH**

Chapter 6, Problem 51.

Determine L_{eq} at terminals ***a-b*** of the circuit in Fig. 6.73.

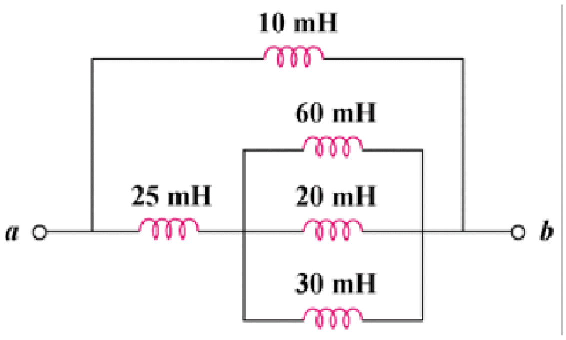


Figure 6.73

Chapter 6, Solution 51.

$$\frac{1}{L} = \frac{1}{60} + \frac{1}{20} + \frac{1}{30} = \frac{1}{10} \qquad L = 10 \text{ mH}$$

$$L_{eq} = 10 \| (25 + 10) = \frac{10x35}{45}$$

= **7.778 mH**

Chapter 6, Problem 52.

Find L_{eq} in the circuit of Fig. 6.74.

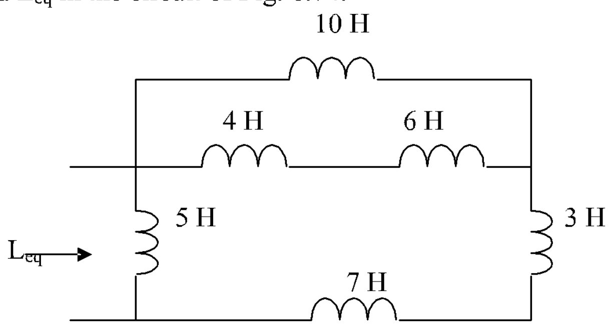


Figure 6.74 For Prob. 6.52.

Chapter 6, Solution 52.

$$L_{eq} = 5//(7+3+10//(4+6)) == 5//(7+3+5)) = \frac{5x15}{20} = \underline{3.75\ \ H}$$

Chapter 6, Problem 53.

Find $\boldsymbol{L_{eq}}$ at the terminals of the circuit in Fig. 6.75.

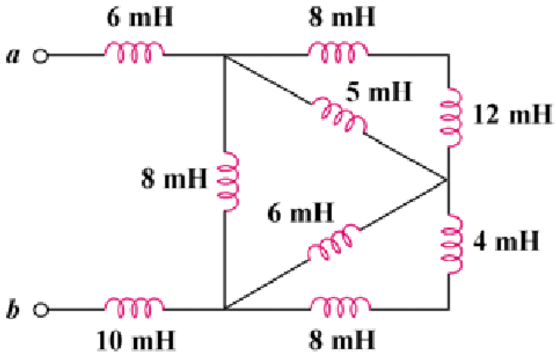


Figure 6.75

Chapter 6, Solution 53.

$$L_{eq} = 6 + 10 + 8\|[5\|(8+12) + 6\|(8+4)]$$

$$= 16 + 8\|(4+4) = 16 + 4$$

L_{eq} = **<u>20 mH</u>**

Chapter 6, Problem 54.

Find the equivalent inductance looking into the terminals of the circuit in Fig. 6.76.

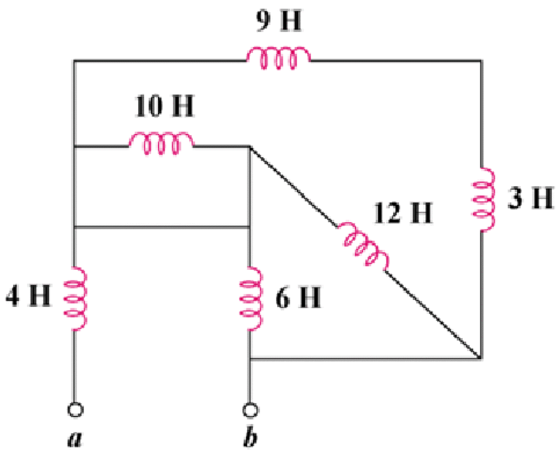


Figure 6.76

Chapter 6, Solution 54.

$$L_{eq} = 4 + (9+3) \| (10 \| 0 + 6 \| 12)$$

$$= 4 + 12 \| (0+4) = 4 + 3$$

L_{eq} = **<u>7H</u>**

Chapter 6, Problem 55.

Find L_{eq} in each of the circuits of Fig. 6.77.

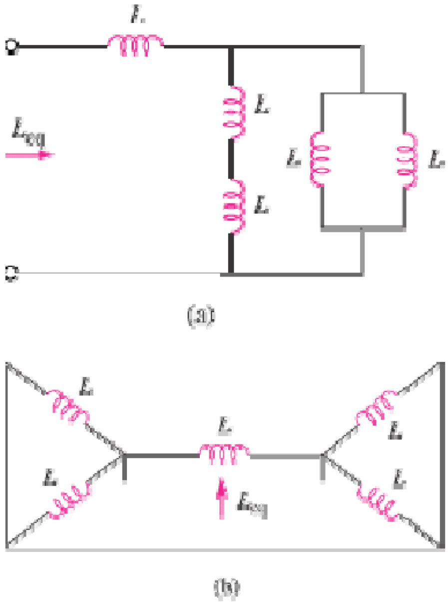


Figure 6.77

Chapter 6, Solution 55.

(a) L//L = 0.5L, L + L = 2L

$$L_{eq} = L + 2L // 0.5L = L + \frac{2Lx0.5L}{2L + 0.5L} = \underline{1.4L} = \mathbf{\underline{1.4\ L}}.$$

(b) L//L = 0.5L, L//L + L//L = L

L_{eq} = L//L = **<u>500 mL</u>**

Chapter 6, Problem 56.

Find L_{eq} in the circuit in Fig. 6.78.

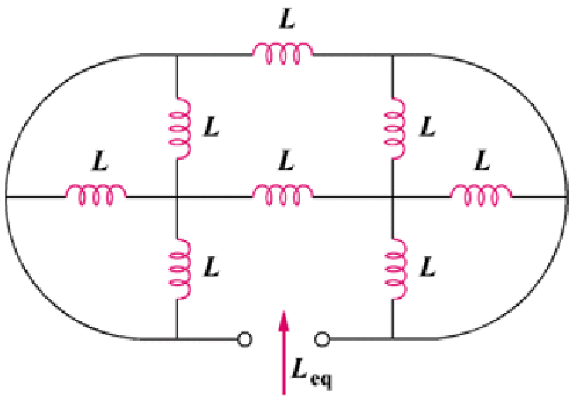
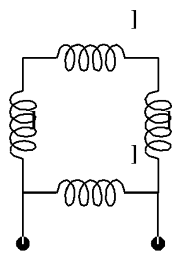


Figure 6.78

Chapter 6, Solution 56.

$$L \| L \| L = \frac{1}{\frac{3}{L}} = \frac{L}{3}$$

Hence the given circuit is equivalent to that shown below:



$$L_{eq} = L \left\| \left(L + \frac{2}{3}L \right) \right. = \frac{Lx\frac{5}{3}L}{L + \frac{5}{3}L} = \underline{\frac{5}{8}L}$$

Chapter 6, Problem 57.

Determine the L_{eq} that can be used to represent the inductive network of Fig. 6.79 at the terminals.

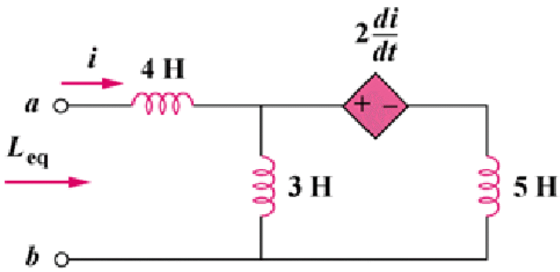


Figure 6.79

Chapter 6, Solution 57.

$$\text{Let } v = L_{eq}\frac{di}{dt} \qquad (1)$$

$$v = v_1 + v_2 = 4\frac{di}{dt} + v_2 \qquad (2)$$

$$i = i_1 + i_2 \longrightarrow i_2 = i - i_1 \qquad (3)$$

$$v_2 = 3\frac{di_1}{dt} \text{ or } \frac{di_1}{dt} = \frac{v_2}{3} \qquad (4)$$

and

$$-v_2 + 2\frac{di}{dt} + 5\frac{di_2}{dt} = 0$$

$$v_2 = 2\frac{di}{dt} + 5\frac{di_2}{dt} \qquad (5)$$

Incorporating (3) and (4) into (5),

$$v_2 = 2\frac{di}{dt} + 5\frac{di}{dt} - 5\frac{di_1}{dt} = 7\frac{di}{dt} - 5\frac{v_2}{3}$$

$$v_2\left(1 + \frac{5}{3}\right) = 7\frac{di}{dt}$$

$$v_2 = \frac{21}{8}\frac{di}{dt}$$

Substituting this into (2) gives

$$v = 4\frac{di}{dt} + \frac{21}{8}\frac{di}{dt}$$

$$= \frac{53}{8}\frac{di}{dt}$$

Comparing this with (1),

$$L_{eq} = \frac{53}{8} = \underline{\mathbf{6.625\ H}}$$

Chapter 6, Problem 58.

The current waveform in Fig. 6.80 flows through a 3-H inductor. Sketch the voltage across the inductor over the interval $0 < t < 6$ s.

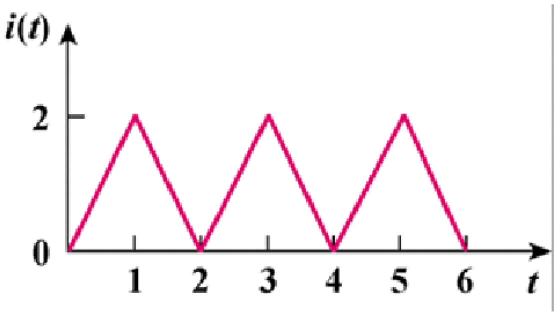
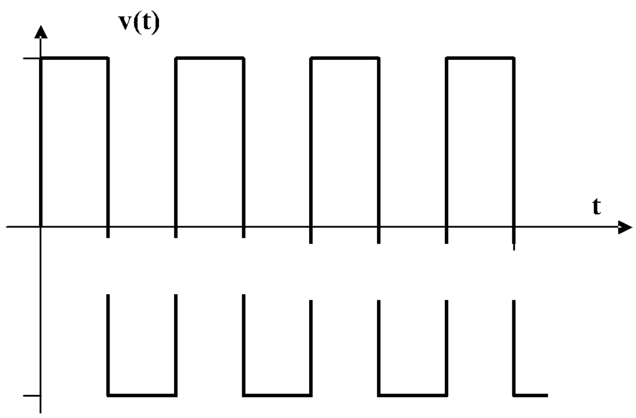


Figure 6.80

Chapter 6, Solution 58.

$$v = L\frac{di}{dt} = 3\frac{di}{dt} = 3 \text{ x slope of i(t).}$$

Thus v is sketched below:



Chapter 6, Problem 59.

(a) For two inductors in series as in Fig. 6.81(a), show that the current-division principle is

$$v_1 = \frac{L_1}{L_1 + L_2} v_s, \qquad v_2 = \frac{L_2}{L_1 + L_2} v_s$$

assuming that the initial conditions are zero.

(b) For two inductors in parallel as in Fig. 6.81(b), show that the current-division principle is

$$i_1 = \frac{L_2}{L_1 + L_2} i_s, \qquad \mathrm{i}_2 = \frac{L_1}{L_1 + L_2} i_s$$

assuming that the initial conditions are zero.

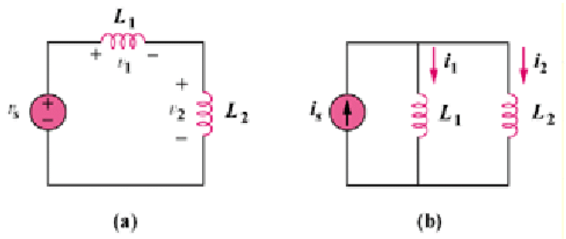


Figure 6.81

Chapter 6, Solution 59.

(a) $v_s = (L_1 + L_2)\frac{di}{dt}$

$\frac{di}{dt} = \frac{v_s}{L_1 + L_2}$

$v_1 = L_1 \frac{di}{dt}, \; v_2 = L_2 \frac{di}{dt}$

$\underline{v_1 = \frac{L_1}{L_1 + L_2} v_s, \; v_L = \frac{L_2}{L_1 + L_2} v_s}$

(b) $v_i = v_2 = L_1 \frac{di_1}{dt} = L_2 \frac{di_2}{dt}$

$i_s = i_1 + i_2$

$\frac{di_s}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} = \frac{v}{L_1} + \frac{v}{L_2} = v\frac{(L_1 + L_2)}{L_1 L_2}$

$i_1 = \frac{1}{L_1}\int v dt = \frac{1}{L_1}\int \frac{L_1 L_2}{L_1 + L_2}\frac{di_s}{dt} dt = \underline{\frac{L_2}{L_1 + L_2} i_s}$

$i_2 = \frac{1}{L_2}\int v dt = \frac{1}{L_2}\int \frac{L_1 L_2}{L_1 + L_2}\frac{di_s}{dt} dt = \underline{\frac{L_1}{L_1 + L_2} i_s}$

Chapter 6, Problem 60.

In the circuit of Fig. 6.82, $i_o(0) = 2$ A. Determine $i_o(t)$ and $v_o(t)$ for $t > 0$.

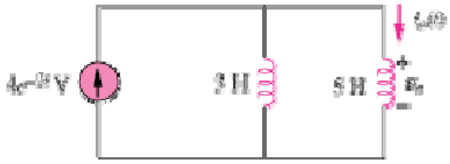


Figure 6.82

Chapter 6, Solution 60

$$L_{eq} = 3//5 = \frac{15}{8}$$

$$v_o = L_{eq}\frac{di}{dt} = \frac{15}{8}\frac{d}{dt}\left(4e^{-2t}\right) = \underline{-15e^{-2t}}$$

$$i_o = \frac{I}{L}\int_0^t v_o(t)dt + i_o(0) = 2 + \frac{1}{5}\int_0^t (-15)e^{-2t}dt = 2 + 1.5e^{-2t}\Big|_0^t = \underline{0.5 + 1.5e^{-2t}} \text{ A}$$

Chapter 6, Problem 61.

Consider the circuit in Fig. 6.83. Find: (a) L_{eq}, $i_1(t)$ and $i_2(t)$ if $i_s = 3e^{-t}$ mA, (b) $v_o(t)$, (c) energy stored in the 20-mH inductor at t=1s.

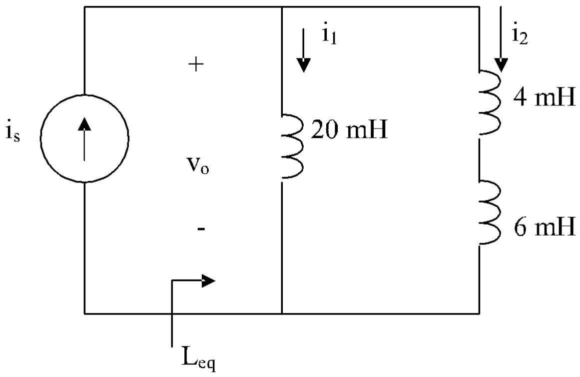


Figure 6.83 For Prob. 6.61.

Chapter 6, Solution 61.

(a) $L_{eq} = 20//(4+6) = 20x10/30 = \underline{6.667 \text{ mH}}$

Using current division,

$$i_1(t) = \frac{10}{10+20} i_s = \underline{e^{-t} \text{ mA}}$$

$$i_2(t) = \underline{2e^{-t} \text{ mA}}$$

(b) $V_o = L_{eq} \frac{di_s}{dt} = \frac{20}{3} x10^{-3}(-3e^{-t}x10^{-3}) = \underline{-20e^{-t} \ \mu\text{V}}$

(c) $w = \frac{1}{2} L i_1^2 = \frac{1}{2} x20x10^{-3} xe^{-2}x10^{-6} = \underline{1.3534 \text{ nJ}}$

Chapter 6, Problem 62.

Consider the circuit in Fig. 6.84. Given that $v(t) = 12e^{-3t}$ mV for $t > 0$ and $i_1(0) = -10$ mA, find: (a) $i_2(0)$, (b) $i_1(t)$ and $i_2(t)$.

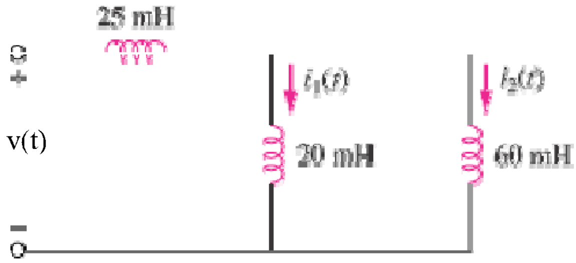


Figure 6.84

Chapter 6, Solution 62.

(a) $L_{eq} = 25 + 20 // 60 = 25 + \dfrac{20x60}{80} = 40$ mH

$$v = L_{eq}\frac{di}{dt} \longrightarrow i = \frac{1}{L_{eq}}\int v(t)dt + i(0) = \frac{10^{-3}}{40x10^{-3}}\int_0^t 12e^{-3t}dt + i(0) = -0.1(e^{-3t} - 1) + i(0)$$

Using current division and the fact that all the currents were zero when the circuit was put together, we get,

$$i_1 = \frac{60}{80}i = \frac{3}{4}i, \qquad i_2 = \frac{1}{4}i$$

$$i_1(0) = \frac{3}{4}i(0) \longrightarrow 0.75i(0) = -0.01 \longrightarrow i(0) = -0.01333$$

$$i_2 = \frac{1}{4}(-0.1e^{-3t} + 0.08667) \text{ A} = -25e^{-3t} + 21.67 \text{ mA}$$

$$i_2(0) = -25 + 21.67 = \underline{-3.33 \text{ mA}}$$

(b) $i_1 = \dfrac{3}{4}(-0.1e^{-3t} + 0.08667) \text{ A} = \underline{-75e^{-3t} + 65 \text{ mA}}$

$i_2 = \underline{-25e^{-3t} + 21.67 \text{ mA}}$

Chapter 6, Problem 63.

In the circuit in Fig. 6.85, sketch v_o.

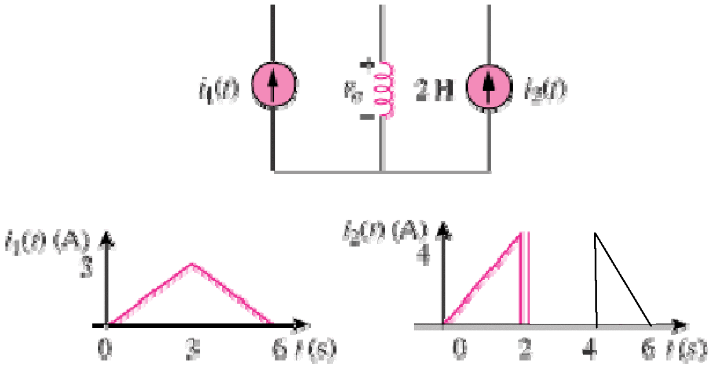


Figure 6.85

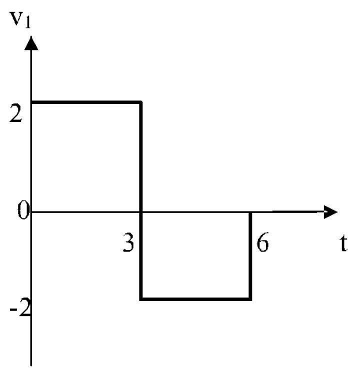
Chapter 6, Solution 63.

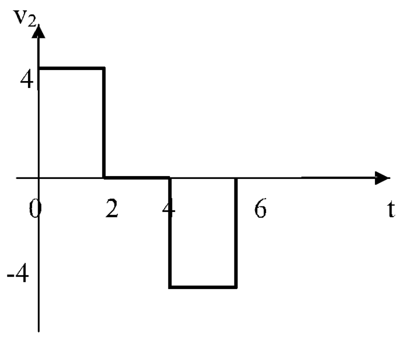
We apply superposition principle and let
$v_o = v_1 + v_2$

where v_1 and v_2 are due to i_1 and i_2 respectively.

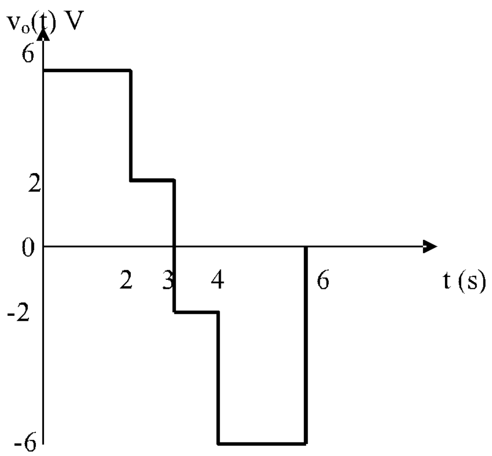
$$v_1 = L\frac{di_1}{dt} = 2\frac{di_1}{dt} = \begin{cases} 2, & 0 < t < 3 \\ -2, & 3 < t < 6 \end{cases}$$

$$v_2 = L\frac{di_2}{dt} = 2\frac{di_2}{dt} = \begin{cases} 4, & 0 < t < 2 \\ 0, & 2 < t < 4 \\ -4, & 4 < t < 6 \end{cases}$$





Adding v_1 and v_2 gives v_o, which is shown below.



Chapter 6, Problem 64.

The switch in Fig. 6.86 has been in position *A* for a long time. At $t = 0$, the switch moves from position *A* to *B*. The switch is a make-before-break type so that there is no interruption in the inductor current. Find:
(a) $i(t)$ for $t > 0$,
(b) *v just after* the switch has been moved to position *B*,
(c) $v(t)$ long after the switch is in position *B*.

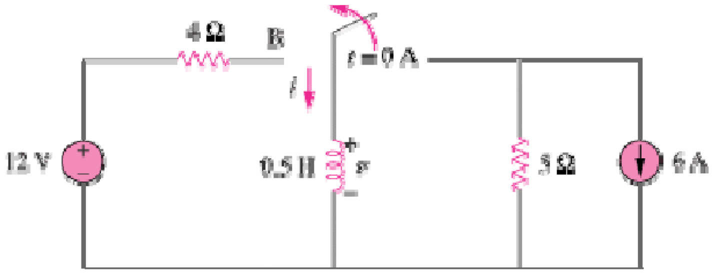


Figure 6.86

Chapter 6, Solution 64.

(a) When the switch is in position A,
i= –6 = i(0)
When the switch is in position B,
$i(\infty) = 12/4 = 3, \qquad \tau = L/R = 1/8$

$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/t} = \underline{3 - 9e^{-8t}\ \text{A}}$

(b) -12 + 4i(0) + v=0, i.e. v=12 – 4i(0) = 36 V

(c) At steady state, the inductor becomes a short circuit so that
v= 0 V

Chapter 6, Problem 65.

The inductors in Fig. 6.87 are initially charged and are connected to the black box at $t = 0$. If $i_1(0) = 4$ A, $i_2(0) = -2$ A, and $v(t) = 50e^{-200t}$ mV, $t \geq 0$\$, find:

(a). the energy initially stored in each inductor,
(b). the total energy delivered to the black box from $t = 0$ to $t = \infty$,
(c). $i_1(t)$ and $i_2(t)$, $t \geq 0$,
(d). $i(t)$, $t \geq 0$.

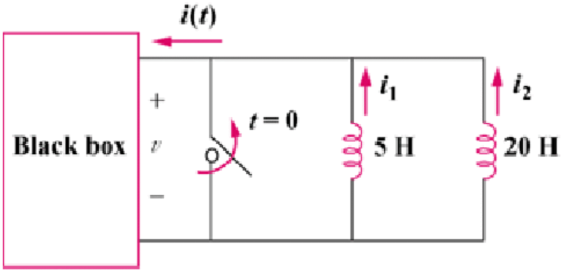


Figure 6.87

Chapter 6, Solution 65.

(a) $w_5 = \frac{1}{2}L_1 i_1^2 = \frac{1}{2}x5x(4)^2 = \underline{\textbf{40 J}}$

$w_{20} = \frac{1}{2}(20)(-2)^2 = \underline{\textbf{40 J}}$

(b) $w = w_5 + w_{20} = \underline{\textbf{80 J}}$

(c) $i_1 = \frac{1}{L_1}\int_0^t -50e^{-200t}dt + i_1(0) = \frac{1}{5}\left(\frac{1}{200}\right)\left(50e^{-200t}x10^{-3}\right)\Big|_0^t + 4$

$= \underline{\textbf{5x10}^{-5}\textbf{(e}^{-200t}\textbf{– 1) + 4 A}}$

$i_2 = \frac{1}{L_2}\int_0^t -50e^{-200t}dt + i_2(0) = \frac{1}{20}\left(\frac{1}{200}\right)\left(50e^{-200t}x10^{-3}\right)\Big|_0^t - 2$

$= \underline{\textbf{1.25x10}^{-5}\textbf{ (e}^{-200t}\textbf{ – 1) – 2 A}}$

(d) $i = i_1 + i_2 = \underline{\textbf{6.25x10}^{-5}\textbf{ (e}^{-200t}\textbf{– 1) + 2 A}}$

Chapter 6, Problem 66.

The current i(t) through a 20-mH inductor is equal, in magnitude, to the voltage across it for all values of time. If i(0) =2 A, find i(t).

Chapter 6, Solution 66.

If v=i, then

$$i = L\frac{di}{dt} \longrightarrow \frac{dt}{L} = \frac{di}{i}$$

Integrating this gives

$$\frac{t}{L} = \ln(i) - \ln(C_o) = \ln\left(\frac{i}{C_o}\right) \rightarrow i = C_o e^{t/L}$$

$i(0) = 2 = C_o$

$$i(t) = 2e^{t/0.02} = \underline{\mathbf{2e^{50t}\ A}}.$$

Chapter 6, Problem 67.

An op amp integrator has ***R***= 50 kΩ and ***C*** = 0.04 μF. If the input voltage is v_i = 10 sin 50***t*** mV, obtain the output voltage.

Chapter 6, Solution 67.

$$v_o = -\frac{1}{RC}\int vi\,dt, \text{ RC} = 50 \times 10^3 \times 0.04 \times 10^{-6} = 2 \times 10^{-3}$$

$$v_o = \frac{-10^3}{2}\int 10\sin 50t\,dt$$

v_o = **<u>100 cos 50t mV</u>**

Chapter 6, Problem 68.

A 10-V dc voltage is applied to an integrator with ***R*** = 50 kΩ, ***C*** = 100 μF at ***t*** = 0. How long will it take for the op amp to saturate if the saturation voltages are +12 V and -12 V? Assume that the initial capacitor voltage was zero.

Chapter 6, Solution 68.

$$v_o = -\frac{1}{RC}\int vi\,dt + v(0), \text{ RC} = 50 \times 10^3 \times 100 \times 10^{-6} = 5$$

$$v_o = -\frac{1}{5}\int_o^t 10dt + 0 = -2t$$

The op amp will saturate at $v_o = \pm 12$

$-12 = -2t \longrightarrow$ **<u>t = 6s</u>**

Chapter 6, Problem 69.

An op amp integrator with $R = 4\ \text{M}\Omega$ and $C = 1\ \mu\text{F}$ has the input waveform shown in Fig. 6.88. Plot the output waveform.

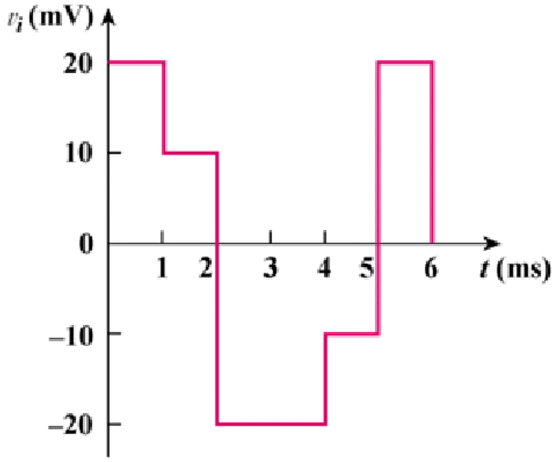


Figure 6.88

Chapter 6, Solution 69.

$RC = 4 \times 10^6 \times 1 \times 10^{-6} = 4$

$$v_o = -\frac{1}{RC}\int v_i dt = -\frac{1}{4}\int v_i dt$$

For $0 < t < 1$, $v_i = 20$, $v_o = -\frac{1}{4}\int_0^t 20dt = -5t$ mV

For $1 < t < 2$, $v_i = 10$, $v_o = -\frac{1}{4}\int_1^t 10dt + v(1) = -2.5(t-1) - 5$

$= -2.5t - 2.5$mV

For $2 < t < 4$, $v_i = -20$, $v_o = +\frac{1}{4}\int_2^t 20dt + v(2) = 5(t-2) - 7.5$

$= 5t - 17.5$ mV

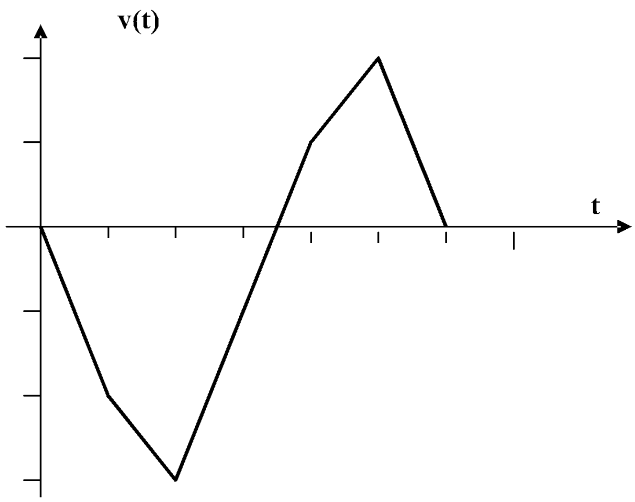
For $4 < t < 5$m, $v_i = -10$, $v_o = \frac{1}{4}\int_4^t 10dt + v(4) = 2.5(t-4) + 2.5$

$= 2.5t - 7.5$ mV

For $5 < t < 6$, $v_i = 20$, $v_o = -\frac{1}{4}\int_5^t 20dt + v(5) = -5(t-5) + 5$

$= -5t + 30$ mV

Thus $v_o(t)$ is as shown below:



Chapter 6, Problem 70.

Using a single op amp, a capacitor, and resistors of 100 kΩ or less, design a circuit to implement

$$v_0 = -50\int_0^t v_i(t)dt$$

Assume $\boldsymbol{v_o} = 0$ at $\boldsymbol{t} = 0$.

Chapter 6, Solution 70.
One possibility is as follows:

$$\frac{1}{RC} = 50$$

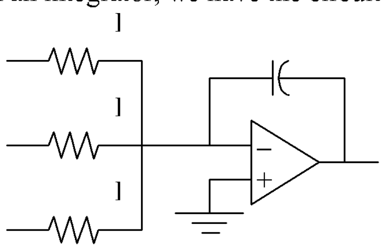
Let R = 100 kΩ, $C = \dfrac{1}{50x100x10^3} = 0.2\mu F$

Chapter 6, Problem 71.
Show how you would use a single op amp to generate

$$v_0 = -\int (v_1 + 4v_2 + 10v_3)dt$$

If the integrating capacitor is $\boldsymbol{C} = 2\ \mu F$, obtain other component values.

Chapter 6, Solution 71.
By combining a summer with an integrator, we have the circuit below:



$$v_o = -\frac{1}{R_1C}\int v_1dt - \frac{1}{R_2C}\int v_2dt - \frac{1}{R_2C}\int v_2dt$$

For the given problem, C = **2μF**,

$R_1C = 1 \longrightarrow R_1 = 1/(C) = 10^6/(2) =$ **500 kΩ**

$R_2C = 1/(4) \longrightarrow R_2 = 1/(4C) = 500k\Omega/(4) =$ **125 kΩ**

$R_3C = 1/(10) \longrightarrow R_3 = 1/(10C) =$ **50 kΩ**

Chapter 6, Problem 72.

At t = 1.5 ms, calculate v_o due to the cascaded integrators in Fig. 6.89. Assume that the integrators are reset to 0 V at t = 0.

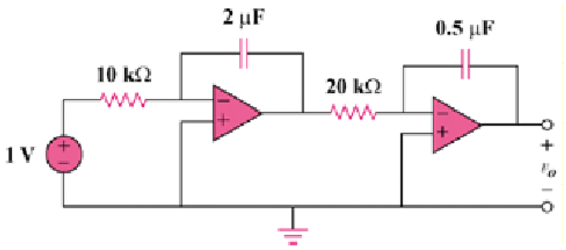


Figure 6.89

Chapter 6, Solution 72.

The output of the first op amp is

$$v_1 = -\frac{1}{RC}\int v_i\,dt = -\frac{1}{10x10^3 x2x10^{-6}}\int_0^t v_i dt = -\frac{100t}{2}$$

$$= -50t$$

$$v_o = -\frac{1}{RC}\int v_i\,dt = -\frac{1}{20x10^3 x0.5x10^{-6}}\int_0^t (-50t)dt$$

$$= 2500t^2$$

At t = 1.5ms,

$$v_o = 2500(1.5)^2 x10^{-6} = \underline{\mathbf{5.625\ mV}}$$

Chapter 6, Problem 73.

Show that the circuit in Fig. 6.90 is a noninverting integrator.

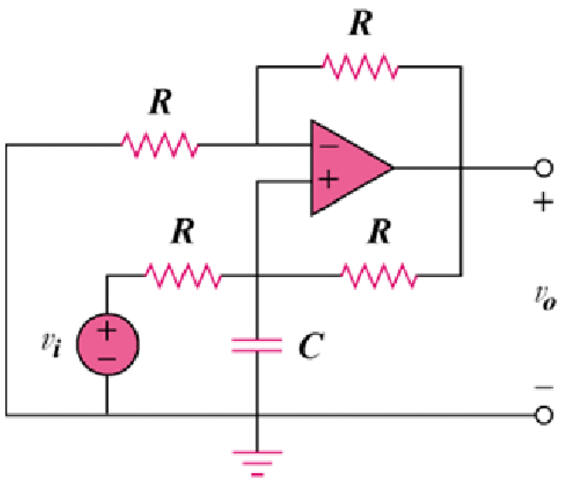


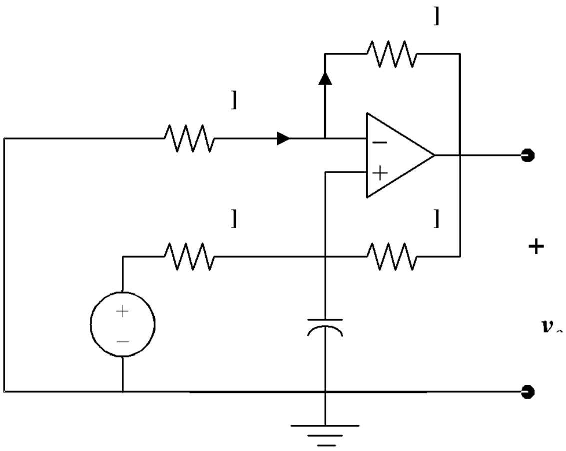
Figure 6.90

Chapter 6, Solution 73.

Consider the op amp as shown below:

Let $v_a = v_b = v$

At node a, $\frac{0-v}{R} = \frac{v-v_o}{R} \longrightarrow 2v - v_o = 0$ (1)



At node b, $\frac{v_i - v}{R} = \frac{v - v_o}{R} + C\frac{dv}{dt}$

$$v_i = 2v - v_o + RC\frac{dv}{dt} \quad (2)$$

Combining (1) and (2),

$$v_i = v_o - v_o + \frac{RC}{2}\frac{dv_o}{dt}$$

or

$$v_o = \frac{2}{RC}\int v_i \, dt$$

showing that the circuit is a noninverting integrator.

Chapter 6, Problem 74.

The triangular waveform in Fig. 6.91(a) is applied to the input of the op amp differentiator in Fig. 6.91(b). Plot the output.

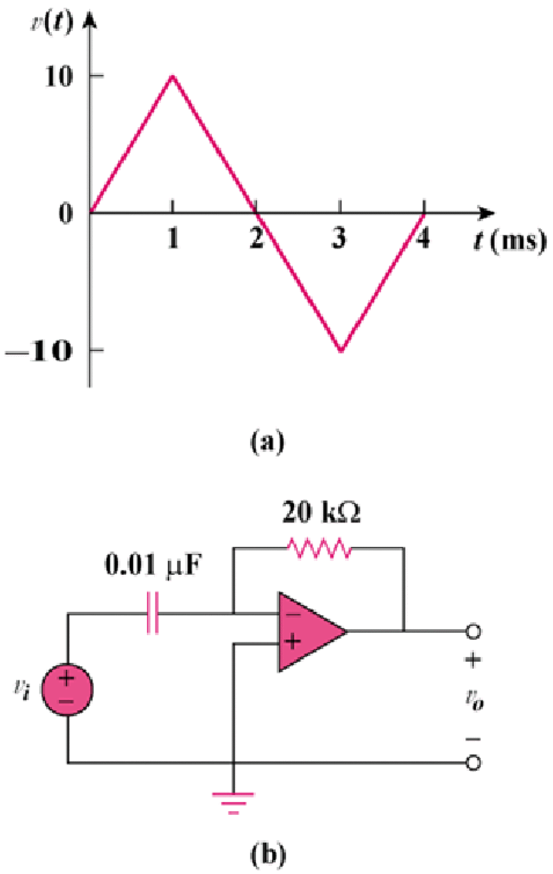


Figure 6.91

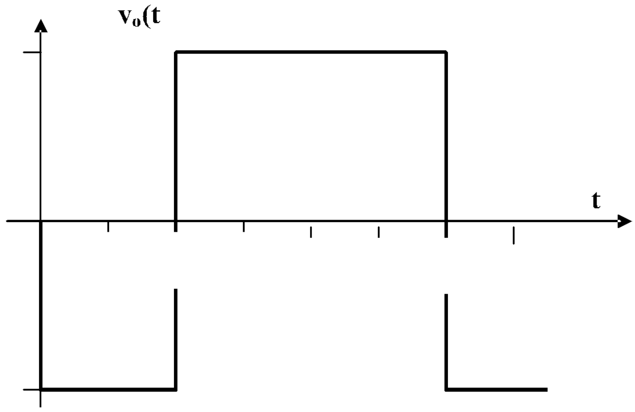
Chapter 6, Solution 74.

RC = 0.01 x 20 x 10^{-3} sec

$$v_o = -RC\frac{dv_i}{dt} = -0.2\frac{dv}{dt}\,\text{msec}$$

$$v_o = \begin{cases} -2V, & 0 < t < 1 \\ 2V, & 1 < t < 3 \\ -2V, & 3 < t < 4 \end{cases}$$

Thus $v_o(t)$ is as sketched below:



Chapter 6, Problem 75.

An op amp differentiator has $\boldsymbol{R}$= 250 kΩ and $\boldsymbol{C}$ = 10 μF.
The input voltage is a ramp $\boldsymbol{r(t)}$ = 12 $\boldsymbol{t}$ mV. Find the output voltage.

Chapter 6, Solution 75.

$$v_0 = -RC\frac{dv_i}{dt}, \quad RC = 250x10^3x10x10^{-6} = 2.5$$

$$v_o = -2.5\frac{d}{dt}(12t) = \underline{\boldsymbol{\textit{-30 mV}}}$$

Chapter 6, Problem 76.

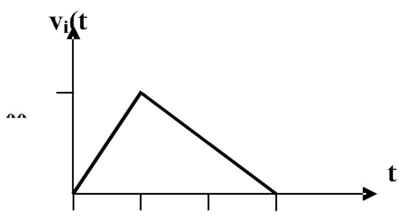
A voltage waveform has the following characteristics: a positive slope of 20 V/s for 5 ms followed by a negative slope of 10 V/s for 10 ms. If the waveform is applied to a differentiator with $\boldsymbol{R}$ = 50 kΩ, $\boldsymbol{C}$ = 10 μF, sketch the output voltage waveform.

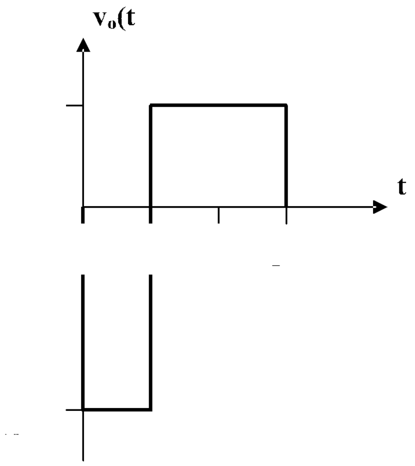
Chapter 6, Solution 76.

$$v_o = -RC\frac{dv_i}{dt}, \quad RC = 50 \text{ x } 10^3 \text{ x } 10 \text{ x } 10^{-6} = 0.5$$

$$v_o = -0.5\frac{dv_i}{dt} = \begin{bmatrix} -10, & 0 < t < 5 \\ 5, & 5 < t < 15 \end{bmatrix}$$

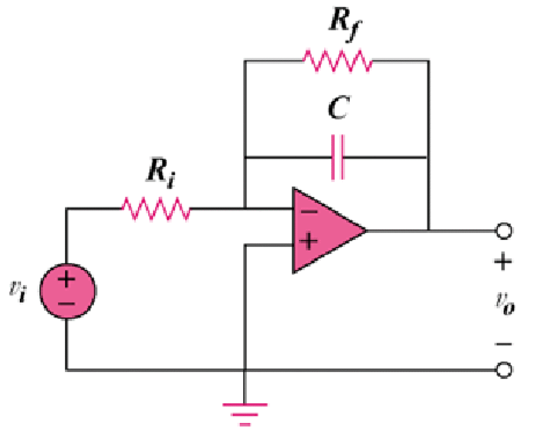
The input is sketched in Fig. (a), while the output is sketched in Fig. (b).



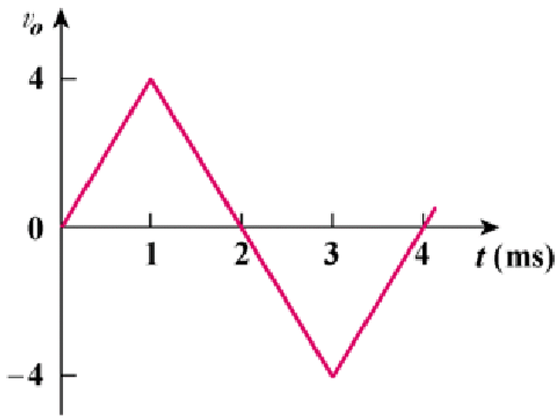


Chapter 6, Problem 77.

The output v_o of the op amp circuit of Fig. 6.92(a) is shown in Fig. 6.92(b). Let $Ri = R_f = 1$ MΩ and $C = 1$ μF. Determine the input voltage waveform and sketch it.



(a)



(b)

Figure 6.92

Chapter 6, Solution 77.

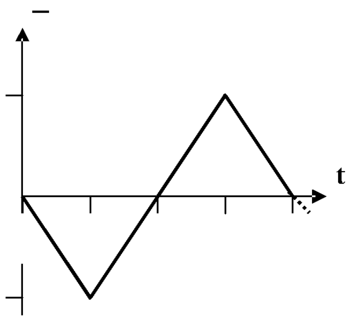
$i = i_R + i_C$

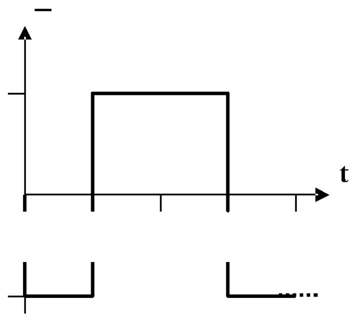
$$\frac{v_i - 0}{R} = \frac{0 - v_0}{R_F} + C\frac{d}{dt}(0 - v_o)$$

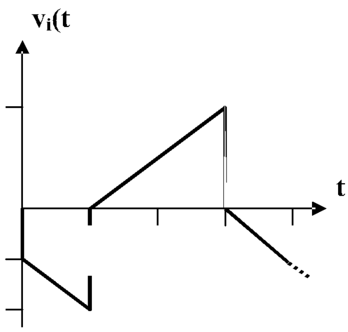
$$R_F C = 10^6 x10^{-6} = 1$$

Hence $v_i = -\left(v_o + \frac{dv_o}{dt}\right)$

Thus v_i is obtained from v_o as shown below:







Chapter 6, Problem 78.

Design an analog computer to simulate

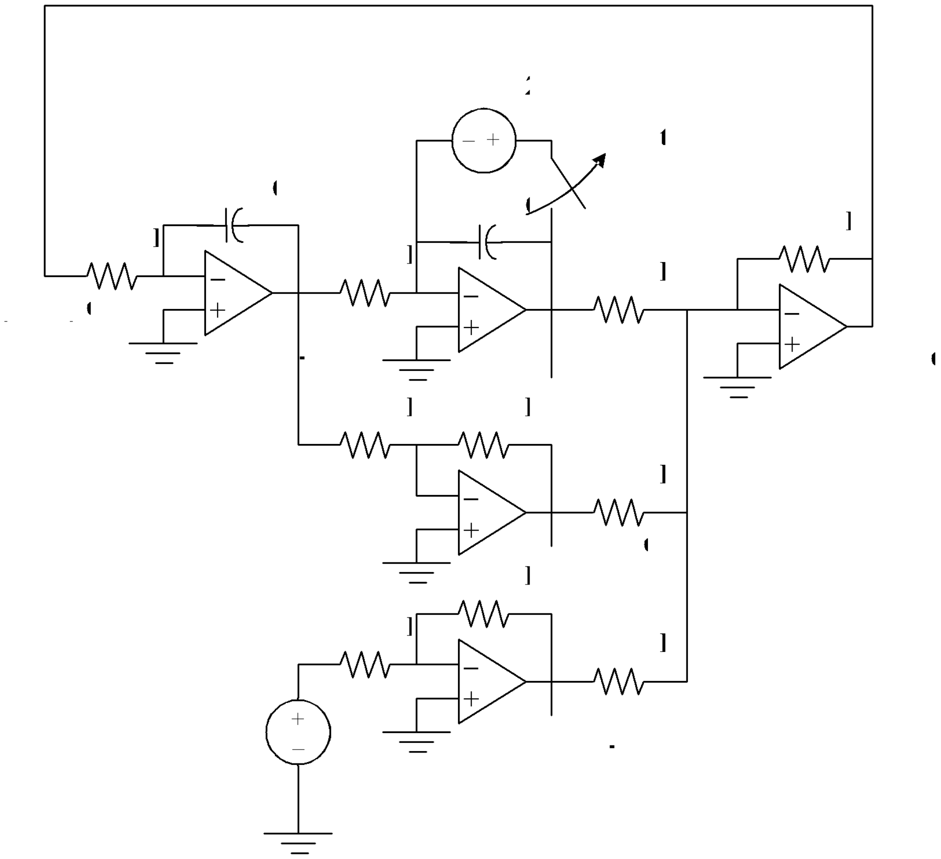
$$\frac{d^2v_0}{dt^2} + 2\frac{dv_0}{dt} + v_0 = 10\sin 2t$$

where $v_0(0) = 2$ and $v'_0(0) = 0$.

Chapter 6, Solution 78.

$$\frac{d^2v_o}{dt} = 10\sin 2t - \frac{2dv_o}{dt} - v_o$$

Thus, by combining integrators with a summer, we obtain the appropriate analog computer as shown below:



Chapter 6, Problem 79.

Design an analog computer circuit to solve the following ordinary differential equation.

$$\frac{dy(t)}{dt} + 4y(t) = f(t)$$

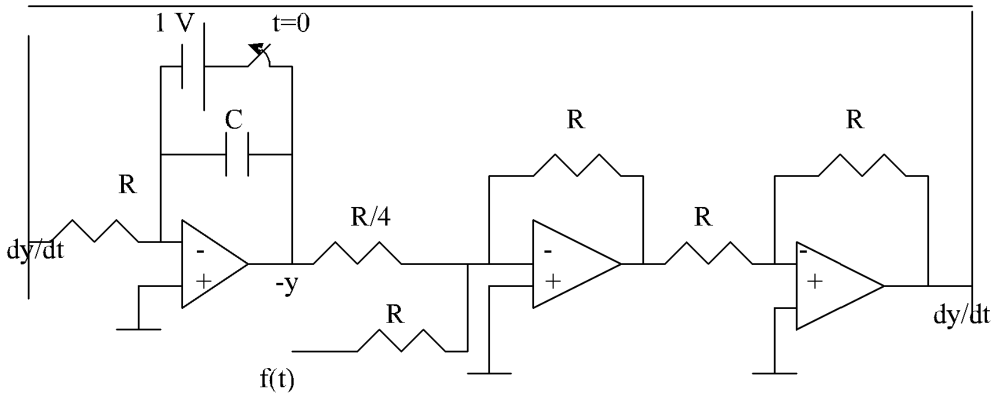
where $y(0) = 1$ V.

Chapter 6, Solution 79.

We can write the equation as

$$\frac{dy}{dt} = f(t) - 4y(t)$$

which is implemented by the circuit below.



Chapter 6, Problem 80.

Figure 6.93 presents an analog computer designed to solve a differential equation. Assuming *f*(*t*) is known, set up the equation for *f*(*t*).

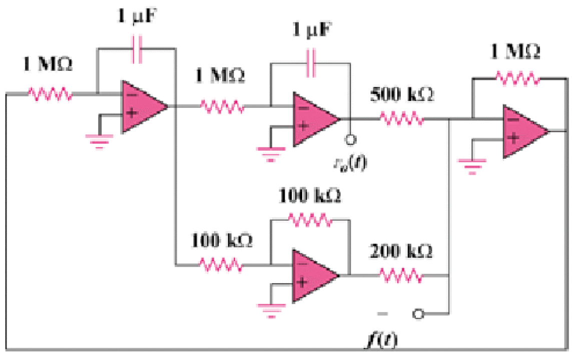


Figure 6.93

Chapter 6, Solution 80.

From the given circuit,

$$\frac{d^2v_o}{dt^2} = f(t) - \frac{1000k\Omega}{5000k\Omega}v_o - \frac{1000k\Omega}{200k\Omega}\frac{dv_o}{dt}$$

or

$$\underline{\frac{d^2v_o}{dt^2} + 5\frac{dv_o}{dt} + 2v_o = f(t)}$$

Chapter 6, Problem 81.

Design an analog computer to simulate the following equation:

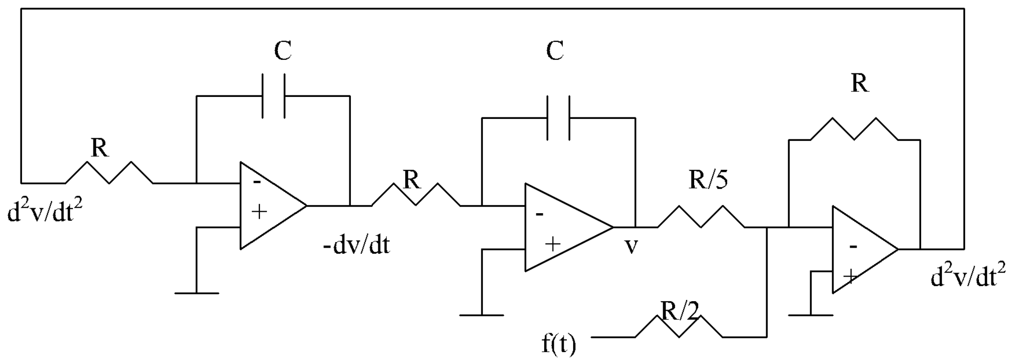
$$\frac{d^2v}{dt^2} + 5v = -2f(t)$$

Chapter 6, Solution 81

We can write the equation as

$$\frac{d^2v}{dt^2} = -5v - 2f(t)$$

which is implemented by the circuit below.



Chapter 6, Problem 82.

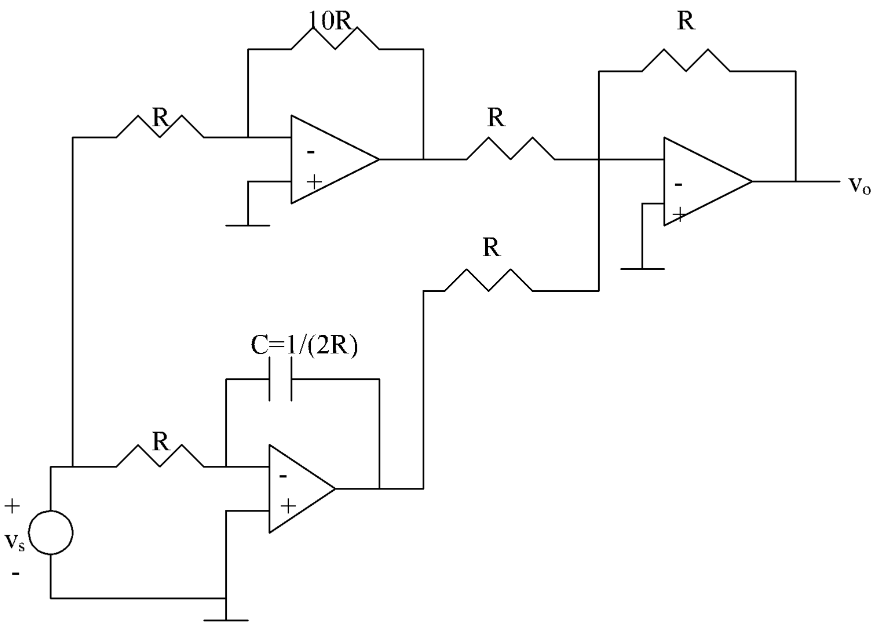
Design an an op amp circuit such that:

$$v_0 = 10v_s + 2\int v_s dt$$

where v_s and v_0 are the input voltage and output voltage respectively.

Chapter 6, Solution 82

The circuit consists of a summer, an inverter, and an integrator. Such circuit is shown below.

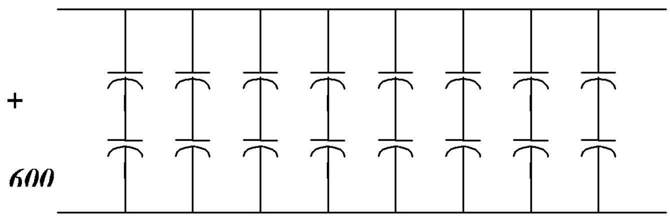


Chapter 6, Problem 83.

Your laboratory has available a large number of 10-μF capacitors rated at 300 V. To design a capacitor bank of 40-μF rated at 600 V, how many 10-μF capacitors are needed and how would you connect them?

Chapter 6, Solution 83.

Since two 10μF capacitors in series gives 5μF, rated at 600V, it requires 8 groups in parallel with each group consisting of two capacitors in series, as shown below:



Answer: **8 groups in parallel with each group made up of 2 capacitors in series.**

Chapter 6, Problem 84.

An 8-mH inductor is used in a fusion power experiment. If the current through the inductor is $i(t) = 5\sin^2 \pi t$ mA, t >0, find the power being delivered to the inductor and the energy stored in it at t=0.5s.

Chapter 6, Solution 84.

$$v = L(di/dt) = 8x10^{-3}x5x2\pi\sin(\pi t)\cos(\pi t)10^{-3} = 40\pi\sin(2\pi t)\ \mu V$$

$$p = vi = 40\pi\sin(2\pi t)5\sin^2(\pi t)10^{-9}\ W,\ \text{at } t=0\ p = \underline{\mathbf{0W}}$$

$$w = \frac{1}{2}Li^2 = \frac{1}{2}x8x10^{-3}x[5\sin^2(\pi/2)x10^{-3}]^2 = 4x25x10^{-9} = \underline{100\ nJ}$$

Chapter 6, Problem 85.

A square-wave generator produces the voltage waveform shown in Fig. 6.94(a). What kind of a circuit component is needed to convert the voltage waveform to the triangular current waveform shown in Fig. 6.94(b)? Calculate the value of the component, assuming that it is initially uncharged.

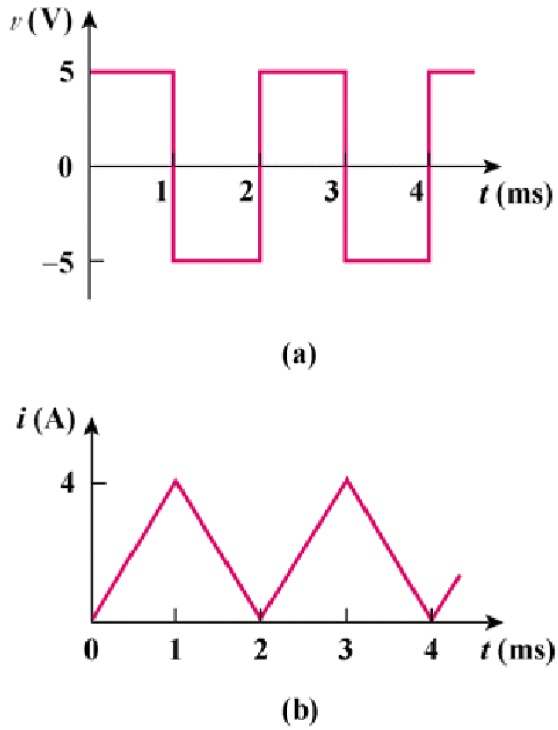


Figure 6.94

Chapter 6, Solution 85.

It is evident that differentiating i will give a waveform similar to v. Hence,

$$v = L\frac{di}{dt}$$

$$i = \begin{cases} 4t, 0 < t < 1ms \\ 8 - 4t, 1 < t < 2ms \end{cases}$$

$$v = L\frac{di}{dt} = \begin{cases} 4000L, 0 < t < 1ms \\ -4000L, 1 < t < 2ms \end{cases}$$

But,
$$v = \begin{cases} 5V, 0 < t < 1ms \\ -5V, 1 < t < 2ms \end{cases}$$

Thus, $4000L = 5 \longrightarrow L = 1.25$ mH in a **<u>1.25 mH inductor</u>**

Chapter 6, Problem 86.

An electric motor can be modeled as a series combination of a 12-Ω resistor and 200-mH inductor. If a current $i(t) = 2te^{-10t}$A flows through the series combination, find the voltage across the combination.

Chapter 6, Solution 86.

$$v = v_R + v_L = Ri + L\frac{di}{dt} = 12x2te^{-10t} + 200x10^{-3}x(-20te^{-10t} + 2e^{-10t}) = \underline{(0.4 - 20t)e^{-10t}\ \text{V}}$$

Chapter 7, Problem 1.

In the circuit shown in Fig. 7.81

$$v(t) = 56e^{-200t}\,\text{V}, \quad t > 0$$

$$i(t) = 8e^{-200t}\,\text{mA}, \quad t > 0$$

(a) Find the values of *R* and *C*.
(b) Calculate the time constant τ .
(c) Determine the time required for the voltage to decay half its initial value at *t* = 0.

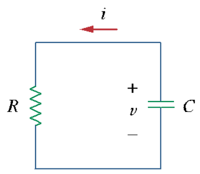


Figure 7.81
For Prob. 7.1

Chapter 7, Solution 1.

(a) τ=RC = 1/200

For the resistor, V=iR= $56e^{-200t} = 8Re^{-200t} x10^{-3} \longrightarrow R = \frac{56}{8} = \underline{7\ \text{k}\Omega}$

$$C = \frac{1}{200R} = \frac{1}{200X7X10^3} = \underline{0.7143\mu F}$$

(b) τ =1/200= **5 ms**
(c) If value of the voltage at = 0 is 56 .

$$\frac{1}{2}x56 = 56e^{-200t} \longrightarrow e^{200t} = 2$$

$$200t_o = \ln 2 \longrightarrow t_o = \frac{1}{200}\ln 2 = \underline{3.466\ \text{ms}}$$

Chapter 7, Problem 2.

Find the time constant for the *RC* circuit in Fig. 7.82.

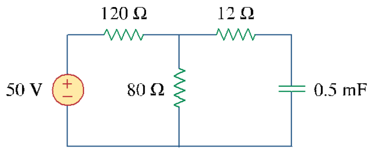


Figure 7.82
For Prob. 7.2.

Chapter 7, Solution 2.

$\tau = R_{th}C$

where R_{th} is the Thevenin equivalent at the capacitor terminals.

$$R_{th} = 120 \,||\, 80 + 12 = 60\ \Omega$$

$$\tau = 60 \times 0.5 \times 10^{-3} = \underline{\mathbf{30\ ms}}$$

Chapter 7, Problem 3.

Determine the time constant for the circuit in Fig. 7.83.

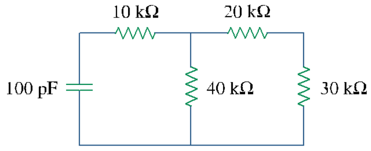


Figure 7.83
For Prob. 7.3.

Chapter 7, Solution 3.

R = 10 +20//(20+30) =10 + 40x50/(40 + 50)=32.22 kΩ

$$\tau = RC = 32.22X10^3 X100X10^{-12} = \underline{3.222\ \mu S}$$

Chapter 7, Problem 4.

The switch in Fig. 7.84 moves instantaneously from *A* to *B* at $t = 0$. Find v for $t > 0$.

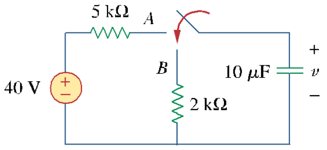


Figure 7.84
For Prob. 7.4.

Chapter 7, Solution 4.

For t<0, $v(0^-)=40$ V.

For t >0. we have a source-free RC circuit.

$$\tau = RC = 2x10^3 x10x10^{-6} = 0.02$$

$$v(t) = v(0)e^{-t/\tau} = \underline{40e^{-50t}\ \text{V}}$$

Chapter 7, Problem 5.

For the circuit shown in Fig. 7.85, find $i(t)$, $t > 0$.

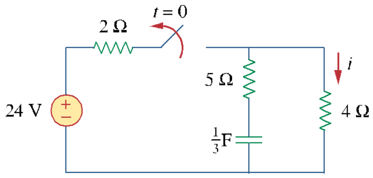


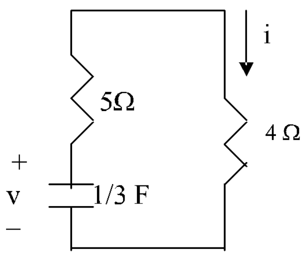
Figure 7.85
For Prob. 7.5.

Chapter 7, Solution 5.

Let v be the voltage across the capacitor.
For t <0,

$$v(0^-) = \frac{4}{2+4}(24) = 16 \text{ V}$$

For t >0, we have a source-free RC circuit as shown below.



$$\tau = RC = (4+5)\frac{1}{3} = 3s$$

$$v(t) = v(0)e^{-t/\tau} = 16e^{-t/3}$$

$$i(t) = -C\frac{dv}{dt} = -\frac{1}{3}(-\frac{1}{3})16e^{-t/3} = \underline{1.778e^{-t/3} \text{ A}}$$

Chapter 7, Problem 6.

The switch in Fig. 7.86 has been closed for a long time, and it opens at $t = 0$. Find $v(t)$ for $t \geq 0$.

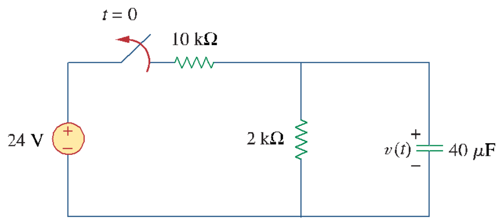


Figure 7.86
For Prob. 7.6.

Chapter 7, Solution 6.

$$v_o = v(0) = \frac{2}{10+2}(24) = 4\,V$$

$$v(t) = v_o e^{-t/\tau}, \quad \tau = RC = 40x10^{-6}x2x10^3 = \frac{2}{25}$$

$$v(t) = \underline{4e^{-12.5t}V}$$

Chapter 7, Problem 7.

Assuming that the switch in Fig. 7.87 has been in position *A* for a long time and is moved to position *B* at $t = 0$, find $v_0(t)$ for $t \geq 0$.

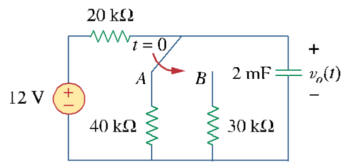


Figure 7.87
For Prob. 7.7.

Chapter 7, Solution 7.

When the switch is at position A, the circuit reaches steady state. By voltage division,

$$v_o(0) = \frac{40}{40+20}(12V) = 8V$$

When the switch is at position B, the circuit reaches steady state. By voltage division,

$$v_o(\infty) = \frac{30}{30+20}(12V) = 7.2V$$

$$R_{Th} = 20k // 30k = \frac{20x30}{50} = 12k\Omega$$

$$\tau = R_{Th}C = 12x10^3 x2x10^{-3} = 24s$$

$$v_o(t) = v_o(\infty) + [v_o(0) - v_o(\infty)]e^{-t/\tau} = 7.2 + (8-7.2)e^{-t/24} = \underline{7.2 + 0.8e^{-t/24} \quad \text{V}}$$

Chapter 7, Problem 8.

For the circuit in Fig. 7.88, if

$v = 10e^{-4t}$ V and $i = 0.2e-4t$ A, $t > 0$

(a) Find R and C.
(b) Determine the time constant.
(c) Calculate the initial energy in the capacitor.
(d) -Obtain the time it takes to dissipate 50 percent of the initial energy.

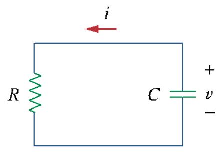


Figure 7.88
For Prob. 7.8.

Chapter 7, Solution 8.

(a) $\tau = RC = \frac{1}{4}$

$-i = C\frac{dv}{dt}$

$-0.2e^{-4t} = C(10)(-4)e^{-4t} \longrightarrow C = \underline{\textbf{5 mF}}$

$R = \frac{1}{4C} = \underline{\textbf{50 }\Omega}$

(b) $\tau = RC = \frac{1}{4} = \underline{\textbf{0.25 s}}$

(c) $w_C(0) = \frac{1}{2}CV_0^2 = \frac{1}{2}(5\times10^{-3})(100) = \underline{\textbf{250 mJ}}$

(d) $w_R = \frac{1}{2}\times\frac{1}{2}CV_0^2 = \frac{1}{2}CV_0^2\left(1-e^{-2t_0/\tau}\right)$

$0.5 = 1-e^{-8t_0} \longrightarrow e^{-8t_0} = \frac{1}{2}$

or $e^{8t_0} = 2$

$t_0 = \frac{1}{8}\ln(2) = \underline{\textbf{86.6 ms}}$

Chapter 7, Problem 9.

The switch in Fig. 7.89 opens at $t = 0$. Find v_0 for $t > 0$

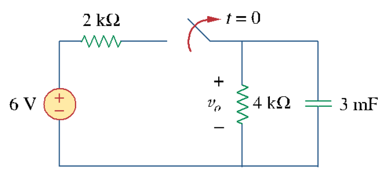


Figure 7.89
For Prob. 7.9.

Chapter 7, Solution 9.

For t < 0, the switch is closed so that

$$v_o(0) = \frac{4}{2+4}(6) = 4 \text{ V}$$

For t >0, we have a source-free RC circuit.

$$\tau = RC = 3x10^{-3}x4x10^{3} = 12s$$

$$v_o(t) = v_o(0)e^{-t/\tau} = \underline{4e^{-t/12} \text{ V}}$$

Chapter 7, Problem 10.

For the circuit in Fig. 7.90, find $v_o(t)$ for $t > 0$. Determine the time necessary for the capacitor voltage to decay to one-third of its value at $t = 0$.

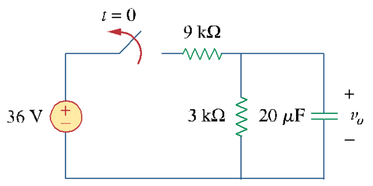


Figure 7.90
For Prob. 7.10.

Chapter 7, Solution 10.

For t<0, $$v(0^-) = \frac{3}{3+9}(36V) = \underline{9\ V}$$

For t>0, we have a source-free RC circuit

$$\tau = RC = 3x10^3 x20x10^{-6} = 0.06s$$

$$v_o(t) = \underline{\mathbf{9e^{-16.667t}\ V}}$$

Let the time be t_o.

$3 = 9e^{-16.667to}$ or $e^{16.667to} = 9/3 = 3$

$$t_o = \ln(3)/16.667 = \underline{\mathbf{65.92\ ms}}.$$

Chapter 7, Problem 11.

For the circuit in Fig. 7.91, find i_0 for $t > 0$.

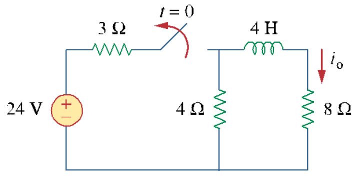
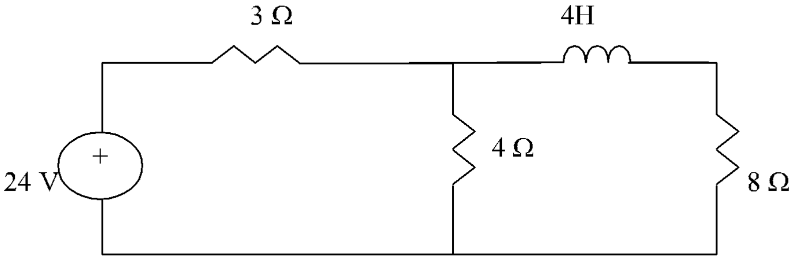
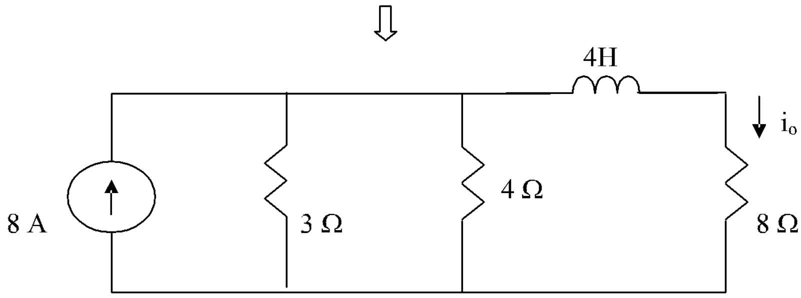


Figure 7.91
For Prob. 7.11.

Chapter 7, Solution 11.

For t<0, we have the circuit shown below.





3//4= 4x3/7=1.7143

$$i_o(0^-) = \frac{1.7143}{1.7143+8}(8) = 1.4118 \text{ A}$$

For t >0, we have a source-free RL circuit.

$$\tau = \frac{L}{R} = \frac{4}{4+8} = 1/3$$

$$i_o(t) = i_o(0)e^{-t/\tau} = \underline{1.4118e^{-3t} \text{ A}}$$

Chapter 7, Problem 12.

The switch in the circuit of Fig. 7.92 has been closed for a long time. At $t = 0$ the switch is opened. Calculate $i(t)$ for $t > 0$.

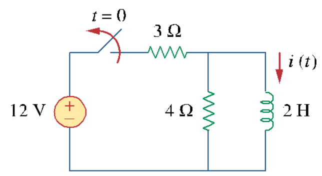
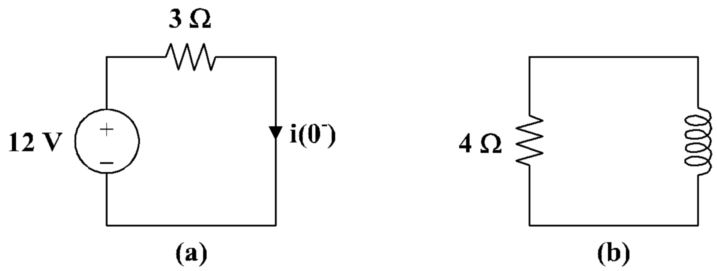


Figure 7.92
For Prob. 7.12.

Chapter 7, Solution 12.

When t < 0, the switch is closed and the inductor acts like a short circuit to dc. The 4 Ω resistor is short-circuited so that the resulting circuit is as shown in Fig. (a).



$$i(0^-) = \frac{12}{3} = 4 \text{ A}$$

Since the current through an inductor cannot change abruptly,

$$i(0) = i(0^-) = i(0^+) = 4 \text{ A}$$

When t > 0, the voltage source is cut off and we have the RL circuit in Fig. (b).

$$\tau = \frac{L}{R} = \frac{2}{4} = 0.5$$

Hence,

$$i(t) = i(0)\, e^{-t/\tau} = \underline{\mathbf{4\, e^{-2t}\ A}}$$

Chapter 7, Problem 13.

In the circuit of Fig. 7.93,

$v(t) = 20e^{-10^3 t}$ V, $\quad t > 0$

$i(t) = 4e^{-10^3 t}$ mA, $\quad t > 0$

(a) Find *R*, *L*, and τ .
(b) Calculate the energy dissipated in the resistance for $0 < t < 0.5$ ms.

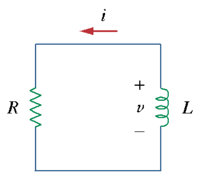


Figure 7.93
For Prob. 7.13.

Chapter 7, Solution 13.

(a) $\tau = \dfrac{1}{10^3} = \underline{1ms}$

$v = iR \longrightarrow 20e^{-1000t} = Rx4e^{-1000t}x10^{-3}$

From this, R = 20/4 kΩ= 5 kΩ

But $\tau = \dfrac{L}{R} = \dfrac{1}{10^3} \qquad \longrightarrow \qquad L = \dfrac{5x1000}{1000} = \underline{5H}$

(b) The energy dissipated in the resistor is

$$w = \int_0^t p\,dt = \int_0^t 80x10^{-3}e^{-2x10^3}dt = -\frac{80x10^{-3}}{2x10^3}e^{-2x10^3 t}\Bigg|_0^{0.5x10^{-3}}$$

$$= 40(1 - e^{-1})\mu J = \underline{25.28\mu J}$$

Chapter 7, Problem 14.

Calculate the time constant of the circuit in Fig. 7.94.

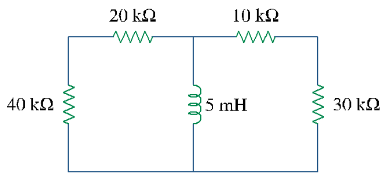


Figure 7.94
For Prob. 7.14.

Chapter 7, Solution 14.

$$R_{Th} = (40+20)//(10+30) = \frac{60x40}{100} = 24\,k\Omega$$

$$\tau = L/R = \frac{5x10^{-3}}{24x10^{3}} = \underline{0.2083\,\mu s}$$

Chapter 7, Problem 15.

Find the time constant for each of the circuits in Fig. 7.95.

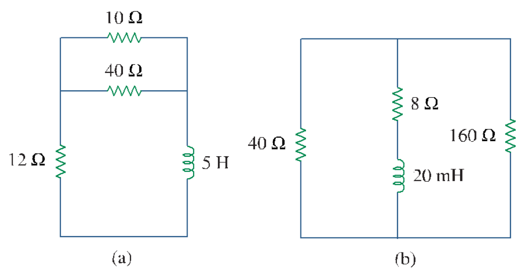


Figure 7.95
For Prob. 7.15.

Chapter 7, Solution 15

(a) $R_{Th} = 12 + 10//40 = 20\Omega$, $\tau = \frac{L}{R_{Th}} = 5/20 = \underline{0.25s}$

(b) $R_{Th} = 40//160 + 8 = 40\Omega$, $\tau = \frac{L}{R_{Th}} = (20x10^{-3})/40 = \underline{0.5\text{ ms}}$

Chapter 7, Problem 16.

Determine the time constant for each of the circuits in Fig. 7.96.

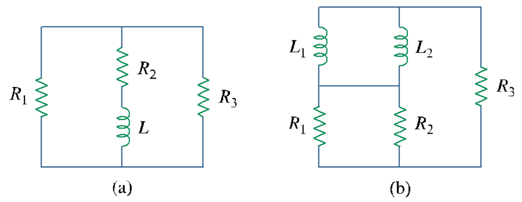


Figure 7.96
For Prob. 7.16.

Chapter 7, Solution 16.

$$\tau = \frac{L_{eq}}{R_{eq}}$$

(a) $L_{eq} = L$ and $R_{eq} = R_2 + \dfrac{R_1R_3}{R_1 + R_3} = \dfrac{R_2(R_1 + R_3) + R_1R_3}{R_1 + R_3}$

$$\tau = \underline{\mathbf{\frac{L(R_1 + R_3)}{R_2(R_1 + R_3) + R_1R_3}}}$$

(b) where $L_{eq} = \dfrac{L_1L_2}{L_1 + L_2}$ and $R_{eq} = R_3 + \dfrac{R_1R_2}{R_1 + R_2} = \dfrac{R_3(R_1 + R_2) + R_1R_2}{R_1 + R_2}$

$$\tau = \underline{\mathbf{\frac{L_1L_2(R_1 + R_2)}{(L_1 + L_2)(R_3(R_1 + R_2) + R_1R_2)}}}$$

Chapter 7, Problem 17.

Consider the circuit of Fig. 7.97. Find $v_0(t)$ if $i(0) = 2$ A and $v(t) = 0$.

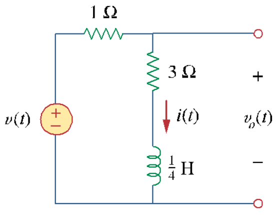


Figure 7.97
For Prob. 7.17.

Chapter 7, Solution 17.

$$i(t) = i(0)\,e^{-t/\tau}, \qquad \tau = \frac{L}{R_{eq}} = \frac{1/4}{4} = \frac{1}{16}$$

$$i(t) = 2\,e^{-16t}$$

$$v_o(t) = 3i + L\frac{di}{dt} = 6\,e^{-16t} + (1/4)(-16)\,2\,e^{-16t}$$

$$v_o(t) = \underline{-2\,e^{-16t}\;u(t)\text{V}}$$

Chapter 7, Problem 18.

For the circuit in Fig. 7.98, determine $v_0(t)$ when $i(0) = 1$ A and $v(t) = 0$.

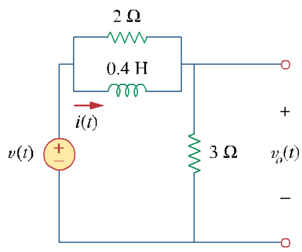
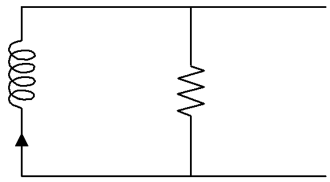


Figure 7.98
For Prob. 7.18.

Chapter 7, Solution 18.

If $v(t) = 0$, the circuit can be redrawn as shown below.



$$R_{eq} = 2 \,\|\, 3 = \frac{6}{5}, \qquad \tau = \frac{L}{R} = \frac{2}{5} \times \frac{5}{6} = \frac{1}{3}$$

$$i(t) = i(0)\,e^{-t/\tau} = e^{-3t}$$

$$v_o(t) = -L\frac{di}{dt} = \frac{-2}{5}(-3)\,e^{-3t} = \underline{\mathbf{1.2\,e^{-3t}\ V}}$$

Chapter 7, Problem 19.

In the circuit of Fig. 7.99, find $i(t)$ for $t > 0$ if $i(0) = 2$ A.

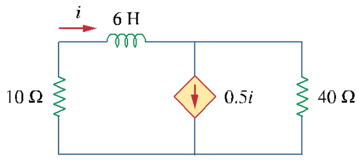
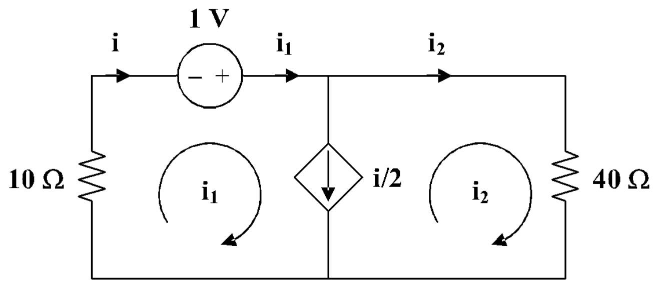


Figure 7.99
For Prob. 7.19.

Chapter 7, Solution 19.



To find R_{th} we replace the inductor by a 1-V voltage source as shown above.

$$10i_1 - 1 + 40i_2 = 0$$

But $\quad i = i_2 + i/2 \quad$ and $\quad i = i_1$

i.e. $\quad i_1 = 2i_2 = i$

$$10i - 1 + 20i = 0 \longrightarrow i = \frac{1}{30}$$

$$R_{th} = \frac{1}{i} = 30\,\Omega$$

$$\tau = \frac{L}{R_{th}} = \frac{6}{30} = 0.2 \text{ s}$$

$$i(t) = \underline{2e^{-5t}u(t) \text{ A}}$$

Chapter 7, Problem 20.

For the circuit in Fig. 7.100,

$v = 120e^{-50t}$ V

and

$i = 30e^{-50t}$ A, $t > 0$

(a) Find L and R.
(b) Determine the time constant.
(c) Calculate the initial energy in the inductor.
(d) What fraction of the initial energy is dissipated in 10 ms?

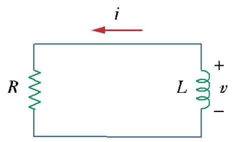


Figure 7.100
For Prob. 7.20.

Chapter 7, Solution 20.

(a) $\tau = \frac{L}{R} = \frac{1}{50} \longrightarrow R = 50L$

$-v = L\frac{di}{dt}$

$-120e^{-50t} = L(30)(-50)e^{-50t} \longrightarrow L = \underline{\textbf{80 mH}}$

$R = 50L = \underline{\textbf{4}\ \boldsymbol{\Omega}}$

(b) $\tau = \frac{L}{R} = \frac{1}{50} = \underline{\textbf{20 ms}}$

(c) $w = \frac{1}{2}Li^2(0) = \frac{1}{2}(0.08)(30)^2 = \underline{\textbf{36J}}$

The value of the energy remaining at 10 ms is given by:

$w_{10} = 0.04(30e^{-0.5})^2 = 0.04(18.196)^2 = 13.24$J.

So, the fraction of the energy dissipated in the first 10 ms is given by:

(36–13.24)/36 = **0.6322** or **63.2%**.

Chapter 7, Problem 21.

In the circuit of Fig. 7.101, find the value of R for which the steady-state energy stored in the inductor will be 1 J.

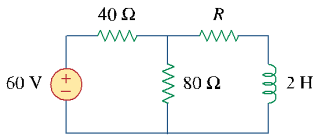
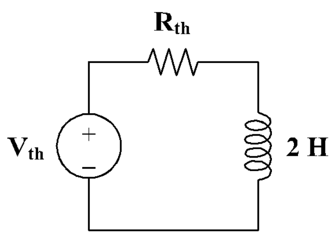


Figure 7.101
For Prob. 7.21.

Chapter 7, Solution 21.

The circuit can be replaced by its Thevenin equivalent shown below.



$$V_{th} = \frac{80}{80+40}(60) = 40 \text{ V}$$

$$R_{th} = 40 \,||\, 80 + R = \frac{80}{3} + R$$

$$I = i(0) = i(\infty) = \frac{V_{th}}{R_{th}} = \frac{40}{80/3 + R}$$

$$w = \frac{1}{2}LI^2 = \frac{1}{2}(2)\left(\frac{40}{R + 80/3}\right)^2 = 1$$

$$\frac{40}{R + 80/3} = 1 \longrightarrow R = \frac{40}{3}$$

$$R = \underline{13.333 \, \Omega}$$

Chapter 7, Problem 22.

Find $i(t)$ and $v(t)$ for $t > 0$ in the circuit of Fig. 7.102 if $i(0) = 10$ A.

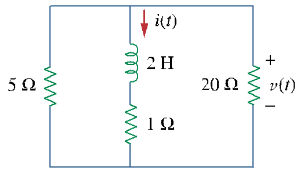


Figure 7.102
For Prob. 7.22.

Chapter 7, Solution 22.

$$i(t) = i(0)e^{-t/\tau}, \qquad \tau = \frac{L}{R_{eq}}$$

$$R_{eq} = 5 \,||\, 20 + 1 = 5\,\Omega, \qquad \tau = \frac{2}{5}$$

$$i(t) = \underline{\mathbf{10\,e^{-2.5t}\ A}}$$

Using current division, the current through the 20 ohm resistor is

$$i_o = \frac{5}{5+20}(-i) = \frac{-i}{5} = -2e^{-2.5t}$$

$$v(t) = 20 i_o = \underline{\mathbf{-40\,e^{-2.5t}\ V}}$$

Chapter 7, Problem 23.

Consider the circuit in Fig. 7.103. Given that $v_0(0) = 2$ V, find v_o and v_x for $t > 0$.

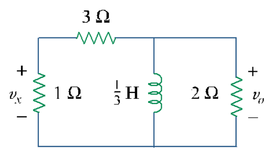


Figure 7.103
For Prob. 7.23.

Chapter 7, Solution 23.

Since the 2 Ω resistor, 1/3 H inductor, and the (3+1) Ω resistor are in parallel, they always have the same voltage.

$$-i = \frac{2}{2} + \frac{2}{3+1} = 1.5 \quad \longrightarrow \quad i(0) = -1.5$$

The Thevenin resistance R_{th} at the inductor's terminals is

$$R_{th} = 2 \,||\, (3+1) = \frac{4}{3}, \qquad \tau = \frac{L}{R_{th}} = \frac{1/3}{4/3} = \frac{1}{4}$$

$$i(t) = i(0)e^{-t/\tau} = -1.5e^{-4t}, \quad t > 0$$

$$v_L = v_o = L\frac{di}{dt} = -1.5(-4)(1/3)e^{-4t}$$

$$v_o = \underline{\mathbf{2e^{-4t}\ V,\quad t > 0}}$$

$$v_x = \frac{1}{3+1}v_L = \underline{\mathbf{0.5e^{-4t}\ V,\quad t > 0}}$$

Chapter 7, Problem 24.

Express the following signals in terms of singularity functions.

(a) $v(t) = \begin{cases} 0, & t < 0 \\ -5, & t > 0 \end{cases}$

(b) $i(t) = \begin{cases} 0, & t < 1 \\ -10, & 1 < t < 3 \\ 10, & 3 < t < 5 \\ 0, & t < 5 \end{cases}$

(c) $x(t) = \begin{cases} t-1 & 1 < t < 2 \\ 1, & 2 < t < 3 \\ 4-t & 3 < t < 4 \\ 0, & \text{Otherwise} \end{cases}$

(d) $y(t) = \begin{cases} 2, & t < 0 \\ -5, & 0 < t < 1 \\ 0, & t < 1 \end{cases}$

Chapter 7, Solution 24.

(a) $v(t) = \underline{\mathbf{-5u(t)}}$

(b) $i(t) = -10[u(t) - u(t-3)] + 10[u(t-3) - u(t-5)]$

$= \underline{\mathbf{-10u(t) + 20u(t-3) - 10u(t-5)}}$

(c) $x(t) = (t-1)[u(t-1) - u(t-2)] + [u(t-2) - u(t-3)]$
$+ (4-t)[u(t-3) - u(t-4)]$
$= (t-1)u(t-1) - (t-2)u(t-2) - (t-3)u(t-3) + (t-4)u(t-4)$
$= \underline{\mathbf{r(t-1) - r(t-2) - r(t-3) + r(t-4)}}$

(d) $y(t) = 2u(-t) - 5[u(t) - u(t-1)]$
$= \underline{\mathbf{2u(-t) - 5u(t) + 5u(t-1)}}$

Chapter 7, Problem 25.

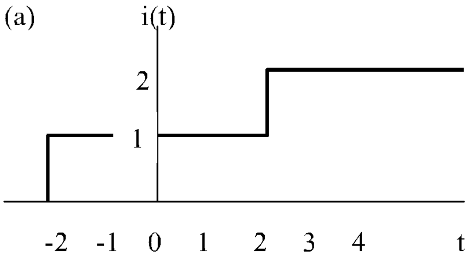
Sketch each of the following waveforms.

(a) $i(t) = u(t - 2) + u(t + 2)$
(b) $v(t) = r(t) - r(t - 3) + 4u(t - 5) - 8u(t - 8)$

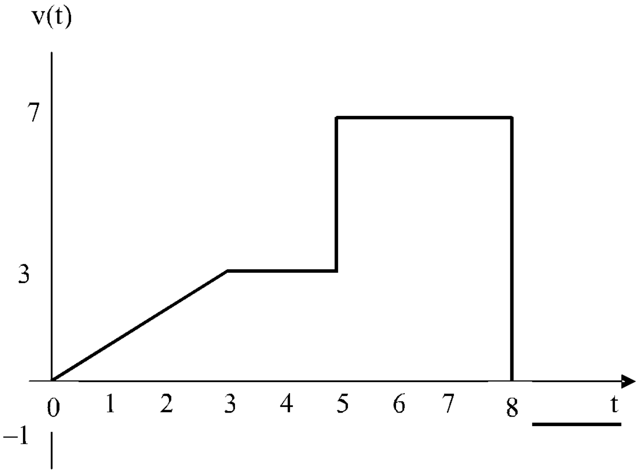
Chapter 7, Solution 25.

The waveforms are sketched below.

(a)



(b)



Chapter 7, Problem 26.

Express the signals in Fig. 7.104 in terms of singularity functions.

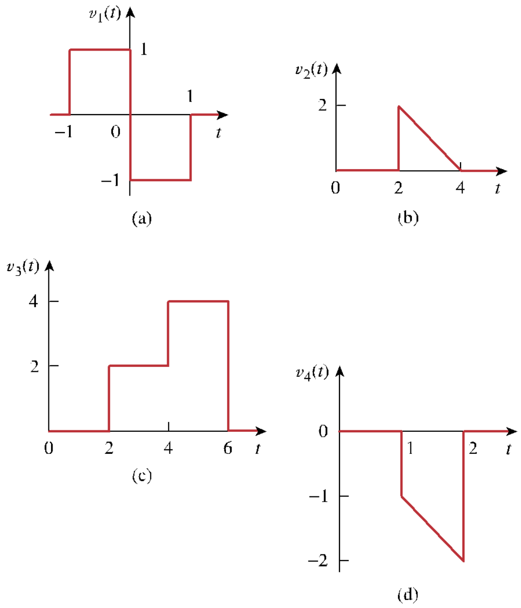


Figure 7.104
For Prob. 7.26.

Chapter 7, Solution 26.

(a) $v_1(t) = u(t+1) - u(t) + [u(t-1) - u(t)]$

$v_1(t) = \underline{\mathbf{u(t+1) - 2u(t) + u(t-1)}}$

(b) $v_2(t) = (4-t)[u(t-2) - u(t-4)]$

$v_2(t) = -(t-4)u(t-2) + (t-4)u(t-4)$

$v_2(t) = \underline{\mathbf{2u(t-2) - r(t-2) + r(t-4)}}$

(c) $v_3(t) = 2[u(t-2) - u(t-4)] + 4[u(t-4) - u(t-6)]$

$v_3(t) = \underline{\mathbf{2u(t-2) + 2u(t-4) - 4u(t-6)}}$

(d) $v_4(t) = -t[u(t-1) - u(t-2)] = -t\,u(t-1) + t\,u(t-2)$

$v_4(t) = (-t+1-1)u(t-1) + (t-2+2)u(t-2)$

$v_4(t) = \underline{\mathbf{-r(t-1) - u(t-1) + r(t-2) + 2u(t-2)}}$

Chapter 7, Problem 27.

Express $v(t)$ in Fig. 7.105 in terms of step functions.

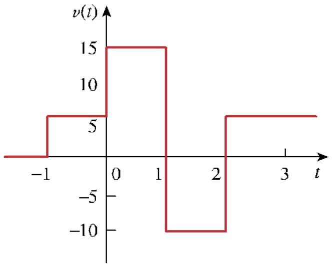


Figure 7.105
For Prob. 7.27.

Chapter 7, Solution 27.

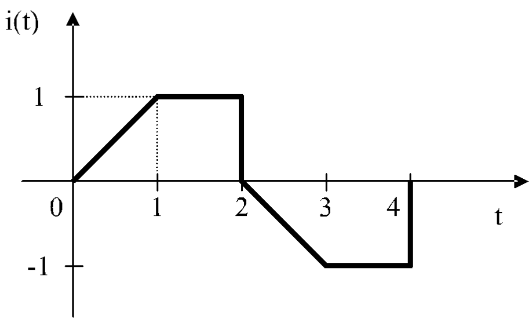
$$v(t) = \underline{\mathbf{5u(t+1)+10u(t)–25u(t–1)+15u(t-2)V}}$$

Chapter 7, Problem 28.

Sketch the waveform represented by
$i(t) = r(t) - r(t-1) - u(t-2) - r(t-2)$
$+ r(t-3) + u(t-4)$

Chapter 7, Solution 28.

$i(t)$ is sketched below.



Chapter 7, Problem 29.

Sketch the following functions:

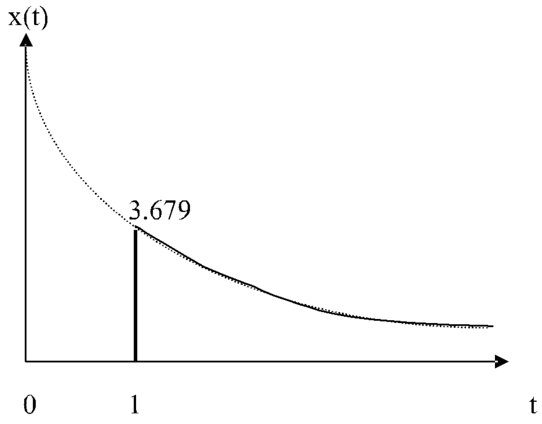
(a) $x(t) = 10e^{-t}\, u(t\text{-}1)$

(b) $y(t) = 10e^{-(t-1)}\, u(t)$

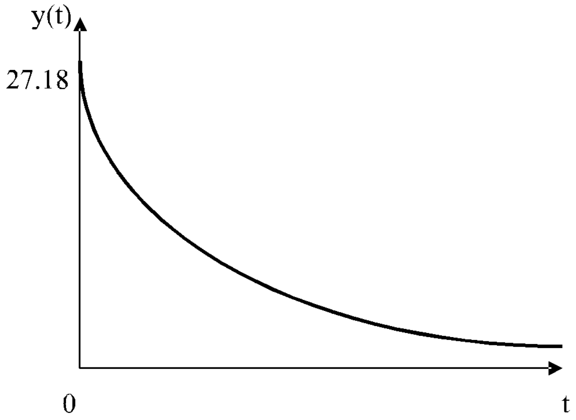
(c) $z(t) = \cos 4t\, \delta\, (t - 1)$

Chapter 7, Solution 29

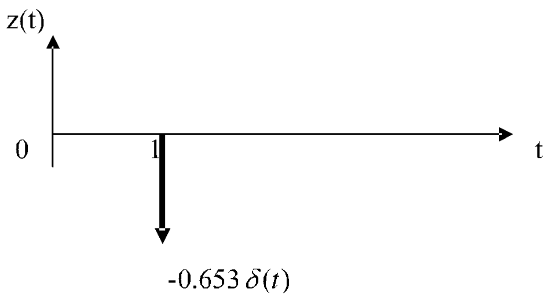
(a)



(b)



(c) $z(t) = \cos 4t\delta(t-1) = \cos 4\delta(t-1) = -0.6536\delta(t-1)$, which is sketched below.



Chapter 7, Problem 30.

Evaluate the following integrals involving the impulse functions:

(a) $\int_{-\infty}^{\infty} 4t^2 \delta(t-1)dt$

(b) $\int_{-\infty}^{\infty} 4t^2 \cos 2\pi t\, \delta(t-0.5)dt$

Chapter 7, Solution 30.

(a) $\int_{-\infty}^{\infty} 4t^2\, \delta(t-1)\, dt = 4t^2\big|_{t=1} = \underline{4}$

(b) $\int_{-\infty}^{\infty} 4t^2 \cos(2\pi t)\, \delta(t-0.5)\, dt = 4t^2 \cos(2\pi t)\big|_{t=0.5} = \cos\pi = \underline{\mathbf{-1}}$

Chapter 7, Problem 31.

Evaluate the following integrals:

(a) $\int_{-\infty}^{\infty} e^{-4t^2} \delta(t-2)dt$

(b) $\int_{-\infty}^{\infty} [5\delta(t) + e^{-t}\, \delta(t) + \cos 2\pi t\, \delta(t)]dt$

Chapter 7, Solution 31.

(a) $\int_{-\infty}^{\infty} \left[e^{-4t^2}\, \delta(t-2) \right] dt = e^{-4t^2}\big|_{t=2} = e^{-16} = \underline{\mathbf{112 \times 10^{-9}}}$

(b) $\int_{-\infty}^{\infty} \left[5\delta(t) + e^{-t}\, \delta(t) + \cos 2\pi t\, \delta(t) \right] dt = \left(5 + e^{-t} + \cos(2\pi t) \right)\big|_{t=0} = 5 + 1 + 1 = \underline{7}$

Chapter 7, Problem 32.

Evaluate the following integrals:

(a) $\int_1^t u(\lambda)d\lambda$

(b) $\int_0^4 r(t-1)dt$

(c) $\int_1^5 (t-6)^2\,\delta(t-2)dt$

Chapter 7, Solution 32.

(a) $\int_1^t u(\lambda)d\lambda = \int_1^t 1d\lambda = \lambda \Big|_1^t = \underline{t-1}$

(b) $\int_0^4 r(t-1)dt = \int_0^1 0dt + \int_1^4 (t-1)dt = \frac{t^2}{2} - t\Big|_1^4 = \underline{4.5}$

(c) $\int_1^5 (t-6)^2\,\delta(t-2)dt = (t-6)^2\Big|_{t=2} = \underline{16}$

Chapter 7, Problem 33.

The voltage across a 10-mH inductor is $20\,\delta\,(t\,\text{-}2)$ mV. Find the inductor current, assuming that the inductor is initially uncharged.

Chapter 7, Solution 33.

$$i(t) = \frac{1}{L}\int_0^t v(t)\ dt + i(0)$$

$$i(t) = \frac{10^{-3}}{10\times 10^{-3}}\int_0^t 20\,\delta(t-2)\ dt + 0$$

$$i(t) = \underline{\mathbf{2\,u(t-2)\ A}}$$

Chapter 7, Problem 34.

Evaluate the following derivatives:

(a) $\frac{d}{dt}[u(t - 1)\, u(t + 1)]$

(b) $\frac{d}{dt}[r(t - 6)\, u(t - 2)]$

(c) $\frac{d}{dt}[\sin 4t u(t - 31)]$

Chapter 7, Solution 34.

(a)
$$\frac{d}{dt}[u(t-1)\,u(t+1)] = \delta(t-1)u(t+1) +$$
$$u(t-1)\delta(t+1) = \delta(t-1)\bullet 1 + 0 \bullet \delta(t+1) = \underline{\delta(t-1)}$$

(b)
$$\frac{d}{dt}[r(t-6)\,u(t-2)] = u(t-6)u(t-2) +$$
$$r(t-6)\delta(t-2) = u(t-6)\bullet 1 + 0 \bullet \delta(t-2) = \underline{u(t-6)}$$

(c)
$$\frac{d}{dt}[\sin 4t\, u(t-3)] = 4\cos 4t\, u(t-3) + \sin 4t\delta(t-3)$$
$$= 4\cos 4t\, u(t-3) + \sin 4x3\delta(t-3)$$
$$= \underline{4\cos 4t\, u(t-3) - 0.5366\delta(t-3)}$$

Chapter 7, Problem 35.

Find the solution to the following differential equations:

(a) $\frac{dv}{dt} + 2v = 0, \qquad v(0) = -1 \text{ V}$

(b) $2\frac{di}{dt} + 3i = 0, \qquad i(0) = 2$

Chapter 7, Solution 35.

(a)
$$v = Ae^{-2t}, \qquad v(0) = A = -1$$
$$v = \underline{-e^{-2t}\ \mathbf{u(t)V}}$$

(b)
$$i = Ae^{3t/2}, \qquad i(0) = A = 2$$
$$i(t) = \underline{2e^{1.5t}\ \mathbf{u(t)A}}$$

Chapter 7, Problem 36.

Solve for v in the following differential equations, subject to the stated initial condition.

(a) $dv / dt + v = u(t)$, $v(0) = 0$
(b) $2\, dv / dt - v = 3u(t)$, $v(0) = -6$

Chapter 7, Solution 36.

(a) $v(t) = A + Be^{-t}, \quad t > 0$

$A = 1, \qquad v(0) = 0 = 1 + B \qquad \text{or} \qquad B = -1$

$v(t) = \underline{\mathbf{1 - e^{-t}\ V, \quad t > 0}}$

(b) $v(t) = A + Be^{t/2}, \quad t > 0$

$A = -3, \qquad v(0) = -6 = -3 + B \qquad \text{or} \qquad B = -3$

$v(t) = \underline{\mathbf{-3\left(1 + e^{t/2}\right) V, \quad t > 0}}$

Chapter 7, Problem 37.

A circuit is described by

$$4\frac{dv}{dt} + v = 10$$

(a) What is the time constant of the circuit?
(b) What is $v(\infty)$ the final value of v?
(c) If $v(0) = 2$ find $v(t)$ for $t \geq 0$.

Chapter 7, Solution 37.

Let $v = v_h + v_p$, $v_p = 10$.

$$\dot{v}_h + \frac{1}{4}v_h = 0 \longrightarrow v_h = Ae^{-t/4}$$

$$v = 10 + Ae^{-0.25t}$$

$$v(0) = 2 = 10 + A \longrightarrow A = -8$$

$$v = 10 - 8e^{-0.25t}$$

(a) $\tau = \underline{4s}$

(b) $v(\infty) = \underline{10}\ \underline{\mathbf{V}}$

(c) $\underline{v = 10 - 8e^{-0.25t}}\ \underline{\mathbf{u(t)V}}$

Chapter 7, Problem 38.

A circuit is described by

$$\frac{di}{dt} + 3i = 2u(t)$$

Find $i(t)$ for $t > 0$ given that $i(0) = 0$.

Chapter 7, Solution 38

Let $i = i_p + i_h$

$$\dot{i}_h + 3i_h = 0 \longrightarrow i_h = Ae^{-3t}u(t)$$

Let $i_p = ku(t)$, $\dot{i}_p = 0$, $3ku(t) = 2u(t) \longrightarrow k = \frac{2}{3}$

$$i_p = \frac{2}{3}u(t)$$

$$i = (Ae^{-3t} + \frac{2}{3})u(t)$$

If $i(0) = 0$, then $A + 2/3 = 0$, i.e. $A = -2/3$. Thus

$$\underline{i = \frac{2}{3}(1 - e^{-3t})u(t)}$$

Chapter 7, Problem 39.

Calculate the capacitor voltage for $t < 0$ and $t > 0$ for each of the circuits in Fig. 7.106.

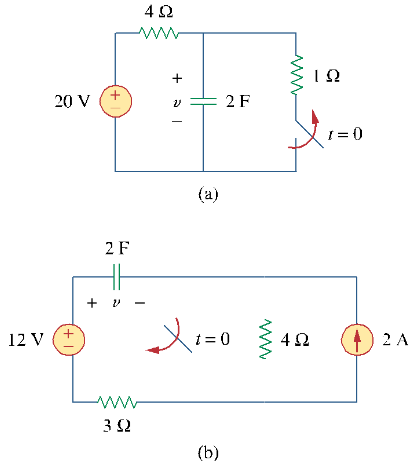


Figure 7.106
For Prob. 7.39.

Chapter 7, Solution 39.

(a) Before t = 0,

$$v(t) = \frac{1}{4+1}(20) = \underline{\mathbf{4\ V}}$$

After t = 0,

$$v(t) = v(\infty) + \left[\, v(0) - v(\infty) \right] e^{-t/\tau}$$

$$\tau = RC = (4)(2) = 8, \quad v(0) = 4, \qquad v(\infty) = 20$$

$$v(t) = 20 + (4 - 20)\,e^{-t/8}$$

$$v(t) = \underline{20 - 16\,e^{-t/8}\ V}$$

(b) Before t = 0, $v = v_1 + v_2$, where v_1 is due to the 12-V source and v_2 is due to the 2-A source.

$$v_1 = 12\ V$$

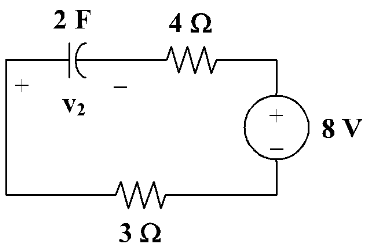
To get v_2, transform the current source as shown in Fig. (a).

$$v_2 = \text{-}8\ V$$

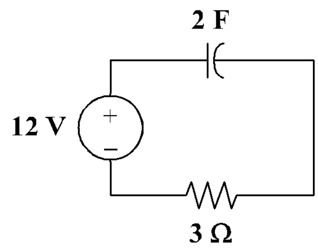
Thus,

$$v = 12 - 8 = \underline{\mathbf{4\ V}}$$

After t = 0, the circuit becomes that shown in Fig. (b).



(a)



(b)

$$v(t) = v(\infty) + \left[\, v(0) - v(\infty) \right] e^{-t/\tau}$$

$$v(\infty) = 12, \qquad v(0) = 4, \qquad \tau = RC = (2)(3) = 6$$

$$v(t) = 12 + (4 - 12)\,e^{-t/6}$$

$$v(t) = \underline{\mathbf{12 - 8\,e^{-t/6}\ V}}$$

Chapter 7, Problem 40.

Find the capacitor voltage for $t < 0$ and $t > 0$ for each of the circuits in Fig. 7.107.

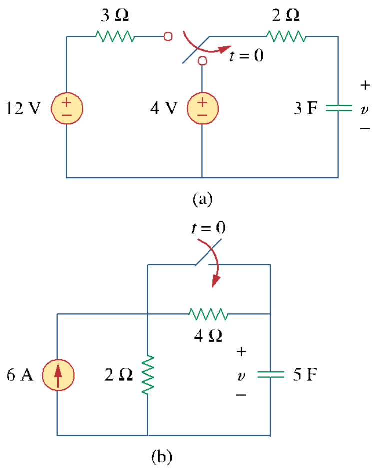


Figure 7.107
For Prob. 7.40.

Chapter 7, Solution 40.

(a) Before t = 0, $v = \underline{\mathbf{12\ V}}$.

After t = 0, $v(t) = v(\infty) + \left[v(0) - v(\infty) \right] e^{-t/\tau}$

$$v(\infty) = 4, \qquad v(0) = 12, \qquad \tau = RC = (2)(3) = 6$$

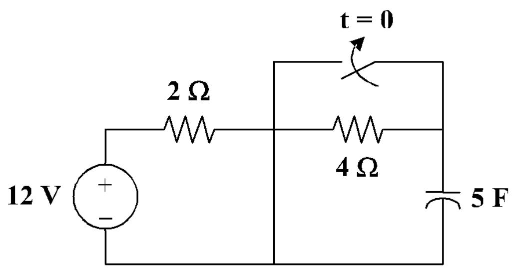
$$v(t) = 4 + (12 - 4)\,e^{-t/6}$$

$$v(t) = \underline{\mathbf{4 + 8\,e^{-t/6}\ V}}$$

(b) Before t = 0, $v = \underline{\mathbf{12\ V}}$.

After t = 0, $v(t) = v(\infty) + \left[v(0) - v(\infty) \right] e^{-t/\tau}$

After transforming the current source, the circuit is shown below.



$$v(0) = 12, \qquad v(\infty) = 12, \qquad \tau = RC = (2)(5) = 10$$

$v = \underline{\mathbf{12\ V}}$

Chapter 7, Problem 41.

For the circuit in Fig. 7.108, find $v(t)$ for $t > 0$.

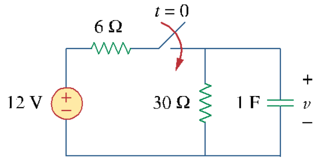


Figure 7.108
For Prob. 7.41.

Chapter 7, Solution 41.

$$v(0) = 0, \qquad v(\infty) = \frac{30}{36}(12) = 10$$

$$R_{eq}C = (6 \,\|\, 30)(1) = \frac{(6)(30)}{36} = 5$$

$$v(t) = v(\infty) + \left[v(0) - v(\infty) \right] e^{-t/\tau}$$

$$v(t) = 10 + (0 - 10)e^{-t/5}$$

$$v(t) = \underline{10(1 - e^{-0.2t})\,u(t)V}$$

Chapter 7, Problem 42.

(a) If the switch in Fig. 7.109 has been open for a long time and is closed at $t = 0$, find $v_o(t)$.

(b) Suppose that the switch has been closed for a long time and is opened at $t = 0$. Find $v_o(t)$.

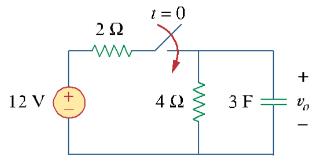


Figure 7.109
For Prob. 7.42.

Chapter 7, Solution 42.

(a) $$v_o(t) = v_o(\infty) + \left[v_o(0) - v_o(\infty)\right] e^{-t/\tau}$$

$$v_o(0) = 0, \qquad v_o(\infty) = \frac{4}{4+2}(12) = 8$$

$$\tau = R_{eq}C_{eq}, \qquad R_{eq} = 2 \,||\, 4 = \frac{4}{3}$$

$$\tau = \frac{4}{3}(3) = 4$$

$$v_o(t) = 8 - 8e^{-t/4}$$

$$v_o(t) = \underline{\mathbf{8(1 - e^{-0.25t})\ V}}$$

(b) For this case, $v_o(\infty) = 0$ so that

$$v_o(t) = v_o(0)e^{-t/\tau}$$

$$v_o(0) = \frac{4}{4+2}(12) = 8, \qquad \tau = RC = (4)(3) = 12$$

$$v_o(t) = \underline{\mathbf{8e^{-t/12}\ V}}$$

Chapter 7, Problem 43.

Consider the circuit in Fig. 7.110. Find $i(t)$ for $t < 0$ and $t > 0$.

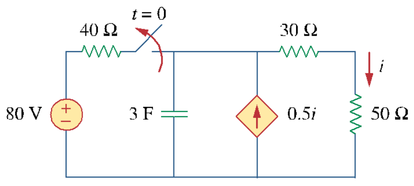
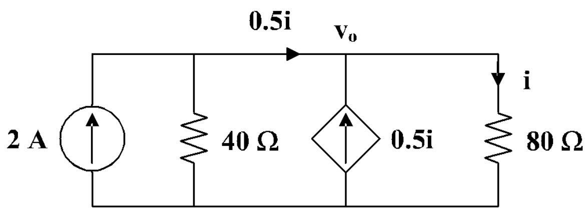


Figure 7.110
For Prob. 7.43.

Chapter 7, Solution 43.

Before t = 0, the circuit has reached steady state so that the capacitor acts like an open circuit. The circuit is equivalent to that shown in Fig. (a) after transforming the voltage source.



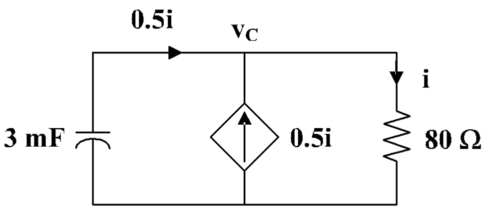
(a)

$$0.5i = 2 - \frac{v_o}{40}, \qquad i = \frac{v_o}{80}$$

Hence, $\frac{1}{2}\frac{v_o}{80} = 2 - \frac{v_o}{40} \longrightarrow v_o = \frac{320}{5} = 64$

$$i = \frac{v_o}{80} = \mathbf{0.8\ A}$$

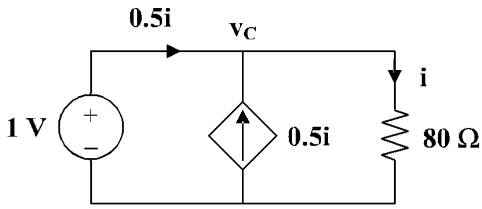
After t = 0, the circuit is as shown in Fig. (b).



(b)

$$v_C(t) = v_C(0)e^{-t/\tau}, \qquad \tau = R_{th}C$$

To find R_{th}, we replace the capacitor with a 1-V voltage source as shown in Fig. (c).



(c)

$$i = \frac{v_C}{80} = \frac{1}{80}, \qquad i_o = 0.5i = \frac{0.5}{80}$$

$$R_{th} = \frac{1}{i_o} = \frac{80}{0.5} = 160\,\Omega, \qquad \tau = R_{th}C = 480$$

$$v_C(0) = 64 \text{ V}$$

$$v_C(t) = 64e^{-t/480}$$

$$0.5i = -i_C = -C\frac{dv_C}{dt} = -3\left(\frac{1}{480}\right)64e^{-t/480}$$

$$i(t) = \underline{0.8e^{-t/480}\ u(t)\text{A}}$$

Chapter 7, Problem 44.

The switch in Fig. 7.111 has been in position *a* for a long time. At $t = 0$ it moves to position *b*. Calculate $i(t)$ for all $t > 0$.

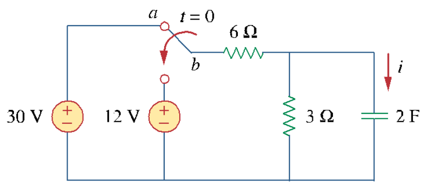


Figure 7.111
For Prob. 7.44.

Chapter 7, Solution 44.

$$R_{eq} = 6 \,||\, 3 = 2\,\Omega, \qquad \tau = RC = 4$$

$$v(t) = v(\infty) + \left[v(0) - v(\infty) \right] e^{-t/\tau}$$

Using voltage division,

$$v(0) = \frac{3}{3+6}(30) = 10\text{ V}, \qquad v(\infty) = \frac{3}{3+6}(12) = 4\text{ V}$$

Thus,

$$v(t) = 4 + (10-4)\,e^{-t/4} = 4 + 6\,e^{-t/4}$$

$$i(t) = C\frac{dv}{dt} = (2)(6)\left(\frac{-1}{4}\right)e^{-t/4} = \underline{\mathbf{-3\,e^{-0.25t}\ A}}$$

Chapter 7, Problem 45.

Find v_o in the circuit of Fig. 7.112 when $v_s = 6u(t)$. Assume that $v_o(0) = 1$ V.

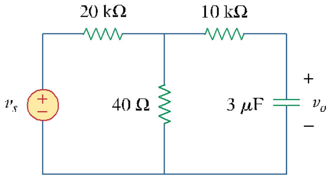
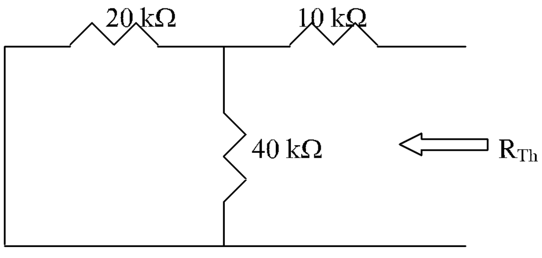


Figure 7.112
For Prob. 7.45.

Chapter 7, Solution 45.

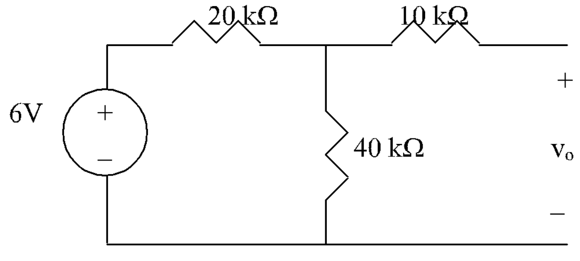
To find R_{Th}, consider the circuit shown below.



$$R_{Th} = 10 + 20 // 40 = 10 + \frac{20x40}{60} = \frac{70}{3} k\Omega$$

$$\tau = R_{Th}C = \frac{70}{3} x10^3 x3x10^{-6} = 0.07$$

To find $v_o(\infty)$, consider the circuit below.



$$v_o(\infty) = \frac{40}{40+20}(6V) = 4V$$

$$v_o(t) = v_o(\infty) + [v_o(0) - v_o(\infty)]e^{-t/\tau} = 4 + (1-4)e^{-t/0.07} = \underline{4 - 3e^{-14.286t}\ \text{V}}\ \textbf{u(t)}$$

Chapter 7, Problem 46.

For the circuit in Fig. 7.113, $i_s(t) = 5u(t)$ Find $v(t)$.

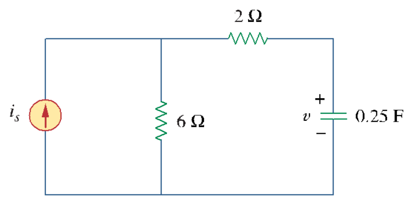


Figure 7.113
For Prob. 7.46.

Chapter 7, Solution 46.

$\tau = R_{Th}C = (2+6)x0.25 = 2s, \qquad v(0) = 0, \qquad v(\infty) = 6i_s = 6x5 = 30$

$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau} = \underline{30(1 - e^{-t/2})}$ V

Chapter 7, Problem 47.

Determine $v(t)$ for $t > 0$ in the circuit of Fig. 7.114 if $v(0) = 0$.

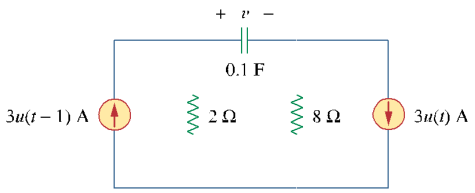


Figure 7.114
For Prob. 7.47.

Chapter 7, Solution 47.

For $t < 0$, $u(t) = 0$, $u(t-1) = 0$, $v(0) = 0$

For $0 < t < 1$, $\tau = RC = (2+8)(0.1) = 1$

$$v(0) = 0, \qquad v(\infty) = (8)(3) = 24$$
$$v(t) = v(\infty) + [v(0) - v(\infty)]\, e^{-t/\tau}$$
$$v(t) = 24(1 - e^{-t})$$

For $t > 1$, $v(1) = 24(1 - e^{-1}) = 15.17$

$$-6 + v(\infty) - 24 = 0 \longrightarrow v(\infty) = 30$$
$$v(t) = 30 + (15.17 - 30)\, e^{-(t-1)}$$
$$v(t) = 30 - 14.83\, e^{-(t-1)}$$

Thus,

$$v(t) = \begin{cases} \mathbf{24(1 - e^{-t})\,V}, & \mathbf{0 < t < 1} \\ \mathbf{30 - 14.83\, e^{-(t-1)}\ V}, & \mathbf{t > 1} \end{cases}$$

Chapter 7, Problem 48.

Find $v(t)$ and $i(t)$ in the circuit of Fig. 7.115.

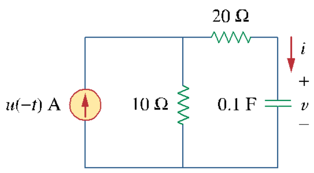


Figure 7.115
For Prob. 7.48.

Chapter 7, Solution 48.

For $t < 0$, $\quad u(-t) = 1, \quad v(0) = 10 \text{ V}$

For $t > 0$, $\quad u(-t) = 0, \quad v(\infty) = 0$

$$R_{th} = 20 + 10 = 30, \quad \tau = R_{th}C = (30)(0.1) = 3$$

$$v(t) = v(\infty) + \left[v(0) - v(\infty) \right] e^{-t/\tau}$$

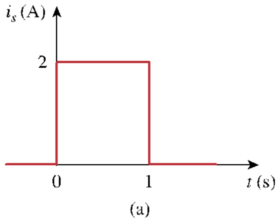
$$v(t) = \underline{\mathbf{10\,e^{-t/3}\ V}}$$

$$i(t) = C\frac{dv}{dt} = (0.1)\left(\frac{-1}{3}\right)10\,e^{-t/3}$$

$$i(t) = \underline{\mathbf{\frac{-1}{3}e^{-t/3}\ A}}$$

Chapter 7, Problem 49.

If the waveform in Fig. 7.116(a) is applied to the circuit of Fig. 7.116(b), find $v(t)$. Assume $v(0) = 0$.



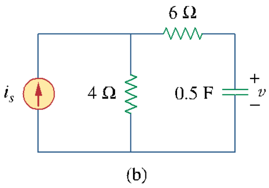


Figure 7.116
For Prob. 7.49 and Review Question 7.10.

Chapter 7, Solution 49.

For $0 < t < 1$, $\quad v(0) = 0, \qquad v(\infty) = (2)(4) = 8$

$$R_{eq} = 4 + 6 = 10, \qquad \tau = R_{eq}C = (10)(0.5) = 5$$

$$v(t) = v(\infty) + \left[\, v(0) - v(\infty)\right] e^{-t/\tau}$$

$$v(t) = 8\left(1 - e^{-t/5}\right) \text{ V}$$

For $t > 1$, $\qquad v(1) = 8\left(1 - e^{-0.2}\right) = 1.45, \qquad v(\infty) = 0$

$$v(t) = v(\infty) + \left[\, v(1) - v(\infty)\right] e^{-(t-1)/\tau}$$

$$v(t) = 1.45\, e^{-(t-1)/5} \text{ V}$$

Thus,

$$v(t) = \begin{cases} \mathbf{8\left(1 - e^{-t/5}\right) V}, & \mathbf{0 < t < 1} \\ \mathbf{1.45\, e^{-(t-1)/5}\ V}, & \mathbf{t > 1} \end{cases}$$

Chapter 7, Problem 50.

* In the circuit of Fig. 7.117, find i_x for $t > 0$. Let $R_1 = R_2 = 1\text{k}\Omega$, $R_3 = 2\text{k}\Omega$, and $C =$ 0.25 mF.

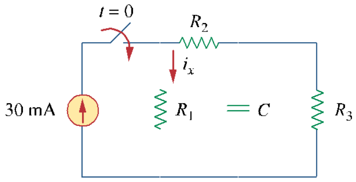


Figure 7.117
For Prob. 7.50.

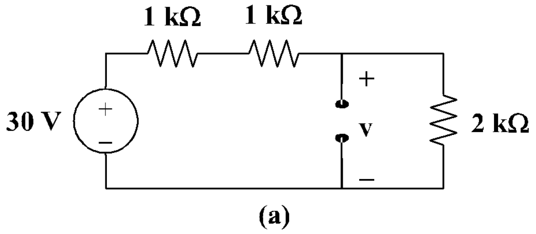
Chapter 7, Solution 50.

For the capacitor voltage,

$$v(t) = v(\infty) + \left[v(0) - v(\infty) \right] e^{-t/\tau}$$

$$v(0) = 0$$

For t > 0, we transform the current source to a voltage source as shown in Fig. (a).



(a)

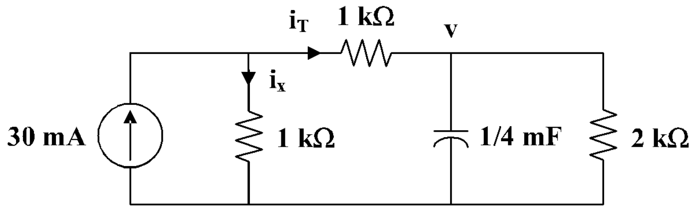
$$v(\infty) = \frac{2}{2+1+1}(30) = 15 \text{ V}$$

$$R_{th} = (1+1) \,||\, 2 = 1 \text{ k}\Omega$$

$$\tau = R_{th} C = 10^3 \times \frac{1}{4} \times 10^{-3} = \frac{1}{4}$$

$$v(t) = 15\left(1 - e^{-4t}\right), \quad t > 0$$

We now obtain i_x from v(t). Consider Fig. (b).



(b)

$$i_x = 30 \text{ mA} - i_T$$

But $\quad i_T = \frac{v}{R_3} + C\frac{dv}{dt}$

$$i_T(t) = 7.5\left(1 - e^{-4t}\right) \text{mA} + \frac{1}{4} \times 10^{-3} (-15)(-4) e^{-4t} \text{ A}$$

$$i_T(t) = 7.5\left(1 + e^{-4t}\right) \text{mA}$$

Thus,

$$i_x(t) = 30 - 7.5 - 7.5 e^{-4t} \text{ mA}$$

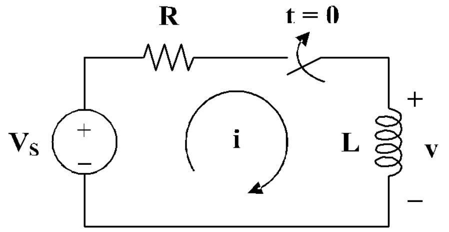
$$i_x(t) = \mathbf{7.5\left(3 - e^{-4t}\right) mA, \quad t > 0}$$

Chapter 7, Problem 51.

Rather than applying the short-cut technique used in Section 7.6, use KVL to obtain Eq. (7.60).

Chapter 7, Solution 51.

Consider the circuit below.



After the switch is closed, applying KVL gives

$$V_S = Ri + L\frac{di}{dt}$$

or $$L\frac{di}{dt} = -R\left(i - \frac{V_S}{R}\right)$$

$$\frac{di}{i - V_S/R} = \frac{-R}{L}dt$$

Integrating both sides,

$$\ln\left(i - \frac{V_S}{R}\right)\Bigg|_{I_0}^{i(t)} = \frac{-R}{L}t$$

$$\ln\left(\frac{i - V_S/R}{I_0 - V_S/R}\right) = \frac{-t}{\tau}$$

or $$\frac{i - V_S/R}{I_0 - V_S/R} = e^{-t/\tau}$$

$$\underline{\mathbf{i(t)} = \frac{\mathbf{V_S}}{\mathbf{R}} + \left(\mathbf{I_0} - \frac{\mathbf{V_S}}{\mathbf{R}}\right)\mathbf{e^{-t/\tau}}}$$

which is the same as Eq. (7.60).

Chapter 7, Problem 52.

For the circuit in Fig. 7.118, find $i(t)$ for $t > 0$.

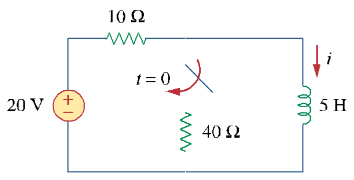


Figure 7.118
For Prob. 7.52.

Chapter 7, Solution 52.

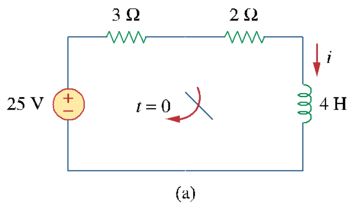
$$i(0) = \frac{20}{10} = 2\text{ A}, \qquad i(\infty) = 2\text{ A}$$

$$i(t) = i(\infty) + \left[\, i(0) - i(\infty)\right] e^{-t/\tau}$$

$$i(t) = \underline{\mathbf{2\ A}}$$

Chapter 7, Problem 53.

Determine the inductor current $i(t)$ for both $t < 0$ and $t > 0$ for each of the circuits in Fig. 7.119.



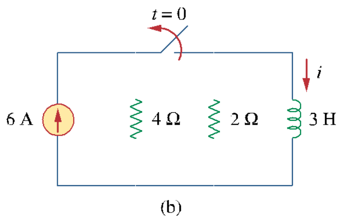


Figure 7.119
For Prob. 7.53.

Chapter 7, Solution 53.

(a) Before t = 0, $i = \frac{25}{3+2} = \underline{\mathbf{5\ A}}$

After t = 0, $i(t) = i(0)e^{-t/\tau}$

$$\tau = \frac{L}{R} = \frac{4}{2} = 2, \qquad i(0) = 5$$

$$i(t) = \underline{5e^{-t/2}\ u(t)A}$$

(b) Before t = 0, the inductor acts as a short circuit so that the 2 Ω and 4 Ω resistors are short-circuited.

$$i(t) = \underline{\mathbf{6\ A}}$$

After t = 0, we have an RL circuit.

$$i(t) = i(0)e^{-t/\tau}, \qquad \tau = \frac{L}{R} = \frac{3}{2}$$

$$i(t) = \underline{6e^{-2t/3}\ u(t)A}$$

Chapter 7, Problem 54.

Obtain the inductor current for both $t < 0$ and $t > 0$ in each of the circuits in Fig. 7.120.

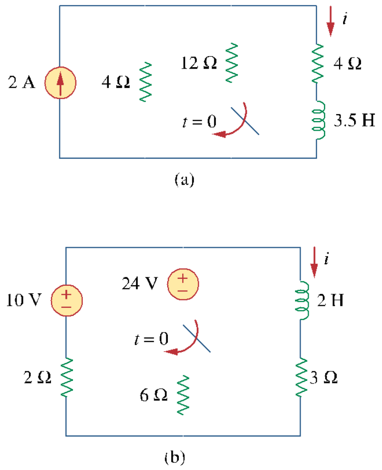


Figure 7.120
For Prob. 7.54.

Chapter 7, Solution 54.

(a) Before t = 0, i is obtained by current division or

$$i(t) = \frac{4}{4+4}(2) = \underline{\mathbf{1\ A}}$$

After t = 0,

$$i(t) = i(\infty) + \left[i(0) - i(\infty)\right] e^{-t/\tau}$$

$$\tau = \frac{L}{R_{eq}}, \qquad R_{eq} = 4 + 4 \,||\, 12 = 7\,\Omega$$

$$\tau = \frac{3.5}{7} = \frac{1}{2}$$

$$i(0) = 1, \qquad i(\infty) = \frac{4\,||\,12}{4 + 4\,||\,12}(2) = \frac{3}{4+3}(2) = \frac{6}{7}$$

$$i(t) = \frac{6}{7} + \left(1 - \frac{6}{7}\right)e^{-2t}$$

$$i(t) = \underline{\mathbf{\frac{1}{7}\left(6 - e^{-2t}\right)\ A}}$$

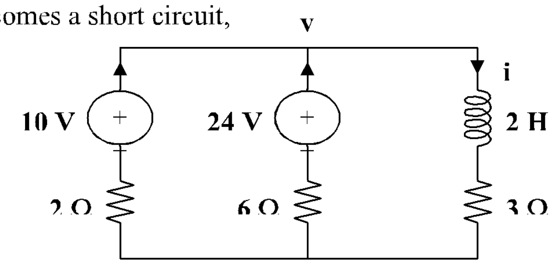
(b) Before t = 0, $i(t) = \frac{10}{2+3} = \underline{\mathbf{2\ A}}$

After t = 0, $R_{eq} = 3 + 6\,||\,2 = 4.5$

$$\tau = \frac{L}{R_{eq}} = \frac{2}{4.5} = \frac{4}{9}$$

$$i(0) = 2$$

To find $i(\infty)$, consider the circuit below, at t = when the inductor becomes a short circuit,



$$\frac{10 - v}{2} + \frac{24 - v}{6} = \frac{v}{3} \longrightarrow v = 9$$

$$i(\infty) = \frac{v}{3} = 3\text{ A}$$

$$i(t) = 3 + (2 - 3)e^{-9t/4}$$

$$i(t) = \underline{\mathbf{3 - e^{-9t/4}\ A}}$$

Chapter 7, Problem 55.

Find $v(t)$ for $t < 0$ and $t > 0$ in the circuit of Fig. 7.121.

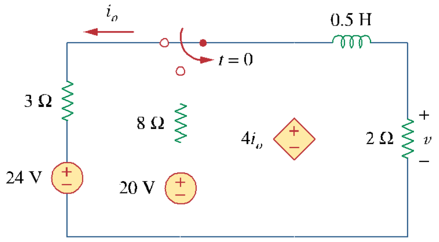
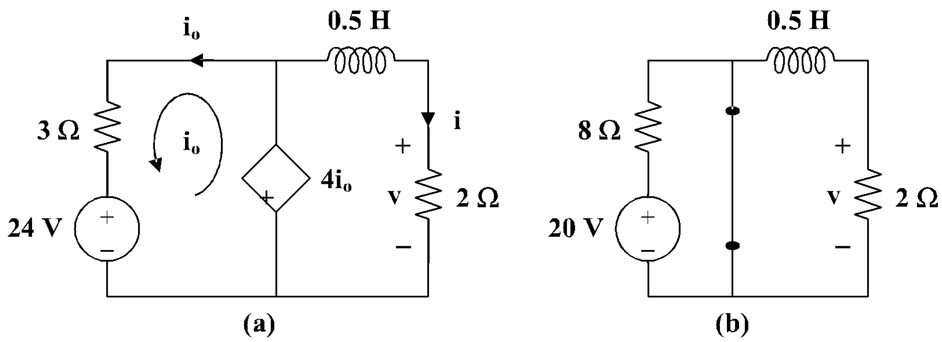


Figure 7.121
For Prob. 7.55.

Chapter 7, Solution 55.

For t < 0, consider the circuit shown in Fig. (a).



$$3i_o + 24 - 4i_o = 0 \longrightarrow i_o = 24$$

$$\underline{v(t) = 4i_o = \mathbf{96\ V}} \qquad i = \frac{v}{2} = 48\ A$$

For t > 0, consider the circuit in Fig. (b).

$$i(t) = i(\infty) + \left[i(0) - i(\infty)\right] e^{-t/\tau}$$

$$i(0) = 48, \qquad i(\infty) = 0$$

$$R_{th} = 2\ \Omega, \qquad \tau = \frac{L}{R_{th}} = \frac{0.5}{2} = \frac{1}{4}$$

$$i(t) = (48)e^{-4t}$$

$$v(t) = 2i(t) = \underline{96e^{-4t}\ u(t)V}$$

Chapter 7, Problem 56.

For the network shown in Fig. 7.122, find $v(t)$ for $t > 0$.

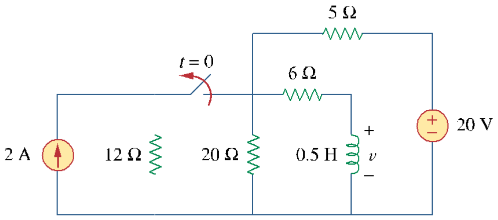


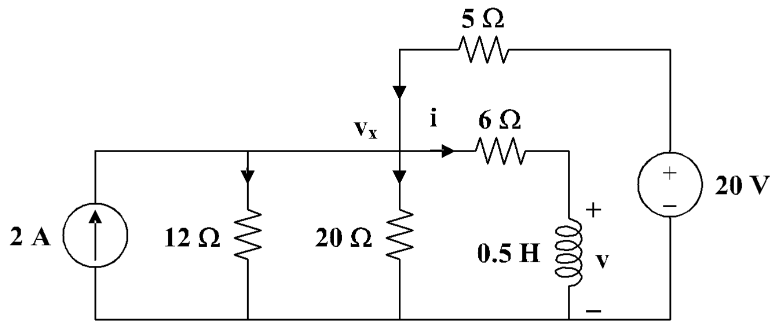
Figure 7.122
For Prob. 7.56.

Chapter 7, Solution 56.

$$R_{eq} = 6 + 20 \| 5 = 10\,\Omega, \qquad \tau = \frac{L}{R} = 0.05$$

$$i(t) = i(\infty) + \left[\, i(0) - i(\infty) \right] e^{-t/\tau}$$

i(0) is found by applying nodal analysis to the following circuit.



$$2 + \frac{20 - v_x}{5} = \frac{v_x}{12} + \frac{v_x}{20} + \frac{v_x}{6} \longrightarrow v_x = 12$$

$$i(0) = \frac{v_x}{6} = 2\text{ A}$$

Since $20 \| 5 = 4$,

$$i(\infty) = \frac{4}{4+6}(4) = 1.6$$

$$i(t) = 1.6 + (2 - 1.6)e^{-t/0.05} = 1.6 + 0.4e^{-20t}$$

$$v(t) = L\frac{di}{dt} = \frac{1}{2}(0.4)(-20)e^{-20t}$$

$$v(t) = \underline{\mathbf{-4\,e^{-20t}\ V}}$$

Chapter 7, Problem 57.

* Find $i_1(t)$ and $i_2(t)$ for $t > 0$ in the circuit of Fig. 7.123.

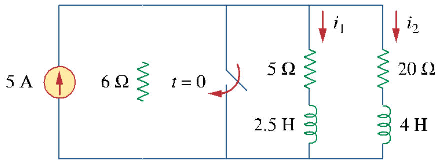
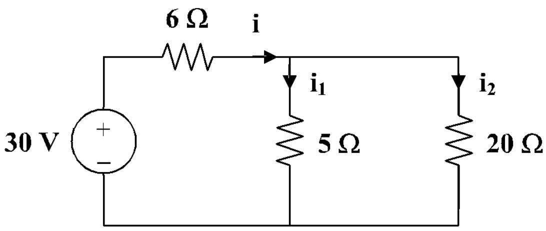


Figure 7.123
For Prob. 7.57.

* An asterisk indicates a challenging problem.

Chapter 7, Solution 57.

At $t = 0^-$, the circuit has reached steady state so that the inductors act like short circuits.



$$i = \frac{30}{6 + 5 \| 20} = \frac{30}{10} = 3, \qquad i_1 = \frac{20}{25}(3) = 2.4, \qquad i_2 = 0.6$$

$$i_1(0) = 2.4 \text{ A}, \qquad i_2(0) = 0.6 \text{ A}$$

For $t > 0$, the switch is closed so that the energies in L_1 and L_2 flow through the closed switch and become dissipated in the 5 Ω and 20 Ω resistors.

$$i_1(t) = i_1(0)e^{-t/\tau_1}, \qquad \tau_1 = \frac{L_1}{R_1} = \frac{2.5}{5} = \frac{1}{2}$$

$$i_1(t) = \underline{2.4e^{-2t}\ u(t)\text{A}}$$

$$i_2(t) = i_2(0)e^{-t/\tau_2}, \qquad \tau_2 = \frac{L_2}{R_2} = \frac{4}{20} = \frac{1}{5}$$

$$i_2(t) = \underline{0.6e^{-5t}\ u(t)\text{A}}$$

Chapter 7, Problem 58.

Rework Prob. 7.17 if $i(0) = 10$ A and $v(t) = 20u(t)$ V.

Chapter 7, Solution 58.

For $t < 0$, $\quad v_o(t) = 0$

For $t > 0$, $\quad i(0) = 10$, $\quad i(\infty) = \frac{20}{1+3} = 5$

$$R_{th} = 1 + 3 = 4\,\Omega, \qquad \tau = \frac{L}{R_{th}} = \frac{1/4}{4} = \frac{1}{16}$$

$$i(t) = i(\infty) + \left[\, i(0) - i(\infty)\right] e^{-t/\tau}$$

$$i(t) = 5\left(1 + e^{-16t}\right) \text{ A}$$

$$v_o(t) = 3i + L\frac{di}{dt} = 15\left(1 + e^{-16t}\right) + \frac{1}{4}(-16)(5)e^{-16t}$$

$$v_o(t) = \underline{\mathbf{15 - 5\,e^{-16t}\ V}}$$

Chapter 7, Problem 59.

Determine the step response $v_0(t)$ to v_s in the circuit of Fig. 7.124.

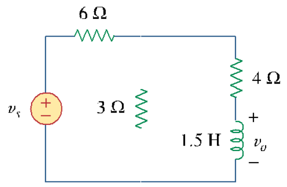


Figure 7.124
For Prob. 7.59.

Chapter 7, Solution 59.

Let I be the current through the inductor.

For $t < 0$, $\quad v_s = 0$, $\quad i(0) = 0$

For $t > 0$, $\quad R_{eq} = 4 + 6 \,||\, 3 = 6$, $\quad \tau = \frac{L}{R_{eq}} = \frac{1.5}{6} = 0.25$

$$i(\infty) = \frac{2}{2+4}(3) = 1$$

$$i(t) = i(\infty) + \left[\, i(0) - i(\infty)\right] e^{-t/\tau}$$

$$i(t) = 1 - e^{-4t}$$

$$v_o(t) = L\frac{di}{dt} = (1.5)(-4)(-e^{-4t})$$

$$v_o(t) = \underline{6\,e^{-4t}u(t)\text{ V}}$$

Chapter 7, Problem 60.

Find $v(t)$ for $t > 0$ in the circuit of Fig. 7.125 if the initial current in the inductor is zero.

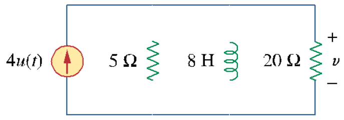


Figure 7.125
For Prob. 7.60.

Chapter 7, Solution 60.

Let I be the inductor current.

For $t < 0$, $\quad u(t) = 0 \longrightarrow i(0) = 0$

For $t > 0$, $\quad R_{eq} = 5 \,||\, 20 = 4\,\Omega, \qquad \tau = \dfrac{L}{R_{eq}} = \dfrac{8}{4} = 2$

$$i(\infty) = 4$$

$$i(t) = i(\infty) + \left[\, i(0) - i(\infty) \right] e^{-t/\tau}$$

$$i(t) = 4\left(1 - e^{-t/2}\right)$$

$$v(t) = L\frac{di}{dt} = (8)(-4)\left(\frac{-1}{2}\right)e^{-t/2}$$

$$v(t) = \underline{\mathbf{16\,e^{-0.5t}\ \ V}}$$

Chapter 7, Problem 61.

In the circuit of Fig. 7.126, i_s changes from 5 A to 10 A at $t = 0$ that is, $i_s = 5u(-t) + 10u(t)$ Find v and i.

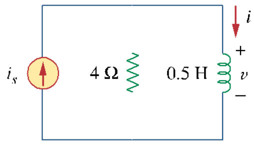
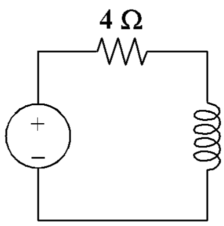


Figure 7.126
For Prob. 7.61.

Chapter 7, Solution 61.

The current source is transformed as shown below.



$$\tau = \frac{L}{R} = \frac{1/2}{4} = \frac{1}{8}, \qquad i(0) = 5, \qquad i(\infty) = 10$$

$$i(t) = i(\infty) + \left[\, i(0) - i(\infty) \right] e^{-t/\tau}$$

$$i(t) = \underline{10 - 5e^{-8t}\ u(t)A}$$

$$v(t) = L\frac{di}{dt} = \left(\frac{1}{2}\right)(-5)(-8)\, e^{-8t}$$

$$v(t) = \underline{20e^{-8t}u(t)\ V}$$

Chapter 7, Problem 62.

For the circuit in Fig. 7.127, calculate $i(t)$ if $i(0) = 0$.

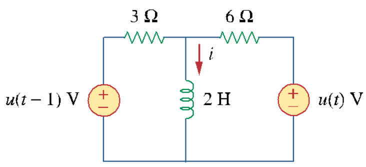


Figure 7.127
For Prob. 7.62.

Chapter 7, Solution 62.

$$\tau = \frac{L}{R_{eq}} = \frac{2}{3 \| 6} = 1$$

For $0 < t < 1$, $u(t-1) = 0$ so that

$$i(0) = 0, \qquad i(\infty) = \frac{1}{6}$$

$$i(t) = \frac{1}{6}\left(1 - e^{-t}\right)$$

For $t > 1$, $\quad i(1) = \frac{1}{6}\left(1 - e^{-1}\right) = 0.1054$

$$i(\infty) = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$$

$$i(t) = 0.5 + (0.1054 - 0.5)e^{-(t-1)}$$

$$i(t) = 0.5 - 0.3946e^{-(t-1)}$$

Thus,

$$i(t) = \begin{cases} \mathbf{\frac{1}{6}\left(1 - e^{-t}\right) A} & \mathbf{0 < t < 1} \\ \mathbf{0.5 - 0.3946e^{-(t-1)}\ A} & \mathbf{t > 1} \end{cases}$$

Chapter 7, Problem 63.

Obtain $v(t)$ and $i(t)$ in the circuit of Fig. 7.128.

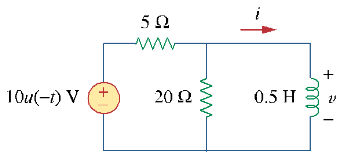


Figure 7.128
For Prob. 7.63.

Chapter 7, Solution 63.

For $t < 0$, $\quad u(-t) = 1, \quad i(0) = \dfrac{10}{5} = 2$

For $t > 0$, $\quad u(-t) = 0, \quad i(\infty) = 0$

$$R_{th} = 5 \,||\, 20 = 4\,\Omega, \qquad \tau = \frac{L}{R_{th}} = \frac{0.5}{4} = \frac{1}{8}$$

$$i(t) = i(\infty) + \left[\, i(0) - i(\infty)\right] e^{-t/\tau}$$

$$i(t) = \underline{2e^{-8t}\ u(t)\text{A}}$$

$$v(t) = L\frac{di}{dt} = \left(\frac{1}{2}\right)(-8)(2)e^{-8t}$$

$$v(t) = \underline{-8e^{-8t}u(t)\ \text{V}}$$

Chapter 7, Problem 64.

Find $v_o(t)$ for $t > 0$ in the circuit of Fig. 7.129.

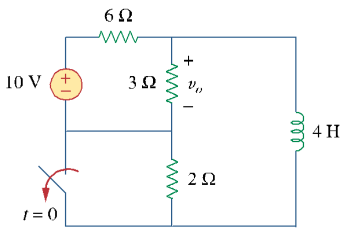
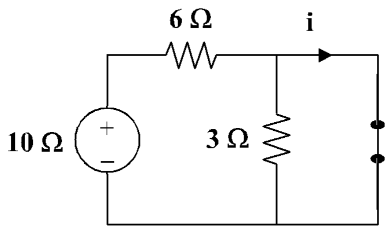


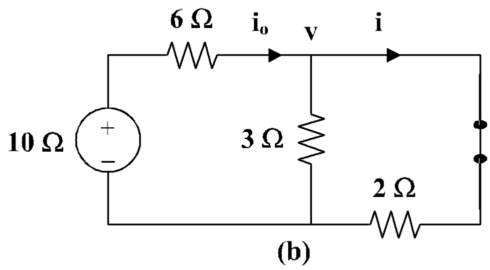
Figure 7.129
For Prob. 7.64.

Chapter 7, Solution 64.

Let i be the inductor current.
For t < 0, the inductor acts like a short circuit and the 3 Ω resistor is short-circuited so that the equivalent circuit is shown in Fig. (a).



(a)



(b)

$$i = i(0) = \frac{10}{6} = 1.667\text{ A}$$

For t > 0, $\quad R_{th} = 2 + 3 \,||\, 6 = 4\,\Omega, \qquad \tau = \frac{L}{R_{th}} = \frac{4}{4} = 1$

To find $i(\infty)$, consider the circuit in Fig. (b).

$$\frac{10 - v}{6} = \frac{v}{3} + \frac{v}{2} \longrightarrow v = \frac{10}{6}$$

$$i = i(\infty) = \frac{v}{2} = \frac{5}{6}$$

$$i(t) = i(\infty) + \left[\, i(0) - i(\infty) \right] e^{-t/\tau}$$

$$i(t) = \frac{5}{6} + \left(\frac{10}{6} - \frac{5}{6}\right) e^{-t} = \frac{5}{6}\left(1 + e^{-t}\right) \text{A}$$

v_o is the voltage across the 4 H inductor and the 2 Ω resistor

$$v_o(t) = 2i + L\frac{di}{dt} = \frac{10}{6} + \frac{10}{6}e^{-t} + (4)\left(\frac{5}{6}\right)(-1)e^{-t} = \frac{10}{6} - \frac{10}{6}e^{-t}$$

$$\underline{v_o(t) = 1.6667\left(1 - e^{-t}\right) V}$$

Chapter 7, Problem 65.

If the input pulse in Fig. 7.130(a) is applied to the circuit in Fig. 7.130(b), determine the response $i(t)$.

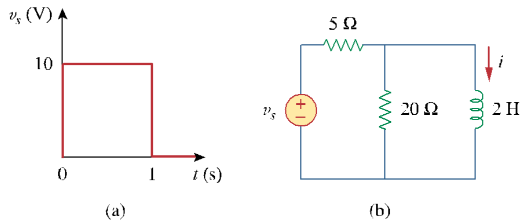
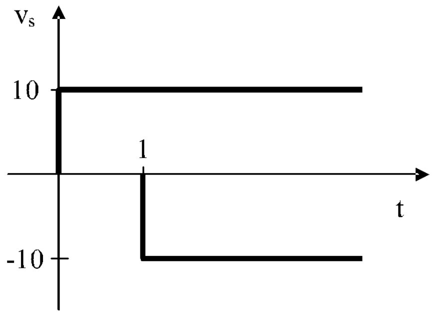


Figure 7.130
For Prob. 7.65.

Chapter 7, Solution 65.

Since $v_s = 10\left[u(t) - u(t-1)\right]$, this is the same as saying that a 10 V source is turned on at t = 0 and a -10 V source is turned on later at t = 1. This is shown in the figure below.



For $0 < t < 1$, $i(0) = 0$, $i(\infty) = \frac{10}{5} = 2$

$$R_{th} = 5 \,||\, 20 = 4, \qquad \tau = \frac{L}{R_{th}} = \frac{2}{4} = \frac{1}{2}$$

$$i(t) = i(\infty) + \left[i(0) - i(\infty)\right] e^{-t/\tau}$$

$$i(t) = 2\left(1 - e^{-2t}\right) \text{ A}$$

$$i(1) = 2\left(1 - e^{-2}\right) = 1.729$$

For $t > 1$, $i(\infty) = 0$ since $v_s = 0$

$$i(t) = i(1)\, e^{-(t-1)/\tau}$$

$$i(t) = 1.729 e^{-2(t-1)} \text{ A}$$

Thus,

$$i(t) = \begin{cases} \mathbf{2\left(1 - e^{-2t}\right) A} & \mathbf{0 < t < 1} \\ \mathbf{1.729\, e^{-2(t-1)} A} & \mathbf{t > 1} \end{cases}$$

Chapter 7, Problem 66.

For the op amp circuit of Fig. 7.131, find v_o. Assume that v_s changes abruptly from 0 to 1 V at $t = 0$.

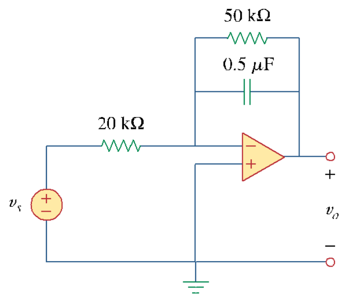


Figure 7.131
For Prob. 7.66.

Chapter 7, Solution 66.

For t<0-, $v_s = 0$ so that $v_o(0) = 0$
Let v be the capacitor voltage
For t>0, $v_s = 1$. At steady state, the capacitor acts like an open circuit so that we have an inverting amplifier

$$v_o(\infty) = -(50k/20k)(1V) = -2.5\text{ V}$$

$$\tau = RC = 50x10^3 x0.5x10^{-6} = 25\text{ ms}$$

$$v_o(t) = v_o(\infty) + (v_o(0) - v_o(\infty))e^{-t/0.025} = \underline{\mathbf{2.5(e^{-40t} - 1)\ V}}.$$

Chapter 7, Problem 67.

If $v(0) = 5$ V, find $v_o(t)$ for $t > 0$ in the op amp circuit of Fig. 7.132. Let $R = 10\text{k}\Omega$ and $C = 1\ \mu\text{F}$.

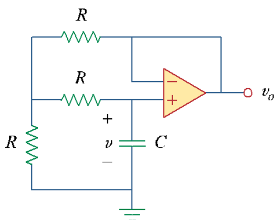
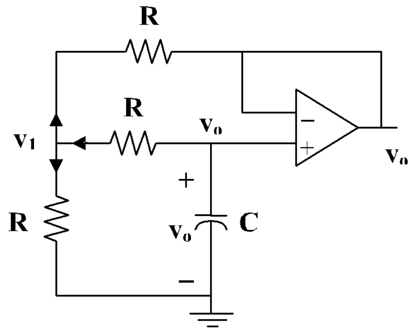


Figure 7.132
For Prob. 7.67.

Chapter 7, Solution 67.

The op amp is a voltage follower so that $v_o = v$ as shown below.



At node 1,

$$\frac{v_o - v_1}{R} = \frac{v_1 - 0}{R} + \frac{v_1 - v_o}{R} \longrightarrow v_1 = \frac{2}{3} v_o$$

At the noninverting terminal,

$$C\frac{dv_o}{dt} + \frac{v_o - v_1}{R} = 0$$

$$-RC\frac{dv_o}{dt} = v_o - v_1 = v_o - \frac{2}{3}v_o = \frac{1}{3}v_o$$

$$\frac{dv_o}{dt} = -\frac{v_o}{3RC}$$

$$v_o(t) = V_T\, e^{-t/3RC}$$

$$V_T = v_o(0) = 5\text{ V}, \qquad \tau = 3RC = (3)(10\times10^3)(1\times10^{-6}) = \frac{3}{100}$$

$$\underline{v_o(t) = 5\,e^{-100t/3}\ u(t)\text{V}}$$

Chapter 7, Problem 68.

Obtain v_o for $t > 0$ in the circuit of Fig. 7.133.

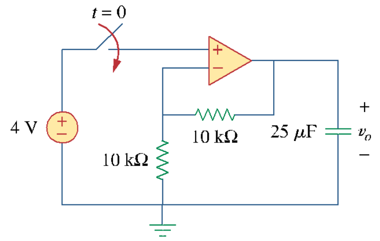


Figure 7.133
For Prob. 7.68.

Chapter 7, Solution 68.

This is a very interesting problem and has both an important ideal solution as well as an important practical solution. Let us look at the ideal solution first. Just before the switch closes, the value of the voltage across the capacitor is zero which means that the voltage at both terminals input of the op amp are each zero. As soon as the switch closes, the output tries to go to a voltage such that the input to the op amp both go to 4 volts. The ideal op amp puts out whatever current is necessary to reach this condition. An infinite (impulse) current is necessary if the voltage across the capacitor is to go to 8 volts in zero time (8 volts across the capacitor will result in 4 volts appearing at the negative terminal of the op amp). So v_o will be equal to **8 volts** for all t > 0.

What happens in a real circuit? Essentially, the output of the amplifier portion of the op amp goes to whatever its maximum value can be. Then this maximum voltage appears across the output resistance of the op amp and the capacitor that is in series with it. This results in an exponential rise in the capacitor voltage to the steady-state value of 8 volts.

$$vC(t) = V_{op\ amp\ max}(1 - e^{-t/(RoutC)}) \text{ volts, for all values of vC less than 8 V,}$$

$$= 8 \text{ V when t is large enough so that the 8 V is reached.}$$

Chapter 7, Problem 69.

For the op amp circuit in Fig. 7.134, find $v_0(t)$ for $t > 0$.

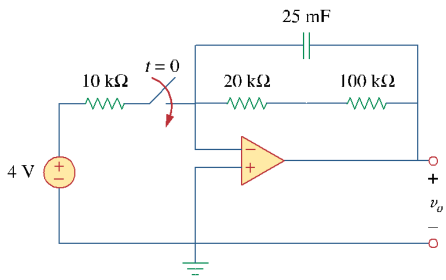


Figure 7.134
For Prob. 7.69.

Chapter 7, Solution 69.

Let v_x be the capacitor voltage.
For $t < 0$, $v_x(0) = 0$

For $t > 0$, the 20 kΩ and 100 kΩ resistors are in series and together, they are in parallel with the capacitor since no current enters the op amp terminals.
As $t \to \infty$, the capacitor acts like an open circuit so that

$$v_o(\infty) = \frac{-4}{10}(20 + 100) = -48$$

$$R_{th} = 20 + 100 = 120\text{ k}\Omega, \qquad \tau = R_{th}C = (120 \times 10^3)(25 \times 10^{-3}) = 3000$$

$$v_o(t) = v_o(\infty) + \left[v_o(0) - v_o(\infty) \right] e^{-t/\tau}$$

$$v_o(t) = -48\left(1 - e^{-t/3000}\right) V = \mathbf{48(e^{-t/3000} - 1)u(t)V}$$

Chapter 7, Problem 70.

Determine v_0 for $t > 0$ when $v_s = 20$ mV in the op amp circuit of Fig. 7.135.

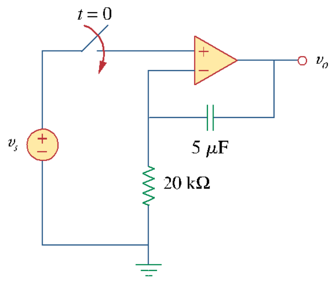


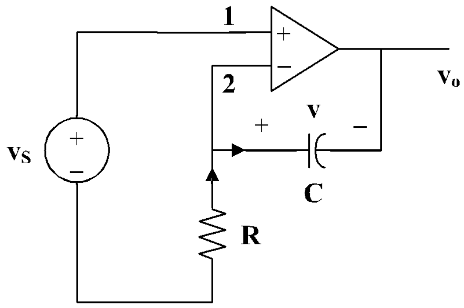
Figure 7.135
For Prob. 7.70.

Chapter 7, Solution 70.

Let v = capacitor voltage.

For t < 0, the switch is open and $v(0) = 0$.

For t > 0, the switch is closed and the circuit becomes as shown below.



$$v_1 = v_2 = v_s \qquad (1)$$

$$\frac{0 - v_s}{R} = C\frac{dv}{dt} \qquad (2)$$

where $v = v_s - v_o \longrightarrow v_o = v_s - v \qquad (3)$

From (1),

$$\frac{dv}{dt} = \frac{v_s}{RC} = 0$$

$$v = \frac{-1}{RC}\int v_s \, dt + v(0) = \frac{-t\,v_s}{RC}$$

Since v is constant,

$$RC = (20\times10^3)(5\times10^{-6}) = 0.1$$

$$v = \frac{-20t}{0.1}\ \text{mV} = -200t\ \text{mV}$$

From (3),

$$v_o = v_s - v = 20 + 200t$$

$$v_o = \underline{\mathbf{20(1+10t)\ mV}}$$

Chapter 7, Problem 71.

For the op amp circuit in Fig. 7.136, suppose $v_0 = 0$ and $v_s = 3$ V. Find $v(t)$ for $t > 0$.

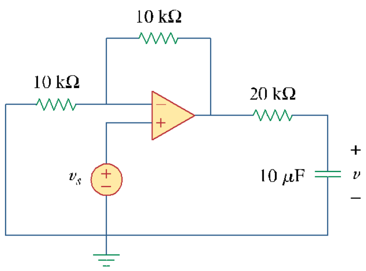
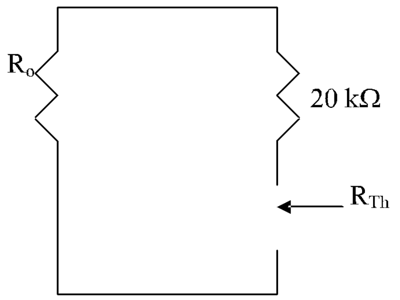


Figure 7.136
For Prob. 7.71.

Chapter 7, Solution 71.

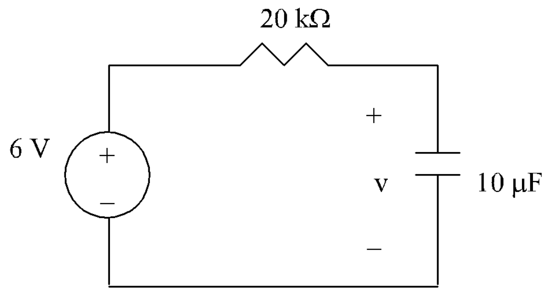
We temporarily remove the capacitor and find the Thevenin equivalent at its terminals. To find R_{Th}, we consider the circuit below.



Since we are assuming an ideal op amp, $R_o = 0$ and R_{Th}=20kΩ . The op amp circuit is a noninverting amplifier. Hence,

$$V_{Th} = (1+\frac{10}{10})v_s = 2v_s = 6V$$

The Thevenin equivalent is shown below.



Thus,

$v(t) = 6(1 - e^{-t/\tau}), t > 0$

where $\tau = R_{TH}C = 20x10^{-3}x10x10^{-6} = 0.2$

$\underline{v(t) = 6(1 - e^{-5t}), t > 0}$ $\underline{\text{V}}$

Chapter 7, Problem 72.

Find i_o in the op amp circuit in Fig. 7.137. Assume that $v(0)$ = -2 V, $R = 10\ \text{k}\Omega$, and $C = 10\ \mu\text{F}$.

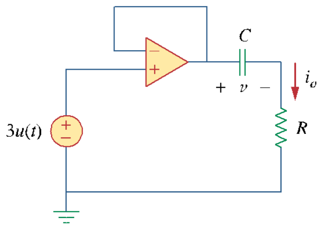
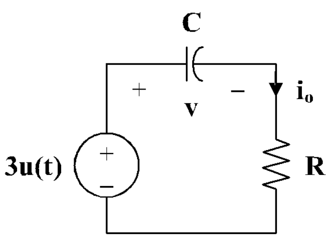


Figure 7.137
For Prob. 7.72.

Chapter 7, Solution 72.

The op amp acts as an emitter follower so that the Thevenin equivalent circuit is shown below.



Hence,

$$v(t) = v(\infty) + \left[\, v(0) - v(\infty) \right] e^{-t/\tau}$$

$$v(0) = \text{-}2\ \text{V}, \quad v(\infty) = 3\ \text{V}, \quad \tau = RC = (10 \times 10^{3})(10 \times 10^{-6}) = 0.1$$

$$v(t) = 3 + (\text{-}2 - 3)\, e^{-10t} = 3 - 5\, e^{-10t}$$

$$i_o = C \frac{dv}{dt} = (10 \times 10^{-6})(\text{-}5)(\text{-}10)\, e^{-10t}$$

$$i_o = \underline{\mathbf{0.5\, e^{-10t}\ mA, \quad t > 0}}$$

Chapter 7, Problem 73.

For the op amp circuit in Fig. 7.138, let $R_1 = 10\ \text{k}\Omega$, $R_f = 20\ \text{k}\Omega$, $C = 20\ \mu\text{F}$, and $v(0) = 1$ V. Find v_0.

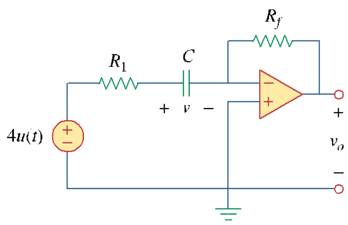
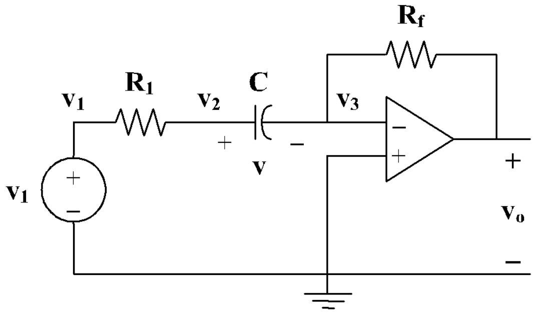


Figure 7.138
For Prob. 7.73.

Chapter 7, Solution 73.

Consider the circuit below.



At node 2,

$$\frac{v_1 - v_2}{R_1} = C\frac{dv}{dt} \qquad (1)$$

At node 3,

$$C\frac{dv}{dt} = \frac{v_3 - v_o}{R_f} \qquad (2)$$

But $v_3 = 0$ and $v = v_2 - v_3 = v_2$. Hence, (1) becomes

$$\frac{v_1 - v}{R_1} = C\frac{dv}{dt}$$

$$v_1 - v = R_1 C\frac{dv}{dt}$$

or $$\frac{dv}{dt} + \frac{v}{R_1 C} = \frac{v_1}{R_1 C}$$

which is similar to Eq. (7.42). Hence,

$$v(t) = \begin{cases} v_T & t < 0 \\ v_1 + (v_T - v_1)e^{-t/\tau} & t > 0 \end{cases}$$

where $v_T = v(0) = 1$ and $v_1 = 4$

$$\tau = R_1 C = (10\times10^3)(20\times10^{-6}) = 0.2$$

$$v(t) = \begin{cases} 1 & t < 0 \\ 4 - 3e^{-5t} & t > 0 \end{cases}$$

From (2),

$$v_o = -R_f C\frac{dv}{dt} = (20\times10^3)(20\times10^{-6})(15e^{-5t})$$

$$v_o = -6e^{-5t}, \quad t > 0$$

$$v_o = \underline{\mathbf{-6e^{-5t}\,u(t)\ V}}$$

Chapter 7, Problem 74.

Determine $v_0(t)$ for $t > 0$ in the circuit of Fig. 7.139. Let $i_s = 10u(t)$ μA and assume that the capacitor is initially uncharged.

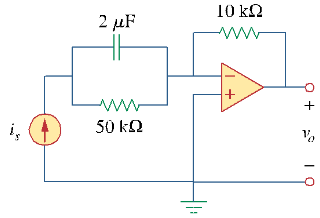
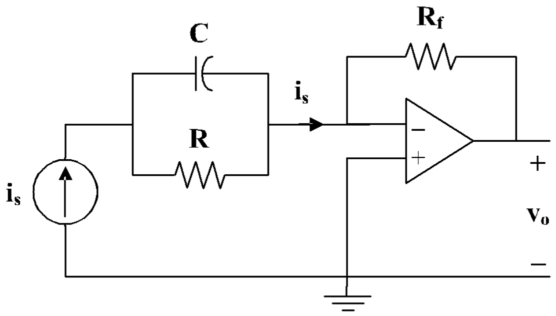


Figure 7.139
For Prob. 7.74.

Chapter 7, Solution 74.

Let v = capacitor voltage. For t < 0, v(0) = 0



For t > 0, $i_s = 10\ \mu A$. Consider the circuit below.

$$i_s = C\frac{dv}{dt} + \frac{v}{R} \qquad (1)$$

$$v(t) = v(\infty) + \left[\, v(0) - v(\infty)\right] e^{-t/\tau} \qquad (2)$$

It is evident from the circuit that

$$\tau = RC = (2\times10^{-6})(50\times10^{3}) = 0.1$$

At steady state, the capacitor acts like an open circuit so that i_s passes through R. Hence,

$$v(\infty) = i_s R = (10\times10^{-6})(50\times10^{3}) = 0.5\ \text{V}$$

Then,

$$v(t) = 0.5\left(1 - e^{-10t}\right)\ \text{V} \qquad (3)$$

But $i_s = \dfrac{0 - v_o}{R_f} \longrightarrow v_o = -i_s R_f \qquad (4)$

Combining (1), (3), and (4), we obtain

$$v_o = \frac{-R_f}{R}v - R_f C\frac{dv}{dt}$$

$$v_o = \frac{-1}{5}v - (10\times10^{3})(2\times10^{-6})\frac{dv}{dt}$$

$$v_o = -0.1 + 0.1e^{-10t} - (2\times10^{-2})(0.5)\left(-10e^{-10t}\right)$$

$$v_o = 0.2e^{-10t} - 0.1$$

$$v_o = \mathbf{0.1\left(2e^{-10t} - 1\right)\ V}$$

Chapter 7, Problem 75.

In the circuit of Fig. 7.140, find v_0 and i_0, given that $v_s = 4u(t)$ V and $v(0) = 1$ V.

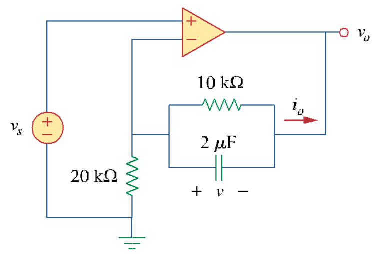


Figure 7.140
For Prob. 7.75.

Chapter 7, Solution 75.

Let v_1 = voltage at the noninverting terminal.

Let v_2 = voltage at the inverting terminal.

For t > 0, $v_1 = v_2 = v_s = 4$

$$\frac{0 - v_s}{R_1} = i_o, \quad R_1 = 20\text{ k}\Omega$$

$$v_o = -i_o R \qquad (1)$$

Also, $i_o = \frac{v}{R_2} + C\frac{dv}{dt}, \qquad R_2 = 10\text{ k}\Omega, \quad C = 2\ \mu\text{F}$

i.e. $$\frac{-v_s}{R_1} = \frac{v}{R_2} + C\frac{dv}{dt} \qquad (2)$$

This is a step response.

$$v(t) = v(\infty) + [\,v(0) - v(\infty)]\,e^{-t/\tau}, \qquad v(0) = 1$$

where $\tau = R_2 C = (10 \times 10^3)(2 \times 10^{-6}) = \frac{1}{50}$

At steady state, the capacitor acts like an open circuit so that i_o passes through R_2. Hence, as $t \to \infty$

$$\frac{-v_s}{R_1} = i_o = \frac{v(\infty)}{R_2}$$

i.e. $$v(\infty) = \frac{-R_2}{R_1} v_s = \frac{-10}{20}(4) = -2$$

$$v(t) = -2 + (1 + 2)\,e^{-50t}$$

$$v(t) = -2 + 3\,e^{-50t}$$

But $v = v_s - v_o$

or $v_o = v_s - v = 4 + 2 - 3\,e^{-50t}$

$$v_o = \underline{6 - 3\,e^{-50t}\ u(t)\text{V}}$$

$$i_o = \frac{-v_s}{R_1} = \frac{-4}{20k} = -0.2\text{ mA}$$

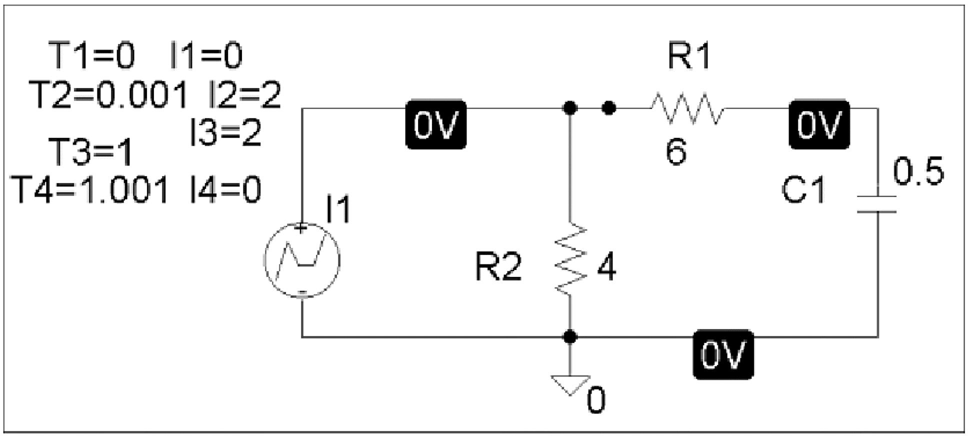
or $i_o = \frac{v}{R_2} + C\frac{dv}{dt} = \underline{\mathbf{-\ 0.2\ mA}}$

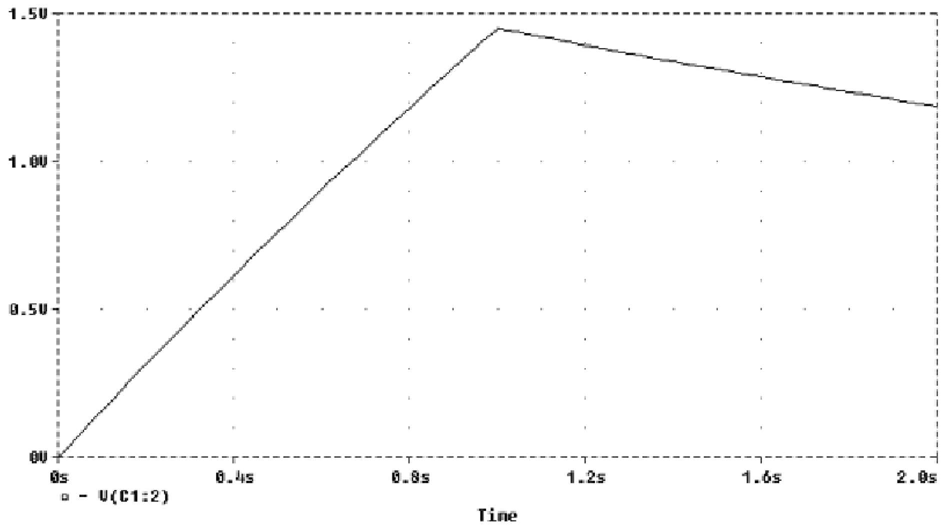
Chapter 7, Problem 76.

Repeat Prob. 7.49 using *PSpice*.

Chapter 7, Solution 76.

The schematic is shown below. For the pulse, we use IPWL and enter the corresponding values as attributes as shown. By selecting Analysis/Setup/Transient, we let Print Step = 25 ms and Final Step = 2 s since the width of the input pulse is 1 s. After saving and simulating the circuit, we select Trace/Add and display –V(C1:2). The plot of V(t) is shown below.





Chapter 7, Problem 77.

The switch in Fig. 7.141 opens at $t = 0$. Use *PSpice* to determine $v(t)$ for $t > 0$.

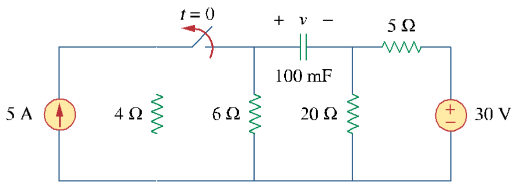
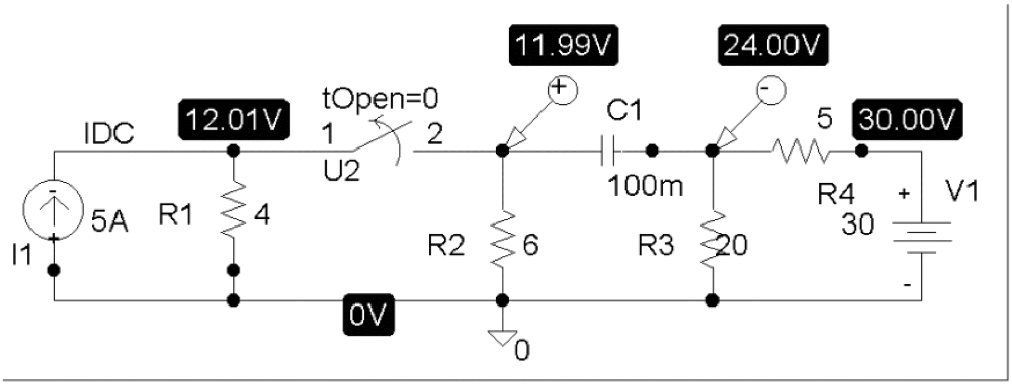
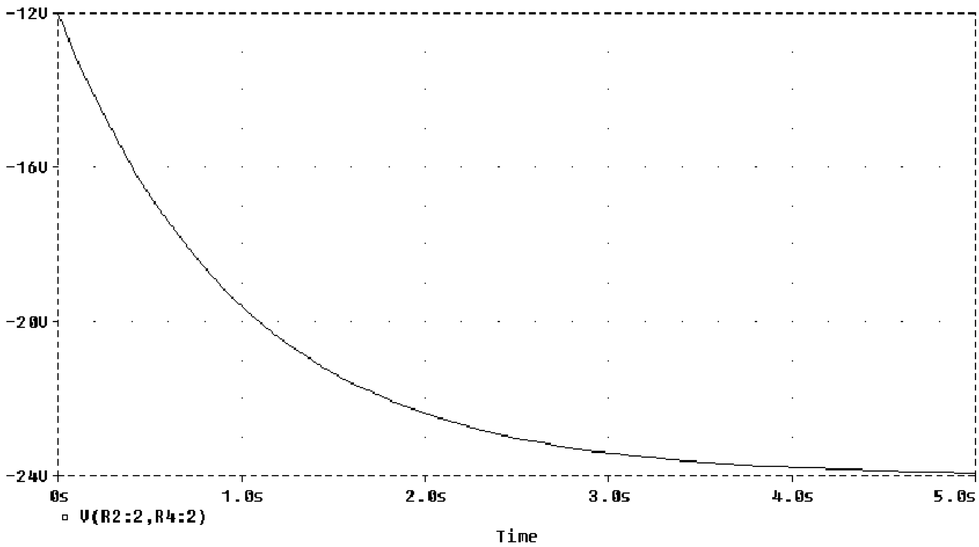


Figure 7.141
For Prob. 7.77.

Chapter 7, Solution 77.

The schematic is shown below. We click Marker and insert Mark Voltage Differential at the terminals of the capacitor to display V after simulation. The plot of V is shown below. Note from the plot that V(0) = 12 V and V(∞) = -24 V which are correct.





Chapter 7, Problem 78.

The switch in Fig. 7.142 moves from position *a* to *b* at $t = 0$. Use *PSpice* to find $i(t)$ for $t > 0$.

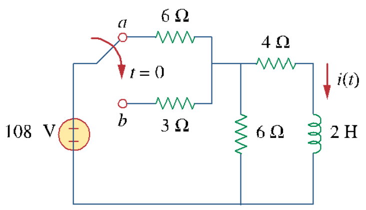


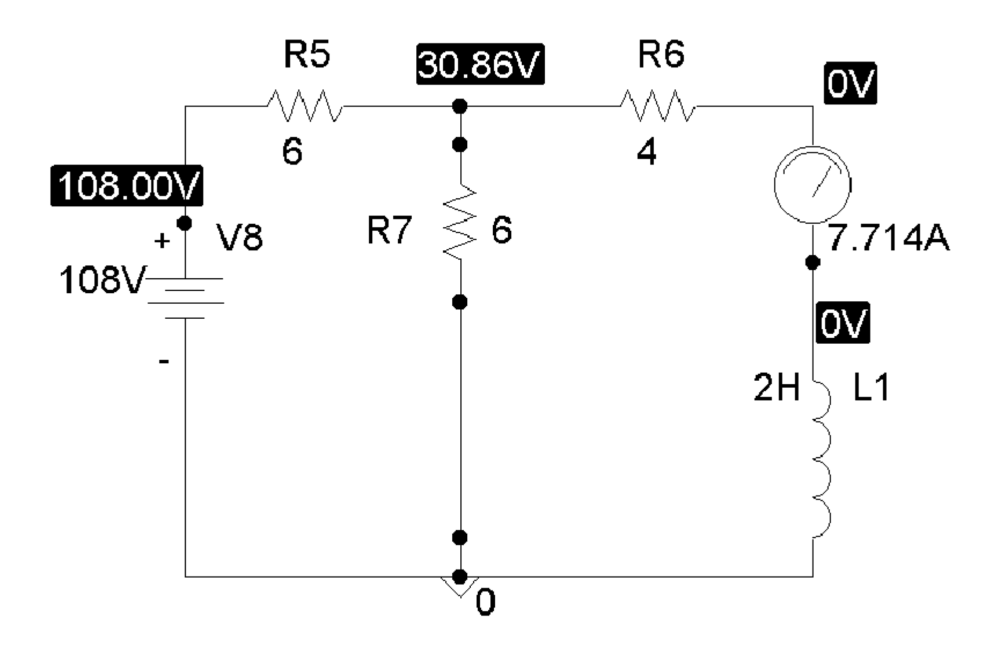
Figure 7.142
For Prob. 7.78.

Chapter 7, Solution 78.

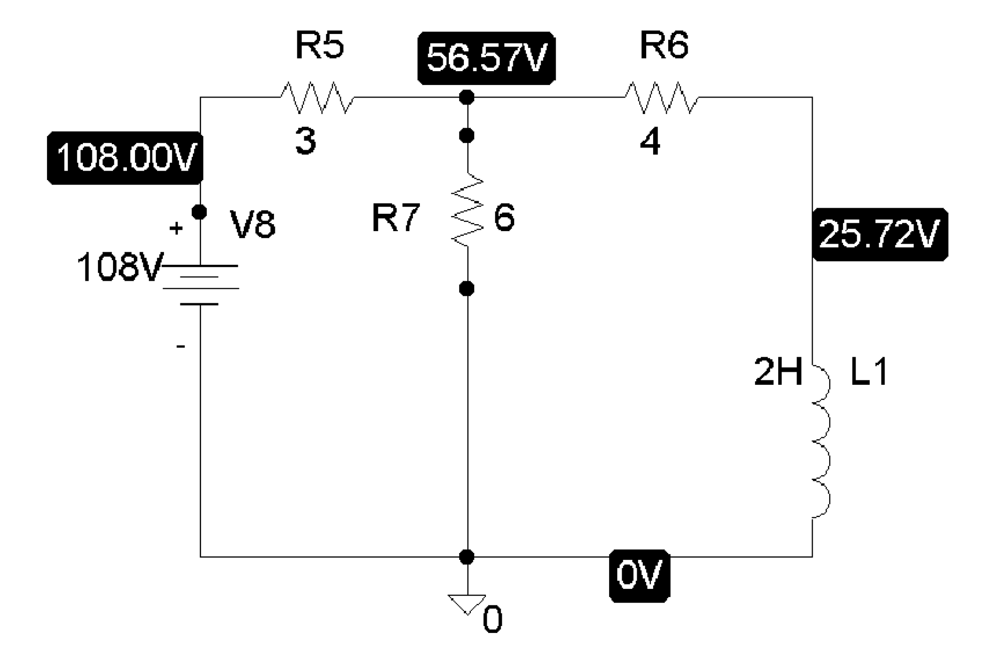
(a) When the switch is in position (a), the schematic is shown below. We insert IPROBE to display i. After simulation, we obtain,

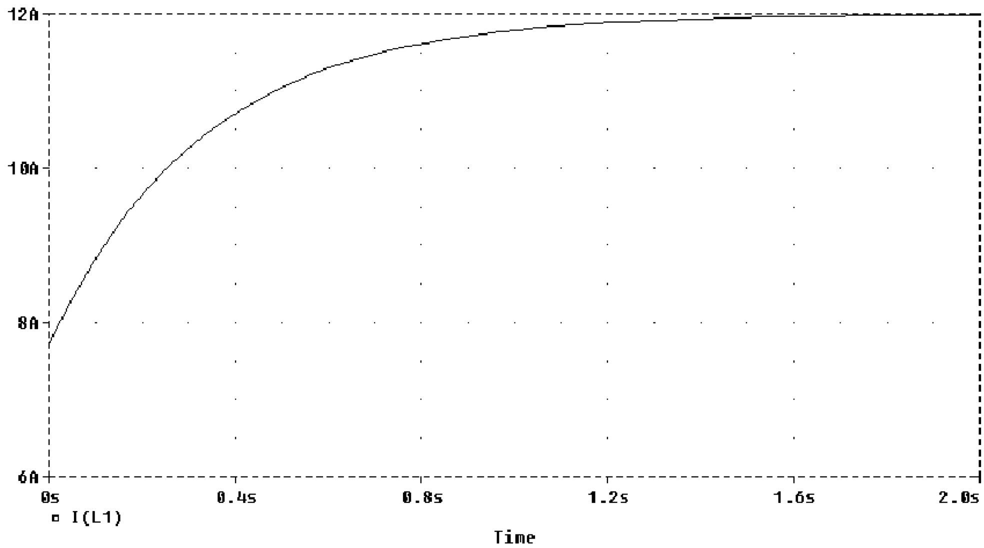
$$i(0) = 7.714 \text{ A}$$

from the display of IPROBE.



(b) When the switch is in position (b), the schematic is as shown below. For inductor I1, we let IC = 7.714. By clicking Analysis/Setup/Transient, we let Print Step = 25 ms and Final Step = 2 s. After Simulation, we click Trace/Add in the probe menu and display I(L1) as shown below. Note that i(∞) = 12A, which is correct.


R5
56.57V
R6
108.00V
3
4
V8
R7
6
108V
25.72V
2H
L1
0V
0


12A
10A
8A
6A
0s
0.4s
0.8s
1.2s
1.6s
2.0s
I(L1)
Time

Chapter 7, Problem 79.

In the circuit of Fig. 7.143, the switch has been in position *a* for a long time but moves instantaneously to position *b* at $t = 0$ Determine $i_0(t)$.

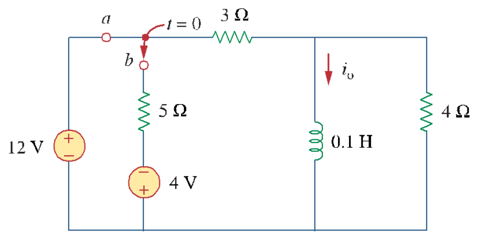


Figure 7.143
For Prob. 7.79.

Chapter 7, Solution 79.

When the switch is in position 1, $i_o(0) = 12/3 = 4A$. When the switch is in position 2,

$$i_o(\infty) = -\frac{4}{5+3} = -0.5 \text{ A}, \qquad R_{Th} = (3+5)//4 = 8/3, \qquad \tau = \frac{L}{R_{Th}} = 3/80$$

$$i_o(t) = i_o(\infty) + [i_o(0) - i_o(\infty)]e^{-t/\tau} = \underline{-0.5 + 4.5e^{-80t/3}}\ u(t)A$$

Chapter 7, Problem 80.

In the circuit of Fig. 7.144, assume that the switch has been in position *a* for a long time, find:

(a) $i_1(0)$, $i_2(0)$, and $v_0(0)$

(b) $i_L(t)$

(c) $i_1(\infty)$, $i_2(\infty)$, and $v_0(\infty)$.

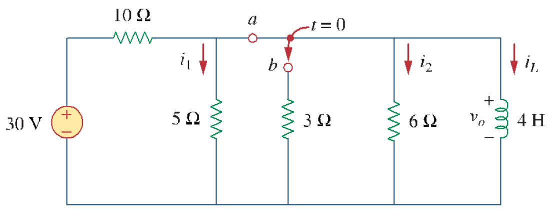


Figure 7.144
For Prob. 7.80.

Chapter 7, Solution 80.

(a) When the switch is in position A, the 5-ohm and 6-ohm resistors are short-circuited so that

$$\underline{i_1(0) = \; i_2(0) = v_o(0) = 0}$$

but the current through the 4-H inductor is $i_L(0) = 30/10 = 3A$.

(b) When the switch is in position B,

$$R_{Th} = 3//6 = 2\Omega, \qquad \tau = \frac{L}{R_{Th}} = 4/2 = 2\,\text{sec}$$

$$i_L(t) = i_L(\infty) + [i_L(0) - i_L(\infty)]e^{-t/\tau} = 0 + 3e^{-t/2} = \underline{3e^{-t/2}\ A}$$

(c) $i_1(\infty) = \dfrac{30}{10+5} = \underline{2\ A}, \qquad i_2(\infty) = -\dfrac{3}{9} i_L(\infty) = \underline{0\ A}$

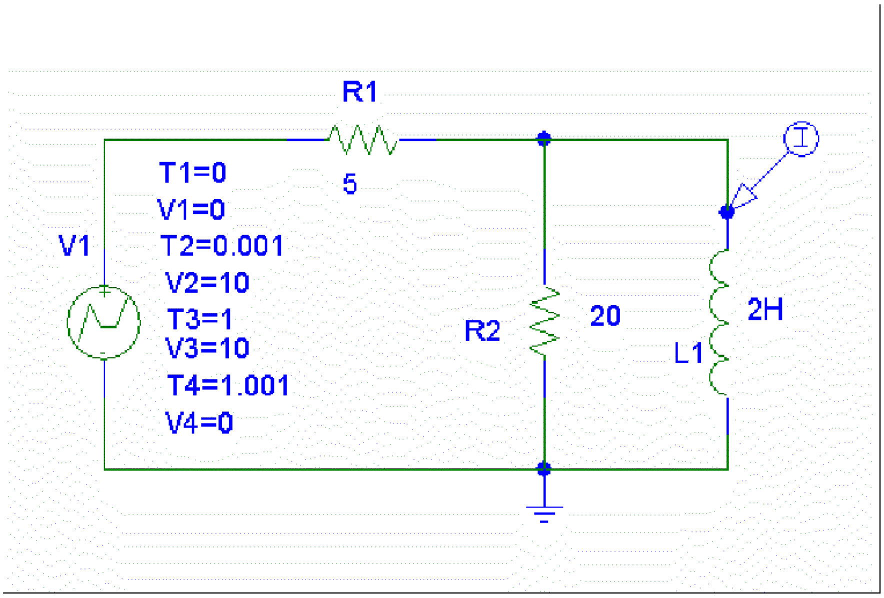
$$v_o(t) = L\frac{di_L}{dt} \longrightarrow \underline{v_o(\infty) = 0\ V}$$

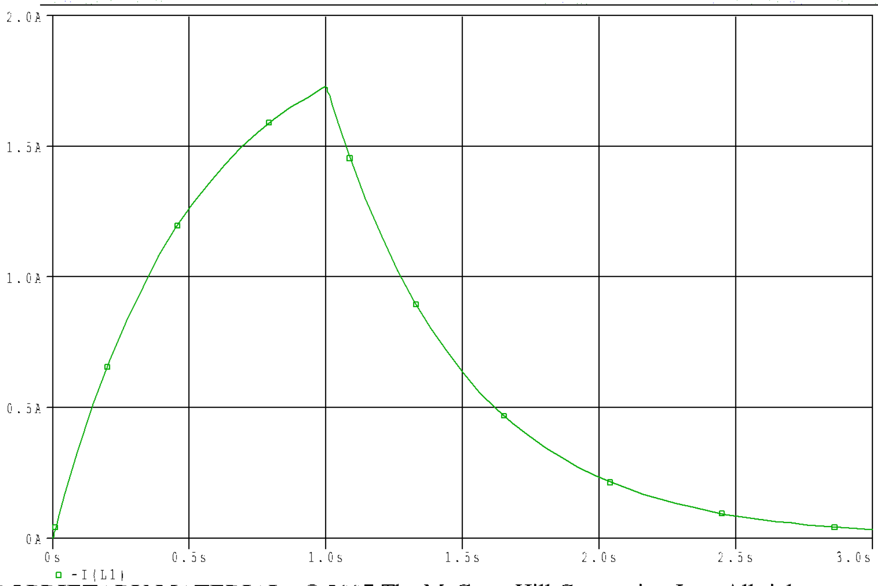
Chapter 7, Problem 81.

Repeat Prob. 7.65 using *PSpice*.

Chapter 7, Solution 81.

The schematic is shown below. We use VPWL for the pulse and specify the attributes as shown. In the Analysis/Setup/Transient menu, we select Print Step = 25 ms and final Step = 3 S. By inserting a current marker at one terminal of LI, we automatically obtain the plot of i after simulation as shown below.





Chapter 7, Problem 82.

In designing a signal-switching circuit, it was found that a 100-μF capacitor was needed for a time constant of 3 ms. What value resistor is necessary for the circuit?

Chapter 7, Solution 82.

$$\tau = RC \longrightarrow R = \frac{\tau}{C} = \frac{3\times 10^{-3}}{100\times 10^{-6}} = \underline{\mathbf{30\ \Omega}}$$

Chapter 7, Problem 83.

An RC circuit consists of a series connection of a 120-V source, a switch, a 34-MΩ resistor, and a 15-μF capacitor. The circuit is used in estimating the speed of a horse running a 4-km racetrack. The switch closes when the horse begins and opens when the horse crosses the finish line. Assuming that the capacitor charges to 85.6 V, calculate the speed of the horse.

Chapter 7, Solution 83.

$$v(\infty) = 120, \qquad v(0) = 0, \qquad \tau = RC = 34x10^{6}x15x10^{-6} = 510s$$

$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau} \qquad \longrightarrow \qquad 85.6 = 120(1 - e^{-t/510})$$

Solving for t gives

$$t = 510\ln 3.488 = 637.16s$$

$$\text{speed} = 4000\text{m}/637.16\text{s} = \underline{\mathbf{6.278m/s}}$$

Chapter 7, Problem 84.

The resistance of a 160-mH coil is 8Ω. Find the time required for the current to build up to 60 percent of its final value when voltage is applied to the coil.

Chapter 7, Solution 84.

Let I_o be the final value of the current. Then

$$i(t) = I_o(1 - e^{-t/\tau}), \qquad \tau = R/L = 0.16/8 = 1/50$$

$$0.6I_o = I_o(1 - e^{-50t}) \qquad \longrightarrow \qquad t = \frac{1}{50}\ln\frac{1}{0.4} = \underline{18.33\text{ ms}}.$$

Chapter 7, Problem 85.

A simple relaxation oscillator circuit is shown in Fig. 7.145. The neon lamp fires when its voltage reaches 75 V and turns off when its voltage drops to 30 V. Its resistance is 120Ω when on and infinitely high when off.

(a) For how long is the lamp on each time the capacitor discharges?
(b) What is the time interval between light flashes?

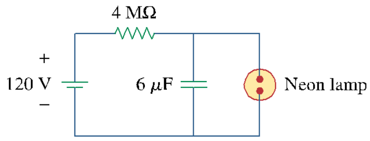


Figure 7.145
For Prob. 7.85.

Chapter 7, Solution 85.

(a) The light is on from 75 volts until 30 volts. During that time we essentially have a 120-ohm resistor in parallel with a 6-µF capacitor.

$v(0) = 75,\ v(\infty) = 0,\ \tau = 120x6x10^{-6} = 0.72$ ms

$v(t_1) = 75\,e^{-t_1/\tau} = 30$ which leads to $t_1 = -0.72\ln(0.4)$ ms = **<u>659.7 µs</u>** of lamp on time.

(b) $\tau = RC = (4\times10^6)(6\times10^{-6}) = 24$ s

Since $v(t) = v(\infty) + [\,v(0) - v(\infty)\,]\,e^{-t/\tau}$

$$v(t_1) - v(\infty) = [\,v(0) - v(\infty)\,]\,e^{-t_1/\tau} \qquad (1)$$

$$v(t_2) - v(\infty) = [\,v(0) - v(\infty)\,]\,e^{-t_2/\tau} \qquad (2)$$

Dividing (1) by (2),

$$\frac{v(t_1) - v(\infty)}{v(t_2) - v(\infty)} = e^{(t_2 - t_1)/\tau}$$

$$t_0 = t_2 - t_1 = \tau\ln\left(\frac{v(t_1) - v(\infty)}{v(t_2) - v(\infty)}\right)$$

$$t_0 = 24\ln\left(\frac{75 - 120}{30 - 120}\right) = 24\ln(2) = \mathbf{\underline{16.636\ s}}$$

Chapter 7, Problem 86.

Figure 7.146 shows a circuit for setting the length of time voltage is applied to the electrodes of a welding machine. The time is taken as how long it takes the capacitor to charge from 0 to 8 V. What is the time range covered by the variable resistor?

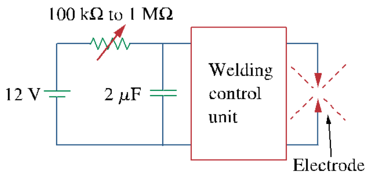


Figure 7.146
For Prob. 7.86.

Chapter 7, Solution 86.

$$v(t) = v(\infty) + \left[v(0) - v(\infty) \right] e^{-t/\tau}$$

$$v(\infty) = 12, \qquad v(0) = 0$$

$$v(t) = 12\left(1 - e^{-t/\tau}\right)$$

$$v(t_0) = 8 = 12\left(1 - e^{-t_0/\tau}\right)$$

$$\frac{8}{12} = 1 - e^{-t_0/\tau} \longrightarrow e^{-t_0/\tau} = \frac{1}{3}$$

$$t_0 = \tau \ln(3)$$

For $R = 100\ \text{k}\Omega$,

$$\tau = RC = (100 \times 10^3)(2 \times 10^{-6}) = 0.2\ \text{s}$$

$$t_0 = 0.2 \ln(3) = 0.2197\ \text{s}$$

For $R = 1\ \text{M}\Omega$,

$$\tau = RC = (1 \times 10^6)(2 \times 10^{-6}) = 2\ \text{s}$$

$$t_0 = 2 \ln(3) = 2.197\ \text{s}$$

Thus,

$$\underline{\mathbf{0.2197\ s < t_0 < 2.197\ s}}$$

Chapter 7, Problem 87.

e2d

A 120-V dc generator energizes a motor whose coil has an inductance of 50 H and a resistance of $100\,\Omega$. A field discharge resistor of $400\,\Omega$ is connected in parallel with the motor to avoid damage to the motor, as shown in Fig. 7.147. The system is at steady state. Find the current through the discharge resistor 100 ms after the breaker is tripped.

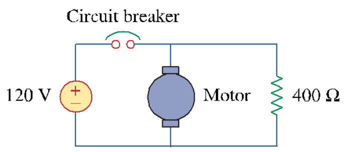


Figure 7.147
For Prob. 7.87.

Chapter 7, Solution 87.

Let i be the inductor current.

For $t < 0$, $\quad i(0^-) = \dfrac{120}{100} = 1.2\text{ A}$

For $t > 0$, we have an RL circuit

$$\tau = \frac{L}{R} = \frac{50}{100+400} = 0.1, \qquad i(\infty) = 0$$

$$i(t) = i(\infty) + \left[\, i(0) - i(\infty)\right] e^{-t/\tau}$$

$$i(t) = 1.2\,e^{-10t}$$

At $t = 100$ ms $= 0.1$ s,

$$i(0.1) = 1.2\,e^{-1} = \underline{\mathbf{441mA}}$$

which is the same as the current through the resistor.

Chapter 7, Problem 88.

The circuit in Fig. 7.148(a) can be designed as an approximate differentiator or an integrator, depending on whether the output is taken across the resistor or the capacitor, and also on the time constant τ = RC of the circuit and the width T of the input pulse in Fig. 7.148(b). The circuit is a differentiator if $\tau \ll T$, say $\tau < 0.1T$, or an integrator if $\tau \gg T$, say $\tau > 10T$.

(a) What is the minimum pulse width that will allow a differentiator output to appear across the capacitor?
(b) If the output is to be an integrated form of the input, what is the maximum value the pulse width can assume?

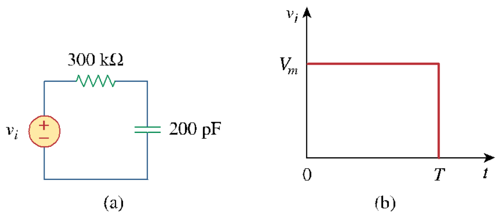


Figure 7.148
For Prob. 7.88.

Chapter 7, Solution 88.

(a) $\tau = RC = (300\times 10^3)(200\times 10^{-12}) = 60\ \mu s$
As a differentiator,
$T > 10\tau = 600\ \mu s = 0.6\ ms$
i.e. $T_{min} = \mathbf{0.6\ ms}$

(b) $\tau = RC = 60\ \mu s$
As an integrator,
$T < 0.1\tau = 6\ \mu s$
i.e. $T_{max} = \mathbf{6\ \mu s}$

Chapter 7, Problem 89.

An *RL* circuit may be used as a differentiator if the output is taken across the inductor and $\tau \ll T$ (say $\tau < 0.1T$), where T is the width of the input pulse. If R is fixed at 200 kΩ determine the maximum value of L required to differentiate a pulse with $T = 10\ \mu$s.

Chapter 7, Solution 89.

$$\text{Since } \tau < 0.1\text{T} = 1\ \mu\text{s}$$

$$\frac{\text{L}}{\text{R}} < 1\ \mu\text{s}$$

$$\text{L} < \text{R} \times 10^{-6} = (200 \times 10^{3})(1 \times 10^{-6})$$

$$\mathbf{L < 200\ mH}$$

Chapter 7, Problem 90.

An attenuator probe employed with oscilloscopes was designed to reduce the magnitude of the input voltage v_i by a factor of 10. As shown in Fig. 7.149, the oscilloscope has internal resistance R_s and capacitance C_s while the probe has an internal resistance R_p. If R_p is fixed at 6 MΩ find R_s and C_s for the circuit to have a time constant of 15 μs.

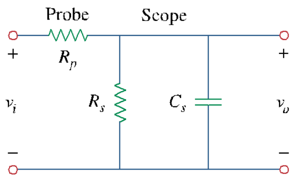
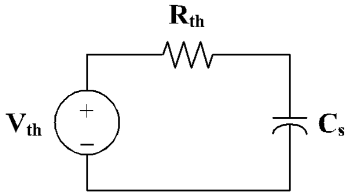


Figure 7.149
For Prob. 7.90.

Chapter 7, Solution 90.

We determine the Thevenin equivalent circuit for the capacitor C_s.

$$V_{th} = \frac{R_s}{R_s + R_p} V_i, \qquad R_{th} = R_s \parallel R_p$$



The Thevenin equivalent is an RC circuit. Since

$$V_{th} = \frac{1}{10} V_i \longrightarrow \frac{1}{10} = \frac{R_s}{R_s + R_p}$$

$$R_s = \frac{1}{9} R_p = \frac{6}{9} = \underline{\mathbf{\frac{2}{3}\ M\Omega}}$$

Also,

$$\tau = R_{th} C_s = 15\ \mu s$$

where $R_{th} = R_p \parallel R_s = \dfrac{6(2/3)}{6 + 2/3} = 0.6\ M\Omega$

$$C_s = \frac{\tau}{R_{th}} = \frac{15 \times 10^{-6}}{0.6 \times 10^{6}} = \underline{\mathbf{25\ pF}}$$

Chapter 7, Problem 91.

The circuit in Fig. 7.150 is used by a biology student to study "frog kick." She noticed that the frog kicked a little when the switch was closed but kicked violently for 5 s when the switch was opened. Model the frog as a resistor and calculate its resistance. Assume that it takes 10 mA for the frog to kick violently.

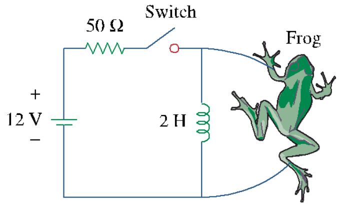


Figure 7.150
For Prob. 7.91.

Chapter 7, Solution 91.

$$i_o(0) = \frac{12}{50} = 240\text{ mA}, \qquad i(\infty) = 0$$

$$i(t) = i(\infty) + \left[\, i(0) - i(\infty) \right] e^{-t/\tau}$$

$$i(t) = 240 e^{-t/\tau}$$

$$\tau = \frac{L}{R} = \frac{2}{R}$$

$$i(t_0) = 10 = 240 e^{-t_0/\tau}$$

$$e^{t_0/\tau} = 24 \longrightarrow t_0 = \tau \ln(24)$$

$$\tau = \frac{t_0}{\ln(24)} = \frac{5}{\ln(24)} = 1.573 = \frac{2}{R}$$

$$R = \frac{2}{1.573} = \underline{\mathbf{1.271\,\Omega}}$$

Chapter 7, Problem 92.

To move a spot of a cathode-ray tube across the screen requires a linear increase in the voltage across the deflection plates, as shown in Fig. 7.151. Given that the capacitance of the plates is 4 nF, sketch the current flowing through the plates.

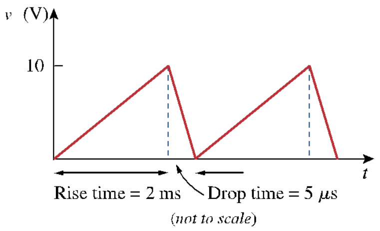


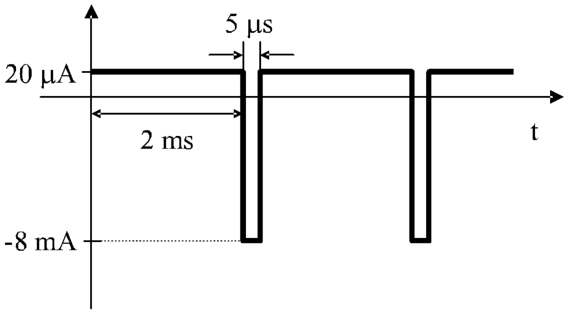
Figure 7.151
For Prob. 7.92.

Chapter 7, Solution 92.

$$i = C\frac{dv}{dt} = 4\times10^{-9}\cdot\begin{cases}\dfrac{10}{2\times10^{-3}} & 0<t<t_R\\ \dfrac{-10}{5\times10^{-6}} & t_R<t<t_D\end{cases}$$

$$i(t) = \begin{cases}20\ \mu A & 0<t<2\ ms\\ -8\ mA & 2\ ms<t<2\ ms+5\ \mu s\end{cases}$$

which is sketched below.



(not to scale)

Chapter 8, Problem 1.

For the circuit in Fig. 8.62, find:

(a) $i(0^+)$ and $v(0^+)$,

(b) $di(0^+)/dt$ and $dv(0^+)/dt$,

(c) $i(\infty)$ and $v(\infty)$.

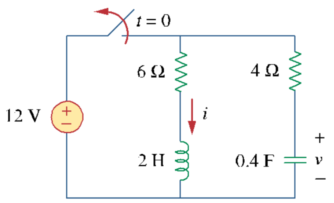
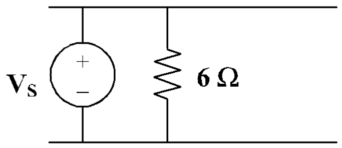


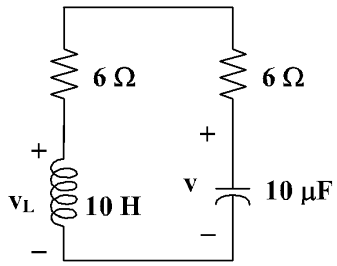
Figure 8.62
For Prob. 8.1.

Chapter 8, Solution 1.

(a) At $t = 0-$, the circuit has reached steady state so that the equivalent circuit is shown in Figure (a).



(a)



(b)

$$i(0-) = 12/6 = 2A, \quad v(0-) = 12V$$

At $t = 0+$, $i(0+) = i(0-) =$ **2A**, $v(0+) = v(0-) =$ **12V**

(b) For $t > 0$, we have the equivalent circuit shown in Figure (b).

$$v_L = Ldi/dt \text{ or } di/dt = v_L/L$$

Applying KVL at $t = 0+$, we obtain,

$$v_L(0+) - v(0+) + 10i(0+) = 0$$

$$v_L(0+) - 12 + 20 = 0, \text{ or } v_L(0+) = -8$$

Hence, $di(0+)/dt = -8/2 =$ **-4 A/s**

Similarly, $i_C = Cdv/dt$, or $dv/dt = i_C/C$

$$i_C(0+) = -i(0+) = -2$$

$$dv(0+)/dt = -2/0.4 = \textbf{-5 V/s}$$

(c) As t approaches infinity, the circuit reaches steady state.

$$i(\infty) = \textbf{0 A}, \quad v(\infty) = \textbf{0 V}$$

Chapter 8, Problem 2.

In the circuit of Fig. 8.63, determine:

(a) $i_R(0^+)$, $i_L(0^+)$, and $i_C(0^+)$,

(b) $di_R(0^+)/dt$, $di_L(0^+)/dt$, and $di_C(0^+)/dt$,

(c) $i_R(\infty)$, $i_L(\infty)$, and $i_C(\infty)$.

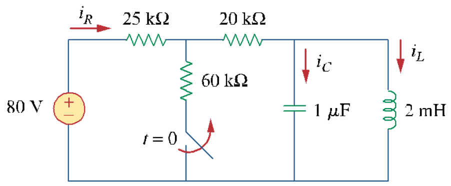
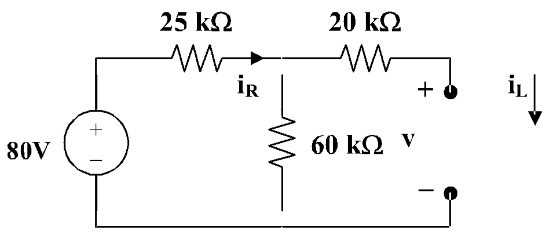


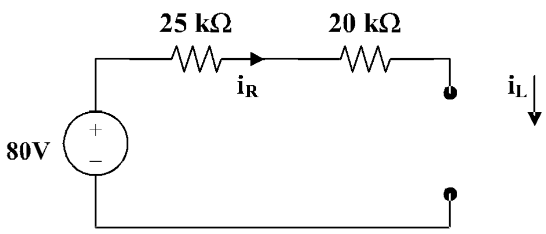
Figure 8.63
For Prob. 8.2.

Chapter 8, Solution 2.

(a) At t = 0-, the equivalent circuit is shown in Figure (a).



(a)



(b)

60||20 = 15 kohms, $i_R(0\text{-}) = 80/(25 + 15) = 2\text{mA}$.

By the current division principle,

$$i_L(0-) = 60(2mA)/(60 + 20) = 1.5 \text{ mA}$$

$$v_C(0-) = 0$$

At t = 0+,

$$v_C(0+) = v_C(0-) = 0$$

$$i_L(0+) = i_L(0-) = \underline{\mathbf{1.5\ mA}}$$

$$80 = i_R(0+)(25 + 20) + v_C(0-)$$

$$i_R(0+) = 80/45k = \underline{\mathbf{1.778\ mA}}$$

But, $i_R = i_C + i_L$

$$1.778 = i_C(0+) + 1.5 \text{ or } i_C(0+) = \underline{\mathbf{0.278\ mA}}$$

(b) $v_L(0+) = v_C(0+) = 0$

But, $v_L = Ldi_L/dt$ and $di_L(0+)/dt = v_L(0+)/L = 0$

$$di_L(0+)/dt = \underline{\mathbf{0}}$$

$$\text{Again, } 80 = 45i_R + v_C$$

$$0 = 45di_R/dt + dv_C/dt$$

But, $dv_C(0+)/dt = i_C(0+)/C = 0.278 \text{ mohms}/1\ \mu F = 278 \text{ V/s}$

Hence, $di_R(0+)/dt = (-1/45)dv_C(0+)/dt = -278/45$

$$di_R(0+)/dt = \underline{\mathbf{-6.1778\ A/s}}$$

$$\text{Also, } i_R = i_C + i_L$$

$$di_R(0+)/dt = di_C(0+)/dt + di_L(0+)/dt$$

$$-6.1788 = di_C(0+)/dt + 0, \text{ or } di_C(0+)/dt = \underline{\mathbf{-6.1788\ A/s}}$$

(c) As t approaches infinity, we have the equivalent circuit in Figure (b).

$$i_R(\infty) = i_L(\infty) = 80/45k = \underline{\mathbf{1.778\ mA}}$$

$$i_C(\infty) = Cdv(\infty)/dt = \underline{\mathbf{0}}.$$

Chapter 8, Problem 3.

Refer to the circuit shown in Fig. 8.64. Calculate:

(a) $i_L(0^+)$, $v_c(0^+)$and $v_R(0^+)$,

(b) $di_L(0^+)/dt$, $dv_c(0^+)/dt$, and $dv_R(0^+)/dt$,

(c) $i_L(\infty)$, $v_c(\infty)$and $v_R(\infty)$

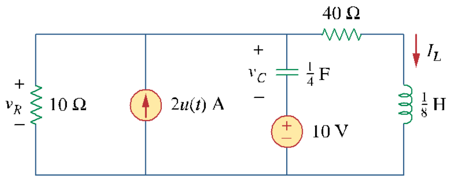


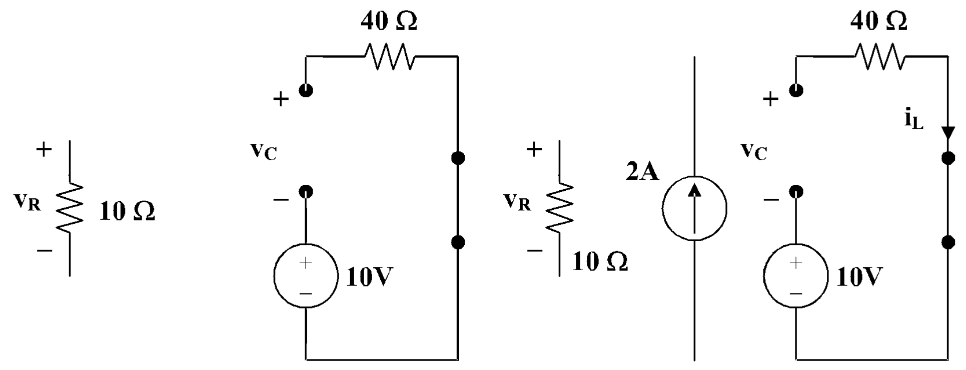
Figure 8.64
For Prob. 8.3.

Chapter 8, Solution 3.

At t = 0^-, u(t) = 0. Consider the circuit shown in Figure (a). $i_L(0^-)$ = 0, and $v_R(0^-)$ = 0. But, $-v_R(0^-) + v_C(0^-) + 10 = 0$, or $v_C(0^-)$ = -10V.

(a) At t = 0^+, since the inductor current and capacitor voltage cannot change abruptly, the inductor current must still be equal to **<u>0A</u>**, the capacitor has a voltage equal to **<u>–10V</u>**. Since it is in series with the +10V source, together they represent a direct short at t = 0^+. This means that the entire 2A from the current source flows through the capacitor and not the resistor. Therefore, $v_R(0^+)$ = **<u>0 V</u>**.

(b) At $t = 0^+$, $v_L(0+) = 0$, therefore $L di_L(0+)/dt = v_L(0^+) = 0$, thus, di_L/dt = **0A/s**, $i_C(0^+) = 2$ A, this means that $dv_C(0^+)/dt = 2/C$ = **8 V/s**. Now for the value of $dv_R(0^+)/dt$. Since $v_R = v_C + 10$, then $dv_R(0^+)/dt = dv_C(0^+)/dt + 0$ = **8 V/s**.



(a) (b)

(c) As t approaches infinity, we end up with the equivalent circuit shown in Figure (b).

$$i_L(\infty) = 10(2)/(40 + 10) = \underline{\textbf{400 mA}}$$

$$v_C(\infty) = 2[10||40] - 10 = 16 - 10 = \underline{\textbf{6V}}$$

$$v_R(\infty) = 2[10||40] = \underline{\textbf{16 V}}$$

Chapter 8, Problem 4.

In the circuit of Fig. 8.65, find:

(a) $v(0^+)$ and $i(0^+)$,
(b) $dv(0^+)/dt$ and $di(0^+)/dt$,
(c) $v(\infty)$ and $i(\infty)$.

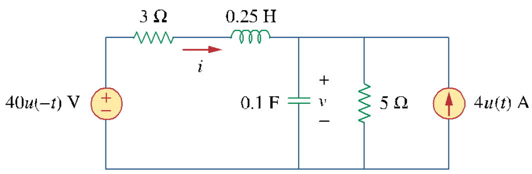


Figure 8.65
For Prob. 8.4.

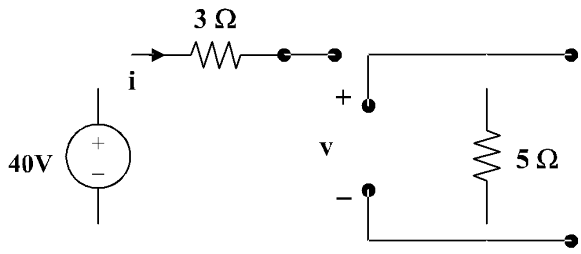
Chapter 8, Solution 4.

(a) At t = 0$^-$, u(-t) = 1 and u(t) = 0 so that the equivalent circuit is shown in Figure (a).

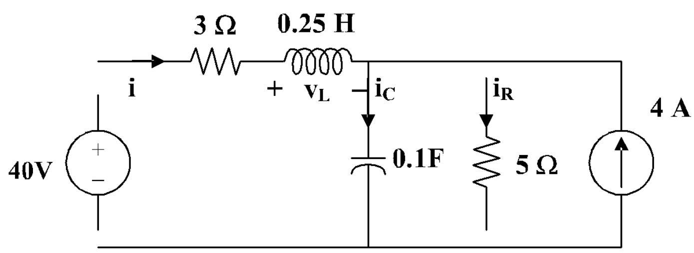
$$i(0^-) = 40/(3 + 5) = 5A, \text{ and } v(0^-) = 5i(0^-) = 25V.$$

Hence, $i(0^+) = i(0^-) =$ **5A**

$v(0^+) = v(0^-) =$ **25V**



(a)



(b)

(b) $i_C = Cdv/dt$ or $dv(0^+)/dt = i_C(0^+)/C$

For $t = 0^+$, $4u(t) = 4$ and $4u(-t) = 0$. The equivalent circuit is shown in Figure (b). Since i and v cannot change abruptly,

$$i_R = v/5 = 25/5 = 5A,\ i(0^+) + 4 = i_C(0^+) + i_R(0^+)$$

$$5 + 4 = i_C(0^+) + 5 \text{ which leads to } i_C(0^+) = 4$$

$$dv(0^+)/dt = 4/0.1 = \underline{\mathbf{40\ V/s}}$$

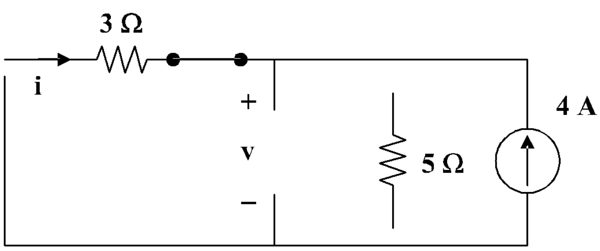
Similarly, $v_L = Ldi/dt$ which leads to $di(0^+)/dt = v_L(0^+)/L$

$$3i(0^+) + v_L(0^+) + v(0^+) = 0$$

$$15 + v_L(0^+) + 25 = 0 \text{ or } v_L(0^+) = -40$$

$$di(0^+)/dt = -40/0.25 = \underline{\mathbf{-160\ A/s}}$$

(c) As t approaches infinity, we have the equivalent circuit in Figure (c).



(c)

$$i(\infty) = -5(4)/(3 + 5) = \underline{\mathbf{-2.5\ A}}$$

$$v(\infty) = 5(4 - 2.5) = \underline{\mathbf{7.5\ V}}$$

Chapter 8, Problem 5.

Refer to the circuit in Fig. 8.66. Determine:

(a) $i(0^+)$ and $v(0^+)$,

(b) $di(0^+)/dt$ and $dv(0^+)/dt$,

(c) $i(\infty)$ and $v(\infty)$.

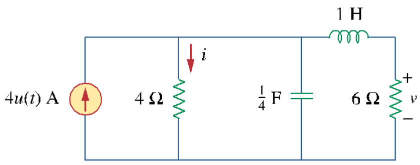


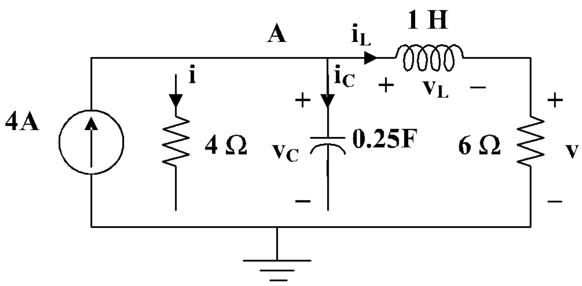
Figure 8.66
For Prob. 8.5.

Chapter 8, Solution 5.

(a) For $t < 0$, $4u(t) = 0$ so that the circuit is not active (all initial conditions = 0).

$$i_L(0-) = 0 \text{ and } v_C(0-) = 0.$$

For $t = 0+$, $4u(t) = 4$. Consider the circuit below.



Since the 4-ohm resistor is in parallel with the capacitor,

$$i(0+) = v_C(0+)/4 = 0/4 = \underline{\mathbf{0\ A}}$$

Also, since the 6-ohm resistor is in series with the inductor,
$v(0+) = 6i_L(0+) = \underline{\mathbf{0V}}$.

(b) $di(0+)/dt = d(v_R(0+)/R)/dt = (1/R)dv_R(0+)/dt = (1/R)dv_C(0+)/dt$

$$= (1/4)4/0.25 \text{ A/s} = \underline{\mathbf{4\ A/s}}$$

$$v = 6i_L \text{ or } dv/dt = 6di_L/dt \text{ and } dv(0+)/dt = 6di_L(0+)/dt = 6v_L(0+)/L = 0$$

$$\text{Therefore } dv(0+)/dt = \underline{\mathbf{0\ V/s}}$$

(c) As t approaches infinity, the circuit is in steady-state.

$$i(\infty) = 6(4)/10 = \underline{\mathbf{2.4\ A}}$$

$$v(\infty) = 6(4 - 2.4) = \underline{\mathbf{9.6\ V}}$$

Chapter 8, Problem 6.

In the circuit of Fig. 8.67, find:

(a) $v_R(0^+)$ and $v_L(0^+)$,
(b) $dv_R(0^+)/dt$ and $dv_L(0^+)/dt$,
(c) $v_R(\infty)$ and $v_L(\infty)$,

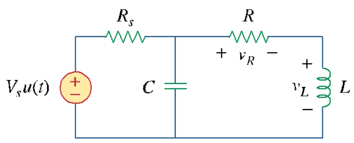


Figure 8.67
For Prob. 8.6.

Chapter 8, Solution 6.

(a) Let i = the inductor current. For $t < 0$, $u(t) = 0$ so that

$$i(0) = 0 \text{ and } v(0) = 0.$$

For $t > 0$, $u(t) = 1$. Since, $v(0+) = v(0-) = 0$, and $i(0+) = i(0-) = 0$.

$$v_R(0+) = Ri(0+) = \underline{\mathbf{0\ V}}$$

Also, since $v(0+) = v_R(0+) + v_L(0+) = 0 = 0 + v_L(0+)$ or $v_L(0+) = \underline{\mathbf{0\ V}}$.
(1)

(b) Since $i(0+) = 0$, $\quad i_C(0+) = V_S/R_S$

But, $\quad i_C = Cdv/dt$ which leads to $dv(0+)/dt = V_S/(CR_S)$ (2)

From (1), $\quad dv(0+)/dt = dv_R(0+)/dt + dv_L(0+)/dt$ (3)

$v_R = iR$ or $dv_R/dt = Rdi/dt$ (4)

But, $\quad v_L = Ldi/dt$, $\quad v_L(0+) = 0 = Ldi(0+)/dt$ and $di(0+)/dt = 0$ (5)

From (4) and (5), $\quad dv_R(0+)/dt = \underline{\mathbf{0\ V/s}}$

From (2) and (3), $\quad dv_L(0+)/dt = dv(0+)/dt = \underline{\mathbf{V_s/(CR_s)}}$

(c) As t approaches infinity, the capacitor acts like an open circuit, while the inductor acts like a short circuit.

$$v_R(\infty) = \underline{\mathbf{[R/(R + R_s)]V_s}}$$

$$v_L(\infty) = \underline{\mathbf{0\ V}}$$

Chapter 8, Problem 7.

A series *RLC* circuit has $R = 10\text{k}\Omega$, $L = 0.1$ mH, and $C = 10\mu$F. What type of damping is exhibited by the circuit?

Chapter 8, Solution 7.

$$\alpha = \frac{R}{2L} = \frac{10x10^3}{2x0.1x10^{-3}} = 50x10^6$$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.1x10^{-3}x10x10^{-6}}} = 3.162x10^4$$

$\alpha > \omega_o \quad \longrightarrow \quad$ overdamped

Chapter 8, Problem 8.

A branch current is described by

$$\frac{d^2i(t)}{dt^2} + 4\frac{di(t)}{dt} + 10i(t) = 0$$

Determine: (a) the characteristic equation, (b) the type of damping exhibited by the circuit, (c) $i(t)$ given that $i(0) = 1$ and $di(0)/dt = 2$.

Chapter 8, Solution 8.

(a) The characteristic equation is $s^2 + 4s + 10 = 0$

(b) $s_{1,2} = \dfrac{-4 \pm \sqrt{16-40}}{2} = -2 \pm j2.45$

This is underdamped case.

(c) $i(t) = (A\cos 2.45t + B\sin 2.45t)e^{-2t}$

$$\frac{di}{dt} = (-2A\cos 2.45t - 2B\sin 2.45t - 2.45A\sin 2.45t + 2.45B\cos 2.45t)e^{-2t}$$

i(0) =1 = A

di(0)/dt = 2 = –2A + 2.45B = –2 + 2.45B or B = 1.6327

i(t) = **$\{\cos(2.45t) + 1.6327\sin(2.45t)\}e^{-2t}$ A**.

Please note that this problem can be checked using MATLAB.

Chapter 8, Problem 9.

The current in an *RLC* circuit is described by $\frac{d^2 i}{dt^2} + 10\frac{di}{dt} + 25i = 0$

If $i(0) = 10$ and $di(0)/dt = 0$ find $i(t)$ for $t > 0$.

Chapter 8, Solution 9.

$s^2 + 10s + 25 = 0$, thus $s_{1,2} = \frac{-10 \pm \sqrt{10 - 10}}{2} = -5$, repeated roots.

$$i(t) = [(A + Bt)e^{-5t}],\ i(0) = 10 = A$$

$$di/dt = [Be^{-5t}] + [-5(A + Bt)e^{-5t}]$$

$$di(0)/dt = 0 = B - 5A = B - 50 \text{ or } B = 50.$$

Therefore, $i(t) = \underline{\mathbf{[(10 + 50t)e^{-5t}]\ A}}$

Chapter 8, Problem 10.

The differential equation that describes the voltage in an *RLC* network is

$$\frac{d^2 v}{dt^2} + 5\frac{dv}{dt} + 4v = 0$$

Given that $v(0) = 0$, $dv(0)/dt = 10$ obtain $v(t)$.

Chapter 8, Solution 10.

$s^2 + 5s + 4 = 0$, thus $s_{1,2} = \frac{-5 \pm \sqrt{25 - 16}}{2} = -4, -1.$

$$v(t) = (Ae^{-4t} + Be^{-t}),\ v(0) = 0 = A + B, \text{ or } B = -A$$

$$dv/dt = (-4Ae^{-4t} - Be^{-t})$$

$$dv(0)/dt = 10 = -4A - B = -3A \text{ or } A = -10/3 \text{ and } B = 10/3.$$

Therefore, $v(t) = \underline{\mathbf{(-(10/3)e^{-4t} + (10/3)e^{-t})\ V}}$

Chapter 8, Problem 11.

The natural response of an *RLC* circuit is described by the differential equation

$$\frac{d^2v}{dt^2}+2\frac{dv}{dt}+v=0$$

for which the initial conditions are $v(0)=10$ and $dv(0)/dt=0$ Solve for $v(t)$

Chapter 8, Solution 11.

$$s^2 + 2s + 1 = 0, \text{ thus } s_{1,2} = \frac{-2\pm\sqrt{4-4}}{2} = -1, \text{ repeated roots.}$$

$$v(t) = [(A + Bt)e^{-t}], \; v(0) = 10 = A$$

$$dv/dt = [Be^{-t}] + [-(A + Bt)e^{-t}]$$

$$dv(0)/dt = 0 = B - A = B - 10 \text{ or } B = 10.$$

Therefore, $v(t)$ = **$[(10 + 10t)e^{-t}]$ V**

Chapter 8, Problem 12.

If $R=20\Omega, L=0.6\text{H}$ what value of *C* will make an *RLC* series circuit:

(a) overdamped,
(b) critically damped,
(c) underdamped?

Chapter 8, Solution 12.

(a) Overdamped when $C > 4L/(R^2) = 4\text{x}0.6/400 = 6\text{x}10^{-3}$, or $C >$ **6 mF**

(b) Critically damped when $C =$ **6 mF**

(c) Underdamped when $C <$ **6mF**

Chapter 8, Problem 13.

For the circuit in Fig. 8.68, calculate the value of *R* needed to have a critically damped response.

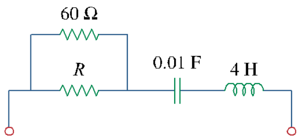


Figure 8.68
For Prob. 8.13.

Chapter 8, Solution 13.

Let $R||60 = R_o$. For a series RLC circuit,

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.01x4}} = 5$$

For critical damping, $\omega_o = \alpha = R_o/(2L) = 5$

or $R_o = 10L = 40 = 60R/(60 + R)$

which leads to $R =$ **<u>120 ohms</u>**

Chapter 8, Problem 14.

The switch in Fig. 8.69 moves from position A to position B at $t = 0$ (please note that the switch must connect to point B before it breaks the connection at A, a make-before-break switch). Find $v(t)$ for $t > 0$

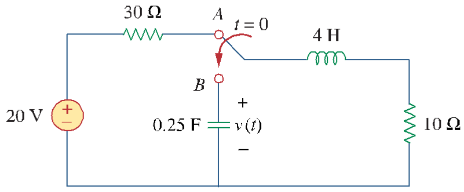


Figure 8.69
For Prob. 8.14.

Chapter 8, Solution 14.

When the switch is in position A, v(0⁻)= 0 and $i_L(0) = \frac{20}{40} = 0.5A$. When the switch is in position B, we have a source-free series RCL circuit.

$$\alpha = \frac{R}{2L} = \frac{10}{2x4} = 1.25$$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{4}x4}} = 1$$

Since $\alpha > \omega_o$, we have overdamped case.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -1.25 \pm \sqrt{1.5625 - 1} = -0.5 \text{ and } -2$$

$$v(t) = Ae^{-2t} + Be^{-0.5t} \qquad (1)$$

$$v(0) = 0 = A + B \qquad (2)$$

$$i_C(0) = C\frac{dv(0)}{dt} = 0.5 \quad \longrightarrow \quad \frac{dv(0)}{dt} = \frac{0.5}{C} = 2$$

But $\frac{dv(t)}{dt} = -2Ae^{-2t} - 0.5Be^{-0.5t}$

$$\frac{dv(0)}{dt} = -2A - 0.5B = 2 \qquad (3)$$

Solving (2) and (3) gives A= –1.3333 and B = 1.3333

$$v(t) = \underline{\mathbf{-1.3333e^{-2t} + 1.3333e^{-0.5t}\ V}}.$$

Chapter 8, Problem 15.

The responses of a series *RLC* circuit are

$v_c(t) = 30 - 10e^{-20t} + 30e^{-10t}$ V

$i_L(t) = 40e^{-20t} - 60e^{-10t}$ mA

where v_c and i_L are the capacitor voltage and inductor current, respectively. Determine the values of *R*, *L*, and *C*.

Chapter 8, Solution 15.

Given that $s_1 = -10$ and $s_2 = -20$, we recall that

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -10, -20$$

Clearly, $\quad s_1 + s_2 = -2\alpha = -30$ or $\alpha = 15 = R/(2L)$ or $R = 60L \qquad (1)$

$$s_1 = -15 + \sqrt{15^2 - \omega_o^2} = -10 \text{ which leads to } 15^2 - \omega_o^2 = 25$$

$$\text{or } \omega_o = \sqrt{225 - 25} = \sqrt{200} = 1/\sqrt{LC}, \text{ thus } LC = 1/200 \qquad (2)$$

Since we have a series RLC circuit, $i_L = i_C = Cdv_C/dt$ which gives,

$$i_L/C = dv_C/dt = [200e^{-20t} - 300e^{-30t}] \text{ or } i_L = 100C[2e^{-20t} - 3e^{-30t}]$$

But, $\quad i$ is also $= 20\{[2e^{-20t} - 3e^{-30t}]x10^{-3}\} = 100C[2e^{-20t} - 3e^{-30t}]$

Therefore, $\quad C = (0.02/10^2) =$ **<u>200 μF</u>**

$L = 1/(200C) =$ **<u>25 H</u>**

$R = 30L =$ **<u>750 ohms</u>**

Chapter 8, Problem 16.

Find $i(t)$ for $t > 0$ in the circuit of Fig. 8.70.

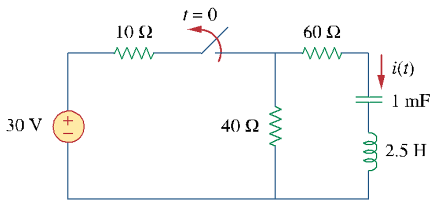


Figure 8.70
For Prob. 8.16.

Chapter 8, Solution 16.

At t = 0, i(0) = 0, $v_C(0)$ = 40x30/50 = 24V

For t > 0, we have a source-free RLC circuit.

$$\alpha = R/(2L) = (40 + 60)/5 = 20 \text{ and } \omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-3}x2.5}} = 20$$

$\omega_o = \alpha$ leads to critical damping

$i(t) = [(A + Bt)e^{-20t}],\ i(0) = 0 = A$

$di/dt = \{[Be^{-20t}] + [-20(Bt)e^{-20t}]\},$

but $di(0)/dt = -(1/L)[Ri(0) + v_C(0)] = -(1/2.5)[0 + 24]$

Hence, B = -9.6 or i(t) = $\underline{\mathbf{[-9.6te^{-20t}]\ A}}$

Chapter 8, Problem 17.

In the circuit of Fig. 8.71, the switch instantaneously moves from position *A* to *B* at $t = 0$ Find $v(t)$ for all $t \geq 0$

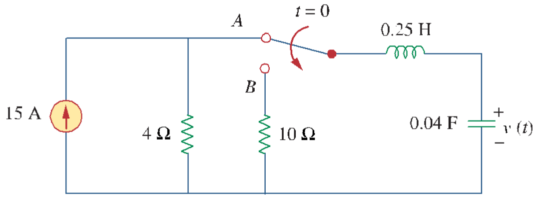


Figure 8.71
For Prob. 8.17.

Chapter 8, Solution 17.

$$i(0) = I_0 = 0, \ v(0) = V_0 = 4x15 = 60$$

$$\frac{di(0)}{dt} = -\frac{1}{L}(RI_0 + V_0) = -4(0+60) = -240$$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{4}\frac{1}{25}}} = 10$$

$$\alpha = \frac{R}{2L} = \frac{10}{2\frac{1}{4}} = 20, \text{ which is} > \omega_o.$$

$$s = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -20 \pm \sqrt{300} = -20 \pm 10\sqrt{3} = -2.679, -37.32$$

$$i(t) = A_1 e^{-2.679t} + A_2 e^{-37.32t}$$

$$i(0) = 0 = A_1 + A_2, \ \frac{di(0)}{dt} = -2.679A_1 - 37.32A_2 = -240$$

$$\text{This leads to } A_1 = -6.928 = -A_2$$

$$i(t) = 6.928\left(e^{-37.32t} - e^{-2.679t}\right)$$

$$\text{Since, } v(t) = \frac{1}{C}\int_0^t i(t)dt + \text{const, and } v(0) = 60\text{V, we get}$$

$$v(t) = \underline{(64.65e^{-2.679t} - 4.641e^{-37.32t})\text{ V}}$$

We note that v(0) = 60.009V and not 60V. This is due to rounding errors since v(t) must go to zero as time goes to infinity. {In other words, the constant of integration must be zero.

Chapter 8, Problem 18.

Find the voltage across the capacitor as a function of time for $t > 0$ for the circuit in Fig. 8.72. Assume steady-state conditions exist at $t = 0^-$

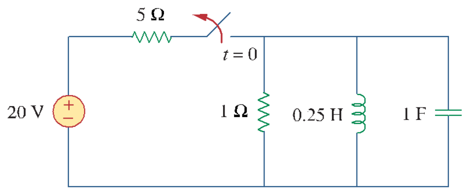


Figure 8.72
For Prob. 8.18.

Chapter 8, Solution 18.

When the switch is off, we have a source-free parallel RLC circuit.

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.25x1}} = 2, \qquad \alpha = \frac{1}{2RC} = 0.5$$

$\alpha < \omega_o \qquad \longrightarrow \qquad$ underdamped case $\quad \omega_d = \sqrt{\omega_o^{\,2} - \alpha^2} = \sqrt{4 - 0.25} = 1.936$

$I_o(0) = i(0)$ = initial inductor current = 20/5 = 4A

$V_o(0) = v(0)$ = initial capacitor voltage = 0 V

$$v(t) = e^{-\alpha t}(A_1 \cos\omega_d t + A_2 \sin\omega_d t) = e^{-0.5\alpha t}(A_1 \cos 1.936t + A_2 \sin 1.936t)$$

$v(0) = 0 = A_1$

$$\frac{dv}{dt} = e^{-0.5\alpha t}(-0.5)(A_1 \cos 1.936t + A_2 \sin 1.936t) + e^{-0.5\alpha t}(-1.936A_1 \sin 1.936t + 1.936A_2 \cos 1.936t)$$

$$\frac{dv(0)}{dt} = -\frac{(V_o + RI_o)}{RC} = -\frac{(0+4)}{1} = -4 = -0.5A_1 + 1.936A_2 \qquad \longrightarrow \qquad A_2 = -2.066$$

Thus,

$$\underline{v(t) = -2.066e^{-0.5t} \sin 1.936t}$$

Chapter 8, Problem 19.

Obtain $v(t)$ for $t > 0$ in the circuit of Fig. 8.73.

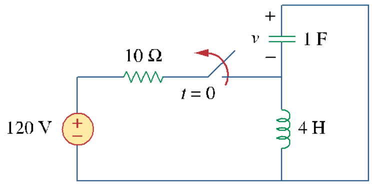
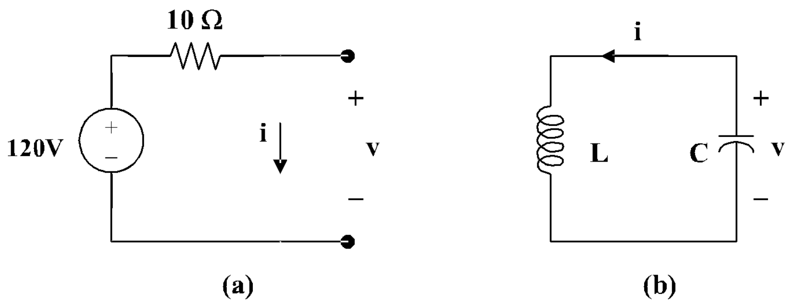


Figure 8.73
For Prob. 8.19.

Chapter 8, Solution 19.

For t < 0, the equivalent circuit is shown in Figure (a).



$$i(0) = 120/10 = 12, \ v(0) = 0$$

For t > 0, we have a series RLC circuit as shown in Figure (b) with R = 0 = α.

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4}} = 0.5 = \omega_d$$

$$i(t) = [A\cos 0.5t + B\sin 0.5t], \ i(0) = 12 = A$$

$$v = -Ldi/dt, \text{ and } -v/L = di/dt = 0.5[-12\sin 0.5t + B\cos 0.5t],$$

which leads to $-v(0)/L = 0 = B$

Hence, $i(t) = 12\cos 0.5t$ A and $v = 0.5$

However, $v = -Ldi/dt = -4(0.5)[-12\sin 0.5t] =$ **24sin0.5t V**

Chapter 8, Problem 20.

The switch in the circuit of Fig. 8.74 has been closed for a long time but is opened at $t = 0$ Determine $i(t)$ for $t > 0$.

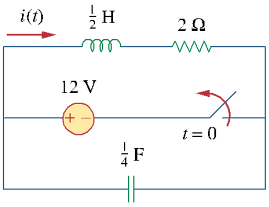
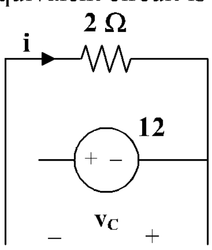


Figure 8.74
For Prob. 8.20.

Chapter 8, Solution 20.

For t < 0, the equivalent circuit is as shown below.



$$v(0) = -12V \text{ and } i(0) = 12/2 = 6A$$

For t > 0, we have a series RLC circuit.

$$\alpha = R/(2L) = 2/(2x0.5) = 2$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{0.5x1/4} = 2\sqrt{2}$$

Since α is less than ω_o, we have an under-damped response.

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = \sqrt{8-4} = 2$$

$$i(t) = (A\cos 2t + B\sin 2t)e^{-2t}$$

$$i(0) = 6 = A$$

$$di/dt = -2(6\cos 2t + B\sin 2t)e^{-2t} + (-2x6\sin 2t + 2B\cos 2t)e^{-\alpha t}$$

$$di(0)/dt = -12 + 2B = -(1/L)[Ri(0) + v_C(0)] = -2[12 - 12] = 0$$

Thus, B = 6 and i(t) = $\underline{\mathbf{(6\cos 2t + 6\sin 2t)e^{-2t}\ A}}$

Chapter 8, Problem 21.

* Calculate $v(t)$ for $t > 0$ in the circuit of Fig. 8.75.

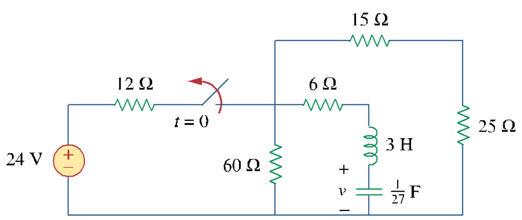


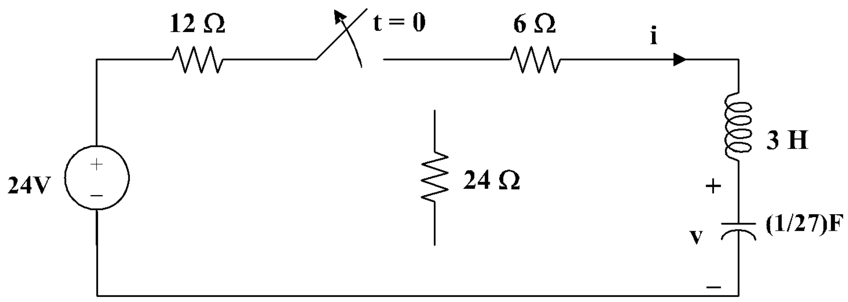
Figure 8.75
For Prob. 8.21.

* An asterisk indicates a challenging problem.

Chapter 8, Solution 21.

By combining some resistors, the circuit is equivalent to that shown below.

60||(15 + 25) = 24 ohms.



At t = 0-,   i(0) = 0, v(0) = 24x24/36 = 16V

For t > 0, we have a series RLC circuit.   R = 30 ohms, L = 3 H, C = (1/27) F

$$\alpha = R/(2L) = 30/6 = 5$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{3x1/27} = 3, \text{ clearly } \alpha > \omega_o \text{ (overdamped response)}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -5 \pm \sqrt{5^2 - 3^2} = -9, -1$$

$$v(t) = [Ae^{-t} + Be^{-9t}],\ v(0) = 16 = A + B \qquad (1)$$

$$i = Cdv/dt = C[-Ae^{-t} - 9Be^{-9t}]$$

$$i(0) = 0 = C[-A - 9B] \text{ or } A = -9B \qquad (2)$$

From (1) and (2),   B = -2 and A = 18.

Hence,   v(t) = **<u>(18e^{-t} – 2e^{-9t}) V</u>**

Chapter 8, Problem 22.

Assuming $R = 2\,\mathrm{k}\Omega$, design a parallel *RLC* circuit that has the characteristic equation

$s^2 + 100s + 10^6 = 0$.

Chapter 8, Solution 22.

Compare the characteristic equation with eq. (8.8), i.e.

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

we obtain

$$\frac{R}{L} = 100 \quad \longrightarrow \quad L = \frac{R}{100} = \frac{2000}{100} = \underline{20H}$$

$$\frac{1}{LC} = 10^6 \quad \rightarrow \quad C = \frac{1}{10^6 L} = \frac{10^{-6}}{20} = \underline{50\text{ nF}}$$

Chapter 8, Problem 23.

For the network in Fig. 8.76, what value of *C* is needed to make the response underdamped with unity damping factor $\alpha = 1$?

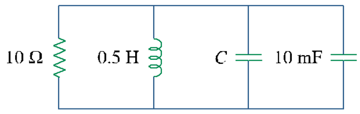


Figure 8.76
For Prob. 8.23.

Chapter 8, Solution 23.

Let $C_o = C + 0.01$. For a parallel RLC circuit,

$$\alpha = 1/(2RC_o), \quad \omega_o = 1/\sqrt{LC_o}$$

$\alpha = 1 = 1/(2RC_o)$, we then have $C_o = 1/(2R) = 1/20 = 50$ mF

$$\omega_o = 1/\sqrt{0.5x0.5} = 6.32 > \alpha \text{ (underdamped)}$$

$C_o = C + 10$ mF $= 50$ mF or **<u>40 mF</u>**

Chapter 8, Problem 24.

The switch in Fig. 8.77 moves from position A to position B at $t = 0$ (please note that the switch must connect to point B before it breaks the connection at A, a make-before-break switch). Determine $i(t)$ for $t > 0$

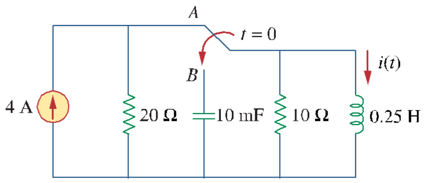


Figure 8.77
For Prob. 8.24.

Chapter 8, Solution 24.
When the switch is in position A, the inductor acts like a short circuit so

$$i(0^-) = 4$$

When the switch is in position B, we have a source-free parallel RCL circuit

$$\alpha = \frac{1}{2RC} = \frac{1}{2x10x10x10^{-3}} = 5$$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{4}x10x10^{-3}}} = 20$$

Since $\alpha < \omega_o$, we have an underdamped case.

$$s_{1,2} = -5 + \sqrt{25 - 400} = -5 + j19.365$$

$$i(t) = e^{-5t}\left(A_1 \cos 19.365t + A_2 \sin 19.365t\right)$$

$$i(0) = 4 = A_1$$

$$v = L\frac{di}{dt} \quad \longrightarrow \quad \frac{di(0)}{dt} = \frac{v(0)}{L} = 0$$

$$\frac{di}{dt} = e^{-5t}\left(-5A_1 \cos 19.365t - 5A_2 \sin 19.365t - 19.365A_1 \sin 19.365t + 19.365A_2 \cos 19.365t\right)$$

$$0 = \frac{di(0)}{dt} = -5A_1 + 19.365A_2 \quad \longrightarrow \quad A_2 = \frac{5A_1}{19.365} = 1.033$$

$$i(t) = \underline{e^{-5t}\left(4\cos 19.365t + 1.033\sin 19.365t\right)}$$

Chapter 8, Problem 25.

In the circuit of Fig. 8.78, calculate $i_o(t)$ and $v_o(t)$ for $t > 0$

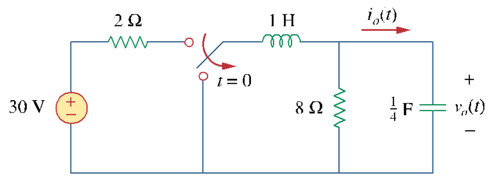


Figure 8.78
For Prob. 8.25.

Chapter 8, Solution 25.

In the circuit in Fig. 8.76, calculate $i_o(t)$ and $v_o(t)$ for t>0.

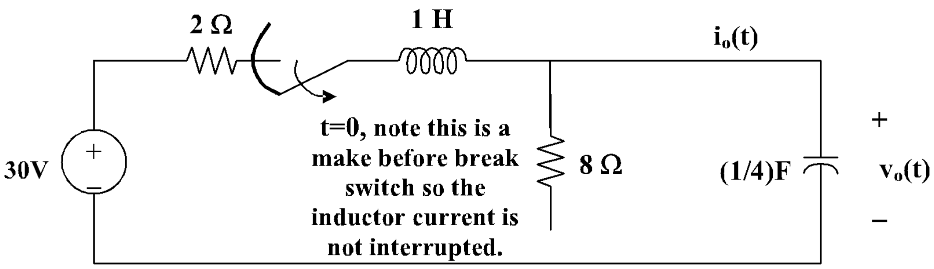


Figure 8.78 For Problem 8.25.

At t = 0^-, $v_o(0) = (8/(2 + 8)(30) = 24$

For t > 0, we have a source-free parallel RLC circuit.

$$\alpha = 1/(2RC) = 1/4$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1x1/4} = 2$$

Since α is less than ω_o, we have an under-damped response.

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = \sqrt{4-(1/16)} = 1.9843$$

$$v_o(t) = (A_1\cos\omega_d t + A_2\sin\omega_d t)e^{-\alpha t}$$

$$v_o(0) = 30(8/(2+8)) = 24 = A_1 \text{ and } i_o(t) = C(dv_o/dt) = 0 \text{ when } t = 0.$$

$$dv_o/dt = -\alpha(A_1\cos\omega_d t + A_2\sin\omega_d t)e^{-\alpha t} + (-\omega_d A_1\sin\omega_d t + \omega_d A_2\cos\omega_d t)e^{-\alpha t}$$

$$\text{at } t = 0, \text{ we get } dv_o(0)/dt = 0 = -\alpha A_1 + \omega_d A_2$$

Thus, $A_2 = (\alpha/\omega_d)A_1 = (1/4)(24)/1.9843 = 3.024$

$v_o(t) =$ **(24cos1.9843t + 3.024sin1.9843t)$e^{-t/4}$ volts**.

$i_0(t) = Cdv/dt = 0.25[-24(1.9843)\sin 1.9843t + 3.024(1.9843)\cos 1.9843t - 0.25(24\cos 1.9843t) - 0.25(3.024\sin 1.9843t)]e^{-t/4}$
= **[0.000131cos1.9843t – 12.095sin1.9843t]$e^{-t/4}$ A**.

Chapter 8, Problem 26.

The step response of an *RLC* circuit is described by

$$\frac{d^2 i}{dt^2} + 2\frac{di}{dt} + 5i = 10$$

Given that $i(0) = 2$ and $di(0)/dt = 4$, solve for $i(t)$

Chapter 8, Solution 26.

$$s^2 + 2s + 5 = 0, \text{ which leads to } s_{1,2} = \frac{-2 \pm \sqrt{4-20}}{2} = -1 \pm j4$$

$$i(t) = I_s + [(A_1\cos 4t + A_2\sin 4t)e^{-t}], \; I_s = 10/5 = 2$$

$$i(0) = 2 = = 2 + A_1, \text{ or } A_1 = 0$$

$$di/dt = [(A_2\cos 4t)e^{-t}] + [(-A_2\sin 4t)e^{-t}] = 4 = 4A_2, \text{ or } A_2 = 1$$

$$i(t) = \underline{\mathbf{2 + \sin 4t e^{-t} \; A}}$$

Chapter 8, Problem 27.

A branch voltage in an *RLC* circuit is described by

$$\frac{d^2 v}{dt^2} + 4\frac{dv}{dt} + 8v = 24$$

If the initial conditions are $v(0) = 0 = dv(0)/dt$, find $v(t)$.

Chapter 8, Solution 27.

$$s^2 + 4s + 8 = 0 \text{ leads to } s = \frac{-4 \pm \sqrt{16-32}}{2} = -2 \pm j2$$

$$v(t) = V_s + (A_1\cos 2t + A_2\sin 2t)e^{-2t}$$

$$8V_s = 24 \text{ means that } V_s = 3$$

$$v(0) = 0 = 3 + A_1 \text{ leads to } A_1 = -3$$

$$dv/dt = -2(A_1\cos 2t + A_2\sin 2t)e^{-2t} + (-2A_1\sin 2t + 2A_2\cos 2t)e^{-2t}$$

$$0 = dv(0)/dt = -2A_1 + 2A_2 \text{ or } A_2 = A_1 = -3$$

$$v(t) = \underline{\mathbf{[3 - 3(\cos 2t + \sin 2t)e^{-2t}] \text{ volts}}}$$

Chapter 8, Problem 28.

A series *RLC* circuit is described by

$$L\frac{d^2i}{dt^2}+R\frac{di}{dt}+\frac{i}{C}=2$$

Find the response when $L=0.5\,\text{H}, R=4\,\Omega$, and $C=0.2\,\text{F}$. Let $i(0)=1, di(0)/dt=0$.

Chapter 8, Solution 28.

The characteristic equation is

$$Ls^2+Rs+\frac{1}{C}=0 \longrightarrow \frac{1}{2}s^2+4s+\frac{1}{0.2}=0 \longrightarrow s^2+8s+10=0$$

$$s_{1,2}=\frac{-8\pm\sqrt{64-40}}{2}= -6.45 \text{ and } -1.5505$$

$$\text{i(t)} = \text{i}_s + \text{Ae}^{-6.45t} + \text{Be}^{-1.5505t}$$

But $\dfrac{I_s}{LC}=2 \longrightarrow I_s=\dfrac{2}{0.5\text{x}0.2}=20$

$$\text{i(t)} = 20 + \text{Ae}^{-6.45t} + \text{Be}^{-1.5505t}$$

$$\text{i(0)} = 1 = 20 + \text{A} + \text{B} \quad \text{or} \quad \text{A} + \text{B} = -19 \qquad (1)$$

$$\frac{\text{di(t)}}{\text{dt}} = -6.45\text{Ae}^{-6.45t} - 1.5505\text{e}^{-1.5505t}$$

$$\text{but } \frac{\text{di(0)}}{\text{dt}} = 0 = -6.45\text{A} - 1.5505\text{B} \qquad (2)$$

Solving (1) and (2) gives A= 6.013, B= –25.013
Hence,

i(t) = **$20 + 6.013e^{-6.45t} - 25.013e^{-1.5505t}$ A**.

Chapter 8, Problem 29.

Solve the following differential equations subject to the specified initial conditions

(a) $d^2v/dt^2 + 4v = 12,\ v(0) = 0,\ dv(0)/dt = 2$

(b) $d^2i/dt^2 + 5\ di/dt + 4i = 8,\ i(0) = -1,\ di(0)/dt = 0$

(c) $d^2v/dt^2 + 2\ dv/dt + v = 3,\ v(0) = 5,\ dv(0)/dt = 1$

(d) $d^2i/dt^2 = 2\ di/dt = 5i = 10,\ i(0) = 4,\ di(0)/dt = -2$

Chapter 8, Solution 29.

(a) $s^2 + 4 = 0$ which leads to $s_{1,2} = \pm j2$ (an undamped circuit)

$$v(t) = V_s + A\cos 2t + B\sin 2t$$

$$4V_s = 12 \text{ or } V_s = 3$$

$$v(0) = 0 = 3 + A \text{ or } A = -3$$

$$dv/dt = -2A\sin 2t + 2B\cos 2t$$

$dv(0)/dt = 2 = 2B$ or $B = 1$, therefore $v(t) =$ **<u>(3 – 3cos2t + sin2t) V</u>**

(b) $s^2 + 5s + 4 = 0$ which leads to $s_{1,2} = -1, -4$

$$i(t) = (I_s + Ae^{-t} + Be^{-4t})$$

$$4I_s = 8 \text{ or } I_s = 2$$

$$i(0) = -1 = 2 + A + B, \text{ or } A + B = -3 \qquad (1)$$

$$di/dt = -Ae^{-t} - 4Be^{-4t}$$

$$di(0)/dt = 0 = -A - 4B, \text{ or } B = -A/4 \qquad (2)$$

From (1) and (2) we get $A = -4$ and $B = 1$

$i(t) =$ **<u>$(2 - 4e^{-t} + e^{-4t})$ A</u>**

(c) $s^2 + 2s + 1 = 0$, $s_{1,2} = -1, -1$

$$v(t) = [V_s + (A + Bt)e^{-t}],\ V_s = 3.$$

$$v(0) = 5 = 3 + A \text{ or } A = 2$$

$$dv/dt = [-(A + Bt)e^{-t}] + [Be^{-t}]$$

$$dv(0)/dt = -A + B = 1 \text{ or } B = 2 + 1 = 3$$

$$v(t) = \underline{\mathbf{[3 + (2 + 3t)e^{-t}]\ V}}$$

(d) $s^2 + 2s + 5 = 0$, $s_{1,2} = -1 + j2, -1 - j2$

$$i(t) = [I_s + (A\cos 2t + B\sin 2t)e^{-t}], \text{ where } 5I_s = 10 \text{ or } I_s = 2$$

$$i(0) = 4 = 2 + A \text{ or } A = 2$$

$$di/dt = [-(A\cos 2t + B\sin 2t)e^{-t}] + [(-2A\sin 2t + 2B\cos 2t)e^{-t}]$$

$$di(0)/dt = -2 = -A + 2B \text{ or } B = 0$$

$$i(t) = \underline{\mathbf{[2 + (2\cos 2t)e^{-t}]\ A}}$$

Chapter 8, Problem 30.

The step responses of a series *RLC* circuit are

$v_C = 40 - 10e^{-2000t} - 10e^{-4000t}$ V, $\quad t > 0$

$i_L(t) = 3e^{-2000t} + 6e^{-4000t}$ mA, $\quad t > 0$

(a) Find *C*. (b) Determine what type of damping is exhibited by the circuit.

Chapter 8, Solution 30.

(a) $$i_L(t) = i_C(t) = C\frac{dv_o}{dt} \qquad (1)$$

$$\frac{dv}{dt} = 2000x10e^{-2000t} + 4000x10e^{-4000t} = 2x10^4(e^{-2000t} + 2e^{-4000t}) \qquad (2)$$

But $i_L(t) = 3[e^{-2000t} + 2e^{-4000t}]x10^{-3}$ (3)

Substituting (2) and (3) into (1), we get

$$2x10^4 xC = 3x10^{-3} \longrightarrow C = 1.5x10^{-7} = \underline{150 \text{ nF}}$$

(b) Since $s_1 = -2000$ and $s_2 = -4000$ are real and negative, it is an **<u>overdamped</u>** case.

Chapter 8, Problem 31.

Consider the circuit in Fig. 8.79. Find $v_L(0^+)$ and $v_C(0^+)$

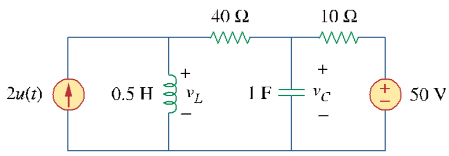


Figure 8.79
For Prob. 8.31.

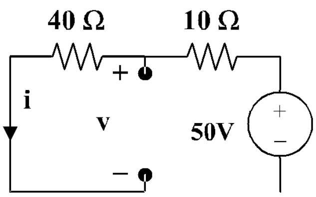
Chapter 8, Solution 31.

For t = 0-, we have the equivalent circuit in Figure (a). For t = 0+, the equivalent circuit is shown in Figure (b). By KVL,

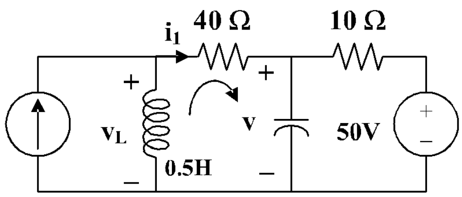
$$v(0+) = v(0-) = 40, \quad i(0+) = i(0-) = 1$$

By KCL, 2 = i(0+) + i_1 = 1 + i_1 which leads to i_1 = 1. By KVL, -v_L + 40i_1 + v(0+) = 0 which leads to $v_L(0+)$ = 40x1 + 40 = 80

$$v_L(0+) = \underline{\mathbf{80\ V}}, \quad v_C(0+) = \underline{\mathbf{40\ V}}$$



(a)



(b)

Chapter 8, Problem 32.

For the circuit in Fig. 8.80, find $v(t)$ for $t > 0$.

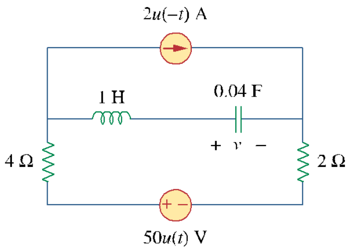
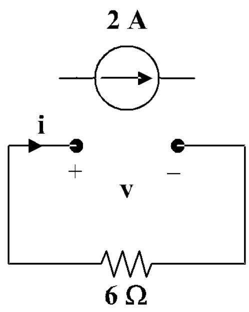


Figure 8.80
For Prob. 8.32.

Chapter 8, Solution 32.

For t = 0-, the equivalent circuit is shown below.



$$i(0-) = 0, \quad v(0-) = -2x6 = -12V$$

For t > 0, we have a series RLC circuit with a step input.

$$\alpha = R/(2L) = 6/2 = 3, \quad \omega_o = 1/\sqrt{LC} = 1/\sqrt{0.04}$$

$$s = -3 \pm \sqrt{9-25} = -3 \pm j4$$

$$\text{Thus,} \quad v(t) = V_f + [(A\cos4t + B\sin4t)e^{-3t}]$$

$$\text{where } V_f = \text{final capacitor voltage} = 50 \text{ V}$$

$$v(t) = 50 + [(A\cos4t + B\sin4t)e^{-3t}]$$

$$v(0) = -12 = 50 + A \text{ which gives } A = -62$$

$$i(0) = 0 = Cdv(0)/dt$$

$$dv/dt = [-3(A\cos4t + B\sin4t)e^{-3t}] + [4(-A\sin4t + B\cos4t)e^{-3t}]$$

$$0 = dv(0)/dt = -3A + 4B \text{ or } B = (3/4)A = -46.5$$

$$v(t) = \underline{\{50 + [(-62\cos4t - 46.5\sin4t)e^{-3t}]\}\text{ V}}$$

Chapter 8, Problem 33.

Find $v(t)$ for $t > 0$ in the circuit of Fig. 8.81.

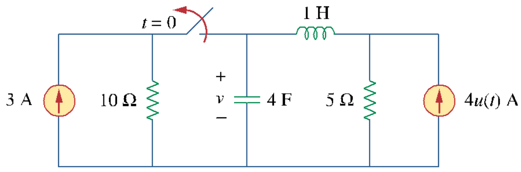
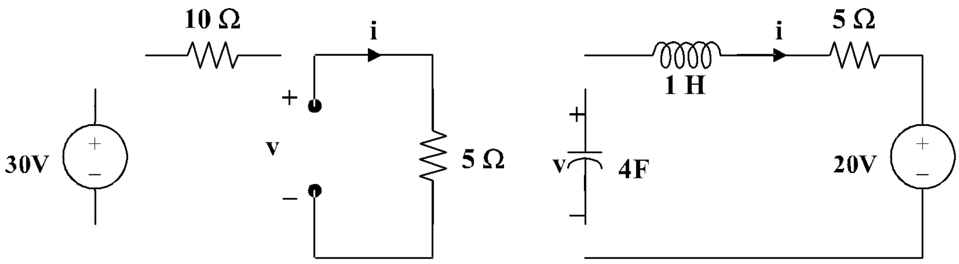


Figure 8.81
For Prob. 8.33.

Chapter 8, Solution 33.

We may transform the current sources to voltage sources. For $t = 0^-$, the equivalent circuit is shown in Figure (a).



(a) (b)

$$i(0) = 30/15 = 2\text{ A}, \quad v(0) = 5x30/15 = 10\text{ V}$$

For t > 0, we have a series RLC circuit, shown in (b).

$$\alpha = R/(2L) = 5/2 = 2.5$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{4} = 0.5, \text{ clearly } \alpha > \omega_o \text{ (overdamped response)}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -2.5 \pm \sqrt{6.25 - 0.25} = -4.95, -0.0505$$

$$v(t) = V_s + [A_1e^{-4.95t} + A_2e^{-0.0505t}], \quad V_s = 20.$$

$$v(0) = 10 = 20 + A_1 + A_2 \quad \text{or}$$
$$A_2 = -10 - A_1 \qquad (1)$$

$$i(0) = Cdv(0)/dt \text{ or } dv(0)/dt = 2/4 = 1/2$$

Hence, $$0.5 = -4.95A_1 - 0.0505A_2 \qquad (2)$$

From (1) and (2), $$0.5 = -4.95A_1 + 0.505(10 + A_1) \text{ or}$$
$$-4.445A_1 = -0.005$$

$$A_1 = 0.001125, \quad A_2 = -10.001$$

$$v(t) = \underline{\mathbf{[20 + 0.001125e^{-4.95t} - 10.001e^{-0.05t}]\ V}}$$

Chapter 8, Problem 34.

Calculate $i(t)$ for $t > 0$ in the circuit of Fig. 8.82.

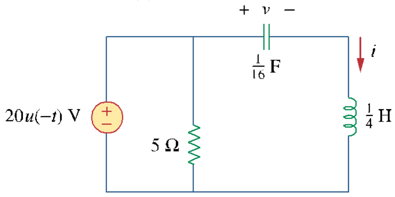


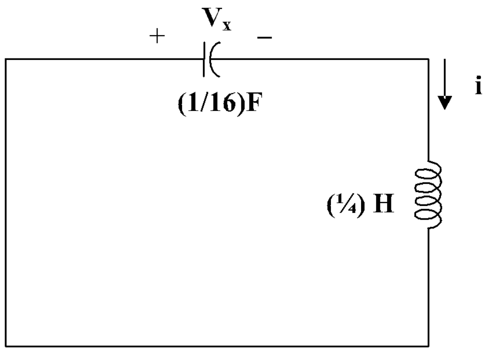
Figure 8.82
For Prob. 8.34.

Chapter 8, Solution 34.

Before t = 0, the capacitor acts like an open circuit while the inductor behaves like a short circuit.

$$i(0) = 0,\ v(0) = 20\text{ V}$$

For t > 0, the LC circuit is disconnected from the voltage source as shown below.



This is a lossless, source-free, series RLC circuit.

$$\alpha \;=\; R/(2L) \;=\; 0,\ \omega_o = 1/\sqrt{LC} \;=\; 1/\sqrt{\frac{1}{16}+\frac{1}{4}} \;=\; 8,\ \ s = \pm j8$$

Since α is less than ω_o, we have an underdamped response. Therefore,

$$i(t) = A_1\cos 8t + A_2\sin 8t \quad \text{where } i(0) = 0 = A_1$$

$$di(0)/dt = (1/L)v_L(0) = -(1/L)v(0) = -4x20 = -80$$

However, $di/dt = 8A_2\cos 8t$, thus, $di(0)/dt = -80 = 8A_2$ which leads to $A_2 = -10$

Now we have $\qquad i(t) \;=\; \underline{\mathbf{-10\sin 8t\ A}}$

Chapter 8, Problem 35.

Determine $v(t)$ for $t > 0$ in the circuit of Fig. 8.83.

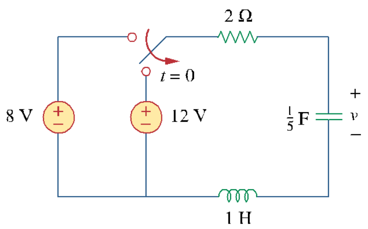


Figure 8.83
For Prob. 8.35.

Chapter 8, Solution 35.

$$\text{At t} = 0\text{-}, \; i_L(0) = 0, \; v(0) = v_C(0) = 8 \text{ V}$$

For t > 0, we have a series RLC circuit with a step input.

$$\alpha = R/(2L) = 2/2 = 1, \; \omega_o = 1/\sqrt{LC} = 1/\sqrt{1/5} = \sqrt{5}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -1 \pm j2$$

$$v(t) = V_s + [(A\cos 2t + B\sin 2t)e^{-t}], \; V_s = 12.$$

$$v(0) = 8 = 12 + A \text{ or } A = -4, \; i(0) = Cdv(0)/dt = 0.$$

$$\text{But } dv/dt = [-(A\cos 2t + B\sin 2t)e^{-t}] + [2(-A\sin 2t + B\cos 2t)e^{-t}]$$

$$0 = dv(0)/dt = -A + 2B \text{ or } 2B = A = -4 \text{ and } B = -2$$

$$v(t) = \underline{\mathbf{\{12 - (4\cos 2t + 2\sin 2t)e^{-t} \text{ V}}}.$$

Chapter 8, Problem 36.

Obtain $v(t)$ and $i(t)$ for $t > 0$ in the circuit of Fig. 8.84.

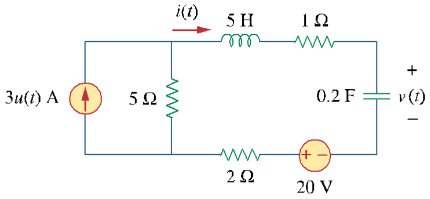
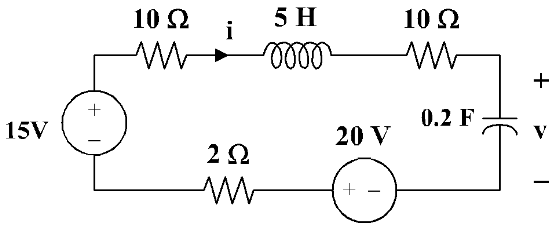


Figure 8.84
For Prob. 8.36.

Chapter 8, Solution 36.
For t = 0-, 3u(t) = 0. Thus, i(0) = 0, and v(0) = 20 V.
For t > 0, we have the series RLC circuit shown below.



$$\alpha = R/(2L) = (2 + 5 + 1)/(2x5) = 0.8$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{5x0.2} = 1$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -0.8 \pm j0.6$$

$$v(t) = V_s + [(A\cos0.6t + B\sin0.6t)e^{-0.8t}]$$

$$V_s = 15 + 20 = 35V \text{ and } v(0) = 20 = 35 + A \text{ or } A = -15$$

$$i(0) = Cdv(0)/dt = 0$$

$$\text{But } dv/dt = [-0.8(A\cos0.6t + B\sin0.6t)e^{-0.8t}] + [0.6(-A\sin0.6t + B\cos0.6t)e^{-0.8t}]$$

$$0 = dv(0)/dt = -0.8A + 0.6B \text{ which leads to } B = 0.8x(-15)/0.6 = -20$$

$$v(t) = \underline{\mathbf{\{35 - [(15\cos0.6t + 20\sin0.6t)e^{-0.8t}]\}\ V}}$$

$$i = Cdv/dt = 0.2\{[0.8(15\cos0.6t + 20\sin0.6t)e^{-0.8t}] + [0.6(15\sin0.6t - 20\cos0.6t)e^{-0.8t}]\}$$

$$i(t) = \underline{\mathbf{[(5\sin0.6t)e^{-0.8t}]\ A}}$$

Chapter 8, Problem 37.

* For the network in Fig. 8.85, solve for $i(t)$ for $t > 0$.

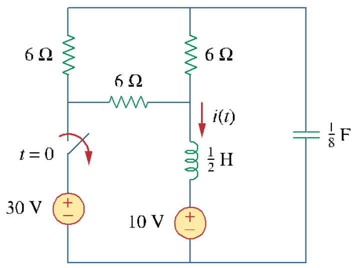
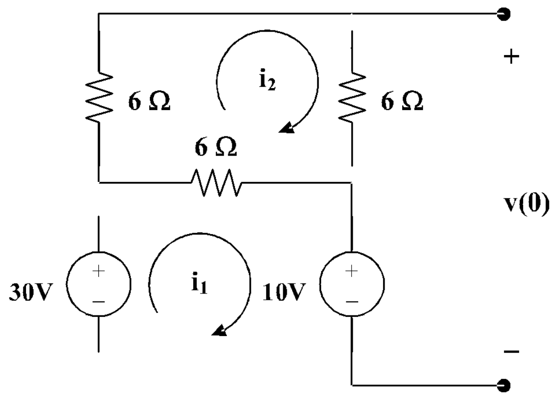


Figure 8.85
For Prob. 8.37.

* An asterisk indicates a challenging problem.

Chapter 8, Solution 37.

For t = 0-, the equivalent circuit is shown below.



$$18i_2 - 6i_1 = 0 \quad \text{or} \quad i_1 = 3i_2 \qquad (1)$$

$$-30 + 6(i_1 - i_2) + 10 = 0 \quad \text{or} \quad i_1 - i_2 = 10/3 \qquad (2)$$

From (1) and (2). $i_1 = 5,\ i_2 = 5/3$

$$i(0) = i_1 = 5\text{A}$$

$$-10 - 6i_2 + v(0) = 0$$

$$v(0) = 10 + 6\text{x}5/3 = 20$$

For $t > 0$, we have a series RLC circuit.

$$R = 6||12 = 4$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{(1/2)(1/8)} = 4$$

$$\alpha = R/(2L) = (4)/(2x(1/2)) = 4$$

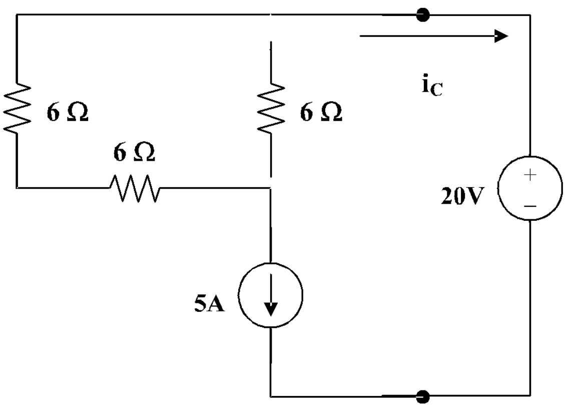
$\alpha = \omega_o$, therefore the circuit is critically damped

$$v(t) = V_s + [(A + Bt)e^{-4t}], \text{ and } V_s = 10$$

$$v(0) = 20 = 10 + A, \text{ or } A = 10$$

$$i_C = Cdv/dt = C[-4(10 + Bt)e^{-4t}] + C[(B)e^{-4t}]$$

To find $i_C(0)$ we need to look at the circuit right after the switch is opened. At this time, the current through the inductor forces that part of the circuit to act like a current source and the capacitor acts like a voltage source. This produces the circuit shown below. Clearly, $i_C(0+)$ must equal $-i_L(0) = -5A$.



$$i_C(0) = -5 = C(-40 + B) \text{ which leads to } -40 = -40 + B \text{ or } B = 0$$

$$i_C = Cdv/dt = (1/8)[-4(10 + 0t)e^{-4t}] + (1/8)[(0)e^{-4t}]$$

$$i_C(t) = [-(1/2)(10)e^{-4t}]$$

$$i(t) = -i_C(t) = \mathbf{\underline{[5e^{-4t}]\ A}}$$

Chapter 8, Problem 38.

Refer to the circuit in Fig. 8.86. Calculate $i(t)$ for $t > 0$

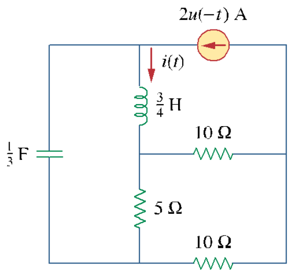
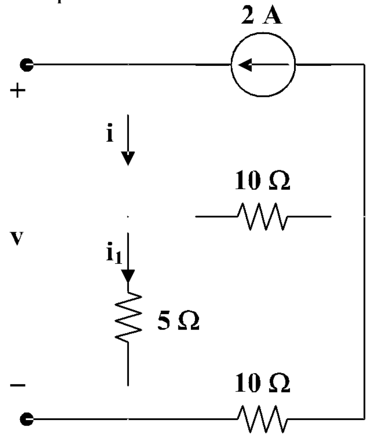


Figure 8.86
For Prob. 8.38.

Chapter 8, Solution 38.

At $t = 0^-$, the equivalent circuit is as shown.



$$i(0) = 2A, \quad i_1(0) = 10(2)/(10 + 15) = 0.8 \text{ A}$$

$$v(0) = 5i_1(0) = 4V$$

For $t > 0$, we have a source-free series RLC circuit.

$$R = 5||(10 + 10) = 4 \text{ ohms}$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{(1/3)(3/4)} = 2$$

$$\alpha = R/(2L) = (4)/(2x(3/4)) = 8/3$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -4.431, \ -0.903$$

$$i(t) = [Ae^{-4.431t} + Be^{-0.903t}]$$

$$i(0) = A + B = 2 \qquad (1)$$

$$di(0)/dt = (1/L)[-Ri(0) + v(0)] = (4/3)(-4x2 + 4) = -16/3 = -5.333$$

$$\text{Hence, } -5.333 = -4.431A - 0.903B \qquad (2)$$

From (1) and (2), A = 1 and B = 1.

$$i(t) = \underline{[e^{-4.431t} + e^{-0.903t}] \text{ A}}$$

Chapter 8, Problem 39.

Determine $v(t)$ for $t > 0$ in the circuit of Fig. 8.87.

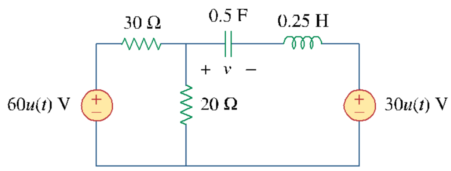
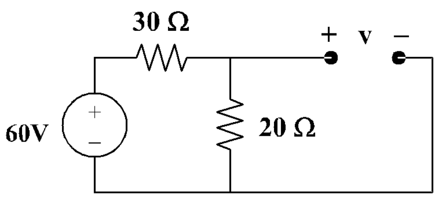


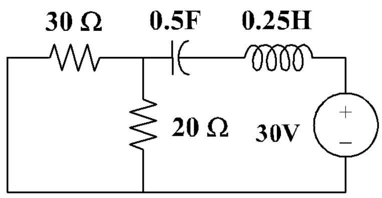
Figure 8.87
For Prob. 8.39.

Chapter 8, Solution 39.

For $t = 0^-$, the equivalent circuit is shown in Figure (a). Where $60u(-t) = 60$ and $30u(t) = 0$.



(a)



(b)

$$v(0) = (20/50)(60) = 24 \text{ and } i(0) = 0$$

For $t > 0$, the circuit is shown in Figure (b).

$$R = 20||30 = 12 \text{ ohms}$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{(1/2)(1/4)} = \sqrt{8}$$

$$\alpha = R/(2L) = (12)/(0.5) = 24$$

Since $\alpha > \omega_o$, we have an overdamped response.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -47.833, \ -0.167$$

Thus, $$v(t) = V_s + [Ae^{-47.833t} + Be^{-0.167t}], \quad V_s = 30$$

$$v(0) = 24 = 30 + A + B \text{ or } -6 = A + B \qquad (1)$$

$$i(0) = Cdv(0)/dt = 0$$

But, $$dv(0)/dt = -47.833A - 0.167B = 0$$

$$B = -286.43A \qquad (2)$$

From (1) and (2), $A = 0.021$ and $B = -6.021$

$$v(t) = \underline{\mathbf{30 + [0.021e^{-47.833t} - 6.021e^{-0.167t}]\ V}}$$

Chapter 8, Problem 40.

The switch in the circuit of Fig. 8.88 is moved from position a to b at $t = 0$. Determine $i(t)$ for $t > 0$.

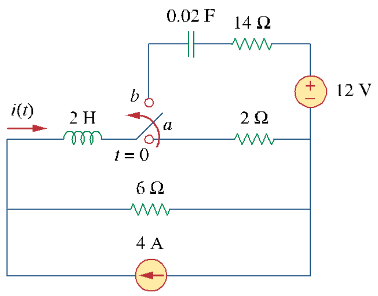
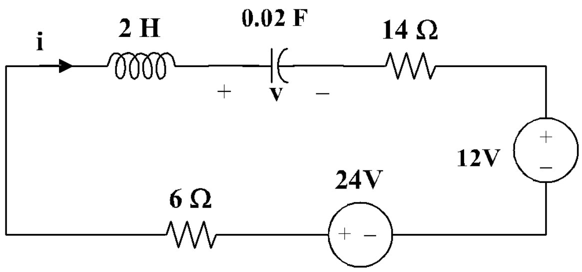


Figure 8.88
For Prob. 8.40.

Chapter 8, Solution 40.

At $t = 0\text{-}$, $v_C(0) = 0$ and $i_L(0) = i(0) = (6/(6+2))4 = 3A$

For $t > 0$, we have a series RLC circuit with a step input as shown below.



$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{2x0.02} = 5$$

$$\alpha = R/(2L) = (6 + 14)/(2x2) = 5$$

Since $\alpha = \omega_o$, we have a critically damped response.

$$v(t) = V_s + [(A + Bt)e^{-5t}], \quad V_s = 24 - 12 = 12V$$

$$v(0) = 0 = 12 + A \text{ or } A = -12$$

$$i = Cdv/dt = C\{[Be^{-5t}] + [-5(A + Bt)e^{-5t}]\}$$

$$i(0) = 3 = C[-5A + B] = 0.02[60 + B] \text{ or } B = 90$$

Thus, $i(t) = 0.02\{[90e^{-5t}] + [-5(-12 + 90t)e^{-5t}]\}$

$$i(t) = \mathbf{\{(3 - 9t)e^{-5t}\}\ A}$$

Chapter 8, Problem 41.

* For the network in Fig. 8.89, find $i(t)$ for $t > 0$.

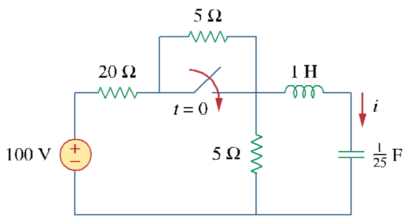


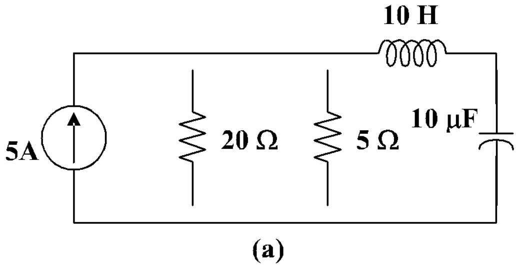
Figure 8.89
For Prob. 8.41.

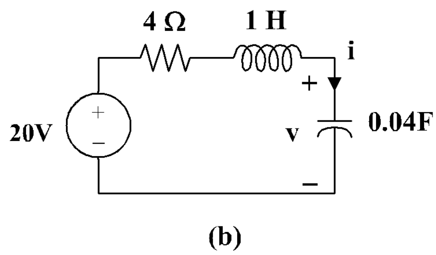
Chapter 8, Solution 41.

At t = 0-, the switch is open. i(0) = 0, and

$$v(0) = 5x100/(20 + 5 + 5) = 50/3$$

For t > 0, we have a series RLC circuit shown in Figure (a). After source transformation, it becomes that shown in Figure (b).





$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1x1/25} = 5$$

$$\alpha = R/(2L) = (4)/(2x1) = 2$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -2 \pm j4.583$$

Thus, $$v(t) = V_s + [(A\cos\omega_d t + B\sin\omega_d t)e^{-2t}],$$

where $\omega_d = 4.583$ and $V_s = 20$

$$v(0) = 50/3 = 20 + A \text{ or } A = -10/3$$

$$i(t) = Cdv/dt = C(-2)\,[(A\cos\omega_d t + B\sin\omega_d t)e^{-2t}] + C\omega_d[(-A\sin\omega_d t + B\cos\omega_d t)e^{-2t}]$$

$$i(0) = 0 = -2A + \omega_d B$$

$$B = 2A/\omega_d = -20/(3x4.583) = -1.455$$

$$i(t) = C\{[(0\cos\omega_d t + (-2B - \omega_d A)\sin\omega_d t)]e^{-2t}\}$$

$$= (1/25)\{[(2.91 + 15.2767)\sin\omega_d t)]e^{-2t}\}$$

$$i(t) = \underline{\mathbf{\{0.7275\sin(4.583t)e^{-2t}\}\ A}}$$

Chapter 8, Problem 42.

* Given the network in Fig. 8.90, find $v(t)$ for $t > 0$.

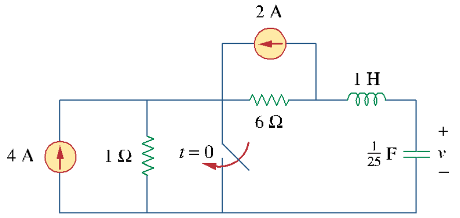
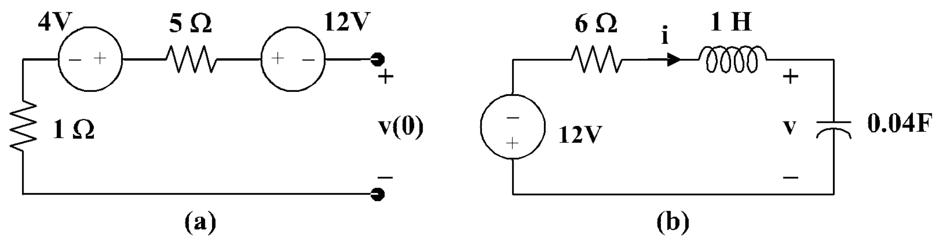


Figure 8.90
For Prob. 8.42.

Chapter 8, Solution 42.

For t = 0-, we have the equivalent circuit as shown in Figure (a).

$$i(0) = i(0) = 0, \text{ and } v(0) = 4 - 12 = -8V$$



For t > 0, the circuit becomes that shown in Figure (b) after source transformation.

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1x1/25} = 5$$

$$\alpha = R/(2L) = (6)/(2) = 3$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -3 \pm j4$$

Thus, $$v(t) = V_s + [(A\cos 4t + B\sin 4t)e^{-3t}], \quad V_s = -12$$

$$v(0) = -8 = -12 + A \text{ or } A = 4$$

$$i = Cdv/dt, \text{ or } i/C = dv/dt = [-3(A\cos 4t + B\sin 4t)e^{-3t}] + [4(-A\sin 4t + B\cos 4t)e^{-3t}]$$

$$i(0) = -3A + 4B \text{ or } B = 3$$

$$v(t) = \underline{\{-12 + [(4\cos 4t + 3\sin 4t)e^{-3t}]\}\ A}$$

Chapter 8, Problem 43.

The switch in Fig. 8.91 is opened at $t = 0$ after the circuit has reached steady state. Choose R and C such that $\alpha = 8$ Np/s and $\omega_d = 30$ rad/s.

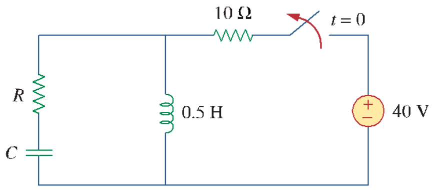


Figure 8.91
For Prob. 8.43.

Chapter 8, Solution 43.

For t>0, we have a source-free series RLC circuit.

$$\alpha = \frac{R}{2L} \longrightarrow R = 2\alpha L = 2x8x0.5 = \underline{8\Omega}$$

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = 30 \longrightarrow \omega_o = \sqrt{900 - 64} = \sqrt{836}$$

$$\omega_o = \frac{1}{\sqrt{LC}} \longrightarrow C = \frac{1}{\omega^2{}_o L} = \frac{1}{836x0.5} = \underline{2.392 \text{ mF}}$$

Chapter 8, Problem 44.

A series RLC circuit has the following parameters: $R = 1$ kΩ, $L = 1$H, and $C = 10$ nF. What type of damping does this circuit exhibit?

Chapter 8, Solution 44.

$$\alpha = \frac{R}{2L} = \frac{1000}{2x1} = 500, \quad \omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{100x10^{-9}}} = 10^4$$

$\omega_o > \alpha \longrightarrow$ **underdamped.**

Chapter 8, Problem 45.

In the circuit of Fig. 8.92, find $v(t)$ and $i(t)$ for $t > 0$. Assume $v(0) = 0$ V and $i(0) = 1$ A.

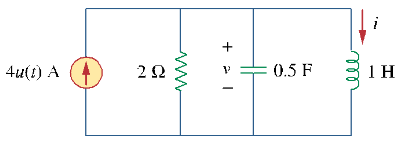


Figure 8.92
For Prob. 8.45.

Chapter 8, Solution 45.

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1x0.5} = \sqrt{2}$$

$$\alpha = 1/(2RC) = (1)/(2x2x0.5) = 0.5$$

Since $\alpha < \omega_o$, we have an underdamped response.

$$s_{1,2} = -\alpha \pm \sqrt{\omega_o^2 - \alpha^2} = -0.5 \pm j1.3229$$

Thus, $$i(t) = I_s + [(A\cos 1.3229t + B\sin 1.3229t)e^{-0.5t}], \quad I_s = 4$$

$$i(0) = 1 = 4 + A \text{ or } A = -3$$

$$v = v_C = v_L = Ldi(0)/dt = 0$$

$$di/dt = [1.3229(-A\sin 1.3229t + B\cos 1.3229t)e^{-0.5t}] + [-0.5(A\cos 1.3229t + B\sin 1.3229t)e^{-0.5t}]$$

$$di(0)/dt = 0 = 1.3229B - 0.5A \text{ or } B = 0.5(-3)/1.3229 = -1.1339$$

Thus, $$i(t) = \underline{\mathbf{\{4 - [(3\cos 1.3229t + 1.1339\sin 1.3229t)e^{-t/2}]\}\ A}}$$

To find v(t) we use $v(t) = v_L(t) = Ldi(t)/dt$.

From above,

$$di/dt = [1.3229(-A\sin 1.3229t + B\cos 1.3229t)e^{-0.5t}] + [-0.5(A\cos 1.323t + B\sin 1.323t)e^{-0.5t}]$$

Thus,

$$v(t) = Ldi/dt = [1.323(-A\sin 1.323t + B\cos 1.323t)e^{-0.5t}] + [-0.5(A\cos 1.323t + B\sin 1.323t)e^{-0.5t}]$$
$$= [1.3229(3\sin 1.3229t - 1.1339\cos 1.3229t)e^{-0.5t}] + [(1.5\cos 1.3229t + 0.5670\sin 1.3229t)e^{-0.5t}]$$

$$v(t) = [(-0\cos 1.323t + 4.536\sin 1.323t)e^{-0.5t}]\ V$$
$$= \underline{\mathbf{[(4.536\sin 1.323t)e^{-t/2}]\ V}}$$

Please note that the term in front of the cos calculates out to $-3.631x10^{-5}$ which is zero for all practical purposes when considering the rounding errors of the terms used to calculate it.

Chapter 8, Problem 46.

Find $i(t)$ for $t > 0$ in the circuit of Fig. 8.93.

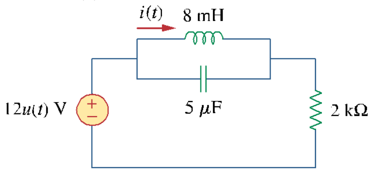
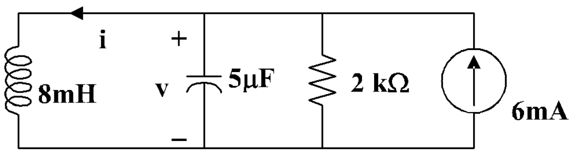


Figure 8.93
For Prob. 8.46.

Chapter 8, Solution 46.

For t = 0-, u(t) = 0, so that v(0) = 0 and i(0) = 0.

For t > 0, we have a parallel RLC circuit with a step input, as shown below.



$$\alpha = 1/(2RC) = (1)/(2x2x10^3 \text{ x}5x10^{-6}) = 50$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{8x10^{-3}x5x10^{-6}} = 5{,}000$$

Since $\alpha < \omega_o$, we have an underdamped response.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} \cong -50 \pm j5{,}000$$

Thus, $i(t) = I_s + [(A\cos 5{,}000t + B\sin 5{,}000t)e^{-50t}], \quad I_s = 6\text{mA}$

$$i(0) = 0 = 6 + A \text{ or } A = -6\text{mA}$$

$$v(0) = 0 = Ldi(0)/dt$$

$$di/dt = [5{,}000(-A\sin 5{,}000t + B\cos 5{,}000t)e^{-50t}] + [-50(A\cos 5{,}000t + B\sin 5{,}000t)e^{-50t}]$$

$$di(0)/dt = 0 = 5{,}000B - 50A \text{ or } B = 0.01(-6) = -0.06\text{mA}$$

Thus, $i(t) = \underline{\mathbf{\{6 - [(6\cos 5{,}000t + 0.06\sin 5{,}000t)e^{-50t}]\}\ mA}}$

Chapter 8, Problem 47.

Find the output voltage $v_o(t)$ in the circuit of Fig. 8.94.

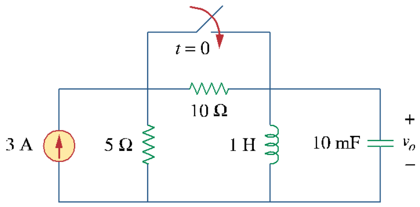


Figure 8.94
For Prob. 8.47.

Chapter 8, Solution 47.

At t = 0-, we obtain, $i_L(0) = 3x5/(10 + 5) = 1A$

and $v_o(0) = 0$.

For t > 0, the 10-ohm resistor is short-circuited and we have a parallel RLC circuit with a step input.

$$\alpha = 1/(2RC) = (1)/(2x5x0.01) = 10$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1x0.01} = 10$$

Since $\alpha = \omega_o$, we have a critically damped response.

$$s_{1,2} = -10$$

Thus, $$i(t) = I_s + [(A + Bt)e^{-10t}], \quad I_s = 3$$

$$i(0) = 1 = 3 + A \text{ or } A = -2$$

$$v_o = Ldi/dt = [Be^{-10t}] + [-10(A + Bt)e^{-10t}]$$

$$v_o(0) = 0 = B - 10A \text{ or } B = -20$$

Thus, $v_o(t) = \underline{\mathbf{(200te^{-10t})\ V}}$

Chapter 8, Problem 48.

Given the circuit in Fig. 8.95, find $i(t)$ and $v(t)$ for $t > 0$.

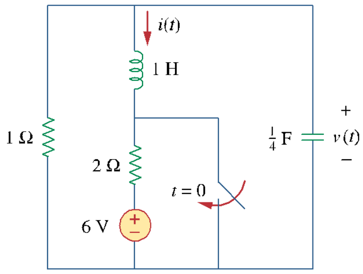


Figure 8.95
For Prob. 8.48.

Chapter 8, Solution 48.

For t = 0-, we obtain i(0) = -6/(1 + 2) = -2 and v(0) = 2x1 = 2.

For t > 0, the voltage is short-circuited and we have a source-free parallel RLC circuit.

$$\alpha = 1/(2RC) = (1)/(2x1x0.25) = 2$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1x0.25} = 2$$

Since $\alpha = \omega_o$, we have a critically damped response.

$$s_{1,2} = -2$$

Thus, $i(t) = [(A + Bt)e^{-2t}]$, $i(0) = -2 = A$

$$v = Ldi/dt = [Be^{-2t}] + [-2(-2 + Bt)e^{-2t}]$$

$$v_o(0) = 2 = B + 4 \text{ or } B = -2$$

Thus, $i(t) =$ **[(-2 - 2t)e^{-2t}] A**

and $v(t) =$ **[(2 + 4t)e^{-2t}] V**

Chapter 8, Problem 49.

Determine $i(t)$ for $t > 0$ in the circuit of Fig. 8.96.

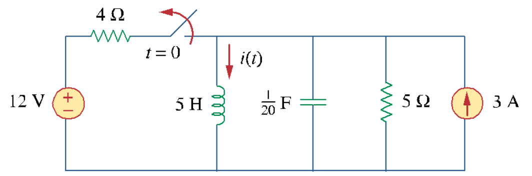


Figure 8.96
For Prob. 8.49.

Chapter 8, Solution 49.

$$\text{For } t = 0^-,\ i(0) = 3 + 12/4 = 6 \text{ and } v(0) = 0.$$

For $t > 0$, we have a parallel *RLC* circuit with a step input.

$$\alpha = 1/(2RC) = (1)/(2x5x0.05) = 2$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{5x0.05} = 2$$

Since $\alpha = \omega_o$, we have a critically damped response.

$$s_{1,2} = -2$$

Thus, $$i(t) = I_s + [(A + Bt)e^{-2t}],\quad I_s = 3$$

$$i(0) = 6 = 3 + A \text{ or } A = 3$$

$$v = Ldi/dt \text{ or } v/L = di/dt = [Be^{-2t}] + [-2(A + Bt)e^{-2t}]$$

$$v(0)/L = 0 = di(0)/dt = B - 2x3 \text{ or } B = 6$$

$$\text{Thus, } i(t) = \underline{\mathbf{\{3 + [(3 + 6t)e^{-2t}]\}\ A}}$$

Chapter 8, Problem 50.

For the circuit in Fig. 8.97, find $i(t)$ for $t > 0$.

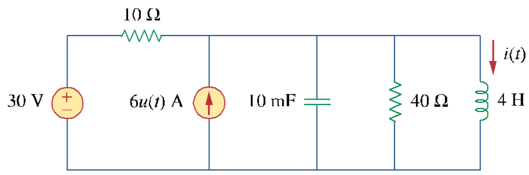
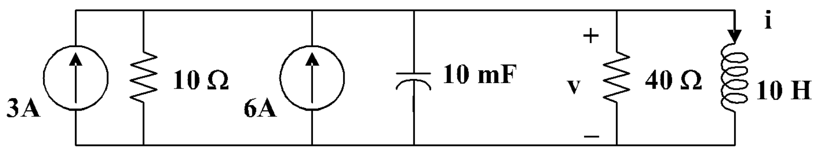


Figure 8.97
For Prob. 8.50.

Chapter 8, Solution 50.

For t = 0-, 4u(t) = 0, v(0) = 0, and i(0) = 30/10 = 3A.

For t > 0, we have a parallel RLC circuit.



$$I_s = 3 + 6 = 9\text{A} \text{ and } R = 10||40 = 8 \text{ ohms}$$

$$\alpha = 1/(2RC) = (1)/(2\text{x}8\text{x}0.01) = 25/4 = 6.25$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{4\text{x}0.01} = 5$$

Since $\alpha > \omega_o$, we have a overdamped response.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -10, -2.5$$

Thus, $$i(t) = I_s + [Ae^{-10t}] + [Be^{-2.5t}], \quad I_s = 9$$

$$i(0) = 3 = 9 + A + B \text{ or } A + B = -6$$

$$di/dt = [-10Ae^{-10t}] + [-2.5Be^{-2.5t}],$$

$$v(0) = 0 = Ldi(0)/dt \text{ or } di(0)/dt = 0 = -10A - 2.5B \text{ or } B = -4A$$

$$\text{Thus, } A = 2 \text{ and } B = -8$$

Clearly, $i(t) = \underline{\{9 + [2e^{-10t}] + [-8e^{-2.5t}]\}\text{ A}}$

Chapter 8, Problem 51.

Find $v(t)$ for $t > 0$ in the circuit of Fig. 8.98.

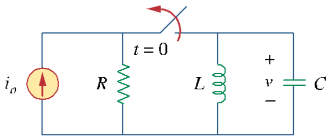


Figure 8.98
For Prob. 8.51.

Chapter 8, Solution 51.

Let i = inductor current and v = capacitor voltage.

At t = 0, v(0) = 0 and i(0) = i_o.

For t > 0, we have a parallel, source-free LC circuit (R = ∞).

$$\alpha = 1/(2RC) = 0 \text{ and } \omega_o = 1/\sqrt{LC} \text{ which leads to } s_{1,2} = \pm j\omega_o$$

$$v = A\cos\omega_o t + B\sin\omega_o t, \quad v(0) = 0 \quad A$$

$$i_C = Cdv/dt = -i$$

$$dv/dt = \omega_o B\sin\omega_o t = -i/C$$

$$dv(0)/dt = \omega_o B = -i_o/C \text{ therefore } B = i_o/(\omega_o C)$$

$$v(t) = \underline{\mathbf{-(i_o/(\omega_o C))\sin\omega_o t\ V\ \ where\ \ \omega_o = 1/\sqrt{LC}}}$$

Chapter 8, Problem 52.

The step response of a parallel *RLC* circuit is

$v = 10 + 20e^{-300t}(\cos 400t - 2\sin 400t)\text{V}, \quad t \geq 0$

when the inductor is 50 mH. Find *R* and *C*.

Chapter 8, Solution 52.

$$\alpha = 300 = \frac{1}{2RC} \qquad (1)$$

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = 400 \quad \longrightarrow \quad \omega_o = \sqrt{400^2 - 300^2} = 264.575 = \frac{1}{\sqrt{LC}} \qquad (2)$$

From (2),

$$C = \frac{1}{(264.575)^2 x 50 x 10^{-3}} = \underline{285.71\mu\text{F}}$$

From (1),

$$R = \frac{1}{2\alpha C} = \frac{1}{2x300}(3500) = \underline{5.833\Omega}$$

Chapter 8, Problem 53.

After being open for a day, the switch in the circuit of Fig. 8.99 is closed at $t = 0$. Find the differential equation describing $i(t)$, $t > 0$.

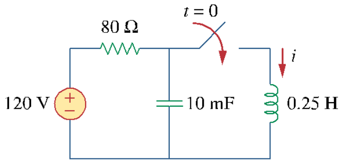
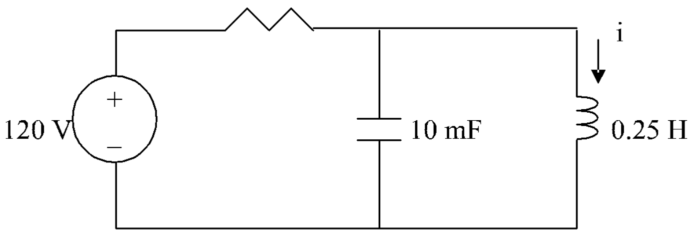


Figure 8.99
For Prob. 8.53.

Chapter 8, Solution 53.

At t<0, $i(0^-) = 0, v_c(0^-) = 120V$

For t >0, we have the circuit as shown below.



$$\frac{120 - V}{R} = C\frac{dv}{dt} + i \longrightarrow 120 = V + RC\frac{dv}{dt} + iR \qquad (1)$$

But $$v_L = v = L\frac{di}{dt} \qquad (2)$$

Substituting (2) into (1) yields

$$120 = L\frac{di}{dt} + RCL\frac{d^2i}{dt^2} + iR \longrightarrow 120 = \frac{1}{4}\frac{di}{dt} + 80x\frac{1}{4}x10x10^{-3}\frac{d^2i}{dt^2} + 80i$$

or

<u>**$(d^2i/dt^2) + 0.125(di/dt) + 400i = 600$**</u>

Chapter 8, Problem 54.

The switch in Fig. 8.100 moves from position A to B at $t = 0$. Determine: (a) $i(0^+)$ and $v(0^+)$, (b) $di(0^+)/dt$, (c) $i(\infty)$ and $v(\infty)$.

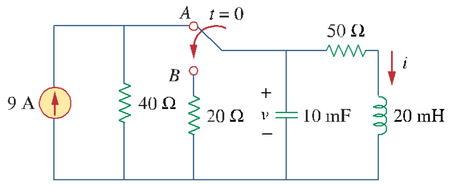
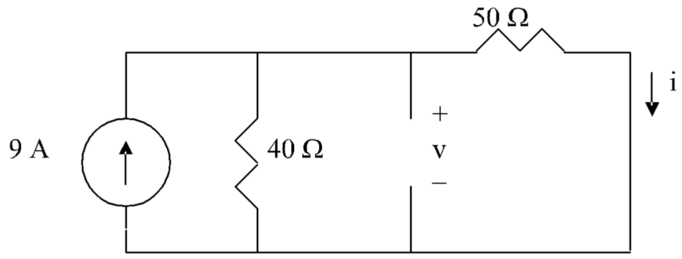


Figure 8.100
For Prob. 8.54.

Chapter 8, Solution 54.

(a) When the switch is at A, the circuit has reached steady state. Under this condition, the circuit is as shown below.



$$i(0^-) = \frac{40}{50+40}(9) = 4A, \qquad v(0^-) = 50i = 50x4 = 200 \text{ V}$$

$$v(0^+) = v(0^-) = \underline{200 \text{ V}}$$

$$i(0^+) = i(0^-) = \underline{4 \text{ A}}$$

(b) $v_L = L\frac{di}{dt} \longrightarrow \frac{di(0)}{dt} = \frac{v_L(0^+)}{L}$

At $t = 0^+$, the right hand loop becomes,

$-200 + 50x4 + v_L(0^+) = 0$ or $v_L(0^+) = 0$ and $(di(0^+)/dt) = \underline{\mathbf{0}}$.

$$i_C = C\frac{dv}{dt} \longrightarrow \frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C}$$

At $t = 0^+$, and looking at the current flowing out of the node at the top of the circuit,

$((200–0)/20) + i_C + 4 = 0$ or $i_C = –14$ A.

Therefore,

$dv(0^+)/dt = –14/0.01 = \underline{\mathbf{–1.4\ kV/s}}$.

(a) When the switch is in position B, the circuit reaches steady state. Since it is source-free, i and v decay to zero with time.

$$\underline{i(\infty) = 0, v(\infty) = 0}$$

Chapter 8, Problem 55.

For the circuit in Fig. 8.101, find $v(t)$ for $t > 0$. Assume that $v(0^+) = 4$ V and $i(0^+) = 2$ A.

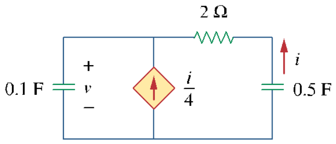


Figure 8.101
For Prob. 8.55.

Chapter 8, Solution 55.

At the top node, writing a KCL equation produces,

$$i/4 + i = C_1 dv/dt, \quad C_1 = 0.1$$

$$5i/4 = C_1 dv/dt = 0.1 dv/dt$$

$$i = 0.08 dv/dt \qquad (1)$$

But, $$v = -(2i + (1/C_2)\int i dt), \quad C_2 = 0.5$$

or $$-dv/dt = 2di/dt + 2i \qquad (2)$$

Substituting (1) into (2) gives,

$$-dv/dt = 0.16d^2v/dt^2 + 0.16dv/dt$$

$$0.16d^2v/dt^2 + 0.16dv/dt + dv/dt = 0, \text{ or } d^2v/dt^2 + 7.25dv/dt = 0$$

Which leads to $s^2 + 7.25s = 0 = s(s + 7.25)$ or $s_{1,2} = 0, -7.25$

$$v(t) = A + Be^{-7.25t} \qquad (3)$$

$$v(0) = 4 = A + B \qquad (4)$$

From (1), $i(0) = 2 = 0.08dv(0+)/dt$ or $dv(0+)/dt = 25$

But, $dv/dt = -7.25Be^{-7.25t}$, which leads to,

$$dv(0)/dt = -7.25B = 25 \text{ or } B = -3.448 \text{ and } A = 4 - B = 4 + 3.448 = 7.448$$

Thus, $v(t) = \underline{\{7.448 - 3.448e^{-7.25t}\}\text{ V}}$

Chapter 8, Problem 56.

In the circuit of Fig. 8.102, find $i(t)$ for $t > 0$.

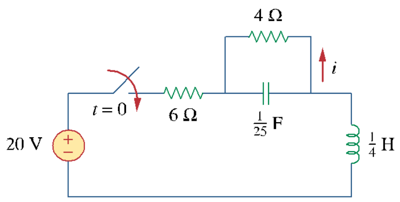
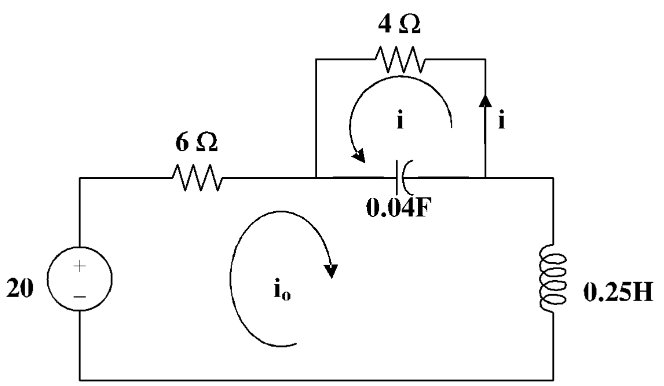


Figure 8.102
For Prob. 8.56.

Chapter 8, Solution 56.

For $t < 0$, $i(0) = 0$ and $v(0) = 0$.

For $t > 0$, the circuit is as shown below.



Applying KVL to the larger loop,

$$-20 + 6i_o + 0.25di_o/dt + 25\int(i_o + i)dt = 0 \quad (1)$$

For the smaller loop, $\quad 4i + 25\int(i + i_o)dt = 0$ or $\int(i + i_o)dt = -0.16i \quad (2)$

Taking the derivative, $\quad 4di/dt + 25(i + i_o) = 0$ or $i_o = -0.16di/dt - i \quad (3)$

and $\quad di_o/dt = -0.16d^2i/dt^2 - di/dt \quad (4)$

From (1), (2), (3), and (4), $\quad -20 - 0.96di/dt - 6i - 0.04d^2i/dt^2 - 0.25di/dt - 4i = 0$

Which becomes, $\quad d^2i/dt^2 + 30.25di/dt + 250i = -500$

This leads to, $s^2 + 30.25s + 250 = 0$

$$\text{or } s_{1,2} = \frac{-30.25 \pm \sqrt{(30.25)^2 - 1000}}{2} = -15.125 \pm j4.608$$

This is clearly an underdamped response.

Thus, $i(t) = I_s + e^{-15.125t}(A_1\cos(4.608t) + A_2\sin(4.608t))$A.

At t = 0, $i_o(0) = 0$ and $i(0) = 0 = I_s + A_1$ or $A_1 = -I_s$. As t approaches infinity, $i_o(\infty)$ = 20/10 = 2A = $-i(\infty)$ or $i(\infty) = -2A = I_s$ and $A_1 = 2$.

In addition, from (3), we get $di(0)/dt = -6.25i_o(0) - 6.25i(0) = 0$.

$di/dt = 0 - 15.125\, e^{-15.125t}(A_1\cos(4.608t) + A_2\sin(4.608t)) + e^{-15.125t}(-A_1 4.608\sin(4.608t) + A_2 4.608\cos(4.608t))$. At t=0, $di(0)/dt = 0 = -15.125A_1 + 4.608A_2 = -30.25 + 4.608A_2$ or $A_2 = 30.25/4.608 = 6.565$.

This leads to,

$$i(t) = \underline{\mathbf{(-2 + e^{-15.125t}(2\cos(4.608t) + 6.565\sin(4.608t))\ A}}$$

Chapter 8, Problem 57.

If the switch in Fig. 8.103 has been closed for a long time before $t = 0$, but is opened at $t = 0$ determine:

(a) the characteristic equation of the circuit,

(b) i_x and v_R for $t > 0$.

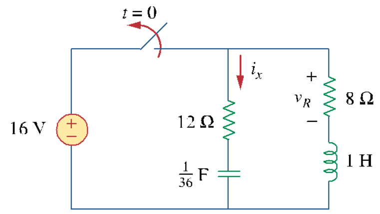


Figure 8.103
For Prob. 8.57.

Chapter 8, Solution 57.

(a) Let v = capacitor voltage and i = inductor current. At t = 0-, the switch is closed and the circuit has reached steady-state.

$$v(0-) = 16V \text{ and } i(0-) = 16/8 = 2A$$

At t = 0+, the switch is open but, v(0+) = 16 and i(0+) = 2.

We now have a source-free RLC circuit.

$$R\ 8 + 12 = 20 \text{ ohms}, \ L = 1H, \ C = 4mF.$$

$$\alpha = R/(2L) = (20)/(2x1) = 10$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1x(1/36)} = 6$$

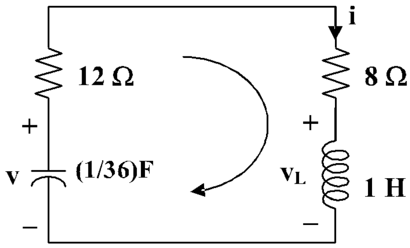
Since $\alpha > \omega_o$, we have a overdamped response.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -18, -2$$

Thus, the characteristic equation is (s + 2)(s + 18) = 0 or $\underline{\mathbf{s^2 + 20s + 36 = 0}}$.

(b) $i(t) = [Ae^{-2t} + Be^{-18t}]$ and $i(0) = 2 = A + B$ (1)

To get di(0)/dt, consider the circuit below at $t = 0+$.



$$-v(0) + 20i(0) + v_L(0) = 0, \text{ which leads to,}$$

$$-16 + 20x2 + v_L(0) = 0 \text{ or } v_L(0) = -24$$

$$Ldi(0)/dt = v_L(0) \text{ which gives } di(0)/dt = v_L(0)/L = -24/1 = -24 \text{ A/s}$$

Hence $-24 = -2A - 18B$ or $12 = A + 9B$ (2)

From (1) and (2), $B = 1.25$ and $A = 0.75$

$$i(t) = [0.75e^{-2t} + 1.25e^{-18t}] = -i_x(t) \text{ or } i_x(t) = \underline{\mathbf{[-0.75e^{-2t} - 1.25e^{-18t}]\ A}}$$

$$v(t) = 8i(t) = \underline{\mathbf{[6e^{-2t} + 10e^{-18t}]\ A}}$$

Chapter 8, Problem 58.

In the circuit of Fig. 8.104, the switch has been in position 1 for a long time but moved to position 2 at $t = 0$ Find:

(a) $v(0^+)$, $dv(0^+)/dt$

(b) $v(t)$ for $t \geq 0$

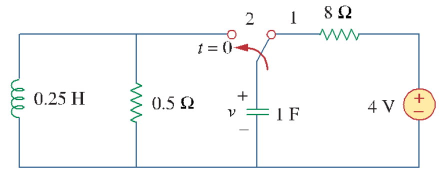


Figure 8.104
For Prob. 8.58.

Chapter 8, Solution 58.

(a) Let i =inductor current, v = capacitor voltage i(0) =0, <u>v(0) = 4</u>

$$\frac{dv(0)}{dt} = -\frac{[v(0) + Ri(0)]}{RC} = -\frac{(4+0)}{0.5} = \underline{-8\text{ V/s}}$$

(b) For $t \geq 0$, the circuit is a source-free RLC parallel circuit.

$$\alpha = \frac{1}{2RC} = \frac{1}{2x0.5x1} = 1, \qquad \omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.25x1}} = 2$$

$$\omega_d = \sqrt{\omega^2{}_o - \alpha^2} = \sqrt{4-1} = 1.732$$

Thus,

$$v(t) = e^{-t}(A_1 \cos 1.732t + A_2 \sin 1.732t)$$

$$v(0) = 4 = A_1$$

$$\frac{dv}{dt} = -e^{-t}A_1 \cos 1.732t - 1.732e^{-t}A_1 \sin 1.732t - e^{-t}A_2 \sin 1.732t + 1.732e^{-t}A_2 \cos 1.732t$$

$$\frac{dv(0)}{dt} = -8 = -A_1 + 1.732A_2 \qquad \longrightarrow \qquad A_2 = -2.309$$

$$v(t) = \underline{e^{-t}(4\cos 1.732t - 2.309\sin 1.732t)}\text{ V}$$

Chapter 8, Problem 59.

The make before break switch in Fig. 8.105 has been in position 1 for $t < 0$. At $t = 0$, it is moved instantaneously to position 2. Determine $v(t)$.

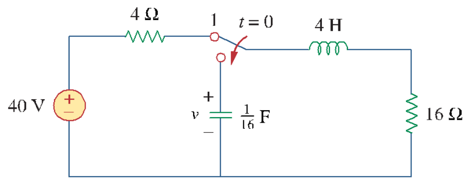


Figure 8.105
For Prob. 8.59.

Chapter 8, Solution 59.

Let i = inductor current and v = capacitor voltage
v(0) = 0, i(0) = 40/(4+16) = 2A
For t>0, the circuit becomes a source-free series RLC with

$$\alpha = \frac{R}{2L} = \frac{16}{2x4} = 2, \qquad \omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4x1/16}} = 2, \quad \longrightarrow \quad \alpha = \omega_o = 2$$

$$i(t) = Ae^{-2t} + Bte^{-2t}$$

i(0) = 2 = A

$$\frac{di}{dt} = -2Ae^{-2t} + Be^{-2t} - 2Bte^{-2t}$$

$$\frac{di(0)}{dt} = -2A + B = -\frac{1}{L}[Ri(0) - v(0)] \quad \longrightarrow \quad -2A + B = -\frac{1}{4}(32 - 0), \qquad B = -4$$

$$i(t) = 2e^{-2t} - 4te^{-2t}$$

$$v = \frac{1}{C}\int_0^t -idt + v(0) = -32\int_0^t e^{-2t}dt + 64\int_0^t te^{-2t}dt = +16e^{-2t}\Big|_0^t + \frac{64}{4}e^{-2t}(-2t-1)\Big|_0^t$$

v = $\underline{\mathbf{-32te^{-2t}\ V}}$.

Checking,

$$v = Ldi/dt + Ri = 4(-4e^{-2t} - 4e^{-2t} + 8e^{-2t}) + 16(2e^{-2t} - 4te^{-2t}) = -32te^{-2t}\ V.$$

Chapter 8, Problem 60.

Obtain i_1 and i_2 for $t > 0$ in the circuit of Fig. 8.106.

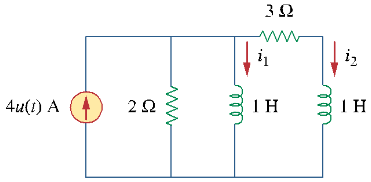


Figure 8.106
For Prob. 8.60.

Chapter 8, Solution 60.

At t = 0-, 4u(t) = 0 so that $i_1(0) = 0 = i_2(0)$ (1)

Applying nodal analysis,

$$4 = 0.5di_1/dt + i_1 + i_2 \quad (2)$$

Also, $i_2 = [1di_1/dt - 1di_2/dt]/3$ or $3i_2 = di_1/dt - di_2/dt$ (3)

Taking the derivative of (2), $0 = d^2i_1/dt^2 + 2di_1/dt + 2di_2/dt$ (4)

From (2) and (3), $di_2/dt = di_1/dt - 3i_2 = di_1/dt - 3(4 - i_1 - 0.5di_1/dt)$

$$= di_1/dt - 12 + 3i_1 + 1.5di_1/dt$$

Substituting this into (4),

$$d^2i_1/dt^2 + 7di_1/dt + 6i_1 = 24 \text{ which gives } s^2 + 7s + 6 = 0 = (s + 1)(s + 6)$$

Thus, $i_1(t) = I_s + [Ae^{-t} + Be^{-6t}]$, $6I_s = 24$ or $I_s = 4$

$$i_1(t) = 4 + [Ae^{-t} + Be^{-6t}] \text{ and } i_1(0) = 4 + [A + B] \quad (5)$$

$$i_2 = 4 - i_1 - 0.5di_1/dt = i_1(t) = 4 + -4 - [Ae^{-t} + Be^{-6t}] - [-Ae^{-t} - 6Be^{-6t}]$$

$$= [-0.5Ae^{-t} + 2Be^{-6t}] \text{ and } i_2(0) = 0 = -0.5A + 2B \quad (6)$$

From (5) and (6), A = -3.2 and B = -0.8

$$i_1(t) = \underline{\mathbf{\{4 + [-3.2e^{-t} - 0.8e^{-6t}]\}\ A}}$$

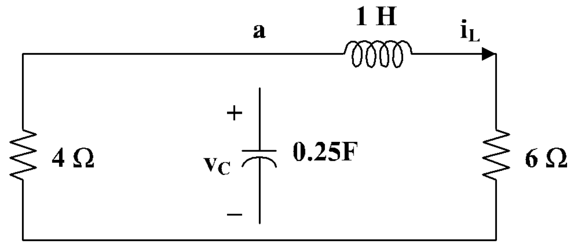
$$i_2(t) = \underline{\mathbf{[1.6e^{-t} - 1.6e^{-6t}]\ A}}$$

Chapter 8, Problem 61.

For the circuit in Prob. 8.5, find i and v for $t > 0$.

Chapter 8, Solution 61.

For t > 0, we obtain the natural response by considering the circuit below.



At node a, $v_C/4 + 0.25dv_C/dt + i_L = 0$ (1)

But, $v_C = 1di_L/dt + 6i_L$ (2)

Combining (1) and (2),

$$(1/4)di_L/dt + (6/4)i_L + 0.25d^2i_L/dt^2 + (6/4)di_L/dt + i_L = 0$$

$$d^2i_L/dt^2 + 7di_L/dt + 10i_L = 0$$

$$s^2 + 7s + 10 = 0 = (s + 2)(s + 5) \text{ or } s_{1,2} = -2, -5$$

$$\text{Thus, } i_L(t) = i_L(\infty) + [Ae^{-2t} + Be^{-5t}],$$

where $i_L(\infty)$ represents the final inductor current = 4(4)/(4 + 6) = 1.6

$$i_L(t) = 1.6 + [Ae^{-2t} + Be^{-5t}] \text{ and } i_L(0) = 1.6 + [A+B] \text{ or } -1.6 = A+B \quad (3)$$

$$di_L/dt = [-2Ae^{-2t} - 5Be^{-5t}]$$

$$\text{and } di_L(0)/dt = 0 = -2A - 5B \text{ or } A = -2.5B \quad (4)$$

From (3) and (4), A = -8/3 and B = 16/15

$$i_L(t) = 1.6 + [-(8/3)e^{-2t} + (16/15)e^{-5t}]$$

$$v(t) = 6i_L(t) = \underline{\mathbf{\{9.6 + [-16e^{-2t} + 6.4e^{-5t}]\}\ V}}$$

$$v_C = 1di_L/dt + 6i_L = [\ (16/3)e^{-2t} - (16/3)e^{-5t}] + \{9.6 + [-16e^{-2t} + 6.4e^{-5t}]\}$$

$$v_C = \{9.6 + [-(32/3)e^{-2t} + 1.0667e^{-5t}]\}$$

$$i(t) = v_C/4 = \underline{\mathbf{\{2.4 + [-2.667e^{-2t} + 0.2667e^{-5t}]\}\ A}}$$

Chapter 8, Problem 62.

Find the response $v_R(t)$ for $t > 0$ in the circuit of Fig. 8.107. Let $R = 3\Omega, L = 2\text{H}$, $C = 1/18\text{F}$.

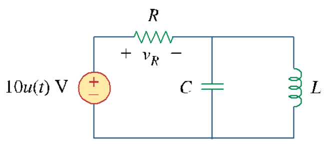


Figure 8.107
For Prob. 8.62.

Chapter 8, Solution 62.

This is a parallel RLC circuit as evident when the voltage source is turned off.

$$\alpha = 1/(2RC) = (1)/(2x3x(1/18)) = 3$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{2x1/18} = 3$$

Since $\alpha = \omega_o$, we have a critically damped response.

$$s_{1,2} = -3$$

Let v(t) = capacitor voltage

Thus, $v(t) = V_s + [(A + Bt)e^{-3t}]$ where $V_s = 0$

But $-10 + v_R + v = 0$ or $v_R = 10 - v$

Therefore v_R = **$10 - [(A + Bt)e^{-3t}]$** where A and B are determined from initial conditions.

Chapter 8, Problem 63.

For the op amp circuit in Fig. 8.108, find the differential equation for $i(t)$.

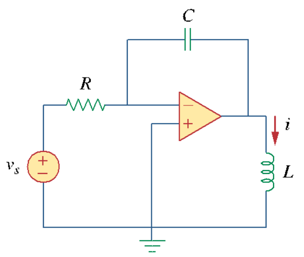


Figure 8.108
For Prob. 8.63.

Chapter 8, Solution 63.

$$\frac{v_s - 0}{R} = C\frac{d(0 - v_o)}{dt} \quad \longrightarrow \quad \frac{v_s}{R} = -C\frac{dv_o}{dt}$$

$$v_o = L\frac{di}{dt} \quad \longrightarrow \quad \frac{dv_o}{dt} = L\frac{d^2i}{dt^2} = -\frac{v_s}{RC}$$

Thus,

$$\underline{\frac{d^2i(t)}{dt^2} = -\frac{v_s}{RCL}}$$

Chapter 8, Problem 64.

For the op amp circuit in Fig. 8.109, derive the differential equation relating v_o to v_s.

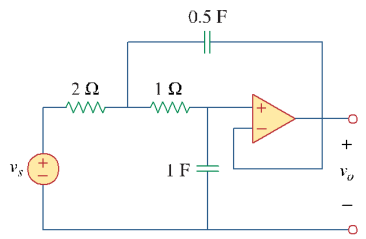
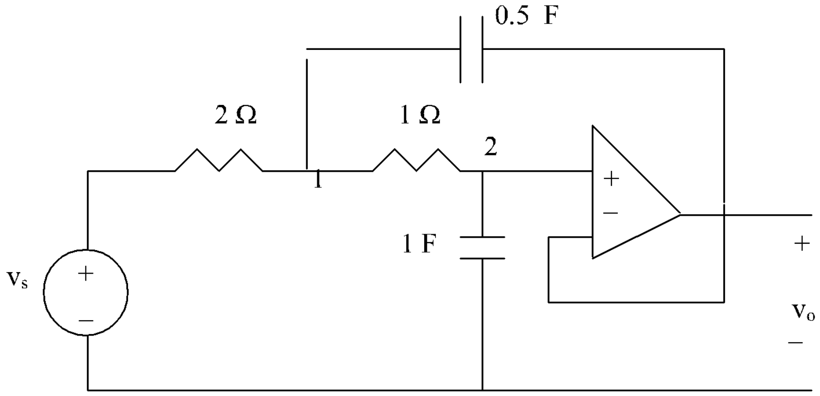


Figure 8.109
For Prob. 8.64.

Chapter 8, Solution 64.

Consider the circuit as shown below.



At node 1,

$$\frac{v_s - v_1}{2} = \frac{v_1 - v_2}{1} + \frac{1}{2}\frac{d}{dt}(v_1 - v_o) \longrightarrow v_s = 3v_1 - 2v_2 + \frac{d}{dt}(v_1 - v_o) \quad (1)$$

At node 2,

$$\frac{v_1 - v_2}{1} = 1\frac{d}{dt}(v_2 - 0) \longrightarrow v_1 - v_2 = \frac{dv_2}{dt} \quad (2)$$

But $v_2 = v_o$ so that (1) and (2) become

$$v_s = 3v_1 - 2v_o + \frac{d}{dt}(v_1 - v_o) \quad (1a)$$

$$v_1 = v_o + \frac{dv_o}{dt} \quad (2a)$$

Substituting (2a) into (1a) gives

$$v_s = 3v_o + 3\frac{dv_o}{dt} - 2v_o + \frac{dv_o}{dt} + \frac{d^2v_o}{dt^2} - \frac{dv_o}{dt}$$

$$\underline{v_s = \frac{d^2v_o}{dt^2} + 3\frac{dv_o}{dt} + v_o}$$

Chapter 8, Problem 65.

Determine the differential equation for the op amp circuit in Fig. 8.110. If $v_1(0^+) = 2$ V and $v_2(0^+) = 0$ V find v_o for $t > 0$. Let $R = 100\text{k}\Omega$ and $C = 1\,\mu\text{F}$.

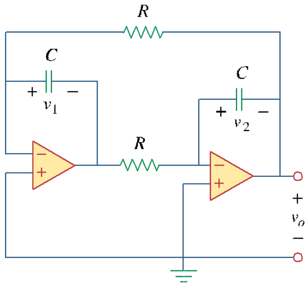


Figure 8.110
For Prob. 8.65.

Chapter 8, Solution 65.
At the input of the first op amp,

$$(v_o - 0)/R = Cd(v_1 - 0)/dt \quad (1)$$

At the input of the second op amp,

$$(-v_1 - 0)/R = Cdv_2/dt \quad (2)$$

Let us now examine our constraints. Since the input terminals are essentially at ground, then we have the following,

$$v_o = -v_2 \text{ or } v_2 = -v_o \quad (3)$$

Combining (1), (2), and (3), eliminating v_1 and v_2 we get,

$$\frac{d^2v_o}{dt^2} - \left(\frac{1}{R^2C^2}\right)v_o = \frac{d^2v_o}{dt^2} - 100v_o = 0$$

Which leads to $s^2 - 100 = 0$

Clearly this produces roots of –10 and +10.

And, we obtain,

$$v_o(t) = (Ae^{+10t} + Be^{-10t})V$$

At $t = 0$, $v_o(0+) = -v_2(0+) = 0 = A + B$, thus $B = -A$

This leads to $v_o(t) = (Ae^{+10t} - Ae^{-10t})V$. Now we can use $v_1(0+) = 2V$.

From (2), $v_1 = -RCdv_2/dt = 0.1dv_o/dt = 0.1(10Ae^{+10t} + 10Ae^{-10t})$

$$v_1(0+) = 2 = 0.1(20A) = 2A \text{ or } A = 1$$

Thus, $v_o(t) = \underline{\mathbf{(e^{+10t} - e^{-10t})V}}$

It should be noted that this circuit is unstable (clearly one of the poles lies in the right-half-plane).

Chapter 8, Problem 66.

Obtain the differential equations for $v_o(t)$ in the op amp circuit of Fig. 8.111.

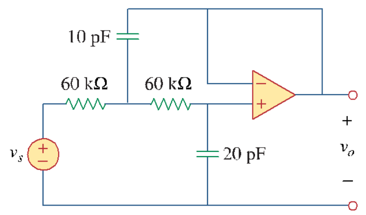
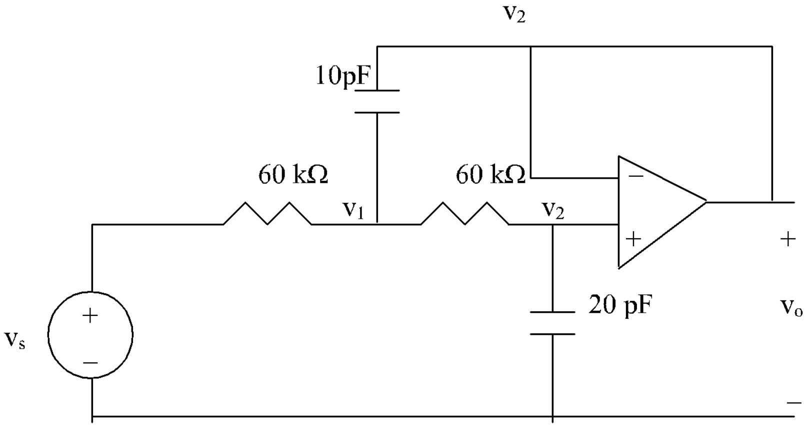


Figure 8.111
For Prob. 8.66.

Chapter 8, Solution 66.

We apply nodal analysis to the circuit as shown below.



At node 1,

$$\frac{v_s - v_1}{60k} = \frac{v_1 - v_2}{60k} + 10pF\frac{d}{dt}(v_1 - v_o)$$

But $v_2 = v_o$

$$v_s = 2v_1 - v_o + 6x10^{-7}\frac{d(v_1 - v_o)}{dt} \qquad (1)$$

At node 2,

$$\frac{v_1 - v_2}{60k} = 20pF\frac{d}{dt}(v_2 - 0), \qquad v_2 = v_o$$

$$v_1 = v_o + 1.2x10^{-6}\frac{dv_o}{dt} \qquad (2)$$

Substituting (2) into (1) gives

$$v_s = 2\left(v_o + 1.2x10^{-6}\frac{dv_o}{dt}\right) - v_o + 6x10^{-7}\left(1.2x10^{-6}\frac{d^2v_o}{dt^2}\right)$$

$$v_s = \underline{v_o + 2.4x10^{-6}(dv_o/dt) + 7.2x10^{-13}(d^2v_o/dt^2)}.$$

Chapter 8, Problem 67.

* In the op amp circuit of Fig. 8.112, determine $v_o(t)$ for $t > 0$. Let $v_{\text{in}} = u(t)\text{V}$, $R_1 = R_2 = 10\,\text{k}\Omega$, $C_1 = C_2 = 100\,\mu\text{F}$.

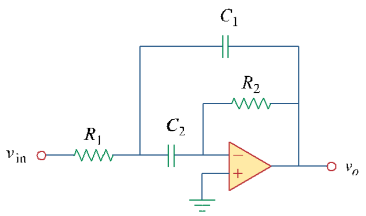


Figure 8.112
For Prob. 8.67.

* An asterisk indicates a challenging problem.

Chapter 8, Solution 67.

At node 1,

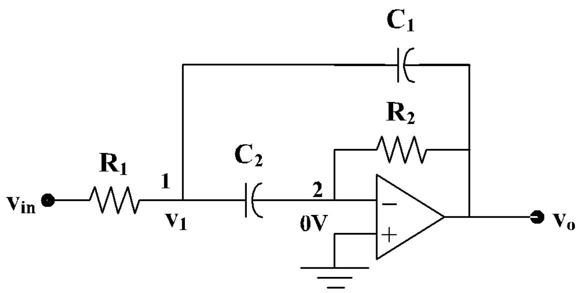
$$\frac{v_{in} - v_1}{R_1} = C_1 \frac{d(v_1 - v_o)}{dt} + C_2 \frac{d(v_1 - 0)}{dt} \qquad (1)$$

At node 2, $$C_2 \frac{d(v_1 - 0)}{dt} = \frac{0 - v_o}{R_2}, \text{ or } \frac{dv_1}{dt} = \frac{-v_o}{C_2 R_2} \qquad (2)$$

From (1) and (2),

$$v_{in} - v_1 = -\frac{R_1 C_1}{C_2 R_2}\frac{dv_o}{dt} - R_1 C_1 \frac{dv_o}{dt} - R_1 \frac{v_o}{R_2}$$

$$v_1 = v_{in} + \frac{R_1 C_1}{C_2 R_2}\frac{dv_o}{dt} + R_1 C_1 \frac{dv_o}{dt} + R_1 \frac{v_o}{R_2} \qquad (3)$$



From (2) and (3),

$$-\frac{v_o}{C_2R_2} = \frac{dv_1}{dt} = \frac{dv_{in}}{dt} + \frac{R_1C_1}{C_2R_2}\frac{dv_o}{dt} + R_1C_1\frac{d^2v_o}{dt^2} + \frac{R_1}{R_2}\frac{dv_o}{dt}$$

$$\frac{d^2v_o}{dt^2} + \frac{1}{R_2}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\frac{dv_o}{dt} + \frac{v_o}{C_1C_2R_2R_1} = -\frac{1}{R_1C_1}\frac{dv_{in}}{dt}$$

But $C_1C_2R_1R_2 = 10^{-4} x10^{-4} x10^4 x10^4 = 1$

$$\frac{1}{R_2}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = \frac{2}{R_2C_1} = \frac{2}{10^4 x10^{-4}} = 2$$

$$\frac{d^2v_o}{dt^2} + 2\frac{dv_o}{dt} + v_o = -\frac{dv_{in}}{dt}$$

Which leads to $s^2 + 2s + 1 = 0$ or $(s + 1)^2 = 0$ and $s = -1, -1$

Therefore, $v_o(t) = [(A + Bt)e^{-t}] + V_f$

As t approaches infinity, the capacitor acts like an open circuit so that

$$V_f = v_o(\infty) = 0$$

$v_{in} = 10u(t)$ mV and the fact that the initial voltages across each capacitor is 0

means that $v_o(0) = 0$ which leads to $A = 0$.

$$v_o(t) = [Bte^{-t}]$$

$$\frac{dv_o}{dt} = [(B - Bt)e^{-t}] \qquad (4)$$

From (2), $$\frac{dv_o(0+)}{dt} = -\frac{v_o(0+)}{C_2R_2} = 0$$

From (1) at t = 0+,

$$\frac{1-0}{R_1} = -C_1\frac{dv_o(0+)}{dt} \text{ which leads to } \frac{dv_o(0+)}{dt} = -\frac{1}{C_1R_1} = -1$$

Substituting this into (4) gives $B = -1$

Thus, $v(t) =$ $\mathbf{-te^{-t}u(t)}$ **V**

Chapter 8, Problem 68.

For the step function $v_s = u(t)$, use *PSpice* to find the response $v(t)$ for $0 < t < 6\,\text{s}$ in the circuit of Fig. 8.113.

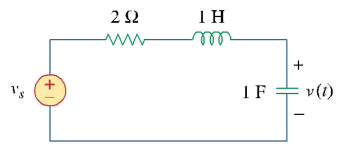
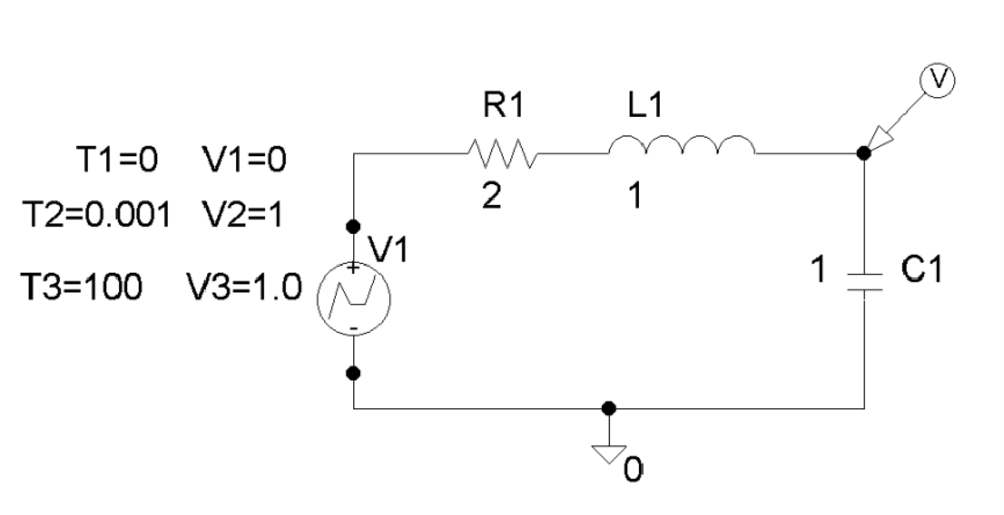
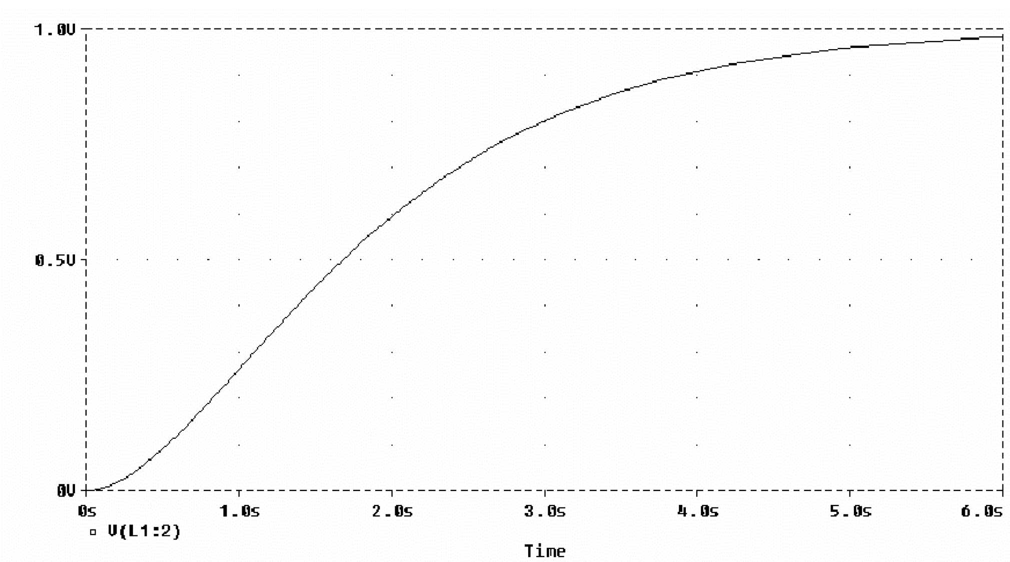


Figure 8.113
For Prob. 8.68.

Chapter 8, Solution 68.

The schematic is as shown below. The unit step is modeled by VPWL as shown. We insert a voltage marker to display V after simulation. We set Print Step = 25 ms and final step = 6s in the transient box. The output plot is shown below.





Chapter 8, Problem 69.

Given the source-free circuit in Fig. 8.114, use *PSpice* to get $i(t)$ for $0 < t < 20\,\text{s}$. Take $v(0) = 30$ V and $i(0) = 2$ A.

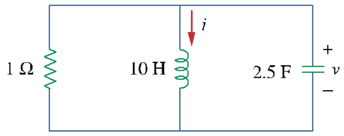
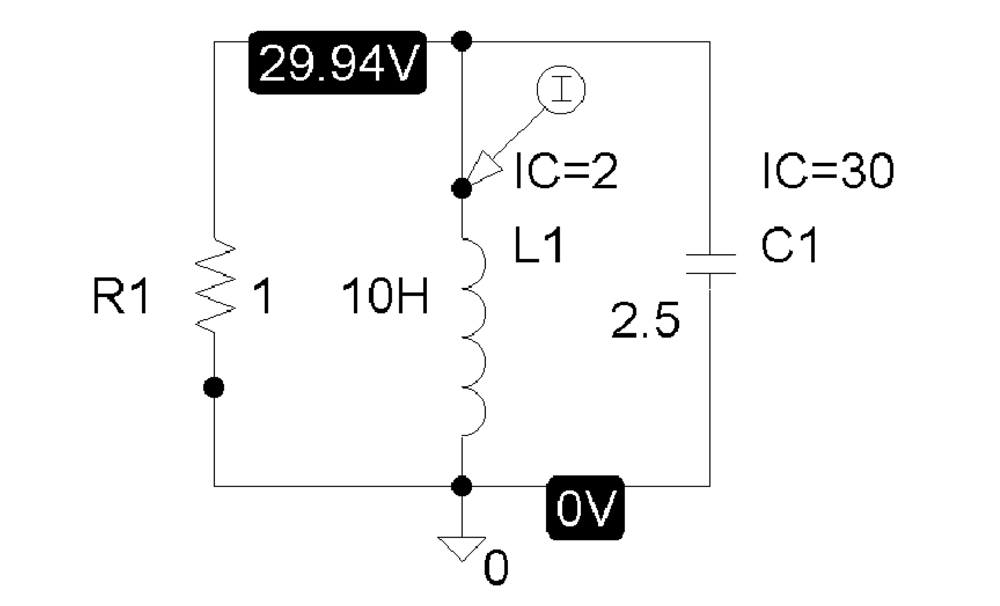
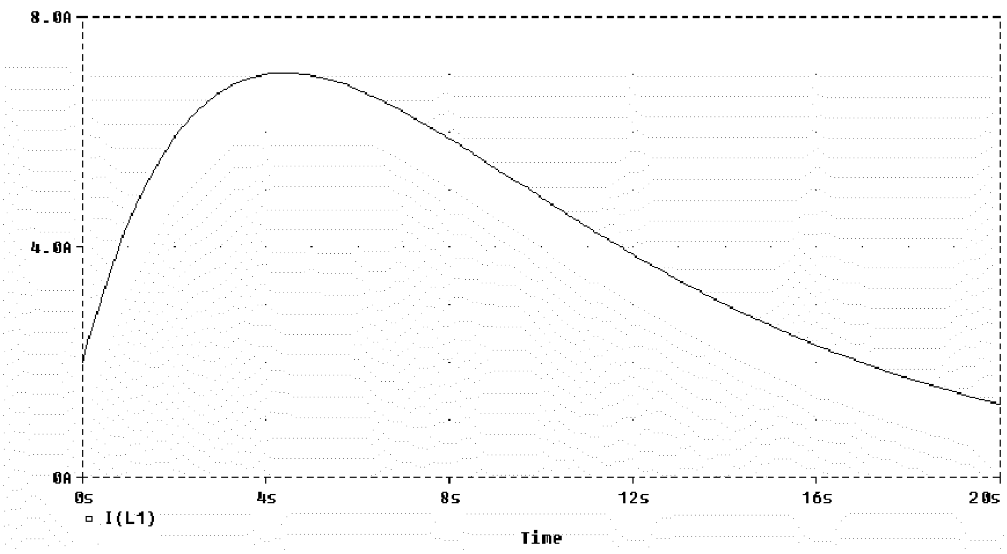


Figure 8.114
For Prob. 8.69.

Chapter 8, Solution 69.

The schematic is shown below. The initial values are set as attributes of L1 and C1. We set Print Step to 25 ms and the Final Time to 20s in the transient box. A current marker is inserted at the terminal of L1 to automatically display i(t) after simulation. The result is shown below.





Chapter 8, Problem 70.

For the circuit in Fig. 8.115, use *PSpice* to obtain $v(t)$t for $0 < t < 4$ s. Assume that the capacitor voltage and inductor current at $t = 0$ are both zero.

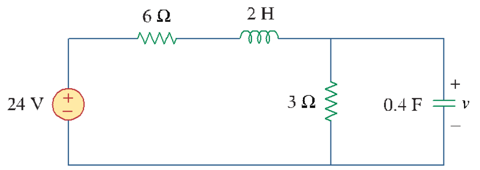
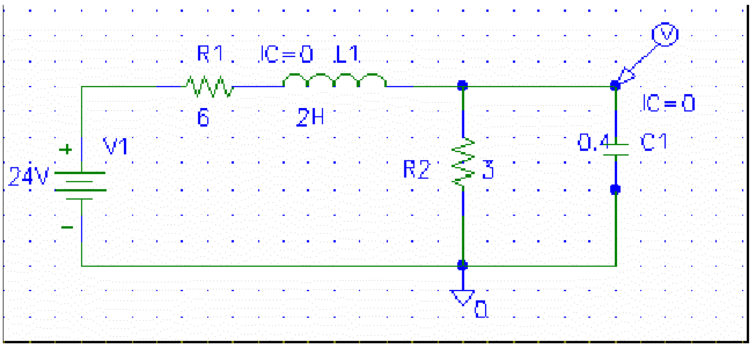


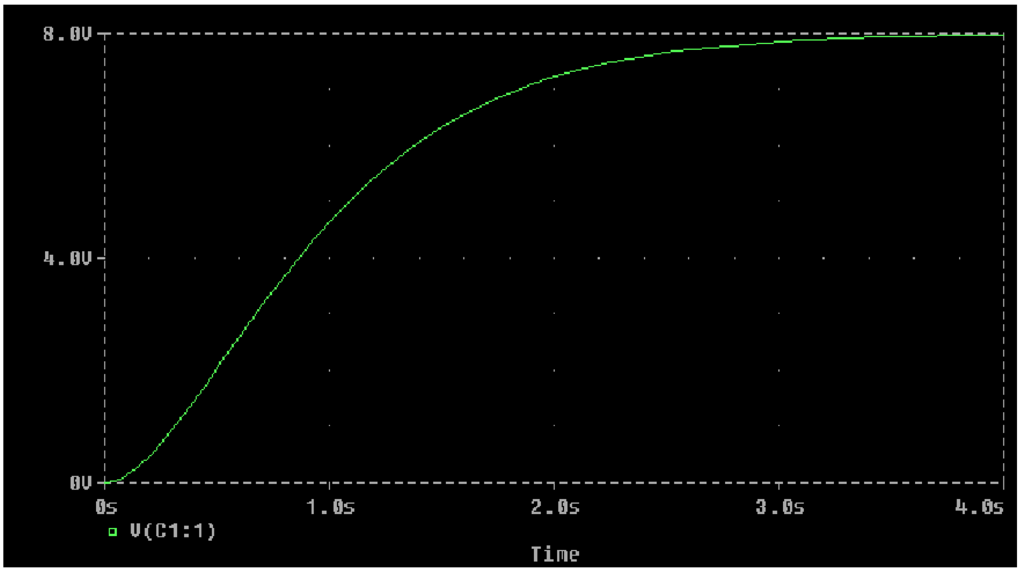
Figure 8.115
For Prob. 8.70.

Chapter 8, Solution 70.

The schematic is shown below.



After the circuit is saved and simulated, we obtain the capacitor voltage v(t) as shown below.



Chapter 8, Problem 71.

Obtain $v(t)$ for $0 < t < 4\,\text{s}$ in the circuit of Fig. 8.116 using *PSpice*.

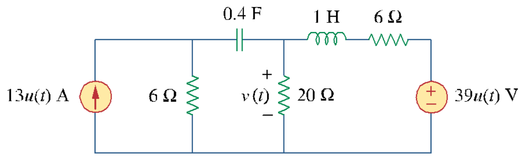
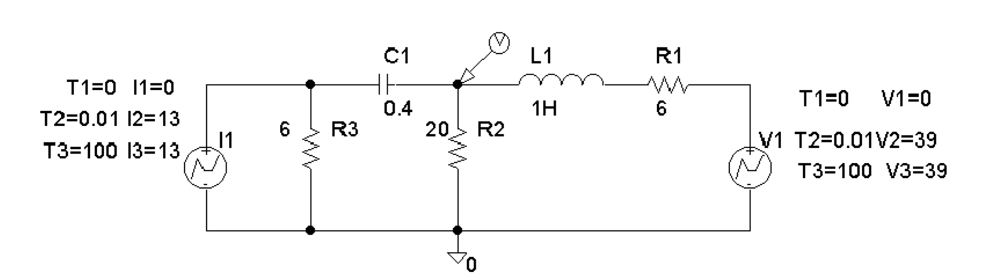
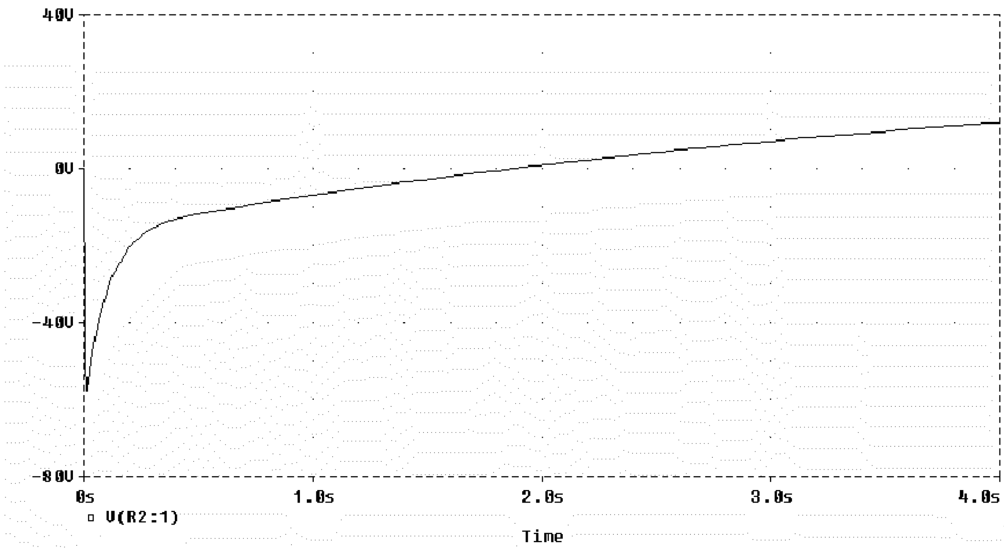


Figure 8.116
For Prob. 8.71.

Chapter 8, Solution 71.

The schematic is shown below. We use VPWL and IPWL to model the 39 u(t) V and 13 u(t) A respectively. We set Print Step to 25 ms and Final Step to 4s in the Transient box. A voltage marker is inserted at the terminal of R2 to automatically produce the plot of v(t) after simulation. The result is shown below.





Chapter 8, Problem 72.

The switch in Fig. 8.117 has been in position 1 for a long time. At $t = 0$, it is switched to position 2. Use *PSpice* to find $i(t)$ for $0 < t < 0.2$ s.

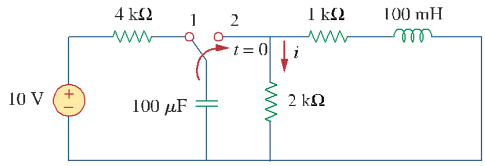
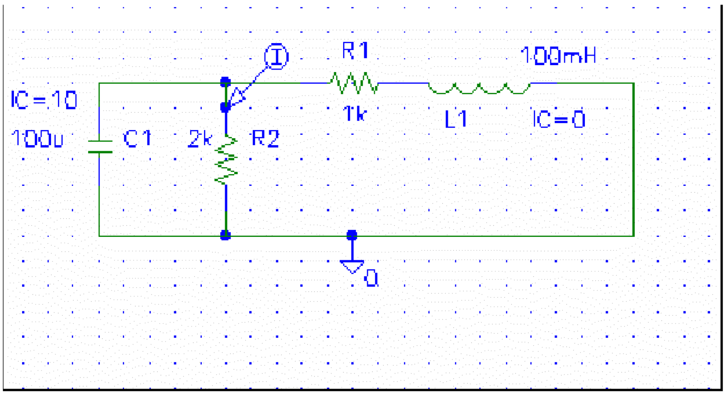


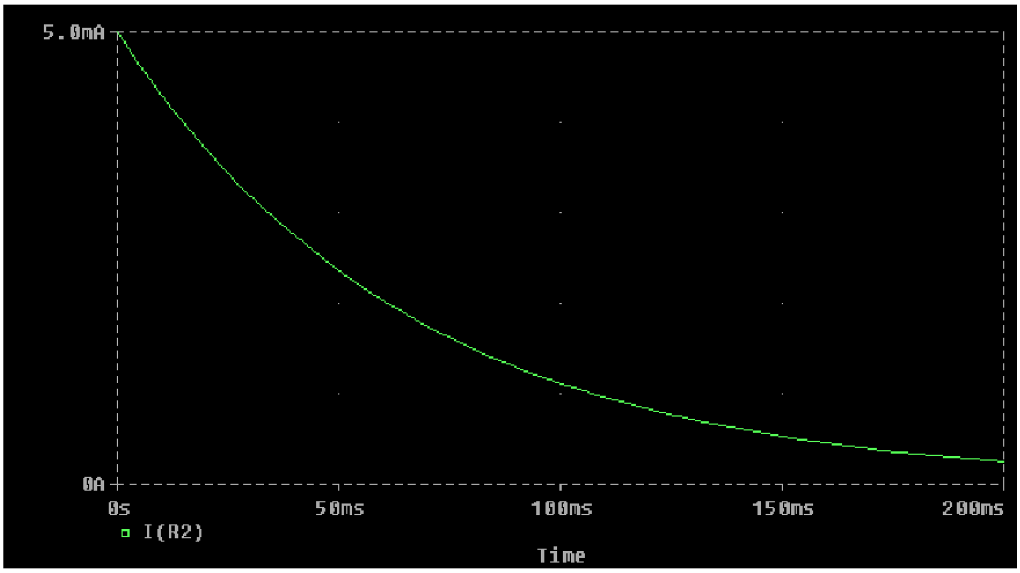
Figure 8.117
For Prob. 8.72.

Chapter 8, Solution 72.

When the switch is in position 1, we obtain IC=10 for the capacitor and IC=0 for the inductor. When the switch is in position 2, the schematic of the circuit is shown below.



When the circuit is simulated, we obtain i(t) as shown below.



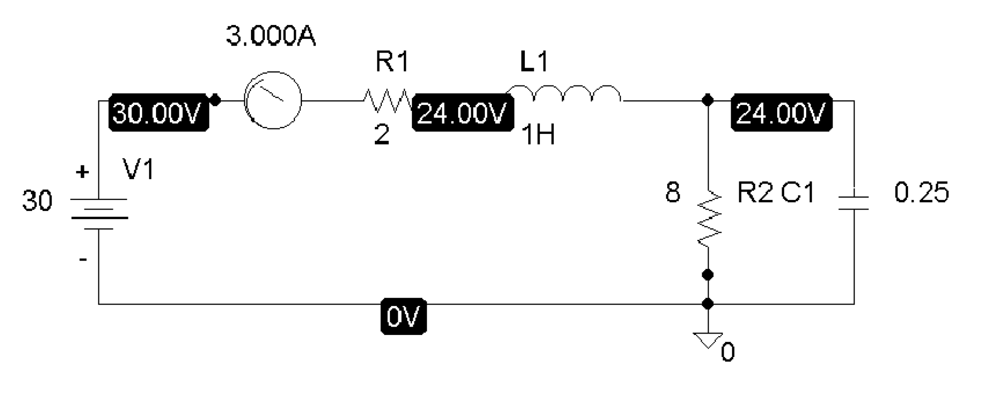
Chapter 8, Problem 73.

Rework Prob. 8.25 using *PSpice*. Plot $v_o(t)$ for $0 < t < 4\,\text{s}$.

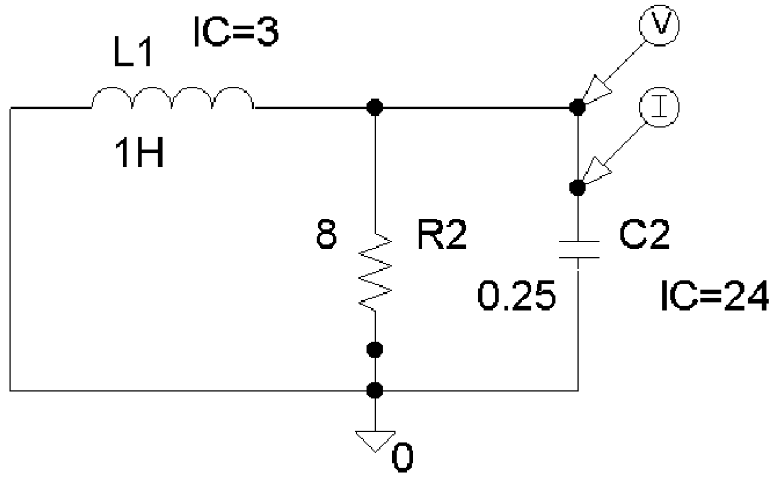
Chapter 8, Solution 73.

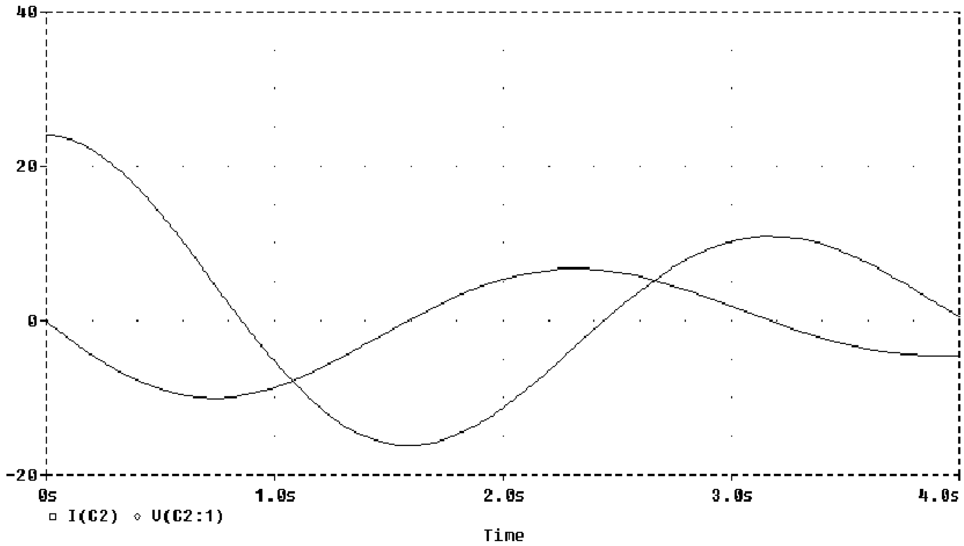
(a) For t < 0, we have the schematic below. When this is saved and simulated, we obtain the initial inductor current and capacitor voltage as

$$i_L(0) = 3\text{ A} \quad \text{and } v_c(0) = 24\text{ V}.$$



(b) For $t > 0$, we have the schematic shown below. To display i(t) and v(t), we insert current and voltage markers as shown. The initial inductor current and capacitor voltage are also incorporated. In the Transient box, we set Print Step = 25 ms and the Final Time to 4s. After simulation, we automatically have $i_o(t)$ and $v_o(t)$ displayed as shown below.





Chapter 8, Problem 74.

The dual is constructed as shown in Fig. 8.118(a). The dual is redrawn as shown in Fig. 8.118(b).

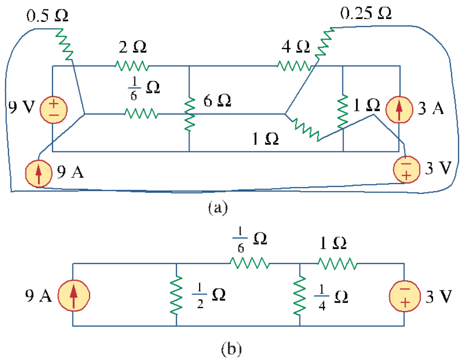
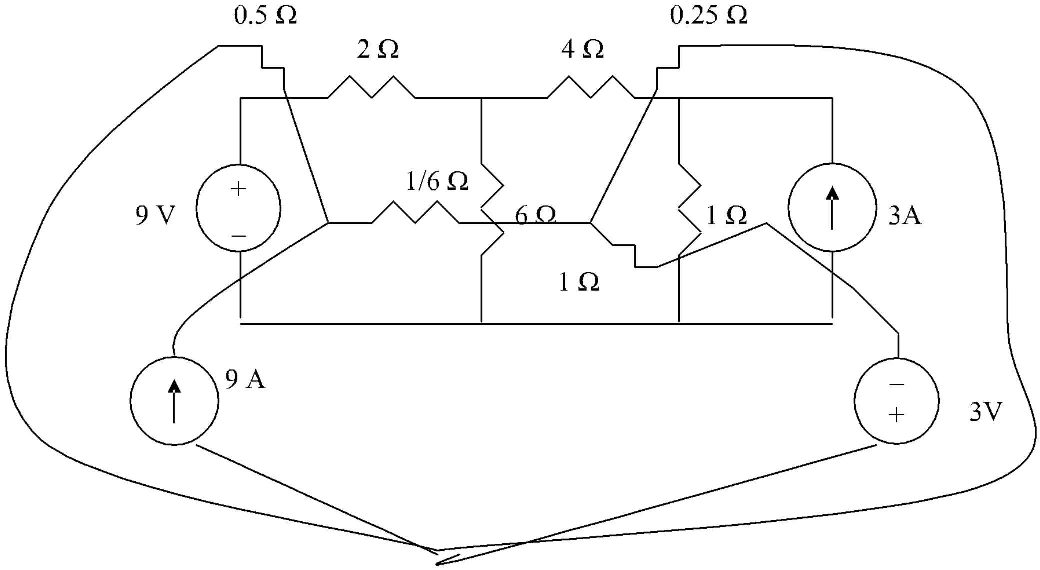
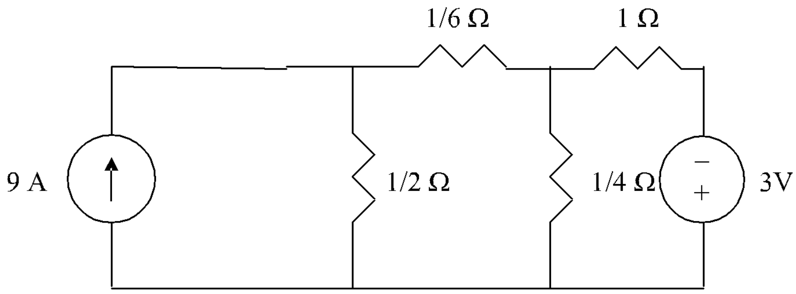


Figure 8.118
For Prob. 8.74.

Chapter 8, Solution 74.
The dual is constructed as shown below.



The dual is redrawn as shown below.



Chapter 8, Problem 75.

Obtain the dual of the circuit in Fig. 8.119.

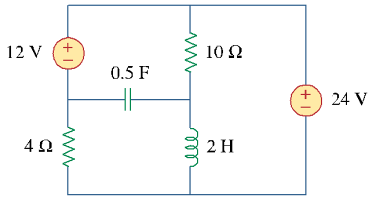
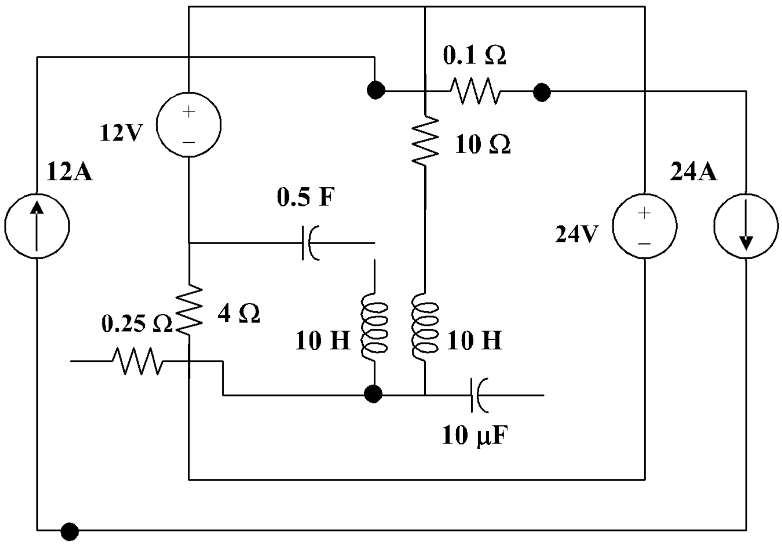


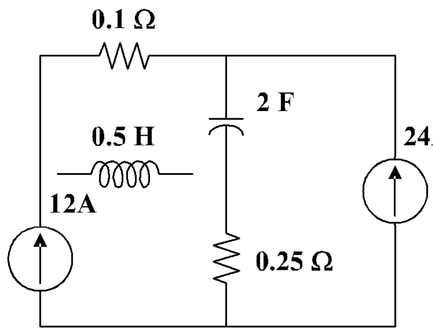
Figure 8.119
For Prob. 8.75.

Chapter 8, Solution 75.

The dual circuit is connected as shown in Figure (a). It is redrawn in Figure (b).



(a)



(b)

Chapter 8, Problem 76.

Find the dual of the circuit in Fig. 8.120.

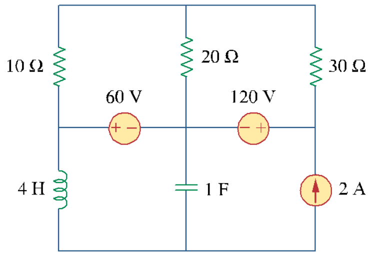
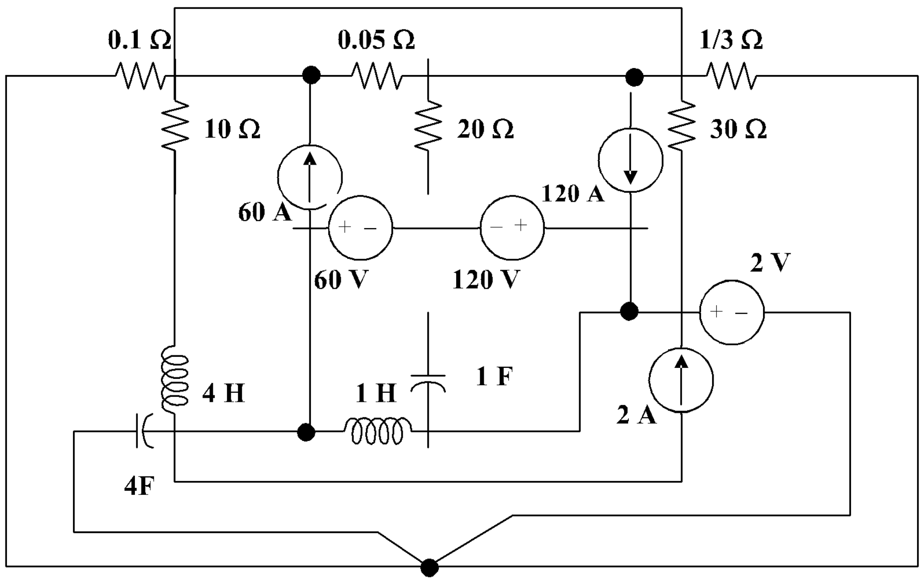


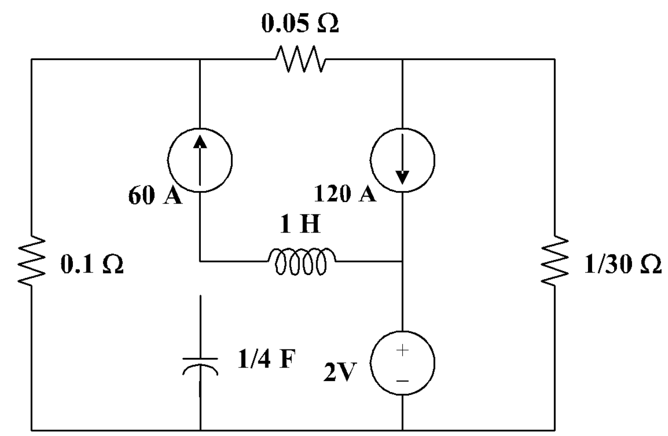
Figure 8.120
For Prob. 8.76.

Chapter 8, Solution 76.

The dual is obtained from the original circuit as shown in Figure (a). It is redrawn in Figure (b).



(a)



(b)

Chapter 8, Problem 77.

Draw the dual of the circuit in Fig. 8.121.

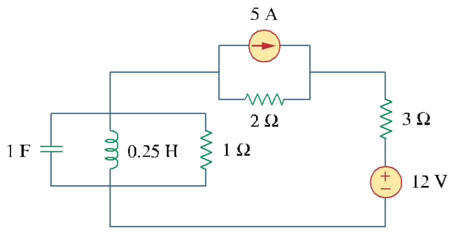
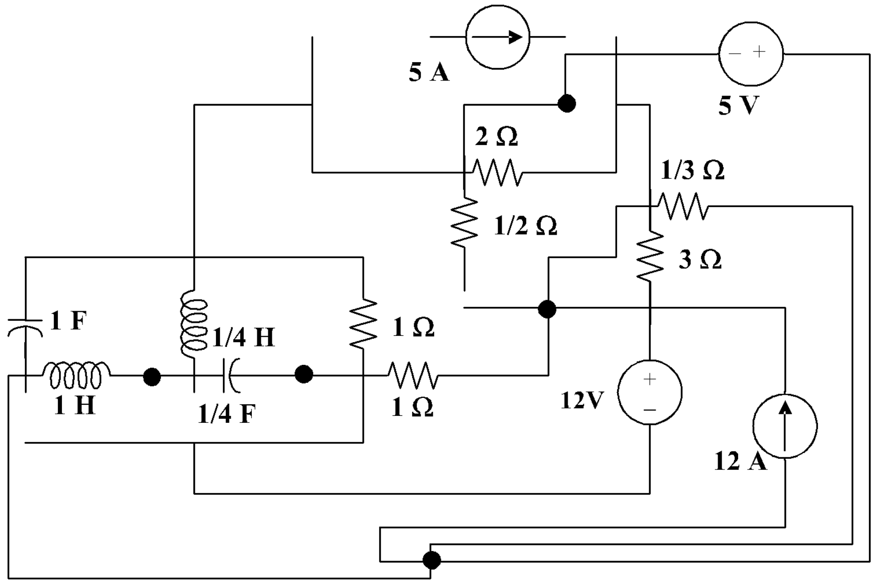


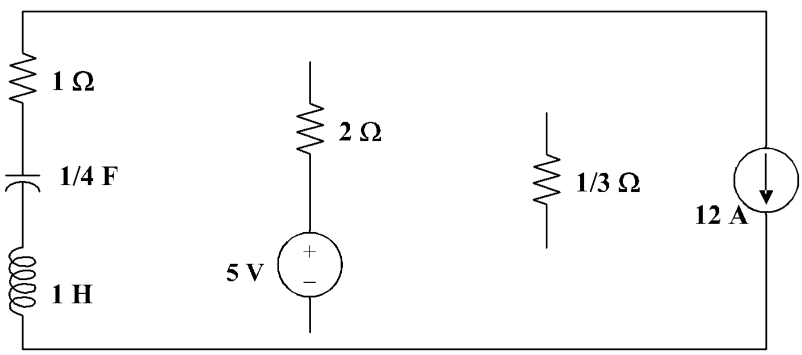
Figure 8.121
For Prob. 8.77.

Chapter 8, Solution 77.

The dual is constructed in Figure (a) and redrawn in Figure (b).



(a)



(b)

Chapter 8, Problem 78.

An automobile airbag igniter is modeled by the circuit in Fig. 8.122. Determine the time it takes the voltage across the igniter to reach its first peak after switching from *A* to *B*. Let $R = 3\Omega, C = 1/30\,\text{F}$, and $L = 60\,\text{mH}$.

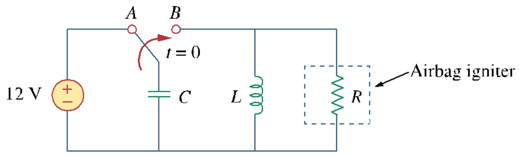


Figure 8.122
For Prob. 8.78.

Chapter 8, Solution 78.

The voltage across the igniter is $v_R = v_C$ since the circuit is a parallel RLC type.

$$v_C(0) = 12, \text{ and } i_L(0) = 0.$$

$$\alpha = 1/(2RC) = 1/(2x3x1/30) = 5$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{60x10^{-3}x1/30} = 22.36$$

$\alpha < \omega_o$ produces an underdamped response.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -5 \pm j21.794$$

$$v_C(t) = e^{-5t}(A\cos 21.794t + B\sin 21.794t) \qquad (1)$$

$$v_C(0) = 12 = A$$

$$dv_C/dt = -5[(A\cos 21.794t + B\sin 21.794t)e^{-5t}]$$

$$+ 21.794[(-A\sin 21.794t + B\cos 21.794t)e^{-5t}] \qquad (2)$$

$$dv_C(0)/dt = -5A + 21.794B$$

But, $dv_C(0)/dt = -[v_C(0) + Ri_L(0)]/(RC) = -(12 + 0)/(1/10) = -120$

Hence, $-120 = -5A + 21.794B$, leads to B $(5x12 - 120)/21.794 = -2.753$

At the peak value, $dv_C(t_o)/dt = 0$, i.e.,

$$0 = A + B\tan 21.794t_o + (A21.794/5)\tan 21.794t_o - 21.794B/5$$

$$(B + A21.794/5)\tan 21.794t_o = (21.794B/5) - A$$

$$\tan 21.794t_o = [(21.794B/5) - A]/(B + A21.794/5) = -24/49.55 = -0.484$$

Therefore, $21.7945t_o = |-0.451|$

$t_o = |-0.451|/21.794 =$ **20.68 ms**

Chapter 8, Problem 79.

A load is modeled as a 250-mH inductor in parallel with a 12-Ω resistor. A capacitor is needed to be connected to the load so that the network is critically damped at 60 Hz. Calculate the size of the capacitor.

Chapter 8, Solution 79.

For critical damping of a parallel RLC circuit,

$$\alpha = \omega_o \quad \longrightarrow \quad \frac{1}{2RC} = \frac{1}{\sqrt{LC}}$$

Hence,

$$C = \frac{L}{4R^2} = \frac{0.25}{4x144} = \underline{434\mu\text{F}}$$

Chapter 8, Problem 80.

eTd

A mechanical system is modeled by a series *RLC* circuit. It is desired to produce an overdamped response with time constants 0.1 ms and 0.5 ms. If a series 50-kΩ resistor is used, find the values of *L* and *C*.

Chapter 8, Solution 80.

$$t_1 = 1/|s_1| = 0.1x10^{-3} \text{ leads to } s_1 = -1000/0.1 = -10,000$$

$$t_2 = 1/|s_2| = 0.5x10^{-3} \text{ leads to } s_1 = -2,000$$

$$s_1 = -\alpha - \sqrt{\alpha^2 - \omega_o^2}$$

$$s_2 = -\alpha + \sqrt{\alpha^2 - \omega_o^2}$$

$$s_1 + s_2 = -2\alpha = -12,000, \text{ therefore } \alpha = 6,000 = R/(2L)$$

$$L = R/12,000 = 50,000/12,000 = \underline{\mathbf{4.167H}}$$

$$s_2 = -\alpha + \sqrt{\alpha^2 - \omega_o^2} = -2,000$$

$$\alpha - \sqrt{\alpha^2 - \omega_o^2} = 2,000$$

$$6,000 - \sqrt{\alpha^2 - \omega_o^2} = 2,000$$

$$\sqrt{\alpha^2 - \omega_o^2} = 4,000$$

$$\alpha^2 - \omega_o^2 = 16x10^6$$

$$\omega_o^2 = \alpha^2 - 16x10^6 = 36x10^6 - 16x10^6$$

$$\omega_o = 10^3\sqrt{20} = 1/\sqrt{LC}$$

$$C = 1/(20x10^6 x4.167) = \underline{\mathbf{12\ nF}}$$

Chapter 8, Problem 81.

An oscillogram can be adequately modeled by a second-order system in the form of a parallel *RLC* circuit. It is desired to give an underdamped voltage across a 200-Ω resistor. If the damping frequency is 4 kHz and the time constant of the envelope is 0.25 s, find the necessary values of *L* and *C*.

Chapter 8, Solution 81.

$$t = 1/\alpha = 0.25 \text{ leads to } \alpha = 4$$

But, $\alpha\ 1/(2RC)$ or, $C = 1/(2\alpha R) = 1/(2x4x200) = \underline{\mathbf{625\ \mu F}}$

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2}$$

$$\omega_o^2 = \omega_d^2 + \alpha^2 = (2\pi 4x10^3)^2 + 16 \cong (2\pi 4x10^3 0^2 = 1/(LC)$$

This results in $L = 1/(64\pi^2 x10^6 x625x10^{-6}) = \underline{\mathbf{2.533\ \mu H}}$

Chapter 8, Problem 82.

The circuit in Fig. 8.123 is the electrical analog of body functions used in medical schools to study convulsions. The analog is as follows:

C_1 = Volume of fluid in a drug

C_2 = Volume of blood stream in a specified region

R_1 = Resistance in the passage of the drug from the input to the blood stream

R_2 = Resistance of the excretion mechanism, such as kidney, etc.

v_0 = Initial concentration of the drug dosage

$v(t)$ = Percentage of the drug in the blood stream

Find $v(t)$ for $t > 0$ given that $C_1 = 0.5\mu\,\text{F}$, $C_2 = 5\mu\,\text{F}, R1 = 5\,\text{M}\Omega$, and $v_0 = 60u(t)\text{V}$.

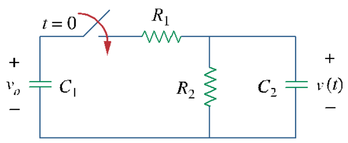
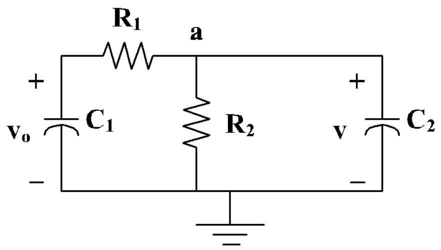


Figure 8.123
For Prob. 8.82.

Chapter 8, Solution 82.

For t = 0-, v(0) = 0.

For t > 0, the circuit is as shown below.



At node a,

$$(v_o - v/R_1 = (v/R_2) + C_2 dv/dt$$

$$v_o = v(1 + R_1/R_2) + R_1C_2\ dv/dt$$

$$60 = (1 + 5/2.5) + (5x10^6\ x5x10^{-6})dv/dt$$

$$60 = 3v + 25dv/dt$$

$$v(t) = V_s + [Ae^{-3t/25}]$$

where $3V_s = 60$ yields $V_s = 20$

$$v(0) = 0 = 20 + A \text{ or } A = -20$$

$$v(t) = \underline{\mathbf{20(1 - e^{-3t/25})V}}$$

Chapter 8, Problem 83.

Figure 8.124 shows a typical tunnel-diode oscillator circuit. The diode is modeled as a nonlinear resistor with $i_D = f(v_D)$ i.e., the diode current is a nonlinear function of the voltage across the diode. Derive the differential equation for the circuit in terms of v and i_D.

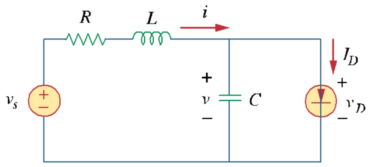


Figure 8.124
For Prob. 8.83.

Chapter 8, Solution 83.

$$i = i_D + Cdv/dt \qquad (1)$$

$$-v_s + iR + Ldi/dt + v = 0 \qquad (2)$$

Substituting (1) into (2),

$$v_s = Ri_D + RCdv/dt + Ldi_D/dt + LCd^2v/dt^2 + v = 0$$

$$LCd^2v/dt^2 + RCdv/dt + Ri_D + Ldi_D/dt = v_s$$

$$\underline{\mathbf{d^2v/dt^2 + (R/L)dv/dt + (R/LC)i_D + (1/C)di_D/dt = v_s/LC}}$$

Chapter 9, Problem 1.

Given the sinusoidal voltage $v(t) = 50 \cos(30t + 10^o)$ V, find: (a) the amplitude V_m,(b) the period T, (c) the frequency f, and (d) $v(t)$ at $t = 10$ ms.

Chapter 9, Solution 1.

(a) V_m = **50 V**.

(b) Period $T = \frac{2\pi}{\omega} = \frac{2\pi}{30} = 0.2094s$ = **209.4ms**

(c) Frequency f = ω/(2π) = 30/(2π) = **4.775 Hz**.

(d) At t=1ms, v(0.01) = 50cos(30x0.01rad + 10˚)
= 50cos(1.72˚ + 10˚) = **44.48 V** and ωt = **0.3 rad**.

Chapter 9, Problem 2.

A current source in a linear circuit has

$i_s = 8 \cos(500\pi t - 25^o)$ A

(a) What is the amplitude of the current?
(b) What is the angular frequency?
(c) Find the frequency of the current.
(d) Calculate i_s at t = 2ms.

Chapter 9, Solution 2.

(a) amplitude = **8 A**

(b) ω = 500π = **1570.8 rad/s**

(c) $f = \frac{\omega}{2\pi}$ = **250 Hz**

(d) $I_s = 8\angle\text{-}25°$ A

$I_s(2\text{ ms}) = 8\cos((500\pi)(2\times10^{-3}) - 25°)$
$= 8\cos(\pi - 25°) = 8\cos(155°)$
$=$ **-7.25 A**

Chapter 9, Problem 3.

Express the following functions in cosine form:

(a) 4 sin (ωt - 30^o)

(b) -2 sin 6t

(c) -10sin($\omega t + 20^o$)

Chapter 9, Solution 3.

(a) 4 sin(ωt – 30°) = 4 cos(ωt – 30° – 90°) = **<u>4 cos(ωt – 120°)</u>**

(b) -2 sin(6t) = **<u>2 cos(6t + 90°)</u>**

(c) -10 sin(ωt + 20°) = 10 cos(ωt + 20° + 90°) = **<u>10 cos(ωt + 110°)</u>**

Chapter 9, Problem 4.

(a) Express v = 8 cos(7t = 15^o) in sine form.

(b) Convert i = -10 sin(3t - 85^o) to cosine form.

Chapter 9, Solution 4.

(a) v = 8 cos(7t + 15°) = 8 sin(7t + 15° + 90°) = **<u>8 sin(7t + 105°)</u>**

(b) i = -10 sin(3t – 85°) = 10 cos(3t – 85° + 90°) = **<u>10 cos(3t + 5°)</u>**

Chapter 9, Problem 5.

Given v_1 = 20 sin(ωt + 60^o) and v_2 = 60 cos(ωt - 10^o) determine the phase angle between the two sinusoids and which one lags the other.

Chapter 9, Solution 5.

v_1 = 20 sin(ωt + 60°) = 20 cos(ωt + 60° – 90°) = 20 cos(ωt – 30°)
v_2 = 60 cos(ωt – 10°)

This indicates that the phase angle between the two signals is **<u>20°</u>** and that **<u>v_1 lags v_2</u>**.

Chapter 9, Problem 6.

For the following pairs of sinusoids, determine which one leads and by how much.

(a) $v(t) = 10 \cos(4t - 60^o)$ and $i(t) = 4 \sin(4t + 50^o)$

(b) $v_1(t) = 4 \cos(377t + 10^o)$ and $v_2(t) = -20 \cos 377t$

(c) $x(t) = 13 \cos 2t + 5 \sin 2t$ and $y(t) = 15 \cos(2t - 11.8^o)$

Chapter 9, Solution 6.

(a) $v(t) = 10 \cos(4t - 60°)$

$i(t) = 4 \sin(4t + 50°) = 4 \cos(4t + 50° - 90°) = 4 \cos(4t - 40°)$

Thus, **<u>i(t) leads v(t) by 20°</u>**.

(b) $v_1(t) = 4 \cos(377t + 10°)$

$v_2(t) = -20 \cos(377t) = 20 \cos(377t + 180°)$

Thus, **<u>$v_2(t)$ leads $v_1(t)$ by 170°</u>**.

(c) $x(t) = 13 \cos(2t) + 5 \sin(2t) = 13 \cos(2t) + 5 \cos(2t - 90°)$

$\mathbf{X} = 13\angle 0° + 5\angle -90° = 13 - j5 = 13.928\angle -21.04°$

$x(t) = 13.928 \cos(2t - 21.04°)$

$y(t) = 15 \cos(2t - 11.8°)$

phase difference $= -11.8° + 21.04° = 9.24°$

Thus, **<u>y(t) leads x(t) by 9.24°</u>**.

Chapter 9, Problem 7.

If $f(\phi) = \cos\phi + j \sin\phi$, show that $f(\phi) = e^{j\phi}$.

Chapter 9, Solution 7.

If $f(\phi) = \cos\phi + j \sin\phi$,

$$\frac{df}{d\phi} = -\sin\phi + j\cos\phi = j(\cos\phi + j\sin\phi) = jf(\phi)$$

$$\frac{df}{f} = j\,d\phi$$

Integrating both sides

$\ln f = j\phi + \ln A$

$f = Ae^{j\phi} = \cos\phi + j \sin\phi$

$f(0) = A = 1$

i.e. **<u>$f(\phi) = e^{j\phi} = \cos\phi + j \sin\phi$</u>**

Chapter 9, Problem 8.

Calculate these complex numbers and express your results in rectangular form:

(a) $\dfrac{15\angle 45^o}{3-j4}+j2$

(b) $\dfrac{8\angle -20^o}{(2+j)(3-j4)}+\dfrac{10}{-5+j12}$

(c) $10+(8\angle 50^o)(5-j12)$

Chapter 9, Solution 8.

(a)
$$\frac{15\angle 45°}{3-j4}+j2 = \frac{15\angle 45°}{5\angle -53.13°}+j2$$
$$= 3\angle 98.13° + j2$$
$$= -0.4245 + j2.97 + j2$$
$$= \underline{\mathbf{-0.4243 + j4.97}}$$

(b)
$$(2+j)(3-j4) = 6-j8+j3+4 = 10-j5 = 11.18\angle -26.57°$$
$$\frac{8\angle -20°}{(2+j)(3-j4)}+\frac{10}{-5+j12} = \frac{8\angle -20°}{11.18\angle -26.57°}+\frac{(-5-j12)(10)}{25+144}$$
$$= 0.7156\angle 6.57° - 0.2958 - j0.71$$
$$= 0.7109 + j0.08188 - 0.2958 - j0.71$$
$$= \underline{\mathbf{0.4151 - j0.6281}}$$

(c)
$$10+(8\angle 50°)(13\angle -68.38°) = 10+104\angle -17.38°$$
$$= \underline{\mathbf{109.25 - j31.07}}$$

Chapter 9, Problem 9.

Evaluate the following complex numbers and leave your results in polar form:

(a) $5\angle 30^o \left(6 - j8 + \frac{3\angle 60^o}{2+j}\right)$

(b) $\frac{(10\angle 60^o)(35\angle -50^o)}{(2+j6)-(5+j)}$

Chapter 9, Solution 9.

(a) $(5\angle 30°)(6 - j8 + 1.1197 + j0.7392) = (5\angle 30°)(7.13 - j7.261)$
$= (5\angle 30°)(10.176\angle -45.52°) =$

$\underline{\mathbf{50.88\angle{-15.52}°}}$.

(b) $\frac{(10\angle 60°)(35\angle -50°)}{(-3+j5) = (5.83\angle 120.96°)} =$ $\underline{\mathbf{60.02\angle{-110.96}°}}$.

Chapter 9, Problem 10.

Given that $z_1 = 6 - j8$, $z_2 = 10\angle\text{-}30^o$, and $z_3 = 8e^{-j120^o}$, find:

(a) $z_1 + z_2 + z_3$

(b) $\frac{z_1 z_2}{z_3}$

Chapter 9, Solution 10.

(a) $z_1 = 6 - j8$, $z_2 = 8.66 - j5$, and $z_3 = -4 - j6.9282$

$z_1 + z_2 + z_3 = \underline{10.66 - j19.93}$

(b) $\frac{z_1 z_2}{z_3} = \underline{9.999 + j7.499}$

Chapter 9, Problem 11.

Find the phasors corresponding to the following signals:

(a) $v(t) = 21\cos(4t - 15^o)$ V
(b) $i(t) = -8\sin(10t + 70^o)$ mA
(c) $v(t) = 120\sin(10t - 50^o)$ V
(d) $i(t) = -60\cos(30t + 10^o)$ mA

Chapter 9, Solution 11.

(a) $V = \underline{21 < -15^o \text{ V}}$

(b) $i(t) = 8\sin(10t + 70^o + 180^o) = 8\cos(10t + 70^o + 180^o - 90^o) = 8\cos(10t + 160^o)$

$I = \underline{8 < 160^o \text{ mA}}$

(c) $v(t) = 120\sin(10^3 t - 50^o) = 120\cos(10^3 t - 50^o - 90^o)$

$V = \underline{120 < -140^o \text{ V}}$

(d) $i(t) = -60\cos(30t + 10^o) = 60\cos(30t + 10^o + 180^o)$

$I = \underline{60 < 190^o \text{ mA}}$

Chapter 9, Problem 12.

Let **X** = $8\angle 40^o$ and and **Y** = $10\ \angle -30^o$ Evaluate the following quantities and express your results in polar form:

(a) **(X + Y)X*** (b) **(X -Y)*** (c) **(X + Y)/X**

Chapter 9, Solution 12.

Let **X** = 8∠40° and Y = 10∠-30°. Evaluate the following quantities and express your results in polar form.

(X + Y)/X*
(X - Y)*
(X + Y)/X

X = 6.128+j5.142; Y = 8.66–j5

(a) $$\mathbf{(X+Y)X^*} = \begin{array}{l}(14.788+j0.142)(8\angle -40^\circ)\\ =(14.789\angle 0.55^\circ)(8\angle -40^\circ)=118.31\angle -39.45^\circ\end{array}$$
= **<u>91.36-j75.17</u>**

(b) **(X - Y)*** = (–2.532+j10.142)* = **<u>(–2.532–j10.142)</u>** = 10.453∠–104.02°

(c) **(X + Y)/X** = (14.789∠0.55°)/(8∠40°) = 1.8486∠–39.45°
= **<u>1.4275–j1.1746</u>**

Chapter 9, Problem 13.

Evaluate the following complex numbers:

(a) $\dfrac{2+j3}{1-j6}+\dfrac{7-j8}{-5+j11}$

(b) $\dfrac{(5\angle 10^o)(10\angle -40^o)}{(4\angle -80^o)(-6\angle 50^o)}$

(c) $\begin{vmatrix} 2+j3 & -j2 \\ -j2 & 8-j5 \end{vmatrix}$

Chapter 9, Solution 13.

(a) $(-0.4324+j0.4054)+(-0.8425-j0.2534)=\underline{-1.2749+j0.1520}$

(b) $\dfrac{50\angle -30^o}{24\angle 150^o}=\underline{-2.0833}=\underline{\mathbf{-2.083}}$

(c) (2+j3)(8-j5) –(-4) = **35 +j14**

Chapter 9, Problem 14.

Simplify the following expressions:

(a) $\dfrac{(5-j6)-(2+j8)}{(-3+j4)(5-j)+(4-j6)}$

(b) $\dfrac{(240\angle 75^o+160\angle -30^o)(60-j80)}{(67+j84)(20\angle 32^o)}$

(c) $\left(\dfrac{10+j20}{3+j4}\right)^2\sqrt{(10+j5)(16-j120)}$

Chapter 9, Solution 14.

(a) $\dfrac{3-\text{j}14}{-7+\text{j}17}=\dfrac{14.318\angle -77.91°}{18.385\angle 112.38°}=0.7788\angle 169.71°=\underline{-0.7663+\text{j}0.13912}$

(b) $\dfrac{(62.116+j231.82+138.56-j80)(60-j80)}{(67+j84)(16.96+j10.5983)}=\dfrac{24186-6944.9}{246.06+j2134.7}=\underline{-1.922-j11.55}$

(c) $(-2+j4)^2\sqrt{(260-j120)}=(20\angle -126.86°)(16.923\angle -12.38°)=$
$338.46\angle -139.24°=\underline{-256.4-j221}$

Chapter 9, Problem 15.

Evaluate these determinants:

(a) $\begin{vmatrix} 10+j6 & 2-j3 \\ -5 & -1+j \end{vmatrix}$

(b) $\begin{vmatrix} 20\angle-30^\circ & -4\angle-10^\circ \\ 16\angle0^\circ & 3\angle45^\circ \end{vmatrix}$

(c) $\begin{vmatrix} 1-j & -j & 0 \\ j & 1 & -j \\ 1 & j & 1+j \end{vmatrix}$

Chapter 9, Solution 15.

(a) $$\begin{vmatrix} 10+\text{j}6 & 2-\text{j}3 \\ \text{-}5 & \text{-}1+\text{j} \end{vmatrix} = \text{-}10 - \text{j}6 + \text{j}10 - 6 + 10 - \text{j}15$$

$$= \underline{\mathbf{-6 - j11}}$$

(b) $$\begin{vmatrix} 20\angle-30^\circ & \text{-}4\angle\text{-}10^\circ \\ 16\angle0^\circ & 3\angle45^\circ \end{vmatrix} = 60\angle15^\circ + 64\angle\text{-}10^\circ$$

$$= 57.96 + \text{j}15.529 + 63.03 - \text{j}11.114$$

$$= \underline{\mathbf{120.99 + j4.415}}$$

(c) $$\begin{vmatrix} 1-\text{j} & -\text{j} & 0 \\ \text{j} & 1 & -\text{j} \\ 1 & \text{j} & 1+\text{j} \\ 1-\text{j} & -\text{j} & 0 \\ \text{j} & 1 & -\text{j} \end{vmatrix} = 1+1+0-1-0+\text{j}^2(1-\text{j})+\text{j}^2(1+\text{j})$$

$$= 1 - 1(1 - \text{j} + 1 + \text{j})$$

$$= 1 - 2 = \underline{\mathbf{-1}}$$

Chapter 9, Problem 16.

Transform the following sinusoids to phasors:

(a) -10 cos $(4t + 75^o)$

(b) 5 sin$(20t - 10^o)$

(c) 4 cos$2t$ + 3 sin $2t$

Chapter 9, Solution 16.

(a) $-10\cos(4t + 75°) = 10\cos(4t + 75° - 180°)$
$= 10\cos(4t - 105°)$
The phasor form is **$\underline{10\angle\text{-}105°}$**

(b) $5\sin(20t - 10°) = 5\cos(20t - 10° - 90°)$
$= 5\cos(20t - 100°)$
The phasor form is **$\underline{5\angle\text{-}100°}$**

(c) $4\cos(2t) + 3\sin(2t) = 4\cos(2t) + 3\cos(2t - 90°)$
The phasor form is $4\angle 0° + 3\angle\text{-}90° = 4 - j3 =$ **$\underline{5\angle\text{-}36.87°}$**

Chapter 9, Problem 17.

Two voltages v_1 and v_2 appear in series so that their sum is $v = v_1 + v_2$. If $v_1 = 10\cos(50t - \pi/3)$V and $v_2 = 12\cos(50t + 30^o)$ V, find v.

Chapter 9, Solution 17.

$$V = V_1 + V_2 = 10 < -60^o + 12 < 30^o = 5 - j8.66 + 10.392 + j6 = 15.62 < -9.805^o$$

$$v = \underline{15.62\cos(50t - 9.805^o)\text{ V}} = \underline{\mathbf{15.62cos(50t–9.8^\circ)\ V}}$$

Chapter 9, Problem 18.

Obtain the sinusoids corresponding to each of the following phasors:

(a) $\mathbf{V}_1 = 60\angle 15^o$ V, $\omega = 1$
(b) $\mathbf{V}_2 = 6 + j8$ V, $\omega = 40$
(c) $\mathbf{I}_1 = 2.8e^{-j\pi/3}$ A, $\omega = 377$
(d) $\mathbf{I}_2 = -0.5 - j1.2$ A, $\omega = 10^3$

Chapter 9, Solution 18.

(a) $v_1(t) = \underline{\mathbf{60\cos(t + 15°)}}$

(b) $\mathbf{V}_2 = 6 + j8 = 10\angle 53.13°$
$v_2(t) = \underline{\mathbf{10\cos(40t + 53.13°)}}$

(c) $i_1(t) = \underline{\mathbf{2.8\cos(377t - \pi/3)}}$

(d) $\mathbf{I}_2 = -0.5 - j1.2 = 1.3\angle 247.4°$
$i_2(t) = \underline{\mathbf{1.3\cos(10^3 t + 247.4°)}}$

Chapter 9, Problem 19.

Using phasors, find:

(a) $3\cos(20t + 10^o) – 5 \cos(20t - 30^o)$
(b) $40 \sin 50t + 30 \cos(50t - 45^o)$
(c) $20 \sin 400t + 10 \cos(400t + 60^o) -5 \sin(400t - 20^o)$

Chapter 9, Solution 19.

(a) $3\angle 10° - 5\angle\text{-}30° = 2.954 + j0.5209 - 4.33 + j2.5$
$= -1.376 + j3.021$
$= 3.32\angle 114.49°$

Therefore, $3 \cos(20t + 10°) - 5 \cos(20t - 30°) = \underline{\mathbf{3.32 \cos(20t + 114.49°)}}$

(b) $40\angle\text{-}90° + 30\angle\text{-}45° = -j40 + 21.21 - j21.21$
$= 21.21 - j61.21$
$= 64.78\angle\text{-}70.89°$

Therefore, $40 \sin(50t) + 30 \cos(50t - 45°) = \underline{\mathbf{64.78 \cos(50t - 70.89°)}}$

(c) Using $\sin\alpha = \cos(\alpha - 90°)$,
$20\angle\text{-}90° + 10\angle 60° - 5\angle\text{-}110° = -j20 + 5 + j8.66 + 1.7101 + j4.699$
$= 6.7101 - j6.641$
$= 9.44\angle\text{-}44.7°$

Therefore, $20 \sin(400t) + 10 \cos(400t + 60°) - 5 \sin(400t - 20°)$
$= \underline{\mathbf{9.44 \cos(400t - 44.7°)}}$

Chapter 9, Problem 20.

A linear network has a current input $4\cos(\omega t + 20^o)$A and a voltage output $10 \cos(\omega t +110^o)$ V. Determine the associated impedance.

Chapter 9, Solution 20.

$$I = 4 < 20^o, \quad V = 10 < 110^o$$

$$Z = \frac{V}{I} = \frac{10 < 110^o}{4 < 20^o} = 2.5 < 90^o = \underline{j2.5\ \Omega}$$

Chapter 9, Problem 21.

Simplify the following:

(a) $f(t) = 5\cos(2t + 15(^o) - 4\sin(2t - 30^o)$
(b) $g(t) = 8\sin t + 4\cos(t + 50^o)$
(c) $h(t) = \int_0^t (10\cos 40t + 50\sin 40t)dt$

Chapter 9, Solution 21.

(a) $F = 5\angle 15^o - 4\angle\underline{-30^o - 90^o} = 6.8296 + j4.758 = 8.3236\angle 34.86^o$

$$\underline{f(t) = 8.324\cos(30t + 34.86^o)}$$

(b) $G = 8\angle -90^o + 4\angle 50^o = 2.571 - j4.9358 = 5.565\angle -62.49^o$

$$\underline{g(t) = 5.565\cos(t - 62.49^o)}$$

(c) $\mathrm{H} = \frac{1}{\mathrm{j}\omega}\left(10\angle 0^o + 50\angle -90^o\right), \quad \omega = 40$

i.e. $\mathrm{H} = 0.25\angle -90^o + 1.25\angle -180^o = -\mathrm{j}0.25 - 1.25 = 1.2748\angle -168.69^o$

h(t) = **1.2748cos(40t – 168.69°)**

Chapter 9, Problem 22.

An alternating voltage is given by $v(t) = 20\cos(5t - 30^o)$ V. Use phasors to find

$$10v(t) + 4\frac{dv}{dt} - 2\int_{-\infty}^{t} v(t)dt$$

Assume that the value of the integral is zero at $t = -\infty$.

Chapter 9, Solution 22.

Let $\mathrm{f(t)} = 10v(t) + 4\frac{dv}{dt} - 2\int_{-\infty}^{t} v(t)dt$

$F = 10V + j\omega 4V - \frac{2V}{j\omega}, \quad \omega = 5, \quad V = 20\angle -30^o$

$\mathrm{F} = 10\mathrm{V} + \mathrm{j}20\mathrm{V} - \mathrm{j}0.4\mathrm{V} = (10 + \mathrm{j}20.4)(17.32 - \mathrm{j}10) = 454.4\angle 33.89^o$

$\underline{\mathrm{f(t)} = 454.4\cos(5\mathrm{t} + 33.89^o)}$

Chapter 9, Problem 23.

Apply phasor analysis to evaluate the following.

(a) $v = 50\cos(\omega t + 30^o) + 30\cos(\omega t + 90^o)$V

(b) $i = 15\cos(\omega t + 45^o) - 10\sin(\omega t + 45^o)$A

Chapter 9, Solution 23.

(a) $V = 50 < 30^o + 30 < 90^o = 43.3 + j25 - j30 = 43.588 < -6.587^o$

$v = \underline{43.588\cos(\omega t - 6.587^o)\ \text{V}}$ = **43.49cos(ωt–6.59°) V**

(b) $I = 15 < 45^o - 10 < 45^o - 90^o = (10.607 + j10.607) - (7.071 - j7.071) = 18.028 < 78.69^o$

$i = \underline{18.028\cos(\omega t + 78.69^o)\ \text{A}}$ = **18.028cos(ωt+78.69°) A**

Chapter 9, Problem 24.

Find $v(t)$ in the following integrodifferential equations using the phasor approach:

(a) $v(t) + \int v\, dt = 10\cos t$

(b) $\frac{dv}{dt} + 5v(t) + 4\int v\, dt = 20\sin(4t + 10^o)$

Chapter 9, Solution 24.

(a)

$$\mathbf{V} + \frac{\mathbf{V}}{j\omega} = 10\angle 0°, \quad \omega = 1$$

$$\mathbf{V}(1 - j) = 10$$

$$\mathbf{V} = \frac{10}{1 - j} = 5 + j5 = 7.071\angle 45°$$

Therefore, v(t) = **7.071 cos(t + 45°)**

(b)

$$j\omega\mathbf{V} + 5\mathbf{V} + \frac{4\mathbf{V}}{j\omega} = 20\angle(10° - 90°), \quad \omega = 4$$

$$\mathbf{V}\left(j4 + 5 + \frac{4}{j4}\right) = 20\angle -80°$$

$$\mathbf{V} = \frac{20\angle -80°}{5 + j3} = 3.43\angle -110.96°$$

Therefore, v(t) = **3.43 cos(4t – 110.96°)**

Chapter 9, Problem 25.

Using phasors, determine $i(t)$ in the following equations:

(a) $2\dfrac{di}{dt} + 3i(t) = 4\cos(2t - 45^o)$

(b) $10\int i\, dt + \dfrac{di}{dt} + 6i(t) = 5\cos(5t + 22^o)$

Chapter 9, Solution 25.

(a)

$$2j\omega\mathbf{I} + 3\mathbf{I} = 4\angle\text{-}45°, \quad \omega = 2$$
$$\mathbf{I}(3 + j4) = 4\angle\text{-}45°$$
$$I = \frac{4\angle\text{-}45°}{3 + j4} = \frac{4\angle\text{-}45°}{5\angle 53.13°} = 0.8\angle\text{-}98.13°$$

Therefore, i(t) = **<u>0.8 cos(2t – 98.13°)</u>**

(b)

$$10\frac{\mathbf{I}}{j\omega} + j\omega\mathbf{I} + 6\mathbf{I} = 5\angle 22°, \quad \omega = 5$$
$$(\text{-}j2 + j5 + 6)\mathbf{I} = 5\angle 22°$$
$$\mathbf{I} = \frac{5\angle 22°}{6 + j3} = \frac{5\angle 22°}{6.708\angle 26.56°} = 0.745\angle\text{-}4.56°$$

Therefore, i(t) = **<u>0.745 cos(5t – 4.56°)</u>**

Chapter 9, Problem 26.

The loop equation for a series RLC circuit gives

$$\frac{di}{dt} + 2i + \int_{-\infty}^{t} i\, dt = \cos 2t$$

Assuming that the value of the integral at $t = -\infty$ is zero, find $i(t)$ using the phasor method.

Chapter 9, Solution 26.

$$j\omega\mathbf{I} + 2\mathbf{I} + \frac{\mathbf{I}}{j\omega} = 1\angle 0°, \quad \omega = 2$$
$$\mathbf{I}\left(j2 + 2 + \frac{1}{j2}\right) = 1$$
$$\mathbf{I} = \frac{1}{2 + j1.5} = 0.4\angle\text{-}36.87°$$

Therefore, i(t) = **<u>0.4 cos(2t – 36.87°)</u>**

Chapter 9, Problem 27.

A parallel *RLC* circuit has the node equation

$$\frac{dv}{dt} = 50v + 100\int v\,dt = 110\cos(377t - 10^o)$$

Determine $v(t)$ using the phasor method. You may assume that the value of the integral at $t = -\infty$ is zero.

Chapter 9, Solution 27.

$$j\omega\mathbf{V} + 50\mathbf{V} + 100\frac{\mathbf{V}}{j\omega} = 110\angle\text{-}10°, \quad \omega = 377$$

$$\mathbf{V}\left(j377 + 50 - \frac{j100}{377}\right) = 110\angle\text{-}10°$$

$$\mathbf{V}\,(380.6\angle 82.45°) = 110\angle\text{-}10°$$

$$\mathbf{V} = 0.289\angle\text{-}92.45°$$

Therefore, v(t) = **0.289 cos(377t – 92.45°)**.

Chapter 9, Problem 28.

Determine the current that flows through an 8-Ω resistor connected to a voltage source $v_s = 110\cos 377t$ V.

Chapter 9, Solution 28.

$$i(t) = \frac{v_s(t)}{R} = \frac{110\cos(377t)}{8} = \mathbf{13.75\ cos(377t)\ A}.$$

Chapter 9, Problem 29.

What is the instantaneous voltage across a 2- μ F capacitor when the current through it is i =4 sin($10^6 t$ +25^o) A?

Chapter 9, Solution 29.

$$\mathbf{Z} = \frac{1}{j\omega C} = \frac{1}{j(10^6)(2 \times 10^{-6})} = -j0.5$$

$$\mathbf{V} = \mathbf{IZ} = (4\angle 25°)(0.5\angle -90°) = 2\angle -65°$$

Therefore v(t) = **2 sin(10^6t – 65°) V**.

Chapter 9, Problem 30.

A voltage $v(t)$ = 100 cos($60t$ + 20^o) V is applied to a parallel combination of a 40-kΩ resistor and a 50- μ F capacitor. Find the steady-state currents through the resistor and the capacitor.

Chapter 9, Solution 30.

Since R and C are in parallel, they have the same voltage across them. For the resistor,

$$V = I_R R \quad \longrightarrow \quad I_R = V/R = \frac{100 < 20^o}{40k} = 2.5 < 20^o \text{ mA}$$

$$\underline{i_R = 2.5\cos(60t + 20^o) \text{ mA}}$$

For the capacitor,

$$i_C = C\frac{dv}{dt} = 50x10^{-6}(-60)x100\sin(60t + 20^o) = \underline{-300\sin(60t + 20^o) \text{ mA}}$$

Chapter 9, Problem 31.

A series *RLC* circuit has $R = 80\ \Omega$, $L = 240$ mH, and $C = 5$ mF. If the input voltage is $v(t) = 10\cos 2t$ find the currrent flowing through the circuit.

Chapter 9, Solution 31.

$$L = 240mH \longrightarrow j\omega L = j2x240x10^{-3} = j0.48$$

$$C = 5mF \longrightarrow \frac{1}{j\omega C} = \frac{1}{j2x5x10^{-3}} = -j100$$

$$Z = 80 + j0.48 - j100 = 80 - j99.52$$

$$I = \frac{V}{Z} = \frac{10 < 0^0}{80 - j99.52} = 0.0783 < 51.206^o$$

$$i(t) = \underline{78.3\cos(2t + 51.206^o)\text{ mA}} = \underline{\mathbf{78.3cos(2t+51.26°)\ mA}}$$

Chapter 9, Problem 32.

For the network in Fig. 9.40, find the load current $\mathbf{I}_L$.

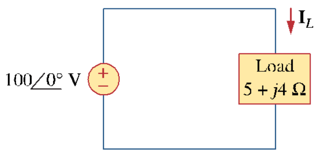


Figure 9.40
For Prob. 9.32.

Chapter 9, Solution 32.

$$I = \frac{V}{Z} = \frac{100 < 0^o}{5 + j4} = 12.195 - 9.756 = \underline{15.62 < -38.66^o\ \text{A}}$$

Chapter 9, Problem 33.

A series *RL* circuit is connected to a 110-V ac source. If the voltage across the resistor is 85 V, find the voltage across the inductor.

Chapter 9, Solution 33.

$$110 = \sqrt{v_R^2 + v_L^2}$$
$$v_L = \sqrt{110^2 - v_R^2}$$
$$v_L = \sqrt{110^2 - 85^2} = \underline{\mathbf{69.82\ V}}$$

Chapter 9, Problem 34.

What value of ω will cause the forced response v_o in Fig. 9.41 to be zero?

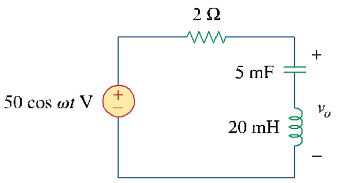


Figure 9.41
For Prob. 9.34.

Chapter 9, Solution 34.

$$v_o = 0 \text{ if } \omega L = \frac{1}{\omega C} \longrightarrow \omega = \frac{1}{\sqrt{LC}}$$

$$\omega = \frac{1}{\sqrt{(5\times10^{-3})(20\times10^{-3})}} = \underline{\mathbf{100\ rad/s}}$$

Chapter 9, Problem 35.

Find current i in the circuit of Fig. 9.42, when $v_s(t) = 50 \cos 200t$ V.

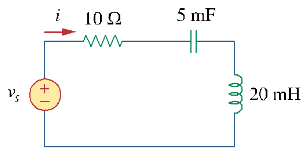


Figure 9.42
For Prob. 9.35.

Chapter 9, Solution 35.

$$v_s(t) = 50\cos 200t \quad \longrightarrow \quad V_s = 50 < 0^o, \omega = 200$$

$$5mF \quad \longrightarrow \quad \frac{1}{j\omega C} = \frac{1}{j200x5x10^{-3}} = -j$$

$$20mH \quad \longrightarrow \quad j\omega L = j20x10^{-3}x200 = j4$$

$$Z_{in} = 10 - j + j4 = 10 + j3$$

$$I = \frac{V_s}{Z_{in}} = \frac{50 < 0^o}{10 + j3} = 4.789 < -16.7^o$$

$$i(t) = \underline{4.789\cos(200t - 16.7^o)\ \text{A}}$$

Chapter 9, Problem 36.

In the circuit of Fig. 9.43, determine i. Let $v_s = 60\cos(200t - 10^o)$V.

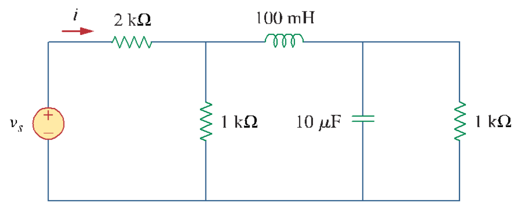


Figure 9.43
For Prob. 9.36.

Chapter 9, Solution 36.

Let Z be the input impedance at the source.

$$100\,\text{mH} \longrightarrow j\omega L = j200x100x10^{-3} = j20$$

$$10\mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j10x10^{-6}x200} = -j500$$

1000//-j500 = 200 –j400
1000//(j20 + 200 –j400) = 242.62 –j239.84

$$Z = 2242.62 - j239.84 = 2255\angle -6.104^o$$

$$I = \frac{60\angle -10^o}{2255\angle -6.104^o} = 26.61\angle -3.896^o \text{ mA}$$

$$i = \underline{266.1\cos(200t - 3.896^o)}\,\text{mA}$$

Chapter 9, Problem 37.

Determine the admittance **Y** for the circuit in Fig. 9.44.

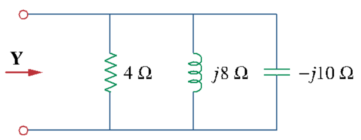


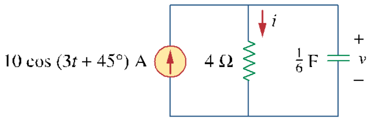
Figure 9.44
For Prob. 9.37.

Chapter 9, Solution 37.

$$Y = \frac{1}{4} + \frac{1}{j8} + \frac{1}{-j10} = \underline{0.25 - j0.025 \text{ S}} = \underline{\mathbf{250\text{–}j25\ mS}}$$

Chapter 9, Problem 38.

Find $i(t)$ and $v(t)$ in each of the circuits of Fig. 9.45.



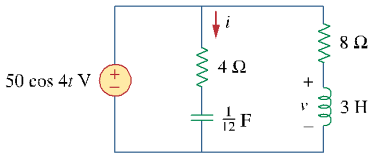


Figure 9.45
For Prob. 9.38.

Chapter 9, Solution 38.

(a) $$\frac{1}{6}\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(3)(1/6)} = -j2$$

$$\mathbf{I} = \frac{-j2}{4 - j2}(10\angle 45°) = 4.472\angle -18.43°$$

Hence, i(t) = **<u>4.472 cos(3t – 18.43°) A</u>**

$$\mathbf{V} = 4\mathbf{I} = (4)(4.472\angle -18.43°) = 17.89\angle -18.43°$$

Hence, v(t) = **<u>17.89 cos(3t – 18.43°) V</u>**

(b) $$\frac{1}{12}\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(4)(1/12)} = -j3$$

$$3\text{ H} \longrightarrow j\omega L = j(4)(3) = j12$$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{50\angle 0°}{4 - j3} = 10\angle 36.87°$$

Hence, i(t) = **<u>10 cos(4t + 36.87°) A</u>**

$$\mathbf{V} = \frac{j12}{8 + j12}(50\angle 0°) = 41.6\angle 33.69°$$

Hence, v(t) = **<u>41.6 cos(4t + 33.69°) V</u>**

Chapter 9, Problem 39.

For the circuit shown in Fig. 9.46, find Z_{eg} and use that to find current **I**. Let ω = 10 rad/s.

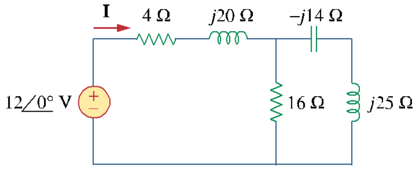


Figure 9.46
For Prob. 9.39.

Chapter 9, Solution 39.

$$Z_{eq} = 4 + j20 + 10//(-j14 + j25) = \underline{9.135 + j27.47\ \ \Omega}$$

$$I = \frac{V}{Z_{eq}} = \frac{12}{9.135 + j27.47} = 0.4145 < -71.605^o$$

$$i(t) = \underline{0.4145\cos(10t - 71.605^o)\ \ A} = \mathbf{\underline{414.5cos(10t–71.6^\circ)\ mA}}$$

Chapter 9, Problem 40.

In the circuit of Fig. 9.47, find i_o when:

(a) $\omega = 1$ rad/s (b) $\omega = 5$ rad/s
(c) $\omega = 10$ rad/s

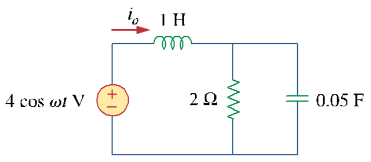


Figure 9.47
For Prob. 9.40.

Chapter 9, Solution 40.

(a) For $\omega = 1$,

$$1\text{ H} \longrightarrow j\omega L = j(1)(1) = j$$

$$0.05\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(1)(0.05)} = \text{-}j20$$

$$\mathbf{Z} = j + 2 \,||\, (\text{-}j20) = j + \frac{\text{-}j40}{2 - j20} = 1.98 + j0.802$$

$$\mathbf{I}_o = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{4\angle 0°}{1.98 + j0.802} = \frac{4\angle 0°}{2.136\angle 22.05°} = 1.872\angle\text{-}22.05°$$

Hence, $i_o(t) =$ **1.872 cos(t – 22.05°) A**

(b) For $\omega = 5$,

$$1\text{ H} \longrightarrow j\omega L = j(5)(1) = j5$$

$$0.05\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(5)(0.05)} = \text{-}j4$$

$$\mathbf{Z} = j5 + 2 \,||\, (\text{-}j4) = j5 + \frac{\text{-}j4}{1 - j2} = 1.6 + j4.2$$

$$\mathbf{I}_o = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{4\angle 0°}{1.6 + j4} = \frac{4\angle 0°}{4.494\angle 69.14°} = 0.89\angle\text{-}69.14°$$

Hence, $i_o(t) =$ **0.89 cos(5t – 69.14°) A**

(c) For $\omega = 10$,

$$1\text{ H} \longrightarrow j\omega L = j(10)(1) = j10$$

$$0.05\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10)(0.05)} = \text{-}j2$$

$$\mathbf{Z} = j10 + 2 \,||\, (\text{-}j2) = j10 + \frac{\text{-}j4}{2 - j2} = 1 + j9$$

$$\mathbf{I}_o = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{4\angle 0°}{1 + j9} = \frac{4\angle 0°}{9.055\angle 83.66°} = 0.4417\angle\text{-}83.66°$$

Hence, $i_o(t) =$ **0.4417 cos(10t – 83.66°) A**

Chapter 9, Problem 41.

Find *v*(*t*) in the *RLC* circuit of Fig. 9.48.

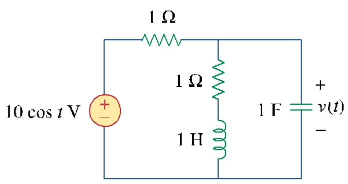


Figure 9.48
For Prob. 9.41.

Chapter 9, Solution 41.

$$\omega = 1,$$

$$1\text{ H} \longrightarrow j\omega L = j(1)(1) = j$$

$$1\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(1)(1)} = \text{-}j$$

$$\mathbf{Z} = 1 + (1 + j) \parallel (\text{-}j) = 1 + \frac{\text{-}j + 1}{1} = 2 - j$$

$$\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}} = \frac{10}{2 - j}, \qquad \mathbf{I}_c = (1 + j)\mathbf{I}$$

$$\mathbf{V} = (\text{-}j)(1 + j)\mathbf{I} = (1 - j)\mathbf{I} = \frac{(1 - j)(10)}{2 - j} = 6.325\angle\text{-}18.43°$$

Thus, $v(t) = \underline{\mathbf{6.325\cos(t - 18.43°)\ V}}$

Chapter 9, Problem 42.

Calculate $v_o(t)$ in the circuit of Fig. 9.49.

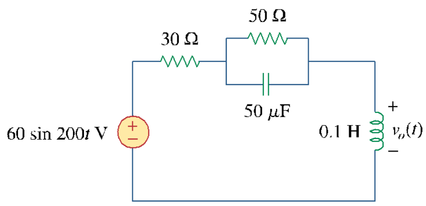


Figure 9.49
For Prob. 9.42.

Chapter 9, Solution 42.

$$\omega = 200$$

$$50\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(200)(50\times 10^{-6})} = \text{-j}100$$

$$0.1\ \text{H} \longrightarrow j\omega L = j(200)(0.1) = j20$$

$$50 \,||\, \text{-j}100 = \frac{(50)(\text{-j}100)}{50 - j100} = \frac{\text{- j}100}{1\text{ - j}2} = 40 - j20$$

$$\mathbf{V}_o = \frac{j20}{j20 + 30 + 40 - j20}(60\angle 0°) = \frac{j20}{70}(60\angle 0°) = 17.14\angle 90°$$

Thus, $v_o(t) =$ **<u>17.14 sin(200t + 90°) V</u>**

or $v_o(t) =$ **<u>17.14 cos(200t) V</u>**

Chapter 9, Problem 43.

Find current $\mathbf{I}_o$ in the circuit shown in Fig. 9.50.

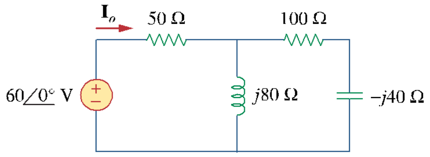


Figure 9.50
For Prob. 9.43.

Chapter 9, Solution 43.

$$Z_{in} = 50 + j80//(100 - j40) = 50 + \frac{j80(100 - j40)}{100 + j40} = 105.71 + j57.93$$

$$I_o = \frac{60 < 0^o}{Z_{in}} = 0.4377 - 0.2411 = \underline{0.4997 < -28.85^o \text{ A}} = \underline{\mathbf{499.7\angle{-28.85^\circ}\ mA}}$$

Chapter 9, Problem 44.

Calculate $i(t)$ in the circuit of Fig. 9.51.

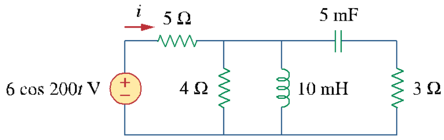


Figure 9.51
For prob. 9.44.

Chapter 9, Solution 44.

$$\omega = 200$$

$$10 \text{ mH} \longrightarrow \quad j\omega L = j(200)(10\times10^{-3}) = j2$$

$$5 \text{ mF} \longrightarrow \quad \frac{1}{j\omega C} = \frac{1}{j(200)(5\times10^{-3})} = -j$$

$$\mathbf{Y} = \frac{1}{4} + \frac{1}{j2} + \frac{1}{3-j} = 0.25 - j0.5 + \frac{3+j}{10} = 0.55 - j0.4$$

$$\mathbf{Z} = \frac{1}{\mathbf{Y}} = \frac{1}{0.55 - j0.4} = 1.1892 + j0.865$$

$$\mathbf{I} = \frac{6\angle0^\circ}{5+\mathbf{Z}} = \frac{6\angle0^\circ}{6.1892 + j0.865} = 0.96\angle{-7.956^\circ}$$

Thus, i(t) = **<u>0.96 cos(200t – 7.956°) A</u>**

Chapter 9, Problem 45.

Find current $\mathbf{I}_o$ in the network of Fig. 9.52.

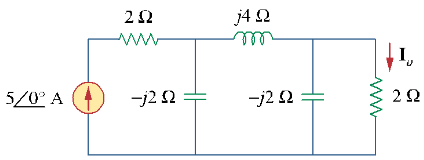
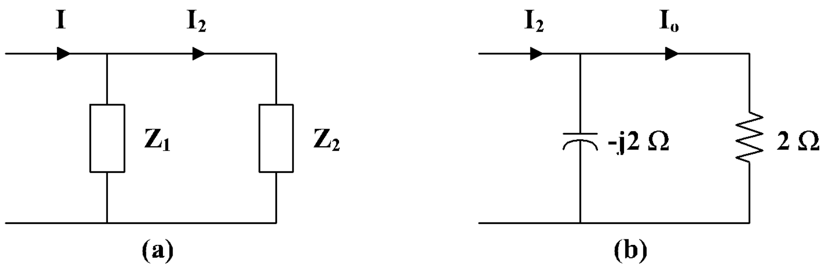


Figure 9.52
For Prob. 9.45.

Chapter 9, Solution 45.

We obtain I_o by applying the principle of current division twice.



$$\mathbf{Z}_1 = -j2, \qquad \mathbf{Z}_2 = j4 + (-j2)\,||\,2 = j4 + \frac{-j4}{2-j2} = 1 + j3$$

$$\mathbf{I}_2 = \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{I} = \frac{-j2}{-j2+1+j3}(5\angle 0°) = \frac{-j10}{1+j}$$

$$\mathbf{I}_o = \frac{-j2}{2-j2}\mathbf{I}_2 = \left(\frac{-j}{1-j}\right)\left(\frac{-j10}{1+j}\right) = \frac{-10}{1+1} = \underline{\mathbf{-5\ A}}$$

Chapter 9, Problem 46.

If $i_s = 5\cos(10t + 40^o)$ A in the circuit of Fig. 9.53, find i_o.

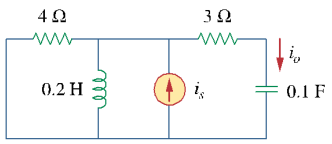


Figure 9.53
For Prob. 9.46.

Chapter 9, Solution 46.

$$i_s = 5\cos(10t + 40°) \longrightarrow \mathbf{I}_s = 5\angle 40°$$

$$0.1\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10)(0.1)} = \text{-}j$$

$$0.2\text{ H} \longrightarrow j\omega L = j(10)(0.2) = j2$$

Let $\mathbf{Z}_1 = 4 \,||\, j2 = \dfrac{j8}{4 + j2} = 0.8 + j1.6$, $\mathbf{Z}_2 = 3 - j$

$$\mathbf{I}_o = \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{I}_s = \frac{0.8 + j1.6}{3.8 + j0.6}(5\angle 40°)$$

$$\mathbf{I}_o = \frac{(1.789\angle 63.43°)(5\angle 40°)}{3.847\angle 8.97°} = 2.325\angle 94.46°$$

Thus, $i_o(t) =$ **<u>2.325 cos(10t + 94.46°) A</u>**

Chapter 9, Problem 47.

In the circuit of Fig. 9.54, determine the value of $i_s(t)$.

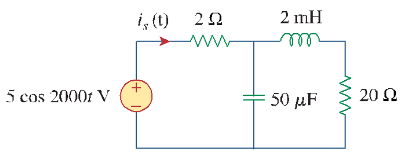
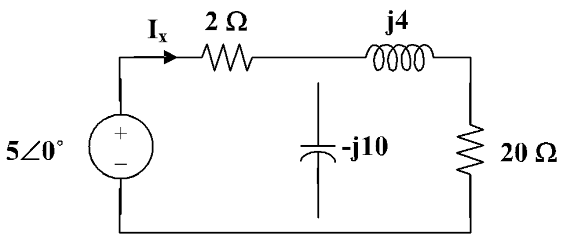


Figure 9.54
For Prob. 9.47.

Chapter 9, Solution 47.

First, we convert the circuit into the frequency domain.



$$I_x = \frac{5}{2+\dfrac{-j10(20+j4)}{-j10+20+j4}} = \frac{5}{2+4.588-j8.626} = \frac{5}{10.854\angle -52.63^\circ} = 0.4607\angle 52.63^\circ$$

$i_s(t)$ = **<u>460.7cos(2000t +52.63°) mA</u>**

Chapter 9, Problem 48.

Given that $v_s(t) = 20\sin(100t - 40^o)$ in Fig. 9.55, determine $i_x(t)$.

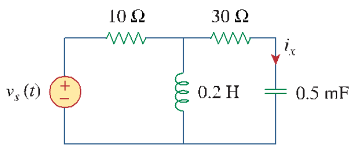
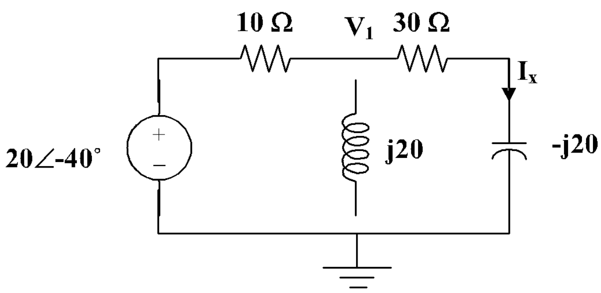


Figure 9.55
For Prob. 9.48.

Chapter 9, Solution 48.

Converting the circuit to the frequency domain, we get:



We can solve this using nodal analysis.

$$\frac{V_1 - 20\angle -40°}{10} + \frac{V_1 - 0}{j20} + \frac{V_1 - 0}{30 - j20} = 0$$

$$V_1(0.1 - j0.05 + 0.02307 + j0.01538) = 2\angle -40°$$

$$V_1 = \frac{2\angle 40°}{0.12307 - j0.03462} = 15.643\angle -24.29°$$

$$I_x = \frac{15.643\angle -24.29°}{30 - j20} = 0.4338\angle 9.4°$$

$$i_x = \underline{0.4338\sin(100t + 9.4°)\,A}$$

Chapter 9, Problem 49.

Find $v_s(t)$ in the circuit of Fig. 9.56 if the current i_x through the 1-Ω resistor is 0.5 sin 200t A.

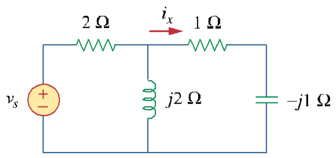
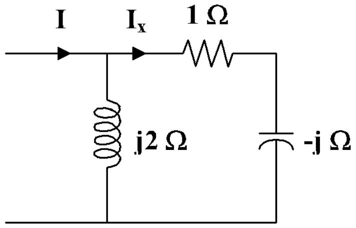


Figure 9.56
For Prob. 9.49.

Chapter 9, Solution 49.

$$\mathbf{Z}_T = 2 + j2 \,||\, (1 - j) = 2 + \frac{(j2)(1-j)}{1+j} = 4$$



$$\mathbf{I}_x = \frac{j2}{j2 + 1 - j}\mathbf{I} = \frac{j2}{1+j}\mathbf{I}, \qquad \text{where } \mathbf{I}_x = 0.5\angle 0° = \frac{1}{2}$$

$$\mathbf{I} = \frac{1+j}{j2}\mathbf{I}_x = \frac{1+j}{j4}$$

$$\mathbf{V}_s = \mathbf{I}\,\mathbf{Z}_T = \frac{1+j}{j4}(4) = \frac{1+j}{j} = 1 - j = 1.414\angle -45°$$

$v_s(t) =$ **1.414 sin(200t – 45°) V**

Chapter 9, Problem 50.

Determine v_x in the circuit of Fig. 9.57. Let $i_s(t) = 5\cos(100t + 40^o)$A.

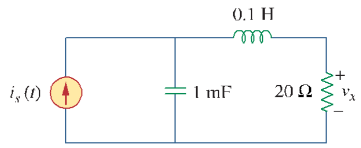
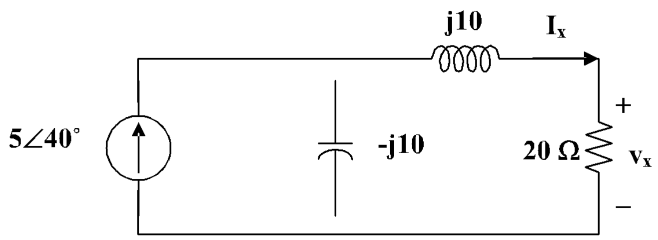


Figure 9.57
For Prob. 9.50.

Chapter 9, Solution 50.

Since ω = 100, the inductor = j100x0.1 = j10 Ω and the capacitor = 1/(j100x10^{-3})
= -j10Ω.



Using the current dividing rule:

$$I_x = \frac{-j10}{-j10 + 20 + j10} 5\angle 40° = -j2.5\angle 40° = 2.5\angle -50°$$

$$V_x = 20I_x = 50\angle -50°$$

$$v_x = \underline{50\cos(100t - 50°)\,V}$$

Chapter 9, Problem 51.

If the voltage v_o across the 2-Ω resistor in the circuit of Fig. 9.58 is 10 cos2t V, obtain i_s.

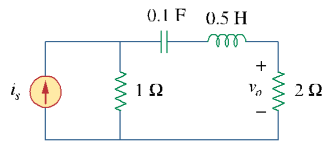


Figure 9.58
For Prob. 9.51.

Chapter 9, Solution 51.

$$0.1\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(2)(0.1)} = \text{-j5}$$

$$0.5\text{ H} \longrightarrow j\omega L = j(2)(0.5) = j$$

The current **I** through the 2-Ω resistor is

$$\mathbf{I} = \frac{1}{1 - j5 + j + 2}\mathbf{I}_s = \frac{\mathbf{I}_s}{3 - j4}, \qquad \text{where } \mathbf{I} = \frac{10}{2}\angle 0° = 5$$

$$\mathbf{I}_S = (5)(3 - j4) = 25\angle\text{-}53.13°$$

Therefore,

$i_s(t) =$ **25 cos(2t – 53.13°) A**

Chapter 9, Problem 52.

If $\mathbf{V}_o = 8\ \angle 30^o$ V in the circuit of Fig. 9.59, find $\mathbf{I}_s$.

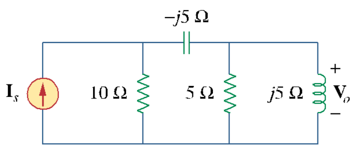
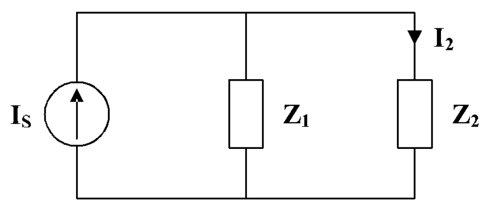


Figure 9.59
For Prob. 9.52.

Chapter 9, Solution 52.

$$5 \,||\, j5 = \frac{j25}{5+j5} = \frac{j5}{1+j} = 2.5 + j2.5$$

$$\mathbf{Z}_1 = 10, \qquad \mathbf{Z}_2 = -j5 + 2.5 + j2.5 = 2.5 - j2.5$$



$$\mathbf{I}_2 = \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{I}_s = \frac{10}{12.5 - j2.5}\mathbf{I}_s = \frac{4}{5-j}\mathbf{I}_s$$

$$\mathbf{V}_o = \mathbf{I}_2\,(2.5 + j2.5)$$

$$8\angle 30^\circ = \left(\frac{4}{5-j}\right)\mathbf{I}_s\,(2.5)(1+j) = \frac{10(1+j)}{5-j}\mathbf{I}_s$$

$$\mathbf{I}_s = \frac{(8\angle 30^\circ)(5-j)}{10(1+j)} = \underline{\mathbf{2.884\angle\text{-}26.31^\circ\ A}}$$

Chapter 9, Problem 53.

Find $\mathbf{I}_o$ in the circuit of Fig. 9.60.

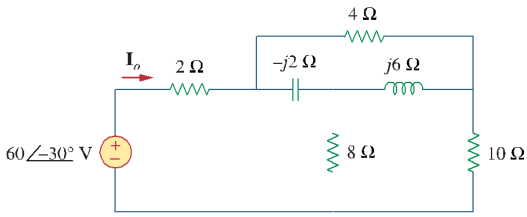
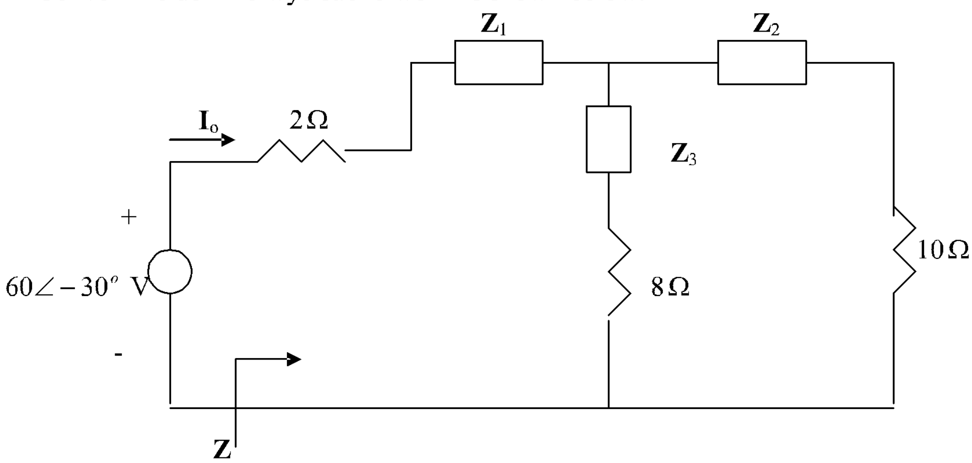


Figure 9.60
For Prob. 9.53.

Chapter 9, Solution 53.

Convert the delta to wye subnetwork as shown below.



$$Z_1 = \frac{-j2x4}{4+j4} = \frac{8\angle-90^\circ}{5.6569\angle45^\circ} = -1-j1, \qquad Z_2 = \frac{j6x4}{4+j4} = 3+j3,$$

$$Z_3 = \frac{12}{4+j4} = 1.5-j1.5$$

$$(Z_3+8)//(Z_2+10) = (9.5-j1.5)//(13+j3) = 5.691\angle0.21^\circ = 5.691+j0.02086$$

$$Z = 2+Z_1+5.691+j0.02086 = 6.691-j0.9791$$

$$I_o = \frac{60\angle-30^o}{Z} = \frac{60\angle-30^o}{6.7623\angle-8.33^o} = \underline{8.873\angle-21.67^o \ \text{A}}$$

Chapter 9, Problem 54.

In the circuit of Fig. 9.61, find $\mathbf{V}_s$ if $\mathbf{I}_o = 2\angle 0^o$ A.

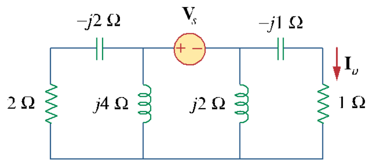
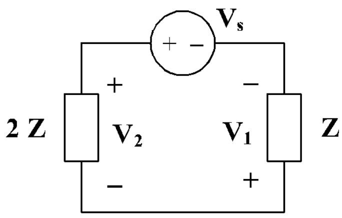


Figure 9.61
For Prob. 9.54.

Chapter 9, Solution 54.

Since the left portion of the circuit is twice as large as the right portion, the equivalent circuit is shown below.



$$\mathbf{V}_1 = \mathbf{I}_o(1-j) = 2(1-j)$$
$$\mathbf{V}_2 = 2\mathbf{V}_1 = 4(1-j)$$
$$\mathbf{V}_s = -\mathbf{V}_1 - \mathbf{V}_2 = -6(1-j)$$
$$\mathbf{V}_s = \underline{\mathbf{8.485\angle{-135^\circ}\ V}}$$

Chapter 9, Problem 55.

ML

* Find **Z** in the network of Fig. 9.62, given that $\mathbf{V}_o = 4\angle 0^o$ V.

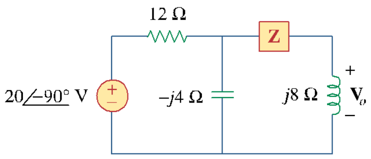
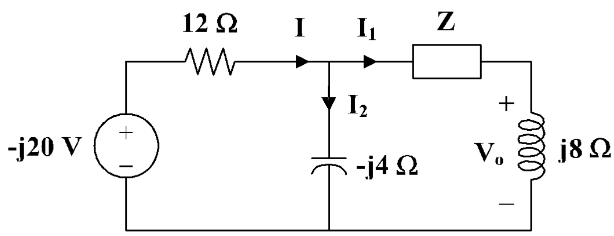


Figure 9.62
For Prob. 9.55.

* An asterisk indicates a challenging problem.

Chapter 9, Solution 55.



$$\mathbf{I}_1 = \frac{\mathbf{V}_o}{j8} = \frac{4}{j8} = \text{-j}0.5$$

$$\mathbf{I}_2 = \frac{\mathbf{I}_1(\mathbf{Z} + j8)}{-j4} = \frac{(\text{-j}0.5)(\mathbf{Z} + j8)}{-j4} = \frac{\mathbf{Z}}{8} + j$$

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = \text{-j}0.5 + \frac{\mathbf{Z}}{8} + j = \frac{\mathbf{Z}}{8} + j0.5$$

$$-j20 = 12\mathbf{I} + \mathbf{I}_1(\mathbf{Z} + j8)$$

$$-j20 = 12\left(\frac{\mathbf{Z}}{8} + \frac{j}{2}\right) + \frac{-j}{2}(\mathbf{Z} + j8)$$

$$-4 - j26 = \mathbf{Z}\left(\frac{3}{2} - j\frac{1}{2}\right)$$

$$\mathbf{Z} = \frac{-4 - j26}{\frac{3}{2} - j\frac{1}{2}} = \frac{26.31\angle 261.25°}{1.5811\angle -18.43°} = 16.64\angle 279.68°$$

Z = **<u>2.798 – j16.403 Ω</u>**

Chapter 9, Problem 56.

At $\omega = 377$ rad/s, find the input impedance of the circuit shown in Fig. 9.63.

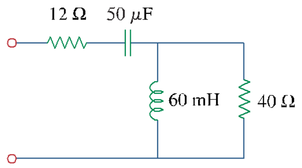


Figure 9.63
For Prob. 9.56.

Chapter 9, Solution 56.

$$50\mu F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j377x50x10^{-6}} = -j53.05$$

$$60mH \longrightarrow j\omega L = j377x60x10^{-3} = j22.62$$

$$Z_{in} = 12 - j53.05 + j22.62 // 40 = \underline{21.692 - j35.91\ \Omega}$$

Chapter 9, Problem 57.

At $\omega = 1$ rad/s, obtain the input admittance in the circuit of Fig. 9.64.

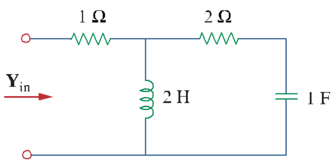


Figure 9.64
For Prob. 9.57.

Chapter 9, Solution 57.

$$2\text{H} \longrightarrow j\omega L = j2$$

$$1\text{F} \longrightarrow \frac{1}{j\omega C} = -j$$

$$Z = 1 + j2 // (2 - j) = 1 + \frac{j2(2 - j)}{j2 + 2 - j} = 2.6 + j1.2$$

$$Y = 1/Z = \underline{0.3171 - j0.1463\ \text{S}}$$

Chapter 9, Problem 58.

Find the equivalent impedance in Fig. 9.65 at $\omega = 10$ krad/s.

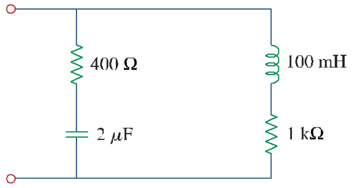


Figure 9.65
For Prob. 9.58.

Chapter 9, Solution 58.

$$2\mu F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j10^4 x2x10^{-6}} = -j50$$

$$100mH \longrightarrow j\omega L = j10^4 x100x10^{-3} = j1000$$

$$Z_{in} = (400 - j50)//(1000 + j1000) = \frac{(400 - j50)(1000 + j1000)}{1400 + j950} = \underline{336.24 + j21.83\ \Omega}$$

Chapter 9, Problem 59.

For the network in Fig. 9.66, find $\mathbf{Z}_{in}$. Let $\omega = 10$ rad/s.

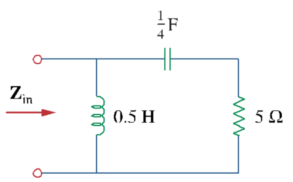


Figure 9.66
For Prob. 9.59.

Chapter 9, Solution 59.

$$0.25F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j10x0.25} = -j0.4$$

$$0.5H \longrightarrow j\omega L = j10x0.5 = j5$$

$$Z_{in} = j5 \| (5 - j0.4) = \frac{(5\angle 90°)(5.016\angle -4.57°)}{6.794\angle 42.61°} = 3.691\angle 42.82°$$

$$= \underline{\mathbf{2.707 + j2.509\ \Omega}}.$$

Chapter 9, Problem 60.

Obtain $\mathbf{Z}_{in}$ for the circuit in Fig. 9.67.

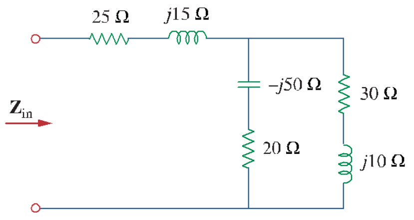


Figure 9.67
For Prob. 9.60.

Chapter 9, Solution 60.

$$Z = (25 + j15) + (20 - j50)//(30 + j10) = 25 + j15 + 26.097 - j5.122 = \underline{51.1 + j9.878\Omega}$$

Chapter 9, Problem 61.

Find $\mathbf{Z}_{eq}$ in the circuit of Fig. 9.68.

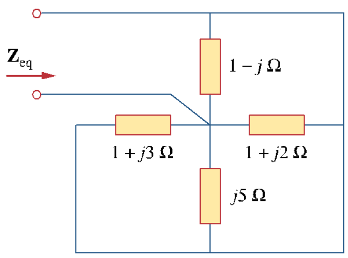


Figure 9.68
For Prob. 9.61.

Chapter 9, Solution 61.

All of the impedances are in parallel.

$$\frac{1}{\mathbf{Z}_{eq}} = \frac{1}{1-j} + \frac{1}{1+j2} + \frac{1}{j5} + \frac{1}{1+j3}$$

$$\frac{1}{\mathbf{Z}_{eq}} = (0.5 + j0.5) + (0.2 - j0.4) + (-j0.2) + (0.1 - j0.3) = 0.8 - j0.4$$

$$\mathbf{Z}_{eq} = \frac{1}{0.8 - j0.4} = \underline{\mathbf{1 + j0.5\ \Omega}}$$

Chapter 9, Problem 62.

For the circuit in Fig. 9.69, find the input impedance $\mathbf{Z}_{in}$ at 10 krad/s.

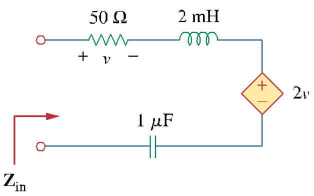
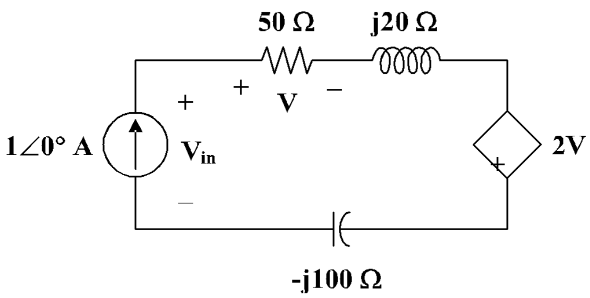


Figure 9.69
For Prob. 9.62.

Chapter 9, Solution 62.

$$2\text{ mH} \longrightarrow j\omega L = j(10\times 10^{3})(2\times 10^{-3}) = j20$$

$$1\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10\times 10^{3})(1\times 10^{-6})} = -j100$$



$$\mathbf{V} = (1\angle 0°)(50) = 50$$

$$\mathbf{V}_{in} = (1\angle 0°)(50 + j20 - j100) + (2)(50)$$
$$\mathbf{V}_{in} = 50 - j80 + 100 = 150 - j80$$

$$\mathbf{Z}_{in} = \frac{\mathbf{V}_{in}}{1\angle 0°} = \underline{\mathbf{150 - j80\ \Omega}}$$

Chapter 9, Problem 63.

ML

For the circuit in Fig. 9.70, find the value of $\mathbf{Z}_T$.

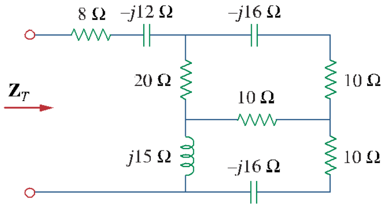
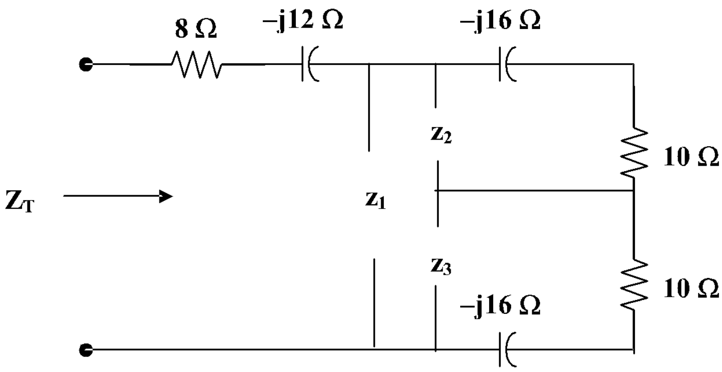


Figure 9.70
For Prob. 9.63.

Chapter 9, Solution 63.
First, replace the wye composed of the 20-ohm, 10-ohm, and j15-ohm impedances with the corresponding delta.

$$z_1 = \frac{200 + j150 + j300}{10} = 20 + j45$$

$$z_2 = \frac{200 + j450}{j15} = 30 - j13.333, \; z_3 = \frac{200 + j450}{20} = 10 + j22.5$$



Now all we need to do is to combine impedances.

$$z_2 \| (10 - j16) = \frac{(30 - j13.333)(10 - j16)}{40 - j29.33} = 8.721 - j8.938$$

$$z_3 \| (10 - j16) = 21.70 - j3.821$$

$$Z_T = 8 - j12 + z_1 \| (8.721 - j8.938 + 21.7 - j3.821) = \underline{34.69 - j6.93\Omega}$$

Chapter 9, Problem 64.

Find $\mathbf{Z}_T$ and $\mathbf{I}$ in the circuit of Fig. 9.71.

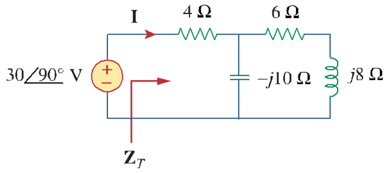


Figure 9.71
For Prob. 9.64.

Chapter 9, Solution 64.

$$Z_T = 4 + \frac{-j10(6+j8)}{6-j2} = \underline{19 - j5\Omega}$$

$$I = \frac{30\angle 90°}{Z_T} = -0.3866 + j1.4767 = \underline{1.527\angle 104.7° \text{ A}}$$

Chapter 9, Problem 65.

Determine $\mathbf{Z}_T$ and $\mathbf{I}$ for the circuit in Fig. 9.72.

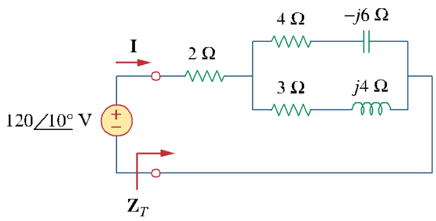


Figure 9.72
For Prob. 9.65.

Chapter 9, Solution 65.

$$\mathbf{Z}_T = 2 + (4 - j6) \,||\, (3 + j4)$$

$$\mathbf{Z}_T = 2 + \frac{(4-j6)(3+j4)}{7-j2}$$

$$\mathbf{Z}_T = \underline{\mathbf{6.83 + j1.094\ \Omega}} = 6.917\angle 9.1°\ \Omega$$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_T} = \frac{120\angle 10°}{6.917\angle 9.1°} = \underline{\mathbf{17.35\angle 0.9°\ A}}$$

Chapter 9, Problem 66.

For the circuit in Fig. 9.73, calculate $\mathbf{Z}_T$ and $\mathbf{V}_{ab}$.

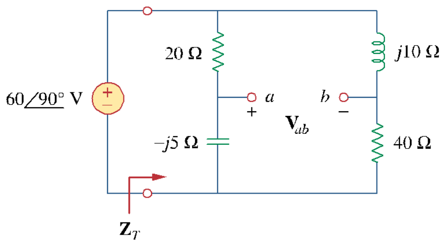


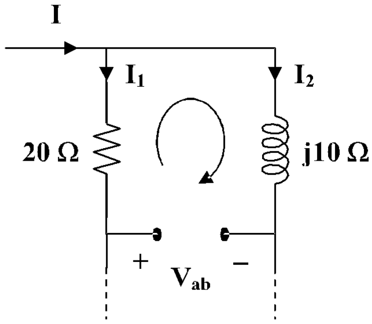
Figure 9.73
For Prob. 9.66.

Chapter 9, Solution 66.

$$\mathbf{Z}_T = (20 - j5) \,||\, (40 + j10) = \frac{(20 - j5)(40 + j10)}{60 + j5} = \frac{170}{145}(12 - j)$$

$$\mathbf{Z}_T = \underline{\mathbf{14.069 - j1.172\ \Omega}} = 14.118\angle\text{-}4.76°$$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_T} = \frac{60\angle 90°}{14.118\angle\text{-}4.76°} = 4.25\angle 94.76°$$



$$\mathbf{I}_1 = \frac{40 + j10}{60 + j5}\mathbf{I} = \frac{8 + j2}{12 + j}\mathbf{I}$$

$$\mathbf{I}_2 = \frac{20 - j5}{60 + j5}\mathbf{I} = \frac{4 - j}{12 + j}\mathbf{I}$$

$$\mathbf{V}_{ab} = \text{-}20\mathbf{I}_1 + j10\mathbf{I}_2$$

$$\mathbf{V}_{ab} = \frac{\text{-}(160 + j40)}{12 + j}\mathbf{I} + \frac{10 + j40}{12 + j}\mathbf{I}$$

$$\mathbf{V}_{ab} = \frac{\text{-}150}{12 + j}\mathbf{I} = \frac{(\text{-}12 + j)(150)}{145}\mathbf{I}$$

$$\mathbf{V}_{ab} = (12.457\angle 175.24°)(4.25\angle 97.76°)$$

$$\mathbf{V}_{ab} = \underline{\mathbf{52.94\angle 273°\ V}}$$

Chapter 9, Problem 67.

At $\omega = 10^3$ rad/s find the input admittance of each of the circuits in Fig. 9.74.

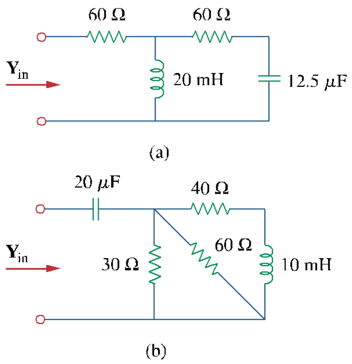


Figure 9.74
For Prob. 9.67.

Chapter 9, Solution 67.

(a) 20 mH $\longrightarrow$ $j\omega L = j(10^3)(20\times 10^{-3}) = j20$

12.5 μF $\longrightarrow$ $\dfrac{1}{j\omega C} = \dfrac{1}{j(10^3)(12.5\times 10^{-6})} = -j80$

$$\mathbf{Z}_{in} = 60 + j20 \,||\, (60 - j80)$$

$$\mathbf{Z}_{in} = 60 + \frac{(j20)(60 - j80)}{60 - j60}$$

$$\mathbf{Z}_{in} = 63.33 + j23.33 = 67.494\angle 20.22°$$

$$\mathbf{Y}_{in} = \frac{1}{\mathbf{Z}_{in}} = \underline{\mathbf{14.8\angle{-20.22°}\ mS}}$$

(b) 10 mH $\longrightarrow$ $j\omega L = j(10^3)(10\times 10^{-3}) = j10$

20 μF $\longrightarrow$ $\dfrac{1}{j\omega C} = \dfrac{1}{j(10^3)(20\times 10^{-6})} = -j50$

$$30 \,||\, 60 = 20$$

$$\mathbf{Z}_{in} = -j50 + 20 \,||\, (40 + j10)$$

$$\mathbf{Z}_{in} = -j50 + \frac{(20)(40 + j10)}{60 + j10}$$

$$\mathbf{Z}_{in} = 13.5 - j48.92 = 50.75\angle -74.56°$$

$$\mathbf{Y}_{in} = \frac{1}{\mathbf{Z}_{in}} = \underline{\mathbf{19.7\angle 74.56°\ mS}} = 5.24 + j18.99\ \text{mS}$$

Chapter 9, Problem 68.

Determine $\mathbf{Y}_{eq}$ for the circuit in Fig. 9.75.

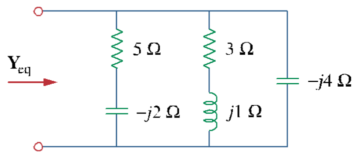


Figure 9.75
For Prob. 9.68.

Chapter 9, Solution 68.

$$\mathbf{Y}_{eq} = \frac{1}{5-j2} + \frac{1}{3+j} + \frac{1}{-j4}$$

$$\mathbf{Y}_{eq} = (0.1724 + j0.069) + (0.3 - j0.1) + (j0.25)$$

$$\mathbf{Y}_{eq} = \underline{\mathbf{0.4724 + j0.219\ S}}$$

Chapter 9, Problem 69.

Find the equivalent admittance $\mathbf{Y}_{eq}$ of the circuit in Fig. 9.76.

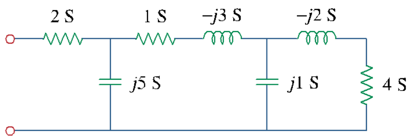


Figure 9.76
For Prob. 9.69.

Chapter 9, Solution 69.

$$\frac{1}{\mathbf{Y}_o} = \frac{1}{4} + \frac{1}{-j2} = \frac{1}{4}(1 + j2)$$

$$\mathbf{Y}_o = \frac{4}{1 + j2} = \frac{(4)(1 - j2)}{5} = 0.8 - j1.6$$

$$\mathbf{Y}_o + j = 0.8 - j0.6$$

$$\frac{1}{\mathbf{Y}_o'} = \frac{1}{1} + \frac{1}{-j3} + \frac{1}{0.8 - j0.6} = (1) + (j0.333) + (0.8 + j0.6)$$

$$\frac{1}{\mathbf{Y}_o'} = 1.8 + j0.933 = 2.028\angle 27.41°$$

$$\mathbf{Y}_o' = 0.4932\angle -27.41° = 0.4378 - j0.2271$$

$$\mathbf{Y}_o' + j5 = 0.4378 + j4.773$$

$$\frac{1}{\mathbf{Y}_{eq}} = \frac{1}{2} + \frac{1}{0.4378 + j4.773} = 0.5 + \frac{0.4378 - j4.773}{22.97}$$

$$\frac{1}{\mathbf{Y}_{eq}} = 0.5191 - j0.2078$$

$$\mathbf{Y}_{eq} = \frac{0.5191 - j0.2078}{0.3126} = \underline{\mathbf{1.661 + j0.6647\ S}}$$

Chapter 9, Problem 70.

ML

Find the equivalent impedance of the circuit in Fig. 9.77.

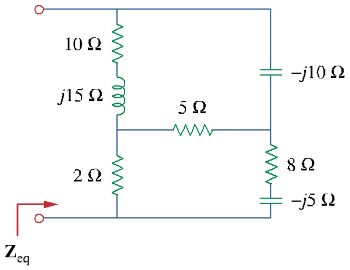
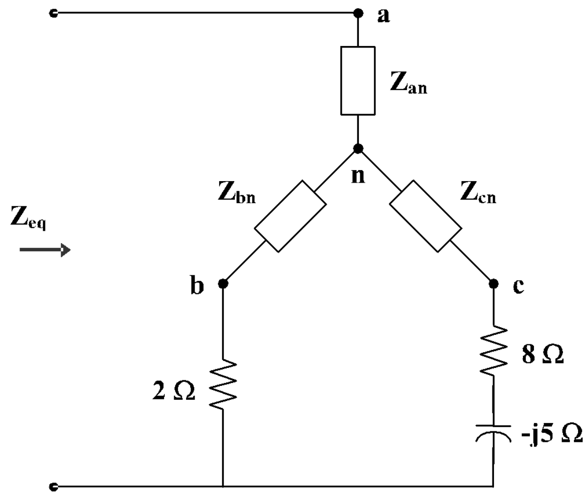


Figure 9.77
For Prob. 9.70.

Chapter 9, Solution 70.

Make a delta-to-wye transformation as shown in the figure below.



$$\mathbf{Z}_{an} = \frac{(-j10)(10+j15)}{5-j10+10+j15} = \frac{(10)(15-j10)}{15+j5} = 7-j9$$

$$\mathbf{Z}_{bn} = \frac{(5)(10+j15)}{15+j5} = 4.5+j3.5$$

$$\mathbf{Z}_{cn} = \frac{(5)(-j10)}{15+j5} = -1-j3$$

$$\mathbf{Z}_{eq} = \mathbf{Z}_{an} + (\mathbf{Z}_{bn} + 2) \,||\, (\mathbf{Z}_{cn} + 8 - j5)$$

$$\mathbf{Z}_{eq} = 7 - j9 + (6.5 + j3.5) \,||\, (7 - j8)$$

$$\mathbf{Z}_{eq} = 7 - j9 + \frac{(6.5+j3.5)(7-j8)}{13.5-j4.5}$$

$$\mathbf{Z}_{eq} = 7 - j9 + 5.511 - j0.2$$

$$\mathbf{Z}_{eq} = 12.51 - j9.2 = \underline{\mathbf{15.53\angle{-36.33}^\circ\ \Omega}}$$

Chapter 9, Problem 71.

Obtain the equivalent impedance of the circuit in Fig. 9.78.

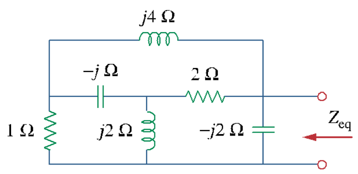
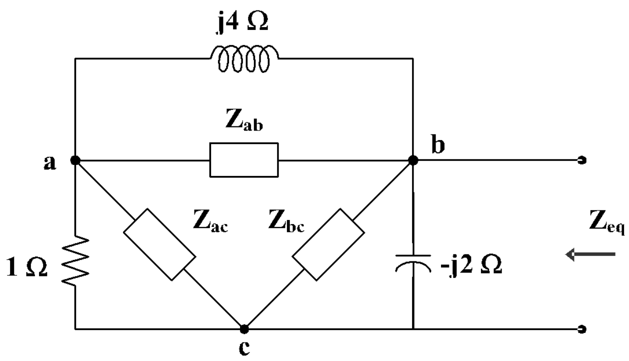


Figure 9.78
For Prob. 9.71.

Chapter 9, Solution 71.

We apply a wye-to-delta transformation.



$$\mathbf{Z}_{ab} = \frac{2 - j2 + j4}{j2} = \frac{2 + j2}{j2} = 1 - j$$

$$\mathbf{Z}_{ac} = \frac{2 + j2}{2} = 1 + j$$

$$\mathbf{Z}_{bc} = \frac{2 + j2}{-j} = -2 + j2$$

$$j4 \,||\, \mathbf{Z}_{ab} = j4 \,||\, (1 - j) = \frac{(j4)(1 - j)}{1 + j3} = 1.6 - j0.8$$

$$1 \,||\, \mathbf{Z}_{ac} = 1 \,||\, (1 + j) = \frac{(1)(1 + j)}{2 + j} = 0.6 + j0.2$$

$$j4 \,||\, \mathbf{Z}_{ab} + 1 \,||\, \mathbf{Z}_{ac} = 2.2 - j0.6$$

$$\frac{1}{\mathbf{Z}_{eq}} = \frac{1}{-j2} + \frac{1}{-2 + j2} + \frac{1}{2.2 - j0.6}$$

$$= j0.5 - 0.25 - j0.25 + 0.4231 + j0.1154$$

$$= 0.173 + j0.3654 = 0.4043\angle 64.66°$$

$\mathbf{Z}_{eq} = 2.473\angle\text{-}64.66°\ \Omega$ = **<u>1.058 – j2.235 Ω</u>**

Chapter 9, Problem 72.

ML

Calculate the value of $\mathbf{Z}_{ab}$ in the network of Fig. 9.79.

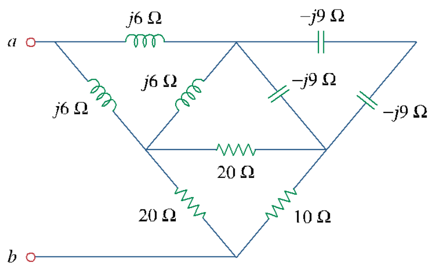
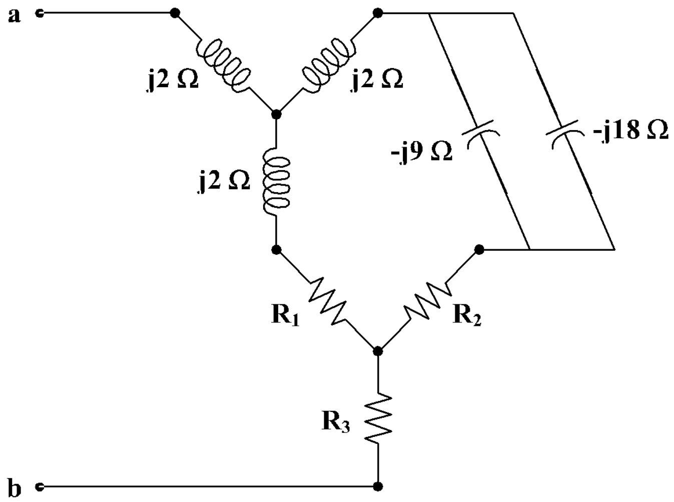


Figure 9.79
For Prob. 9.72.

Chapter 9, Solution 72.

Transform the delta connections to wye connections as shown below.



$$-j9 \,||\, -j18 = -j6,$$

$$R_1 = \frac{(20)(20)}{20+20+10} = 8\,\Omega, \qquad R_2 = \frac{(20)(10)}{50} = 4\,\Omega,$$

$$R_3 = \frac{(20)(10)}{50} = 4\,\Omega$$

$$\mathbf{Z}_{ab} = j2 + (j2+8) \,||\, (j2 - j6 + 4) + 4$$

$$\mathbf{Z}_{ab} = 4 + j2 + (8 + j2) \,||\, (4 - j4)$$

$$\mathbf{Z}_{ab} = 4 + j2 + \frac{(8+j2)(4-j4)}{12 - j2}$$

$$\mathbf{Z}_{ab} = 4 + j2 + 3.567 - j1.4054$$

$$\mathbf{Z}_{ab} = \underline{\mathbf{7.567 + j0.5946\ \Omega}}$$

Chapter 9, Problem 73.

Determine the equivalent impedance of the circuit in Fig. 9.80.

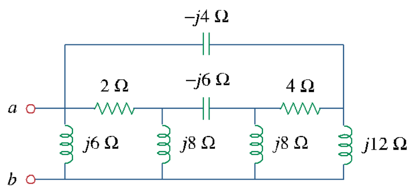
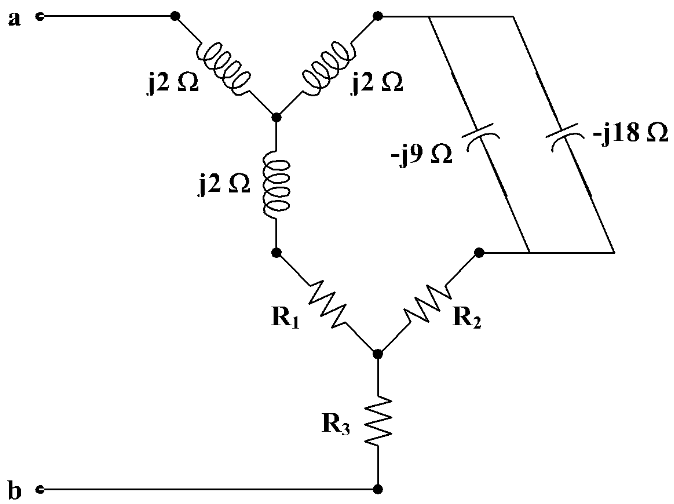


Figure 9.80
For Prob. 9.73.

Chapter 9, Solution 73.

Transform the delta connection to a wye connection as in Fig. (a) and then transform the wye connection to a delta connection as in Fig. (b).



$$\mathbf{Z}_1 = \frac{(j8)(-j6)}{j8 + j8 - j6} = \frac{48}{j10} = -j4.8$$
$$\mathbf{Z}_2 = \mathbf{Z}_1 = -j4.8$$
$$\mathbf{Z}_3 = \frac{(j8)(j8)}{j10} = \frac{-64}{j10} = j6.4$$

$$(2 + \mathbf{Z}_1)(4 + \mathbf{Z}_2) + (4 + \mathbf{Z}_2)(\mathbf{Z}_3) + (2 + \mathbf{Z}_1)(\mathbf{Z}_3) =$$
$$(2 - j4.8)(4 - j4.8) + (4 - j4.8)(j6.4) + (2 - j4.8)(j6.4) = 46.4 + j9.6$$

$$\mathbf{Z}_a = \frac{46.4 + j9.6}{j6.4} = 1.5 - j7.25$$
$$\mathbf{Z}_b = \frac{46.4 + j9.6}{4 - j4.8} = 3.574 + j6.688$$
$$\mathbf{Z}_c = \frac{46.4 + j9.6}{2 - j4.8} = 1.727 + j8.945$$

$$j6 \,||\, \mathbf{Z}_b = \frac{(6\angle 90°)(7.583\angle 61.88°)}{3.574 + j12.688} = 07407 + j3.3716$$
$$-j4 \,||\, \mathbf{Z}_a = \frac{(-j4)(1.5 - j7.25)}{1.5 - j11.25} = 0.186 - j2.602$$
$$j12 \,||\, \mathbf{Z}_c = \frac{(12\angle 90°)(9.11\angle 79.07°)}{1.727 + j20.945} = 0.5634 + j5.1693$$

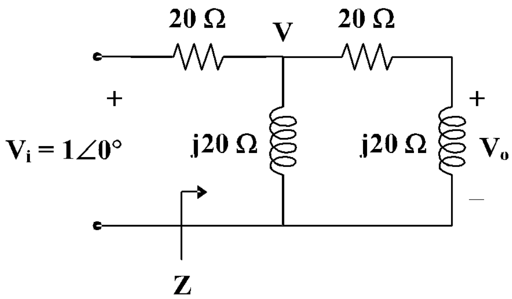
$$\mathbf{Z}_{eq} = (j6 \,||\, \mathbf{Z}_b) \,||\, (-j4 \,||\, \mathbf{Z}_a + j12 \,||\, \mathbf{Z}_c)$$
$$\mathbf{Z}_{eq} = (0.7407 + j3.3716) \,||\, (0.7494 + j2.5673)$$
$$\mathbf{Z}_{eq} = 1.508\angle 75.42°\ \Omega = \mathbf{0.3796 + j1.46\ \Omega}$$

Chapter 9, Problem 74.

Design an *RL* circuit to provide a 90° leading phase shift.

Chapter 9, Solution 74.

One such RL circuit is shown below.



We now want to show that this circuit will produce a 90° phase shift.

$$\mathbf{Z} = j20 \,||\, (20 + j20) = \frac{(j20)(20 + j20)}{20 + j40} = \frac{-20 + j20}{1 + j2} = 4(1 + j3)$$

$$\mathbf{V} = \frac{\mathbf{Z}}{\mathbf{Z} + 20}\mathbf{V}_i = \frac{4 + j12}{24 + j12}(1\angle 0°) = \frac{1 + j3}{6 + j3} = \frac{1}{3}(1 + j)$$

$$\mathbf{V}_o = \frac{j20}{20 + j20}\mathbf{V} = \left(\frac{j}{1 + j}\right)\left(\frac{1}{3}(1 + j)\right) = \frac{j}{3} = 0.3333\angle 90°$$

This shows that the output leads the input by 90°.

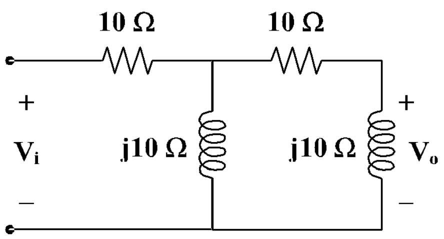
Chapter 9, Problem 75.

e2d

Design a circuit that will transform a sinusoidal voltage input to a cosinusoidal voltage output.

Chapter 9, Solution 75.

Since $\cos(\omega t) = \sin(\omega t + 90°)$, we need a phase shift circuit that will cause the output to lead the input by 90°. **<u>This is achieved by the RL circuit shown below, as explained in the previous problem</u>**.



This can also be obtained by an RC circuit.

Chapter 9, Problem 76.

e2d

For the following pairs of signals, determine if v_1 leads or lags v_2 and by how much.

(a) $v_1 = 10\cos(5t - 20^o)$, $v_2 = 8\sin 5t$

(b) $v_1 = 19\cos(2t - 90^o)$, $v_2 = 6\sin 2t$

(c) $v_1 = -4\cos 10t$, $v_2 = 15\sin 10t$

Chapter 9, Solution 76.

(a) $v_2 = 8\sin 5t = 8\cos(5t - 90^o)$

v_1 leads v_2 by 70°.

(b) $v_2 = 6\sin 2t = 6\cos(2t - 90^o)$

v_1 leads v_2 by 180°.

(c) $v_1 = -4\cos 10t = 4\cos(10t + 180^o)$

$v_2 = 15\sin 10t = 15\cos(10t - 90^o)$

v_1 leads v_2 by 270°.

Chapter 9, Problem 77.

Refer to the *RC* circuit in Fig. 9.81.

(a) Calculate the phase shift at 2 MHz.

(b) Find the frequency where the phase shift is 45^o.

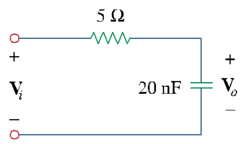


Figure 9.81
For Prob. 9.77.

Chapter 9, Solution 77.

(a) $$\mathbf{V}_o = \frac{-jX_c}{R - jX_c}\mathbf{V}_i$$

where $$X_c = \frac{1}{\omega C} = \frac{1}{(2\pi)(2\times 10^6)(20\times 10^{-9})} = 3.979$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{-j3.979}{5 - j3.979} = \frac{3.979}{\sqrt{5^2 + 3.979^2}}\angle(-90° + \tan^{-1}(3.979/5))$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{3.979}{\sqrt{25 + 15.83}}\angle(-90° - 38.51°)$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_i} = 0.6227\angle -51.49°$$

Therefore, the phase shift is <u>**51.49° lagging**</u>

(b) $$\theta = -45° = -90° + \tan^{-1}(X_c/R)$$

$$45° = \tan^{-1}(X_c/R) \longrightarrow R = X_c = \frac{1}{\omega C}$$

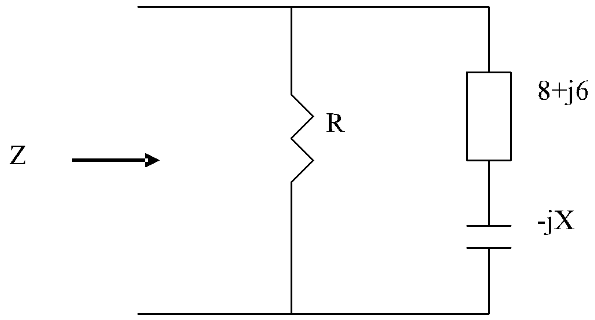
$$\omega = 2\pi f = \frac{1}{RC}$$

$$f = \frac{1}{2\pi RC} = \frac{1}{(2\pi)(5)(20\times 10^{-9})} = \underline{\mathbf{1.5915\ MHz}}$$

Chapter 9, Problem 78.

A coil with impedance $8 + j6\ \Omega$ is connected in series with a capacitive reactance *X*. The series combination is connected in parallel with a resistor *R*. Given that the equivalent impedance of the resulting circuit is $5\angle 0^o\ \Omega$ find the value of *R* and *X*.

Chapter 9, Solution 78.



$$Z = R//[8 + j(6 - X)] = \frac{R[8 + j(6 - X)]}{R + 8 + j(6 - X)} = 5$$

i.e 8R + j6R – jXR = 5R + 40 + j30 –j5X

Equating real and imaginary parts:

8R = 5R + 40 which leads to **R=<u>13.333Ω</u>**
6R-XR =30-5X which leads to **X= <u>6 Ω.</u>**

Chapter 9, Problem 79.

(a) Calculate the phase shift of the circuit in Fig. 9.82.
(b) State whether the phase shift is leading or lagging (output with respect to input).
(c) Determine the magnitude of the output when the input is 120 V.

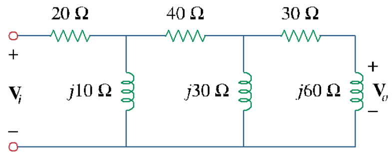
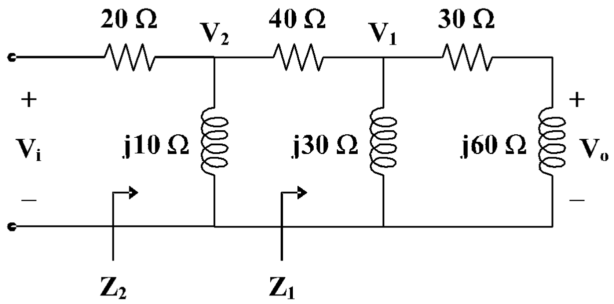


Figure 9.82
For Prob. 9.79.

Chapter 9, Solution 79.

(a) Consider the circuit as shown.



$$\mathbf{Z}_1 = j30 \,||\, (30 + j60) = \frac{(j30)(30 + j60)}{30 + j90} = 3 + j21$$

$$\mathbf{Z}_2 = j10 \,||\, (40 + \mathbf{Z}_1) = \frac{(j10)(43 + j21)}{43 + j31} = 1.535 + j8.896 = 9.028\angle 80.21°$$

Let $\mathbf{V}_i = 1\angle 0°$.

$$\mathbf{V}_2 = \frac{\mathbf{Z}_2}{\mathbf{Z}_2 + 20}\mathbf{V}_i = \frac{(9.028\angle 80.21°)(1\angle 0°)}{21.535 + j8.896}$$

$$\mathbf{V}_2 = 0.3875\angle 57.77°$$

$$\mathbf{V}_1 = \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + 40}\mathbf{V}_2 = \frac{3 + j21}{43 + j21}\mathbf{V}_2 = \frac{(21.213\angle 81.87°)(0.3875\angle 57.77°)}{47.85\angle 26.03°}$$

$$\mathbf{V}_1 = 0.1718\angle 113.61°$$

$$\mathbf{V}_o = \frac{j60}{30 + j60}\mathbf{V}_1 = \frac{j2}{1 + j2}\mathbf{V}_1 = \frac{2}{5}(2 + j)\mathbf{V}_1$$

$$\mathbf{V}_o = (0.8944\angle 26.56°)(0.1718\angle 113.6°)$$

$$\mathbf{V}_o = 0.1536\angle 140.2°$$

Therefore, the phase shift is **140.2°**

(b) The phase shift is **leading**.

(c) If $\mathbf{V}_i = 120$ V, then

$$\mathbf{V}_o = (120)(0.1536\angle 140.2°) = 18.43\angle 140.2° \text{ V}$$

and the magnitude is **18.43 V**.

Chapter 9, Problem 80.

Consider the phase-shifting circuit in Fig. 9.83. Let $\mathbf{V}_i = 120$ V operating at 60 Hz. Find:

(a) $\mathbf{V}_o$ when R is maximum

(b) $\mathbf{V}_o$ when R is minimum

(c) the value of R that will produce a phase shift of 45^o

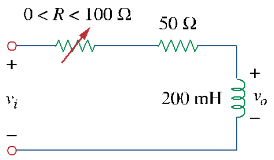


Figure 9.83
For Prob. 9.80.

Chapter 9, Solution 80.

$$200 \text{ mH} \longrightarrow \quad j\omega L = j(2\pi)(60)(200\times10^{-3}) = j75.4\,\Omega$$

$$\mathbf{V}_o = \frac{j75.4}{R + 50 + j75.4}\mathbf{V}_i = \frac{j75.4}{R + 50 + j75.4}(120\angle 0°)$$

(a) When $R = 100\,\Omega$,

$$\mathbf{V}_o = \frac{j75.4}{150 + j75.4}(120\angle 0°) = \frac{(75.4\angle 90°)(120\angle 0°)}{167.88\angle 26.69°}$$

$$\mathbf{V}_o = \underline{\mathbf{53.89\angle 63.31°\ V}}$$

(b) When $R = 0\,\Omega$,

$$\mathbf{V}_o = \frac{j75.4}{50 + j75.4}(120\angle 0°) = \frac{(75.4\angle 90°)(120\angle 0°)}{90.47\angle 56.45°}$$

$$\mathbf{V}_o = \underline{\mathbf{100\angle 33.55°\ V}}$$

(c) To produce a phase shift of 45°, the phase of $\mathbf{V}_o = 90° + 0° - \alpha = 45°$.
Hence, α = phase of $(R + 50 + j75.4) = 45°$.
For α to be 45°, $R + 50 = 75.4$
Therefore, $R = \underline{\mathbf{25.4\ \Omega}}$

Chapter 9, Problem 81.

The ac bridge in Fig. 9.37 is balanced when $R_1 = 400\ \Omega$, $R_2 = 600\ \Omega$, $R_3 = 1.2\text{k}\Omega$, and $C_2 = 0.3\ \mu\text{F}$. Find R_x and C_x. Assume R_2 and C_2 are in series.

Chapter 9, Solution 81.

Let $\mathbf{Z}_1 = \text{R}_1$, $\mathbf{Z}_2 = \text{R}_2 + \dfrac{1}{\text{j}\omega\text{C}_2}$, $\mathbf{Z}_3 = \text{R}_3$, and $\mathbf{Z}_x = \text{R}_x + \dfrac{1}{\text{j}\omega\text{C}_x}$.

$$\mathbf{Z}_x = \frac{\mathbf{Z}_3}{\mathbf{Z}_1}\mathbf{Z}_2$$

$$\text{R}_x + \frac{1}{\text{j}\omega\text{C}_x} = \frac{\text{R}_3}{\text{R}_1}\left(\text{R}_2 + \frac{1}{\text{j}\omega\text{C}_2}\right)$$

$$\text{R}_x = \frac{\text{R}_3}{\text{R}_1}\text{R}_2 = \frac{1200}{400}(600) = \underline{\mathbf{1.8\ k\Omega}}$$

$$\frac{1}{\text{C}_x} = \left(\frac{\text{R}_3}{\text{R}_1}\right)\left(\frac{1}{\text{C}_2}\right) \longrightarrow \text{C}_x = \frac{\text{R}_1}{\text{R}_3}\text{C}_2 = \left(\frac{400}{1200}\right)(0.3\times10^{-6}) = \underline{\mathbf{0.1\ \mu F}}$$

Chapter 9, Problem 82.

A capacitance bridge balances when $R_1 = 100\,\Omega$, and $R_2 = 2\text{k}\Omega$ and $C_s = 40\ \mu\text{F}$. What is C_x the capacitance of the capacitor under test?

Chapter 9, Solution 82.

$$\text{C}_x = \frac{\text{R}_1}{\text{R}_2}\text{C}_s = \left(\frac{100}{2000}\right)(40\times10^{-6}) = \underline{\mathbf{2\ \mu F}}$$

Chapter 9, Problem 83.

An inductive bridge balances when $R_1 = 1.2\text{k}\Omega$, $R_2 = 500\,\Omega$, and $L_s = 250$ mH. What is the value of L_x, the inductance of the inductor under test?

Chapter 9, Solution 83.

$$\text{L}_x = \frac{\text{R}_2}{\text{R}_1}\text{L}_s = \left(\frac{500}{1200}\right)(250\times10^{-3}) = \underline{\mathbf{104.17\ mH}}$$

Chapter 9, Problem 84.

The ac bridge shown in Fig. 9.84 is known as a *Maxwell bridge* and is used for accurate measurement of inductance and resistance of a coil in terms of a standard capacitance C_s. Show that when the bridge is balanced,

$$L_x = R_2 R_3 C_s \qquad \text{and} \qquad R_x = \frac{R_2}{R_1} R_3$$

Find L_x and R_x for $R_1 = 40\text{k}\Omega$, $R_2 = 1.6\text{k}\Omega$, $R_3 = 4\text{k}\Omega$, and $C_s = 0.45\ \mu\text{F}$.

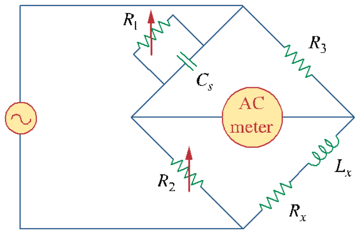


Figure 9.84
Maxwell bridge; For Prob. 9.84.

Chapter 9, Solution 84.

Let $\mathbf{Z}_1 = R_1 \parallel \dfrac{1}{j\omega C_s}$, $\mathbf{Z}_2 = R_2$, $\mathbf{Z}_3 = R_3$, and $\mathbf{Z}_x = R_x + j\omega L_x$.

$$\mathbf{Z}_1 = \frac{\dfrac{R_1}{j\omega C_s}}{R_1 + \dfrac{1}{j\omega C_s}} = \frac{R_1}{j\omega R_1 C_s + 1}$$

Since $\mathbf{Z}_x = \dfrac{\mathbf{Z}_3}{\mathbf{Z}_1}\mathbf{Z}_2$,

$$R_x + j\omega L_x = R_2 R_3 \frac{j\omega R_1 C_s + 1}{R_1} = \frac{R_2 R_3}{R_1}(1 + j\omega R_1 C_s)$$

Equating the real and imaginary components,

$$\underline{\mathbf{R_x = \frac{R_2 R_3}{R_1}}}$$

$\omega L_x = \dfrac{R_2 R_3}{R_1}(\omega R_1 C_s)$ implies that

$$\underline{\mathbf{L_x = R_2 R_3 C_s}}$$

Given that $R_1 = 40\ \text{k}\Omega$, $R_2 = 1.6\ \text{k}\Omega$, $R_3 = 4\ \text{k}\Omega$, and $C_s = 0.45\ \mu\text{F}$

$$R_x = \frac{R_2 R_3}{R_1} = \frac{(1.6)(4)}{40}\ \text{k}\Omega = 0.16\ \text{k}\Omega = \underline{\mathbf{160\ \Omega}}$$

$$L_x = R_2 R_3 C_s = (1.6)(4)(0.45) = \underline{\mathbf{2.88\ H}}$$

Chapter 9, Problem 85.

The ac bridge circuit of Fig. 9.85 is called a *Wien bridge*. It is used for measuring the frequency of a source. Show that when the bridge is balanced,

$$f = \frac{1}{2\pi\sqrt{R_2 R_4 C_2 C_4}}$$

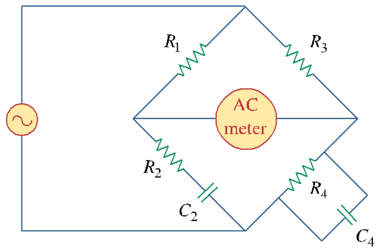


Figure 9.85
Wein bridge; For Prob. 9.85.

Chapter 9, Solution 85.

Let $\mathbf{Z}_1 = R_1$, $\mathbf{Z}_2 = R_2 + \dfrac{1}{j\omega C_2}$, $\mathbf{Z}_3 = R_3$, and $\mathbf{Z}_4 = R_4 \,||\, \dfrac{1}{j\omega C_4}$.

$$\mathbf{Z}_4 = \frac{R_4}{j\omega R_4 C_4 + 1} = \frac{-jR_4}{\omega R_4 C_4 - j}$$

Since $\mathbf{Z}_4 = \dfrac{\mathbf{Z}_3}{\mathbf{Z}_1}\mathbf{Z}_2 \longrightarrow \mathbf{Z}_1\mathbf{Z}_4 = \mathbf{Z}_2\mathbf{Z}_3$,

$$\frac{-jR_4R_1}{\omega R_4 C_4 - j} = R_3\left(R_2 - \frac{j}{\omega C_2}\right)$$

$$\frac{-jR_4R_1(\omega R_4 C_4 + j)}{\omega^2 R_4^2 C_4^2 + 1} = R_3R_2 - \frac{jR_3}{\omega C_2}$$

Equating the real and imaginary components,

$$\frac{R_1R_4}{\omega^2 R_4^2 C_4^2 + 1} = R_2R_3 \qquad (1)$$

$$\frac{\omega R_1 R_4^2 C_4}{\omega^2 R_4^2 C_4^2 + 1} = \frac{R_3}{\omega C_2} \qquad (2)$$

Dividing (1) by (2),

$$\frac{1}{\omega R_4 C_4} = \omega R_2 C_2$$

$$\omega^2 = \frac{1}{R_2 C_2 R_4 C_4}$$

$$\omega = 2\pi f = \frac{1}{\sqrt{R_2 C_2 R_4 C_4}}$$

$$\mathbf{f = \frac{1}{2\pi\sqrt{R_2 R_4 C_2 C_4}}}$$

Chapter 9, Problem 86.

The circuit shown in Fig. 9.86 is used in a television receiver. What is the total impedance of this circuit?

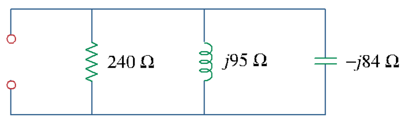


Figure 9.86
For Prob. 9.86.

Chapter 9, Solution 86.

$$\mathbf{Y} = \frac{1}{240} + \frac{1}{j95} + \frac{1}{-j84}$$

$$\mathbf{Y} = 4.1667 \times 10^{-3} - j0.01053 + j0.0119$$

$$\mathbf{Z} = \frac{1}{\mathbf{Y}} = \frac{1000}{4.1667 + j1.37} = \frac{1000}{4.3861\angle 18.2^\circ}$$

$$\mathbf{Z} = \underline{\mathbf{228\angle\text{-}18.2^\circ\ \Omega}}$$

Chapter 9, Problem 87.

The network in Fig. 9.87 is part of the schematic describing an industrial electronic sensing device. What is the total impedance of the circuit at 2 kHz?

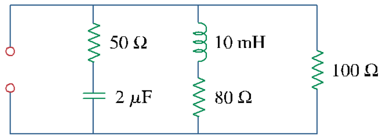


Figure 9.87
For Prob. 9.87.

Chapter 9, Solution 87.

$$\mathbf{Z}_1 = 50 + \frac{1}{j\omega C} = 50 + \frac{-j}{(2\pi)(2\times10^3)(2\times10^{-6})}$$

$$\mathbf{Z}_1 = 50 - j39.79$$

$$\mathbf{Z}_2 = 80 + j\omega L = 80 + j(2\pi)(2\times10^3)(10\times10^{-3})$$

$$\mathbf{Z}_2 = 80 + j125.66$$

$$\mathbf{Z}_3 = 100$$

$$\frac{1}{\mathbf{Z}} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3}$$

$$\frac{1}{\mathbf{Z}} = \frac{1}{100} + \frac{1}{50 - j39.79} + \frac{1}{80 + j125.66}$$

$$\frac{1}{\mathbf{Z}} = 10^{-3}\,(10 + 12.24 + j9.745 + 3.605 - j5.663)$$

$$= (25.85 + j4.082)\times10^{-3}$$

$$= 26.17\times10^{-3}\,\angle 8.97°$$

$\mathbf{Z}$ = **$\underline{38.21\angle\text{-}8.97°\ \Omega}$**

Chapter 9, Problem 88.

A series audio circuit is shown in Fig. 9.88.

(a) What is the impedance of the circuit?

(b) If the frequency were halved, what would be the impedance of the circuit?

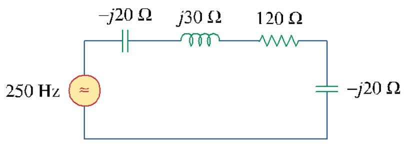


Figure 9.88
For Prob. 9.88.

Chapter 9, Solution 88.

(a) $\mathbf{Z} = -j20 + j30 + 120 - j20$

$\mathbf{Z} = \underline{\mathbf{120 - j10\ \Omega}}$

(b) If the frequency were halved, $\frac{1}{\omega C} = \frac{1}{2\pi f C}$ would cause the capacitive impedance to double, while $\omega L = 2\pi f L$ would cause the inductive impedance to halve. Thus,

$\mathbf{Z} = -j40 + j15 + 120 - j40$

$\mathbf{Z} = \underline{\mathbf{120 - j65\ \Omega}}$

Chapter 9, Problem 89.

An industrial load is modeled as a series combination of a capacitance and a resistance as shown in Fig. 9.89. Calculate the value of an inductance *L* across the series combination so that the net impedance is resistive at a frequency of 50 kHz.

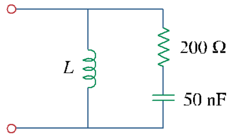


Figure 9.89
For Prob. 9.89.

Chapter 9, Solution 89.

$$\mathbf{Z}_{in} = j\omega L \,||\, \left(R + \frac{1}{j\omega C}\right)$$

$$\mathbf{Z}_{in} = \frac{j\omega L\left(R + \frac{1}{j\omega C}\right)}{R + j\omega L + \frac{1}{j\omega C}} = \frac{\frac{L}{C} + j\omega L R}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$\mathbf{Z}_{in} = \frac{\left(\frac{L}{C} + j\omega L R\right)\left(R - j\left(\omega L - \frac{1}{\omega C}\right)\right)}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

To have a resistive impedance, $\text{Im}(\mathbf{Z}_{in}) = 0$. Hence,

$$\omega L R^2 - \left(\frac{L}{C}\right)\left(\omega L - \frac{1}{\omega C}\right) = 0$$

$$\omega R^2 C = \omega L - \frac{1}{\omega C}$$

$$\omega^2 R^2 C^2 = \omega^2 LC - 1$$

$$L = \frac{\omega^2 R^2 C^2 + 1}{\omega^2 C}$$

Now we can solve for L.

$$L = R^2 C + 1/(\omega^2 C)$$

$= (200^2)(50\text{x}10^{-9}) + 1/((2\pi\text{x}50{,}000)^2(50\text{x}10^{-9})$
$= 2\text{x}10^{-3} + 0.2026\text{x}10^{-3} =$ **2.203 mH**.

Checking, converting the series resistor and capacitor into a parallel combination, gives 220.3Ω in parallel with -j691.9Ω. The value of the parallel inductance is $\omega L = 2\pi\text{x}50{,}000\text{x}2.203\text{x}10^{-3} = 692.1\Omega$ which we need to have if we are to cancel the effect of the capacitance. The answer checks.

Chapter 9, Problem 90.

An industrial coil is modeled as a series combination of an inductance *L* and resistance *R*, as shown in Fig. 9.90. Since an ac voltmeter measures only the magnitude of a sinusoid, the following measurements are taken at 60 Hz when the circuit operates in the steady state:

$|\mathbf{V}_s| = 145$ V, $|\mathbf{V}_1| = 50$ V, $|\mathbf{V}_o| = 110$ V

Use these measurements to determine the values of *L* and *R*.

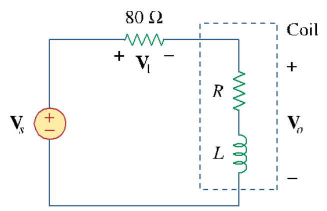


Figure 9.90
For Prob. 9.90.

Chapter 9, Solution 90.

Let $\mathbf{V}_s = 145\angle 0°$, $X = \omega L = (2\pi)(60)\,L = 377L$

$$\mathbf{I} = \frac{\mathbf{V}_s}{80 + R + jX} = \frac{145\angle 0°}{80 + R + jX}$$

$$\mathbf{V}_1 = 80\,\mathbf{I} = \frac{(80)(145)}{80 + R + jX}$$

$$50 = \left| \frac{(80)(145)}{80 + R + jX} \right| \qquad (1)$$

$$\mathbf{V}_o = (R + jX)\,\mathbf{I} = \frac{(R + jX)(145\angle 0°)}{80 + R + jX}$$

$$110 = \left| \frac{(R + jX)(145)}{80 + R + jX} \right| \qquad (2)$$

From (1) and (2),

$$\frac{50}{110} = \frac{80}{\left| R + jX \right|}$$

$$\left| R + jX \right| = (80)\left(\frac{11}{5}\right)$$

$$R^2 + X^2 = 30976 \qquad (3)$$

From (1),

$$\left| 80 + R + jX \right| = \frac{(80)(145)}{50} = 232$$

$$6400 + 160R + R^2 + X^2 = 53824$$

$$160R + R^2 + X^2 = 47424 \qquad (4)$$

Subtracting (3) from (4),

$160R = 16448 \longrightarrow$ R = **<u>102.8 Ω</u>**

From (3),

$$X^2 = 30976 - 10568 = 20408$$

$X = 142.86 = 377L \longrightarrow$ L = **<u>0.3789 H</u>**

Chapter 9, Problem 91.

Figure 9.91 shows a parallel combination of an inductance and a resistance. If it is desired to connect a capacitor in series with the parallel combination such that the net impedance is resistive at 10 MHz, what is the required value of *C*?

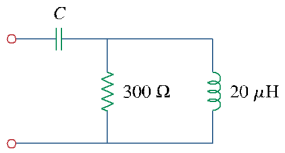


Figure 9.91
For Prob. 9.91.

Chapter 9, Solution 91.

$$\mathbf{Z}_{in} = \frac{1}{j\omega C} + R \parallel j\omega L$$

$$\mathbf{Z}_{in} = \frac{-j}{\omega C} + \frac{j\omega LR}{R + j\omega L}$$

$$= \frac{-j}{\omega C} + \frac{\omega^2 L^2 R + j\omega LR^2}{R^2 + \omega^2 L^2}$$

To have a resistive impedance, $\text{Im}(\mathbf{Z}_{in}) = 0$.
Hence,

$$\frac{-1}{\omega C} + \frac{\omega LR^2}{R^2 + \omega^2 L^2} = 0$$

$$\frac{1}{\omega C} = \frac{\omega LR^2}{R^2 + \omega^2 L^2}$$

$$C = \frac{R^2 + \omega^2 L^2}{\omega^2 LR^2}$$

where $\omega = 2\pi f = 2\pi \times 10^7$

$$C = \frac{9 \times 10^4 + (4\pi^2 \times 10^{14})(400 \times 10^{-12})}{(4\pi^2 \times 10^{14})(20 \times 10^{-6})(9 \times 10^4)}$$

$$C = \frac{9 + 16\pi^2}{72\pi^2} \text{ nF}$$

C = <u>**235 pF**</u>

Chapter 9, Problem 92.

A transmission line has a series impedance of $\mathbf{Z} = 100\angle 75^o\ \Omega$ and a shunt admittance of $\mathbf{Y} = 450\angle 48^o\ \mu\text{S}$. Find: (a) the characteristic impedance $\mathbf{Z}_o = \sqrt{\mathbf{Z}/\mathbf{Y}}$
(b) the propagation constant $\gamma = \sqrt{\mathbf{ZY}}$.

Chapter 9, Solution 92.

(a) $Z_o = \sqrt{\dfrac{Z}{Y}} = \sqrt{\dfrac{100\angle 75^o}{450\angle 48^o x10^{-6}}} = \underline{471.4\angle 13.5^o\,\Omega}$

(b) $\gamma = \sqrt{ZY} = \sqrt{100\angle 75^o x 450\angle 48^o x10^{-6}} = \underline{0.2121\angle 61.5^o}$

Chapter 9, Problem 93.

A power transmission system is modeled as shown in Fig. 9.92. Given the following;

Source voltage	$\mathbf{V}_s = 115\ \angle 0^o$ V,
Source impedance	$\mathbf{Z}_s = 1 + j0.5\,\Omega$,
Line impedance	$\mathbf{Z}_\ell = 0.4 + j0.3\,\Omega$,
Load impedance	$\mathbf{Z}_L = 23.2 + j18.9\,\Omega$,
find the load current	$\mathbf{I}_L$.

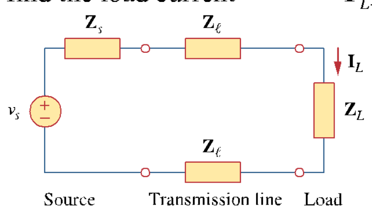


Figure 9.92
For Prob. 9.93.

Chapter 9, Solution 93.

$$\mathbf{Z} = \mathbf{Z}_s + 2\mathbf{Z}_\ell + \mathbf{Z}_L$$
$$\mathbf{Z} = (1 + 0.8 + 23.2) + j(0.5 + 0.6 + 18.9)$$
$$\mathbf{Z} = 25 + j20$$

$$\mathbf{I}_L = \frac{\mathbf{V}_S}{\mathbf{Z}} = \frac{115\angle 0°}{32.02\angle 38.66°}$$
$$\mathbf{I}_L = \underline{\mathbf{3.592\angle\text{-}38.66°\ A}}$$

Chapter 10, Problem 1.

Determine *i* in the circuit of Fig. 10.50.

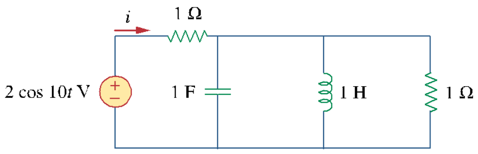


Figure 10.50
For Prob. 10.1.

Chapter 10, Solution 1.

We first determine the input impedance.

$$1H \longrightarrow j\omega L = j1x10 = j10$$

$$1F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j10x1} = -j0.1$$

$$Z_{in} = 1 + \left(\frac{1}{j10} + \frac{1}{-j0.1} + \frac{1}{1}\right)^{-1} = 1.0101 - j0.1 = 1.015 < -5.653^o$$

$$I = \frac{2 < 0^o}{1.015 < -5.653^o} = 1.9704 < 5.653^o$$

$$i(t) = \underline{1.9704\cos(10t + 5.653^o)\ A} = \underline{\mathbf{1.9704cos(10t+5.65^\circ)\ A}}$$

Chapter 10, Problem 2.

Solve for $\mathbf{V}_o$ in Fig. 10.51, using nodal analysis.

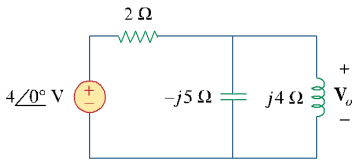
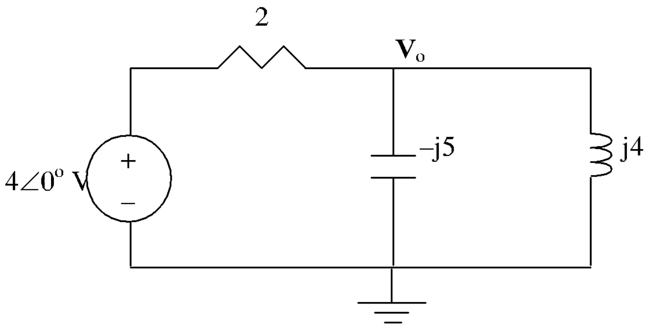


Figure 10.51
For Prob. 10.2.

Chapter 10, Solution 2.

Consider the circuit shown below.



At the main node,

$$\frac{4-V_o}{2}=\frac{V_o}{-j5}+\frac{V_o}{j4} \quad \longrightarrow \quad 40=V_o(10+j)$$

$$V_o=\frac{40}{10-j}=\underline{3.98<5.71^o \ \ \text{A}}$$

Chapter 10, Problem 3.

Determine v_o in the circuit of Fig. 10.52.

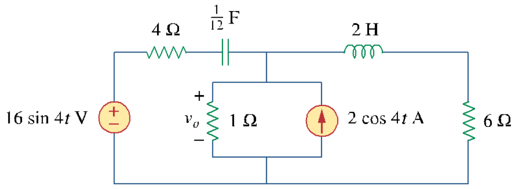


Figure 10.52
For Prob. 10.3.

Chapter 10, Solution 3.

$$\omega = 4$$

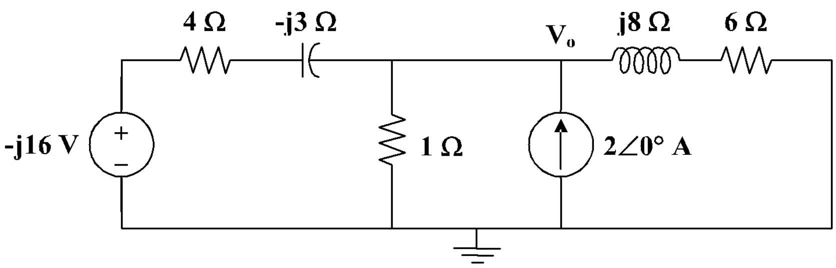
$$2\cos(4t) \longrightarrow 2\angle 0°$$

$$16\sin(4t) \longrightarrow 16\angle -90° = -j16$$

$$2\text{ H} \longrightarrow j\omega L = j8$$

$$1/12\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(4)(1/12)} = -j3$$

The circuit is shown below.



Applying nodal analysis,

$$\frac{-j16 - V_o}{4 - j3} + 2 = \frac{V_o}{1} + \frac{V_o}{6 + j8}$$

$$\frac{-j16}{4 - j3} + 2 = \left(1 + \frac{1}{4 - j3} + \frac{1}{6 + j8}\right) V_o$$

$$V_o = \frac{3.92 - j2.56}{1.22 + j0.04} = \frac{4.682\angle -33.15°}{1.2207\angle 1.88°} = 3.835\angle -35.02°$$

Therefore, $\quad v_o(t) =$ **3.835 cos(4t – 35.02°) V**

Chapter 10, Problem 4.

Determine i_1 in the circuit of Fig. 10.53.

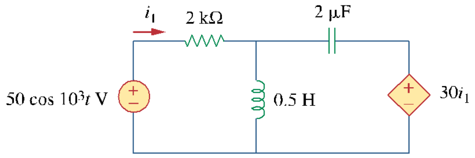


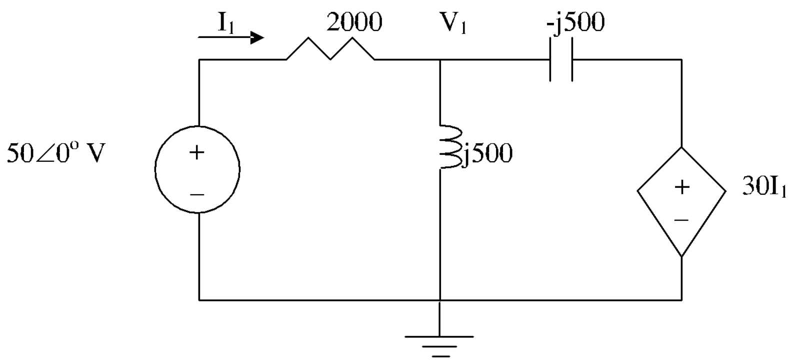
Figure 10.53
For Prob. 10.4.

Chapter 10, Solution 4.

$$0.5H \longrightarrow j\omega L = j0.5x10^3 = j500$$

$$2\mu F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j10^3 x2x10^{-6}} = -j500$$

Consider the circuit as shown below.



At node 1,

$$\frac{50 - V_1}{2000} + \frac{30I_1 - V_1}{-j500} = \frac{V_1}{j500}$$

But $\quad I_1 = \dfrac{50 - V_1}{2000}$

$$50 - V_1 + j4x30(\frac{50 - V_1}{2000}) + j4V_1 - j4V_1 = 0 \quad \rightarrow \quad V_1 = 50$$

$$I_1 = \frac{50 - V_1}{2000} = 0$$

$$i_1(t) = \underline{0\ A}$$

Chapter 10, Problem 5.

ps

Find i_o in the circuit of Fig. 10.54.

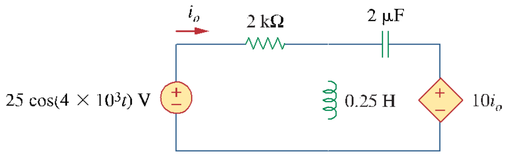


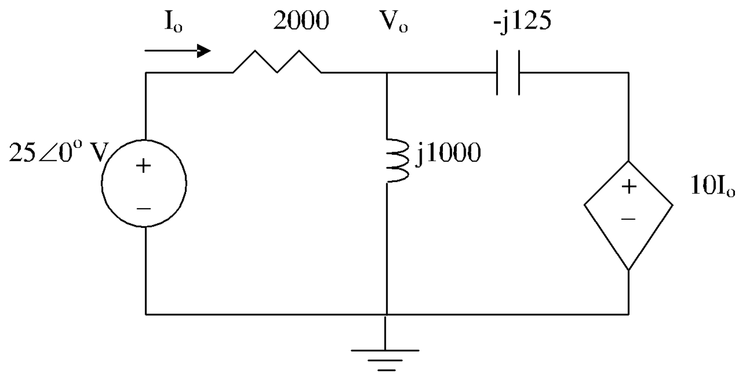
Figure 10.54
For Prob. 10.5.

Chapter 10, Solution 5.

$$0.25H \longrightarrow j\omega L = j0.25x4x10^3 = j1000$$

$$2\mu F \longrightarrow 1/j\omega C = \frac{1}{j4x10^3x2x10^{-6}} = -j125$$

Consider the circuit as shown below.



At node V_o,

$$\frac{V_o - 25}{2000} + \frac{V_o - 0}{j1000} + \frac{V_o - 10I_o}{-j125} = 0$$
$$V_o - 25 - j2V_o + j16V_o - j160I_o = 0$$
$$(1 + j14)V_o - j160I_o = 25$$

But $I_o = (25–V_o)/2000$

$$(1 + j14)V_o - j2 + j0.08V_o = 25$$
$$V_o = \frac{25 + j2}{1 + j14.08} = \frac{25.08\angle 4.57°}{14.115\angle 58.94°}1.7768\angle -81.37°$$

Now to solve for i_o,

$$I_o = \frac{25 - V_o}{2000} = \frac{25 - 0.2666 + j1.7567}{2000} = 12.367 + j0.8784\,\text{mA}$$
$$= 12.398\angle 4.06°$$

i_o = **$12.398\cos(4x10^3t + 4.06^\circ)$ mA**.

Chapter 10, Problem 6.

Determine $\mathbf{V}_x$ in Fig. 10.55.

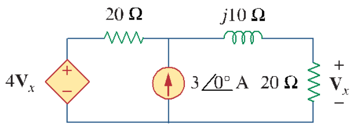


Figure 10.55
For Prob. 10.6.

Chapter 10, Solution 6.

Let V_o be the voltage across the current source. Using nodal analysis we get:

$$\frac{V_o - 4V_x}{20} - 3 + \frac{V_o}{20 + j10} = 0 \quad \text{where} \quad V_x = \frac{20}{20 + j10} V_o$$

Combining these we get:

$$\frac{V_o}{20} - \frac{4V_o}{20 + j10} - 3 + \frac{V_o}{20 + j10} = 0 \rightarrow (1 + j0.5 - 3)V_o = 60 + j30$$

$$V_o = \frac{60 + j30}{-2 + j0.5} \text{ or } V_x = \frac{20(3)}{-2 + j0.5} = \underline{\mathbf{29.11\angle{-166^\circ}\ V}}.$$

Chapter 10, Problem 7.

Use nodal analysis to find **V** in the circuit of Fig. 10.56.

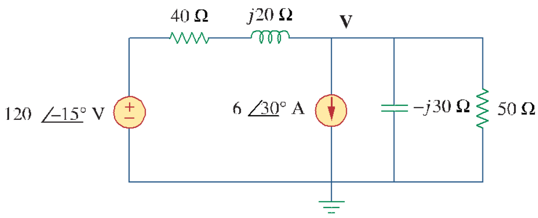


Figure 10.56
For Prob. 10.7.

Chapter 10, Solution 7.

At the main node,

$$\frac{120\angle -15^o - V}{40 + j20} = 6\angle 30^o + \frac{V}{-j30} + \frac{V}{50} \quad \longrightarrow \quad \frac{115.91 - j31.058}{40 + j20} - 5.196 - j3 = V\left(\frac{1}{40 + j20} + \frac{j}{30} + \frac{1}{50}\right)$$

$$V = \frac{-3.1885 - j4.7805}{0.04 + j0.0233} = \underline{124.08\angle -154^o \ \text{V}}$$

Chapter 10, Problem 8.

Use nodal analysis to find current i_o in the circuit of Fig. 10.57. Let $i_s = 6\cos(200t + 15°)$ A.

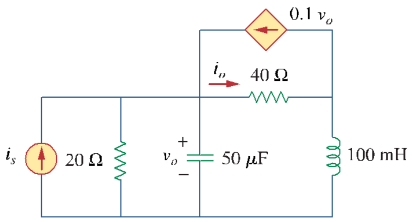


Figure 10.57
For Prob. 10.8.

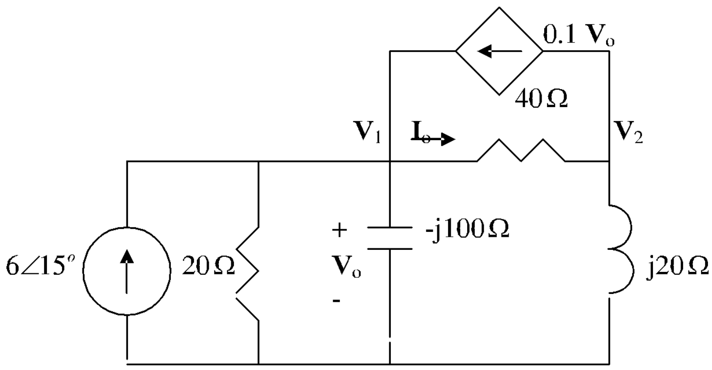
Chapter 10, Solution 8.

$\omega = 200$,

$$100\text{mH} \longrightarrow j\omega L = j200\text{x}0.1 = j20$$

$$50\mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j200\text{x}50\text{x}10^{-6}} = -j100$$

The frequency-domain version of the circuit is shown below.



At node 1,

$$6\angle 15^{o} + 0.1V_1 = \frac{V_1}{20} + \frac{V_1}{-j100} + \frac{V_1 - V_2}{40}$$

or $$5.7955 + j1.5529 = (-0.025 + j0.01)V_1 - 0.025V_2 \qquad (1)$$

At node 2,

$$\frac{V_1 - V_2}{40} = 0.1V_1 + \frac{V_2}{j20} \longrightarrow 0 = 3V_1 + (1 - j2)V_2 \qquad (2)$$

From (1) and (2),

$$\begin{bmatrix} (-0.025 + j0.01) & -0.025 \\ 3 & (1 - j2) \end{bmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} (5.7955 + j1.5529) \\ 0 \end{pmatrix} \quad \text{or} \quad AV = B$$

Using MATLAB,

V = inv(A)*B

leads to $V_1 = -70.63 - j127.23, \qquad V_2 = -110.3 + j161.09$

$$I_o = \frac{V_1 - V_2}{40} = 7.276\angle -82.17^{o}$$

Thus,

$$\underline{i_o(t) = 7.276\cos(200t - 82.17^{o})\ \text{A}}$$

Chapter 10, Problem 9.

Use nodal analysis to find v_o in the circuit of Fig. 10.58.

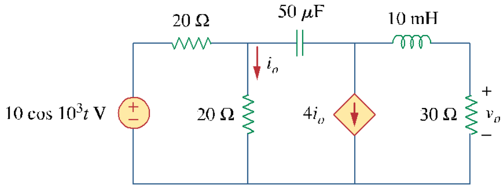
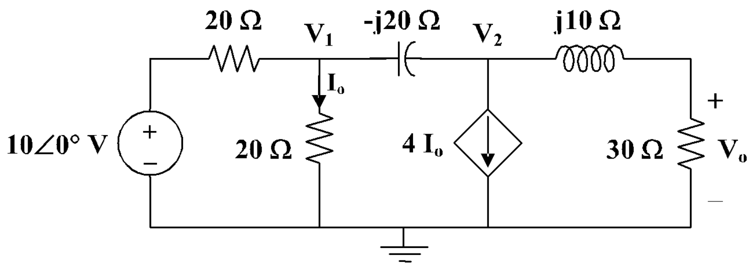


Figure 10.58
For Prob. 10.9.

Chapter 10, Solution 9.

$$10\cos(10^3 t) \longrightarrow 10\angle 0°, \quad \omega = 10^3$$
$$10 \text{ mH} \longrightarrow j\omega L = j10$$
$$50\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10^3)(50\times 10^{-6})} = -j20$$

Consider the circuit shown below.



At node 1,

$$\frac{10 - \mathbf{V}_1}{20} = \frac{\mathbf{V}_1}{20} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j20}$$
$$10 = (2 + j)\mathbf{V}_1 - j\mathbf{V}_2 \qquad (1)$$

At node 2,

$$\frac{\mathbf{V}_1 - \mathbf{V}_2}{-j20} = (4)\frac{\mathbf{V}_1}{20} + \frac{\mathbf{V}_2}{30 + j10}, \text{ where } \mathbf{I}_o = \frac{\mathbf{V}_1}{20} \text{ has been substituted.}$$
$$(-4 + j)\mathbf{V}_1 = (0.6 + j0.8)\mathbf{V}_2$$
$$\mathbf{V}_1 = \frac{0.6 + j0.8}{-4 + j}\mathbf{V}_2 \qquad (2)$$

Substituting (2) into (1)

$$10 = \frac{(2 + j)(0.6 + j0.8)}{-4 + j}\mathbf{V}_2 - j\mathbf{V}_2$$

or
$$\mathbf{V}_2 = \frac{170}{0.6 - j26.2}$$

$$\mathbf{V}_o = \frac{30}{30 + j10}\mathbf{V}_2 = \frac{3}{3 + j}\cdot\frac{170}{0.6 - j26.2} = 6.154\angle 70.26°$$

Therefore, $v_o(t) =$ **6.154 cos(10^3 t + 70.26°) V**

Chapter 10, Problem 10.

Use nodal analysis to find v_o in the circuit of Fig. 10.59. Let $\omega = 2$ krad/s.

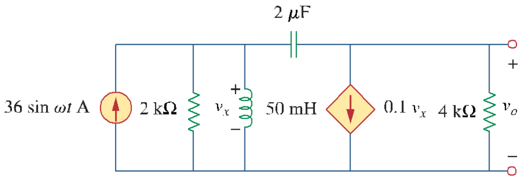


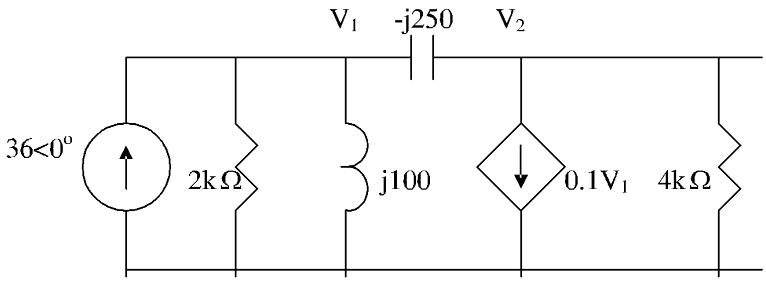
Figure 10.59
For Prob. 10.10.

Chapter 10, Solution 10.

$$50\text{ mH} \longrightarrow j\omega L = j2000\text{x}50\text{x}10^{-3} = j100, \qquad \omega = 2000$$

$$2\mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j2000\text{x}2\text{x}10^{-6}} = -j250$$

Consider the frequency-domain equivalent circuit below.



At node 1,

$$36 = \frac{V_1}{2000} + \frac{V_1}{j100} + \frac{V_1 - V_2}{-j250} \longrightarrow 36 = (0.0005 - j0.006)V_1 - j0.004V_2 \qquad (1)$$

At node 2,

$$\frac{V_1 - V_2}{-j250} = 0.1V_1 + \frac{V_2}{4000} \longrightarrow 0 = (0.1 - j0.004)V_1 + (0.00025 + j0.004)V_2 \quad (2)$$

Solving (1) and (2) gives

$$V_o = V_2 = -535.6 + j893.5 = 8951.1\angle 93.43^o$$

v_o (t) = **<u>8.951 sin(2000t +93.43°) kV</u>**

Chapter 10, Problem 11.

Apply nodal analysis to the circuit in Fig. 10.60 and determine $\mathbf{I}_o$.

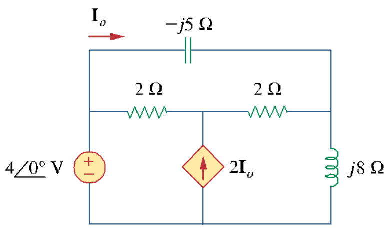
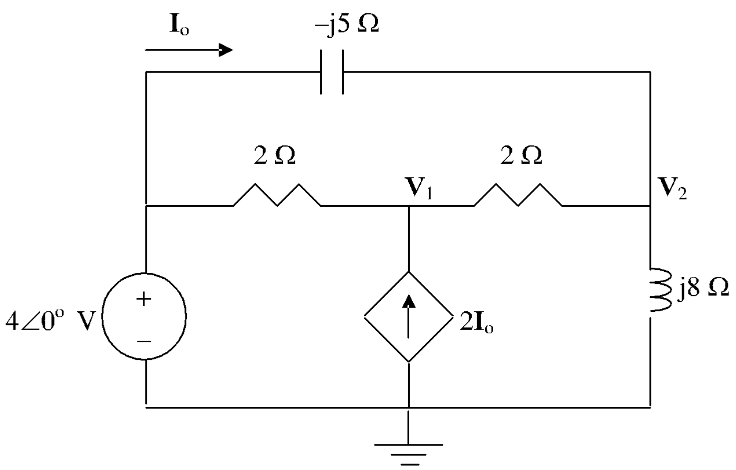


Figure 10.60
For Prob. 10.11.

Chapter 10, Solution 11.

Consider the circuit as shown below.



At node 1,

$$\frac{V_1 - 4}{2} - 2I_o + \frac{V_1 - V_2}{2} = 0$$

$$V_1 - 0.5V_2 - 2I_o = 2$$

But, $I_o = (4–V_2)/(–j5) = –j0.2V_2 + j0.8$

Now the first node equation becomes,

$$V_1 – 0.5V_2 + j0.4V_2 – j1.6 = 2 \text{ or}$$
$$V_1 + (–0.5+j0.4)V_2 = 2 + j1.6$$

At node 2,

$$\frac{V_2 - V_1}{2} + \frac{V_2 - 4}{-j5} + \frac{V_2 - 0}{j8} = 0$$

$$–0.5V_1 + (0.5 + j0.075)V_2 = j0.8$$

Using MATLAB to solve this, we get,

```
>> Y=[1,(-0.5+0.4i);-0.5,(0.5+0.075i)]

Y =

  1.0000          -0.5000 + 0.4000i
 -0.5000           0.5000 + 0.0750i

>> I=[(2+1.6i);0.8i]

I =

  2.0000 + 1.6000i
       0 + 0.8000i

>> V=inv(Y)*I

V =

  4.8597 + 0.0543i
  4.9955 + 0.9050i
```

$I_o = –j0.2V_2 + j0.8 = –j0.9992 + 0.01086 + j0.8 = 0.01086 – j0.1992$

$= \underline{\mathbf{199.5\angle 86.89^\circ \text{ mA}}}$.

Chapter 10, Problem 12.

By nodal analysis, find i_o in the circuit of Fig. 10.61.

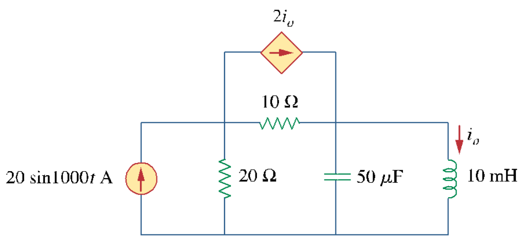


Figure 10.61
For Prob. 10.12.

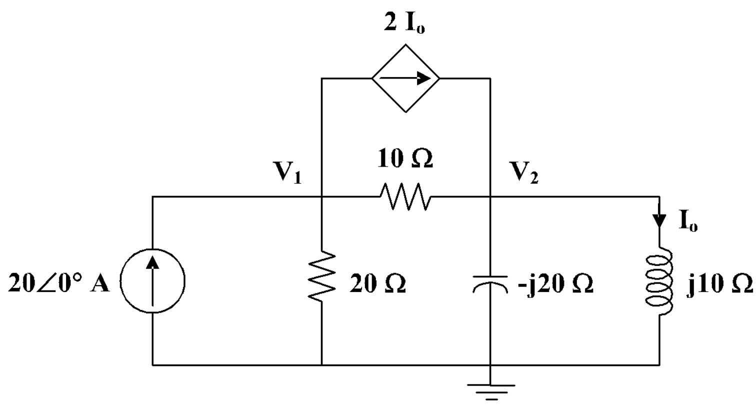
Chapter 10, Solution 12.

$$20\sin(1000t) \longrightarrow 20\angle 0°, \quad \omega = 1000$$

$$10 \text{ mH} \longrightarrow j\omega L = j10$$

$$50\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10^3)(50\times 10^{-6})} = -j20$$

The frequency-domain equivalent circuit is shown below.



At node 1,

$$20 = 2\mathbf{I}_o + \frac{\mathbf{V}_1}{20} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{10},$$ where

$$\mathbf{I}_o = \frac{\mathbf{V}_2}{j10}$$

$$20 = \frac{2\mathbf{V}_2}{j10} + \frac{\mathbf{V}_1}{20} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{10}$$

$$400 = 3\mathbf{V}_1 - (2 + j4)\mathbf{V}_2 \qquad (1)$$

At node 2,

$$\frac{2\mathbf{V}_2}{j10} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{10} = \frac{\mathbf{V}_2}{-j20} + \frac{\mathbf{V}_2}{j10}$$

$$j2\mathbf{V}_1 = (-3 + j2)\mathbf{V}_2$$

or $$\mathbf{V}_1 = (1 + j1.5)\mathbf{V}_2 \qquad (2)$$

Substituting (2) into (1),

$$400 = (3 + j4.5)\mathbf{V}_2 - (2 + j4)\mathbf{V}_2 = (1 + j0.5)\mathbf{V}_2$$

$$\mathbf{V}_2 = \frac{400}{1 + j0.5}$$

$$\mathbf{I}_o = \frac{\mathbf{V}_2}{j10} = \frac{40}{j(1 + j0.5)} = 35.74\angle -116.6°$$

Therefore, $i_o(t) =$ **35.74 sin(1000t – 116.6°) A**

Chapter 10, Problem 13.

Determine $\mathbf{V}_x$ in the circuit of Fig. 10.62 using any method of your choice.

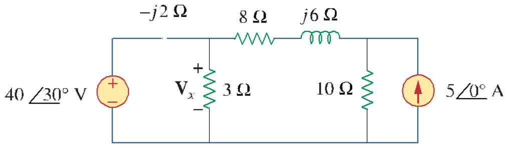
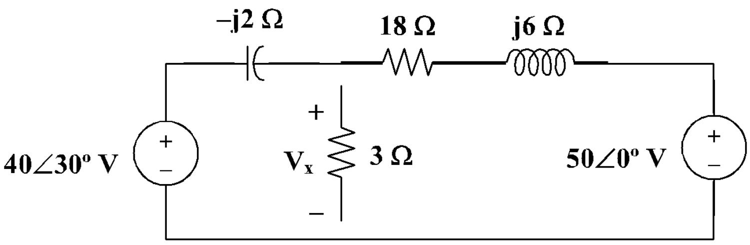


Figure 10.62
For Prob. 10.13.

Chapter 10, Solution 13.

Nodal analysis is the best approach to use on this problem. We can make our work easier by doing a source transformation on the right hand side of the circuit.



$$\frac{V_x - 40\angle 30^\circ}{-j2} + \frac{V_x}{3} + \frac{V_x - 50}{18 + j6} = 0$$

which leads to V_x = **$\underline{29.36\angle 62.88^\circ \text{ A}}$**.

Chapter 10, Problem 14.

Calculate the voltage at nodes 1 and 2 in the circuit of Fig. 10.63 using nodal analysis.

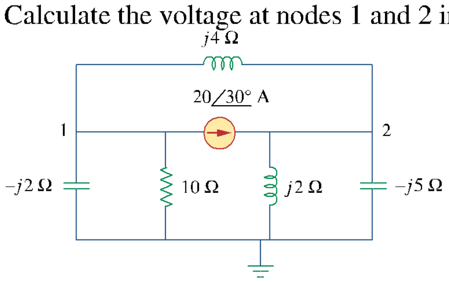


Figure 10.63
For Prob. 10.14.

Chapter 10, Solution 14.

At node 1,

$$\frac{0-\mathbf{V}_1}{-j2}+\frac{0-\mathbf{V}_1}{10}+\frac{\mathbf{V}_2-\mathbf{V}_1}{j4}=20\angle 30°$$

$$-(1+j2.5)\,\mathbf{V}_1-j2.5\,\mathbf{V}_2=173.2+j100 \qquad (1)$$

At node 2,

$$\frac{\mathbf{V}_2}{j2}+\frac{\mathbf{V}_2}{-j5}+\frac{\mathbf{V}_2-\mathbf{V}_1}{j4}=20\angle 30°$$

$$-j5.5\,\mathbf{V}_2+j2.5\,\mathbf{V}_1=173.2+j100 \qquad (2)$$

Equations (1) and (2) can be cast into matrix form as

$$\begin{bmatrix} 1+j2.5 & j2.5 \\ j2.5 & -j5.5 \end{bmatrix}\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}=\begin{bmatrix} -200\angle 30° \\ 200\angle 30° \end{bmatrix}$$

$$\Delta=\begin{vmatrix} 1+j2.5 & j2.5 \\ j2.5 & -j5.5 \end{vmatrix}=20-j5.5=20.74\angle -15.38°$$

$$\Delta_1=\begin{vmatrix} -200\angle 30° & j2.5 \\ 200\angle 30° & -j5.5 \end{vmatrix}=j3(200\angle 30°)=600\angle 120°$$

$$\Delta_2=\begin{vmatrix} 1+j2.5 & -200\angle 30° \\ j2.5 & 200\angle 30° \end{vmatrix}=(200\angle 30°)(1+j5)=1020\angle 108.7°$$

$$\mathbf{V}_1=\frac{\Delta_1}{\Delta}=28.93\angle 135.38°$$

$$\mathbf{V}_2=\frac{\Delta_2}{\Delta}=49.18\angle 124.08°$$

Chapter 10, Problem 15.

Solve for the current **I** in the circuit of Fig. 10.64 using nodal analysis.

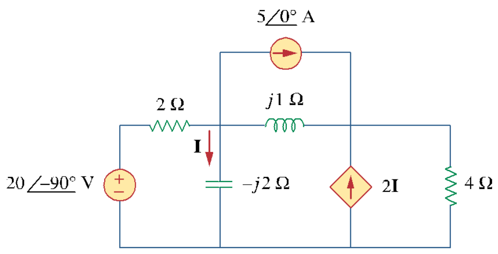
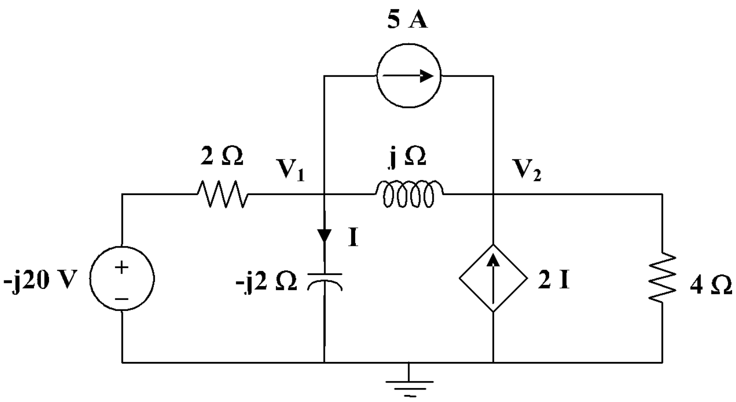


Figure 10.64
For Prob. 10.15.

Chapter 10, Solution 15.

We apply nodal analysis to the circuit shown below.



At node 1,

$$\frac{\text{-}\,j20 - \mathbf{V}_1}{2} = 5 + \frac{\mathbf{V}_1}{\text{-}\,j2} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j}$$

$$\text{-}5 - j10 = (0.5 - j0.5)\,\mathbf{V}_1 + j\mathbf{V}_2 \qquad (1)$$

At node 2,

$$5 + 2\mathbf{I} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j} = \frac{\mathbf{V}_2}{4},$$

where $\mathbf{I} = \dfrac{\mathbf{V}_1}{\text{-}\,j2}$

$$\mathbf{V}_2 = \frac{5}{0.25 - j}\mathbf{V}_1 \qquad (2)$$

Substituting (2) into (1),

$$\text{-}5 - j10 - \frac{j5}{0.25 - j} = 0.5\,(1 - j)\,\mathbf{V}_1$$

$$(1 - j)\,\mathbf{V}_1 = \text{-}10 - j20 - \frac{j40}{1 - j4}$$

$$(\sqrt{2}\angle\text{-}45°)\,\mathbf{V}_1 = \text{-}10 - j20 + \frac{160}{17} - \frac{j40}{17}$$

$$\mathbf{V}_1 = 15.81\angle 313.5°$$

$$\mathbf{I} = \frac{\mathbf{V}_1}{\text{-}\,j2} = (0.5\angle 90°)(15.81\angle 313.5°)$$

$$\mathbf{I} = \underline{\mathbf{7.906\angle 43.49°\ A}}$$

Chapter 10, Problem 16.

Use nodal analysis to find $\mathbf{V}_x$ in the circuit shown in Fig. 10.65.

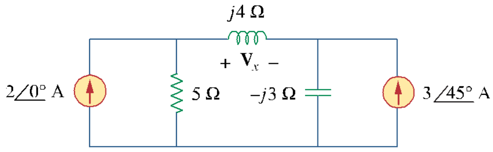
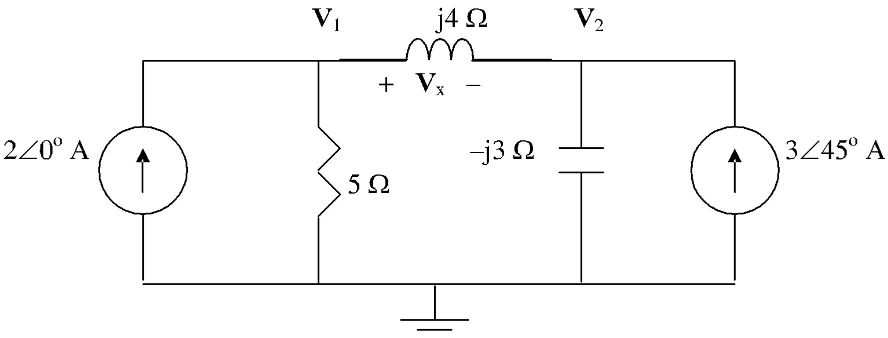


Figure 10.65
For Prob. 10.16.

Chapter 10, Solution 16.

Consider the circuit as shown in the figure below.



At node 1,

$$-2 + \frac{V_1 - 0}{5} + \frac{V_1 - V_2}{j4} = 0 \qquad (1)$$

$$(0.2 - j0.25)V_1 + j0.25V_2 = 2$$

At node 2,

$$\frac{V_2 - V_1}{j4} + \frac{V_2 - 0}{-j3} - 3\angle 45° = 0 \qquad (2)$$

$$j0.25V_1 + j0.08333V_2 = 2.121 + j2.121$$

In matrix form, (1) and (2) become

$$\begin{bmatrix} 0.2 - j0.25 & j0.25 \\ j0.25 & j0.08333 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 2.121 + j2.121 \end{bmatrix}$$

Solving this using MATLAB, we get,

```
>> Y=[(0.2-0.25i),0.25i;0.25i,0.08333i]

Y =

  0.2000 - 0.2500i       0 + 0.2500i
       0 + 0.2500i       0 + 0.0833i

>> I=[2;(2.121+2.121i)]

I =

  2.0000
  2.1210 + 2.1210i

>> V=inv(Y)*I

V =

  5.2793 - 5.4190i
  9.6145 - 9.1955i
```

$V_s = V_1 - V_2 = -4.335 + j3.776 =$ **$5.749\angle 138.94°$ V**.

Chapter 10, Problem 17.

By nodal analysis, obtain current $\mathbf{I}_o$ in the circuit of Fig. 10.66.

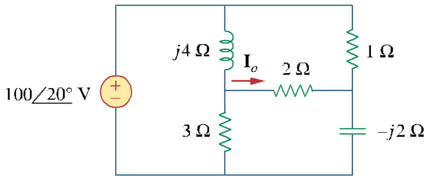
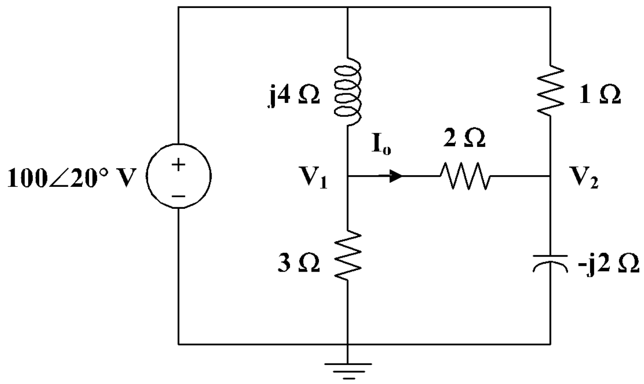


Figure 10.66
For Prob. 10.17.

Chapter 10, Solution 17.

Consider the circuit below.



At node 1,

$$\frac{100\angle 20° - \mathbf{V}_1}{j4} = \frac{\mathbf{V}_1}{3} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{2}$$

$$100\angle 20° = \frac{\mathbf{V}_1}{3}(3 + j10) - j2\mathbf{V}_2$$

(1)

At node 2,

$$\frac{100\angle 20° - \mathbf{V}_2}{1} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{2} = \frac{\mathbf{V}_2}{-j2}$$

$$100\angle 20° = -0.5\mathbf{V}_1 + (1.5 + j0.5)\mathbf{V}_2$$

(2)

From (1) and (2),

$$\begin{bmatrix} 100\angle 20° \\ 100\angle 20° \end{bmatrix} = \begin{bmatrix} -0.5 & 0.5(3 + j) \\ 1 + j10/3 & -j2 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} -0.5 & 1.5 + j0.5 \\ 1 + j10/3 & -j2 \end{vmatrix} = 0.1667 - j4.5$$

$$\Delta_1 = \begin{vmatrix} 100\angle 20° & 1.5 + j0.5 \\ 100\angle 20° & -j2 \end{vmatrix} = -55.45 - j286.2$$

$$\Delta_2 = \begin{vmatrix} -0.5 & 100\angle 20° \\ 1 + j10/3 & 100\angle 20° \end{vmatrix} = -26.95 - j364.5$$

$$\mathbf{V}_1 = \frac{\Delta_1}{\Delta} = 64.74\angle -13.08°$$

$$\mathbf{V}_2 = \frac{\Delta_2}{\Delta} = 81.17\angle -6.35°$$

$$\mathbf{I}_o = \frac{\mathbf{V}_1 - \mathbf{V}_2}{2} = \frac{\Delta_1 - \Delta_2}{2\Delta} = \frac{-28.5 + j78.31}{0.3333 - j9}$$

$\mathbf{I}_o$ = <u>**9.25∠-162.12° A**</u>

Chapter 10, Problem 18.

Use nodal analysis to obtain $\mathbf{V}_o$ in the circuit of Fig. 10.67 below.

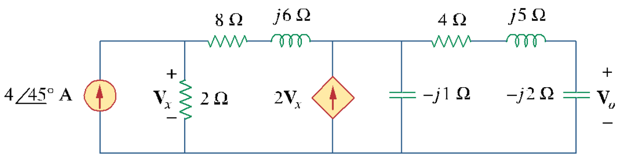
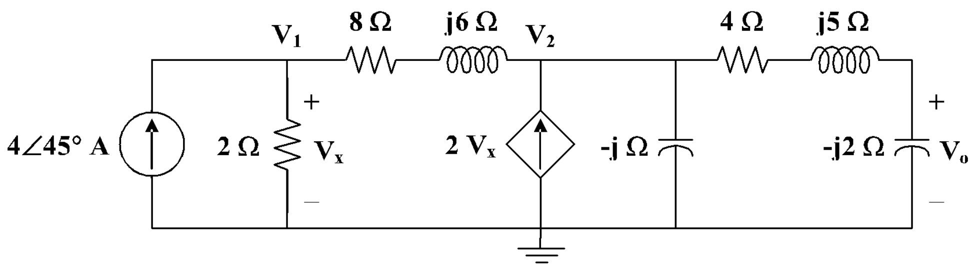


Figure 10.67
For Prob. 10.18.

Chapter 10, Solution 18.

Consider the circuit shown below.



At node 1,

$$4\angle 45° = \frac{\mathbf{V}_1}{2} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{8 + j6}$$

$$200\angle 45° = (29 - j3)\,\mathbf{V}_1 - (4 - j3)\,\mathbf{V}_2$$

(1)

At node 2,

$$\frac{\mathbf{V}_1 - \mathbf{V}_2}{8 + j6} + 2\mathbf{V}_x = \frac{\mathbf{V}_2}{-j} + \frac{\mathbf{V}_2}{4 + j5 - j2}, \qquad \text{where } \mathbf{V}_x = \mathbf{V}_1$$

$$(104 - j3)\,\mathbf{V}_1 = (12 + j41)\,\mathbf{V}_2$$

$$\mathbf{V}_1 = \frac{12 + j41}{104 - j3}\mathbf{V}_2$$

(2)

Substituting (2) into (1),

$$200\angle 45° = (29 - j3)\frac{(12 + j41)}{104 - j3}\mathbf{V}_2 - (4 - j3)\,\mathbf{V}_2$$

$$200\angle 45° = (14.21\angle 89.17°)\,\mathbf{V}_2$$

$$\mathbf{V}_2 = \frac{200\angle 45°}{14.21\angle 89.17°}$$

$$\mathbf{V}_o = \frac{-j2}{4 + j5 - j2}\mathbf{V}_2 = \frac{-j2}{4 + j3}\mathbf{V}_2 = \frac{-6 - j8}{25}\mathbf{V}_2$$

$$\mathbf{V}_o = \frac{10\angle 233.13°}{25} \cdot \frac{200\angle 45°}{14.21\angle 89.17°}$$

$$\mathbf{V}_o = \underline{\mathbf{5.63\angle 189°\ V}}$$

Chapter 10, Problem 19.

Obtain $\mathbf{V}_o$ in Fig. 10.68 using nodal analysis.

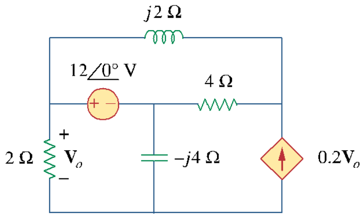
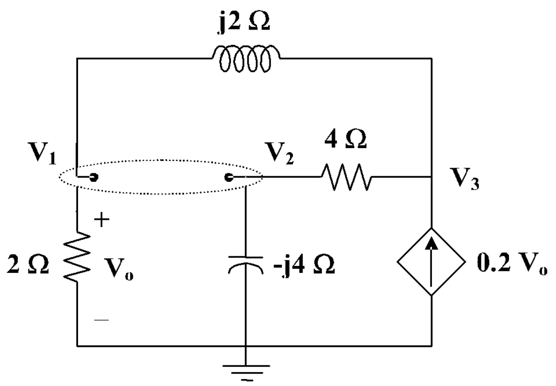


Figure 10.68
For Prob. 10.19.

Chapter 10, Solution 19.

We have a supernode as shown in the circuit below.



Notice that $\mathbf{V}_o = \mathbf{V}_1$.

At the supernode,

$$\frac{\mathbf{V}_3 - \mathbf{V}_2}{4} = \frac{\mathbf{V}_2}{-j4} + \frac{\mathbf{V}_1}{2} + \frac{\mathbf{V}_1 - \mathbf{V}_3}{j2}$$

$$0 = (2 - j2)\mathbf{V}_1 + (1 + j)\mathbf{V}_2 + (-1 + j2)\mathbf{V}_3 \qquad (1)$$

At node 3,

$$0.2\mathbf{V}_1 + \frac{\mathbf{V}_1 - \mathbf{V}_3}{j2} = \frac{\mathbf{V}_3 - \mathbf{V}_2}{4}$$

$$(0.8 - j2)\mathbf{V}_1 + \mathbf{V}_2 + (-1 + j2)\mathbf{V}_3 = 0 \qquad (2)$$

Subtracting (2) from (1),

$$0 = 1.2\mathbf{V}_1 + j\mathbf{V}_2 \qquad (3)$$

But at the supernode,

$$\mathbf{V}_1 = 12\angle 0° + \mathbf{V}_2$$

or

$$\mathbf{V}_2 = \mathbf{V}_1 - 12 \qquad (4)$$

Substituting (4) into (3),

$$0 = 1.2\mathbf{V}_1 + j(\mathbf{V}_1 - 12)$$

$$\mathbf{V}_1 = \frac{j12}{1.2 + j} = \mathbf{V}_o$$

$$\mathbf{V}_o = \frac{12\angle 90°}{1.562\angle 39.81°}$$

$$\mathbf{V}_o = \underline{\mathbf{7.682\angle 50.19°\ V}}$$

Chapter 10, Problem 20.

Refer to Fig. 10.69. If $v_s(t) = V_m \sin \omega t$ and $v_o(t) = A \sin(\omega t + \phi)$ derive the expressions for A and ϕ

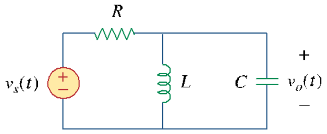
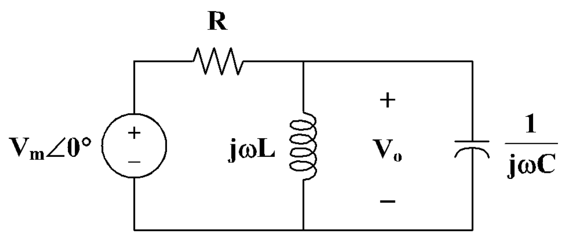


Figure 10.69
For Prob. 10.20.

Chapter 10, Solution 20.

The circuit is converted to its frequency-domain equivalent circuit as shown below.



Let $\quad \mathbf{Z} = j\omega L \,||\, \dfrac{1}{j\omega C} = \dfrac{\frac{L}{C}}{j\omega L + \frac{1}{j\omega C}} = \dfrac{j\omega L}{1-\omega^2 LC}$

$$\mathbf{V}_o = \frac{\mathbf{Z}}{R+\mathbf{Z}} V_m = \frac{\frac{j\omega L}{1-\omega^2 LC}}{R + \frac{j\omega L}{1-\omega^2 LC}} V_m = \frac{j\omega L}{R(1-\omega^2 LC) + j\omega L} V_m$$

$$\mathbf{V}_o = \frac{\omega L\, V_m}{\sqrt{R^2(1-\omega^2 LC)^2 + \omega^2 L^2}} \angle \left(90° - \tan^{-1}\frac{\omega L}{R(1-\omega^2 LC)}\right)$$

If $\quad \mathbf{V}_o = A\angle\phi$, then

$$A = \underline{\frac{\boldsymbol{\omega L\, V_m}}{\sqrt{\mathbf{R^2(1-\omega^2 LC)^2 + \omega^2 L^2}}}}$$

and $\quad \phi = \underline{\mathbf{90° - \tan^{-1}\dfrac{\omega L}{R(1-\omega^2 LC)}}}$

Chapter 10, Problem 21.

For each of the circuits in Fig. 10.70, find $\mathbf{V}_o/\mathbf{V}_i$ for $\omega = 0, \omega \to \infty$, and $\omega^2 = 1/LC$.

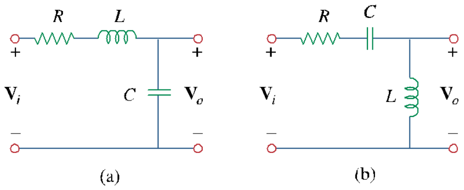


Figure 10.70
For Prob. 10.21.

Chapter 10, Solution 21.

(a) $$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{\dfrac{1}{j\omega C}}{R + j\omega L + \dfrac{1}{j\omega C}} = \frac{1}{1 - \omega^2 LC + j\omega RC}$$

At $\omega = 0$, $$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1}{1} = \underline{\mathbf{1}}$$

As $\omega \to \infty$, $$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \underline{\mathbf{0}}$$

At $\omega = \dfrac{1}{\sqrt{LC}}$, $$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1}{jRC \cdot \dfrac{1}{\sqrt{LC}}} = \underline{\frac{\mathbf{-j}}{\mathbf{R}}\sqrt{\frac{\mathbf{L}}{\mathbf{C}}}}$$

(b) $$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{j\omega L}{R + j\omega L + \dfrac{1}{j\omega C}} = \frac{-\omega^2 LC}{1 - \omega^2 LC + j\omega RC}$$

At $\omega = 0$, $$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \underline{\mathbf{0}}$$

As $\omega \to \infty$, $$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1}{1} = \underline{\mathbf{1}}$$

At $\omega = \dfrac{1}{\sqrt{LC}}$, $$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{-1}{jRC \cdot \dfrac{1}{\sqrt{LC}}} = \underline{\frac{\mathbf{j}}{\mathbf{R}}\sqrt{\frac{\mathbf{L}}{\mathbf{C}}}}$$

Chapter 10, Problem 22.

For the circuit in Fig. 10.71, determine $\mathbf{V}_o/\mathbf{V}_s$.

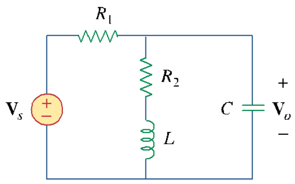
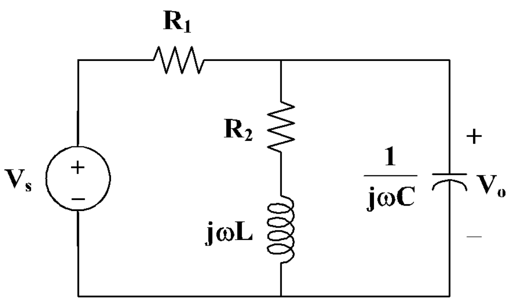


Figure 10.71
For Prob. 10.22.

Chapter 10, Solution 22.

Consider the circuit in the frequency domain as shown below.



Let $\quad \mathbf{Z} = (R_2 + j\omega L) \,||\, \frac{1}{j\omega C}$

$$\mathbf{Z} = \frac{\frac{1}{j\omega C}(R_2 + j\omega L)}{R_2 + j\omega L + \frac{1}{j\omega C}} = \frac{R_2 + j\omega L}{1 + j\omega R_2 - \omega^2 LC}$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{\mathbf{Z}}{\mathbf{Z} + R_1} = \frac{\frac{R_2 + j\omega L}{1 - \omega^2 LC + j\omega R_2 C}}{R_1 + \frac{R_2 + j\omega L}{1 - \omega^2 LC + j\omega R_2 C}}$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{\mathbf{R_2 + j\omega L}}{\mathbf{R_1 + R_2 - \omega^2 LCR_1 + j\omega(L + R_1 R_2 C)}}$$

Chapter 10, Problem 23.

Using nodal analysis obtain **V** in the circuit of Fig. 10.72.

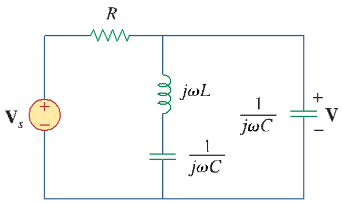


Figure 10.72
For Prob. 10.23.

Chapter 10, Solution 23.

$$\frac{V - V_S}{R} + \frac{V}{j\omega L + \frac{1}{j\omega C}} + j\omega CV = 0$$

$$V + \frac{j\omega RCV}{-\omega^2 LC + 1} + j\omega RCV = V_S$$

$$\left(\frac{1 - \omega^2 LC + j\omega RC + j\omega RC - j\omega^3 RLC^2}{1 - \omega^2 LC}\right) V = V_S$$

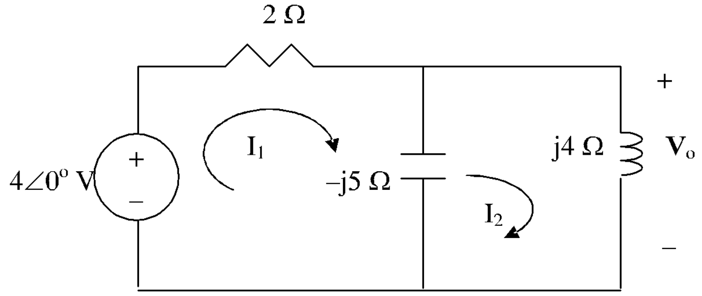
$$V = \underline{\frac{(1 - \omega^2 LC)V_S}{1 - \omega^2 LC + j\omega RC(2 - \omega^2 LC)}}$$

Chapter 10, Problem 24.

Use mesh analysis to find $\mathbf{V}_o$ in the circuit of Prob. 10.2.

Chapter 10, Solution 24.

Consider the circuit as shown below.



For mesh 1,

$$4 = (2 - j5)I_1 + j5I_1 \qquad (1)$$

For mesh 2,

$$0 = j5I_1 + (j4 - j5)I_2 \quad \longrightarrow \quad I_1 = \frac{1}{5}I_2 \qquad (2)$$

Substituting (2) into (1),

$$4 = (2 - j5)\frac{1}{5}I_2 + j5I_2 \quad \longrightarrow \quad I_2 = \frac{1}{0.1 + j}$$

$$V_o = j4I_2 = \frac{j4}{0.1 + j} = \underline{3.98 < 5.71^o \text{ V}}$$

Chapter 10, Problem 25.

ML

Solve for i_o in Fig. 10.73 using mesh analysis.

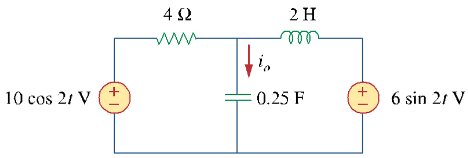


Figure 10.73
For Prob. 10.25.

Chapter 10, Solution 25.

$$\omega = 2$$

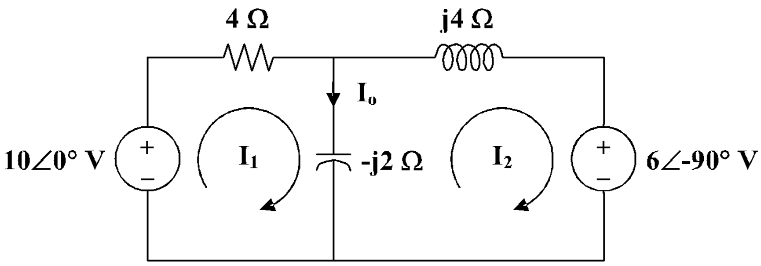
$$10\cos(2t) \longrightarrow 10\angle 0°$$

$$6\sin(2t) \longrightarrow 6\angle -90° = -j6$$

$$2\text{ H} \longrightarrow j\omega L = j4$$

$$0.25\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(2)(1/4)} = -j2$$

The circuit is shown below.



For loop 1,

$$-10 + (4 - j2)\mathbf{I}_1 + j2\mathbf{I}_2 = 0$$

$$5 = (2 - j)\mathbf{I}_1 + j\mathbf{I}_2 \qquad (1)$$

For loop 2,

$$j2\mathbf{I}_1 + (j4 - j2)\mathbf{I}_2 + (-j6) = 0$$

$$\mathbf{I}_1 + \mathbf{I}_2 = 3 \qquad (2)$$

In matrix form (1) and (2) become

$$\begin{bmatrix} 2-j & j \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

$$\Delta = 2(1-j), \qquad \Delta_1 = 5 - j3, \qquad \Delta_2 = 1 - j3$$

$$\mathbf{I}_o = \mathbf{I}_1 - \mathbf{I}_2 = \frac{\Delta_1 - \Delta_2}{\Delta} = \frac{4}{2(1-j)} = 1 + j = 1.414\angle 45°$$

Therefore, $i_o(t) =$ **1.4142 cos(2t + 45°) A**

Chapter 10, Problem 26.

Use mesh analysis to find current i_o in the circuit of Fig. 10.74.

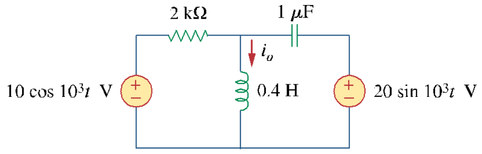


Figure 10.74
For Prob. 10.26.

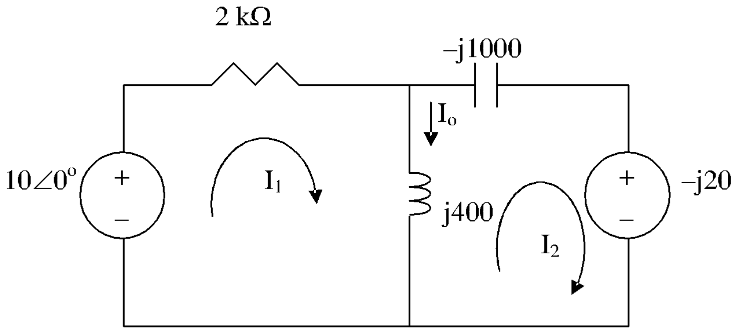
Chapter 10, Solution 26.

$$0.4H \longrightarrow j\omega L = j10^3 x0.4 = j400$$

$$1\mu F \longrightarrow 1/j\omega C = \frac{1}{j10^3 x10^{-6}} = -j1000$$

$$20\sin 10^3 t = 20\cos(10^3 t - 90^o) \longrightarrow 20 < -90 = -j20$$

The circuit becomes that shown below.



For loop 1,

$$-10 + (12000 + j400)I_1 - j400I_2 = 0 \longrightarrow 1 = (200 + j40)I_1 - j40I_2 \ (1)$$

For loop 2,

$$-j20 + (j400 - j1000)I_2 - j400I_1 = 0 \longrightarrow -12 = 40I_1 + 60I_2$$

(2)

In matrix form, (1) and (2) become

$$\begin{bmatrix} 1 \\ -12 \end{bmatrix} = \begin{bmatrix} 200 + j40 & -j40 \\ 40 & 60 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Solving this leads to

I_1 =0.0025-j0.0075, I_2= -0.035+j0.005

$$I_o = I_1 - I_2 = 0.0375 - j0.0125 = 39.5 < -18.43 \text{ mA}$$

$$i_o = 39.5\cos(10^3 t - 18.43^o) \text{ mA}$$

Chapter 10, Problem 27.

Using mesh analysis, find $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit of Fig. 10.75.

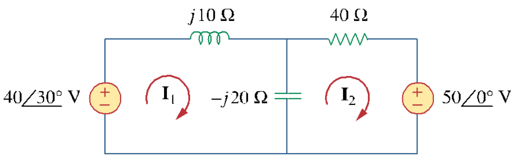


Figure 10.75
For Prob. 10.27.

Chapter 10, Solution 27.

For mesh 1,

$$-40\angle 30° + (j10 - j20)\mathbf{I}_1 + j20\mathbf{I}_2 = 0$$

$$4\angle 30° = -j\mathbf{I}_1 + j2\mathbf{I}_2 \qquad (1)$$

For mesh 2,

$$50\angle 0° + (40 - j20)\mathbf{I}_2 + j20\mathbf{I}_1 = 0$$

$$5 = -j2\mathbf{I}_1 - (4 - j2)\mathbf{I}_2 \qquad (2)$$

From (1) and (2),

$$\begin{bmatrix} 4\angle 30° \\ 5 \end{bmatrix} = \begin{bmatrix} -j & j2 \\ -j2 & -(4-j2) \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

$$\Delta = -2 + 4j = 4.472\angle 116.56°$$

$$\Delta_1 = -(4\angle 30°)(4 - j2) - j10 = 21.01\angle 211.8°$$

$$\Delta_2 = -j5 + 8\angle 120° = 4.44\angle 154.27°$$

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \underline{\mathbf{4.698\angle 95.24°\ A}}$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \underline{\mathbf{0.9928\angle 37.71°\ A}}$$

Chapter 10, Problem 28.

ML

In the circuit of Fig. 10.76, determine the mesh currents i_1 and i_2. Let $v_1 = 10\cos 4t$ **V** and $v_2 = 20\cos(4t - 30°)$V.

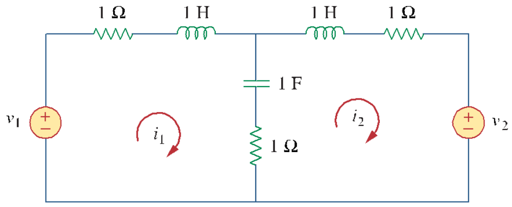


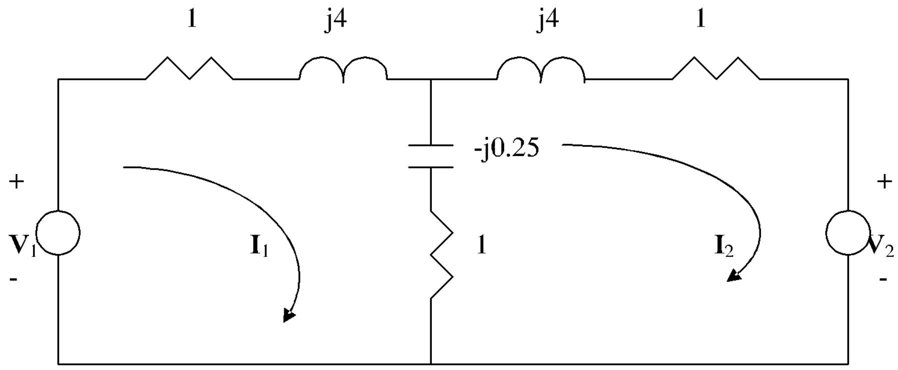
Figure 10.76
For Prob. 10.28.

Chapter 10, Solution 28.

$$1\text{H} \longrightarrow j\omega L = j4, \qquad 1\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j1x4} = -j0.25$$

The frequency-domain version of the circuit is shown below, where

$$V_1 = 10\angle 0^o, \qquad V_2 = 20\angle -30^o \; .$$



$$V_1 = 10\angle 0^o, \qquad V_2 = 20\angle -30^o$$

Applying mesh analysis,

$$10 = (2 + j3.75)I_1 - (1 - j0.25)I_2 \qquad (1)$$

$$-20\angle -30^o = -(1 - j0.25)I_1 + (2 + j3.75)I_2 \qquad (2)$$

From (1) and (2), we obtain

$$\begin{pmatrix} 10 \\ -17.32 + j10 \end{pmatrix} = \begin{pmatrix} 2 + j3.75 & -1 + j0.25 \\ -1 + j0.25 & 2 + j3.75 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

Solving this leads to

$$I_1 = 2.741\angle -41.07^o, \qquad I_2 = 4.114\angle 92^o$$

Hence,

$i_1(t) =$ **<u>2.741cos(4t–41.07°)A</u>**, $i_2(t) =$ **<u>4.114cos(4t+92°)A</u>**.

Chapter 10, Problem 29.

By using mesh analysis, find $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit depicted in Fig. 10.77.

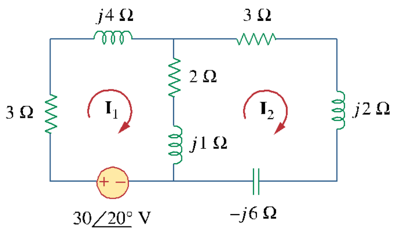


Figure 10.77
For Prob. 10.29.

Chapter 10, Solution 29.

For mesh 1,

$$(5+j5)\mathbf{I}_1 - (2+j)\mathbf{I}_2 - 30\angle 20° = 0$$
$$30\angle 20° = (5+j5)\mathbf{I}_1 - (2+j)\mathbf{I}_2 \qquad (1)$$

For mesh 2,

$$(5+j3-j6)\mathbf{I}_2 - (2+j)\mathbf{I}_1 = 0$$
$$0 = -(2+j)\mathbf{I}_1 + (5-j3)\mathbf{I}_2 \qquad (2)$$

From (1) and (2),

$$\begin{bmatrix} 30\angle 20° \\ 0 \end{bmatrix} = \begin{bmatrix} 5+j5 & -(2+j) \\ -(2+j) & 5-j3 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

$$\Delta = 37 + j6 = 37.48\angle 9.21°$$
$$\Delta_1 = (30\angle 20°)(5.831\angle -30.96°) = 175\angle -10.96°$$
$$\Delta_2 = (30\angle 20°)(2.356\angle 26.56°) = 67.08\angle 46.56°$$

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \underline{\mathbf{4.67\angle{-20.17}°\ A}}$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \underline{\mathbf{1.79\angle 37.35°\ A}}$$

Chapter 10, Problem 30.

Use mesh analysis to find v_o in the circuit of Fig. 10.78. Let $v_{s1} = 120\cos(100t + 90°)$ V, $v_{s2} = 80 \cos 100t$ V.

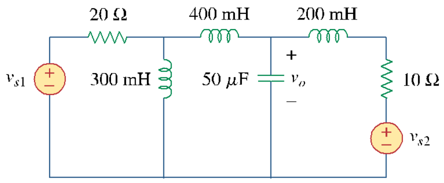


Figure 10.78
For Prob. 10.30.

Chapter 10, Solution 30.

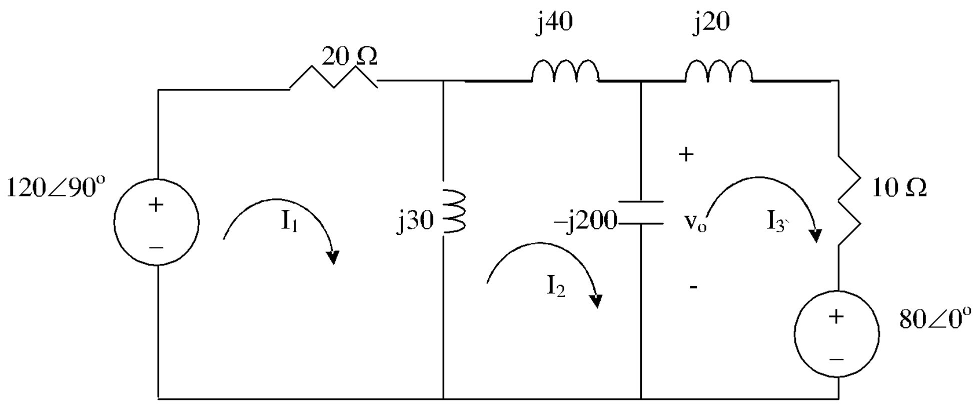
$$300mH \longrightarrow j\omega L = j100x300x10^{-3} = j30$$

$$200mH \longrightarrow j\omega L = j100x200x10^{-3} = j20$$

$$400mH \longrightarrow j\omega L = j100x400x10^{-3} = j40$$

$$50\mu F \longrightarrow 1/j\omega C = \frac{1}{j100x50x10^{-6}} = -j200$$

The circuit becomes that shown below.



For mesh 1,

$$-120 < 90^o + (20 + j30)I_1 - j30I_2 = 0 \quad \longrightarrow \quad j120 = (20 + j30)I_1 - j30I_2 \qquad (1)$$

For mesh 2,

$$-j30I_1 + (j30 + j40 - j200)I_2 + j200I_3 = 0 \quad \longrightarrow \quad 0 = -3I_1 - 13I_2 + 20I_3 \qquad (2)$$

For mesh 3,

$$80 + j200I_2 + (10 - j180)I_3 = 0 \rightarrow -8 = j20I_2 + (1 - j18)I_3 \qquad (3)$$

We put (1) to (3) in matrix form.

$$\begin{bmatrix} 2+j3 & -j3 & 0 \\ -3 & -13 & 20 \\ 0 & j20 & 1-j18 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} j12 \\ 0 \\ -8 \end{bmatrix}$$

This is an excellent candidate for MATLAB.

```
>> Z=[(2+3i),-3i,0;-3,-13,20;0,20i,(1-18i)]

Z =

  2.0000 + 3.0000i       0 - 3.0000i       0
 -3.0000           -13.0000           20.0000
       0                 0 +20.0000i   1.0000 -18.0000i

>> V=[12i;0;-8]

V =

       0 +12.0000i
       0
 -8.0000

>> I=inv(Z)*V

I =

  2.0557 + 3.5651i
  0.4324 + 2.1946i
  0.5894 + 1.9612i
```

$V_o = -j200(I_2 - I_3) = -j200(-0.157+j0.2334) = 46.68 + j31.4 = 56.26\angle 33.93^\circ$

v_o = **<u>56.26cos(100t + 33.93° V</u>**.

Chapter 10, Problem 31.

Use mesh analysis to determine current $\mathbf{I}_o$ in the circuit of Fig. 10.79 below.

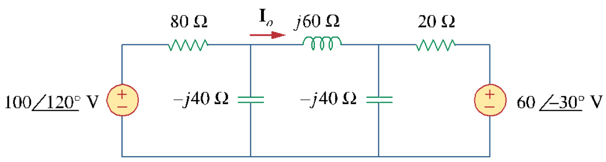
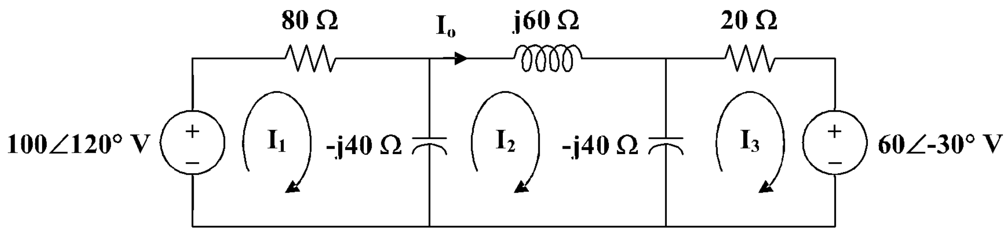


Figure 10.79
For Prob. 10.31.

Chapter 10, Solution 31.

Consider the network shown below.



For loop 1,

$$-100\angle 120° + (80 - j40)\mathbf{I}_1 + j40\mathbf{I}_2 = 0$$

$$10\angle 20° = 4(2 - j)\mathbf{I}_1 + j4\mathbf{I}_2 \qquad (1)$$

For loop 2,

$$j40\mathbf{I}_1 + (j60 - j80)\mathbf{I}_2 + j40\mathbf{I}_3 = 0$$

$$0 = 2\mathbf{I}_1 - \mathbf{I}_2 + 2\mathbf{I}_3 \qquad (2)$$

For loop 3,

$$60\angle -30° + (20 - j40)\mathbf{I}_3 + j40\mathbf{I}_2 = 0$$

$$-6\angle -30° = j4\mathbf{I}_2 + 2(1 - j2)\mathbf{I}_3 \qquad (3)$$

From (2),

$$2\mathbf{I}_3 = \mathbf{I}_2 - 2\mathbf{I}_1$$

Substituting this equation into (3),

$$-6\angle -30° = -2(1 - j2)\mathbf{I}_1 + (1 + j2)\mathbf{I}_2 \qquad (4)$$

From (1) and (4),

$$\begin{bmatrix} 10\angle 120° \\ -6\angle -30° \end{bmatrix} = \begin{bmatrix} 4(2-j) & j4 \\ -2(1-j2) & 1+j2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 8 - j4 & -j4 \\ -2 + j4 & 1 + j2 \end{vmatrix} = 32 + j20 = 37.74\angle 32°$$

$$\Delta_2 = \begin{vmatrix} 8 - j4 & 10\angle 120° \\ -2 + j4 & -6\angle -30° \end{vmatrix} = -4.928 + j82.11 = 82.25\angle 93.44°$$

$$\mathbf{I}_o = \mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \underline{\mathbf{2.179\angle 61.44°\ A}}$$

Chapter 10, Problem 32.

Determine $\mathbf{V}_o$ and $\mathbf{I}_o$ in the circuit of Fig. 10.80 using mesh analysis.

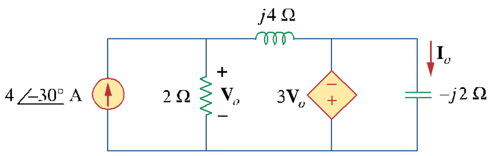
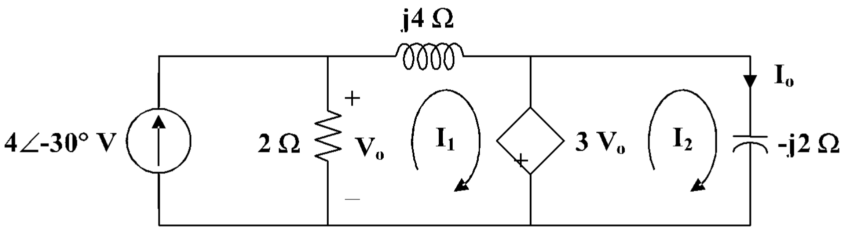


Figure 10.80
For Prob. 10.32.

Chapter 10, Solution 32.

Consider the circuit below.



For mesh 1,

$$(2+j4)\mathbf{I}_1 - 2(4\angle -30°) + 3\mathbf{V}_o = 0$$

where $\mathbf{V}_o = 2(4\angle -30° - \mathbf{I}_1)$

Hence,

$$(2+j4)\mathbf{I}_1 - 8\angle -30° + 6(4\angle -30° - \mathbf{I}_1) = 0$$

$$4\angle -30° = (1-j)\mathbf{I}_1$$

or $\mathbf{I}_1 = 2\sqrt{2}\angle 15°$

$$\mathbf{I}_o = \frac{3\mathbf{V}_o}{-j2} = \frac{3}{-j2}(2)(4\angle -30° - \mathbf{I}_1)$$

$$\mathbf{I}_o = j3(4\angle -30° - 2\sqrt{2}\angle 15°)$$

$$\mathbf{I}_o = \underline{\mathbf{8.485\angle 15°\ A}}$$

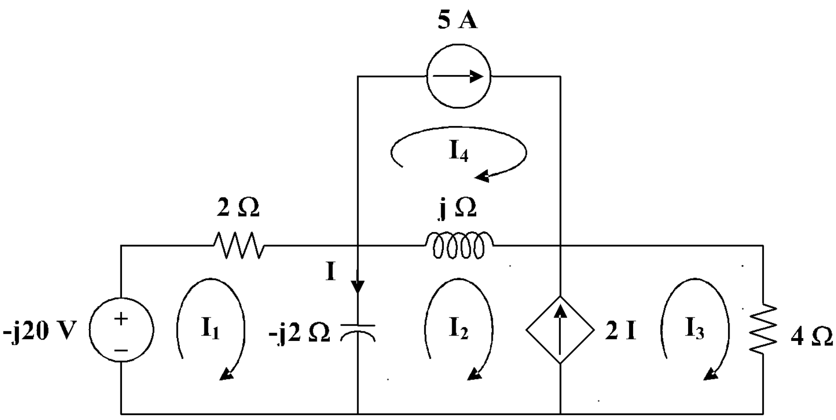
$$\mathbf{V}_o = \frac{-j2\mathbf{I}_o}{3} = \underline{\mathbf{5.657\angle\text{-}75°\ V}}$$

Chapter 10, Problem 33.

Compute **I** in Prob. 10.15 using mesh analysis.

Chapter 10, Solution 33.

Consider the circuit shown below.



For mesh 1,

$$j20 + (2 - j2)\mathbf{I}_1 + j2\mathbf{I}_2 = 0$$

$$(1 - j)\mathbf{I}_1 + j\mathbf{I}_2 = -j10 \qquad (1)$$

For the supermesh,

$$(j - j2)\mathbf{I}_2 + j2\mathbf{I}_1 + 4\mathbf{I}_3 - j\mathbf{I}_4 = 0 \qquad (2)$$

Also,

$$\mathbf{I}_3 - \mathbf{I}_2 = 2\mathbf{I} = 2(\mathbf{I}_1 - \mathbf{I}_2)$$

$$\mathbf{I}_3 = 2\mathbf{I}_1 - \mathbf{I}_2 \qquad (3)$$

For mesh 4,

$$\mathbf{I}_4 = 5 \qquad (4)$$

Substituting (3) and (4) into (2),

$$(8 + j2)\mathbf{I}_1 - (-4 + j)\mathbf{I}_2 = j5 \qquad (5)$$

Putting (1) and (5) in matrix form,

$$\begin{bmatrix} 1 - j & j \\ 8 + j2 & 4 - j \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} -j10 \\ j5 \end{bmatrix}$$

$$\Delta = -3 - j5, \qquad \Delta_1 = -5 + j40, \qquad \Delta_2 = -15 + j85$$

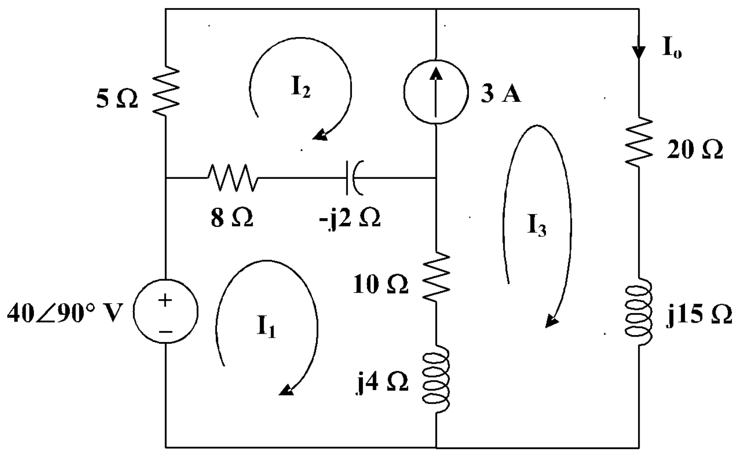
$$\mathbf{I} = \mathbf{I}_1 - \mathbf{I}_2 = \frac{\Delta_1 - \Delta_2}{\Delta} = \frac{10 - j45}{-3 - j5} = \underline{\mathbf{7.906\angle 43.49° \ A}}$$

Chapter 10, Problem 34.

Use mesh analysis to find $\mathbf{I}_o$ in Fig. 10.28 (for Example 10.10).

Chapter 10, Solution 34.

The circuit is shown below.



For mesh 1,

$$-j40 + (18 + j2)\mathbf{I}_1 - (8 - j2)\mathbf{I}_2 - (10 + j4)\mathbf{I}_3 = 0 \qquad (1)$$

For the supermesh,

$$(13 - j2)\mathbf{I}_2 + (30 + j19)\mathbf{I}_3 - (18 + j2)\mathbf{I}_1 = 0 \qquad (2)$$

Also,

$$\mathbf{I}_2 = \mathbf{I}_3 - 3 \qquad (3)$$

Adding (1) and (2) and incorporating (3),

$$-j40 + 5(\mathbf{I}_3 - 3) + (20 + j15)\mathbf{I}_3 = 0$$

$$\mathbf{I}_3 = \frac{3 + j8}{5 + j3} = 1.465\angle 38.48°$$

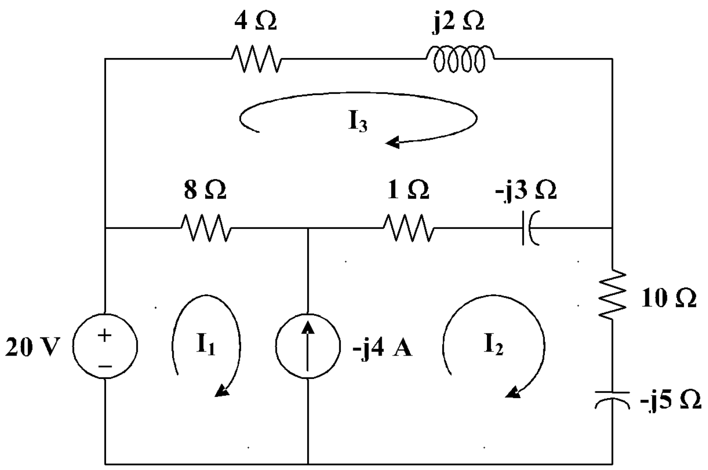
$$\mathbf{I}_o = \mathbf{I}_3 = \underline{\mathbf{1.465\angle 38.48°\ A}}$$

Chapter 10, Problem 35.

Calculate $\mathbf{I}_o$ in Fig. 10.30 (for Practice Prob. 10.10) using mesh analysis.

Chapter 10, Solution 35.

Consider the circuit shown below.



For the supermesh,

$$-20 + 8\mathbf{I}_1 + (11 - j8)\mathbf{I}_2 - (9 - j3)\mathbf{I}_3 = 0 \qquad (1)$$

Also,

$$\mathbf{I}_1 = \mathbf{I}_2 + j4 \qquad (2)$$

For mesh 3,

$$(13 - j)\mathbf{I}_3 - 8\mathbf{I}_1 - (1 - j3)\mathbf{I}_2 = 0 \qquad (3)$$

Substituting (2) into (1),

$$(19 - j8)\mathbf{I}_2 - (9 - j3)\mathbf{I}_3 = 20 - j32 \qquad (4)$$

Substituting (2) into (3),

$$-(9 - j3)\mathbf{I}_2 + (13 - j)\mathbf{I}_3 = j32 \qquad (5)$$

From (4) and (5),

$$\begin{bmatrix} 19 - j8 & -(9 - j3) \\ -(9 - j3) & 13 - j \end{bmatrix} \begin{bmatrix} \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 20 - j32 \\ j32 \end{bmatrix}$$

$$\Delta = 167 - j69, \qquad \Delta_2 = 324 - j148$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{324 - j148}{167 - j69} = \frac{356.2\angle -24.55°}{180.69\angle -22.45°}$$

$\mathbf{I}_2 =$ **1.971∠-2.1° A**

Chapter 10, Problem 36.

Compute $\mathbf{V}_o$ in the circuit of Fig. 10.81 using mesh analysis.

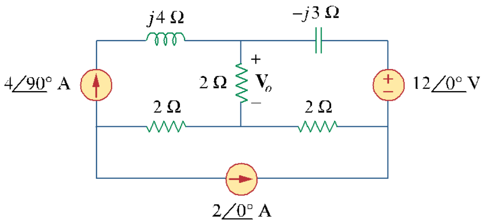
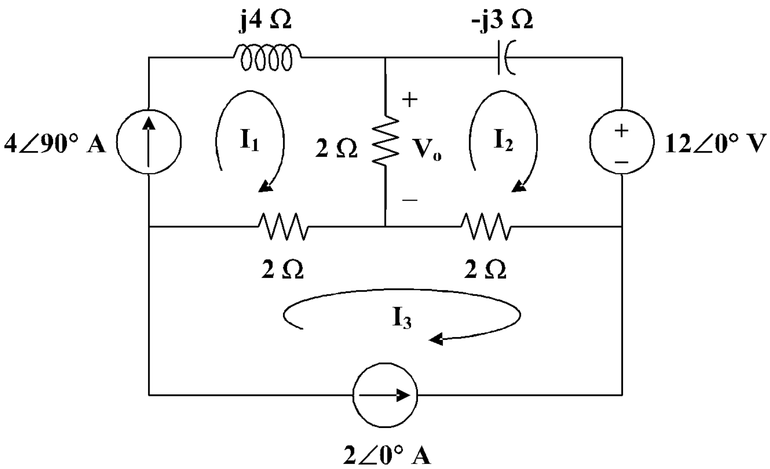


Figure 10.81
For Prob. 10.36.

Chapter 10, Solution 36.

Consider the circuit below.



Clearly,

$$\mathbf{I}_1 = 4\angle 90° = j4 \qquad \text{and} \qquad \mathbf{I}_3 = -2$$

For mesh 2,

$$(4 - j3)\mathbf{I}_2 - 2\mathbf{I}_1 - 2\mathbf{I}_3 + 12 = 0$$

$$(4 - j3)\mathbf{I}_2 - j8 + 4 + 12 = 0$$

$$\mathbf{I}_2 = \frac{-16 + j8}{4 - j3} = -3.52 - j0.64$$

Thus,

$$\mathbf{V}_o = 2(\mathbf{I}_1 - \mathbf{I}_2) = (2)(3.52 + j4.64) = 7.04 + j9.28$$

$$\mathbf{V}_o = \underline{\mathbf{11.648\angle 52.82°\ V}}$$

Chapter 10, Problem 37.

Use mesh analysis to find currents $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ in the circuit of Fig. 10.82.

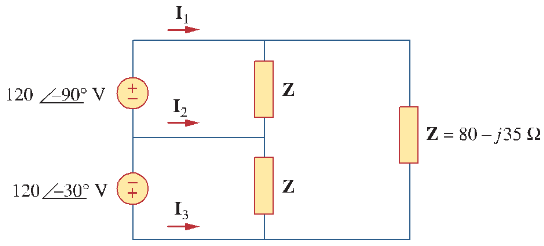
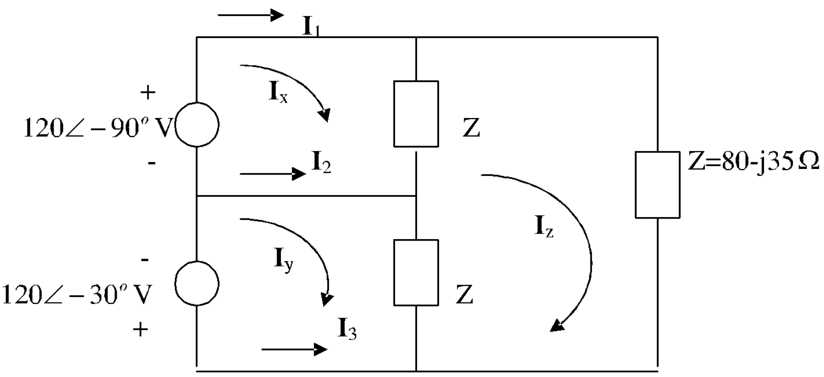


Figure 10.82
For Prob. 10.37.

Chapter 10, Solution 37.



For mesh x,

$$ZI_x - ZI_z = -j120 \qquad (1)$$

For mesh y,

$$ZI_y - ZI_z = -120\angle 30^o = -103.92 + j60 \qquad (2)$$

For mesh z,

$$-ZI_x - ZI_y + 3ZI_z = 0 \qquad (3)$$

Putting (1) to (3) together leads to the following matrix equation:

$$\begin{pmatrix} (80-j35) & 0 & (-80+j35) \\ 0 & (80-j35) & (-80+j35) \\ (-80+j35) & (-80+j35) & (240-j105) \end{pmatrix}\begin{pmatrix} I_x \\ I_y \\ I_z \end{pmatrix} = \begin{pmatrix} -j120 \\ -103.92+j60 \\ 0 \end{pmatrix} \longrightarrow \quad AI = B$$

Using MATLAB, we obtain

$$I = inv(A)*B = \begin{pmatrix} -0.2641 - j2.366 \\ -2.181 - j0.954 \\ -0.815 - j1.1066 \end{pmatrix}$$

$$I_1 = I_x = -0.2641 - j2.366 = \underline{2.38\angle -96.37^o} \text{ A}$$

$$I_2 = I_y - I_x = -1.9167 + j1.4116 = \underline{2.38\angle 143.63^o} \text{ A}$$

$$I_3 = -I_y = 2.181 + j0.954 = \underline{2.38\angle 23.63^o \text{ A}}$$

Chapter 10, Problem 38.

Using mesh analysis, obtain $\mathbf{I}_o$ in the circuit shown in Fig. 10.83.

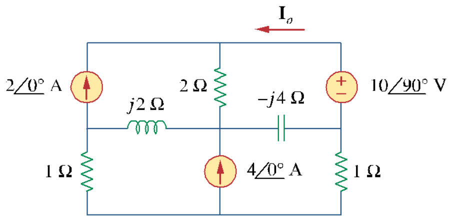
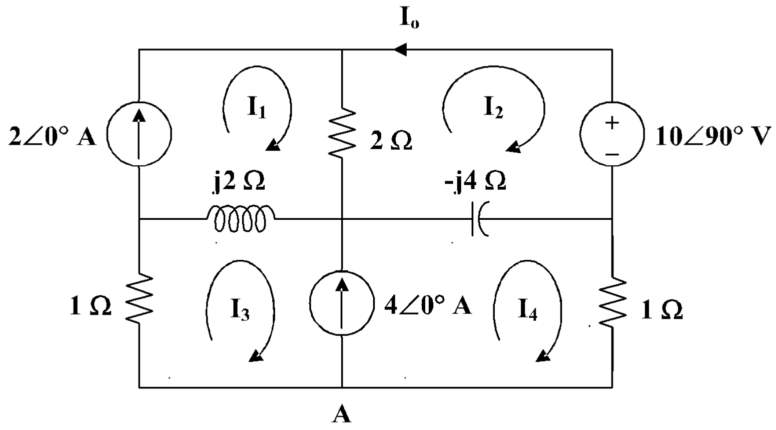


Figure 10.83
For Prob. 10.38.

Chapter 10, Solution 38.

Consider the circuit below.



Clearly,

$$\mathbf{I}_1 = 2 \qquad (1)$$

For mesh 2,

$$(2 - j4)\mathbf{I}_2 - 2\mathbf{I}_1 + j4\mathbf{I}_4 + 10\angle 90° = 0 \qquad (2)$$

Substitute (1) into (2) to get

$$(1 - j2)\mathbf{I}_2 + j2\mathbf{I}_4 = 2 - j5$$

For the supermesh,

$$(1 + j2)\mathbf{I}_3 - j2\mathbf{I}_1 + (1 - j4)\mathbf{I}_4 + j4\mathbf{I}_2 = 0$$

$$j4\mathbf{I}_2 + (1 + j2)\mathbf{I}_3 + (1 - j4)\mathbf{I}_4 = j4 \qquad (3)$$

At node A,

$$\mathbf{I}_3 = \mathbf{I}_4 - 4 \qquad (4)$$

Substituting (4) into (3) gives

$$j2\mathbf{I}_2 + (1 - j)\mathbf{I}_4 = 2(1 + j3) \qquad (5)$$

From (2) and (5),

$$\begin{bmatrix} 1 - j2 & j2 \\ j2 & 1 - j \end{bmatrix} \begin{bmatrix} \mathbf{I}_2 \\ \mathbf{I}_4 \end{bmatrix} = \begin{bmatrix} 2 - j5 \\ 2 + j6 \end{bmatrix}$$

$$\Delta = 3 - j3, \qquad \Delta_1 = 9 - j11$$

$$\mathbf{I}_o = -\mathbf{I}_2 = \frac{-\Delta_1}{\Delta} = \frac{-(9 - j11)}{3 - j3} = \frac{1}{3}(-10 + j)$$

$\mathbf{I}_o$ = **3.35∠174.3° A**

Chapter 10, Problem 39.

Find $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{I}_3$, and $\mathbf{I}_x$ in the circuit of Fig. 10.84.

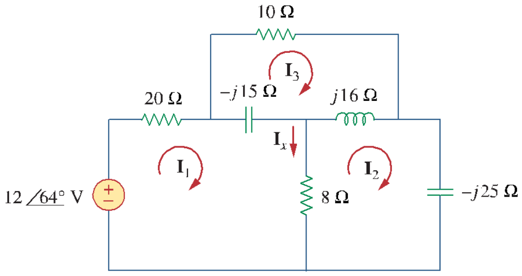


Figure 10.84
For Prob. 10.39.

Chapter 10, Solution 39.

For mesh 1,

$$(28 - j15)I_1 - 8I_2 + j15I_3 = 12\angle 64^o \qquad (1)$$

For mesh 2,

$$-8I_1 + (8 - j9)I_2 - j16I_3 = 0 \qquad (2)$$

For mesh 3,

$$j15I_1 - j16I_2 + (10 + j)I_3 = 0 \qquad (3)$$

In matrix form, (1) to (3) can be cast as

$$\begin{pmatrix} (28 - j15) & -8 & j15 \\ -8 & (8 - j9) & -j16 \\ j15 & -j16 & (10 + j) \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 12\angle 64^o \\ 0 \\ 0 \end{pmatrix} \quad \text{or} \quad AI = B$$

Using MATLAB,

$$I = inv(A)*B$$

$$I_1 = -0.128 + j0.3593 = \underline{0.3814\angle 109.6^o \text{ A}}$$

$$I_2 = -0.1946 + j0.2841 = \underline{0.3443\angle 124.4^o \text{ A}}$$

$$I_3 = 0.0718 - j0.1265 = \underline{0.1455\angle -60.42^o \text{ A}}$$

$$I_x = I_1 - I_2 = 0.0666 + j0.0752 = \underline{0.1005\angle 48.5^o \text{ A}}$$

Chapter 10, Problem 40.

Find i_o in the circuit shown in Fig. 10.85 using superposition.

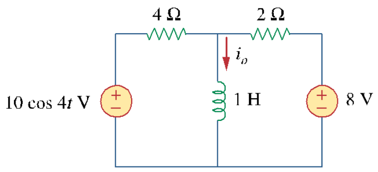
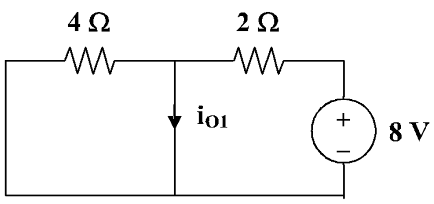


Figure 10.85
For Prob. 10.40.

Chapter 10, Solution 40.

Let $i_O = i_{O1} + i_{O2}$, where i_{O1} is due to the dc source and i_{O2} is due to the ac source. For i_{O1}, consider the circuit in Fig. (a).

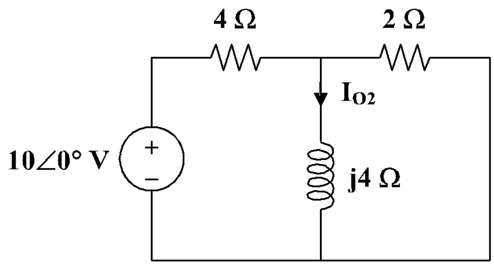


(a)

Clearly,

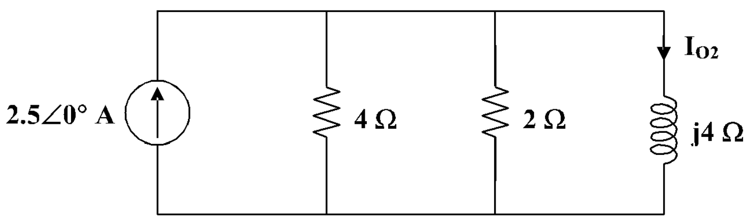
$$i_{O1} = 8/2 = 4 \text{ A}$$

For i_{O2}, consider the circuit in Fig. (b).



(b)

If we transform the voltage source, we have the circuit in Fig. (c), where $4 \parallel 2 = 4/3\,\Omega$.



(c)

By the current division principle,

$$\mathbf{I}_{O2} = \frac{4/3}{4/3 + j4}(2.5\angle 0°)$$

$$\mathbf{I}_{O2} = 0.25 - j0.75 = 0.79\angle -71.56°$$

Thus,
$$i_{O2} = 0.79\cos(4t - 71.56°) \text{ A}$$

Therefore,

$$i_O = i_{O1} + i_{O2} = \underline{\mathbf{4 + 0.79\ cos(4t - 71.56°)\ A}}$$

Chapter 10, Problem 41.

Find v_o for the circuit in Fig. 10.86, assuming that $v_s = 6 \cos 2t + 4 \sin 4t$ V.

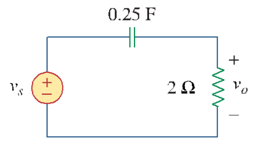


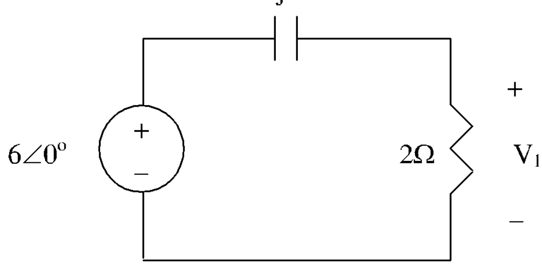
Figure 10.86
For Prob. 10.41.

Chapter 10, Solution 41.

We apply superposition principle. We let

$$v_o = v_1 + v_2$$

where v_1 and v_2 are due to the sources 6cos2t and 4sin4t respectively. To find v_1, consider the circuit below.



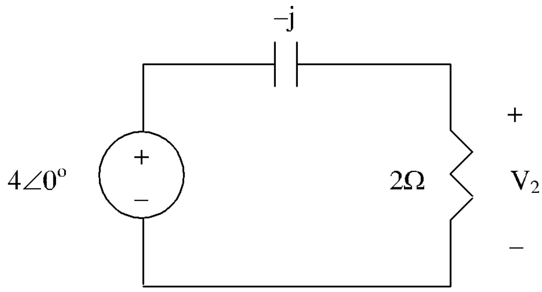
$$1/4F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j2x1/4} = -j2$$

$$V_1 = \frac{2}{2 - j2}(6) = 3 + j3 = 4.2426 < 45^o$$

Thus,

$$v_1 = 4.2426\cos(2t + 45^o)$$

To get v_2, consider the circuit below



$$1/4F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j4x1/4} = -j1$$

$$V_2 = \frac{2}{2 - j}(4) = 3.2 + j1.6 = 3.578 < 26.56^o$$

$$v_2 = 3.578\sin(4t + 26.56^o)$$

Hence,

v_o = **<u>4.243cos(2t + 45°) + 3.578sin(4t + 25.56°) V</u>**.

Chapter 10, Problem 42.

Solve for $\mathbf{I}_o$ in the circuit of Fig. 10.87.

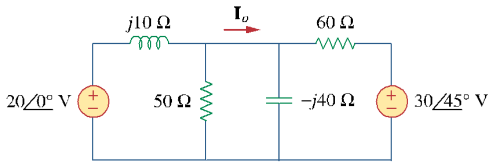
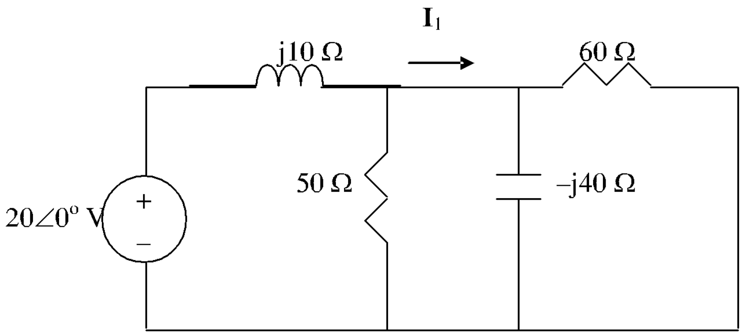


Figure 10.87
For Prob. 10.42.

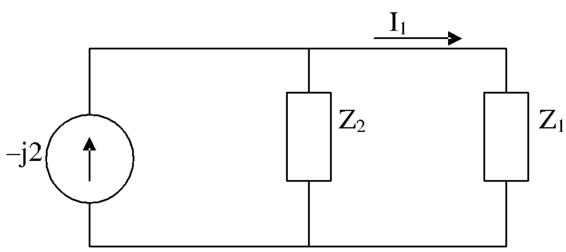
Chapter 10, Solution 42.

Let $I_o = I_1 + I_2$

where I_1 and I_2 are due to 20<0° and 30<45° sources respectively. To get I_1, we use the circuit below.



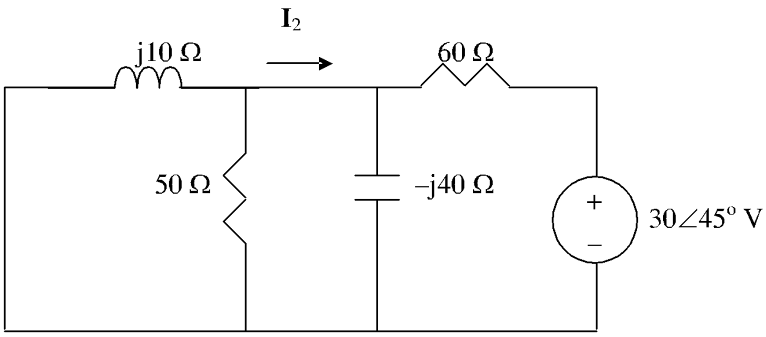
Let Z_1 = -j40//60 = 18.4615 –j27.6927, Z_2 = j10//50=1.9231 + j9.615
Transforming the voltage source to a current source leads to the circuit below.



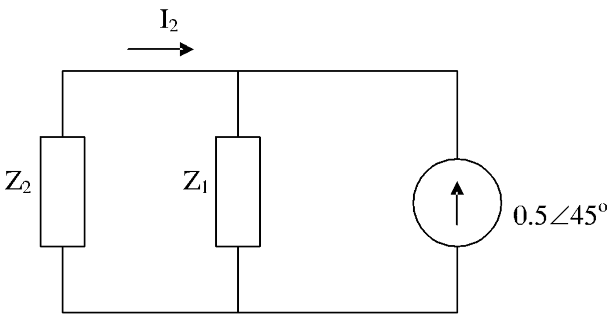
Using current division,

$$I_1 = \frac{Z_2}{Z_1 + Z_2}(-j2) = 0.6217 + j0.3626$$

To get I_2, we use the circuit below.



After transforming the voltage source, we obtain the circuit below.



Using current division,

$$I_2 = \frac{-Z_1}{Z_1 + Z_2}(0.5 < 45^o) = -0.5275 - j0.3077$$

Hence,

$$I_o = I_1 + I_2 = 0.0942 + j0.0509 = \underline{0.109 < 30^o\ \text{A}}$$

Chapter 10, Problem 43.

Using the superposition principle, find i_x in the circuit of Fig. 10.88.

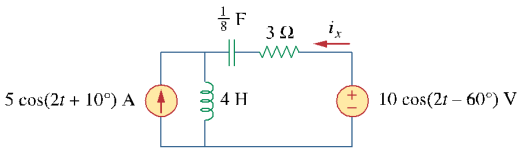


Figure 10.88
For Prob. 10.43.

Chapter 10, Solution 43.

Let $\mathbf{I}_x = \mathbf{I}_1 + \mathbf{I}_2$, where $\mathbf{I}_1$ is due to the voltage source and $\mathbf{I}_2$ is due to the current source.

$$\omega = 2$$

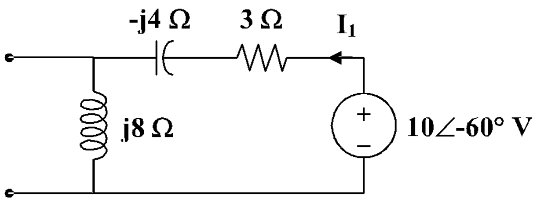
$$5\cos(2t + 10°) \longrightarrow 5\angle 10°$$

$$10\cos(2t - 60°) \longrightarrow 10\angle\text{-}60°$$

$$4\text{ H} \longrightarrow j\omega L = j8$$

$$\frac{1}{8}\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(2)(1/8)} = \text{-}j4$$

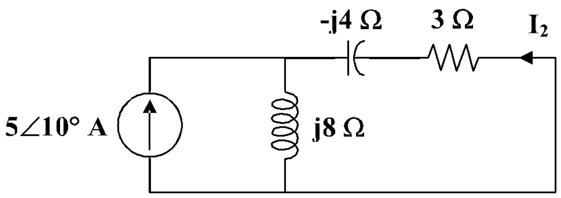
For $\mathbf{I}_1$, consider the circuit in Fig. (a).



(a)

$$\mathbf{I}_1 = \frac{10\angle\text{-}60°}{3 + j8 - j4} = \frac{10\angle\text{-}60°}{3 + j4}$$

For $\mathbf{I}_2$, consider the circuit in Fig. (b).



(b)

$$\mathbf{I}_2 = \frac{\text{-}j8}{3 + j8 - j4}(5\angle 10°) = \frac{\text{-}j40\angle 10°}{3 + j4}$$

$$\mathbf{I}_x = \mathbf{I}_1 + \mathbf{I}_2 = \frac{1}{3 + j4}(10\angle\text{-}60° - j40\angle 10°)$$

$$\mathbf{I}_x = \frac{49.51\angle\text{-}76.04°}{5\angle 53.13°} = 9.902\angle\text{-}129.17°$$

Therefore, i_x = **9.902 cos(2t – 129.17°) A**

Chapter 10, Problem 44.

Use the superposition principle to obtain v_x in the circuit of Fig. 10.89. Let $v_s = 50\sin 2t$ V and $i_s = 12\cos(6t + 10°)$A.

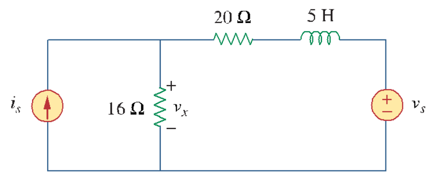


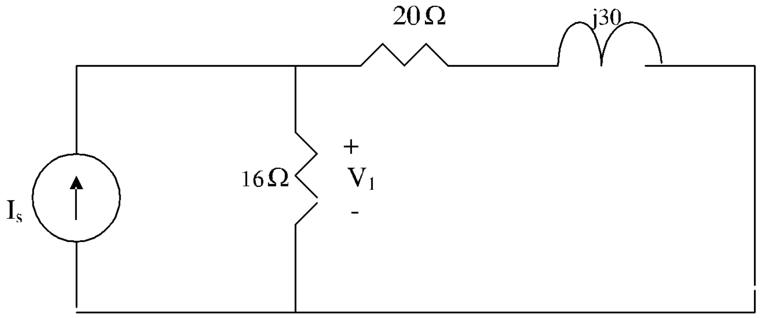
Figure 10.89
For Prob. 10.44.

Chapter 10, Solution 44.

Let $v_x = v_1 + v_2$, where v_1 and v_2 are due to the current source and voltage source respectively.

For v_1, $\omega = 6$, $5\,H \longrightarrow j\omega L = j30$

The frequency-domain circuit is shown below.

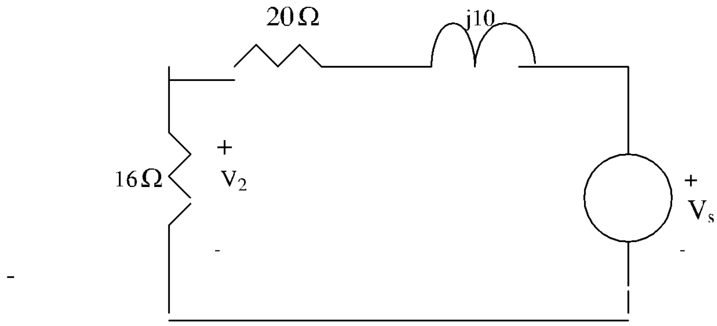


Let $Z = 16 // (20 + j30) = \dfrac{16(20 + j30)}{36 + j30} = 11.8 + j3.497 = 12.31\angle 16.5^o$

$V_1 = I_s Z = (12\angle 10^o)(12.31\angle 16.5^o) = 147.7\angle 26.5^o \longrightarrow v_1 = 147.7\cos(6t + 26.5^o)$ V

For v_2, $\omega = 2$, $5\,H \longrightarrow j\omega L = j10$

The frequency-domain circuit is shown below.



Using voltage division,

$$V_2 = \frac{16}{16 + 20 + j10} V_s = \frac{16(50\angle 0^o)}{36 + j10} = 21.41\angle -15.52^o \longrightarrow v_2 = 21.41\sin(2t - 15.52^o)\text{ V}$$

Thus,

$$v_x = \underline{147.7\cos(6t + 26.5^o) + 21.41\sin(2t - 15.52^o)\text{ V}}$$

Chapter 10, Problem 45.

Use superposition to find $i(t)$ in the circuit of Fig. 10.90.

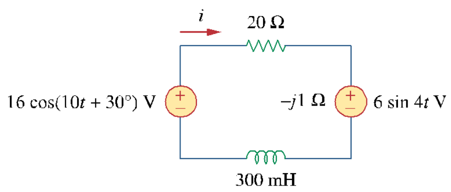
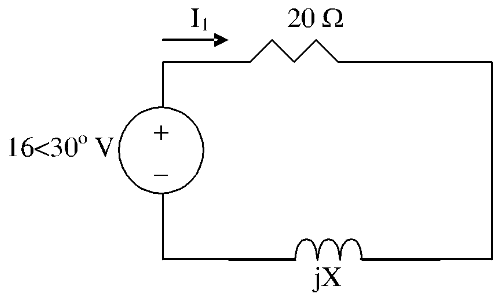


Figure 10.90
For Prob. 10.45.

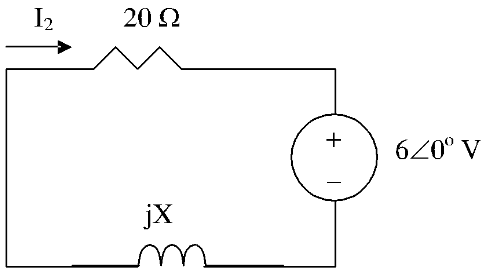
Chapter 10, Solution 45.

Let $i = i_1 + i_2$, where i_1 and i_2 are due to 16cos(10t +30°) and 6sin4t sources respectively. To find i_1 , consider the circuit below.



$$X = \omega L = 10x300x10^{-3} = 3$$
$$I_1 = \frac{16 < 30^o}{20 + j3} = 0.7911$$
$$i_1 = 0.7911\cos(10t + 21.47^o)\ \text{A}$$

To find i_2 , consider the circuit below.



$$X = \omega L = 4x300x10^{-3} = 1.2$$
$$I_2 = -\frac{6 < 0^o}{20 + j1.2} = 0.2995 < 176.6^o$$
$$i_1 = 0.2995\sin(4t + 176.6^o)\ \text{A}$$

Thus,

$$i = i_1 + i_2 = \underline{0.7911\cos(10t + 21.47^o) + 0.2995\sin(4t + 176.6^o)\ \text{A}}$$

= **<u>791.1cos(10t+21.47˚)+299.5sin(4t+176.6˚) mA</u>**

Chapter 10, Problem 46.
Solve for $v_o(t)$ in the circuit of Fig. 10.91 using the superposition principle.

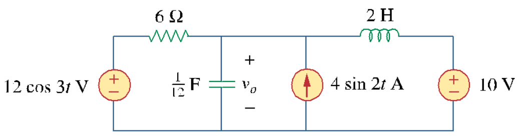
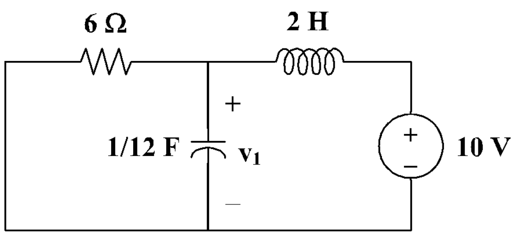


Figure 10.91
For Prob. 10.46.

Chapter 10, Solution 46.

Let $v_o = v_1 + v_2 + v_3$, where v_1, v_2, and v_3 are respectively due to the 10-V dc source, the ac current source, and the ac voltage source. For v_1 consider the circuit in Fig. (a).



(a)

The capacitor is open to dc, while the inductor is a short circuit. Hence,

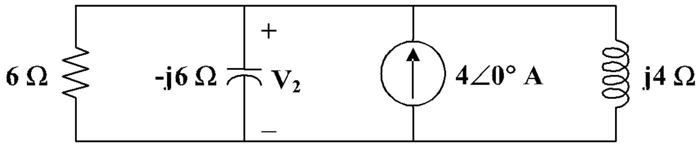
$$v_1 = 10 \text{ V}$$

For v_2, consider the circuit in Fig. (b).

$$\omega = 2$$

$$2 \text{ H} \longrightarrow j\omega L = j4$$

$$\frac{1}{12} \text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(2)(1/12)} = -j6$$



(b)

Applying nodal analysis,

$$4 = \frac{\mathbf{V}_2}{6} + \frac{\mathbf{V}_2}{-j6} + \frac{\mathbf{V}_2}{j4} = \left(\frac{1}{6} + \frac{j}{6} - \frac{j}{4}\right)\mathbf{V}_2$$

$$\mathbf{V}_2 = \frac{24}{1 - j0.5} = 21.45\angle 26.56°$$

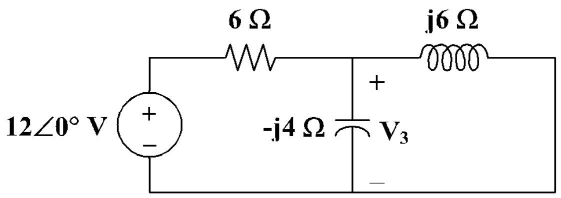
Hence, $v_2 = 21.45\sin(2t + 26.56°)$ V

For v_3, consider the circuit in Fig. (c).

$$\omega = 3$$

$$2\text{ H} \longrightarrow j\omega L = j6$$

$$\frac{1}{12}\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(3)(1/12)} = -j4$$



(c)

At the non-reference node,

$$\frac{12 - \mathbf{V}_3}{6} = \frac{\mathbf{V}_3}{-j4} + \frac{\mathbf{V}_3}{j6}$$

$$\mathbf{V}_3 = \frac{12}{1 + j0.5} = 10.73\angle -26.56°$$

Hence, $v_3 = 10.73\cos(3t - 26.56°)$ V

Therefore, $v_o =$ **<u>10 + 21.45 sin(2t + 26.56°) + 10.73 cos(3t – 26.56°) V</u>**

Chapter 10, Problem 47.

Determine i_o in the circuit of Fig. 10.92, using the superposition principle.

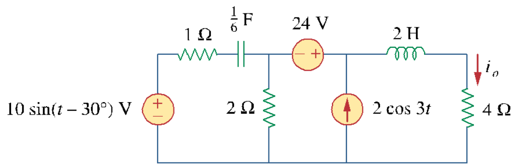
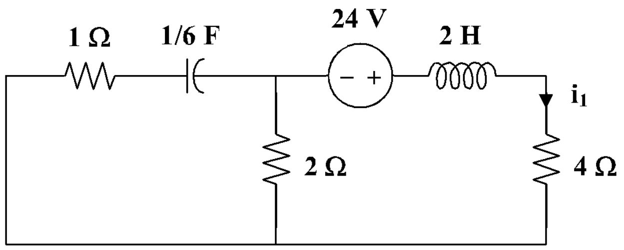


Figure 10.92
For Prob. 10.47.

Chapter 10, Solution 47.

Let $i_o = i_1 + i_2 + i_3$, where i_1, i_2, and i_3 are respectively due to the 24-V dc source, the ac voltage source, and the ac current source. For i_1, consider the circuit in Fig. (a).



Since the capacitor is an open circuit to dc,

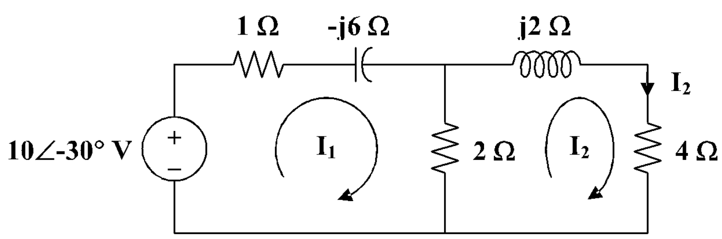
$$i_1 = \frac{24}{4+2} = 4 \text{ A}$$

For i_2, consider the circuit in Fig. (b).

$$\omega = 1$$

$$2\text{ H} \longrightarrow j\omega L = j2$$

$$\frac{1}{6}\text{F} \longrightarrow \frac{1}{j\omega C} = \text{-}j6$$



(b)

For mesh 1,

$$\text{-}10\angle\text{-}30° + (3 - j6)\mathbf{I}_1 - 2\mathbf{I}_2 = 0$$

$$10\angle\text{-}30° = 3(1 - 2j)\mathbf{I}_1 - 2\mathbf{I}_2 \qquad (1)$$

For mesh 2,

$$0 = \text{-}2\mathbf{I}_1 + (6 + j2)\mathbf{I}_2$$

$$\mathbf{I}_1 = (3 + j)\mathbf{I}_2 \qquad (2)$$

Substituting (2) into (1)

$$10\angle -30° = 13 - j15\mathbf{I}_2$$

$$\mathbf{I}_2 = 0.504\angle 19.1°$$

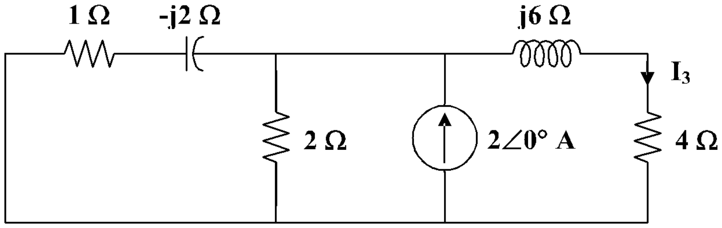
Hence, $$i_2 = 0.504\sin(t + 19.1°)\text{ A}$$

For i_3, consider the circuit in Fig. (c).

$$\omega = 3$$

$$2\text{ H} \longrightarrow j\omega L = j6$$

$$\frac{1}{6}\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(3)(1/6)} = -j2$$



(c)

$$2 \,||\, (1 - j2) = \frac{2(1 - j2)}{3 - j2}$$

Using current division,

$$\mathbf{I}_3 = \frac{\frac{2(1-j2)}{3-j2}\cdot(2\angle 0°)}{4 + j6 + \frac{2(1-j2)}{3-j2}} = \frac{2(1-j2)}{13 + j3}$$

$$\mathbf{I}_3 = 0.3352\angle -76.43°$$

Hence $$i_3 = 0.3352\cos(3t - 76.43°)\text{ A}$$

Therefore, $i_o =$ **<u>4 + 0.504 sin(t + 19.1°) + 0.3352 cos(3t – 76.43°) A</u>**

Chapter 10, Problem 48.

Find i_o in the circuit of Fig. 10.93 using superposition.

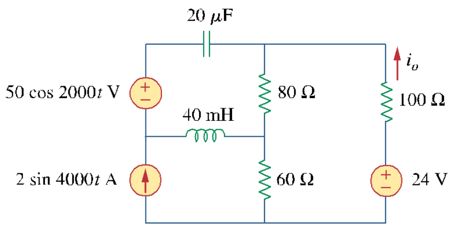


Figure 10.93
For Prob. 10.48.

Chapter 10, Solution 48.

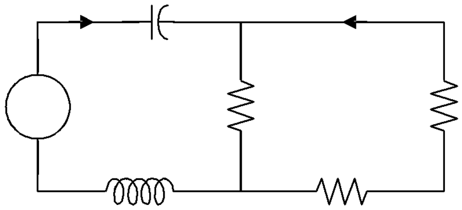
Let $i_O = i_{O1} + i_{O2} + i_{O3}$, where i_{O1} is due to the ac voltage source, i_{O2} is due to the dc voltage source, and i_{O3} is due to the ac current source. For i_{O1}, consider the circuit in Fig. (a).

$$\omega = 2000$$

$$50\cos(2000t) \longrightarrow 50\angle 0°$$

$$40 \text{ mH} \longrightarrow j\omega L = j(2000)(40\times 10^{-3}) = j80$$

$$20\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(2000)(20\times 10^{-6})} = \text{-}j25$$



$$80 \,||\, (60+100) = 160/3$$

$$\mathbf{I} = \frac{50}{160/3 + j80 - j25} = \frac{30}{32 + j33}$$

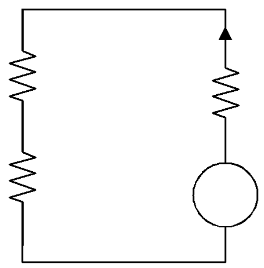
Using current division,

$$\mathbf{I}_{O1} = \frac{-80\mathbf{I}}{80+160} = \frac{-1}{3}\mathbf{I} = \frac{10\angle 180°}{46\angle 45.9°}$$

$$\mathbf{I}_{O1} = 0.217\angle 134.1°$$

Hence, $i_{O1} = 0.217\cos(2000t + 134.1°)$ A

For i_{O2}, consider the circuit in Fig. (b).



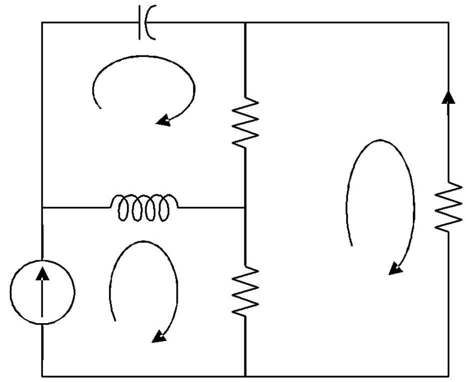
$$i_{O2} = \frac{24}{80+60+100} = 0.1 \text{ A}$$

For i_{O3}, consider the circuit in Fig. (c).

$\omega = 4000$

$2\cos(4000t) \longrightarrow 2\angle 0°$

$40 \text{ mH} \longrightarrow j\omega L = j(4000)(40\times 10^{-3}) = j160$



$$20\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(4000)(20\times 10^{-6})} = -j12.5$$

For mesh 1,

$$\mathbf{I}_1 = 2 \qquad (1)$$

For mesh 2,

$$(80 + j160 - j12.5)\mathbf{I}_2 - j160\mathbf{I}_1 - 80\mathbf{I}_3 = 0$$

Simplifying and substituting (1) into this equation yields

$$(8 + j14.75)\mathbf{I}_2 - 8\mathbf{I}_3 = j32 \qquad (2)$$

For mesh 3,

$$240\mathbf{I}_3 - 60\mathbf{I}_1 - 80\mathbf{I}_2 = 0$$

Simplifying and substituting (1) into this equation yields

$$\mathbf{I}_2 = 3\mathbf{I}_3 - 1.5 \qquad (3)$$

Substituting (3) into (2) yields

$$(16 + j44.25)\mathbf{I}_3 = 12 + j54.125$$

$$\mathbf{I}_3 = \frac{12 + j54.125}{16 + j44.25} = 1.1782\angle 7.38°$$

$$\mathbf{I}_{O3} = -\mathbf{I}_3 = -1.1782\angle 7.38°$$

Hence, $\quad i_{O3} = -1.1782\sin(4000t + 7.38°)\ \text{A}$

Therefore, $\quad i_O =$ **<u>0.1 + 0.217 cos(2000t + 134.1°) – 1.1782 sin(4000t + 7.38°) A</u>**

Chapter 10, Problem 49.

Using source transformation, find *i* in the circuit of Fig. 10.94.

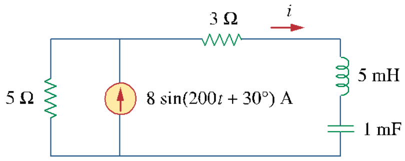


Figure 10.94
For Prob. 10.49.

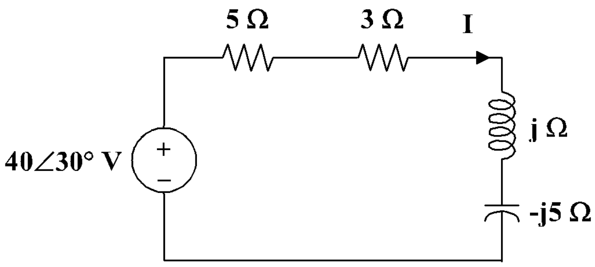
Chapter 10, Solution 49.

$$8\sin(200t+30°) \longrightarrow 8\angle 30°, \quad \omega = 200$$

$$5\text{ mH} \longrightarrow j\omega L = j(200)(5\times 10^{-3}) = j$$

$$1\text{ mF} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(200)(1\times 10^{-3})} = -j5$$

After transforming the current source, the circuit becomes that shown in the figure below.



$$\mathbf{I} = \frac{40\angle 30°}{5+3+j-j5} = \frac{40\angle 30°}{8-j4} = 4.472\angle 56.56°$$

i = **4.472 sin(200t + 56.56°) A**

Chapter 10, Problem 50.

Use source transformation to find v_o in the circuit of Fig. 10.95.

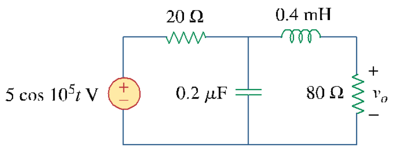


Figure 10.95
For Prob. 10.50.

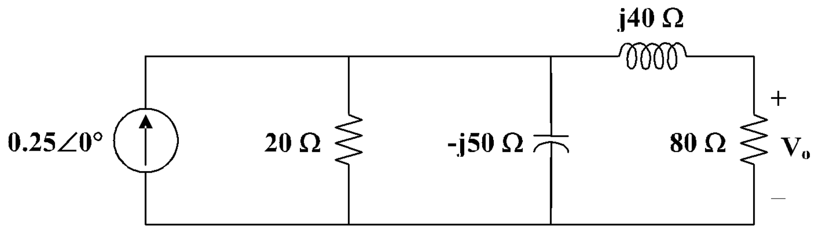
Chapter 10, Solution 50.

$$5\cos(10^5 t) \longrightarrow 5\angle 0°, \quad \omega = 10^5$$

$$0.4 \text{ mH} \longrightarrow j\omega L = j(10^5)(0.4\times 10^{-3}) = j40$$

$$0.2\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10^5)(0.2\times 10^{-6})} = -j50$$

After transforming the voltage source, we get the circuit in Fig. (a).

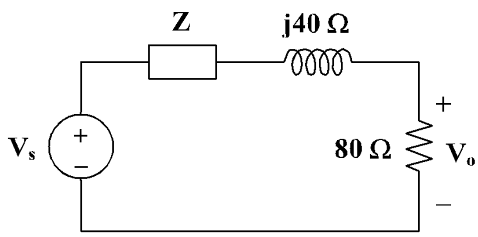


(a)

Let $\mathbf{Z} = 20 \,||\, -j50 = \dfrac{-j100}{2 - j5}$

and $\mathbf{V}_S = (0.25\angle 0°)\,\mathbf{Z} = \dfrac{-j25}{2 - j5}$

With these, the current source is transformed to obtain the circuit in Fig.(b).



(b)

By voltage division,

$$\mathbf{V}_o = \frac{80}{\mathbf{Z} + 80 + j40}\mathbf{V}_S = \frac{80}{\dfrac{-j100}{2 - j5} + 80 + j40} \cdot \frac{-j25}{2 - j5}$$

$$\mathbf{V}_o = \frac{8(-j25)}{36 - j42} = 3.615\angle -40.6°$$

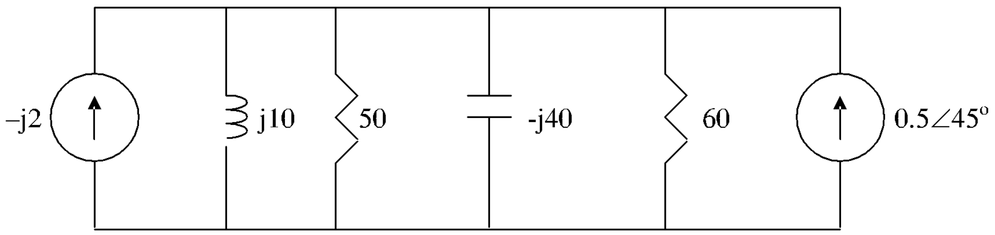
Therefore, $v_o =$ **$3.615 \cos(10^5 t - 40.6°)$ V**

Chapter 10, Problem 51.

Use source transformation to find $\mathbf{I}_o$ in the circuit of Prob. 10.42.

Chapter 10, Solution 51.

Transforming the voltage sources into current sources, we have the circuit as shown below.



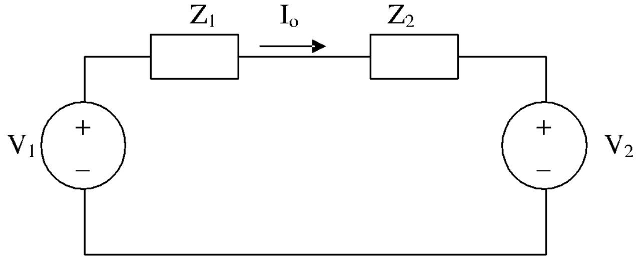
$$\text{Let} \quad Z_1 = j10 // 50 = \frac{j10x50}{50 + j10} = 1.9231 + j9.615$$

$$V_1 = -j2Z_1 = 19.231 - j3.846$$

$$\text{Let} \quad Z_2 = -j40 // 60 = \frac{-j40x60}{60 - j40} = 18.4615 - j27.6923$$

$$V_2 = Z_2 x0.5 < 45^o = 16.315 - 3.263$$

Transforming the current sources to voltage sources leads to the circuit below.



Applying KVL to the loop gives

$$-V_1 + I_o(Z_1 + Z_2) + V_2 = 0 \quad \longrightarrow \quad I_o = \frac{V_1 - V_2}{Z_1 + Z_2}$$

$$I_o = \frac{19.231 - j3.846 - 16.316 + j3.263}{1.9231 + j9.615 + 18.4615 - j27.6923} = \underline{0.1093 < 30^o \text{ A}} = \underline{\mathbf{109.3\angle 30^\circ \text{ mA}}}$$

Chapter 10, Problem 52.

Use the method of source transformation to find $\mathbf{I}_x$ in the circuit of Fig. 10.96.

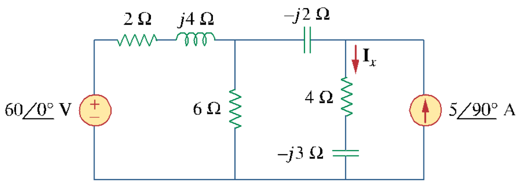


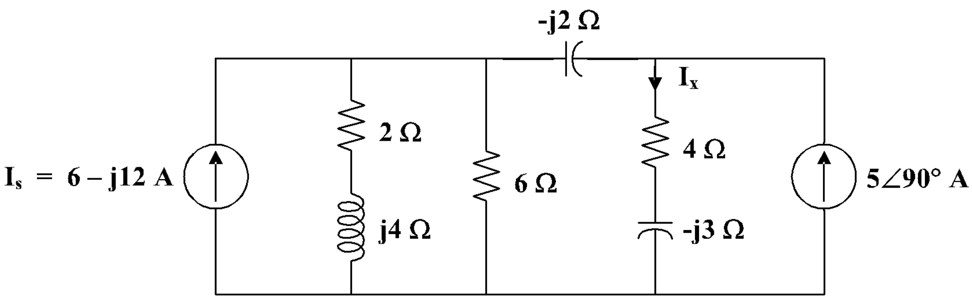
Figure 10.96
For Prob. 10.52.

Chapter 10, Solution 52.

We transform the voltage source to a current source.

$$\mathbf{I}_s = \frac{60\angle 0^\circ}{2 + j4} = 6 - j12$$

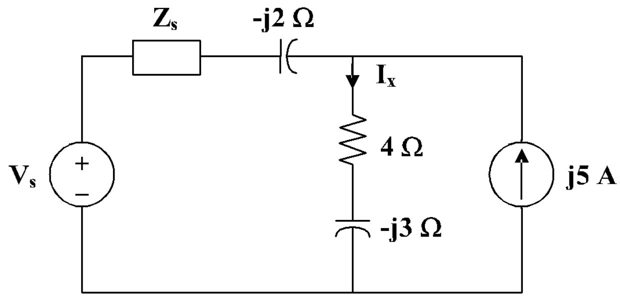
The new circuit is shown in Fig. (a).



(a)

Let
$$\mathbf{Z}_s = 6 \,||\, (2 + j4) = \frac{6(2 + j4)}{8 + j4} = 2.4 + j1.8$$
$$\mathbf{V}_s = \mathbf{I}_s\,\mathbf{Z}_s = (6 - j12)(2.4 + j1.8) = 36 - j18 = 18(2 - j)$$

With these, we transform the current source on the left hand side of the circuit to a voltage source. We obtain the circuit in Fig. (b).

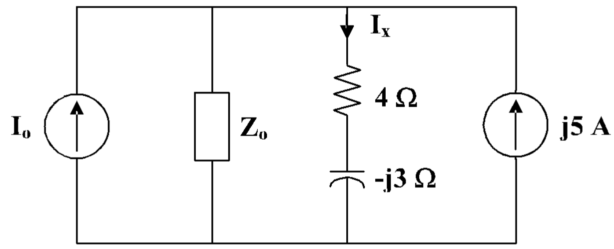


(b)

Let $\mathbf{Z}_o = \mathbf{Z}_s - j2 = 2.4 - j0.2 = 0.2(12 - j)$

$$\mathbf{I}_o = \frac{\mathbf{V}_s}{\mathbf{Z}_o} = \frac{18(2-j)}{0.2(12-j)} = 15.517 - j6.207$$

With these, we transform the voltage source in Fig. (b) to a current source. We obtain the circuit in Fig. (c).



(c)

Using current division,

$$\mathbf{I}_x = \frac{\mathbf{Z}_o}{\mathbf{Z}_o + 4 - j3}(\mathbf{I}_o + j5) = \frac{2.4 - j0.2}{6.4 - j3.2}(15.517 - j1.207)$$

$$\mathbf{I}_x = 5 + j1.5625 = \underline{\mathbf{5.238\angle 17.35° \text{ A}}}$$

Chapter 10, Problem 53.

Use the concept of source transformation to find $\mathbf{V}_o$ in the circuit of Fig. 10.97.

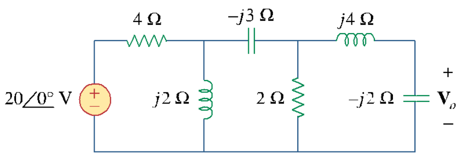
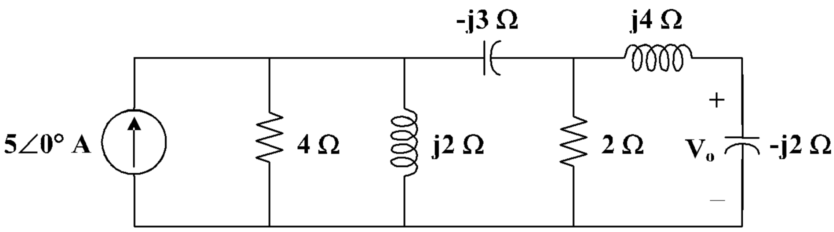


Figure 10.97
For Prob. 10.53.

Chapter 10, Solution 53.

We transform the voltage source to a current source to obtain the circuit in Fig. (a).

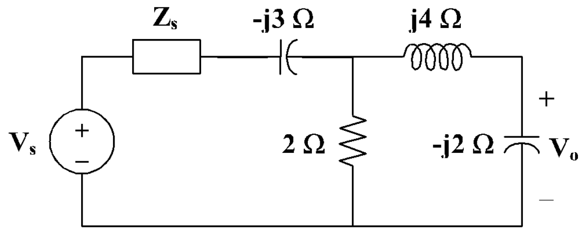


(a)

Let $$\mathbf{Z}_s = 4 \,||\, j2 = \frac{j8}{4 + j2} = 0.8 + j1.6$$

$$\mathbf{V}_s = (5\angle 0°)\,\mathbf{Z}_s = (5)(0.8 + j1.6) = 4 + j8$$

With these, the current source is transformed so that the circuit becomes that shown in Fig. (b).

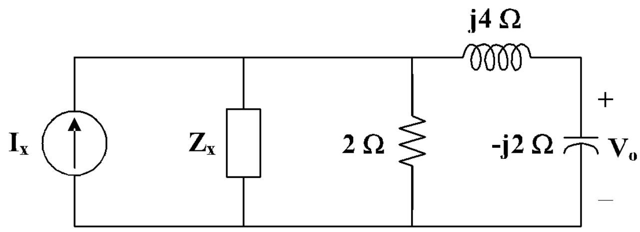


(b)

Let $\quad \mathbf{Z}_x = \mathbf{Z}_s - j3 = 0.8 - j1.4$

$$\mathbf{I}_x = \frac{\mathbf{V}_s}{\mathbf{Z}_s} = \frac{4 + j8}{0.8 - j1.4} = -3.0769 + j4.6154$$

With these, we transform the voltage source in Fig. (b) to obtain the circuit in Fig. (c).

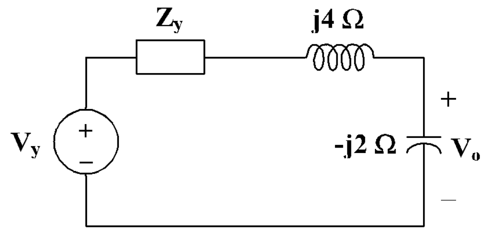


(c)

Let $\quad \mathbf{Z}_y = 2 \,||\, \mathbf{Z}_x = \frac{1.6 - j2.8}{2.8 - j1.4} = 0.8571 - j0.5714$

$$\mathbf{V}_y = \mathbf{I}_x \mathbf{Z}_y = (-3.0769 + j4.6154) \cdot (0.8571 - j0.5714) = j5.7143$$

With these, we transform the current source to obtain the circuit in Fig. (d).
Using current division,



(d)

$$\mathbf{V}_o = \frac{-j2}{\mathbf{Z}_y + j4 - j2}\mathbf{V}_y = \frac{-j2(j5.7143)}{0.8571 - j0.5714 + j4 - j2} = \underline{\mathbf{(3.529 - j5.883)\ V}}$$

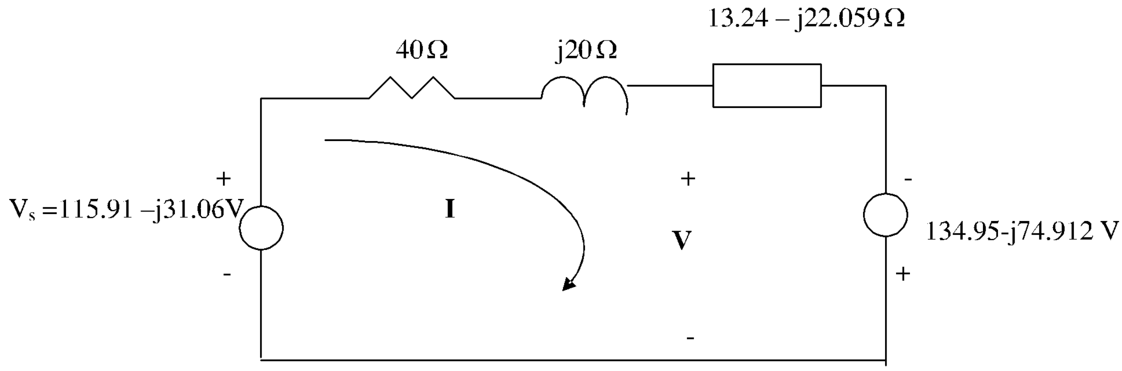
Chapter 10, Problem 54.

Rework Prob. 10.7 using source transformation.

Chapter 10, Solution 54.

$$50\,//(-j30) = \frac{50x(-j30)}{50-j30} = 13.24 - j22.059$$

We convert the current source to voltage source and obtain the circuit below.



Applying KVL gives

-115.91 + j31.058 + (53.24-j2.059)I -134.95 + j74.912 = 0

or $I = \frac{-250.86 + j105.97}{53.24 - j2.059} = -4.7817 + j1.8055$

But $-V_S + (40 + j20)I + V = 0 \quad \longrightarrow \quad V = V_S - (40 + j20)I$

$$V = 115.91 - j31.05 - (40 + j20)(-4.7817 + j1.8055) = \underline{124.06\angle -154^o\ V}$$

which agrees with the result in Prob. 10.7.

Chapter 10, Problem 55.

Find the Thevenin and Norton equivalent circuits at terminals *a-b* for each of the circuits in Fig. 10.98.

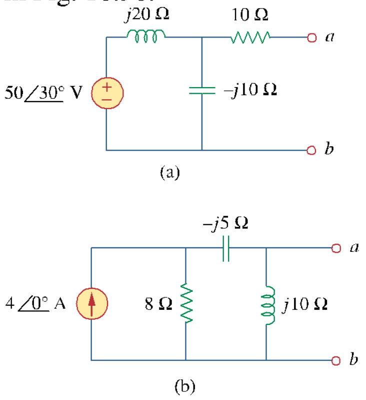
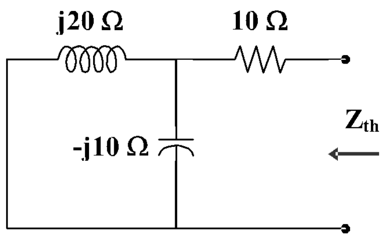


Figure 10.98
For Prob. 10.55.

Chapter 10, Solution 55.

(a) To find $\mathbf{Z}_{th}$, consider the circuit in Fig. (a).

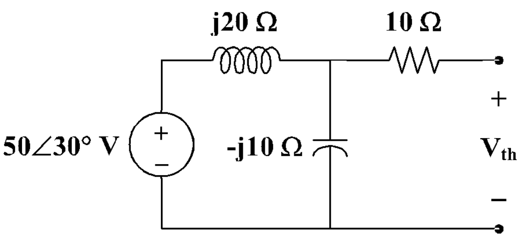


(a)

$$\mathbf{Z}_N = \mathbf{Z}_{th} = 10 + j20 \,||\, (-j10) = 10 + \frac{(j20)(-j10)}{j20 - j10}$$

$$= 10 - j20 = \underline{\mathbf{22.36\angle{-63.43°}\ \Omega}}$$

To find $\mathbf{V}_{th}$, consider the circuit in Fig. (b).

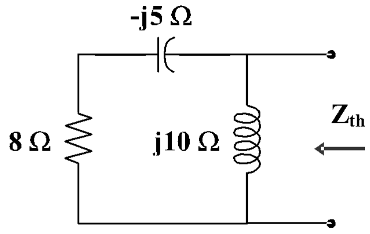


(b)

$$\mathbf{V}_{th} = \frac{-j10}{j20 - j10}(50\angle 30°) = \underline{\mathbf{-50\angle 30°\ V}}$$

$$\mathbf{I}_N = \frac{\mathbf{V}_{th}}{\mathbf{Z}_{th}} = \frac{-50\angle 30°}{22.36\angle -63.43°} = \underline{\mathbf{2.236\angle 273.4°\ A}}$$

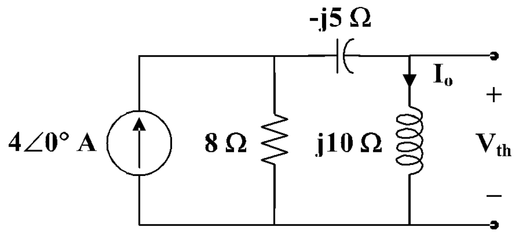
(b) To find $\mathbf{Z}_{th}$, consider the circuit in Fig. (c).



(c)

$$\mathbf{Z}_N = \mathbf{Z}_{th} = j10 \parallel (8 - j5) = \frac{(j10)(8 - j5)}{j10 + 8 - j5} = \underline{\mathbf{10\angle 26°\ \Omega}}$$

To obtain $\mathbf{V}_{th}$, consider the circuit in Fig. (d).



(d)

By current division,

$$\mathbf{I}_o = \frac{8}{8 + j10 - j5}(4\angle 0°) = \frac{32}{8 + j5}$$

$$\mathbf{V}_{th} = j10\,\mathbf{I}_o = \frac{j320}{8 + j5} = \underline{\mathbf{33.92\angle 58°\ V}}$$

$$\mathbf{I}_N = \frac{\mathbf{V}_{th}}{\mathbf{Z}_{th}} = \frac{33.92\angle 58°}{10\angle 26°} = \underline{\mathbf{3.392\angle 32°\ A}}$$

Chapter 10, Problem 56.

For each of the circuits in Fig. 10.99, obtain Thevenin and Norton equivalent circuits at terminals *a-b*.

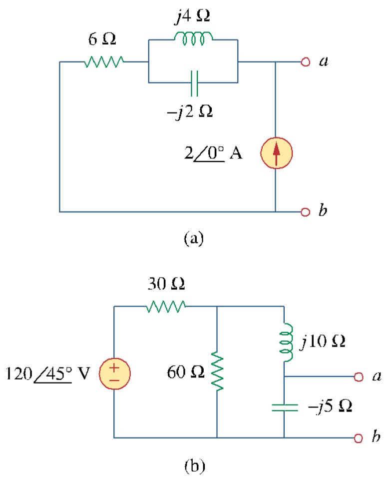
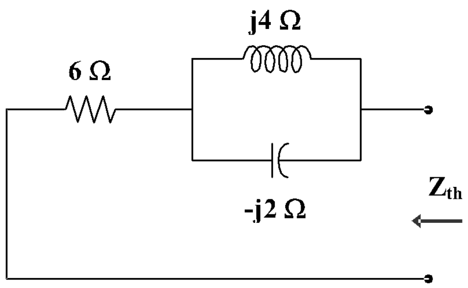


Figure 10.99

For Prob. 10.56.

Chapter 10, Solution 56.

(a) To find $\mathbf{Z}_{th}$, consider the circuit in Fig. (a).



(a)

$$\mathbf{Z}_N = \mathbf{Z}_{th} = 6 + j4 \,||\, (-j2) = 6 + \frac{(j4)(-j2)}{j4 - j2} = 6 - j4$$

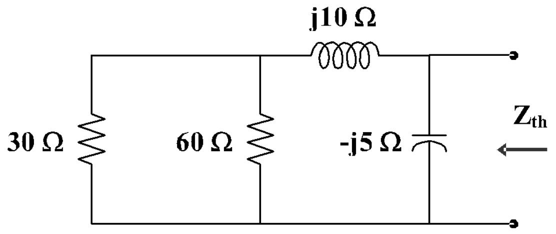
$$= \underline{\mathbf{7.211\angle{-33.69°}\ \Omega}}$$

By placing short circuit at terminals a-b, we obtain,

$\mathbf{I}_N = \underline{\mathbf{2\angle 0°\ A}}$

$$\mathbf{V}_{th} = \mathbf{Z}_{th}\,\mathbf{I}_{th} = (7.211\angle{-33.69°})(2\angle 0°) = \underline{\mathbf{14.422\angle{-33.69°}\ V}}$$

(b) To find $\mathbf{Z}_{th}$, consider the circuit in Fig. (b).



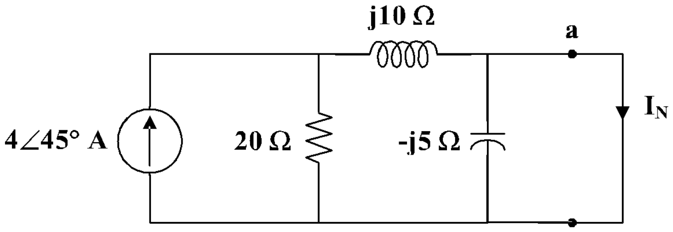
(b)

$$30 \parallel 60 = 20$$

$$\mathbf{Z}_N = \mathbf{Z}_{th} = \text{-j5} \parallel (20 + \text{j10}) = \frac{(\text{-j5})(20 + \text{j10})}{20 + \text{j5}}$$

$$= \underline{\mathbf{5.423\angle\text{-}77.47° \ \Omega}}$$

To find $\mathbf{V}_{th}$ and $\mathbf{I}_N$, we transform the voltage source and combine the 30 Ω and 60 Ω resistors. The result is shown in Fig. (c).



(c)

$$\mathbf{I}_N = \frac{20}{20 + \text{j10}}(4\angle 45°) = \frac{2}{5}(2 - \text{j})(4\angle 45°)$$

$$= \underline{\mathbf{3.578\angle 18.43° \ A}}$$

$$\mathbf{V}_{th} = \mathbf{Z}_{th}\,\mathbf{I}_N = (5.423\angle\text{-}77.47°)(3.578\angle 18.43°)$$

$$= \underline{\mathbf{19.4\angle\text{-}59° \ V}}$$

Chapter 10, Problem 57.

Find the Thevenin and Norton equivalent circuits for the circuit shown in Fig. 10.100.

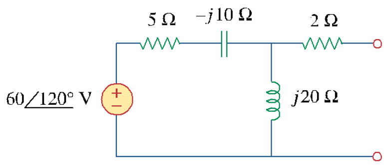
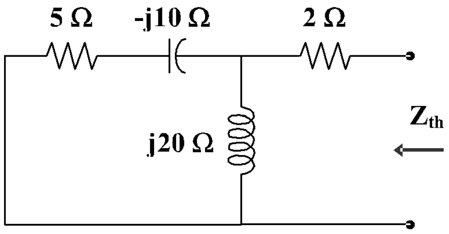


Figure 10.100
For Prob. 10.57.

Chapter 10, Solution 57.

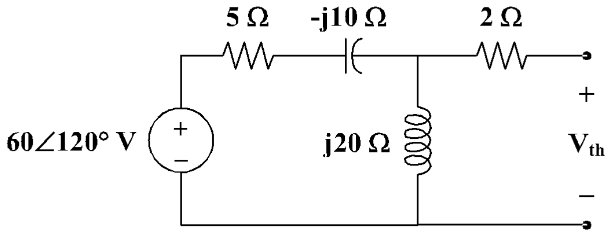
To find $\mathbf{Z}_{th}$, consider the circuit in Fig. (a).



(a)

$$\mathbf{Z}_N = \mathbf{Z}_{th} = 2 + j20 \,||\, (5 - j10) = 2 + \frac{(j20)(5 - j10)}{5 + j10}$$
$$= 18 - j12 = \underline{\mathbf{21.63\angle\text{-}33.7^\circ\ \Omega}}$$

To find $\mathbf{V}_{th}$, consider the circuit in Fig. (b).



(b)

$$\mathbf{V}_{th} = \frac{j20}{5 - j10 + j20}(60\angle 120^\circ) = \frac{j4}{1 + j2}(60\angle 120^\circ)$$
$$= \underline{\mathbf{107.3\angle 146.56^\circ\ V}}$$

$$\mathbf{I}_N = \frac{\mathbf{V}_{th}}{\mathbf{Z}_{th}} = \frac{107.3\angle 146.56^\circ}{21.633\angle\text{-}33.7^\circ} = \underline{\mathbf{4.961\angle\text{-}179.7^\circ\ A}}$$

Chapter 10, Problem 58.

For the circuit depicted in Fig. 10.101, find the Thevenin equivalent circuit at terminals *a-b*.

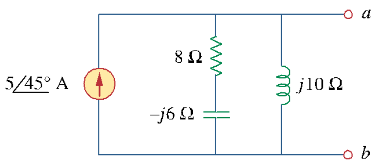
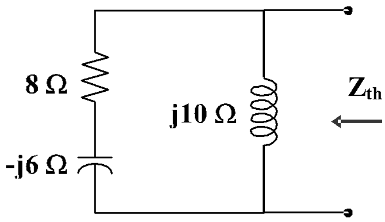


Figure 10.101

For Prob. 10.58.

Chapter 10, Solution 58.

Consider the circuit in Fig. (a) to find $\mathbf{Z}_{th}$.

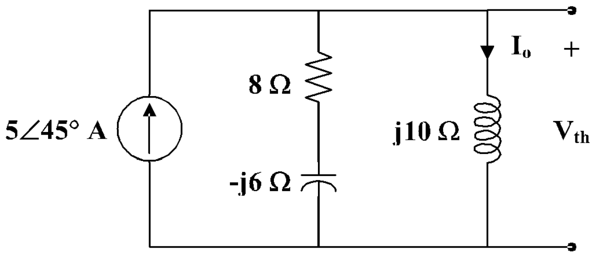


(a)

$$\mathbf{Z}_{th} = j10 \parallel (8 - j6) = \frac{(j10)(8 - j6)}{8 + j4} = 5(2 + j)$$

$$= \underline{\mathbf{11.18\angle 26.56^\circ\ \Omega}}$$

Consider the circuit in Fig. (b) to find $\mathbf{V}_{th}$.



(b)

$$\mathbf{I}_o = \frac{8 - j6}{8 - j6 + j10}(5\angle 45^\circ) = \frac{4 - j3}{4 + j2}(5\angle 45^\circ)$$

$$\mathbf{V}_{th} = j10\mathbf{I}_o = \frac{(j10)(4 - j3)(5\angle 45^\circ)}{(2)(2 + j)} = \underline{\mathbf{55.9\angle 71.56^\circ\ V}}$$

Chapter 10, Problem 59.

Calculate the output impedance of the circuit shown in Fig. 10.102.

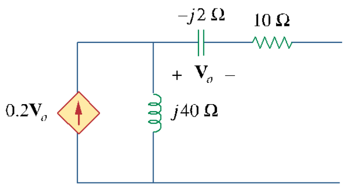
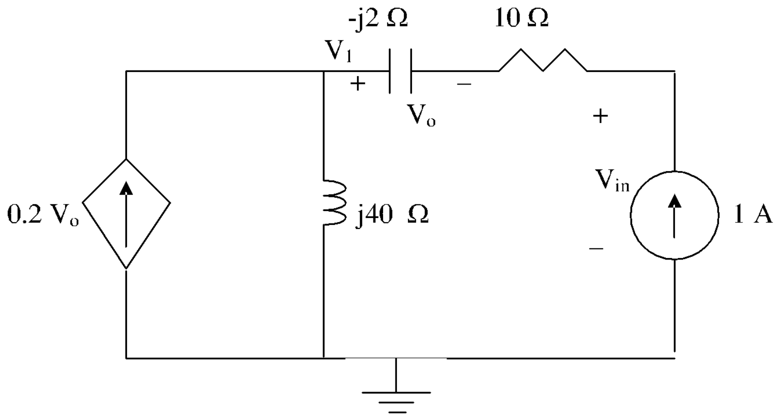


Figure 10.102
For Prob. 10.59.

Chapter 10, Solution 59.

Insert a 1-A current source at the output as shown below.



$$0.2V_o + 1 = \frac{V_1}{j40}$$

But $V_o = -1(-j2) = j2$

$$j2x0.2 + 1 = \frac{V_1}{j40} \longrightarrow V_1 = -16 + j40$$

$V_{in} = V_1 - V_o + 10 = -6 + j38 = 1xZ_{in}$

Z_{in} = **<u>–6 + j38 Ω</u>**.

Chapter 10, Problem 60.

Find the Thevenin equivalent of the circuit in Fig. 10.103 as seen from:
(a) terminals *a*-*b*   (b) terminals *c*-*d*

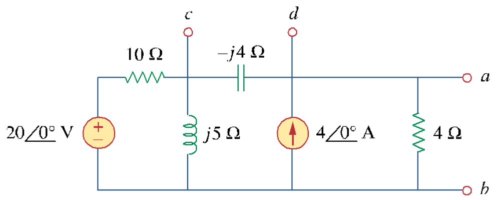
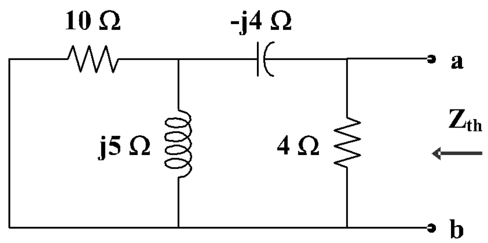


Figure 10.103
For Prob. 10.60.

Chapter 10, Solution 60.

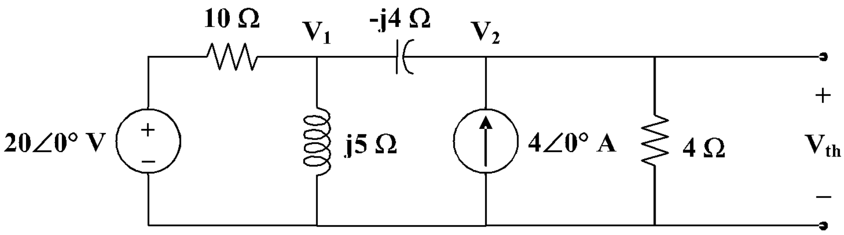
(a) To find $\mathbf{Z}_{th}$, consider the circuit in Fig. (a).



(a)

$$\mathbf{Z}_{th} = 4 \,||\, (-j4 + 10 \,||\, j5) = 4 \,||\, (-j4 + 2 + j4)$$
$$\mathbf{Z}_{th} = 4 \,||\, 2 = \underline{\mathbf{1.333\ \Omega}}$$

To find $\mathbf{V}_{th}$, consider the circuit in Fig. (b).



(b)

At node 1,

$$\frac{20 - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{j5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j4}$$

$$(1 + j0.5)\,\mathbf{V}_1 - j2.5\,\mathbf{V}_2 = 20 \qquad (1)$$

At node 2,

$$4 + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j4} = \frac{\mathbf{V}_2}{4}$$

$$\mathbf{V}_1 = (1 - j)\,\mathbf{V}_2 + j16 \qquad (2)$$

Substituting (2) into (1) leads to

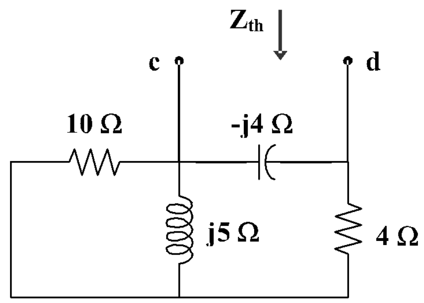
$$28 - j16 = (1.5 - j3)\,\mathbf{V}_2$$

$$\mathbf{V}_2 = \frac{28 - j16}{1.5 - j3} = 8 + j5.333$$

Therefore,

$$\mathbf{V}_{th} = \mathbf{V}_2 = \underline{\mathbf{9.615\angle 33.69^\circ\ V}}$$

(b) To find $\mathbf{Z}_{th}$, consider the circuit in Fig. (c).



(c)

$$\mathbf{Z}_{th} = -j4 \,||\, (4 + 10 \,||\, j5) = -j4 \,||\left(4 + \frac{j10}{2 + j}\right)$$

$$\mathbf{Z}_{th} = -j4 \,||\, (6 + j4) = \frac{-j4}{6}(6 + j4) = \underline{\mathbf{2.667 - j4\ \Omega}}$$

To find $\mathbf{V}_{th}$, we will make use of the result in part (a).

$$\mathbf{V}_2 = 8 + j5.333 = (8/3)(3 + j2)$$

$$\mathbf{V}_1 = (1 - j)\,\mathbf{V}_2 + j16 = j16 + (8/3)(5 - j)$$

$$\mathbf{V}_{th} = \mathbf{V}_1 - \mathbf{V}_2 = 16/3 + j8 = \underline{\mathbf{9.614\angle 56.31^\circ\ V}}$$

Chapter 10, Problem 61.

Find the Thevenin equivalent at terminals *a*-*b* of the circuit in Fig. 10.104.

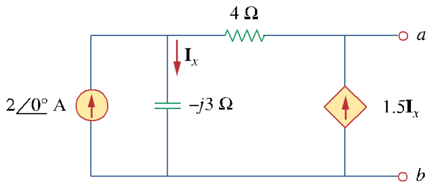
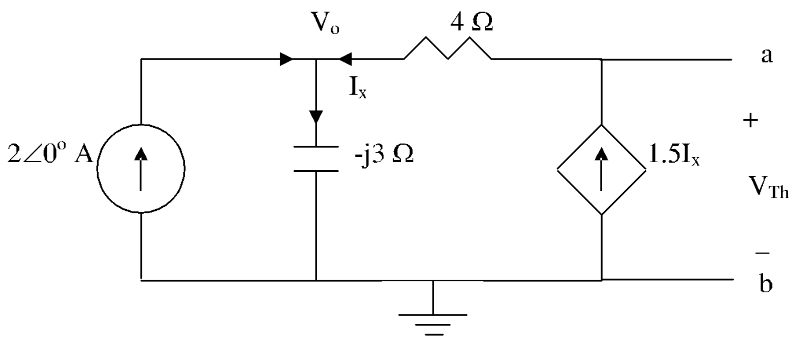


Figure 10.104
For Prob. 10.61.

Chapter 10, Solution 61.

To find V_{Th}, consider the circuit below

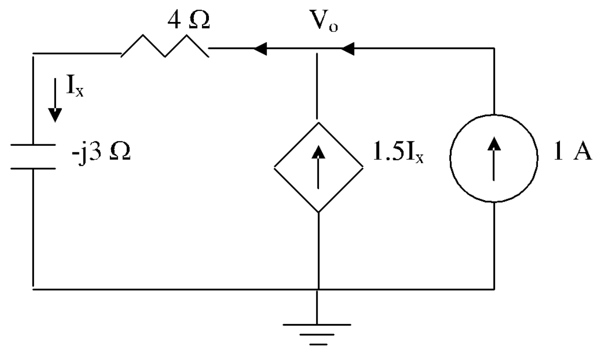


$2 + 1.5I_x = I_x \longrightarrow I_x = -4$

But $\quad V_o = -j3I_x = j12$

$V_{Th} = V_o + 6I_x = \underline{j12 - 24 \text{ V}}$

To find Z_{Th}, consider the circuit shown below.



$1+1.5\, I_x = I_x \Rightarrow \quad I_x = -2$

$-V_o + I_x(4 - j3) = 0 \longrightarrow V_o = -8 + j6$

$$Z_{Th} = \frac{V_o}{1} = \underline{-8 + j6\ \Omega}$$

Chapter 10, Problem 62.

ps

Using Thevenin's theorem, find v_o in the circuit of Fig. 10.105.

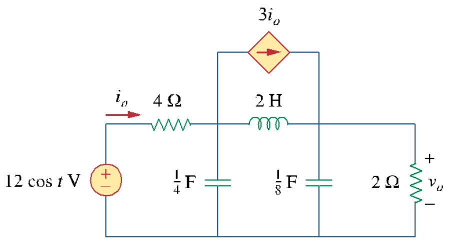


Figure 10.105
For Prob. 10.62.

Chapter 10, Solution 62.

First, we transform the circuit to the frequency domain.

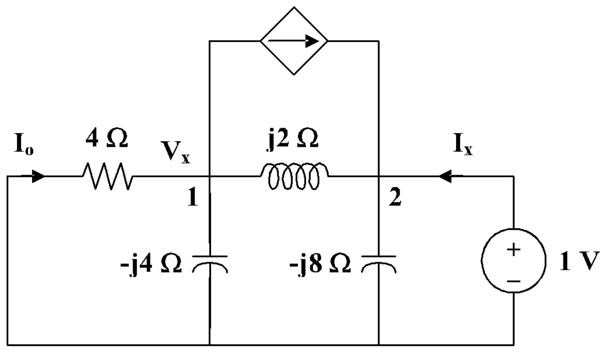
$$12\cos(t) \longrightarrow 12\angle 0°, \quad \omega = 1$$

$$2\text{ H} \longrightarrow j\omega L = j2$$

$$\frac{1}{4}\text{ F} \longrightarrow \frac{1}{j\omega C} = \text{-}j4$$

$$\frac{1}{8}\text{ F} \longrightarrow \frac{1}{j\omega C} = \text{-}j8$$

To find $\mathbf{Z}_{th}$, consider the circuit in Fig. (a).



(a)

At node 1,

$$\frac{\mathbf{V}_x}{4} + \frac{\mathbf{V}_x}{\text{-}j4} + 3\mathbf{I}_o = \frac{1 - \mathbf{V}_x}{j2}, \qquad \text{where } \mathbf{I}_o = \frac{\text{-}\mathbf{V}_x}{4}$$

Thus,
$$\frac{\mathbf{V}_x}{\text{-}j4} - \frac{2\mathbf{V}_x}{4} = \frac{1 - \mathbf{V}_x}{j2}$$

$$\mathbf{V}_x = 0.4 + j0.8$$

At node 2,

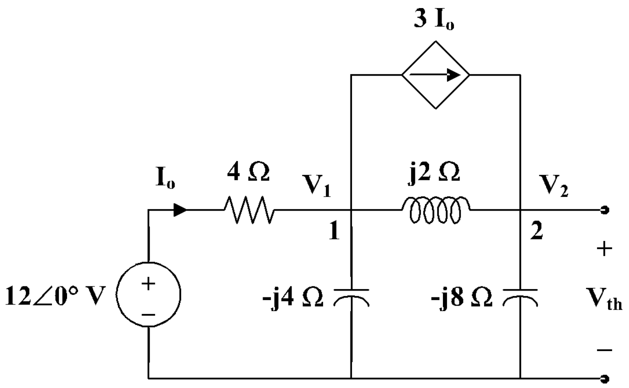
$$\mathbf{I}_x + 3\mathbf{I}_o = \frac{1}{\text{-}j8} + \frac{1 - \mathbf{V}_x}{j2}$$

$$\mathbf{I}_x = (0.75 + j0.5)\mathbf{V}_x - j\frac{3}{8}$$

$$\mathbf{I}_x = \text{-}0.1 + j0.425$$

$$\mathbf{Z}_{th} = \frac{1}{\mathbf{I}_x} = \text{-}0.5246 - j2.229 = 2.29\angle\text{-}103.24°\ \Omega$$

To find $\mathbf{V}_{th}$, consider the circuit in Fig. (b).



(b)

At node 1,

$$\frac{12-\mathbf{V}_1}{4} = 3\mathbf{I}_o + \frac{\mathbf{V}_1}{-j4} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j2}, \qquad \text{where } \mathbf{I}_o = \frac{12-\mathbf{V}_1}{4}$$

$$24 = (2+j)\mathbf{V}_1 - j2\mathbf{V}_2 \qquad (1)$$

At node 2,

$$\frac{\mathbf{V}_1 - \mathbf{V}_2}{j2} + 3\mathbf{I}_o = \frac{\mathbf{V}_2}{-j8}$$

$$72 = (6+j4)\mathbf{V}_1 - j3\mathbf{V}_2 \qquad (2)$$

From (1) and (2),

$$\begin{bmatrix} 24 \\ 72 \end{bmatrix} = \begin{bmatrix} 2+j & -j2 \\ 6+j4 & -j3 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

$$\Delta = -5 + j6, \qquad \Delta_2 = -j24$$

$$\mathbf{V}_{th} = \mathbf{V}_2 = \frac{\Delta_2}{\Delta} = 3.073\angle -219.8°$$

Thus,

$$\mathbf{V}_o = \frac{2}{2+\mathbf{Z}_{th}}\mathbf{V}_{th} = \frac{(2)(3.073\angle -219.8°)}{1.4754 - j2.229}$$

$$\mathbf{V}_o = \frac{6.146\angle -219.8°}{2.673\angle -56.5°} = 2.3\angle -163.3°$$

Therefore, $v_o =$ **2.3 cos(t – 163.3°) V**

Chapter 10, Problem 63.

Obtain the Norton equivalent of the circuit depicted in Fig. 10.106 at terminals *a*-*b*.

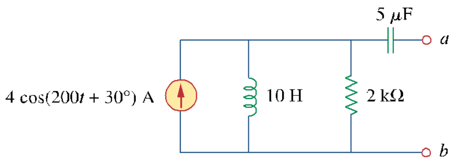


Figure 10.106
For Prob. 10.63.

Chapter 10, Solution 63.

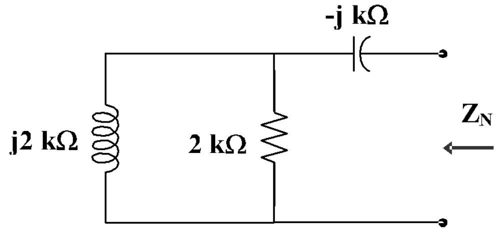
Transform the circuit to the frequency domain.

$$4\cos(200t + 30°) \longrightarrow 4\angle 30°, \quad \omega = 200$$

$$10\text{ H} \longrightarrow j\omega L = j(200)(10) = j2\text{ k}\Omega$$

$$5\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(200)(5\times 10^{-6})} = -j\text{ k}\Omega$$

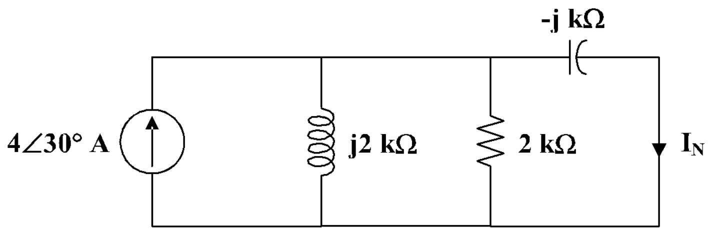
$\mathbf{Z}_N$ is found using the circuit in Fig. (a).



(a)

$$\mathbf{Z}_N = -j + 2 \,||\, j2 = -j + 1 + j = 1\text{ k}\Omega$$

We find $\mathbf{I}_N$ using the circuit in Fig. (b).



(b)

$$j2 \,||\, 2 = 1 + j$$

By the current division principle,

$$\mathbf{I}_N = \frac{1+j}{1+j-j}(4\angle 30°) = 5.657\angle 75°$$

Therefore,

i_N = <u>**5.657 cos(200t + 75°) A**</u>

$\mathbf{Z}_N$ = <u>**1 kΩ**</u>

Chapter 10, Problem 64.

ps

For the circuit shown in Fig. 10.107, find the Norton equivalent circuit at terminals *a-b*.

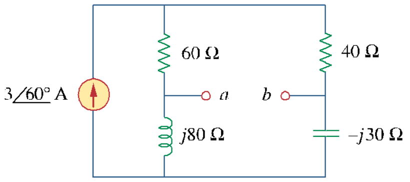
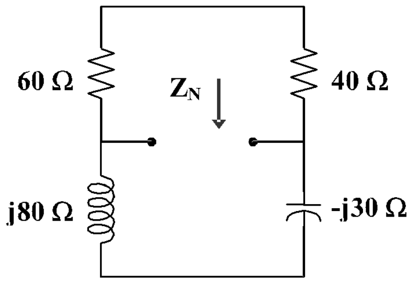


Figure 10.107
For Prob. 10.64.

Chapter 10, Solution 64.

$\mathbf{Z}_N$ is obtained from the circuit in Fig. (a).

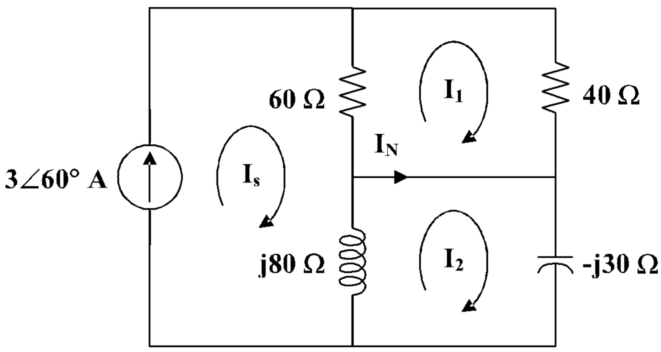


(a)

$$\mathbf{Z}_N = (60 + 40) \,||\, (j80 - j30) = 100 \,||\, j50 = \frac{(100)(j50)}{100 + j50}$$

$$\mathbf{Z}_N = 20 + j40 = \underline{\mathbf{44.72\angle 63.43° \ \Omega}}$$

To find $\mathbf{I}_N$, consider the circuit in Fig. (b).



(b)

$$\mathbf{I}_s = 3\angle 60°$$

For mesh 1,

$$100\mathbf{I}_1 - 60\mathbf{I}_s = 0$$

$$\mathbf{I}_1 = 1.8\angle 60°$$

For mesh 2,

$$(j80 - j30)\mathbf{I}_2 - j80\mathbf{I}_s = 0$$

$$\mathbf{I}_2 = 4.8\angle 60°$$

$I_N = I_2 - I_1 = \underline{\mathbf{3\angle 60° \ A}}$

Chapter 10, Problem 65.

Compute i_o in Fig. 10.108 using Norton's theorem.

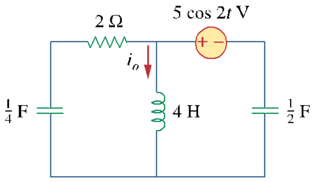


Figure 10.108
For Prob. 10.65.

Chapter 10, Solution 65.

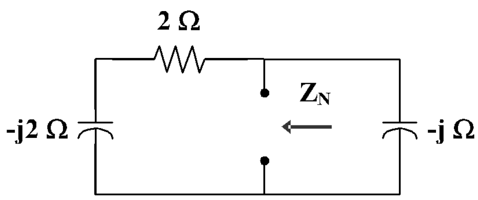
$$5\cos(2t) \longrightarrow 5\angle 0°, \quad \omega = 2$$

$$4\text{ H} \longrightarrow j\omega L = j(2)(4) = j8$$

$$\frac{1}{4}\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(2)(1/4)} = -j2$$

$$\frac{1}{2}\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(2)(1/2)} = -j$$

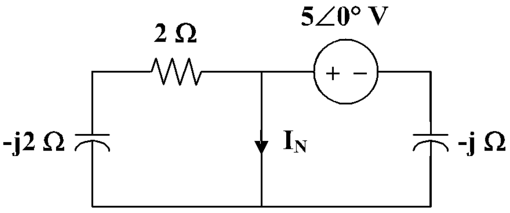
To find $\mathbf{Z}_N$, consider the circuit in Fig. (a).



(a)

$$\mathbf{Z}_N = -j \parallel (2 - j2) = \frac{-j(2-j2)}{2-j3} = \frac{1}{13}(2 - j10)$$

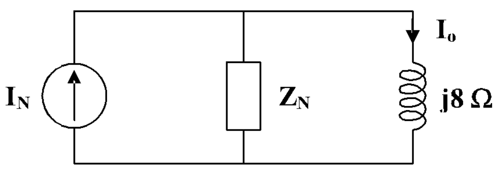
To find $\mathbf{I}_N$, consider the circuit in Fig. (b).



(b)

$$\mathbf{I}_N = \frac{5\angle 0°}{-j} = j5$$

The Norton equivalent of the circuit is shown in Fig. (c).



(c)

Using current division,

$$\mathbf{I}_o = \frac{\mathbf{Z}_N}{\mathbf{Z}_N + j8}\mathbf{I}_N = \frac{(1/13)(2 - j10)(j5)}{(1/13)(2 - j10) + j8} = \frac{50 + j10}{2 + j94}$$

$$\mathbf{I}_o = 0.1176 - j0.5294 = 0542\angle -77.47°$$

Therefore, i_o = **<u>542 cos(2t – 77.47°) mA</u>**

Chapter 10, Problem 66.

ps

At terminals *a-b*, obtain Thevenin and Norton equivalent circuits for the network depicted in Fig. 10.109. Take $\omega = 10$ rad/s.

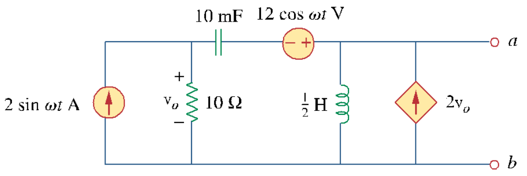


Figure 10.109
For Prob. 10.66.

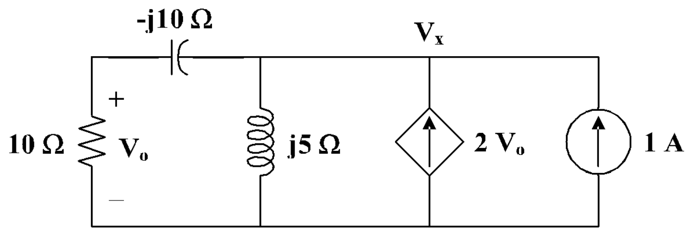
Chapter 10, Solution 66.

$$\omega = 10$$

$$0.5\text{ H} \longrightarrow j\omega L = j(10)(0.5) = j5$$

$$10\text{ mF} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10)(10\times 10^{-3})} = -j10$$

To find $\mathbf{Z}_{th}$, consider the circuit in Fig. (a).



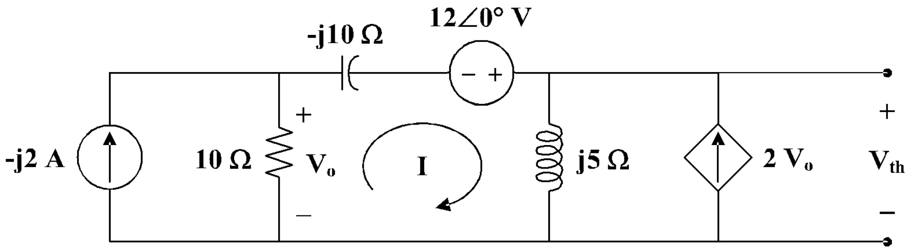
(a)

$$1 + 2\mathbf{V}_o = \frac{\mathbf{V}_x}{j5} + \frac{\mathbf{V}_x}{10 - j10}, \qquad \text{where} \quad \mathbf{V}_o = \frac{10\mathbf{V}_x}{10 - j10}$$

$$1 + \frac{19\mathbf{V}_x}{10 - j10} = \frac{\mathbf{V}_x}{j5} \longrightarrow \mathbf{V}_x = \frac{-10 + j10}{21 + j2}$$

$$\mathbf{Z}_N = \mathbf{Z}_{th} = \frac{\mathbf{V}_x}{1} = \frac{14.142\angle 135°}{21.095\angle 5.44°} = \underline{\mathbf{0.67\angle 129.56°\ \Omega}}$$

To find $\mathbf{V}_{th}$ and $\mathbf{I}_N$, consider the circuit in Fig. (b).



(b)

$$(10 - j10 + j5)\mathbf{I} - (10)(\text{-}j2) + j5(2\mathbf{V}_o) - 12 = 0$$

where $$\mathbf{V}_o = (10)(\text{-}j2 - \mathbf{I})$$

Thus,

$$(10 - j105)\mathbf{I} = \text{-}188 - j20$$

$$\mathbf{I} = \frac{188 + j20}{\text{-}10 + j105}$$

$$\mathbf{V}_{th} = j5(\mathbf{I} + 2\mathbf{V}_o) = j5(-19\mathbf{I} - j40) = -j95\mathbf{I} + 200$$

$$\mathbf{V}_{th} = \frac{-j95(188 + j20)}{\text{-}10 + j105} + 200 = 29.73 + j1.8723$$

$$\mathbf{V}_{th} = \underline{\mathbf{29.79\angle 3.6^\circ\ V}}$$

$$\mathbf{I}_N = \frac{\mathbf{V}_{th}}{\mathbf{Z}_{th}} = \frac{29.79\angle 3.6^\circ}{0.67\angle 129.56^\circ} = \underline{\mathbf{44.46\angle -125.96^\circ\ A}}$$

Chapter 10, Problem 67.

Find the Thevenin and Norton equivalent circuits at terminals *a*-*b* in the circuit of Fig. 10.110.

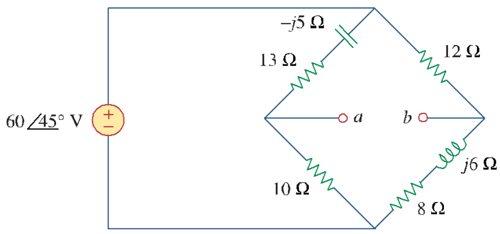


Figure 10.110
For Prob. 10.67.

Chapter 10, Solution 67.

$$Z_N = Z_{Th} = 10//(13 - j5) + 12//(8 + j6) = \frac{10(13 - j5)}{23 - j5} + \frac{12(8 + j6)}{20 + j6} = \underline{11.243 + j1.079\Omega}$$

$$V_a = \frac{10}{23 - j5}(60\angle 45^o) = 13.78 + j21.44, \qquad V_b = \frac{(8 + j6)}{20 + j6}(60\angle 45^o) = 12.069 + j26.08\Omega$$

$$V_{Th} = V_a - V_b = \underline{1.711 - j4.64 = 4.945\angle -69.76^o} \text{ V},$$

$$I_N = \frac{V_{Th}}{Z_{Th}} = \frac{4.945\angle -69.76°}{11.295\angle 5.48°} = \underline{0.4378\angle -75.24^o} \text{ A}$$

Chapter 10, Problem 68.

Find the Thevenin equivalent at terminals *a-b* in the circuit of Fig. 10.111.

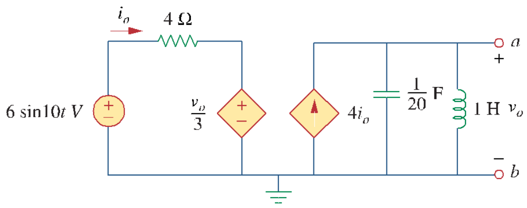


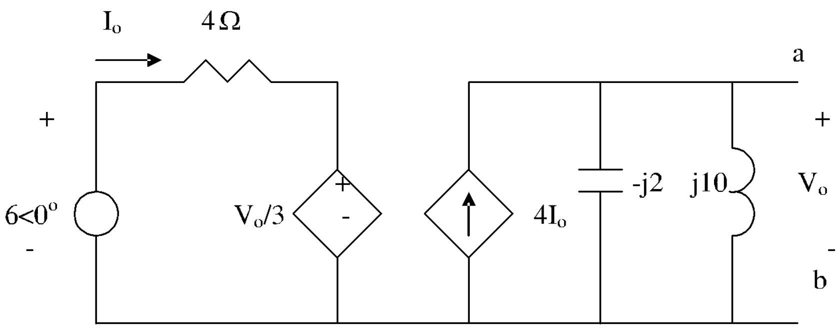
Figure 10.111
For Prob. 10.68.

Chapter 10, Solution 68.

$$1\text{H} \longrightarrow j\omega L = j10x1 = j10$$

$$\frac{1}{20}\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j10x\frac{1}{20}} = -j2$$

We obtain V_{Th} using the circuit below.



$$j10//(-j2) = \frac{j10(-j2)}{j10 - j2} = -j2.5$$

$$V_o = 4I_o x(-j2.5) = -j10I_o \qquad (1)$$

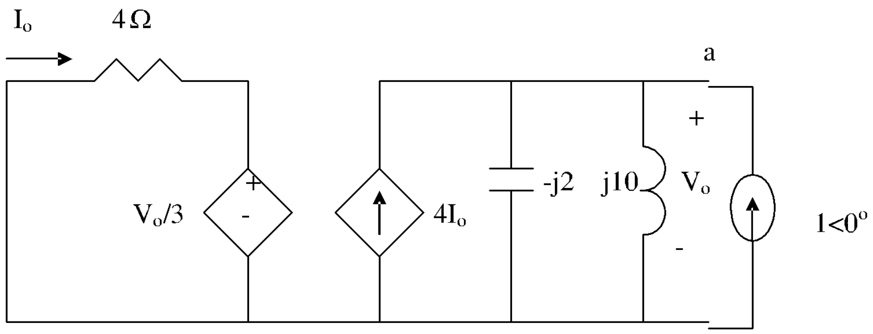
$$-6 + 4I_o + \frac{1}{3}V_o = 0 \qquad (2)$$

Combining (1) and (2) gives

$$I_o = \frac{6}{4 - j10/3}, \qquad V_{Th} = V_o = -j10 I_o = \frac{-j60}{4 - j10/3} = 11.52\angle -50.19^o$$

$$\underline{v_{Th} = 11.52\sin(10t - 50.19^o)}$$

To find $R_{Th,}$ we insert a 1-A source at terminals a-b, as shown below.



$$4I_o + \frac{1}{3}V_o = 0 \qquad \longrightarrow \qquad I_o = -\frac{V_o}{12}$$

$$1 + 4I_o = \frac{V_o}{-j2} + \frac{V_o}{j10}$$

Combining the two equations leads to

$$V_o = \frac{1}{0.333 + j0.4} = 1.2293 - j1.4766$$

$$Z_{Th} = \frac{V_o}{1} = \underline{1.2293 - 1.477\Omega}$$

Chapter 10, Problem 69.

For the differentiator shown in Fig. 10.112, obtain $\mathbf{V}_o/\mathbf{V}_s$. Find $v_o(t)$ when $v_s(t) = \mathbf{V}_m \sin \omega t$ and $\omega = 1/RC$.

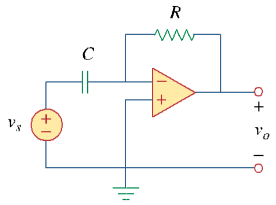


Figure 10.112
For Prob. 10.69.

Chapter 10, Solution 69.

This is an inverting op amp so that

$$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{-\mathbf{Z}_f}{\mathbf{Z}_i} = \frac{-R}{1/j\omega C} = \underline{\mathbf{-j\omega RC}}$$

When $\mathbf{V}_s = V_m$ and $\omega = 1/RC$,

$$\mathbf{V}_o = -j \cdot \frac{1}{RC} \cdot RC \cdot V_m = -jV_m = V_m \angle -90°$$

Therefore,

$$v_o(t) = V_m \sin(\omega t - 90°) = \underline{\mathbf{-\ V_m \cos(\omega t)}}$$

Chapter 10, Problem 70.

The circuit in Fig. 10.113 is an integrator with a feedback resistor. Calculate $v_o(t)$ if $v_s = 2\cos 4\times10^4 t$ V.

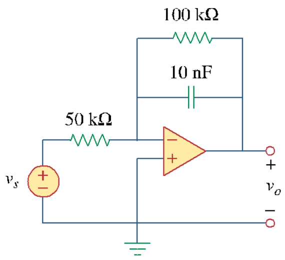


Figure 10.113
For Prob. 10.70.

Chapter 10, Solution 70.

This may also be regarded as an inverting amplifier.

$$2\cos(4\times10^4 t) \longrightarrow 2\angle 0°, \quad \omega = 4\times10^4$$

$$10\text{ nF} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(4\times10^4)(10\times10^{-9})} = \text{-j}2.5\text{ k}\Omega$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{-\mathbf{Z}_f}{\mathbf{Z}_i}$$

where $\mathbf{Z}_i = 50\text{ k}\Omega$ and $\mathbf{Z}_f = 100\text{k} \parallel (\text{-j}2.5\text{k}) = \frac{-\text{j}100}{40-\text{j}}\text{ k}\Omega$.

Thus, $$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{\text{j}2}{40-\text{j}}$$

If $\mathbf{V}_s = 2\angle 0°$,

$$\mathbf{V}_o = \frac{\text{j}4}{40-\text{j}} = \frac{4\angle 90°}{40.01\angle -1.43°} = 0.1\angle 91.43°$$

Therefore,

$$v_o(t) = \underline{\mathbf{0.1\cos(4x10^4 t + 91.43°)\ V}}$$

Chapter 10, Problem 71.

Find v_o in the op amp circuit of Fig. 10.114.

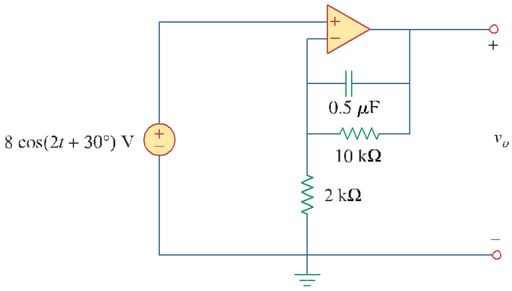


Figure 10.114
For Prob. 10.71.

Chapter 10, Solution 71.

$$8\cos(2t+30^o) \longrightarrow 8\angle 30^o$$

$$0.5\mu F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j2x0.5x10^{-6}} = -j1M\Omega$$

At the inverting terminal,

$$\frac{V_o - 8\angle 30^o}{-j1000k} + \frac{V_o - 8\angle 30^o}{10k} = \frac{8\angle 30^o}{2k} \longrightarrow$$

$$V_o(1 - j100) = 8\angle 30 + 800\angle -60° + 4000\angle -60°$$

$$V_o = \frac{6.928 + j4 + 2400 - j4157}{1 - j100} = \frac{4800\angle -59.9°}{100\angle -89.43°} = 48\angle 29.53^o$$

$v_o(t)$ = **<u>$48\cos(2t + 29.53^o)$ V</u>**

Chapter 10, Problem 72.

Compute $i_o(t)$ in the op amp circuit in Fig. 10.115 if $v_s = 4\cos 10^4 t$ V.

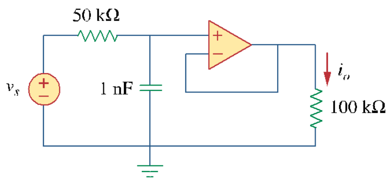


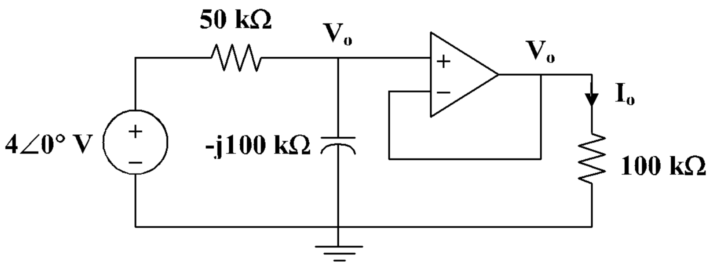
Figure 10.115
For Prob. 10.72.

Chapter 10, Solution 72.

$$4\cos(10^4 t) \longrightarrow 4\angle 0°, \quad \omega = 10^4$$

$$1 \text{ nF} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10^4)(10^{-9})} = -j100 \text{ k}\Omega$$

Consider the circuit as shown below.



At the noninverting node,

$$\frac{4 - \mathbf{V}_o}{50} = \frac{\mathbf{V}_o}{-j100} \longrightarrow \mathbf{V}_o = \frac{4}{1 + j0.5}$$

$$\mathbf{I}_o = \frac{\mathbf{V}_o}{100k} = \frac{4}{(100)(1 + j0.5)} \text{mA} = 35.78\angle -26.56° \ \mu\text{A}$$

Therefore,

$$i_o(t) = \underline{\mathbf{35.78 \cos(10^4 t - 26.56°)\ \mu A}}$$

Chapter 10, Problem 73.

If the input impedance is defined as $\mathbf{Z}_{in} = \mathbf{V}_s / \mathbf{I}_s$ find the input impedance of the op amp circuit in Fig. 10.116 when $R_1 = 10\,\mathrm{k}\Omega, R_2 = 20\,\mathrm{k}\Omega, C_1 = 10\,\mathrm{nF}$, and $\omega = 5000\,\mathrm{rad/s}$.

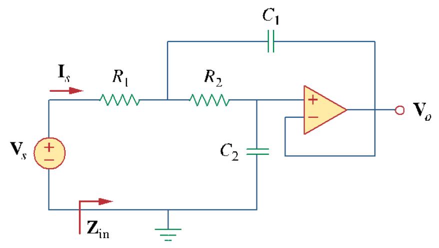


Figure 10.116
For Prob. 10.73.

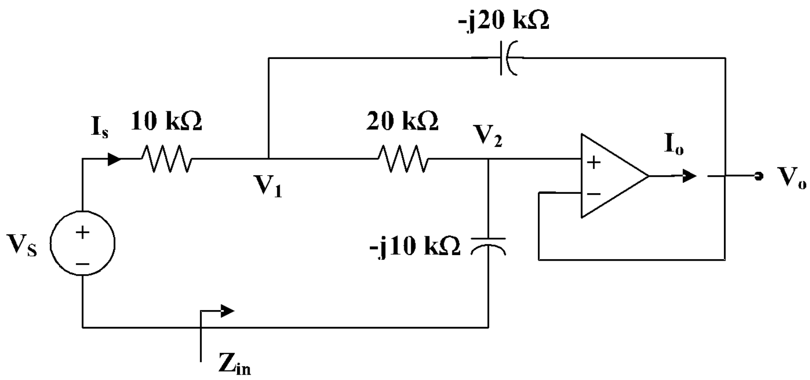
Chapter 10, Solution 73.

As a voltage follower, $\mathbf{V}_2 = \mathbf{V}_o$

$$C_1 = 10\text{ nF} \longrightarrow \frac{1}{j\omega C_1} = \frac{1}{j(5\times 10^3)(10\times 10^{-9})} = -j20\text{ k}\Omega$$

$$C_2 = 20\text{ nF} \longrightarrow \frac{1}{j\omega C_2} = \frac{1}{j(5\times 10^3)(20\times 10^{-9})} = -j10\text{ k}\Omega$$

Consider the circuit in the frequency domain as shown below.



At node 1,

$$\frac{\mathbf{V}_s - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1 - \mathbf{V}_o}{-j20} + \frac{\mathbf{V}_1 - \mathbf{V}_o}{20}$$

$$2\mathbf{V}_s = (3 + j)\mathbf{V}_1 - (1 + j)\mathbf{V}_o \qquad (1)$$

At node 2,

$$\frac{\mathbf{V}_1 - \mathbf{V}_o}{20} = \frac{\mathbf{V}_o - 0}{-j10}$$

$$\mathbf{V}_1 = (1 + j2)\mathbf{V}_o \qquad (2)$$

Substituting (2) into (1) gives

$$2\mathbf{V}_s = j6\mathbf{V}_o \quad \text{or} \quad \mathbf{V}_o = -j\frac{1}{3}\mathbf{V}_s$$

$$\mathbf{V}_1 = (1 + j2)\mathbf{V}_o = \left(\frac{2}{3} - j\frac{1}{3}\right)\mathbf{V}_s$$

$$\mathbf{I}_s = \frac{\mathbf{V}_S - \mathbf{V}_1}{10k} = \frac{(1/3)(1 + j)}{10k}\mathbf{V}_S$$

$$\frac{\mathbf{I}_S}{\mathbf{V}_S} = \frac{1 + j}{30k}$$

$$\mathbf{Z}_{in} = \frac{\mathbf{V}_S}{\mathbf{I}_S} = \frac{30k}{1 + j} = 15(1 - j)\,k$$

$\mathbf{Z}_{in}$ = <u>**21.21∠–45° kΩ**</u>

Chapter 10, Problem 74.

Evaluate the voltage gain $\mathbf{A}_v = \mathbf{V}_o/\mathbf{V}_s$ in the op amp circuit of Fig. 10.117. Find $\mathbf{A}_v$ at $\omega = 0, \omega \to \infty, \omega = 1/R_1C_1$, and $\omega = 1/R_2C_2$.

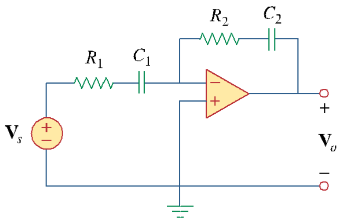


Figure 10.117
For Prob. 10.74.

Chapter 10, Solution 74.

$$\mathbf{Z}_i = R_1 + \frac{1}{j\omega C_1}, \qquad \mathbf{Z}_f = R_2 + \frac{1}{j\omega C_2}$$

$$\mathbf{A}_v = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{-\mathbf{Z}_f}{\mathbf{Z}_i} = -\frac{R_2 + \dfrac{1}{j\omega C_2}}{R_1 + \dfrac{1}{j\omega C_1}} = -\left(\frac{C_1}{C_2}\right)\left(\frac{1 + j\omega R_2 C_2}{1 + j\omega R_1 C_1}\right)$$

At $\omega = 0$, $\qquad \mathbf{A}_v = -\dfrac{\mathbf{C}_1}{\mathbf{C}_2}$

As $\omega \to \infty$, $\qquad \mathbf{A}_v = -\dfrac{\mathbf{R}_2}{\mathbf{R}_1}$

At $\omega = \dfrac{1}{R_1 C_1}$, $\qquad \mathbf{A}_v = -\left(\dfrac{\mathbf{C}_1}{\mathbf{C}_2}\right)\left(\dfrac{1 + j\mathbf{R}_2\mathbf{C}_2/\mathbf{R}_1\mathbf{C}_1}{1 + j}\right)$

At $\omega = \dfrac{1}{R_2 C_2}$, $\qquad \mathbf{A}_v = -\left(\dfrac{\mathbf{C}_1}{\mathbf{C}_2}\right)\left(\dfrac{1 + j}{1 + j\mathbf{R}_1\mathbf{C}_1/\mathbf{R}_2\mathbf{C}_2}\right)$

Chapter 10, Problem 75.

In the op amp circuit of Fig. 10.118, find the closed-loop gain and phase shift of the output voltage with respect to the input voltage if $C_1 = C_2 = 1\text{ nF}$, $R_1 = R_2 = 100\text{ k}\Omega$, $R_3 = 20\text{ k}\Omega$, $R_4 = 40\text{ k}\Omega$, and $\omega = 2000$ rad/s.

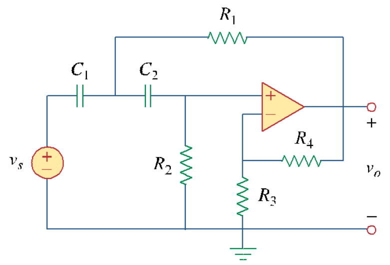


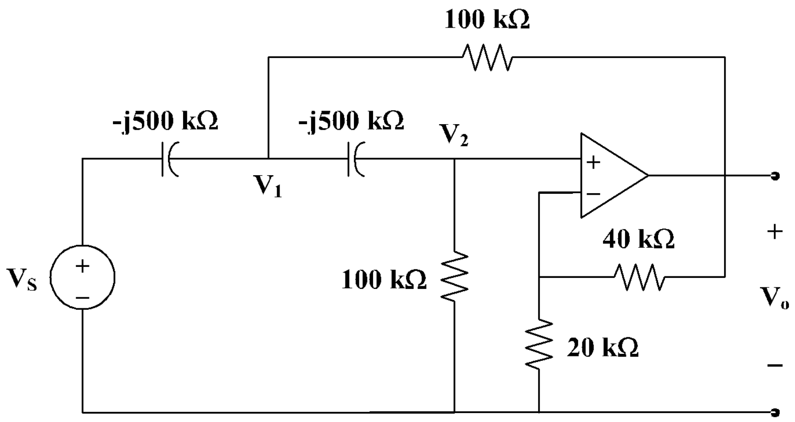
Figure 10.118
For Prob. 10.75.

Chapter 10, Solution 75.

$\omega = 2\times 10^3$

$$C_1 = C_2 = 1\text{ nF} \longrightarrow \frac{1}{j\omega C_1} = \frac{1}{j(2\times 10^3)(1\times 10^{-9})} = \text{-}j500\text{ k}\Omega$$

Consider the circuit shown below.



Let $\mathbf{V}_s$ = 10V.

At node 1,

$$[(\mathbf{V}_1 - 10)/(-j500k)] + [(\mathbf{V}_1 - \mathbf{V}_o)/10^5] + [(\mathbf{V}_1 - \mathbf{V}_2)/(-j500k)] = 0$$

or $(1+j0.4)\mathbf{V}_1 - j0.2\mathbf{V}_2 - \mathbf{V}_o = j2$ (1)

At node 2,

$$[(\mathbf{V}_2 - \mathbf{V}_1)/(-j5)] + (\mathbf{V}_2 - 0) = 0$$

or $-j0.2\mathbf{V}_1 + (1+j0.2)\mathbf{V}_2 = 0$ or $\mathbf{V}_1 = (1-j5)\mathbf{V}_2$ (2)

But

$$\mathbf{V}_2 = \frac{R_3}{R_3 + R_4}\mathbf{V}_o = \frac{\mathbf{V}_o}{3} \qquad (3)$$

From (2) and (3),

$\mathbf{V}_1 = (0.3333 - j1.6667)\mathbf{V}_o$ (4)

Substituting (3) and (4) into (1),

$$(1+j0.4)(0.3333-j1.6667)\mathbf{V}_o - j0.06667\mathbf{V}_o - \mathbf{V}_o = j2$$

$$(1.077\angle 21.8^\circ)(1.6997\angle -78.69^\circ) = 1.8306\angle -56.89^\circ = 1 - j1.5334$$

Thus,

$$(1-j1.5334)\mathbf{V}_o - j0.06667\mathbf{V}_o - \mathbf{V}_o = j2$$

and, $V_o = j2/(-j1.6601) = -1.2499 = 1.2499\angle 180^\circ$ V

Since $\mathbf{V}_s = 10$,

$$\mathbf{V}_o/\mathbf{V}_s = \underline{\mathbf{0.12499\angle 180^\circ}}.$$

Checking with MATLAB.

```
>> Y=[1+0.4i,-0.2i,-1;1,-1+5i,0;0,-3,1]

Y =

   1.0000 + 0.4000i        0 - 0.2000i  -1.0000
   1.0000            -1.0000 + 5.0000i        0
        0            -3.0000             1.0000

>> I=[2i;0;0]

I =

        0 + 2.0000i
        0
        0

>> V=inv(Y)*I

V =

  -0.4167 + 2.0833i
  -0.4167
  -1.2500 + 0.0000i (this last term is vo)
```

and, the answer checks.

Chapter 10, Problem 76.

Determine $\mathbf{V}_o$ and $\mathbf{I}_o$ in the op amp circuit of Fig. 10.119.

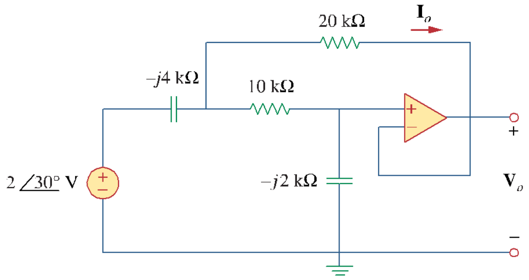


Figure 10.119
For Prob. 10.76.

Chapter 10, Solution 76.

Let the voltage between the -jkΩ capacitor and the 10kΩ resistor be $\mathbf{V}_1$.

$$\frac{2\angle 30^o - V_1}{-j4k} = \frac{V_1 - V_o}{10k} + \frac{V_1 - V_o}{20k} \longrightarrow \qquad (1)$$

$$2\angle 30^o = (1 - j0.6)V_1 + j0.6V_o$$
$$= 1.7321 + j1$$

Also,

$$\frac{V_1 - V_o}{10k} = \frac{V_o}{-j2k} \longrightarrow V_1 = (1 + j5)V_o \qquad (2)$$

Solving (2) into (1) yields

$$2\angle 30° = (1 - j0.6)(1 + j5)V_o + j0.6V_o = (1 + 3 - j0.6 + j5 + j6)V_o$$
$$= (4+j5)V_o$$
$$V_o = \frac{2\angle 30°}{6.403\angle 51.34°} = \underline{0.3124\angle -21.34^o \text{ V}}$$

```
>> Y=[1-0.6i,0.6i;1,-1-0.5i]

Y =

   1.0000 - 0.6000i        0 + 0.6000i
   1.0000             -1.0000 - 5.0000i

>> I=[1.7321+1i;0]

I =

   1.7321 + 1.0000i
        0

>> V=inv(Y)*I

V =

   0.8593 + 1.3410i
   0.2909 - 0.1137i
```

$0.2909 - 0.1137i = V_o = 0.3123\angle{-21.35}°V$. Answer checks.

Chapter 10, Problem 77.

Compute the closed-loop gain $\mathbf{V}_o/\mathbf{V}_s$ for the op amp circuit of Fig. 10.120.

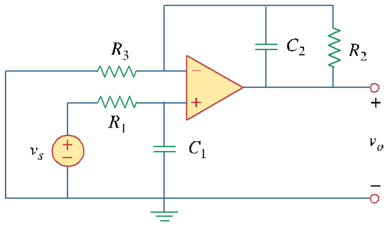
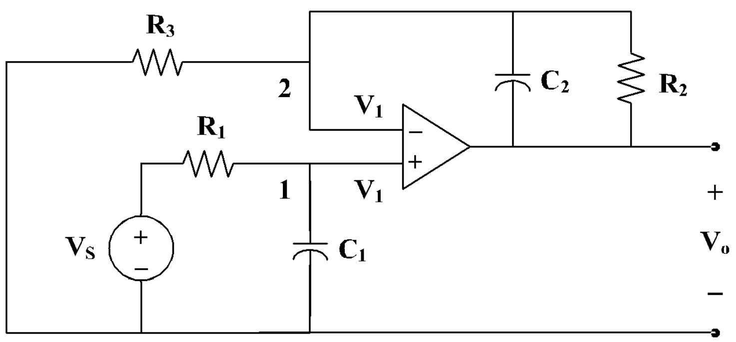


Figure 10.120
For Prob. 10.77.

Chapter 10, Solution 77.

Consider the circuit below.



At node 1,

$$\frac{\mathbf{V}_s - \mathbf{V}_1}{R_1} = j\omega C\mathbf{V}_1$$

$$\mathbf{V}_s = (1 + j\omega R_1 C_1)\mathbf{V}_1 \qquad (1)$$

At node 2,

$$\frac{0 - \mathbf{V}_1}{R_3} = \frac{\mathbf{V}_1 - \mathbf{V}_o}{R_2} + j\omega C_2 (\mathbf{V}_1 - \mathbf{V}_o)$$

$$\mathbf{V}_1 = (\mathbf{V}_o - \mathbf{V}_1)\left(\frac{R_3}{R_2} + j\omega C_2 R_3\right)$$

$$\mathbf{V}_o = \left(1 + \frac{1}{(R_3/R_2) + j\omega C_2 R_3}\right)\mathbf{V}_1 \qquad (2)$$

From (1) and (2),

$$\mathbf{V}_o = \frac{\mathbf{V}_s}{1 + j\omega R_1 C_1}\left(1 + \frac{R_2}{R_3 + j\omega C_2 R_2 R_3}\right)$$

$$\underline{\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{\mathbf{R_2 + R_3 + j\omega C_2 R_2 R_3}}{\mathbf{(1 + j\omega R_1 C_1)(R_3 + j\omega C_2 R_2 R_3)}}}$$

Chapter 10, Problem 78.

Determine $v_o(t)$ in the op amp circuit in Fig. 10.121 below.

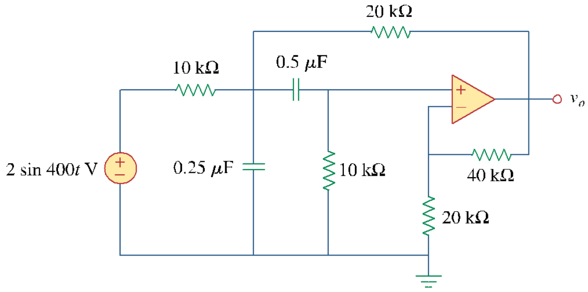


Figure 10.121
For Prob. 10.78.

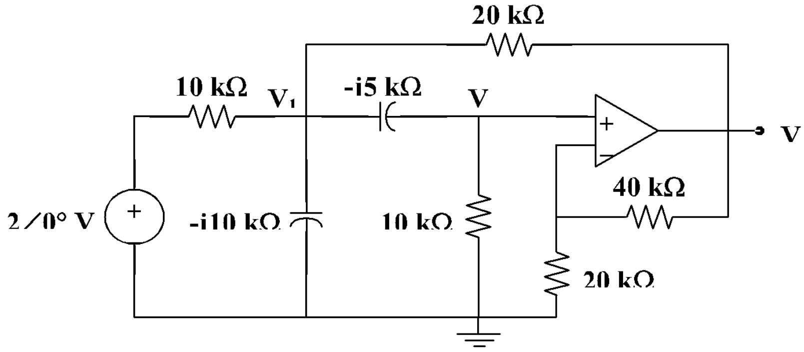
Chapter 10, Solution 78.

$$2\sin(400t) \longrightarrow 2\angle 0°, \quad \omega = 400$$

$$0.5\ \mu F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(400)(0.5\times 10^{-6})} = -j5\ k\Omega$$

$$0.25\ \mu F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(400)(0.25\times 10^{-6})} = -j10\ k\Omega$$

Consider the circuit as shown below.



At node 1,

$$\frac{2-\mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j10} + \frac{\mathbf{V}_1-\mathbf{V}_2}{-j5} + \frac{\mathbf{V}_1-\mathbf{V}_o}{20}$$

$$4 = (3+j6)\mathbf{V}_1 - j4\mathbf{V}_2 - \mathbf{V}_o \qquad (1)$$

At node 2,

$$\frac{\mathbf{V}_1-\mathbf{V}_2}{-j5} = \frac{\mathbf{V}_2}{10}$$

$$\mathbf{V}_1 = (1-j0.5)\mathbf{V}_2 \qquad (2)$$

But

$$\mathbf{V}_2 = \frac{20}{20+40}\mathbf{V}_o = \frac{1}{3}\mathbf{V}_o \qquad (3)$$

From (2) and (3),

$$\mathbf{V}_1 = \frac{1}{3}\cdot(1-j0.5)\mathbf{V}_o \qquad (4)$$

Substituting (3) and (4) into (1) gives

$$4 = (3+j6)\cdot\frac{1}{3}\cdot(1-j0.5)\mathbf{V}_O - j\frac{4}{3}\mathbf{V}_O - \mathbf{V}_O = \left(1+j\frac{1}{6}\right)\mathbf{V}_O$$

$$\mathbf{V}_O = \frac{24}{6+j} = 3.945\angle -9.46°$$

Therefore,

$v_o(t) =$ **3.945 sin(400t – 9.46°) V**

Chapter 10, Problem 79.

For the op amp circuit in Fig. 10.122, obtain $v_o(t)$.

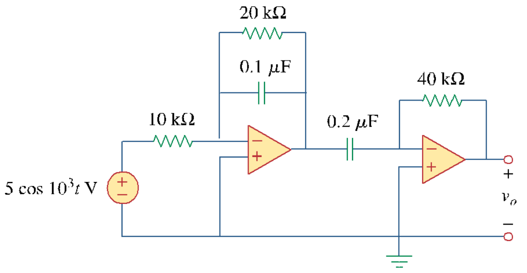


Figure 10.122
For Prob. 10.79.

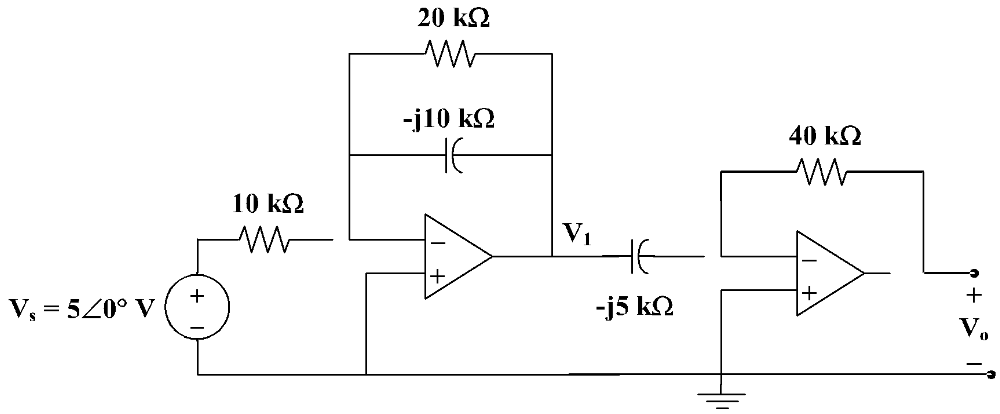
Chapter 10, Solution 79.

$$5\cos(1000t) \longrightarrow 5\angle 0°, \quad \omega = 1000$$

$$0.1\ \mu F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(1000)(0.1\times 10^{-6})} = -j10\ k\Omega$$

$$0.2\ \mu F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(1000)(0.2\times 10^{-6})} = -j5\ k\Omega$$

Consider the circuit shown below.



Since each stage is an inverter, we apply $\mathbf{V}_o = \frac{-\mathbf{Z}_f}{\mathbf{Z}_i}\mathbf{V}_i$ to each stage.

$$\mathbf{V}_o = \frac{-40}{-j5}\mathbf{V}_1 \qquad (1)$$

and

$$\mathbf{V}_1 = \frac{-20\,||\,(-j10)}{10}\mathbf{V}_s \qquad (2)$$

From (1) and (2),

$$\mathbf{V}_o = \left(\frac{-j8}{10}\right)\left(\frac{-(20)(-j10)}{20-j10}\right)5\angle 0°$$

$$\mathbf{V}_o = 16(2+j) = 35.78\angle 26.56°$$

Therefore, $v_o(t) =$ **35.78 cos(1000t + 26.56°) V**

Chapter 10, Problem 80.

Obtain $v_o(t)$ for the op amp circuit in Fig. 10.123 if $v_s = 4\cos(1000t - 60°)$ V.

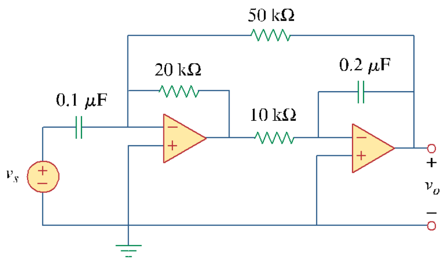


Figure 10.123
For Prob. 10.80.

Chapter 10, Solution 80.

$$4\cos(1000t - 60°) \longrightarrow 4\angle -60°, \quad \omega = 1000$$

$$0.1\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(1000)(0.1\times 10^{-6})} = -j10\ \text{k}\Omega$$

$$0.2\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(1000)(0.2\times 10^{-6})} = -j5\ \text{k}\Omega$$

The two stages are inverters so that

$$\mathbf{V}_o = \left(\frac{20}{-j10}\cdot(4\angle -60°) + \frac{20}{50}\mathbf{V}_o\right)\left(\frac{-j5}{10}\right)$$

$$= \frac{-j}{2}\cdot(j2)\cdot(4\angle -60°) + \frac{-j}{2}\cdot\frac{2}{5}\mathbf{V}_o$$

$$(1 + j/5)\,\mathbf{V}_o = 4\angle -60°$$

$$\mathbf{V}_o = \frac{4\angle -60°}{1 + j/5} = 3.922\angle -71.31°$$

Therefore, $\quad v_o(t) =$ **3.922 cos(1000t – 71.31°) V**

Chapter 10, Problem 81.

ps

Use *PSpice* to determine $\mathbf{V}_o$ in the circuit of Fig. 10.124. Assume $\omega = 1$ rad/s.

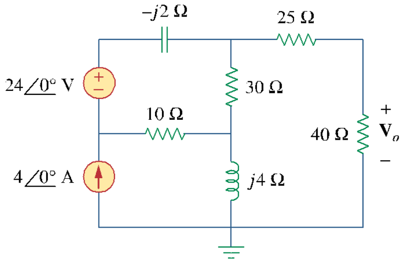


Figure 10.124
For Prob. 10.81.

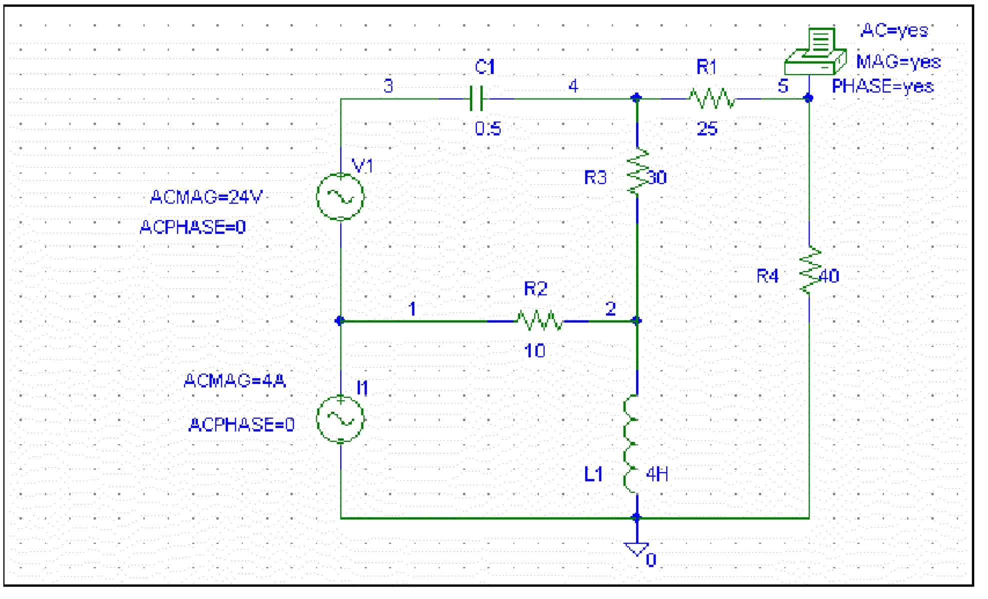
Chapter 10, Solution 81.

We need to get the capacitance and inductance corresponding to –j2 Ω and j4 Ω.

$$-j2 \longrightarrow C = \frac{1}{\omega X_c} = \frac{1}{1x2} = 0.5F$$

$$j4 \longrightarrow L = \frac{X_L}{\omega} = 4H$$

The schematic is shown below.



When the circuit is simulated, we obtain the following from the output file.

FREQ	VM(5)	VP(5)
1.592E-01	1.127E+01	-1.281E+02

From this, we obtain

$$V_o = \underline{\mathbf{11.27\angle 128.1^o\ V}}.$$

Chapter 10, Problem 82.

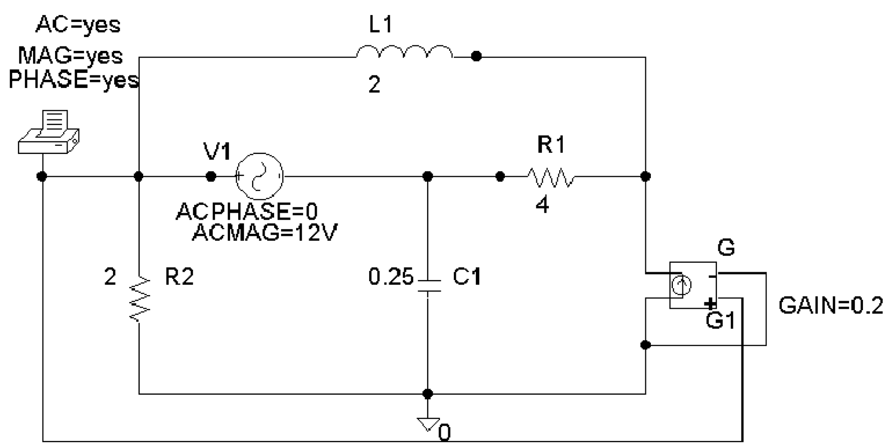
Solve Prob. 10.19 using *PSpice*.

Chapter 10, Solution 82.

The schematic is shown below. We insert PRINT to print V_o in the output file. For AC Sweep, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, we print out the output file which includes:

FREQ	VM($N_0001)	VP($N_0001)
1.592 E-01	7.684 E+00	5.019 E+01

which means that V_o = **7.684∠50.19° V**



Chapter 10, Problem 83.
Use *PSpice* to find $v_o(t)$ in the circuit of Fig. 10.125. Let $i_s = 2\cos(10_3 t)$A.

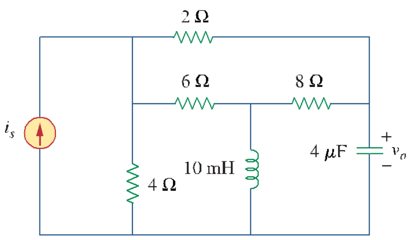
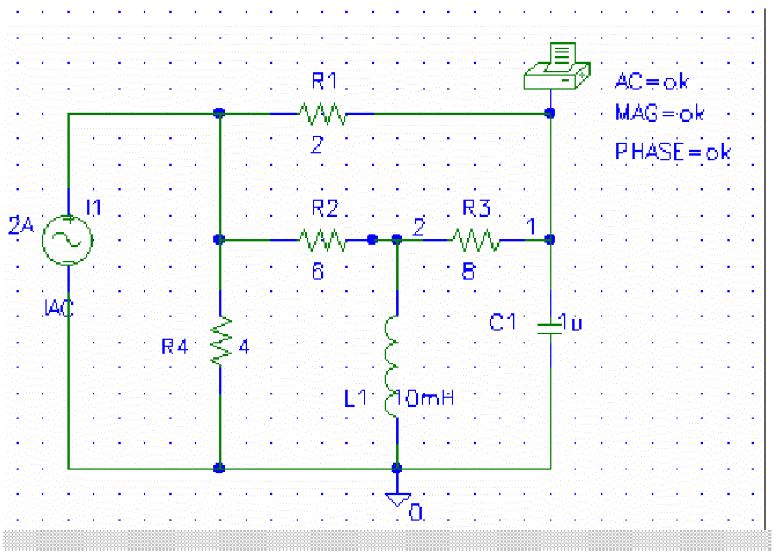


Figure 10.125
For Prob. 10.83.

Chapter 10, Solution 83.

The schematic is shown below. The frequency is $f = \omega / 2\pi = \dfrac{1000}{2\pi} = 159.15$



When the circuit is saved and simulated, we obtain from the output file

FREQ	VM(1)	VP(1)
1.592E+02	6.611E+00	-1.592E+02

Thus,

$$v_o = \underline{\mathbf{6.611\cos(1000t - 159.2^\circ)\ V}}$$

Chapter 10, Problem 84.

Obtain $\mathbf{V}_o$ in the circuit of Fig. 10.126 using *PSpice*.

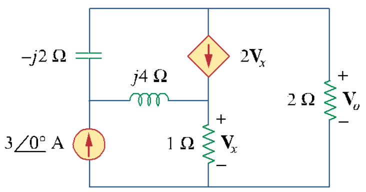


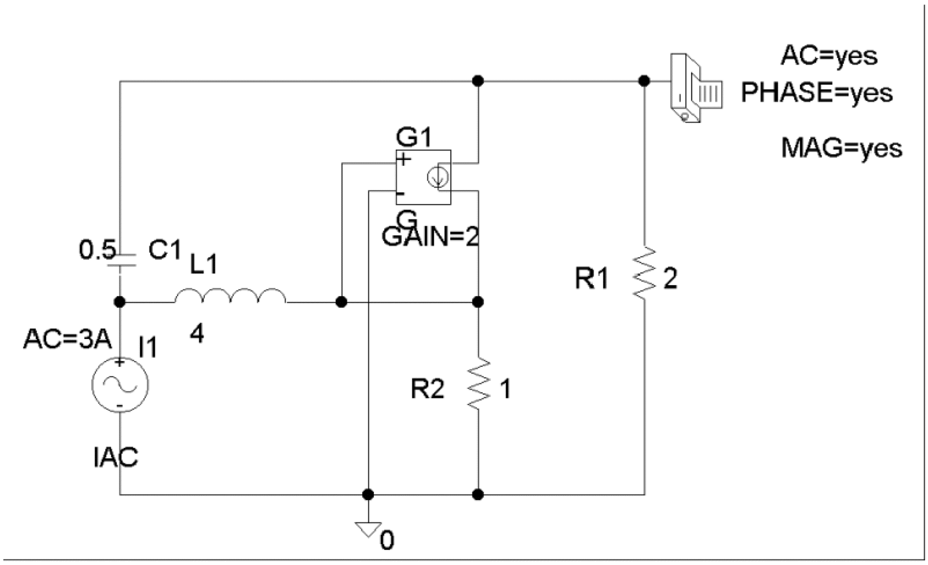
Figure 10.126
For Prob. 10.84.

Chapter 10, Solution 84.

The schematic is shown below. We set PRINT to print V_o in the output file. In AC Sweep box, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, we obtain the output file which includes:

FREQ	VM($N_0003)	VP($N_0003)
1.592 E-01	1.664 E+00	-1.646 E+02

Namely, V_o = **$1.664\angle{-146.4^o}$ V**



Chapter 10, Problem 85.

Use *PSpice* to find $\mathbf{V}_o$ in the circuit of Fig. 10.127.

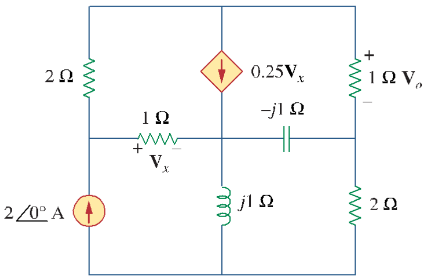
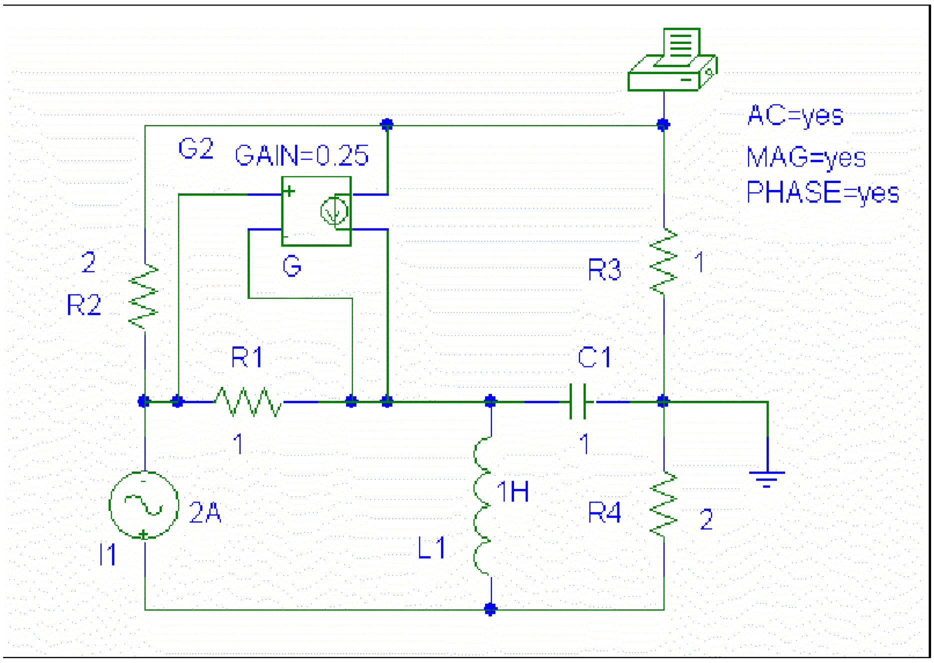


Figure 10.127
For Prob. 10.85.

Chapter 10, Solution 85.

The schematic is shown below. We let $\omega = 1$ rad/s so that L=1H and C=1F.



When the circuit is saved and simulated, we obtain from the output file

```
FREQ        VM($N_0001) VP($N_0001)
1.592E-01   4.471E-01   1.437E+01
```

From this, we conclude that

$$V_o = \underline{\mathbf{447.1\angle 14.37^\circ\ mV}}$$

Checking using MATLAB and nodal analysis we get,

```
>> Y=[1.5,-0.25,-0.25,0;0,1.25,-1.25,1i;-0.5,-1,1.5,0;0,1i,0,0.5-1i]

Y =

  1.5000          -0.2500          -0.2500               0
       0           1.2500          -1.2500          0 + 1.0000i
 -0.5000          -1.0000           1.5000               0
       0          0 + 1.0000i        0          0.5000 - 1.0000i

>> I=[0;0;2;-2]

I =

   0
   0
   2
  -2

>> V=inv(Y)*I

V =

  0.4331 + 0.1110i = Vo = 0.4471∠14.38°, answer checks.
  0.6724 + 0.3775i
  1.9260 + 0.2887i
 -0.1110 - 1.5669i
```

Chapter 10, Problem 86.

Use *PSpice* to find $\mathbf{V}_1$, $\mathbf{V}_2$, and $\mathbf{V}_3$ in the network of Fig. 10.128.

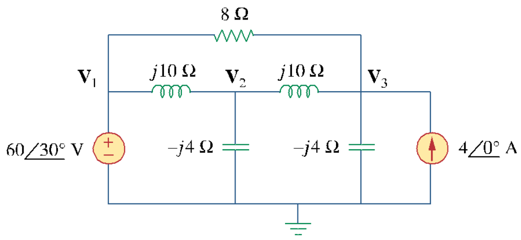


Figure 10.128
For Prob. 10.86.

Chapter 10, Solution 86.

The schematic is shown below. We insert three pseudocomponent PRINTs at nodes 1, 2, and 3 to print V_1, V_2, and V_3, into the output file. Assume that w = 1, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After saving and simulating the circuit, we obtain the output file which includes:

FREQ VM($N_0002) VP($N_0002)

1.592 E-01 6.000 E+01 3.000 E+01

FREQ VM($N_0003) VP($N_0003)

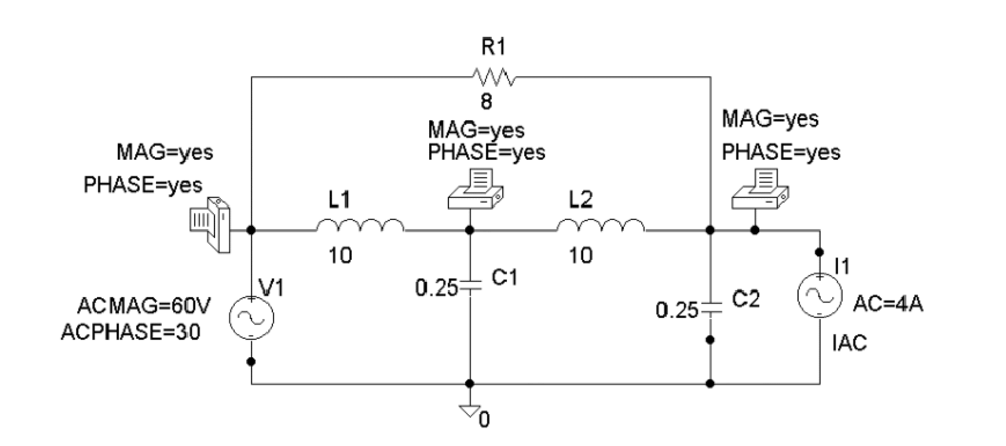
1.592 E-01 2.367 E+02 -8.483 E+01

FREQ VM($N_0001) VP($N_0001)

1.592 E-01 1.082 E+02 1.254 E+02

Therefore,

$$V_1 = \underline{\mathbf{60\angle 30^{o}\ V}} \quad V_2 = \underline{\mathbf{236.7\angle -84.83^{o}\ V}} \quad V_3 = \underline{\mathbf{108.2\angle 125.4^{o}\ V}}$$



Chapter 10, Problem 87.

Determine $\mathbf{V}_1$, $\mathbf{V}_2$, and $\mathbf{V}_3$ in the circuit of Fig. 10.129 using *PSpice*.

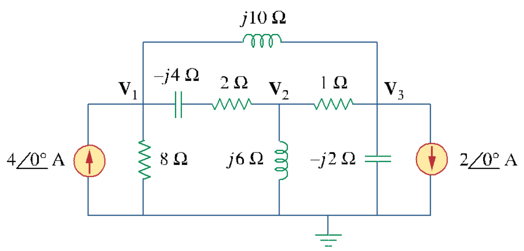


Figure 10.129
For Prob. 10.87.

Chapter 10, Solution 87.
The schematic is shown below. We insert three PRINTs at nodes 1, 2, and 3. We set Total Pts = 1, Start Freq = 0.1592, End Freq = 0.1592 in the AC Sweep box. After simulation, the output file includes:

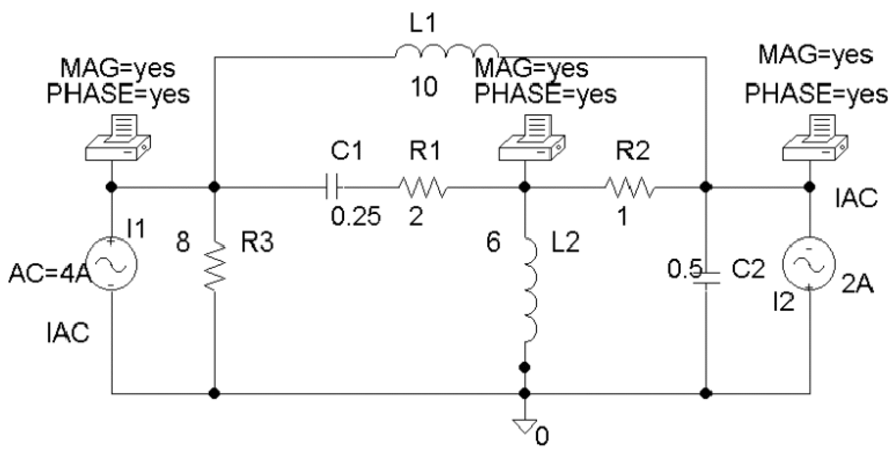
FREQ VM($N_0004) VP($N_0004)
1.592 E-01 1.591 E+01 1.696 E+02

FREQ VM($N_0001) VP($N_0001)
1.592 E-01 5.172 E+00 -1.386 E+02

FREQ VM($N_0003) VP($N_0003)
1.592 E-01 2.270 E+00 -1.524 E+02

Therefore,

$$V_1 = \underline{\mathbf{15.91\angle 169.6^o\ V}} \quad V_2 = \underline{\mathbf{5.172\angle -138.6^o\ V}} \quad V_3 = \underline{\mathbf{2.27\angle -152.4^o\ V}}$$



Chapter 10, Problem 88.

Use *PSpice* to find v_o and i_o in the circuit of Fig. 10.130 below.

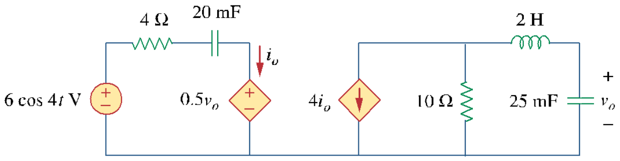


Figure 10.130
For Prob. 10.88.

Chapter 10, Solution 88.

The schematic is shown below. We insert IPRINT and PRINT to print I_o and V_o in the output file. Since w = 4, f = w/2π = 0.6366, we set Total Pts = 1, Start Freq = 0.6366, and End Freq = 0.6366 in the AC Sweep box. After simulation, the output file includes:

FREQ VM($N_0002) VP($N_0002)

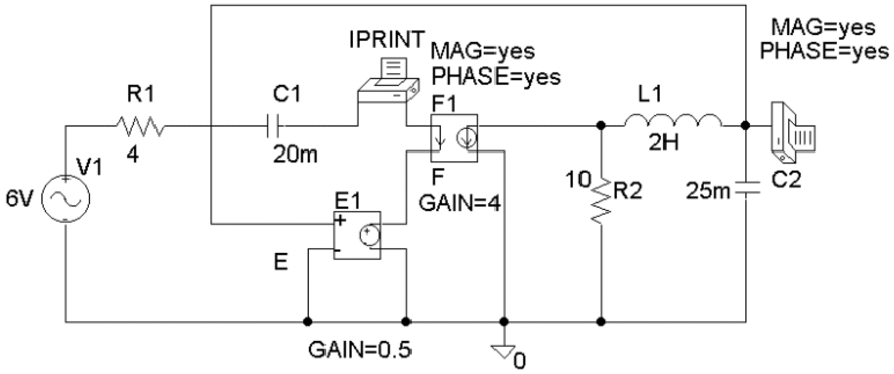
6.366 E-01 3.496 E+01 1.261 E+01

FREQ IM(V_PRINT2) IP (V_PRINT2)

6.366 E-01 8.912 E-01 -8.870 E+01

Therefore, $V_o = 34.96\angle 12.6^o$ V, $I_o = 0.8912\angle -88.7^o$ A

v_o = **<u>34.96 cos(4t + 12.6^o)V</u>**, i_o = **<u>0.8912cos(4t - 88.7^o)A</u>**



Chapter 10, Problem 89.

The op amp circuit in Fig. 10.131 is called an *inductance simulator*. Show that the input impedance is given by

$$\mathbf{Z}_{in} = \frac{\mathbf{V}_{in}}{\mathbf{I}_{in}} = j\omega L_{eq}$$

where

$$L_{eq} = \frac{R_1 R_3 R_4}{R_2} C$$

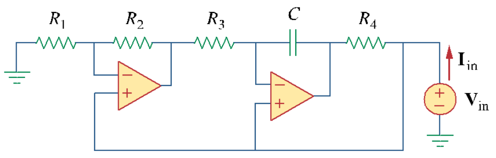
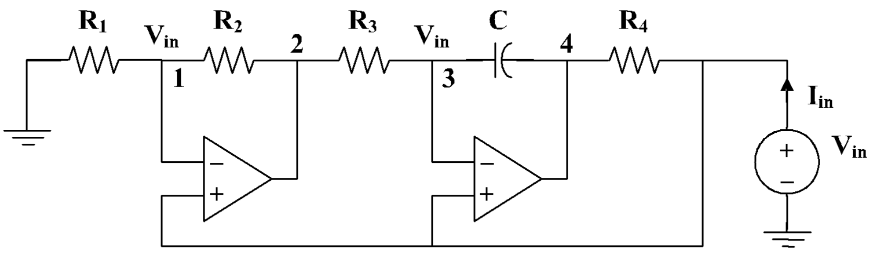


Figure 10.131
For Prob. 10.89.

Chapter 10, Solution 89.

Consider the circuit below.



At node 1,

$$\frac{0-\mathbf{V}_{in}}{R_1} = \frac{\mathbf{V}_{in}-\mathbf{V}_2}{R_2}$$

$$-\mathbf{V}_{in} + \mathbf{V}_2 = \frac{R_2}{R_1}\mathbf{V}_{in} \qquad (1)$$

At node 3,

$$\frac{\mathbf{V}_2-\mathbf{V}_{in}}{R_3} = \frac{\mathbf{V}_{in}-\mathbf{V}_4}{1/j\omega C}$$

$$-\mathbf{V}_{in} + \mathbf{V}_4 = \frac{\mathbf{V}_{in}-\mathbf{V}_2}{j\omega CR_3} \qquad (2)$$

From (1) and (2),

$$-\mathbf{V}_{in} + \mathbf{V}_4 = \frac{-R_2}{j\omega CR_3R_1}\mathbf{V}_{in}$$

Thus,

$$\mathbf{I}_{in} = \frac{\mathbf{V}_{in}-\mathbf{V}_4}{R_4} = \frac{R_2}{j\omega CR_3R_1R_4}\mathbf{V}_{in}$$

$$\mathbf{Z}_{in} = \frac{\mathbf{V}_{in}}{\mathbf{I}_{in}} = \frac{j\omega CR_1R_3R_4}{R_2} = j\omega L_{eq}$$

where $\mathbf{L_{eq} = \dfrac{R_1R_3R_4C}{R_2}}$

Chapter 10, Problem 90.

Figure 10.132 shows a Wien-bridge network. Show that the frequency at which the phase shift between the input and output signals is zero is $f = \frac{1}{2}\pi RC$, and that the necessary gain is $\mathbf{A}_v = \mathbf{V}_o / \mathbf{V}_i = 3$ at that frequency.

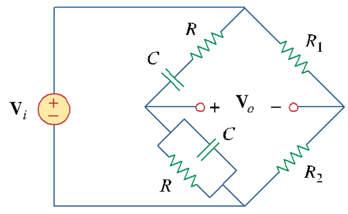


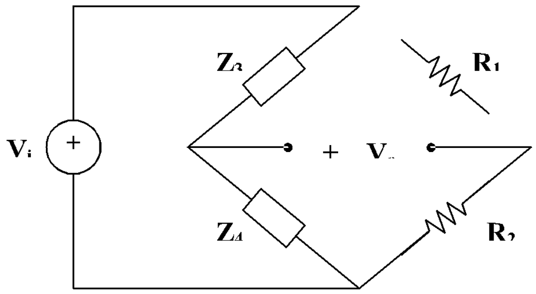
Figure 10.132
For Prob. 10.90.

Chapter 10, Solution 90.

Let $\mathbf{Z}_4 = R \parallel \dfrac{1}{j\omega C} = \dfrac{R}{1 + j\omega RC}$

$$\mathbf{Z}_3 = R + \frac{1}{j\omega C} = \frac{1 + j\omega RC}{j\omega C}$$

Consider the circuit shown below.



$$\mathbf{V}_o = \frac{\mathbf{Z}_4}{\mathbf{Z}_3 + \mathbf{Z}_4}\mathbf{V}_i - \frac{R_2}{R_1 + R_2}\mathbf{V}_i$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{\dfrac{R}{1 + j\omega C}}{\dfrac{R}{1 + j\omega C} + \dfrac{1 + j\omega RC}{j\omega C}} - \frac{R_2}{R_1 + R_2}$$

$$= \frac{j\omega RC}{j\omega RC + (1 + j\omega RC)^2} - \frac{R_2}{R_1 + R_2}$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{j\omega RC}{1 - \omega^2 R^2 C^2 + j3\omega RC} - \frac{R_2}{R_1 + R_2}$$

For $\mathbf{V}_o$ and $\mathbf{V}_i$ to be in phase, $\dfrac{\mathbf{V}_o}{\mathbf{V}_i}$ must be purely real. This happens when

$$1 - \omega^2 R^2 C^2 = 0$$

$$\omega = \frac{1}{RC} = 2\pi f$$

or $$f = \frac{1}{2\pi RC}$$

At this frequency, $\mathbf{A}_v = \dfrac{\mathbf{V}_o}{\mathbf{V}_i} = \dfrac{1}{3} - \dfrac{R_2}{R_1 + R_2}$

Chapter 10, Problem 91.

Consider the oscillator in Fig. 10.133.
(a) Determine the oscillation frequency.
(b) Obtain the minimum value of *R* for which oscillation takes place.

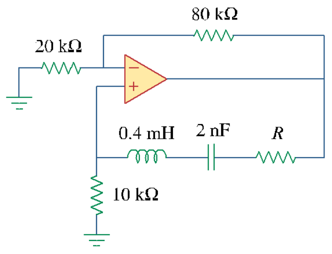


Figure 10.133
For Prob. 10.91.

Chapter 10, Solution 91.

(a) Let $\mathbf{V}_2$ = voltage at the noninverting terminal of the op amp

$\mathbf{V}_o$ = output voltage of the op amp

$\mathbf{Z}_p = 10\text{ k}\Omega = R_o$

$$\mathbf{Z}_s = R + j\omega L + \frac{1}{j\omega C}$$

As in Section 10.9,

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{\mathbf{Z}_p}{\mathbf{Z}_s + \mathbf{Z}_p} = \frac{R_o}{R + R_o + j\omega L - \dfrac{j}{\omega C}}$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{\omega C R_o}{\omega C(R + R_o) + j(\omega^2 LC - 1)}$$

For this to be purely real,

$$\omega_o^2 LC - 1 = 0 \longrightarrow \omega_o = \frac{1}{\sqrt{LC}}$$

$$f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(0.4\times 10^{-3})(2\times 10^{-9})}}$$

f_o = $\underline{\textbf{180 kHz}}$

(b) At oscillation,

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{\omega_o C R_o}{\omega_o C(R + R_o)} = \frac{R_o}{R + R_o}$$

This must be compensated for by

$$\mathbf{A}_v = \frac{\mathbf{V}_o}{\mathbf{V}_2} = 1 + \frac{80}{20} = 5$$

$$\frac{R_o}{R + R_o} = \frac{1}{5} \longrightarrow R = 4R_o = \underline{\mathbf{40\ k\Omega}}$$

Chapter 10, Problem 92.

The oscillator circuit in Fig. 10.134 uses an ideal op amp.
(a) Calculate the minimum value of R_o that will cause oscillation to occur.
(b) Find the frequency of oscillation.

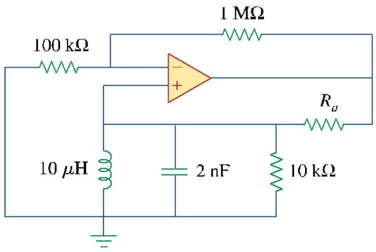


Figure 10.134
For Prob. 10.92.

Chapter 10, Solution 92.

Let $\mathbf{V}_2$ = voltage at the noninverting terminal of the op amp

$\mathbf{V}_o$ = output voltage of the op amp

$\mathbf{Z}_s = R_o$

$$\mathbf{Z}_p = j\omega L \,||\, \frac{1}{j\omega C} \,||\, R = \frac{1}{\dfrac{1}{R} + j\omega C + \dfrac{1}{j\omega L}} = \frac{\omega RL}{\omega L + jR(\omega^2 LC - 1)}$$

As in Section 10.9,

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{\mathbf{Z}_p}{\mathbf{Z}_s + \mathbf{Z}_p} = \frac{\dfrac{\omega RL}{\omega L + jR(\omega^2 LC - 1)}}{R_o + \dfrac{\omega RL}{\omega L + jR(\omega^2 LC - 1)}}$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{\omega RL}{\omega RL + \omega R_o L + jR_o R\,(\omega^2 LC - 1)}$$

For this to be purely real,

$$\omega_o^2 LC = 1 \longrightarrow f_o = \frac{1}{2\pi\sqrt{LC}}$$

(a) At $\omega = \omega_o$,

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{\omega_o RL}{\omega_o RL + \omega_o R_o L} = \frac{R}{R + R_o}$$

This must be compensated for by

$$\mathbf{A}_v = \frac{\mathbf{V}_o}{\mathbf{V}_2} = 1 + \frac{R_f}{R_o} = 1 + \frac{1000k}{100k} = 11$$

Hence,

$$\frac{R}{R + R_o} = \frac{1}{11} \longrightarrow R_o = 10R = \underline{\mathbf{100\ k\Omega}}$$

(b) $$f_o = \frac{1}{2\pi\sqrt{(10\times10^{-6})(2\times10^{-9})}}$$

f_o = **<u>1.125 MHz</u>**

Chapter 10, Problem 93.

Figure 10.135 shows a *Colpitts oscillator*. Show that the oscillation frequency is

$$f_o = \frac{1}{2\pi\sqrt{LC_T}}$$

where $C_T = C_1C_2/(C_1 + C_2)$. Assume $R_i >> X_{C_2}$

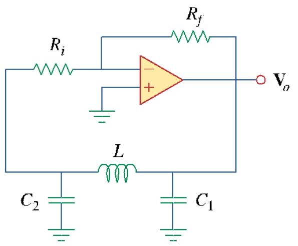


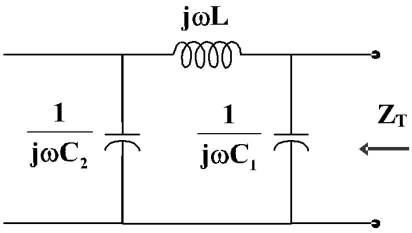
Figure 10.135

A Colpitts oscillator; for Prob. 10.93.

(*Hint:* Set the imaginary part of the impedance in the feedback circuit equal to zero.)

Chapter 10, Solution 93.

As shown below, the impedance of the feedback is



$$\mathbf{Z}_T = \frac{1}{j\omega C_1} \,||\, \left(j\omega L + \frac{1}{j\omega C_2} \right)$$

$$\mathbf{Z}_T = \frac{\frac{-j}{\omega C_1}\left(j\omega L + \frac{-j}{\omega C_2} \right)}{\frac{-j}{\omega C_1} + j\omega L + \frac{-j}{\omega C_2}} = \frac{\frac{1}{\omega} - \omega L C_2}{j(C_1 + C_2 - \omega^2 L C_1 C_2)}$$

In order for $\mathbf{Z}_T$ to be real, the imaginary term must be zero; i.e.

$$C_1 + C_2 - \omega_o^2 L C_1 C_2 = 0$$

$$\omega_o^2 = \frac{C_1 + C_2}{L C_1 C_2} = \frac{1}{L C_T}$$

$$\mathbf{f_o} = \frac{\mathbf{1}}{\mathbf{2\pi\sqrt{LC_T}}}$$

Chapter 10, Problem 94.

Design a Colpitts oscillator that will operate at 50 kHz.

Chapter 10, Solution 94.

If we select $C_1 = C_2 = 20$ nF

$$C_T = \frac{C_1 C_2}{C_1 + C_2} = \frac{C_1}{2} = 10 \text{ nF}$$

Since $f_o = \dfrac{1}{2\pi\sqrt{LC_T}}$,

$$L = \frac{1}{(2\pi f)^2 C_T} = \frac{1}{(4\pi^2)(2500\times10^6)(10\times10^{-9})} = 10.13 \text{ mH}$$

$$X_c = \frac{1}{\omega C_2} = \frac{1}{(2\pi)(50\times10^3)(20\times10^{-9})} = 159\ \Omega$$

We may select $R_i = 20\text{ k}\Omega$ and $R_f \geq R_i$, say $R_f = 20\text{ k}\Omega$.

Thus,

$C_1 = C_2 =$ **20 nF**, $L =$ **10.13 mH** $R_f = R_i =$ **20 kΩ**

Chapter 10, Problem 95.
Figure 10.136 shows a *Hartley oscillator*. Show that the frequency of oscillation is

$$f_o = \frac{1}{2\pi\sqrt{C(L_1 + L_2)}}$$

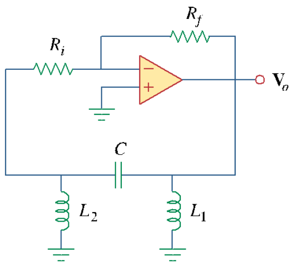
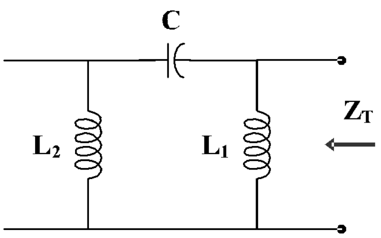


Figure 10.136
A Hartley oscillator; For Prob. 10.95.

Chapter 10, Solution 95.
First, we find the feedback impedance.



$$\mathbf{Z}_T = j\omega L_1 \parallel \left(j\omega L_2 + \frac{1}{j\omega C} \right)$$

$$\mathbf{Z}_T = \frac{j\omega L_1\left(j\omega L_2 - \frac{j}{\omega C}\right)}{j\omega L_1 + j\omega L_2 - \frac{j}{\omega C}} = \frac{\omega^2 L_1 C(1 - \omega L_2)}{j(\omega^2 C(L_1 + L_2) - 1)}$$

In order for $\mathbf{Z}_T$ to be real, the imaginary term must be zero; i.e.

$$\omega_o^2 C(L_1 + L_2) - 1 = 0$$

$$\omega_o = 2\pi f_o = \frac{1}{C(L_1 + L_2)}$$

$$\mathbf{f_o = \frac{1}{2\pi\sqrt{C(L_1 + L_2)}}}$$

Chapter 10, Problem 96.

Refer to the oscillator in Fig. 10.137.

(a) Show that

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{1}{3 + j(\omega L / R - R / \omega L)}$$

(b) Determine the oscillation frequency f_o.

(c) Obtain the relationship between R_1 and R_2 in order for oscillation to occur.

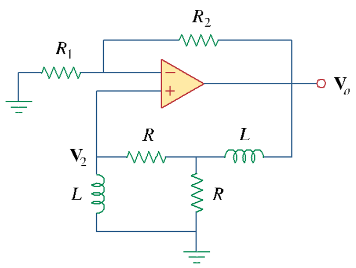
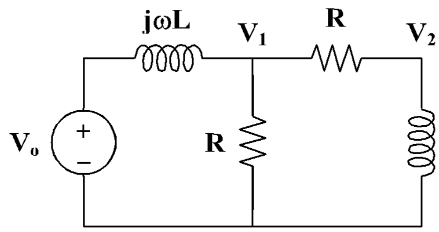


Figure 10.137
For Prob. 10.96.

Chapter 10, Solution 96.

(a) Consider the feedback portion of the circuit, as shown below.



$$\mathbf{V}_2 = \frac{j\omega L}{R + j\omega L}\mathbf{V}_1 \quad \longrightarrow \quad \mathbf{V}_1 = \frac{R + j\omega L}{j\omega L}\mathbf{V}_2 \qquad (1)$$

Applying KCL at node 1,

$$\frac{\mathbf{V}_o - \mathbf{V}_1}{j\omega L} = \frac{\mathbf{V}_1}{R} + \frac{\mathbf{V}_1}{R + j\omega L}$$

$$\mathbf{V}_o - \mathbf{V}_1 = j\omega L\,\mathbf{V}_1\left(\frac{1}{R} + \frac{1}{R + j\omega L}\right)$$

$$\mathbf{V}_o = \mathbf{V}_1\left(1 + \frac{j2\omega RL - \omega^2 L^2}{R\,(R + j\omega L)}\right)$$

(2)

From (1) and (2),

$$\mathbf{V}_o = \left(\frac{R + j\omega L}{j\omega L}\right)\left(1 + \frac{j2\omega RL - \omega^2 L^2}{R\,(R + j\omega L)}\right)\mathbf{V}_2$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_2} = \frac{R^2 + j\omega RL + j2\omega RL - \omega^2 L^2}{j\omega RL}$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{1}{3 + \dfrac{R^2 - \omega^2 L^2}{j\omega RL}}$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{\mathbf{1}}{\mathbf{3 + j(\omega L/R - R/\omega L)}}$$

(b) Since the ratio $\frac{\mathbf{V}_2}{\mathbf{V}_o}$ must be real,

$$\frac{\omega_o L}{R} - \frac{R}{\omega_o L} = 0$$

$$\omega_o L = \frac{R^2}{\omega_o L}$$

$$\omega_o = 2\pi f_o = \frac{R}{L}$$

$$\underline{\mathbf{f_o = \frac{R}{2\pi L}}}$$

(c) When $\omega = \omega_o$

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{1}{3}$$

This must be compensated for by $\mathbf{A}_v = 3$. But

$$\mathbf{A}_v = 1 + \frac{R_2}{R_1} = 3$$

$$\underline{\mathbf{R_2 = 2R_1}}$$

Chapter 11, Problem 1.

If $v(t)$ = 160 cos 50t V and $i(t)$ = –20 sin(50t – 30°) A, calculate the instantaneous power and the average power.

Chapter 11, Solution 1.

$$v(t) = 160\cos(50t)$$
$$i(t) = -20\sin(50t - 30°) = 2\cos(50t - 30° + 180° - 90°)$$
$$i(t) = 20\cos(50t + 60°)$$

$$p(t) = v(t)i(t) = (160)(20)\cos(50t)\cos(50t + 60°)$$
$$p(t) = 1600\left[\cos(100t + 60°) + \cos(60°)\right] \text{ W}$$
$$p(t) = \underline{\mathbf{800 + 1600\cos(100t + 60°)\ W}}$$

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) = \frac{1}{2}(160)(20)\cos(60°)$$
$$P = \underline{\mathbf{800\ W}}$$

Chapter 11, Problem 2.

Given the circuit in Fig. 11.35, find the average power supplied or absorbed by each element.

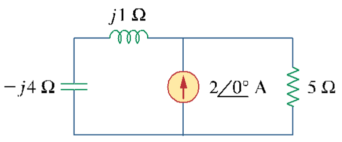
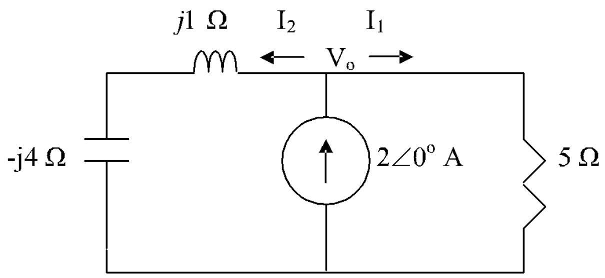


Figure 11.35
For Prob. 11.2.

Chapter 11, Solution 2.

Using current division,



$$I_1 = \frac{j1 - j4}{5 + j1 - j4}(2) = \frac{-j6}{5 - j3}$$

$$I_2 = \frac{5}{5 + j1 - j4}(2) = \frac{10}{5 - j3}$$

For the inductor and capacitor, the average power is zero. For the resistor,

$$P = \frac{1}{2}|I_1|^2 R = \frac{1}{2}(1.029)^2(5) = 2.647 \text{ W}$$

$$V_o = 5I_1 = -2.6471 - j4.4118$$

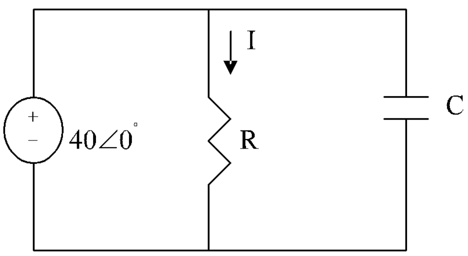
$$S = \frac{1}{2}V_o I^* = \frac{1}{2}(-2.6471 - j4.4118)x2 = -2.6471 - j4.4118$$

Hence the average power supplied by the current source is **2.647 W**.

Chapter 11, Problem 3.

A load consists of a 60-Ω resistor in parallel with a 90 μF capacitor. If the load is connected to a voltage source $v_s(t) = 40 \cos 2000t$, find the average power delivered to the load.

Chapter 11, Solution 3.



$$90\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j90x10^{-6}x2x10^{3}} = -j5.5556$$

$$I = 40/60 = 0.6667\text{A} \quad \text{or} \quad I_{rms} = 0.6667/1.4142 = 0.4714\text{A}$$

The average power delivered to the load is the same as the average power absorbed by the resistor which is

$$P_{avg} = |I_{rms}|^2 60 = \underline{\mathbf{13.333\ W}}.$$

Chapter 11, Problem 4.

Find the average power dissipated by the resistances in the circuit of Fig. 11.36. Additionally, verify the conservation of power.

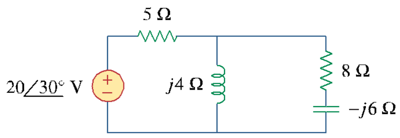
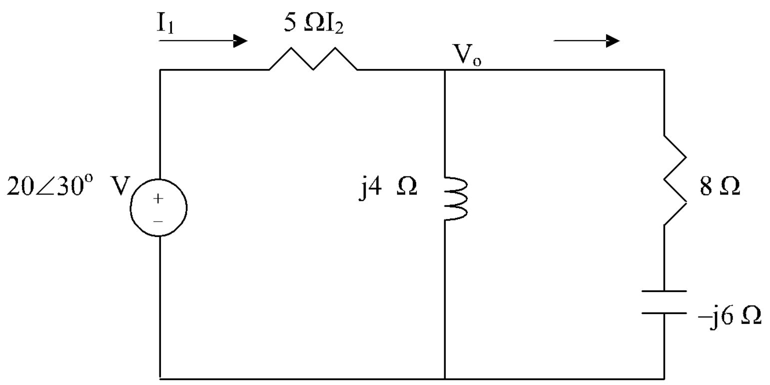


Figure 11.36
For Prob. 11.4.

Chapter 11, Solution 4.

We apply nodal analysis. At the main node,



$$\frac{20<30^o - V_o}{5} = \frac{V_o}{j4} + \frac{V_o}{8-j6} \longrightarrow V_o = 5.152 + j10.639$$

For the 5-Ω resistor,

$$I_1 = \frac{20<30^o - V_o}{5} = 2.438 < -3.0661^o \text{ A}$$

The average power dissipated by the resistor is

$$P_1 = \frac{1}{2}|I_1|^2 R_1 = \frac{1}{2}x2.438^2 x5 = \underline{14.86 \text{ W}}$$

For the 8-Ω resistor,

$$I_2 = \frac{V_o}{8-j} = 1.466 < 71.29^o$$

The average power dissipated by the resistor is

$$P_2 = \frac{1}{2}|I_2|^2 R_2 = \frac{1}{2}x1.466^2 x8 = \underline{8.5966 \text{ W}}$$

The complex power supplied is

$$S = \frac{1}{2}V_s I_1^* = \frac{1}{2}(20<30^o)(2.438<3.0661^o) = 20.43 + j13.30 \text{ VA}$$

Adding P_1 and P_2 gives the real part of S, showing the conservation of power.

Chapter 11, Problem 5.

Assuming that $v_s = 8\cos(2t - 40^\circ)$ V in the circuit of Fig. 11.37, find the average power delivered to each of the passive elements.

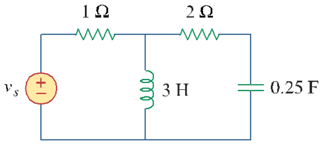
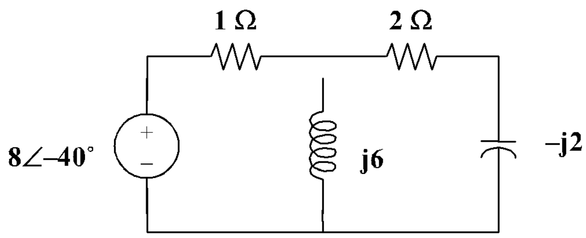


Figure 11.37
For Prob. 11.5.

Chapter 11, Solution 5.

Converting the circuit into the frequency domain, we get:



$$I_{1\Omega} = \frac{8\angle -40^\circ}{1 + \dfrac{j6(2 - j2)}{j6 + 2 - j2}} = 1.6828\angle -25.38^\circ$$

$$P_{1\Omega} = \frac{1.6828^2}{2} 1 = \underline{1.4159\text{ W}}$$

$$P_{3H} = P_{0.25F} = \underline{\mathbf{0}}$$

$$|I_{2\Omega}| = \left| \frac{j6}{j6 + 2 - j2} 1.6828\angle -25.38^\circ \right| = 2.258$$

$$P_{2\Omega} = \frac{2.258^2}{2} 2 = \underline{5.097\text{ W}}$$

Chapter 11, Problem 6.

For the circuit in Fig. 11.38, $i_s = 6\cos 10^3 t$ A. Find the average power absorbed by the 50-Ω resistor.

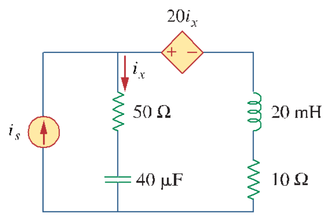


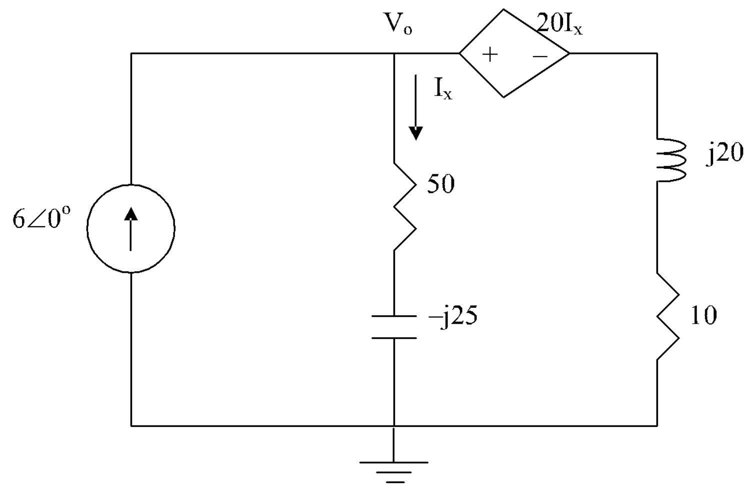
Figure 11.38
For Prob. 11.6.

Chapter 11, Solution 6.

20 mH $\longrightarrow$ $j\omega L = j10^3 x20x10^{-3} = j20$

$$40\mu F \rightarrow \frac{1}{j\omega C} = \frac{1}{j10^3 x40x10^{-6}} = -j25$$

We apply nodal analysis to the circuit below.



$$-6 + \frac{V_o - 20I_x}{10 + j20} + \frac{V_o - 0}{50 - j25} = 0$$

But $I_x = \frac{V_o}{50 - j25}$. Substituting this and solving for V_o leads

$$\left(\frac{1}{10 + j20} - \frac{20}{(10 + j20)} \frac{1}{(50 - j25)} + \frac{1}{50 - j25}\right)V_o = 6$$

$$\left(\frac{1}{22.36\angle 63.43^\circ} - \frac{20}{(22.36\angle 63.43^\circ)(55.9\angle -26.57^\circ)} + \frac{1}{55.9\angle -26.57^\circ}\right)V_o = 6$$

$$(0.02 - j0.04 - 0.012802 + j0.009598 + 0.016 + j0.008)V_o = 6$$

$(0.0232 - j0.0224)V_o = 6$ or $V_o = 6/(0.03225\angle -43.99^\circ = 186.05\angle 43.99^\circ$

For power, all we need is the magnitude of the rms value of I_x.

$|I_x| = 186.05/55.9 = 3.328$ and $|I_x|_{rms} = 3.328/1.4142 = 2.353$

We can now calculate the average power absorbed by the 50-Ω resistor.

$P_{avg} = (2.353)^2 x50 =$ **276.8 W**.

Chapter 11, Problem 7.

Given the circuit of Fig. 11.39, find the average power absorbed by the 10-Ω resistor.

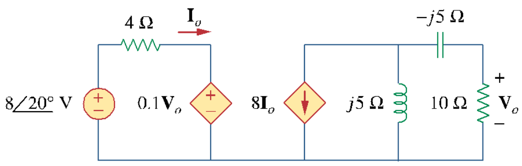


Figure 11.39
For Prob. 11.7.

Chapter 11, Solution 7.

Applying KVL to the left-hand side of the circuit,

$$8\angle 20° = 4\mathbf{I}_o + 0.1\mathbf{V}_o \qquad (1)$$

Applying KCL to the right side of the circuit,

$$8\mathbf{I}_o + \frac{\mathbf{V}_1}{j5} + \frac{\mathbf{V}_1}{10 - j5} = 0$$

But, $\mathbf{V}_o = \frac{10}{10 - j5}\mathbf{V}_1 \longrightarrow \mathbf{V}_1 = \frac{10 - j5}{10}\mathbf{V}_o$

Hence, $8\mathbf{I}_o + \frac{10 - j5}{j50}\mathbf{V}_o + \frac{\mathbf{V}_o}{10} = 0$

$$\mathbf{I}_o = j0.025\mathbf{V}_o \qquad (2)$$

Substituting (2) into (1),

$$8\angle 20° = 0.1\mathbf{V}_o(1 + j)$$

$$\mathbf{V}_o = \frac{80\angle 20°}{1 + j}$$

$$\mathbf{I}_1 = \frac{\mathbf{V}_o}{10} = \frac{8}{\sqrt{2}}\angle -25°$$

$$P = \frac{1}{2}|\mathbf{I}_1|^2 R = \left(\frac{1}{2}\right)\left(\frac{64}{2}\right)(10) = \underline{\mathbf{160W}}$$

Chapter 11, Problem 8.

In the circuit of Fig. 11.40, determine the average power absorbed by the 40-Ω resistor.

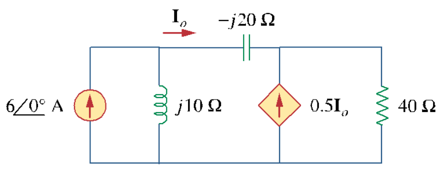
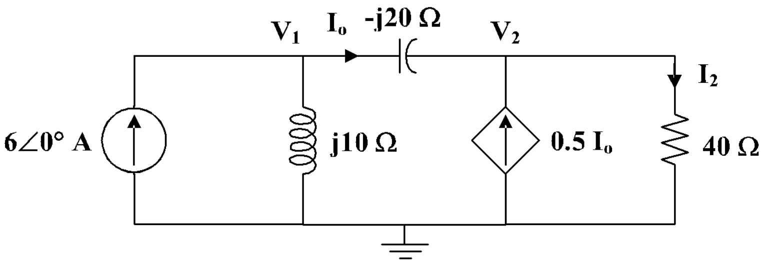


Figure 11.40
For Prob. 11.8.

Chapter 11, Solution 8.

We apply nodal analysis to the following circuit.



At node 1,

$$6 = \frac{\mathbf{V}_1}{j10} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j20} \quad \mathbf{V}_1 = j120 - \mathbf{V}_2 \qquad (1)$$

At node 2,

$$0.5\mathbf{I}_o + \mathbf{I}_o = \frac{\mathbf{V}_2}{40}$$

But, $\mathbf{I}_o = \dfrac{\mathbf{V}_1 - \mathbf{V}_2}{-j20}$

Hence, $\dfrac{1.5(\mathbf{V}_1 - \mathbf{V}_2)}{-j20} = \dfrac{\mathbf{V}_2}{40}$

$$3\mathbf{V}_1 = (3 - j)\mathbf{V}_2 \qquad (2)$$

Substituting (1) into (2),

$$j360 - 3\mathbf{V}_2 - 3\mathbf{V}_2 + j\mathbf{V}_2 = 0$$

$$\mathbf{V}_2 = \frac{j360}{6 - j} = \frac{360}{37}(-1 + j6)$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_2}{40} = \frac{9}{37}(-1 + j6)$$

$$P = \frac{1}{2}|\mathbf{I}_2|^2 R = \frac{1}{2}\left(\frac{9}{\sqrt{37}}\right)^2 (40) = \underline{\mathbf{43.78\ W}}$$

Chapter 11, Problem 9.

For the op amp circuit in Fig. 11.41, $\mathbf{V}_s = 10\angle 30°$ V rms. Find the average power absorbed by the 20-kΩ resistor.

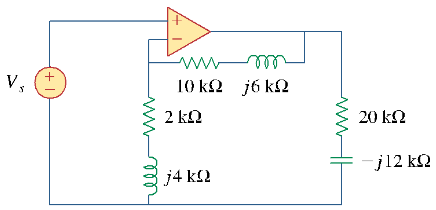


Figure 11.41
For Prob. 11.9.

Chapter 11, Solution 9.
This is a non-inverting op amp circuit. At the output of the op amp,

$$V_o = \left(1 + \frac{Z_2}{Z_1}\right)V_s = \left(1 + \frac{(10 + j6)x10^3}{(2 + j4)x10^3}\right)(8.66 + j5) = 20.712 + j28.124$$

The current through the 20-kς resistor is

$$I_o = \frac{V_o}{20k - j12k} = 0.1411 + j1.491 \text{ mA}$$

$$P = |I_o|^2 R = (1.4975)^2 x10^{-6} x20x10^3 = \underline{44.85 \text{ mW}}$$

Chapter 11, Problem 10.

In the op amp circuit in Fig. 11.42, find the total average power absorbed by the resistors.

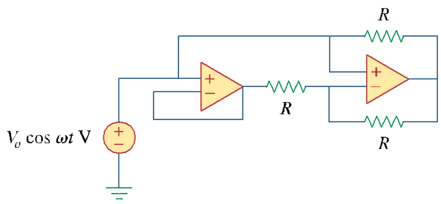


Figure 11.42
For Prob. 11.10.

Chapter 11, Solution 10.

No current flows through each of the resistors. Hence, for each resistor, P = $\underline{\textbf{0 W}}$. It should be noted that the input voltage will appear at the output of each of the op amps.

Chapter 11, Problem 11.

For the network in Fig. 11.43, assume that the port impedance is

$$\mathbf{Z}_{ab} = \frac{R}{\sqrt{1+\omega^2 R^2 C^2}} \angle -\tan^{-1} \omega RC$$

Find the average power consumed by the network when $R = 10\ \text{k}\Omega$, $C = 200\ \text{nF}$, and $i = 2\ \sin(377t + 22°)$ mA.

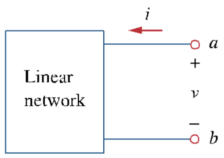


Figure 11.43
For Prob. 11.11.

Chapter 11, Solution 11.

$$\omega = 377, \qquad R = 10^4, \qquad C = 200 \times 10^{-9}$$
$$\omega RC = (377)(10^4)(200 \times 10^{-9}) = 0.754$$
$$\tan^{-1}(\omega RC) = 37.02°$$

$$Z_{ab} = \frac{10k}{\sqrt{1+(0.754)^2}} \angle -37.02° = 7.985\angle -37.02°\ \text{k}\Omega$$

$$i(t) = 2\sin(377t + 22°) = 2\cos(377t - 68°)\ \text{mA}$$
$$I = 2\angle -68°$$

$$S = I_{rms}^2 Z_{ab} = \left(\frac{2\times 10^{-3}}{\sqrt{2}}\right)^2 (7.985\angle -37.02°)\times 10^3$$
$$S = 15.97\angle -37.02°\ \text{mVA}$$

$$P = |S|\cos(37.02) = \underline{\mathbf{12.751\ mW}}$$

Chapter 11, Problem 12.

For the circuit shown in Fig. 11.44, determine the load impedance **Z** for maximum power transfer (to **Z**). Calculate the maximum power absorbed by the load.

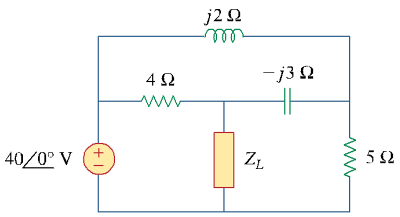
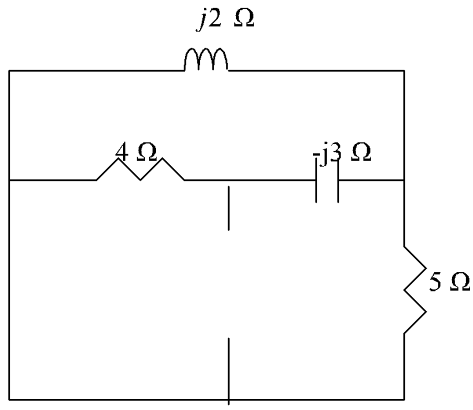


Figure 11.44
For Prob. 11.12.

Chapter 11, Solution 12.

We find the Thevenin impedance using the circuit below.



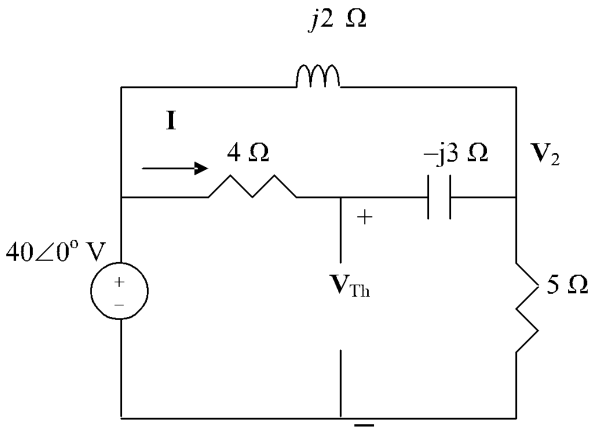
We note that the inductor is in parallel with the 5-Ω resistor and the combination is in series with the capacitor. That whole combination is in parallel with the 4-Ω resistor.

Thus,

$$Z_{Thev} = \frac{4\left(-j3+\frac{5xj2}{5+j2}\right)}{4-j3+\frac{5xj2}{5+j2}} = \frac{4(0.6896-j1.2758)}{4.69-j1.2758} = \frac{4(1.4502\angle-61.61°)}{4.86\angle-15.22°}$$
$$= 1.1936\angle-46.39°$$

$Z_{Thev} = 0.8233 - j0.8642$ or $Z_L = $ **0.8233 + j0.8642Ω**.

We obtain V_{Th} using the circuit below. We apply nodal analysis.



$$\frac{V_2-40}{4-j3}+\frac{V_2-40}{j2}+\frac{V_2-0}{5}=0$$
$$(0.16+j0.12-j0.5+0.2)V_2 = (0.16+j0.12-j0.5)40$$
$$(0.5235\angle-46.55°)V_2 = (0.4123\angle-67.17°)40$$

Thus, $V_2 = 31.5\angle{-20.62}°V = 29.48 - j11.093V$

$I = (40 - V_2)/(4 - j3) = (40 - 29.48 + j11.093)/(4 - j3)$
$= 15.288\angle46.52°/5\angle{-36.87}° = 3.058\angle83.39° = 0.352 + j3.038$

$V_{Thev} = 40 - 4I = 40 - 1.408 - j12.152 = 38.59 - j12.152V$
$= 40.46\angle{-17.479}°V$

We can check our value of V_{Thev} by letting $V_1 = V_{Thev}$. Now we can use nodal analysis to solve for V_1.

At node 1,

$$\frac{V_1 - 40}{4} + \frac{V_1 - V_2}{-j3} + \frac{V_2 - 0}{5} = 0 \rightarrow (0.25 + j0.3333)V_1 + (0.2 - j0.3333)V_2 = 10$$

At node 2,

$$\frac{V_2 - V_1}{-j3} + \frac{V_2 - 40}{j2} = 0 \rightarrow -j0.3333V_1 + (-j0.1667)V_2 = -j20$$

```
>> Z=[(0.25+0.3333i),-0.3333i;-0.3333i,(0.2-0.1667i)]

Z =

  0.2500 + 0.3333i        0 - 0.3333i
       0 - 0.3333i   0.2000 - 0.1667i

>> I=[10;-20i]

I =

  10.0000
       0 -20.0000i

>> V=inv(Z)*I

V =

  38.5993 -12.1459i
  29.4890 -11.0952i
```

Please note, these values check with the ones obtained above.

To calculate the maximum power to the load,

$$|I_L|_{rms} = (40.46/(2x0.8233))/1.4141 = 17.376A$$

$$P_{avg} = (|I_L|_{rms})^2 0.8233 = \underline{\mathbf{248.58\ W}}.$$

Chapter 11, Problem 13.

The Thevenin impedance of a source is $\mathbf{Z}_{\text{Th}} = 120 + j60\,\Omega$, while the peak Thevenin voltage is $\mathbf{V}_{\text{Th}} = 110 + j0\text{ V}$. Determine the maximum available average power from the source.

Chapter 11, Solution 13.

For maximum power transfer to the load, $Z_L = 120 - j60\Omega$.

$I_{Lrms} = 110/(240x1.4142) = 0.3241A$

$$P_{avg} = |I_{Lrms}|^2 120 = \underline{\mathbf{12.605\ W}}.$$

Chapter 11, Problem 14.

It is desired to transfer maximum power to the load **Z** in the circuit of Fig. 11.45. Find **Z** and the maximum power. Let $i_s = 5\cos 40t$ A .

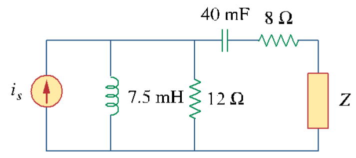


Figure 11.45
For Prob. 11.14.

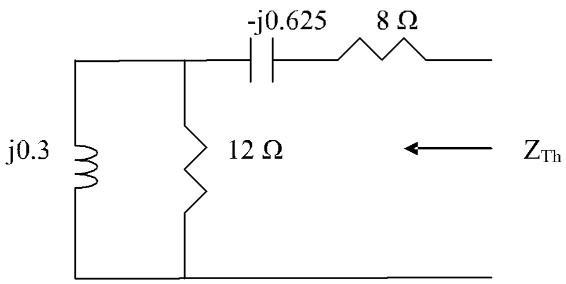
Chapter 11, Solution 14.

We find the Thevenin equivalent at the terminals of Z.

$$40 \text{ mF} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j40x40x10^{-3}} = j0.625$$

$$7.5 \text{ mH} \longrightarrow j\omega L = j40x7.5x10^{-3} = j0.3$$

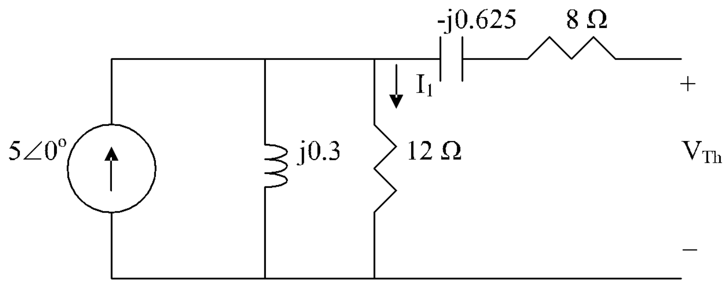
To find Z_{Th}, consider the circuit below.



$$Z_{Th} = 8 - j0.625 + 12 // j0.3 = 8 - j0.625 + \frac{12x0.3}{12+0.3} = 8.0075 - j0.3252$$

$Z_L = (Z_{Thev})^* =$ **8.008 + j0.3252Ω**.

To find V_{Th}, consider the circuit below.



By current division,

$$I_1 = 5(j0.3)/(12+j0.3) = 1.5\angle 90^\circ/12.004\angle 1.43^\circ = 0.12496\angle 88.57^\circ$$

$$= 0.003118 + j0.12492\text{A}$$

$$V_{Thev\ rms} = 12I_1/\sqrt{2} = 1.0603\angle 88.57^\circ\text{V}$$

$$I_{Lrms} = 1.0603\angle 88.57^\circ/2(8.008) = 66.2\angle 88.57^\circ\text{mA}$$

$$P_{avg} = |I_{Lrms}|^2 8.008 = \mathbf{35.09\ mW}.$$

Chapter 11, Problem 15.

In the circuit of Fig. 11.46, find the value of $\mathbf{Z}_L$ that will absorb the maximum power and the value of the maximum power.

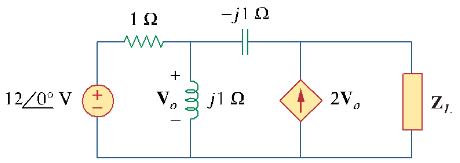
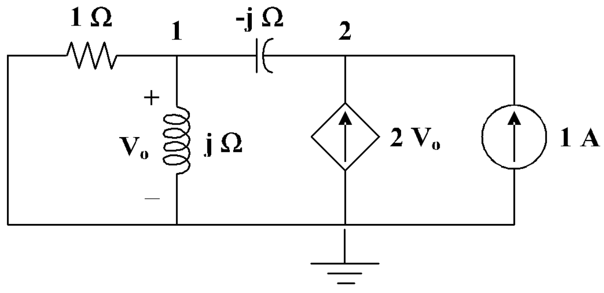


Figure 11.46
For Prob. 11.15.

Chapter 11, Solution 15.

To find $\mathbf{Z}_{Th}$, insert a 1-A current source at the load terminals as shown in Fig. (a).



(a)

At node 1,

$$\frac{\mathbf{V}_o}{1} + \frac{\mathbf{V}_o}{j} = \frac{\mathbf{V}_2 - \mathbf{V}_o}{-j} \longrightarrow \mathbf{V}_o = j\mathbf{V}_2 \qquad (1)$$

At node 2,

$$1 + 2\mathbf{V}_o = \frac{\mathbf{V}_2 - \mathbf{V}_o}{-j} \longrightarrow 1 = j\mathbf{V}_2 - (2+j)\mathbf{V}_o \qquad (2)$$

Substituting (1) into (2),

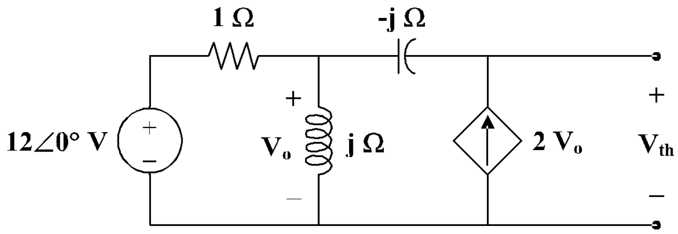
$$1 = j\mathbf{V}_2 - (2+j)(j)\mathbf{V}_2 = (1-j)\mathbf{V}_2$$

$$\mathbf{V}_2 = \frac{1}{1-j}$$

$$\mathbf{Z}_{Th} = \frac{\mathbf{V}_2}{1} = \frac{1+j}{2} = 0.5 + j0.5$$

$$\mathbf{Z}_L = \mathbf{Z}_{Th}^* = \underline{\mathbf{0.5 - j0.5\,\Omega}}$$

We now obtain $\mathbf{V}_{Th}$ from Fig. (b).



(b)

$$2\mathbf{V}_o + \frac{12 - \mathbf{V}_o}{1} = \frac{\mathbf{V}_o}{j}$$

$$\mathbf{V}_o = \frac{-12}{1 + j}$$

$$-\mathbf{V}_o - (-j \times 2\mathbf{V}_o) + \mathbf{V}_{Th} = 0$$

$$\mathbf{V}_{Th} = (1 - j2)\mathbf{V}_o = \frac{(-12)(1 - j2)}{1 + j}$$

$$P_{max} = \frac{|\mathbf{V}_{Th}|^2}{8R_L} = \frac{\left(\frac{12\sqrt{5}}{\sqrt{2}}\right)^2}{(8)(0.5)} = \underline{\mathbf{90\ W}}$$

Chapter 11, Problem 16.

For the circuit of Fig. 11.47, find the maximum power delivered to the load $\mathbf{Z}_L$.

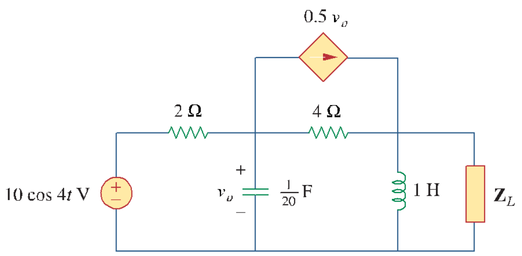
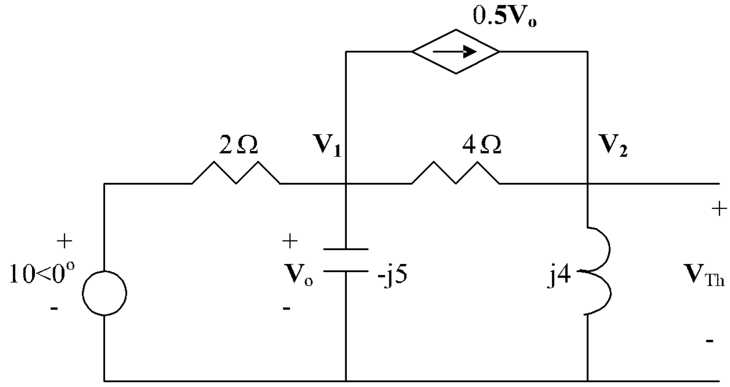


Figure 11.47
For Prob. 11.16.

Chapter 11, Solution 16.

$$\omega = 4, \quad 1\text{H} \longrightarrow j\omega L = j4, \quad 1/20\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j4x1/20} = -j5$$

We find the Thevenin equivalent at the terminals of Z_L. To find V_{Th}, we use the circuit shown below.



At node 1,

$$\frac{10 - V_1}{2} = \frac{V_1}{-j5} + 0.5V_1 + \frac{V_1 - V_2}{4} \longrightarrow 5 = V_1(1.25 + j0.2) - 0.25V_2 \qquad (1)$$

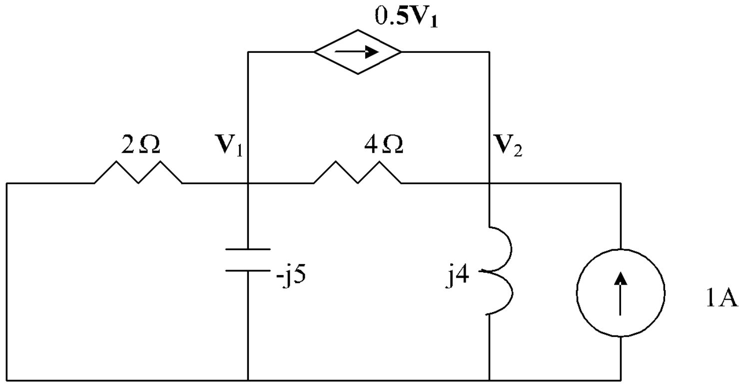
At node 2,

$$\frac{V_1 - V_2}{4} + 0.25V_1 = \frac{V_2}{j4} \longrightarrow 0 = 0.5V_1 + V_2(-0.25 + j0.25) \qquad (2)$$

Solving (1) and (2) leads to

$V_{Th} = V_2 = 6.1947 + j7.0796 = 9.4072\angle 48.81^o$

To obtain R_{Th}, consider the circuit shown below. We replace Z_L by a 1-A current source.



At node 1,

$$\frac{V_1}{2} + \frac{V_1}{-j5} + 0.25V_1 + \frac{V_1 - V_2}{4} = 0 \longrightarrow 0 = V_1(1 + j0.2) - 0.25V_2$$
(3)

At node 2,

$$1 + \frac{V_1 - V_2}{4} + 0.25V_1 = \frac{V_2}{j4} \longrightarrow -1 = 0.5V_1 + V_2(-0.25 + j0.25) \qquad (4)$$

Solving (1) and (2) gives

$$Z_{Th} = \frac{V_2}{1} = 1.9115 + j3.3274 = 3.8374\angle 60.12^o$$

$$P_{max} = \frac{|V_{Th}|^2}{8R_{Th}} = \frac{9.4072^2}{8x1.9115} = \underline{5.787 \text{ W}}$$

Chapter 11, Problem 17.

Calculate the value of $\mathbf{Z}_L$ in the circuit of Fig. 11.48 in order for $\mathbf{Z}_L$ to receive maximum average power. What is the maximum average power received by $\mathbf{Z}_L$?

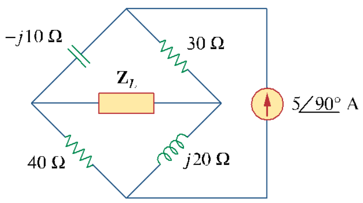
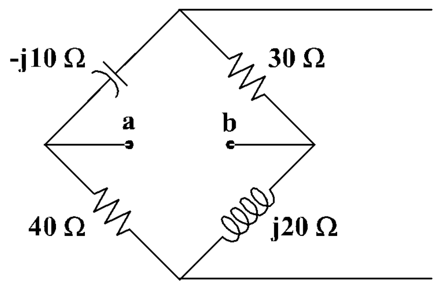


Figure 11.48
For Prob. 11.17.

Chapter 11, Solution 17.

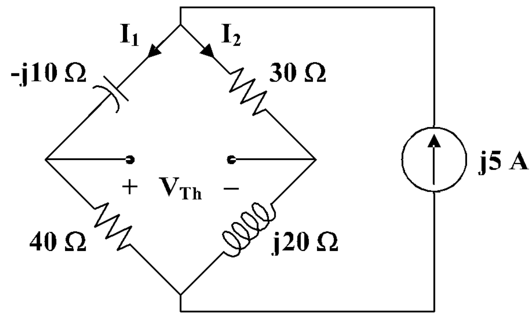
We find R_{Th} at terminals a-b following Fig. (a).



(a)

$$\mathbf{Z}_{Th} = -j10 + 30 \,||\, j20 + 40 = \frac{(30 - j10)(40 + j20)}{70 + j10} = \underline{\mathbf{20\ \Omega}} = Z_L$$

We obtain $\mathbf{V}_{Th}$ from Fig. (b).



(b)

Using current division,

$$\mathbf{I}_1 = \frac{30 + j20}{70 + j10}(j5) = -1.1 + j2.3$$

$$\mathbf{I}_2 = \frac{40 - j10}{70 + j10}(j5) = 1.1 + j2.7$$

$$\mathbf{V}_{Th} = 30\mathbf{I}_2 + j10\mathbf{I}_1 = 10 + j70$$

$$P_{max} = \frac{\left|\mathbf{V}_{Th}\right|^2}{8R_L} = \frac{5000}{(8)(20)} = \underline{\mathbf{31.25\ W}}$$

Chapter 11, Problem 18.

Find the value of $\mathbf{Z}_L$ in the circuit of Fig. 11.49 for maximum power transfer.

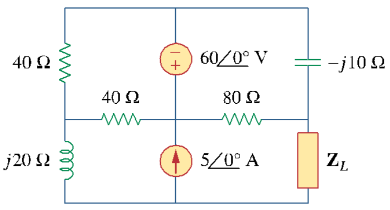
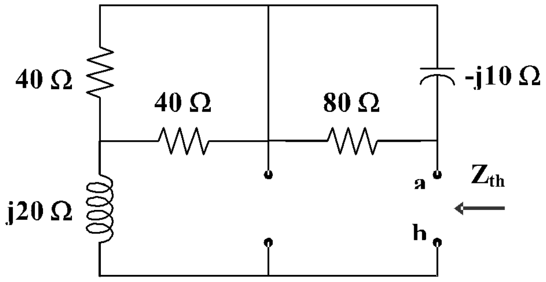


Figure 11.49
For Prob. 11.18.

Chapter 11, Solution 18.

We find $\mathbf{Z}_{Th}$ at terminals a-b as shown in the figure below.



$$\mathbf{Z}_{Th} = j20 + 40 \,||\, 40 + 80 \,||\, (\text{-}j10) = j20 + 20 + \frac{(80)(\text{-}j10)}{80 - j10}$$

$$\mathbf{Z}_{Th} = 21.23 + j10.154$$

$$\mathbf{Z}_L = \mathbf{Z}_{Th}^{*} = \underline{\mathbf{21.23 - j10.15\,\Omega}}$$

Chapter 11, Problem 19.

The variable resistor *R* in the circuit of Fig. 11.50 is adjusted until it absorbs the maximum average power. Find *R* and the maximum average power absorbed.

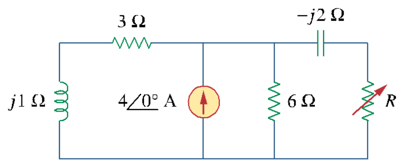


Figure 11.50
For Prob. 11.19.

Chapter 11, Solution 19.

At the load terminals,

$$\mathbf{Z}_{Th} = \text{-j}2 + 6 \,||\, (3+\text{j}) = \text{-j}2 + \frac{(6)(3+\text{j})}{9+\text{j}}$$

$$\mathbf{Z}_{Th} = 2.049 - \text{j}1.561$$

$$R_L = \left|\mathbf{Z}_{Th}\right| = \underline{\mathbf{2.576\Omega}}$$

To get $\mathbf{V}_{Th}$, let $\mathbf{Z} = 6 \,||\, (3+\text{j}) = 2.049 + \text{j}0.439$.

By transforming the current sources, we obtain

$$\mathbf{V}_{Th} = (4\angle 0°)\,\mathbf{Z} = 8.196 + \text{j}1.756$$

$$P_{max} = \left|\frac{8.382}{2.049 - \text{j}1.561 + 2.576}\right|^2 \frac{2.576}{2} = \underline{\mathbf{3.798\ W}}$$

Chapter 11, Problem 20.

The load resistance R_L in Fig. 11.51 is adjusted until it absorbs the maximum average power. Calculate the value of R_L and the maximum average power.

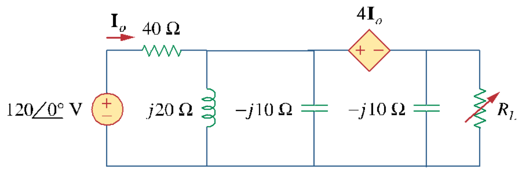
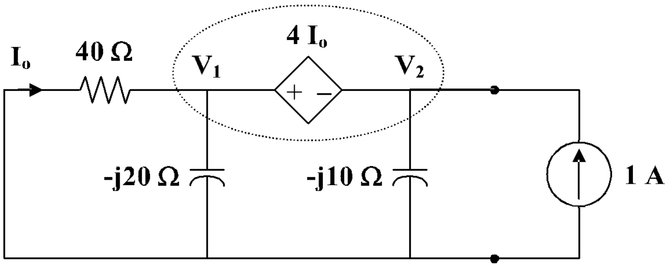


Figure 11.51
For Prob. 11.20.

Chapter 11, Solution 20.

Combine j20 Ω and -j10 Ω to get $j20 \,||\, -j10 = -j20$.

To find $\mathbf{Z}_{Th}$, insert a 1-A current source at the terminals of R_L, as shown in Fig. (a).



(a)

At the supernode,

$$1 = \frac{\mathbf{V}_1}{40} + \frac{\mathbf{V}_1}{-j20} + \frac{\mathbf{V}_2}{-j10}$$

$$40 = (1 + j2)\,\mathbf{V}_1 + j4\,\mathbf{V}_2 \qquad (1)$$

Also, $\mathbf{V}_1 = \mathbf{V}_2 + 4\mathbf{I}_o$, where $\mathbf{I}_o = \frac{-\mathbf{V}_1}{40}$

$$1.1\mathbf{V}_1 = \mathbf{V}_2 \longrightarrow \mathbf{V}_1 = \frac{\mathbf{V}_2}{1.1} \qquad (2)$$

Substituting (2) into (1),

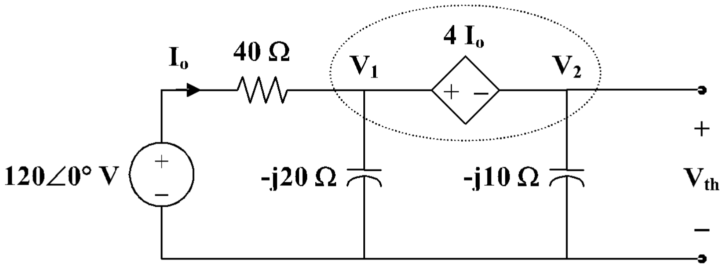
$$40 = (1 + j2)\left(\frac{\mathbf{V}_2}{1.1}\right) + j4\mathbf{V}_2$$

$$\mathbf{V}_2 = \frac{44}{1 + j6.4}$$

$$\mathbf{Z}_{Th} = \frac{\mathbf{V}_2}{1} = 1.05 - j6.71\,\Omega$$

$$R_L = \left|\mathbf{Z}_{Th}\right| = \underline{\mathbf{6.792\,\Omega}}$$

To find $\mathbf{V}_{Th}$, consider the circuit in Fig. (b).



(b)

At the supernode,

$$\frac{120 - \mathbf{V}_1}{40} = \frac{\mathbf{V}_1}{-j20} + \frac{\mathbf{V}_2}{-j10}$$

$$120 = (1 + j2)\mathbf{V}_1 + j4\mathbf{V}_2 \qquad (3)$$

Also, $\mathbf{V}_1 = \mathbf{V}_2 + 4\mathbf{I}_o$, where $\mathbf{I}_o = \dfrac{120 - \mathbf{V}_1}{40}$

$$\mathbf{V}_1 = \frac{\mathbf{V}_2 + 12}{1.1} \qquad (4)$$

Substituting (4) into (3),

$$109.09 - j21.82 = (0.9091 + j5.818)\mathbf{V}_2$$

$$\mathbf{V}_{Th} = \mathbf{V}_2 = \frac{109.09 - j21.82}{0.9091 + j5.818} = 18.893\angle -92.43°$$

$$P_{max} = \left|\frac{18.893}{1.05 - j6.71 + 6.792}\right|^2 \frac{6.792}{2} = \underline{\mathbf{11.379\ W}}$$

Chapter 11, Problem 21.

Assuming that the load impedance is to be purely resistive, what load should be connected to terminals *a-b* of the circuits in Fig. 11.52 so that the maximum power is transferred to the load?

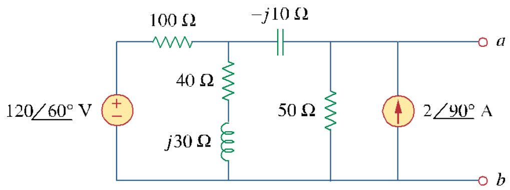
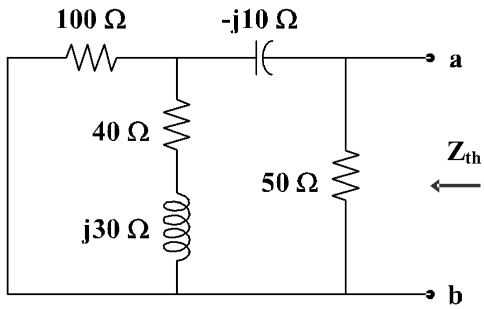


Figure 11.52
For Prob. 11.21.

Chapter 11, Solution 21.

We find $\mathbf{Z}_{Th}$ at terminals a-b, as shown in the figure below.



$$\mathbf{Z}_{Th} = 50 \,||\, [-j10 + 100 \,||\, (40 + j30)]$$

$$\text{where} \quad 100 \,||\, (40 + j30) = \frac{(100)(40 + j30)}{140 + j30} = 31.707 + j14.634$$

$$\mathbf{Z}_{Th} = 50 \,||\, (31.707 + j4.634) = \frac{(50)(31.707 + j4.634)}{81.707 + j4.634}$$

$$\mathbf{Z}_{Th} = 19.5 + j1.73$$

$$R_L = \left| \mathbf{Z}_{Th} \right| = \mathbf{19.58\ \Omega}$$

Chapter 11, Problem 22.

Find the rms value of the offset sine wave shown in Fig. 11.53.

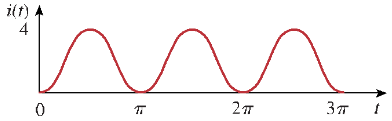


Figure 11.53
For Prob. 11.22.

Chapter 11, Solution 22.

$$i(t) = 4\sin t, \qquad 0 < t < \pi$$

$$I^2_{rms} = \frac{1}{\pi}\int_0^{\pi} 16\sin^2 t dt = \frac{16}{\pi}\left(\frac{t}{2} - \frac{\sin 2t}{4}\right)\Big|_0^{\pi} = \frac{16}{\pi}\left(\frac{\pi}{2} - 0\right) = 8$$

$I_{rms} = \sqrt{8} = \underline{2.828 \text{ A}}$

Chapter 11, Problem 23.

Determine the rms value of the voltage shown in Fig. 11.54.

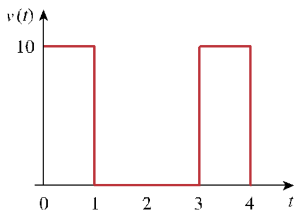


Figure 11.54
For Prob. 11.23.

Chapter 11, Solution 23.

$$V^2_{rms} = \frac{1}{T}\int_0^T v^2(t)dt = \frac{1}{3}\int_0^1 10^2 dt = \frac{100}{3}$$

$$V_{rms} = \mathbf{\underline{5.774 \text{ V}}}$$

Chapter 11, Problem 24.

Determine the rms value of the waveform in Fig. 11.55.

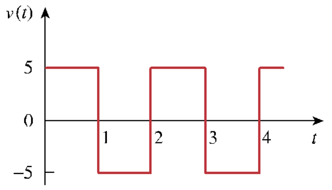


Figure 11.55
For Prob. 11.24.

Chapter 11, Solution 24.

$$T = 2, \qquad v(t) = \begin{cases} 5, & 0 < t < 1 \\ -5, & 1 < t < 2 \end{cases}$$

$$V_{rms}^2 = \frac{1}{2}\left[\int_0^1 5^2\,dt + \int_1^2 (-5)^2\,dt\right] = \frac{25}{2}[1+1] = 25$$

$$V_{rms} = \underline{\mathbf{5\ V}}$$

Chapter 11, Problem 25.

Find the rms value of the signal shown in Fig. 11.56.

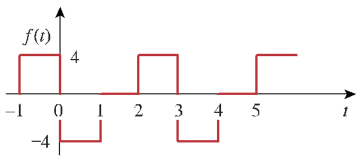


Figure 11.56
For Prob. 11.25.

Chapter 11, Solution 25.

$$f_{rms}^2 = \frac{1}{T}\int_0^T f^2(t)dt = \frac{1}{3}\left[\int_0^1 (-4)^2\,dt + \int_1^2 0dt + \int_2^3 4^2\,dt\right]$$

$$= \frac{1}{3}[16 + 0 + 16] = \frac{32}{3}$$

$$f_{rms} = \sqrt{\frac{32}{3}} = \underline{3.266}$$

Chapter 11, Problem 26.

Find the effective value of the voltage waveform in Fig. 11.57.

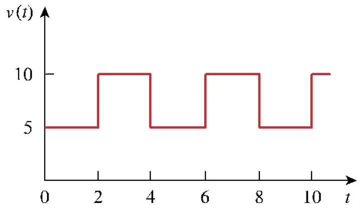


Figure 11.57
For Prob. 11.26.

Chapter 11, Solution 26.

$$T = 4, \qquad v(t) = \begin{cases} 5 & 0 < t < 2 \\ 10 & 2 < t < 4 \end{cases}$$

$$V_{rms}^2 = \frac{1}{4}\left[\int_0^2 5^2\ dt + \int_2^4 (10)^2\ dt\right] = \frac{1}{4}[50 + 200] = 62.5$$

$$V_{rms} = \underline{\mathbf{7.906\ V}}$$

Chapter 11, Problem 27.

Calculate the rms value of the current waveform of Fig. 11.58.

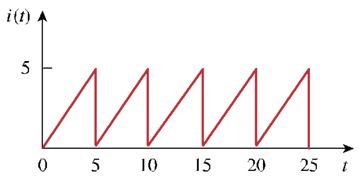


Figure 11.58
For Prob. 11.27.

Chapter 11, Solution 27.

$$T = 5, \qquad i(t) = t, \quad 0 < t < 5$$

$$I_{rms}^2 = \frac{1}{5}\int_0^5 t^2\ dt = \frac{1}{5}\cdot\frac{t^3}{3}\Big|_0^5 = \frac{125}{15} = 8.333$$

$$I_{rms} = \underline{\mathbf{2.887\ A}}$$

Chapter 11, Problem 28.

Find the rms value of the voltage waveform of Fig. 11.59 as well as the average power absorbed by a 2-Ω resistor when the voltage is applied across the resistor.

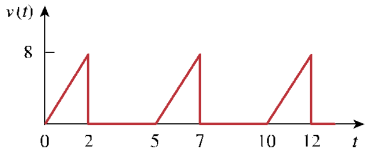


Figure 11.59
For Prob. 11.28.

Chapter 11, Solution 28.

$$V_{rms}^2 = \frac{1}{5}\left[\int_0^2 (4t)^2\, dt + \int_2^5 0^2\, dt\right]$$

$$V_{rms}^2 = \frac{1}{5}\cdot\frac{16t^3}{3}\Big|_0^2 = \frac{16}{15}(8) = 8.533$$

$$V_{rms} = \underline{\mathbf{2.92\ V}}$$

$$P = \frac{V_{rms}^2}{R} = \frac{8.533}{2} = \underline{\mathbf{4.267\ W}}$$

Chapter 11, Problem 29.

Calculate the effective value of the current waveform in Fig. 11.60 and the average power delivered to a 12-Ω resistor when the current runs through the resistor.

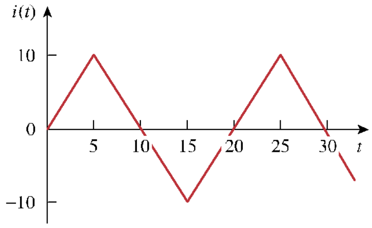


Figure 11.60
For Prob. 11.29.

Chapter 11, Solution 29.

$$T = 20, \qquad i(t) = \begin{cases} 20 - 2t & 5 < t < 15 \\ -40 + 2t & 15 < t < 25 \end{cases}$$

$$I_{eff}^2 = \frac{1}{20}\left[\int_5^{15} (20-2t)^2\,dt + \int_{15}^{25} (-40+2t)^2\,dt\right]$$

$$I_{eff}^2 = \frac{1}{5}\left[\int_5^{15} (100 - 20t + t^2)\,dt + \int_{15}^{25} (t^2 - 40t + 400)\,dt\right]$$

$$I_{eff}^2 = \frac{1}{5}\left[\left(100t - 10t^2 + \frac{t^3}{3}\right)\Big|_5^{15} + \left(\frac{t^3}{3} - 20t^2 + 400t\right)\Big|_{15}^{25}\right]$$

$$I_{eff}^2 = \frac{1}{5}[83.33 + 83.33] = 33.332$$

$$I_{eff} = \underline{\mathbf{5.773\ A}}$$

$$P = I_{eff}^2 R = \underline{\mathbf{400\ W}}$$

Chapter 11, Problem 30.

Compute the rms value of the waveform depicted in Fig. 11.61.

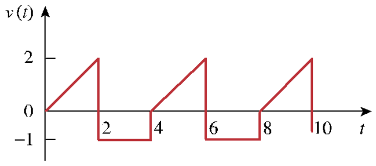


Figure 11.61
For Prob. 11.30.

Chapter 11, Solution 30.

$$v(t) = \begin{cases} t & 0 < t < 2 \\ -1 & 2 < t < 4 \end{cases}$$

$$V_{rms}^2 = \frac{1}{4}\left[\int_0^2 t^2\, dt + \int_2^4 (-1)^2\, dt\right] = \frac{1}{4}\left[\frac{8}{3} + 2\right] = 1.1667$$

$$V_{rms} = \underline{\mathbf{1.08\ V}}$$

Chapter 11, Problem 31.

Find the rms value of the signal shown in Fig. 11.62.

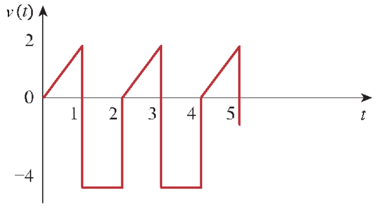


Figure 11.62
For Prob. 11.31.

Chapter 11, Solution 31.

$$V^2{}_{rms} = \frac{1}{2}\int_0^2 v(t)dt = \frac{1}{2}\left[\int_0^1 (2t)^2 dt + \int_1^2 (-4)^2 dt\right] = \frac{1}{2}\left[\frac{4}{3} + 16\right] = 8.6667$$

$$V_{rms} = \underline{2.944\ \text{V}}$$

Chapter 11, Problem 32.

Obtain the rms value of the current waveform shown in Fig. 11.63.

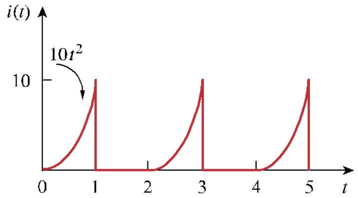


Figure 11.63
For Prob. 11.32.

Chapter 11, Solution 32.

$$I_{rms}^2 = \frac{1}{2}\left[\int_0^1 (10t^2)^2 \ dt + \int_1^2 0 \ dt \right]$$

$$I_{rms}^2 = 50\int_0^1 t^4 \ dt = 50 \cdot \frac{t^5}{5}\Big|_0^1 = 10$$

$$I_{rms} = \underline{\mathbf{3.162 \ A}}$$

Chapter 11, Problem 33.

Determine the rms value for the waveform in Fig. 11.64.

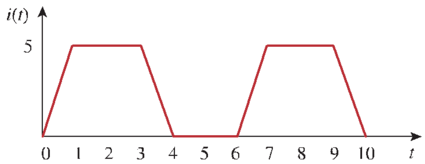


Figure 11.64
For Prob. 11.33.

Chapter 11, Solution 33.

$$I_{rms}^2 = \frac{1}{T}\int_0^T i^2(t)dt = \frac{1}{6}\left[\int_0^1 25t^2dt + \int_1^3 25dt + \int_3^4 (-5t+20)^2 dt \right]$$

$$I_{rms}^2 = \frac{1}{6}\left[25\frac{t^3}{3}\Big|_0^1 + 25(3-1) + (25\frac{t^3}{3} - 100t^2 + 400t)\Big|_3^4 \right] = 11.1056$$

$$\underline{I_{rms} = 3.3325 \text{ A}} = \underline{\mathbf{3.332 \ A}}$$

Chapter 11, Problem 34.

Find the effective value of $f(t)$ defined in Fig. 11.65.

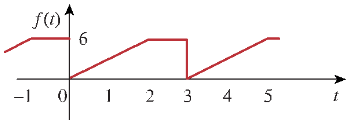


Figure 11.65
For Prob. 11.34.

Chapter 11, Solution 34.

$$f_{rms}^2 = \frac{1}{T}\int_0^T f^2(t)dt = \frac{1}{3}\left[\int_0^2 (3t)^2 dt + \int_2^3 6^2 dt\right]$$

$$= \frac{1}{3}\left[\left.\frac{9t^3}{3}\right|_0^2 + 36\right] = 20$$

$$f_{rms} = \sqrt{20} = \underline{4.472}$$

Chapter 11, Problem 35.

One cycle of a periodic voltage waveform is depicted in Fig. 11.66. Find the effective value of the voltage. Note that the cycle starts at $t = 0$ and ends at $t = 6$ s.

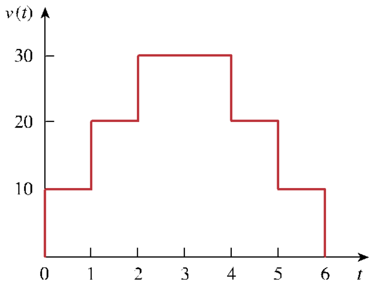


Figure 11.66
For Prob. 11.35.

Chapter 11, Solution 35.

$$V_{rms}^2 = \frac{1}{6}\left[\int_0^1 10^2\, dt + \int_1^2 20^2\, dt + \int_2^4 30^2\, dt + \int_4^5 20^2\, dt + \int_5^6 10^2\, dt\right]$$

$$V_{rms}^2 = \frac{1}{6}[100 + 400 + 1800 + 400 + 100] = 466.67$$

$$V_{rms} = \underline{\mathbf{21.6\ V}}$$

Chapter 11, Problem 36.

Calculate the rms value for each of the following functions:

(a) $i(t) = 10$ A (b) $v(t) = 4 + 3\cos 5t$ V

(c) $i(t) = 8 - 6\sin 2t$ A (d) $v(t) = 5\sin t + 4\cos t$ V

Chapter 11, Solution 36.

(a) $I_{rms} = \underline{10\text{ A}}$

(b) $V^2_{rms} = 4^2 + \left(\frac{3}{\sqrt{2}}\right)^2 \longrightarrow V_{rms} = \sqrt{16 + \frac{9}{2}} = \underline{4.528\text{ V}}$ (checked)

(c) $I_{rms} = \sqrt{64 + \frac{36}{2}} = \underline{9.055\text{ A}}$

$V_{rms} = \sqrt{\frac{25}{2} + \frac{16}{2}} = \underline{4.528\text{ V}}$

Chapter 11, Problem 37.

Calculate the rms value of the sum of these three currents:

$i_1 = 8$, $i_2 = 4\sin(t + 10^\circ)$, $i_3 = 6\cos(2t + 30^\circ)$ A

Chapter 11, Solution 37.

$$i = i_1 + i_2 + i_3 = 8 + 4\sin(t + 10^o) + 6\cos(2t + 30^o)$$

$$I_{rms} = \sqrt{I^2_{1rms} + I^2_{2rms} + I^2_{3rms}} = \sqrt{64 + \frac{16}{2} + \frac{36}{2}} = \sqrt{90} = \underline{9.487\text{ A}}$$

Chapter 11, Problem 38.

For the power system in Fig. 11.67, find: (a) the average power, (b) the reactive power, (c) the power factor. Note that 220 V is an rms value.

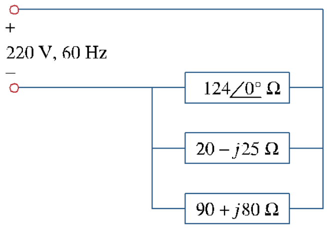


Figure 11.67
For Prob. 11.38.

Chapter 11, Solution 38.

$$S_1 = \frac{V^2}{Z_1^*} = \frac{220^2}{124} = 390.32$$

$$S_2 = \frac{V^2}{Z_2^*} = \frac{220^2}{20+j25} = 944.4 - j1180.5$$

$$S_3 = \frac{V^2}{Z_3^*} = \frac{220^2}{90-j80} = 300 + j267.03$$

$$S = S_1 + S_2 + S_3 = 1634.7 - j913.47 = 1872.6 < -29.196^o \text{ VA}$$

(a) P = Re(S) = **<u>1634.7 W</u>**

(b) Q = Im (S) = **<u>913.47 VA (leading)</u>**

(c) pf = cos (29.196°) = **<u>0.8732</u>**

Chapter 11, Problem 39.

An ac motor with impedance $\mathbf{Z}_L = 4.2 + j3.6\ \Omega$ is supplied by a 220-V, 60-Hz source. (a) Find pf, P, and Q. (b) Determine the capacitor required to be connected in parallel with the motor so that the power factor is corrected to unity.

Chapter 11, Solution 39.

(a) $Z_L = 4.2 + j3.6 = 5.5317\angle 40.6^o$

pf = cos 40.6 = <u>0.7592</u>

$$S = \frac{V_{rms}^2}{Z^*} = \frac{220^2}{5.5317\angle -40.6^o} = 6.643 + j5.694 \text{ kVA}$$

P = 6.643 kW

Q = 5.695 kVAR

(b) $C = \dfrac{P(\tan\theta_1 - \tan\theta_2)}{\omega V_{rms}^2} = \dfrac{6.643x10^3(\tan 40.6^o - \tan 0^o)}{2\pi x60x220^2} = \underline{312\ \mu\text{F}}$,

{It is important to note that this capacitor will see a peak voltage of 220$\sqrt{2}$ = 311.08V, this means that the specifications on the capacitor must be at least this or greater!}

Chapter 11, Problem 40.

A load consisting of induction motors is drawing 80 kW from a 220-V, 60-Hz power line at a pf of 0.72 lagging. Find the capacitance of a capacitor required to raise the pf to 0.92.

Chapter 11, Solution 40.

$pf1 = 0.72 = \cos\theta_1 \longrightarrow \theta_1 = 43.94^0$

$pf2 = 0.92 = \cos\theta_2 \longrightarrow \theta_2 = 23.07^0$

$$C = \frac{P(\tan\theta_1 - \tan\theta_2)}{\omega V_{rms}^2} = \frac{80x10^3(0.9637 - 0.4259)}{2\pi x60x(220)^2} = \underline{2.4\text{ mF}},$$

{Again, we need to note that this capacitor will be exposed to a peak voltage of 311.08V and must be rated to at least this level, preferably higher!}

Chapter 11, Problem 41.

Obtain the power factor for each of the circuits in Fig. 11.68. Specify each power factor as leading or lagging.

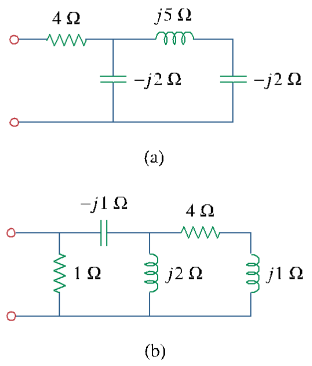


Figure 11.68
For Prob. 11.41.

Chapter 11, Solution 41.

(a) $$-j2 \parallel (j5 - j2) = -j2 \parallel -j3 = \frac{(-j2)(-j3)}{j} = -j6$$

$$\mathbf{Z}_T = 4 - j6 = 7.211\angle -56.31°$$

$$\text{pf} = \cos(-56.31°) = \underline{\mathbf{0.5547 \quad (leading)}}$$

(b) $$j2 \parallel (4 + j) = \frac{(j2)(4 + j)}{4 + j3} = 0.64 + j1.52$$

$$\mathbf{Z} = 1 \parallel (0.64 + j1.52 - j) = \frac{0.64 + j0.44}{1.64 + j0.44} = 0.4793\angle 21.5°$$

$$\text{pf} = \cos(21.5°) = \underline{\mathbf{0.9304 \quad (lagging)}}$$

Chapter 11, Problem 42.

A 110-V rms, 60-Hz source is applied to a load impedance **Z**. The apparent power entering the load is 120 VA at a power factor of 0.707 lagging.

(a) Calculate the complex power.
(b) Find the rms current supplied to the load.
(c) Determine **Z**.
(d) Assuming that $\mathbf{Z} = R + j\omega L$, find the values of R and L.

Chapter 11, Solution 42.

(a) S=120, $pf = 0.707 = \cos\theta \longrightarrow \theta = 45^o$

$$S = S\cos\theta + jS\sin\theta = \underline{84.84 + j84.84 \text{ VA}}$$

(b) $S = V_{rms} I_{rms} \longrightarrow I_{rms} = \frac{S}{V_{rms}} = \frac{120}{110} = \underline{1.091 \text{ A rms}}$

(c) $S = I_{rms}^2 Z \longrightarrow Z = \frac{S}{I_{rms}^2} = \underline{71.278 + j71.278\ \Omega}$

(d) If Z = R + jωL, then R = **<u>71.278 Ω</u>**

$$\omega L = 2\pi f L = 71.278 \longrightarrow L = \frac{71.278}{2\pi x 60} = \underline{0.1891 \text{ H}}$$

Chapter 11, Problem 43.

The voltage applied to a 10-Ω resistor is

$v(t) = 5 + 3\cos(t + 10°) + \cos(2t + 30°)$ V

(a) Calculate the rms value of the voltage.
(b) Determine the average power dissipated in the resistor.

Chapter 11, Solution 43.

(a) $V_{rms} = \sqrt{V^2{}_{1rms} + V^2{}_{2rms} + V^2{}_{3rms}} = \sqrt{25 + \frac{9}{2} + \frac{1}{2}} = \sqrt{30} = \underline{5.477 \text{ V}}$

(b) $P = \frac{V^2{}_{rms}}{R} = 30/10 = \underline{3 \text{ W}}$

Chapter 11, Problem 44.

Find the complex power delivered by v_s to the network in Fig. 11.69. Let $v_s = 100 \cos 2000t$ V.

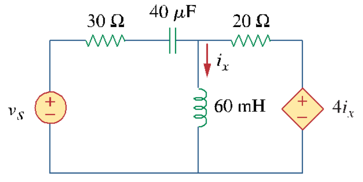


Figure 11.69
For Prob. 11.44.

Chapter 11, Solution 44.

$$40\mu F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j2000x40x10^{-6}} = -j12.5$$

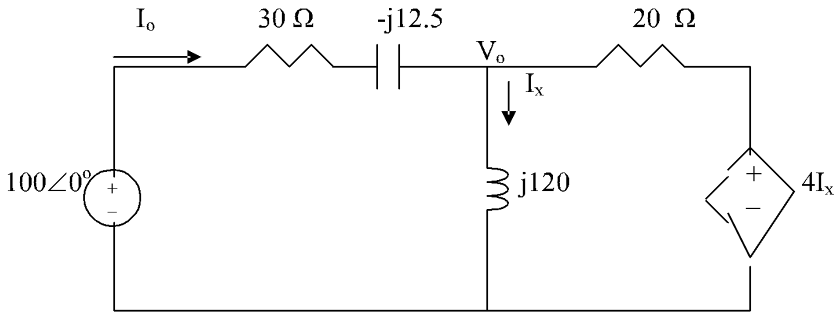
$$60mH \longrightarrow j\omega L = j2000x60x10^{-3} = j120$$

We apply nodal analysis to the circuit shown below.

$$\frac{100 - V_o}{30 - j12.5} + \frac{4I_x - V_o}{20} = \frac{V_o}{j120}$$

But $I_x = \dfrac{V_o}{j120}$. Solving for V_o leads to

$$V_o = 2.9563 + j1.126$$



$$I_o = \frac{100 - V_o}{30 - j12.5} = 2.7696 + j1.1165$$

$$S = \frac{1}{2}V_s I_o^* = \frac{1}{2}(100)(2.7696 - j.1165) = \underline{138.48 - j55.825 \text{ VA}}$$

Chapter 11, Problem 45.

The voltage across a load and the current through it are given by

$v(\text{t}) = 20 + 60 \cos 100t$ V
$i(\text{t}) = 1 - 0.5 \sin 100t$ A

Find:

(a) the rms values of the voltage and of the current
(b) the average power dissipated in the load

Chapter 11, Solution 45.

(a) $V^2_{rms} = 20^2 + \dfrac{60^2}{2} = 2200 \quad \longrightarrow \quad V_{rms} = \underline{46.9\text{ V}}$

$$I_{rms} = \sqrt{1^2 + \frac{0.5^2}{2}} = \sqrt{1.125} = \underline{1.061A}$$

(b) p(t) = v(t)i(t) = 20 + 60cos100t – 10sin100t – 30(sin100t)(cos100t); clearly the average power = **20W**.

Chapter 11, Problem 46.

For the following voltage and current phasors, calculate the complex power, apparent power, real power, and reactive power. Specify whether the pf is leading or lagging.

(a) $\mathbf{V} = 220\angle 30^\circ$ V rms, $\mathbf{I} = 0.5\angle 60^\circ$ A rms

(b) $\mathbf{V} = 250\angle -10^\circ$ V rms,
$\mathbf{I} = 6.2\angle -25^\circ$ A rms

(c) $\mathbf{V} = 80\angle 0^\circ$ V rms, $\mathbf{I} = 2.4\angle -15^\circ$ A rms

(d) $\mathbf{V} = 160\angle 45^\circ$ V rms, $\mathbf{I} = 8.5\angle 90^\circ$ A rms

Chapter 11, Solution 46.

(a) $\mathbf{S} = \mathbf{V}\mathbf{I}^* = (220\angle 30°)(0.5\angle -60°) = 110\angle -30°$

$\mathbf{S} = \underline{\mathbf{95.26 - j55\ VA}}$

Apparent power = $\underline{\mathbf{110\ VA}}$

Real power = $\underline{\mathbf{95.26\ W}}$

Reactive power = $\underline{\mathbf{55\ VAR}}$

pf is **leading** because current leads voltage

(b) $\mathbf{S} = \mathbf{V}\mathbf{I}^* = (250\angle -10°)(6.2\angle 25°) = 1550\angle 15°$

$\mathbf{S} = \underline{\mathbf{1497.2 + j401.2\ VA}}$

Apparent power = $\underline{\mathbf{1550\ VA}}$

Real power = $\underline{\mathbf{1497.2\ W}}$

Reactive power = $\underline{\mathbf{401.2\ VAR}}$

pf is **lagging** because current lags voltage

(c) $\mathbf{S} = \mathbf{V}\mathbf{I}^* = (120\angle 0°)(2.4\angle 15°) = 288\angle 15°$

$\mathbf{S} = \underline{\mathbf{278.2 + j74.54\ VA}}$

Apparent power = $\underline{\mathbf{288\ VA}}$

Real power = $\underline{\mathbf{278.2\ W}}$

Reactive power = $\underline{\mathbf{74.54\ VAR}}$

pf is **lagging** because current lags voltage

(d) $\mathbf{S} = \mathbf{V}\mathbf{I}^* = (160\angle 45°)(8.5\angle -90°) = 1360\angle -45°$

$\mathbf{S} = \underline{\mathbf{961.7 - j961.7\ VA}}$

Apparent power = $\underline{\mathbf{1360\ VA}}$

Real power = $\underline{\mathbf{961.7\ W}}$

Reactive power = $\underline{\mathbf{-961.7\ VAR}}$

pf is **leading** because current leads voltage

Chapter 11, Problem 47.

For each of the following cases, find the complex power, the average power, and the reactive power:

(a) $v(t) = 112\cos(\omega t + 10^\circ)$ V,
$i(t) = 4\cos(\omega t - 50^\circ)$ A

(b) $v(t) = 160\cos 377t$ V,
$i(t) = 4\cos(377t + 45^\circ)$ A

(c) $\mathbf{V} = 80\angle 60^\circ$ V rms, $\mathbf{Z} = 50\angle 30^\circ\ \Omega$

(d) $\mathbf{I} = 10\angle 60^\circ$ A rms, $\mathbf{Z} = 100\angle 45^\circ\ \Omega$

Chapter 11, Solution 47.

(a) $\mathbf{V} = 112\angle 10^\circ, \qquad \mathbf{I} = 4\angle -50^\circ$

$$\mathbf{S} = \frac{1}{2}\mathbf{V}\mathbf{I}^* = 224\angle 60^\circ = \underline{\mathbf{112 + j194\ VA}}$$

Average power = $\underline{\textbf{112 W}}$

Reactive power = $\underline{\textbf{194 VAR}}$

(b) $\mathbf{V} = 160\angle 0^\circ, \qquad \mathbf{I} = 4\angle 45^\circ$

$$\mathbf{S} = \frac{1}{2}\mathbf{V}\mathbf{I}^* = 320\angle -45^\circ = \underline{\mathbf{226.3 - j226.3}}$$

Average power = $\underline{\textbf{226.3 W}}$
Reactive power = $\underline{\textbf{–226.3 VAR}}$

(c) $$\mathbf{S} = \frac{|\mathbf{V}|^2}{\mathbf{Z}^*} = \frac{(80)^2}{50\angle -30^\circ} = 128\angle 30^\circ = \underline{\mathbf{110.85 + j64}}$$

Average power = $\underline{\textbf{110.85 W}}$
Reactive power = $\underline{\textbf{64 VAR}}$

(d) $$\mathbf{S} = |\mathbf{I}|^2\mathbf{Z} = (100)(100\angle 45^\circ) = \underline{\mathbf{7.071 + j7.071\ kVA}}$$

Average power = $\underline{\textbf{7.071 kW}}$

Reactive power = $\underline{\textbf{7.071 kVAR}}$

Chapter 11, Problem 48.

Determine the complex power for the following cases:

(a) $P = 269$ W, $Q = 150$ VAR (capacitive)
(b) $Q = 2000$ VAR, pf = 0.9 (leading)
(c) $S = 600$ VA, $Q = 450$ VAR (inductive)
(d) $V_{rms} = 220$ V, $P = 1$ kW,
$|\mathbf{Z}| = 40\ \Omega$ (inductive)

Chapter 11, Solution 48.

(a) $S = P - jQ = \underline{\mathbf{269 - j150\ VA}}$

(b) $\text{pf} = \cos\theta = 0.9 \longrightarrow \theta = 25.84°$

$$Q = S\sin\theta \longrightarrow S = \frac{Q}{\sin\theta} = \frac{2000}{\sin(25.84°)} = 4588.31$$

$P = S\cos\theta = 4129.48$

$S = \underline{\mathbf{4129 - j2000\ VA}}$

(c) $Q = S\sin\theta \longrightarrow \sin\theta = \dfrac{Q}{S} = \dfrac{450}{600} = 0.75$

$\theta = 48.59$, $\quad \text{pf} = 0.6614$

$P = S\cos\theta = (600)(0.6614) = 396.86$

$S = \underline{\mathbf{396.9 + j450\ VA}}$

(d) $$S = \frac{|\mathbf{V}|^2}{|\mathbf{Z}|} = \frac{(220)^2}{40} = 1210$$

$P = S\cos\theta \longrightarrow \cos\theta = \dfrac{P}{S} = \dfrac{1000}{1210} = 0.8264$

$\theta = 34.26°$

$Q = S\sin\theta = 681.25$

$S = \underline{\mathbf{1000 + j681.2\ VA}}$

Chapter 11, Problem 49.

Find the complex power for the following cases:

(a) $P = 4$ kW, pf = 0.86 (lagging)
(b) $S = 2$ kVA, $P = 1.6$ kW (capacitive)
(c) $\mathbf{V}_{rms} = 208\angle 20°$ V, $\mathbf{I}_{rms} = 6.5\angle -50°$ A
(d) $\mathbf{V}_{rms} = 120\angle 30°$ V, $\mathbf{Z} = 40 + j60\,\Omega$

Chapter 11, Solution 49.

(a)
$$\mathbf{S} = 4 + j\frac{4}{0.86}\sin(\cos^{-1}(0.86))\text{ kVA}$$
$$\mathbf{S} = \underline{\mathbf{4 + j2.373\ kVA}}$$

(b)
$$\text{pf} = \frac{P}{S} = \frac{1.6}{2}0.8 = \cos\theta \longrightarrow \sin\theta = 0.6$$

$$\mathbf{S} = 1.6 - j2\sin\theta = \underline{\mathbf{1.6 - j1.2\ kVA}}$$

(c)
$$\mathbf{S} = \mathbf{V}_{rms}\,\mathbf{I}^*_{rms} = (208\angle 20°)(6.5\angle 50°)\text{ VA}$$
$$\mathbf{S} = 1.352\angle 70° = \underline{\mathbf{0.4624 + j1.2705\ kVA}}$$

(d)
$$\mathbf{S} = \frac{|\mathbf{V}|^2}{\mathbf{Z}^*} = \frac{(120)^2}{40 - j60} = \frac{14400}{72.11\angle\text{-}56.31°}$$
$$\mathbf{S} = 199.7\angle 56.31° = \underline{\mathbf{110.77 + j166.16\ VA}}$$

Chapter 11, Problem 50.

Obtain the overall impedance for the following cases:

(a) P = 1000 W, pf = 0.8(leading),
V_{rms} = 220 V

(b) P = 1500 W, Q = 2000 VAR (inductive),
I_{rms} = 12 A

(c) $\mathbf{S} = 4500\angle 60°$ VA, $\mathbf{V} = 120\angle 45°$ V

Chapter 11, Solution 50.

(a)
$$\mathbf{S} = P - jQ = 1000 - j\frac{1000}{0.8}\sin(\cos^{-1}(0.8))$$
$$\mathbf{S} = 1000 - j750$$

But, $$\mathbf{S} = \frac{|\mathbf{V}_{rms}|^2}{\mathbf{Z}^*}$$

$$\mathbf{Z}^* = \frac{|\mathbf{V}_{rms}|^2}{\mathbf{S}} = \frac{(220)^2}{1000 - j750} = 30.98 + j23.23$$
$$\mathbf{Z} = \underline{\mathbf{30.98 - j23.23\ \Omega}}$$

(b)
$$\mathbf{S} = |\mathbf{I}_{rms}|^2 \mathbf{Z}$$
$$\mathbf{Z} = \frac{\mathbf{S}}{|\mathbf{I}_{rms}|^2} = \frac{1500 + j2000}{(12)^2} = \underline{\mathbf{10.42 + j13.89\ \Omega}}$$

(c)
$$\mathbf{Z}^* = \frac{|\mathbf{V}_{rms}|^2}{\mathbf{S}} = \frac{|\mathbf{V}|^2}{2\mathbf{S}} = \frac{(120)^2}{(2)(4500\angle 60°)} = 1.6\angle -60°$$
$$\mathbf{Z} = 1.6\angle 60° = \underline{\mathbf{0.8 + j1.386\ \Omega}}$$

Chapter 11, Problem 51.

For the entire circuit in Fig. 11.70, calculate:

(a) the power factor
(b) the average power delivered by the source
(c) the reactive power
(d) the apparent power
(e) the complex power

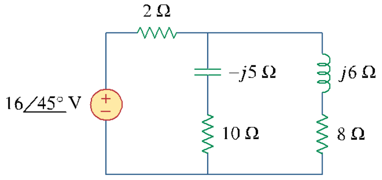


Figure 11.70
For Prob. 11.51.

Chapter 11, Solution 51.

(a) $$\mathbf{Z}_T = 2 + (10 - j5) \,||\, (8 + j6)$$

$$\mathbf{Z}_T = 2 + \frac{(10 - j5)(8 + j6)}{18 + j} = 2 + \frac{110 + j20}{18 + j}$$

$$\mathbf{Z}_T = 8.152 + j0.768 = 8.188\angle 5.382°$$

$$\text{pf} = \cos(5.382°) = \underline{\mathbf{0.9956 \quad (lagging)}}$$

(b) $$\mathbf{S} = \frac{1}{2}\mathbf{V}\mathbf{I}^* = \frac{|\mathbf{V}|^2}{2\mathbf{Z}^*} = \frac{(16)^2}{(2)(8.188\angle -5.382°)}$$

$$\mathbf{S} = 15.63\angle 5.382°$$

$$P = S\cos\theta = \underline{\mathbf{15.56\ W}}$$

(c) $$Q = S\sin\theta = \underline{\mathbf{1.466\ VAR}}$$

(d) $$S = |\mathbf{S}| = \underline{\mathbf{15.63\ VA}}$$

(e) $$\mathbf{S} = 15.63\angle 5.382° = \underline{\mathbf{15.56 + j1.466\ VA}}$$

Chapter 11, Problem 52.

In the circuit of Fig. 11.71, device *A* receives 2 kW at 0.8 pf lagging, device *B* receives 3 kVA at 0.4 pf leading, while device *C* is inductive and consumes 1 kW and receives 500 VAR.

(a) Determine the power factor of the entire system.
(b) Find **I** given that $\mathbf{V}_s = 120\angle 45° \text{ V rms}$.

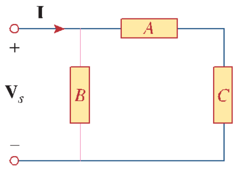


Figure 11.71
For Prob. 11.52.

Chapter 11, Solution 52.

$$S_A = 2000 + j\frac{2000}{0.8}0.6 = 2000 + j1500$$

$$S_B = 3000x0.4 - j3000x0.9165 = 1200 - j2749$$

$$S_C = 1000 + j500$$

$$S = S_A + S_B + S_C = 4200 - j749$$

(a) $$\text{pf} = \frac{4200}{\sqrt{4200^2 + 749^2}} = \underline{0.9845 \text{ leading.}}$$

(b) $$S = V_{rms}I^*_{rms} \longrightarrow I^*_{rms} = \frac{4200 - j749}{120\angle 45°} = 35.55\angle -55.11°$$

$$I_{rms} = \underline{\mathbf{35.55\angle 55.11° \ A}}.$$

Chapter 11, Problem 53.

In the circuit of Fig. 11.72, load *A* receives 4 kVA at 0.8 pf leading. Load *B* receives 2.4 kVA at 0.6 pf lagging. Box *C* is an inductive load that consumes 1 kW and receives 500 VAR.

(a) Determine **I**.
(b) Calculate the power factor of the combination.

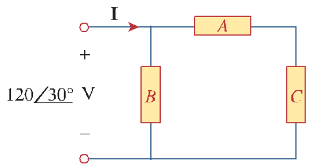


Figure 11.72
For Prob. 11.53.

Chapter 11, Solution 53.

$$S = S_A + S_B + S_C = 4000(0.8\text{–}j0.6) + 2400(0.6+j0.8) + 1000 + j500$$

$$= 5640 + j20 = 5640\angle 0.2^\circ$$

(a)

$$I^*_{rms} = \frac{S_B}{V_{rms}} + \frac{S_A + S_C}{V_{rms}} = \frac{S}{V_{rms}} = \frac{5640\angle 0.2^\circ}{\frac{120\angle 30^\circ}{\sqrt{2}}} = 66.46\angle -29.8^\circ$$

$$I = \sqrt{2}\text{x}66.46\angle 29.88^\circ = \underline{93.97\angle 29.8^\circ\,\text{A}}$$

(b) pf = cos(0.2°) ≈ **<u>1.0 lagging</u>**.

Chapter 11, Problem 54.

For the network in Fig. 11.73, find the complex power absorbed by each element.

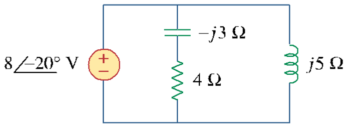


Figure 11.73
For Prob. 11.54.

Chapter 11, Solution 54.

Consider the circuit shown below.

$$\mathbf{I}_1 = \frac{8\angle -20°}{4 - j3} = 1.6\angle 16.87°$$

$$\mathbf{I}_2 = \frac{8\angle -20°}{j5} = 1.6\angle -110°$$

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = (-0.5472 - j1.504) + (1.531 + j0.4643)$$
$$\mathbf{I} = 0.9839 - j1.04 = 1.432\angle -46.58°$$

For the source,

$$\mathbf{S} = \frac{1}{2}\mathbf{V}\mathbf{I}^* = \frac{1}{2}(8\angle -20°)(1.432\angle 46.58°)$$
$$\mathbf{S} = 5.728\angle 26.58° = \underline{\mathbf{5.12 + j2.56\ VA}}$$

For the capacitor,

$$\mathbf{S} = \frac{1}{2}\left|\mathbf{I}_1\right|^2 \mathbf{Z}_c = \frac{1}{2}(1.6)^2(-j3) = \underline{\mathbf{-\ j3.84\ VA}}$$

For the resistor,

$$\mathbf{S} = \frac{1}{2}\left|\mathbf{I}_1\right|^2 \mathbf{Z}_R = \frac{1}{2}(1.6)^2(4) = \underline{\mathbf{5.12\ VA}}$$

For the inductor,

$$\mathbf{S} = \frac{1}{2}\left|\mathbf{I}_2\right|^2 \mathbf{Z}_L = \frac{1}{2}(1.6)^2(j5) = \underline{\mathbf{j6.4\ VA}}$$

Chapter 11, Problem 55.

Find the complex power absorbed by each of the five elements in the circuit of Fig. 11.74.

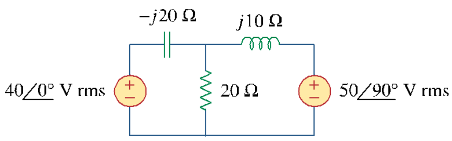
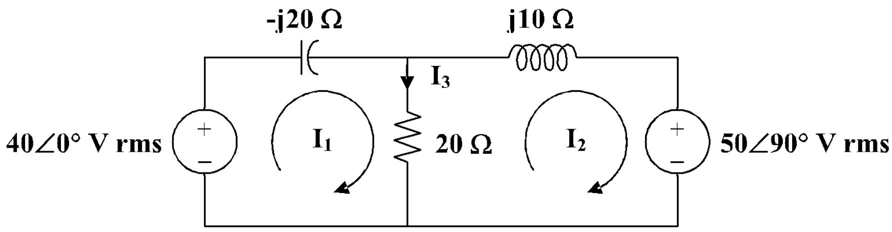


Figure 11.74
For Prob. 11.55.

Chapter 11, Solution 55.

We apply mesh analysis to the following circuit.



For mesh 1,

$$40 = (20 - j20)I_1 - 20I_2$$

$$2 = (1 - j)I_1 - I_2 \qquad (1)$$

For mesh 2,

$$-j50 = (20 + j10)I_2 - 20I_1$$

$$-j5 = -2I_1 + (2 + j)I_2 \qquad (2)$$

Putting (1) and (2) in matrix form,

$$\begin{bmatrix} 2 \\ -j5 \end{bmatrix} = \begin{bmatrix} 1-j & -1 \\ -2 & 2+j \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta = 1 - j, \qquad \Delta_1 = 4 - j3, \qquad \Delta_2 = -1 - j5$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{4 - j3}{1 - j} = \frac{1}{2}(7 + j) = 3.535\angle 8.13°$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{-1 - j5}{1 - j} = 2 - j3 = 3.605\angle -56.31°$$

$$I_3 = I_1 - I_2 = (3.5 + j0.5) - (2 - j3) = 1.5 + j3.5 = 3.808\angle 66.8°$$

For the 40-V source,

$$\mathbf{S} = -\mathbf{V}\mathbf{I}_1^* = -(40)\left(\frac{1}{2}\cdot(7 - j)\right) = \underline{\mathbf{-140 + j20\ VA}}$$

For the capacitor,

$$\mathbf{S} = \left|\mathbf{I}_1\right|^2 \mathbf{Z}_c = \underline{\mathbf{-j250\ VA}}$$

For the resistor,

$$\mathbf{S} = \left|\mathbf{I}_3\right|^2 R = \underline{\mathbf{290\ VA}}$$

For the inductor,

$$\mathbf{S} = \left|\mathbf{I}_2\right|^2 \mathbf{Z}_L = \underline{\mathbf{j130\ VA}}$$

For the j50-V source,

$$\mathbf{S} = \mathbf{V}\mathbf{I}_2^* = (j50)(2 + j3) = \underline{\mathbf{-150 + j100\ VA}}$$

Chapter 11, Problem 56.

PS ML Obtain the complex power delivered by the source in the circuit of Fig. 11.75.

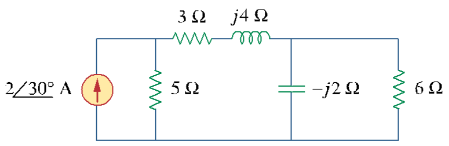
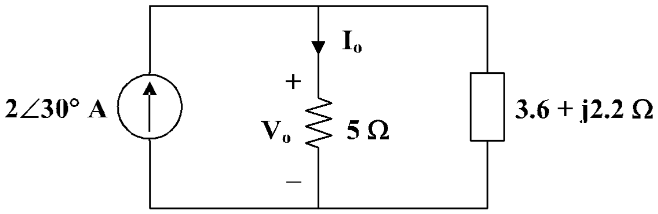


Figure 11.75
For Prob. 11.56.

Chapter 11, Solution 56.

$$- j2 \parallel 6 = \frac{(6)(-j2)}{6 - j2} = 0.6 - j1.8$$
$$3 + j4 + (-j2) \parallel 6 = 3.6 + j2.2$$

The circuit is reduced to that shown below.



$$\mathbf{I}_o = \frac{3.6 + j2.2}{8.6 + j2.2}(2\angle 30°) = 0.95\angle 47.08°$$

$$\mathbf{V}_o = 5\mathbf{I}_o = 4.75\angle 47.08°$$

$$\mathbf{S} = \frac{1}{2}\mathbf{V}_o\mathbf{I}_s^* = \frac{1}{2}\cdot(4.75\angle 47.08°)(2\angle -30°)$$
$$\mathbf{S} = 4.75\angle 17.08° = \underline{\mathbf{4.543 + j1.396\ VA}}$$

Chapter 11, Problem 57.

PS ML For the circuit in Fig. 11.76, find the average, reactive, and complex power delivered by the dependent current source.

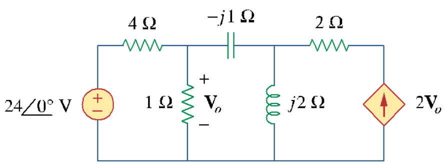
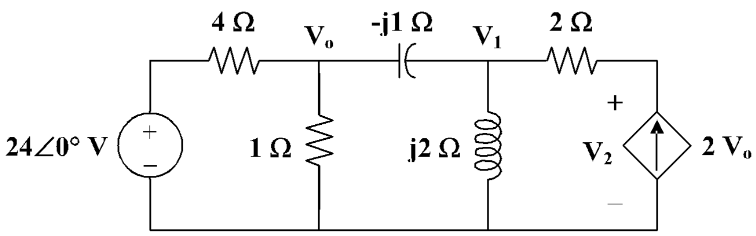


Figure 11.76
For Prob. 11.57.

Chapter 11, Solution 57.

Consider the circuit as shown below.



At node o,

$$\frac{24-\mathbf{V}_o}{4}=\frac{\mathbf{V}_o}{1}+\frac{\mathbf{V}_o-\mathbf{V}_1}{-j}$$

$$24=(5+j4)\mathbf{V}_o-j4\mathbf{V}_1 \qquad (1)$$

At node 1,

$$\frac{\mathbf{V}_o-\mathbf{V}_1}{-j}+2\mathbf{V}_o=\frac{\mathbf{V}_1}{j2}$$

$$\mathbf{V}_1=(2-j4)\mathbf{V}_o \qquad (2)$$

Substituting (2) into (1),

$$24=(5+j4-j8-16)\mathbf{V}_o$$

$$\mathbf{V}_o=\frac{-24}{11+j4}, \qquad \mathbf{V}_1=\frac{(-24)(2-j4)}{11+j4}$$

The voltage across the dependent source is

$$\mathbf{V}_2=\mathbf{V}_1+(2)(2\mathbf{V}_o)=\mathbf{V}_1+4\mathbf{V}_o$$

$$\mathbf{V}_2=\frac{-24}{11+j4}\cdot(2-j4+4)=\frac{(-24)(6-j4)}{11+j4}$$

$$\mathbf{S}=\frac{1}{2}\mathbf{V}_2\mathbf{I}^*=\frac{1}{2}\mathbf{V}_2(2\mathbf{V}_o^*)$$

$$\mathbf{S}=\frac{(-24)(6-j4)}{11+j4}\cdot\frac{-24}{11-j4}=\left(\frac{576}{137}\right)(6-j4)$$

$$\mathbf{S}=\underline{\mathbf{25.23-j16.82\ VA}}$$

Chapter 11, Problem 58.

Obtain the complex power delivered to the 10-kΩ resistor in Fig. 11.77 below.

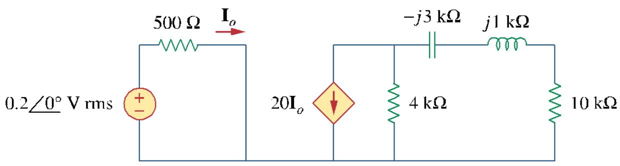
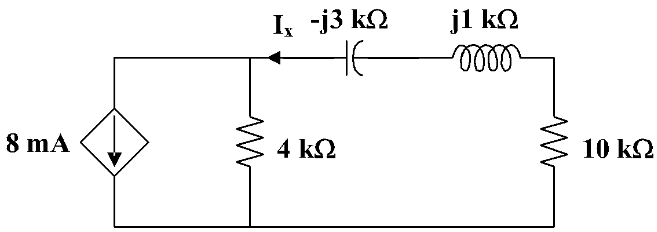


Figure 11.77
For Prob. 11.58.

Chapter 11, Solution 58.



From the left portion of the circuit,

$$\mathbf{I}_o = \frac{0.2}{500} = 0.4 \text{ mA}$$

$$20\mathbf{I}_o = 8 \text{ mA}$$

From the right portion of the circuit,

$$\mathbf{I}_x = \frac{4}{4 + 10 + j - j3}(8 \text{ mA}) = \frac{16}{7 - j} \text{ mA}$$

$$\mathbf{S} = \left| \mathbf{I}_x \right|^2 R = \frac{(16 \times 10^{-3})^2}{50} \cdot (10 \times 10^3)$$

$$\mathbf{S} = \underline{\mathbf{51.2 \text{ mVA}}}$$

Chapter 11, Problem 59.

ML Calculate the reactive power in the inductor and capacitor in the circuit of Fig. 11.78.

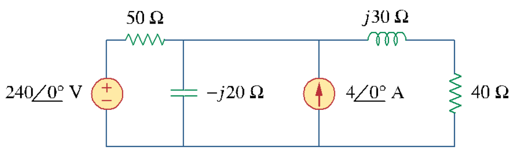


Figure 11.78
For Prob. 11.59.

Chapter 11, Solution 59.

Let $\mathbf{V}_o$ represent the voltage across the current source and then apply nodal analysis to the circuit and we get:

$$4 + \frac{240 - \mathbf{V}_o}{50} = \frac{\mathbf{V}_o}{-j20} + \frac{\mathbf{V}_o}{40 + j30}$$

$$88 = (0.36 + j0.38)\mathbf{V}_o$$

$$\mathbf{V}_o = \frac{88}{0.36 + j0.38} = 168.13\angle -46.55°$$

$$\mathbf{I}_1 = \frac{\mathbf{V}_o}{-j20} = 8.41\angle 43.45°$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_o}{40 + j30} = 3.363\angle -83.42°$$

Reactive power in the inductor is

$$\mathbf{S} = \frac{1}{2}|\mathbf{I}_2|^2\,\mathbf{Z}_L = \frac{1}{2}\cdot(3.363)^2(j30) = \underline{\mathbf{j169.65\ VAR}}$$

Reactive power in the capacitor is

$$\mathbf{S} = \frac{1}{2}|\mathbf{I}_1|^2\,\mathbf{Z}_c = \frac{1}{2}\cdot(8.41)^2(-j20) = \underline{\mathbf{-\ j707.3\ VAR}}$$

Chapter 11, Problem 60.

For the circuit in Fig. 11.79, find $\mathbf{V}_o$ and the input power factor.

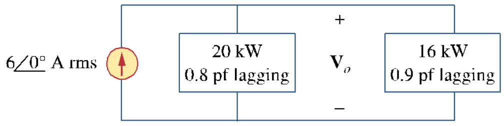


Figure 11.79
For Prob. 11.60.

Chapter 11, Solution 60.

$$S_1 = 20 + j\frac{20}{0.8}\sin(\cos^{-1}(0.8)) = 20 + j15$$

$$S_2 = 16 + j\frac{16}{0.9}\sin(\cos^{-1}(0.9)) = 16 + j7.749$$

$$S = S_1 + S_2 = 36 + j22.749 = 42.585\angle 32.29°$$

But $\quad S = V_o I^* = 6V_o$

$$V_o = \frac{S}{6} = \underline{\mathbf{7.098\angle 32.29°}}$$

$$pf = \cos(32.29°) = \underline{\mathbf{0.8454 \quad (lagging)}}$$

Chapter 11, Problem 61.

Given the circuit in Fig. 11.80, find $\mathbf{I}_o$ and the overall complex power supplied.

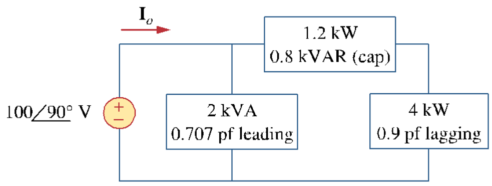
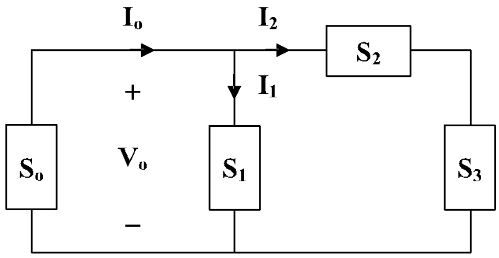


Figure 11.80
For Prob. 11.61.

Chapter 11, Solution 61.

Consider the network shown below.



$$\mathbf{S}_2 = 1.2 - j0.8 \text{ kVA}$$

$$\mathbf{S}_3 = 4 + j\frac{4}{0.9}\sin(\cos^{-1}(0.9)) = 4 + j1.937 \text{ kVA}$$

Let $\mathbf{S}_4 = \mathbf{S}_2 + \mathbf{S}_3 = 5.2 + j1.137 \text{ kVA}$

But $\mathbf{S}_4 = \frac{1}{2}\mathbf{V}_o\mathbf{I}_2^*$

$$\mathbf{I}_2^* = \frac{2\mathbf{S}_4}{\mathbf{V}_o} = \frac{(2)(5.2 + j1.137)\times 10^3}{100\angle 90°} = 22.74 - j104$$

$$\mathbf{I}_2 = 22.74 + j104$$

Similarly, $\mathbf{S}_1 = \sqrt{2} - j\frac{\sqrt{2}}{0.707}\sin(\cos^{-1}(0.707)) = \sqrt{2}(1 - j) \text{ kVA}$

But $\mathbf{S}_1 = \frac{1}{2}\mathbf{V}_o\mathbf{I}_1^*$

$$\mathbf{I}_1^* = \frac{2\mathbf{S}_1}{\mathbf{V}_o} = \frac{(2.8284 - j2.8284)\times 10^3}{j100} = -28.284 - j28.284$$

$$\mathbf{I}_1 = -28.28 + j28.28$$

$$\mathbf{I}_o = \mathbf{I}_1 + \mathbf{I}_2 = -5.54 + j132.28 = \underline{132.4\angle 92.4°\text{A}}$$

$$\mathbf{S}_o = \frac{1}{2}\mathbf{V}_o\mathbf{I}_o^*$$

$$\mathbf{S}_o = \frac{1}{2}\cdot(100\angle 90°)(132.4\angle -92.4°) \text{ VA}$$

$$\mathbf{S}_o = \underline{\mathbf{6.62\angle -2.4° \text{ kVA}}}$$

Chapter 11, Problem 62.

For the circuit in Fig. 11.81, find $\mathbf{V}_s$.

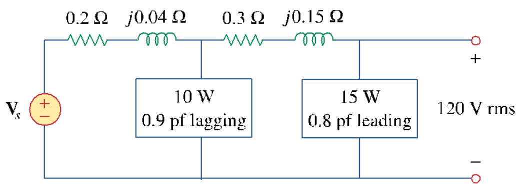
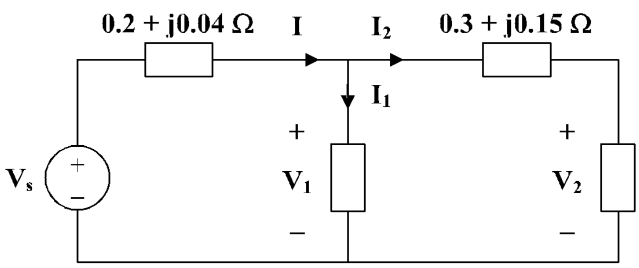


Figure 11.81
For Prob. 11.62.

Chapter 11, Solution 62.

Consider the circuit below.



$$\mathbf{S}_2 = 15 - j\frac{15}{0.8}\sin(\cos^{-1}(0.8)) = 15 - j11.25$$

But $\quad \mathbf{S}_2 = \mathbf{V}_2\mathbf{I}_2^*$

$$\mathbf{I}_2^* = \frac{\mathbf{S}_2}{\mathbf{V}_2} = \frac{15 - j11.25}{120}$$

$$\mathbf{I}_2 = 0.125 + j0.09375$$

$$\mathbf{V}_1 = \mathbf{V}_2 + \mathbf{I}_2(0.3 + j0.15)$$

$$\mathbf{V}_1 = 120 + (0.125 + j0.09375)(0.3 + j0.15)$$

$$\mathbf{V}_1 = 120.02 + j0.0469$$

$$\mathbf{S}_1 = 10 + j\frac{10}{0.9}\sin(\cos^{-1}(0.9)) = 10 + j4.843$$

But $\quad \mathbf{S}_1 = \mathbf{V}_1\mathbf{I}_1^*$

$$\mathbf{I}_1^* = \frac{\mathbf{S}_1}{\mathbf{V}_1} = \frac{11.111\angle 25.84°}{120.02\angle 0.02°}$$

$$\mathbf{I}_1 = 0.093\angle -25.82° = 0.0837 - j0.0405$$

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = 0.2087 + j0.053$$

$$\mathbf{V}_s = \mathbf{V}_1 + \mathbf{I}(0.2 + j0.04)$$

$$\mathbf{V}_s = (120.02 + j0.0469) + (0.2087 + j0.053)(0.2 + j0.04)$$

$$\mathbf{V}_s = 120.06 + j0.0658$$

$$\mathbf{V}_s = \underline{\mathbf{120.06\angle 0.03°\ V}}$$

Chapter 11, Problem 63.

Find $\mathbf{I}_o$ in the circuit of Fig. 11.82.

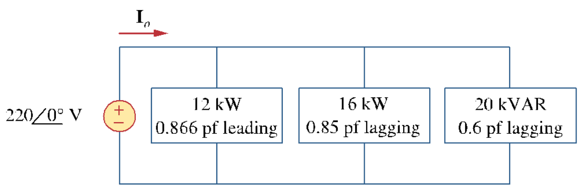


Figure 11.82
For Prob. 11.63.

Chapter 11, Solution 63.

Let $\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3$.

$$\mathbf{S}_1 = 12 - j\frac{12}{0.866}\sin(\cos^{-1}(0.866)) = 12 - j6.929$$

$$\mathbf{S}_2 = 16 + j\frac{16}{0.85}\sin(\cos^{-1}(0.85)) = 16 + j9.916$$

$$\mathbf{S}_3 = \frac{(20)(0.6)}{\sin(\cos^{-1}(0.6)} + j20 = 15 + j20$$

$$\mathbf{S} = 43 + j22.987 = \frac{1}{2}\mathbf{V}\mathbf{I}_o^*$$

$$\mathbf{I}_o^* = \frac{2\mathbf{S}}{\mathbf{V}} = \frac{2(43 + j22.99)x10^3}{220} = 390.9 + j209 = 443.3\angle 28.13°$$

$$\mathbf{I}_o = \underline{\mathbf{443.3\angle{-28.13}^\circ A}}$$

Chapter 11, Problem 64.

Determine $\mathbf{I}_s$ in the circuit of Fig. 11.83, if the voltage source supplies 2.5 kW and 0.4 kVAR (leading).

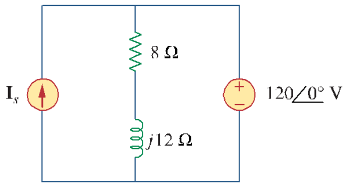
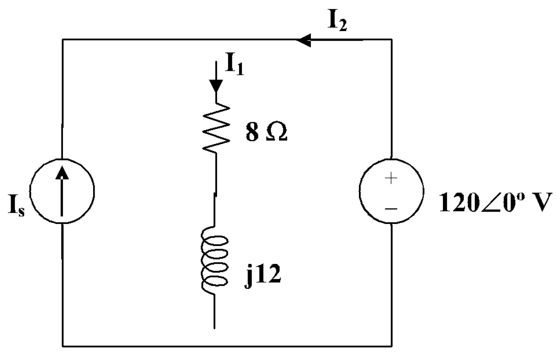


Figure 11.83
For Prob. 11.64.

Chapter 11, Solution 64.



$$I_s + I_2 = I_1 \text{ or } I_s = I_1 - I_2$$

$$I_1 = \frac{120}{8 + j12} = 4.615 - j6.923$$

But, $S = VI_2^* \longrightarrow I_2^* = \frac{S}{V} = \frac{2500 - j400}{120} = 20.83 - j3.333$

$$\text{or } I_2 = 20.83 + j3.333$$

$I_s = I_1 - I_2 = -16.22 - j10.256 =$ **$\underline{19.19\angle{-147.69^\circ}\text{ A}}$**.

Chapter 11, Problem 65.

In the op amp circuit of Fig. 11.84, $v_s = 4 \cos 10^4 t$ V.~norm~&] Find the average power delivered to the 50-kΩ resistor.

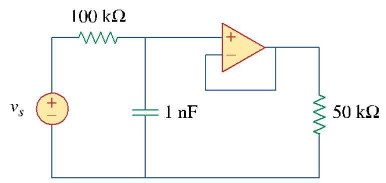


Figure 11.84
For Prob. 11.65.

Chapter 11, Solution 65.

$$C = 1 \text{ nF} \longrightarrow \frac{1}{j\omega C} = \frac{-j}{10^4 \times 10^{-9}} = -j100 \text{ k}\Omega$$

At the noninverting terminal,

$$\frac{4\angle 0° - \mathbf{V}_o}{100} = \frac{\mathbf{V}_o}{-j100} \longrightarrow \mathbf{V}_o = \frac{4}{1+j}$$

$$\mathbf{V}_o = \frac{4}{\sqrt{2}}\angle -45°$$

$$v_o(t) = \frac{4}{\sqrt{2}}\cos(10^4 t - 45°)$$

$$P = \frac{V_{rms}^2}{R} = \left(\frac{4}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}}\right)^2 \left(\frac{1}{50 \times 10^3}\right) \text{W}$$

$$P = \underline{\mathbf{80\ \mu W}}$$

Chapter 11, Problem 66.

Obtain the average power absorbed by the 6-kΩ resistor in the op amp circuit in Fig. 11.85.

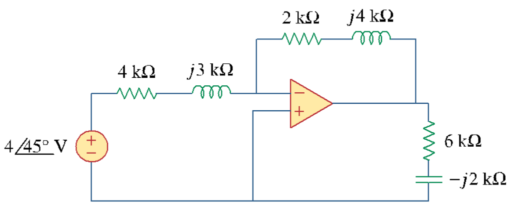


Figure 11.85
For Prob. 11.66.

Chapter 11, Solution 66.

As an inverter,

$$\mathbf{V}_o = \frac{-\mathbf{Z}_f}{\mathbf{Z}_i}\mathbf{V}_s = \frac{-(2+j4)}{4+j3}\cdot(4\angle 45°)$$

$$\mathbf{I}_o = \frac{\mathbf{V}_o}{6-j2}\text{ mA} = \frac{-(2+j4)(4\angle 45°)}{(6-j2)(4+j3)}\text{ mA}$$

The power absorbed by the 6-kΩ resistor is

$$P = \frac{1}{2}|\mathbf{I}_o|^2 R = \frac{1}{2}\cdot\left(\frac{\sqrt{20}\times 4}{\sqrt{40}\times 5}\right)^2 \times 10^{-6}\times 6\times 10^3$$

$$P = \underline{\mathbf{0.96\ mW}}$$

Chapter 11, Problem 67.

For the op amp circuit in Fig. 11.86, calculate:

(a) the complex power delivered by the voltage source
(b) the average power dissipated in the 12-Ω resistor

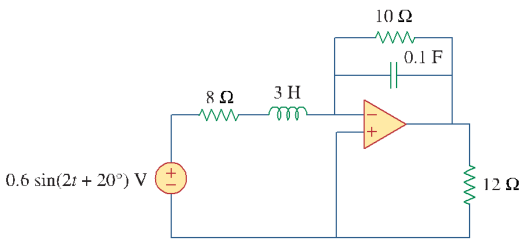


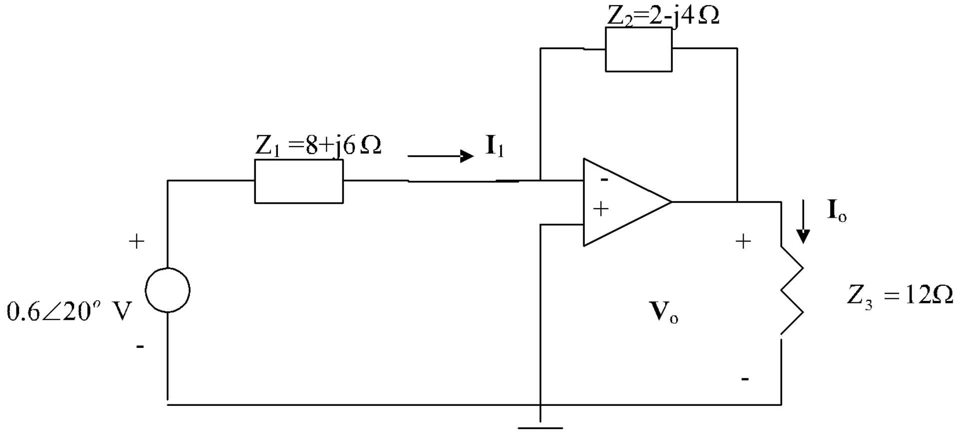
Figure 11.86
For Prob. 11.67.

Chapter 11, Solution 67.

$$\omega = 2, \quad 3H \longrightarrow j\omega L = j6, \quad 0.1F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j2x0.1} = -j5$$

$$10//(-j5) = \frac{-j50}{10-j5} = 2 - j4$$

The frequency-domain version of the circuit is shown below.



(a) $I_1 = \frac{0.6\angle 20^o - 0}{8+j6} = \frac{0.5638 + j0.2052}{8+j6} = 0.06\angle -16.87^o$

$S = \frac{1}{2} V_s I^*_1 = (0.3\angle 20^o)(0.06\angle +16.87^o) = \underline{14.4 + j10.8 \text{ mVA}} = \underline{18\angle 36.86^o \text{ mVA}}$

(b) $V_o = -\frac{Z_2}{Z_1} V_s, \quad I_o = \frac{V_o}{Z_3} = -\frac{(2-j4)}{12(8+j6)}(0.6\angle 20^o) = 0.0224\angle 99.7^o$

$$P = \frac{1}{2} |I_o|^2 R = 0.5(0.0224)^2(12) = \underline{2.904 \text{ mW}}$$

Chapter 11, Problem 68.

Compute the complex power supplied by the current source in the series *RLC* circuit in Fig. 11.87.

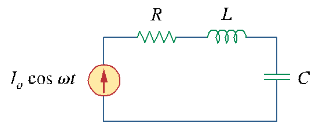


Figure 11.87
For Prob. 11.68.

Chapter 11, Solution 68.

Let $\mathbf{S} = \mathbf{S}_R + \mathbf{S}_L + \mathbf{S}_c$

where $\mathbf{S}_R = P_R + jQ_R = \frac{1}{2} I_o^2 R + j0$

$$\mathbf{S}_L = P_L + jQ_L = 0 + j\frac{1}{2} I_o^2 \omega L$$

$$\mathbf{S}_c = P_c + jQ_c = 0 - j\frac{1}{2} I_o^2 \cdot \frac{1}{\omega C}$$

Hence,
$$\mathbf{S} = \underline{\frac{1}{2} \mathbf{I}_o^2 \left[\mathbf{R} + \mathbf{j}\left(\omega \mathbf{L} - \frac{1}{\omega \mathbf{C}} \right) \right]}$$

Chapter 11, Problem 69.

Refer to the circuit shown in Fig. 11.88.

(a) What is the power factor?
(b) What is the average power dissipated?
(c) What is the value of the capacitance that will give a unity power factor when connected to the load?

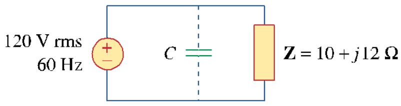


Figure 11.88
For Prob. 11.69.

Chapter 11, Solution 69.

(a) Given that $\mathbf{Z} = 10 + j12$

$$\tan\theta = \frac{12}{10} \longrightarrow \theta = 50.19°$$

$$\text{pf} = \cos\theta = \underline{\mathbf{0.6402}}$$

(b) $$\mathbf{S} = \frac{|\mathbf{V}|^2}{2\mathbf{Z}^*} = \frac{(120)^2}{(2)(10 - j12)} = 295.12 + j354.09$$

The average power absorbed $= P = \text{Re}(\mathbf{S}) = \underline{\mathbf{295.1\ W}}$

(c) For unity power factor, $\theta_1 = 0°$, which implies that the reactive power due to the capacitor is $Q_c = 354.09$

But $$Q_c = \frac{V^2}{2X_c} = \frac{1}{2}\omega C V^2$$

$$C = \frac{2Q_c}{\omega V^2} = \frac{(2)(354.09)}{(2\pi)(60)(120)^2} = \underline{\mathbf{130.4\ \mu F}}$$

Chapter 11, Problem 70.

An 880-VA, 220-V, 50-Hz load has a power factor of 0.8 lagging. What value of parallel capacitance will correct the load power factor to unity?

Chapter 11, Solution 70.

$$\text{pf} = \cos\theta = 0.8 \quad \longrightarrow \quad \sin\theta = 0.6$$
$$Q = S\sin\theta = (880)(0.6) = 528$$

If the power factor is to be unity, the reactive power due to the capacitor is

$$Q_c = Q = 528 \text{ VAR}$$

$$\text{But} \quad Q = \frac{V_{rms}^2}{X_c} = \frac{1}{2}\omega C V^2 \quad \longrightarrow \quad C = \frac{2Q_c}{\omega V^2}$$

$$C = \frac{(2)(528)}{(2\pi)(50)(220)^2} = \underline{\mathbf{69.45\ \mu F}}$$

Chapter 11, Problem 71.

Three loads are connected in parallel to a $120\angle 0°$ V rms source. Load 1 absorbs 60 kVAR at pf = 0.85 lagging, load 2 absorbs 90 kW and 50 kVAR leading, and load 3 absorbs 100 kW at pf = 1. (a) Find the equivalent impedance. (b) Calculate the power factor of the parallel combination. (c) Determine the current supplied by the source.

Chapter 11, Solution 71.

(a) For load 1,

$Q_1 = 60$ kVAR, pf = 0.85 or $\theta_1 = 31.79°$

$Q_1 = S_1 \sin\theta_1 = 60\text{k}$ or $S_1 = 113.89\text{k}$ and $P_1 = 113.89\cos(31.79) = 96.8\text{kW}$

$\mathbf{S}_1 = 96.8 + j60$ kVA

For load 2, $\mathbf{S}_2 = 90 - j50$ kVA

For load 3, $\mathbf{S}_3 = 100$ kVA

Hence,

$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3 = 286.8 + j10\text{kVA} = 287\angle 2°\text{kVA}$

But $\quad \mathbf{S} = (V_{rms})^2/Z^*$ or $Z^* = 120^2/287\angle 2°\text{k} = 0.05017\angle -2°$

Thus, $\quad Z = \underline{\mathbf{0.05017\angle 2°\Omega}}$ or $\underline{\mathbf{0.05014 + j0.0017509\Omega}}$.

(b) From above, pf = cos2° = $\underline{\mathbf{0.9994}}$.

(c) $I_{rms} = V_{rms}/Z = 120/0.05017\angle 2° = \underline{\mathbf{2.392\angle -2°kA}}$ or $\underline{\mathbf{2.391 - j0.08348kA}}$.

Chapter 11, Problem 72.

Two loads connected in parallel draw a total of 2.4 kW at 0.8 pf lagging from a 120-V rms, 60-Hz line. One load absorbs 1.5 kW at a 0.707 pf lagging. Determine: (a) the pf of the second load, (b) the parallel element required to correct the pf to 0.9 lagging for the two loads.

Chapter 11, Solution 72.

(a) $P = S\cos\theta_1 \quad\longrightarrow\quad S = \dfrac{P}{\cos\theta_1} = \dfrac{2.4}{0.8} = 3.0 \text{ kVA}$

$pf = 0.8 = \cos\theta_1 \quad\longrightarrow\quad \theta_1 = 36.87^o$

$Q = S\sin\theta_1 = 3.0\sin 36.87^o = 1.8 \text{ kVAR}$

Hence, $\mathrm{S} = 2.4 + \mathrm{j}1.8 \text{ kVA}$

$S_1 = \dfrac{P_1}{\cos\theta} = \dfrac{1.5}{0.707} = 2.122 \text{ kVA}$

$pf = 0.707 = \cos\theta \quad\longrightarrow\quad \theta = 45^o$

$Q_1 = P_1 = 1.5 \text{ kVAR} \quad\longrightarrow\quad \mathrm{S}_1 = 1.5 + j1.5 \text{ kVA}$

Since, $S = S_1 + S_2 \quad\longrightarrow\quad S_2 = S - S_1 = (2.4 + j1.8) - (1.5 + j1.5) = 0.9 + j0.3 \text{ kVA}$

$S_2 = 0.9497 < 18.43^o$

pf = cos 18.43° = <u>0.9487</u>

(b) $pf = 0.9 = \cos\theta_2 \quad\longrightarrow\quad \theta_2 = 25.84^o$

$$C = \frac{P(\tan\theta_1 - \tan\theta_2)}{\omega V_{rms}^2} = \frac{2400(\tan 36.87 - \tan 25.84)}{2\pi x 60 x (120)^2} = \underline{117.5\ \mu\text{F}}$$

Chapter 11, Problem 73.

A 240-V rms 60-Hz supply serves a load that is 10 kW (resistive), 15 kVAR (capacitive), and 22 kVAR (inductive). Find:

(a) the apparent power
(b) the current drawn from the supply
(c) the kVAR rating and capacitance required to improve the power factor to 0.96 lagging
(d) the current drawn from the supply under the new power-factor conditions

Chapter 11, Solution 73.

(a) $\mathbf{S} = 10 - j15 + j22 = 10 + j7 \text{ kVA}$

$S = |\mathbf{S}| = \sqrt{10^2 + 7^2} = \underline{\mathbf{12.21\ kVA}}$

(b) $\mathbf{S} = \mathbf{V}\mathbf{I}^* \longrightarrow \mathbf{I}^* = \frac{\mathbf{S}}{\mathbf{V}} = \frac{10{,}000 + j7{,}000}{240}$

$\mathbf{I} = 41.667 - j29.167 = \underline{\mathbf{50.86\angle\text{-}35°\ A}}$

(c) $\theta_1 = \tan^{-1}\left(\frac{7}{10}\right) = 35°, \qquad \theta_2 = \cos^{-1}(0.96) = 16.26°$

$Q_c = P_1 [\tan\theta_1 - \tan\theta_2] = 10[\tan(35°) - \tan(16.26°)]$

$Q_c = \underline{\mathbf{4.083\ kVAR}}$

$C = \frac{Q_c}{\omega V_{rms}^2} = \frac{4083}{(2\pi)(60)(240)^2} = \underline{\mathbf{188.03\ \mu F}}$

(d) $\mathbf{S}_2 = P_2 + jQ_2, \qquad P_2 = P_1 = 10 \text{ kW}$

$Q_2 = Q_1 - Q_c = 7 - 4.083 = 2.917 \text{ kVAR}$

$\mathbf{S}_2 = 10 + j2.917 \text{ kVA}$

But $\mathbf{S}_2 = \mathbf{V}\mathbf{I}_2^*$

$\mathbf{I}_2^* = \frac{\mathbf{S}_2}{\mathbf{V}} = \frac{10{,}000 + j2917}{240}$

$\mathbf{I}_2 = 41.667 - j12.154 = \underline{\mathbf{43.4\angle\text{-}16.26°\ A}}$

Chapter 11, Problem 74.

A 120-V rms 60-Hz source supplies two loads connected in parallel, as shown in Fig. 11.89.

(a) Find the power factor of the parallel combination.
(b) Calculate the value of the capacitance connected in parallel that will raise the power factor to unity.

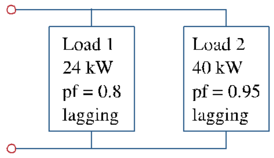


Figure 11.89
For Prob. 11.74.

Chapter 11, Solution 74.

(a) $\theta_1 = \cos^{-1}(0.8) = 36.87°$

$$S_1 = \frac{P_1}{\cos\theta_1} = \frac{24}{0.8} = 30 \text{ kVA}$$

$$Q_1 = S_1 \sin\theta_1 = (30)(0.6) = 18 \text{ kVAR}$$

$$\mathbf{S}_1 = 24 + j18 \text{ kVA}$$

$$\theta_2 = \cos^{-1}(0.95) = 18.19°$$

$$S_2 = \frac{P_2}{\cos\theta_2} = \frac{40}{0.95} = 42.105 \text{ kVA}$$

$$Q_2 = S_2 \sin\theta_2 = 13.144 \text{ kVAR}$$

$$\mathbf{S}_2 = 40 + j13.144 \text{ kVA}$$

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 = 64 + j31.144 \text{ kVA}$$

$$\theta = \tan^{-1}\left(\frac{31.144}{64}\right) = 25.95°$$

$$\text{pf} = \cos\theta = \underline{\mathbf{0.8992}}$$

(b) $\theta_2 = 25.95°$, $\theta_1 = 0°$

$$Q_c = P[\tan\theta_2 - \tan\theta_1] = 64[\tan(25.95°) - 0] = 31.144 \text{ kVAR}$$

$$C = \frac{Q_c}{\omega V_{rms}^2} = \frac{31{,}144}{(2\pi)(60)(120)^2} = \underline{\mathbf{5.74\ mF}}$$

Chapter 11, Problem 75.

Consider the power system shown in Fig. 11.90. Calculate:

(a) the total complex power

(b) the power factor

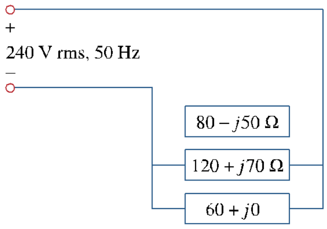


Figure 11.90
For Prob. 11.75.

Chapter 11, Solution 75.

(a) $$\mathbf{S}_1 = \frac{|\mathbf{V}|^2}{\mathbf{Z}_1^*} = \frac{(240)^2}{80 + j50} = \frac{5760}{8 + j5} = 517.75 - j323.59 \text{ VA}$$

$$\mathbf{S}_2 = \frac{(240)^2}{120 - j70} = \frac{5760}{12 - j7} = 358.13 + j208.91 \text{ VA}$$

$$\mathbf{S}_3 = \frac{(240)^2}{60} = 960 \text{ VA}$$

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3 = \underline{1835.9 - j114.68 \text{ VA}}$$

(b) $$\theta = \tan^{-1}\left(\frac{114.68}{1835.88}\right) = 3.574°$$

$$\text{pf} = \cos\theta = \underline{\mathbf{0.998}} \text{ \{leading\}}$$

(c) Since the circuit already has a leading power factor, near unity, no compensation is necessary.

Chapter 11, Problem 76.

Obtain the wattmeter reading of the circuit in Fig. 11.91.

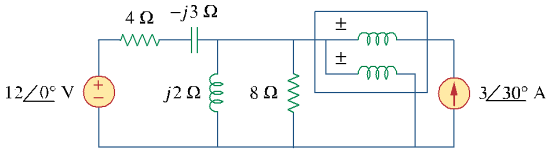
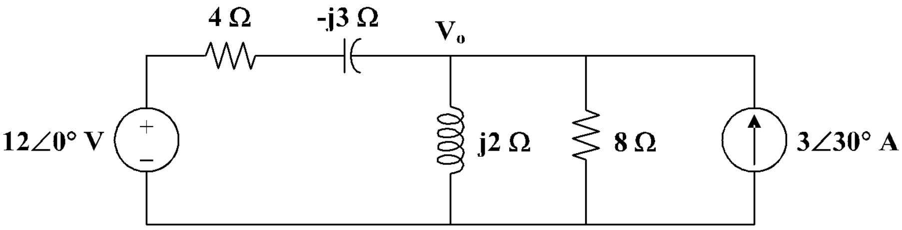


Figure 11.91
For Prob. 11.76.

Chapter 11, Solution 76.

The wattmeter reads the real power supplied by the current source. Consider the circuit below.



$$3\angle 30° + \frac{12 - \mathbf{V}_o}{4 - j3} = \frac{\mathbf{V}_o}{j2} + \frac{\mathbf{V}_o}{8}$$

$$\mathbf{V}_o = \frac{36.14 + j23.52}{2.28 - j3.04} = 0.7547 + j11.322 = 11.347\angle 86.19°$$

$$\mathbf{S} = \frac{1}{2}\mathbf{V}_o\mathbf{I}_o^* = \frac{1}{2}\cdot(11.347\angle 86.19°)(3\angle -30°)$$

$$\mathbf{S} = 17.021\angle 56.19°$$

P = Re(**S**) = **9.471 W**

Chapter 11, Problem 77.

What is the reading of the wattmeter in the network of Fig. 11.92?

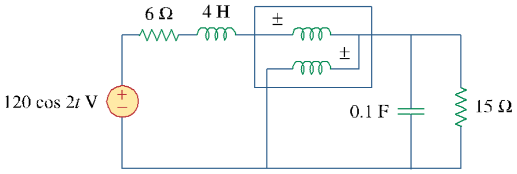


Figure 11.92
For Prob. 11.77.

Chapter 11, Solution 77.

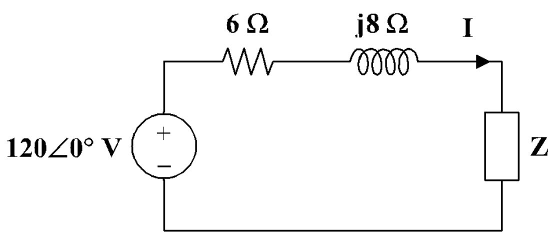
The wattmeter measures the power absorbed by the parallel combination of 0.1 F and 150 Ω.

$$120\cos(2t) \longrightarrow 120\angle 0°, \qquad \omega = 2$$

$$4\text{ H} \longrightarrow j\omega L = j8$$

$$0.1\text{ F} \longrightarrow \frac{1}{j\omega C} = \text{-}j5$$

Consider the following circuit.



$$\mathbf{Z} = 15 \,||\, (\text{-}j5) = \frac{(15)(\text{-}j5)}{15 - j5} = 1.5 - j4.5$$

$$\mathbf{I} = \frac{120}{(6 + j8) + (1.5 - j4.5)} = 14.5\angle\text{-}25.02°$$

$$\mathbf{S} = \frac{1}{2}\mathbf{V}\,\mathbf{I}^* = \frac{1}{2}\left|\mathbf{I}\right|^2 \mathbf{Z} = \frac{1}{2}\cdot(14.5)^2(1.5 - j4.5)$$

$$\mathbf{S} = 157.69 - j473.06 \text{ VA}$$

The wattmeter reads

$$P = \text{Re}(\mathbf{S}) = \underline{\mathbf{157.69\ W}}$$

Chapter 11, Problem 78.

Find the wattmeter reading of the circuit shown in Fig. 11.93.

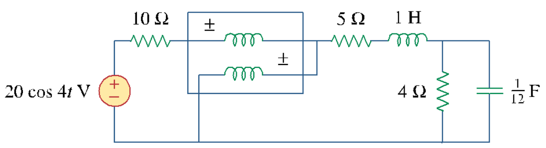


Figure 11.93
For Prob. 11.78.

Chapter 11, Solution 78.

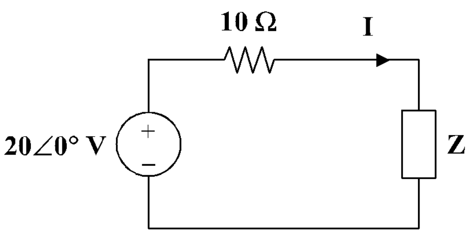
The wattmeter reads the power absorbed by the element to its right side.

$$2\cos(4t) \longrightarrow 2\angle 0°, \qquad \omega = 4$$

$$1\text{ H} \longrightarrow j\omega L = j4$$

$$\frac{1}{12}\text{F} \longrightarrow \frac{1}{j\omega C} = -j3$$

Consider the following circuit.



$$\mathbf{Z} = 5 + j4 + 4 \,||\, -j3 = 5 + j4 + \frac{(4)(-j3)}{4 - j3}$$

$$\mathbf{Z} = 6.44 + j2.08$$

$$\mathbf{I} = \frac{20}{16.44 + j2.08} = 1.207\angle -7.21°$$

$$\mathbf{S} = \frac{1}{2}|\mathbf{I}|^2\,\mathbf{Z} = \frac{1}{2}\cdot(1.207)^2(6.44 + j2.08)$$

$$\text{P} = \text{Re}(\mathbf{S}) = \underline{\mathbf{4.691\ W}}$$

Chapter 11, Problem 79.

Determine the wattmeter reading of the circuit in Fig. 11.94.

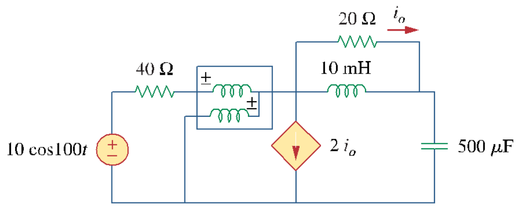


Figure 11.94
For Prob. 11.79.

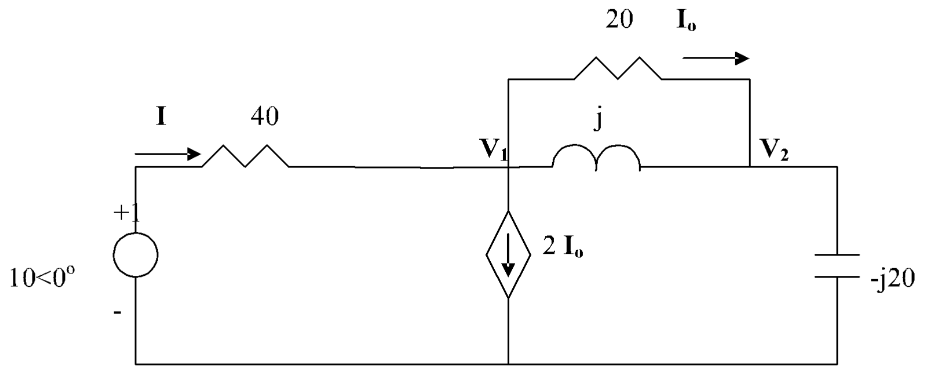
Chapter 11, Solution 79.

The wattmeter reads the power supplied by the source and partly absorbed by the 40-Ω resistor.

$\omega = 100$,

$$10\,\text{mH} \longrightarrow j100x10x10^{-3} = j, \quad 500\mu F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j100x500x10^{-6}} = -j20$$

The frequency-domain circuit is shown below.



At node 1,

$$\frac{10 - V_1}{40} = 2I_o + \frac{V_1 - V_2}{j} + \frac{V_1 - V_2}{20} = \frac{3(V_1 - V_2)}{20} + \frac{V_1 - V_2}{j} \longrightarrow \qquad (1)$$

$$10 = (7 - j40)V_1 + (-6 + j40)V_2$$

At node 2,

$$\frac{V_1 - V_2}{j} + \frac{V_1 - V_2}{20} = \frac{V_2}{-j20} \longrightarrow 0 = (20 + j)V_1 - (19 + j)V_2 \qquad (2)$$

Solving (1) and (2) yields $V_1 = 1.5568 - j4.1405$

$$I = \frac{10 - V_1}{40} = 0.2111 + j0.1035, \quad S = \frac{1}{2}V_1 I^* = -0.04993 - j0.5176$$

$$P = \text{Re}(S) = \underline{\mathbf{50\ mW}}.$$

Chapter 11, Problem 80.

The circuit of Fig. 11.95 portrays a wattmeter connected into an ac network.

(a) Find the load current.

(b) Calculate the wattmeter reading.

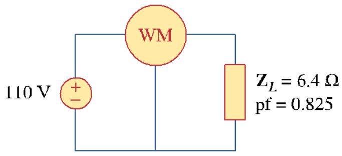


Figure 11.95
For Prob. 11.80.

Chapter 11, Solution 80.

(a) $\mathbf{I} = \dfrac{\mathbf{V}}{\mathbf{Z}} = \dfrac{110}{6.4} = \underline{\mathbf{17.19\ A}}$

(b) $\mathbf{S} = \dfrac{\mathbf{V}^2}{\mathbf{Z}} = \dfrac{(110)^2}{6.4} = 1890.625$

$\cos\theta = \text{pf} = 0.825 \longrightarrow \theta = 34.41°$

$P = S\cos\theta = 1559.76 \cong \underline{\mathbf{1.6\ kW}}$

Chapter 11, Problem 81.

A 120-V rms, 60-Hz electric hair dryer consumes 600 W at a lagging pf of 0.92. Calculate the rms-valued current drawn by the dryer.

Chapter 11, Solution 81.

P = 600 W, $pf = 0.92 \longrightarrow \theta = 23.074^o$

$P = S\cos\theta \longrightarrow S = \dfrac{P}{0.92} = 652.17 \text{ VA}$

S = P+jQ = 600 + j652.17sin23.09° = 600 +j255.6

But $S = V_{rms} I^*_{rms}$.

$I^*_{rms} = \dfrac{S}{V_{rms}} = \dfrac{600 + j255.6}{120}$

$I_{rms} = 5 - j2.13 = \underline{\mathbf{5.435\angle{-23.07}^\circ A}}$.

Chapter 11, Problem 82.

A 240-V rms 60-Hz source supplies a parallel combination of a 5-kW heater and a 30-kVA induction motor whose power factor is 0.82. Determine:

(a) the system apparent power
(b) the system reactive power
(c) the kVA rating of a capacitor required to adjust the system power factor to 0.9 lagging
(d) the value of the capacitor required

Chapter 11, Solution 82.

(a) $P_1 = 5{,}000, \qquad Q_1 = 0$

$$P_2 = 30{,}000x0.82 = 24{,}600, \qquad Q_2 = 30{,}000\sin(\cos^{-1} 0.82) = 17{,}171$$

$$\overline{S} = \overline{S}_1 + \overline{S}_2 = (P_1 + P_2) + j(Q_1 + Q_2) = 29{,}600 + j17{,}171$$

$$S = |\overline{S}| = \underline{34.22 \text{ kVA}}$$

(b) $$Q = \underline{\mathbf{17.171 \text{ kVAR}}}$$

(c) $$pf = \frac{P}{S} = \frac{29{,}600}{34{,}220} = 0.865$$

$$Q_c = P(\tan\theta_1 - \tan\theta_2)$$
$$= 29{,}600\left[\tan(\cos^{-1} 0.865) - \tan(\cos^{-1} 0.9)\right] = \underline{2833 \text{ VAR}}$$

(c) $$C = \frac{Q_c}{\omega V^2_{rms}} = \frac{2833}{2\pi x60x240^2} = \underline{130.46\mu \text{ F}}$$

Chapter 11, Problem 83.

Oscilloscope measurements indicate that the voltage across a load and the current through it are, respectively, $210\angle 60^\circ$ V and $8\angle 25^\circ$ A. Determine:

(a) the real power
(b) the apparent power
(c) the reactive power
(d) the power factor

Chapter 11, Solution 83.

(a) $\bar{S} = \frac{1}{2}VI^* = \frac{1}{2}(210\angle 60^o)(8\angle -25^o) = 840\angle 35^o$

$$P = S\cos\theta = 840\cos 35^o = \underline{688.1\text{ W}}$$

(b) S = **<u>840 VA</u>**

(c) $Q = S\sin\theta = 840\sin 35^o = \underline{481.8\text{ VAR}}$

(d) $pf = P/S = \cos 35^o = \underline{0.8191\text{ (lagging)}}$

Chapter 11, Problem 84.

A consumer has an annual consumption of 1200 MWh with a maximum demand of 2.4 MVA. The maximum demand charge is $30 per kVA per annum, and the energy charge per kWh is 4 cents.

(a) Determine the annual cost of energy.
(b) Calculate the charge per kWh with a flat-rate tariff if the revenue to the utility company is to remain the same as for the two-part tariff.

Chapter 11, Solution 84.

(a) Maximum demand charge $= 2{,}400 \times 30 = \$72{,}000$
Energy cost $= \$0.04 \times 1{,}200 \times 10^3 = \$48{,}000$
Total charge = **<u>$120,000</u>**

(b) To obtain $120,000 from 1,200 MWh will require a flat rate of

$$\frac{\$120{,}000}{1{,}200 \times 10^3}\text{ per kWh} = \underline{\mathbf{\$0.10\ per\ kWh}}$$

Chapter 11, Problem 85.

A regular household system of a single-phase three-wire circuit allows the operation of both 120-V and 240-V, 60-Hz appliances. The household circuit is modeled as shown in Fig. 11.96. Calculate:

(a) the currents $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_n$
(b) the total complex power supplied
(c) the overall power factor of the circuit

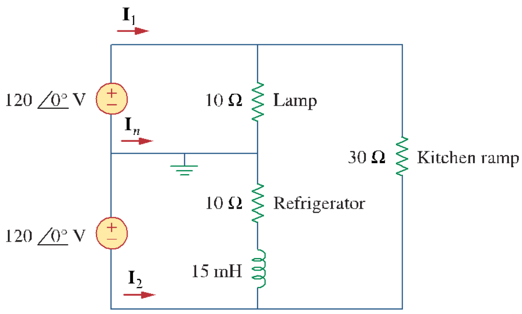
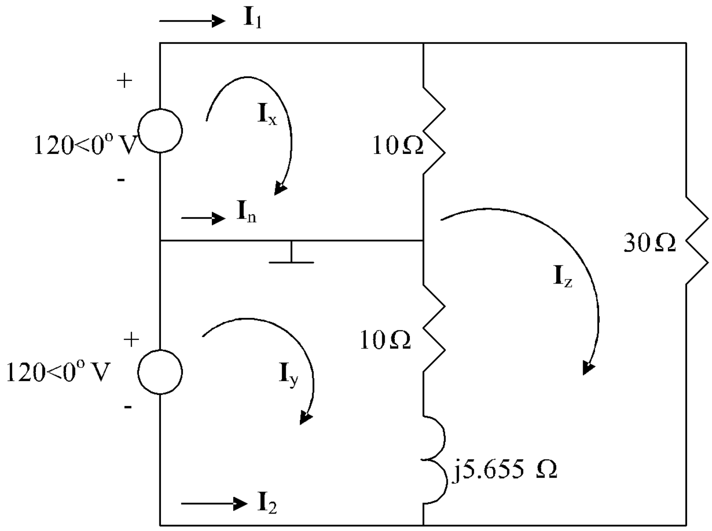


Figure 11.96
For Prob. 11.85.

Chapter 11, Solution 85.

(a) 15 mH $\longrightarrow$ $j2\pi x60x15x10^{-3} = j5.655$

We apply mesh analysis as shown below.



For mesh x,

$120 = 10\,\mathbf{I}_x - 10\,\mathbf{I}_z$ (1)

For mesh y,

$120 = (10+j5.655)\,\mathbf{I}_y - (10+j5.655)\,\mathbf{I}_z$ (2)

For mesh z,

$0 = -10\,\mathbf{I}_x - (10+j5.655)\,\mathbf{I}_y + (50+j5.655)\,\mathbf{I}_z$ (3)

Solving (1) to (3) gives

$\mathbf{I}_x = 20$, $\mathbf{I}_y = 17.09 - j5.142$, $\mathbf{I}_z = 8$

Thus,

$\mathbf{I}_1 = \mathbf{I}_x = \underline{20\text{ A}}$

$\mathbf{I}_2 = -\mathbf{I}_y = -17.09 + j5.142 = \underline{17.85\angle 163.26^o\text{ A}}$

$\mathbf{I}_n = \mathbf{I}_y - \mathbf{I}_x = -2.91 - j5.142 = \underline{5.907\angle -119.5^o\text{ A}}$

(b) $\overline{S_1} = (120)I^{\bullet}{}_x = 120x20 = 2400$, $\overline{S_2} = (120)I^{\bullet}{}_y = 2051 + j617$

$\overline{S} = \overline{S_1} + \overline{S_2} = \underline{4451 + j617\text{ VA}}$

(c) pf = P/S = 4451/4494 = **0.9904** (lagging)

Chapter 11, Problem 86.

e2d

A transmitter delivers maximum power to an antenna when the antenna is adjusted to represent a load of 75-Ω resistance in series with an inductance of 4 μ H. If the transmitter operates at 4.12 MHz, find its internal impedance.

Chapter 11, Solution 86.

For maximum power transfer

$$\mathbf{Z}_L = \mathbf{Z}_{Th}^* \longrightarrow \mathbf{Z}_i = \mathbf{Z}_{Th} = \mathbf{Z}_L^*$$

$$\mathbf{Z}_L = R + j\omega L = 75 + j(2\pi)(4.12\times10^{6})(4\times10^{-6})$$

$$\mathbf{Z}_L = 75 + j103.55\,\Omega$$

$$\mathbf{Z}_i = \underline{\mathbf{75 - j103.55\,\Omega}}$$

Chapter 11, Problem 87.

In a TV transmitter, a series circuit has an impedance of 3kΩ and a total current of 50 mA. If the voltage across the resistor is 80 V, what is the power factor of the circuit?

Chapter 11, Solution 87.

$$\mathbf{Z} = R \pm jX$$

$$\mathbf{V}_R = \mathbf{I}R \longrightarrow R = \frac{\mathbf{V}_R}{\mathbf{I}} = \frac{80}{50\times10^{-3}} = 1.6\text{ k}\Omega$$

$$|\mathbf{Z}|^2 = R^2 + X^2 \longrightarrow X^2 = |\mathbf{Z}|^2 - R^2 = (3)^2 - (1.6)^2$$

$$X = 2.5377\text{ k}\Omega$$

$$\theta = \tan^{-1}\left(\frac{X}{R}\right) = \tan^{-1}\left(\frac{2.5377}{1.6}\right) = 57.77°$$

$$\text{pf} = \cos\theta = \underline{\mathbf{0.5333}}$$

Chapter 11, Problem 88.

A certain electronic circuit is connected to a 110-V ac line. The root-mean-square value of the current drawn is 2 A, with a phase angle of 55º.

(a) Find the true power drawn by the circuit.
(b) Calculate the apparent power.

Chapter 11, Solution 88.

(a) $\quad S = (110)(2\angle 55°) = 220\angle 55°$

$$P = S\cos\theta = 220\cos(55°) = \underline{\mathbf{126.2\ W}}$$

(b) $\quad S = |\mathbf{S}| = \underline{\mathbf{220\ VA}}$

Chapter 11, Problem 89.

e2d

An industrial heater has a nameplate that reads: 210 V 60 Hz 12 kVA 0.78 pf lagging
Determine:

(a) the apparent and the complex power
(b) the impedance of the heater

Chapter 11, Solution 89.

(a) $\quad \text{Apparent power} = S = \underline{\mathbf{12\ kVA}}$

$$P = S\cos\theta = (12)(0.78) = 9.36\ \text{kW}$$
$$Q = S\sin\theta = 12\sin(\cos^{-1}(0.78)) = 7.51\ \text{kVAR}$$

$$\mathbf{S} = P + jQ = \underline{\mathbf{9.36 + j7.51\ kVA}}$$

(b) $$\mathbf{S} = \frac{|\mathbf{V}|^2}{\mathbf{Z}^*} \longrightarrow \mathbf{Z}^* = \frac{|\mathbf{V}|^2}{\mathbf{S}} = \frac{(210)^2}{(9.36 + j7.51)\times 10^3} = 2.866 - j2.3$$

$$\mathbf{Z} = \underline{\mathbf{2.866 + j2.3\ \Omega}}$$

Chapter 11, Problem 90.

* e⊘d A 2000-kW turbine-generator of 0.85 power factor operates at the rated load. An additional load of 300 kW at 0.8 power factor is added.What kVAR of capacitors is required to operate the turbine-generator but keep it from being overloaded?

* An asterisk indicates a challenging problem.

Chapter 11, Solution 90

Original load :

$$P_1 = 2000 \text{ kW}, \qquad \cos\theta_1 = 0.85 \longrightarrow \theta_1 = 31.79°$$

$$S_1 = \frac{P_1}{\cos\theta_1} = 2352.94 \text{ kVA}$$

$$Q_1 = S_1 \sin\theta_1 = 1239.5 \text{ kVAR}$$

Additional load :

$$P_2 = 300 \text{ kW}, \qquad \cos\theta_2 = 0.8 \longrightarrow \theta_2 = 36.87°$$

$$S_2 = \frac{P_2}{\cos\theta_2} = 375 \text{ kVA}$$

$$Q_2 = S_2 \sin\theta_2 = 225 \text{ kVAR}$$

Total load :

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 = (P_1 + P_2) + j(Q_1 + Q_2) = P + jQ$$

$$P = 2000 + 300 = 2300 \text{ kW}$$

$$Q = 1239.5 + 225 = 1464.5 \text{ kVAR}$$

The minimum operating pf for a 2300 kW load and not exceeding the kVA rating of the generator is

$$\cos\theta = \frac{P}{S_1} = \frac{2300}{2352.94} = 0.9775$$

or $\theta = 12.177°$

The maximum load kVAR for this condition is

$$Q_m = S_1 \sin\theta = 2352.94 \sin(12.177°)$$

$$Q_m = 496.313 \text{ kVAR}$$

The capacitor must supply the difference between the total load kVAR (i.e. Q) and the permissible generator kVAR (i.e. Q_m). Thus,

$$Q_c = Q - Q_m = \underline{\mathbf{968.2 \text{ kVAR}}}$$

Chapter 11, Problem 91.

The nameplate of an electric motor has the following information:

Line voltage: 220 V rms

Line current: 15 A rms

Line frequency: 60 Hz

Power: 2700 W

Determine the power factor (lagging) of the motor. Find the value of the capacitance *C* that must be connected across the motor to raise the pf to unity.

Chapter 11, Solution 91

Original load :

$$P_1 = 2000\text{ kW}, \qquad \cos\theta_1 = 0.85 \longrightarrow \theta_1 = 31.79°$$

$$S_1 = \frac{P_1}{\cos\theta_1} = 2352.94\text{ kVA}$$

$$Q_1 = S_1 \sin\theta_1 = 1239.5\text{ kVAR}$$

Additional load :

$$P_2 = 300\text{ kW}, \qquad \cos\theta_2 = 0.8 \longrightarrow \theta_2 = 36.87°$$

$$S_2 = \frac{P_2}{\cos\theta_2} = 375\text{ kVA}$$

$$Q_2 = S_2 \sin\theta_2 = 225\text{ kVAR}$$

Total load :

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 = (P_1 + P_2) + j(Q_1 + Q_2) = P + jQ$$

$$P = 2000 + 300 = 2300\text{ kW}$$

$$Q = 1239.5 + 225 = 1464.5\text{ kVAR}$$

The minimum operating pf for a 2300 kW load and not exceeding the kVA rating of the generator is

$$\cos\theta = \frac{P}{S_1} = \frac{2300}{2352.94} = 0.9775$$

or $\quad \theta = 12.177°$

The maximum load kVAR for this condition is

$$Q_m = S_1 \sin\theta = 2352.94\sin(12.177°)$$

$$Q_m = 496.313\text{ kVAR}$$

The capacitor must supply the difference between the total load kVAR (i.e. Q) and the permissible generator kVAR (i.e. Q_m). Thus,

$$Q_c = Q - Q_m = \mathbf{968.2\text{ kVAR}}$$

Chapter 11, Problem 92.

As shown in Fig. 11.97, a 550-V feeder line supplies an industrial plant consisting of a motor drawing 60 kW at 0.75 pf (inductive), a capacitor with a rating of 20 kVAR, and lighting drawing 20 kW.

(a) -Calculate the total reactive power and apparent power absorbed by the plant.
(b) Determine the overall pf.
(c) Find the current in the feeder line.

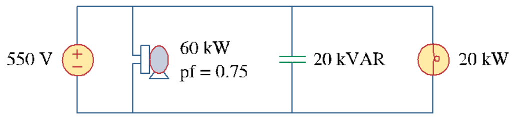


Figure 11.97
For Prob. 11.92.

Chapter 11, Solution 92

(a) Apparent power drawn by the motor is

$$S_m = \frac{P}{\cos\theta} = \frac{60}{0.75} = 80 \text{ kVA}$$

$$Q_m = \sqrt{S^2 - P^2} = \sqrt{(80)^2 - (60)^2} = 52.915 \text{ kVAR}$$

Total real power

$$P = P_m + P_c + P_L = 60 + 0 + 20 = 80 \text{ kW}$$

Total reactive power

$$Q = Q_m + Q_c + Q_L = 52.915 - 20 + 0 = \underline{\mathbf{32.91\ kVAR}}$$

Total apparent power

$$S = \sqrt{P^2 + Q^2} = \underline{\mathbf{86.51\ kVA}}$$

(b) $$\text{pf} = \frac{P}{S} = \frac{80}{86.51} = \underline{\mathbf{0.9248}}$$

(c) $$I = \frac{S}{V} = \frac{86510}{550} = \underline{\mathbf{157.3\ A}}$$

Chapter 11, Problem 93.

A factory has the following four major loads:

- A motor rated at 5 hp, 0.8 pf lagging (1 hp = 0.7457 kW).
- A heater rated at 1.2 kW, 1.0 pf.
- Ten 120-W lightbulbs.
- A synchronous motor rated at 1.6 kVAR, 0.6 pf leading.

(a) Calculate the total real and reactive power.
(b) Find the overall power factor.

Chapter 11, Solution 93

(a) $P_1 = (5)(0.7457) = 3.7285 \text{ kW}$

$$S_1 = \frac{P_1}{pf} = \frac{3.7285}{0.8} = 4.661 \text{ kVA}$$

$$Q_1 = S_1 \sin(\cos^{-1}(0.8)) = 2.796 \text{ kVAR}$$

$$\mathbf{S}_1 = 3.7285 + j2.796 \text{ kVA}$$

$$P_2 = 1.2 \text{ kW}, \qquad Q_2 = 0 \text{ VAR}$$

$$\mathbf{S}_2 = 1.2 + j0 \text{ kVA}$$

$$P_3 = (10)(120) = 1.2 \text{ kW}, \qquad Q_3 = 0 \text{ VAR}$$

$$\mathbf{S}_3 = 1.2 + j0 \text{ kVA}$$

$$Q_4 = 1.6 \text{ kVAR}, \qquad \cos\theta_4 = 0.6 \longrightarrow \sin\theta_4 = 0.8$$

$$S_4 = \frac{Q_4}{\sin\theta_4} = 2 \text{ kVA}$$

$$P_4 = S_4 \cos\theta_4 = (2)(0.6) = 1.2 \text{ kW}$$

$$\mathbf{S}_4 = 1.2 - j1.6 \text{ kVA}$$

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3 + \mathbf{S}_4$$

$$\mathbf{S} = 7.3285 + j1.196 \text{ kVA}$$

Total real power = **7.328 kW**
Total reactive power = **1.196 kVAR**

(b) $$\theta = \tan^{-1}\left(\frac{1.196}{7.3285}\right) = 9.27°$$

$$pf = \cos\theta = \mathbf{0.987}$$

Chapter 11, Problem 94.

A 1-MVA substation operates at full load at 0.7 power factor. It is desired to improve the power factor to 0.95 by installing capacitors. Assume that new substation and distribution facilities cost $120 per kVA installed, and capacitors cost $30 per kVA installed.

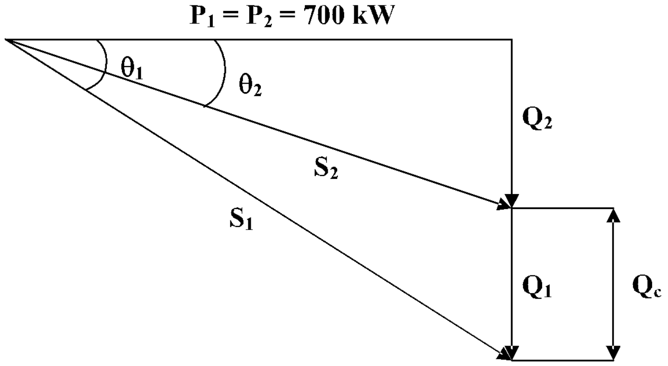
(a) Calculate the cost of capacitors needed.
(b) Find the savings in substation capacity released.
(c) Are capacitors economical for releasing the amount of substation capacity?

Chapter 11, Solution 94

$$\cos\theta_1 = 0.7 \longrightarrow \theta_1 = 45.57°$$
$$S_1 = 1 \text{ MVA} = 1000 \text{ kVA}$$
$$P_1 = S_1 \cos\theta_1 = 700 \text{ kW}$$
$$Q_1 = S_1 \sin\theta_1 = 714.14 \text{ kVAR}$$

For improved pf,

$$\cos\theta_2 = 0.95 \longrightarrow \theta_2 = 18.19°$$
$$P_2 = P_1 = 700 \text{ kW}$$
$$S_2 = \frac{P_2}{\cos\theta_2} = \frac{700}{0.95} = 736.84 \text{ kVA}$$
$$Q_2 = S_2 \sin\theta_2 = 230.08 \text{ kVAR}$$



(a) Reactive power across the capacitor

$$Q_c = Q_1 - Q_2 = 714.14 - 230.08 = 484.06 \text{ kVAR}$$

Cost of installing capacitors $= \$30 \times 484.06 =$ **$14,521.80**

(b) Substation capacity released $= S_1 - S_2$

$$= 1000 - 736.84 = 263.16 \text{ kVA}$$

Saving in cost of substation and distribution facilities

$= \$120 \times 263.16 =$ **$31,579.20**

(c) **Yes**, because (a) is greater than (b). Additional system capacity obtained by using capacitors costs only 46% as much as new substation and distribution facilities.

Chapter 11, Problem 95.

e7d A coupling capacitor is used to block dc current from an amplifier as shown in Fig. 11.98(a). The amplifier and the capacitor act as the source, while the speaker is the load as in Fig. 11.98(b).

(a) At what frequency is maximum power transferred to the speaker?
(b) If V_s = 4.6 V rms, how much power is delivered to the speaker at that frequency?

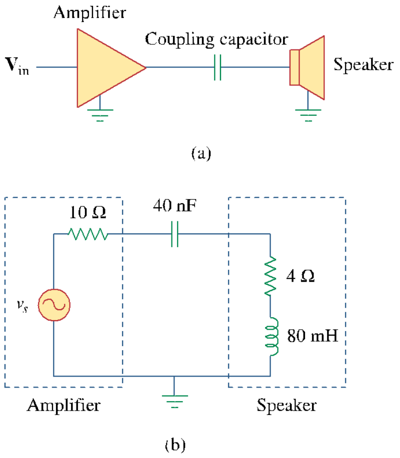


Figure 11.98
For Prob. 11.95.

Chapter 11, Solution 95

(a) Source impedance $\mathbf{Z}_s = R_s - jX_c$

Load impedance $\mathbf{Z}_L = R_L + jX_2$

For maximum load transfer

$$\mathbf{Z}_L = \mathbf{Z}_s^* \longrightarrow R_s = R_L, \quad X_c = X_L$$

$$X_c = X_L \longrightarrow \frac{1}{\omega C} = \omega L$$

or $$\omega = \frac{1}{\sqrt{LC}} = 2\pi f$$

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(80\times10^{-3})(40\times10^{-9})}} = \underline{\mathbf{2.814\ kHz}}$$

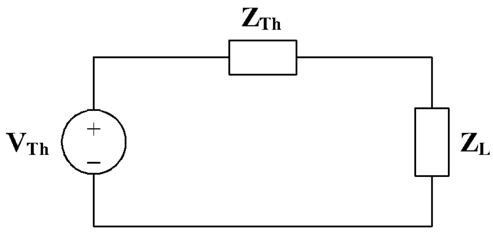
(b) $$P = \left(\frac{V_s}{(10+4)}\right)^2 4 = \left(\frac{4.6}{14}\right)^2 4 = \underline{\mathbf{431.8\ mW}} \quad \text{(since } V_s \text{ is in rms)}$$

Chapter 11, Problem 96.

e2d A power amplifier has an output impedance of $40 + j8\ \Omega$ It produces a no-load output voltage of 146 V at 300 Hz.

(a) Determine the impedance of the load that achieves maximum power transfer.
(b) Calculate the load power under this matching condition.

Chapter 11, Solution 96



(a) $V_{Th} = 146\text{ V}, \quad 300\text{ Hz}$

$Z_{Th} = 40 + j8\ \Omega$

$$Z_L = Z_{Th}^* = \mathbf{40 - j8\ \Omega}$$

(b) $$P = \frac{|V_{Th}|^2}{8R_{Th}} = \frac{(146)^2}{(8)(40)} = \mathbf{66.61\ W}$$

Chapter 11, Problem 97.

A power transmission system is modeled as shown in Fig. 11.99. If $\mathbf{V}_s = 240\angle 0^\circ$ rms, find the average power absorbed by the load.

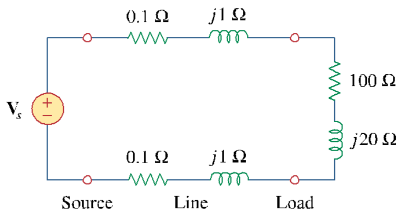


Figure 11.99
For Prob. 11.97.

Chapter 11, Solution 97

$$Z_T = (2)(0.1 + j) + (100 + j20) = 100.2 + j22\ \Omega$$

$$I = \frac{V_s}{Z_T} = \frac{240}{100.2 + j22}$$

$$P = |I|^2 R_L = 100|I|^2 = \frac{(100)(240)^2}{(100.2)^2 + (22)^2} = \underline{\mathbf{547.3\ W}}$$

Chapter 12, Problem 1.

If $\mathbf{V}_{ab}$ = 400 V in a balanced Y-connected three-phase generator, find the phase voltages, assuming the phase sequence is:

(a) *abc* (b) *acb*

Chapter 12, Solution 1.

(a) If $\mathbf{V}_{ab} = 400$, then

$$\mathbf{V}_{an} = \frac{400}{\sqrt{3}}\angle -30° = \underline{\mathbf{231\angle -30°\ V}}$$

$$\mathbf{V}_{bn} = \underline{\mathbf{231\angle -150°\ V}}$$

$$\mathbf{V}_{cn} = \underline{\mathbf{231\angle -270°\ V}}$$

(b) For the acb sequence,

$$\mathbf{V}_{ab} = \mathbf{V}_{an} - \mathbf{V}_{bn} = V_p\angle 0° - V_p\angle 120°$$

$$\mathbf{V}_{ab} = V_p\left(1 + \frac{1}{2} - j\frac{\sqrt{3}}{2}\right) = V_p\sqrt{3}\angle -30°$$

i.e. in the acb sequence, $\mathbf{V}_{ab}$ lags $\mathbf{V}_{an}$ by 30°.

Hence, if $\mathbf{V}_{ab} = 400$, then

$$\mathbf{V}_{an} = \frac{400}{\sqrt{3}}\angle 30° = \underline{\mathbf{231\angle 30°\ V}}$$

$$\mathbf{V}_{bn} = \underline{\mathbf{231\angle 150°\ V}}$$

$$\mathbf{V}_{cn} = \underline{\mathbf{231\angle -90°\ V}}$$

Chapter 12, Problem 2.

What is the phase sequence of a balanced three-phase circuit for which $\mathbf{V}_{an} = 160\angle 30°$ V and $\mathbf{V}_{cn} = 160\angle -90°$ V? Find $\mathbf{V}_{bn}$.

Chapter 12, Solution 2.

Since phase c lags phase a by 120°, this is an **<u>acb sequence</u>**.

$$\mathbf{V}_{bn} = 160\angle(30° + 120°) = \underline{\mathbf{160\angle 150°\ V}}$$

Chapter 12, Problem 3.
Determine the phase sequence of a balanced three-phase circuit in which
$\mathbf{V}_{bn} = 208\angle 130°$ V and $\mathbf{V}_{cn} = 208\angle 10°$ V. Obtain $\mathbf{V}_{an}$.

Chapter 12, Solution 3.

Since $\mathbf{V}_{bn}$ leads $\mathbf{V}_{cn}$ by 120°, this is an **abc sequence**.

$$\mathbf{V}_{an} = 208\angle(130° + 120°) = \underline{\mathbf{208\angle 250°\ V}}$$

Chapter 12, Problem 4.
A three-phase system with *abc* sequence and $\mathbf{V}_L = 200$ V feeds a Y-connected load with $Z_L = 40\angle 30°\Omega$. Find the line currents.

Chapter 12, Solution 4.

$$V_L = 200 = \sqrt{3}V_p \quad \longrightarrow \quad V_p = \frac{200}{\sqrt{3}}$$

$$I_a = \frac{V_{an}}{Z_Y} = \frac{200 < 0^o}{\sqrt{3}x40 < 30^o} = \underline{2.887 < -30^o \text{ A}}$$

$$I_b = I_a < -120^o = \underline{2.887 < -150^o \text{ A}}$$

$$I_c = I_a < +120^o = \underline{2.887 < 90^o \text{ A}}$$

Chapter 12, Problem 5.

For a Y-connected load, the time-domain expressions for three line-to-neutral voltages at the terminals are:

$v_{AN} = 150 \cos(\omega t + 32^o)$ V

$v_{BN} = 150 \cos(\omega t - 88^o)$ V

$v_{CN} = 150 \cos(\omega t + 152^o)$ V

Write the time-domain expressions for the line-to-line voltages v_{AN}, v_{BC}, and v_{CA} .

Chapter 12, Solution 5.

$$V_{AB} = \sqrt{3}V_p < 30^o = \sqrt{3}x150 < 32^o + 30^o = 260 < 62^o$$

Thus,

$$v_{AB} = \underline{260\cos(\omega t + 62^o) \text{ V}}$$

Using abc sequence,

$$v_{BC} = \underline{260\cos(\omega t - 58^o) \text{ V}}$$

$$v_{CA} = \underline{260\cos(\omega t + 182^o) \text{ V}}$$

Chapter 12, Problem 6.

For the Y-Y circuit of Fig. 12.41, find the line currents, the line voltages, and the load voltages.

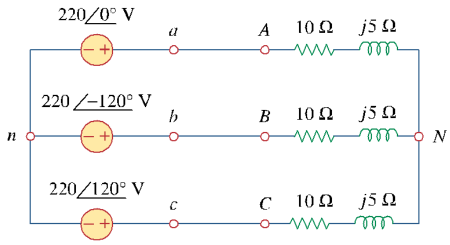


Figure 12.41
For Prob. 12.6.

Chapter 12, Solution 6.

$$\mathbf{Z}_Y = 10 + j5 = 11.18\angle 26.56°$$

The line currents are

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y} = \frac{220\angle 0°}{11.18\angle 26.56°} = \underline{\mathbf{19.68\angle -26.56°\ A}}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120° = \underline{\mathbf{19.68\angle -146.56°\ A}}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle 120° = \underline{\mathbf{19.68\angle 93.44°\ A}}$$

The line voltages are

$$\mathbf{V}_{ab} = 220\sqrt{3}\angle 30° = \underline{\mathbf{381\angle 30°\ V}}$$

$$\mathbf{V}_{bc} = \underline{\mathbf{381\angle -90°\ V}}$$

$$\mathbf{V}_{ca} = \underline{\mathbf{381\angle -210°\ V}}$$

The load voltages are

$$\mathbf{V}_{AN} = \mathbf{I}_a\mathbf{Z}_Y = \mathbf{V}_{an} = \underline{\mathbf{220\angle 0°\ V}}$$

$$\mathbf{V}_{BN} = \mathbf{V}_{bn} = \underline{\mathbf{220\angle -120°\ V}}$$

$$\mathbf{V}_{CN} = \mathbf{V}_{cn} = \underline{\mathbf{220\angle 120°\ V}}$$

Chapter 12, Problem 7.

Obtain the line currents in the three-phase circuit of Fig. 12.42 on the next page.

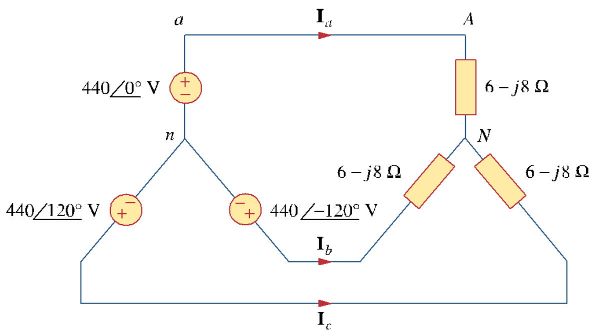
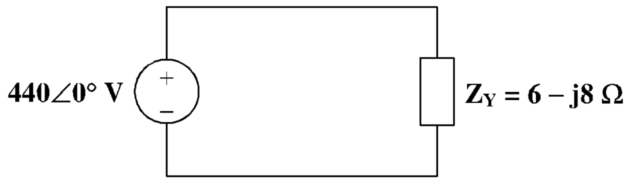


Figure 12.42
For Prob. 12.7.

Chapter 12, Solution 7.

This is a balanced Y-Y system.



Using the per-phase circuit shown above,

$$\mathbf{I}_a = \frac{440\angle 0°}{6 - j8} = \underline{\mathbf{44\angle 53.13°\ A}}$$

$$\mathbf{I}_b = \mathbf{I}_a \angle -120° = \underline{\mathbf{44\angle -66.87°\ A}}$$

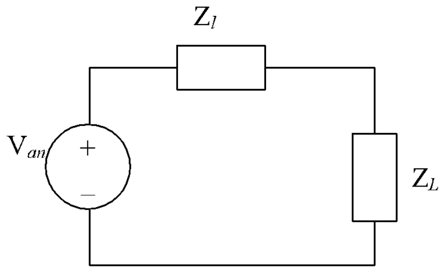
$$\mathbf{I}_c = \mathbf{I}_a \angle 120° = \underline{\mathbf{44\angle 173.13°\ A}}$$

Chapter 12, Problem 8.

In a balanced three-phase Y-Y system, the source is an *abc* sequence of voltages and $\mathbf{V}_{an}$ = 100 $\angle 20°$ V rms. The line impedance per phase is $0.6 + j1.2\ \Omega$, while the per-phase impedance of the load is $10 + j14\ \Omega$. Calculate the line currents and the load voltages.

Chapter 12, Solution 8.

Consider the per phase equivalent circuit shown below.



$$I_a = \frac{V_{an}}{Z_L + Z_\ell} = \frac{100 < 20^o}{10.6 + j15.2} = \underline{\mathbf{5.396\angle -35.1^\circ\ A}}$$

$$I_b = I_a < -120^o = \underline{\mathbf{5.396\angle -155.1^\circ\ A}}$$

$$I_c = I_a < +120^o = \underline{\mathbf{5.396\angle 84.9^\circ\ A}}$$

$$V_{La} = I_a Z_L = (4.4141 - j3.1033)(10 + j14) = \underline{\mathbf{92.83\angle 19.35^\circ\ A}}$$

$$V_{Lb} = V_{La} < -120^o = \underline{\mathbf{92.83\angle -100.65^\circ\ A}}$$

$$V_{Lc} = V_{La} < +120^o = \underline{\mathbf{92.83\angle 139.35^\circ\ A}}$$

Chapter 12, Problem 9.

A balanced Y-Y four-wire system has phase voltages

$\mathbf{V}_{an} = 120\angle 0°$ $\mathbf{V}_{bn} = 120\angle -120°$

$\mathbf{V}_{cn} = 120\angle 120°$ V

The load impedance per phase is $19 + j13\ \Omega$, and the line impedance per phase is $1 + j2\ \Omega$. Solve for the line currents and neutral current.

Chapter 12, Solution 9.

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_L + \mathbf{Z}_Y} = \frac{120\angle 0°}{20 + j15} = \underline{\mathbf{4.8\angle -36.87°\ A}}$$

$$\mathbf{I}_b = \mathbf{I}_a \angle -120° = \underline{\mathbf{4.8\angle -156.87°\ A}}$$

$$\mathbf{I}_c = \mathbf{I}_a \angle 120° = \underline{\mathbf{4.8\angle 83.13°\ A}}$$

As a balanced system, $\mathbf{I}_n = \underline{\mathbf{0\ A}}$

Chapter 12, Problem 10.

For the circuit in Fig. 12.43, determine the current in the neutral line.

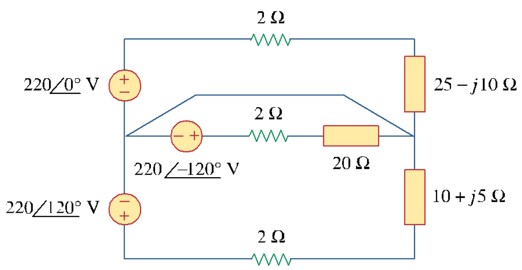


Figure 12.43
For Prob. 12.10.

Chapter 12, Solution 10.

Since the neutral line is present, we can solve this problem on a per-phase basis.

For phase a,

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_A + 2} = \frac{220\angle 0°}{27 - j10} = \frac{220}{28.79\angle -20.32°} = 7.642\angle 20.32°$$

For phase b,

$$\mathbf{I}_b = \frac{\mathbf{V}_{bn}}{\mathbf{Z}_B + 2} = \frac{220\angle -120°}{22} = 10\angle -120°$$

For phase c,

$$\mathbf{I}_c = \frac{\mathbf{V}_{cn}}{\mathbf{Z}_C + 2} = \frac{220\angle 120°}{12 + j5} = \frac{220\angle 120°}{13\angle 22.62°} = 16.923\angle 97.38°$$

The current in the neutral line is

$$\mathbf{I}_n = -(\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c) \text{ or } -\mathbf{I}_n = \mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c$$

$$-\mathbf{I}_n = (7.166 + j2.654) + (-5 - j8.667) + (-2.173 + j16.783)$$

$$\mathbf{I}_n = 0.007 - j10.77 = \underline{\mathbf{10.77\angle 90°A}}$$

Chapter 12, Problem 11.

In the Y-Δ system shown in Fig. 12.44, the source is a positive sequence with $\mathbf{V}_{an}$ = 120 $\angle 0°$ V and phase impedance $\mathbf{Z}_p = 2 - j3\ \Omega$. Calculate the line voltage $\mathbf{V}_L$ and the line current $\mathbf{I}_L$.

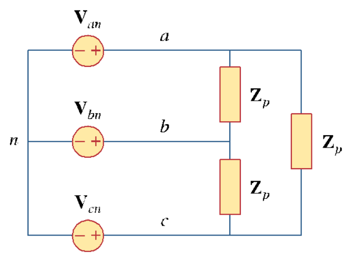


Figure 12.44
For Prob. 12.11.

Chapter 12, Solution 11.

$$V_{AB} = V_{ab} = \sqrt{3}V_p < 30^o = \sqrt{3}(120) < 30^o$$

$$V_L = |V_{ab}| = \sqrt{3}x120 = \underline{207.85\ \text{V}}$$

$$I_{AB} = \frac{V_{AB}}{Z_A} = \frac{\sqrt{3}V_p < 30^o}{2 - j3}$$

$$I_a = I_{AB}\sqrt{3} < -30^o = \frac{3V_p < 0^o}{2 - j3} = \frac{3x120}{2 - j3} = 55.385 + j83.07$$

$$I_L = |I_a| = \underline{99.846\ \ \text{A}}$$

Chapter 12, Problem 12.

Solve for the line currents in the Y-Δ circuit of Fig. 12.45. Take $\mathbf{Z}_\Delta = 60\angle 45°\Omega$.

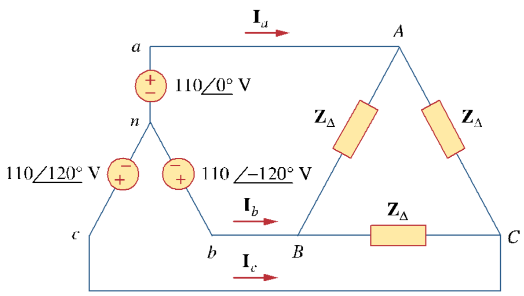
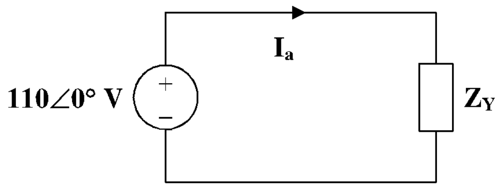


Figure 12.45
For Prob. 12.12.

Chapter 12, Solution 12.

Convert the delta-load to a wye-load and apply per-phase analysis.



$$\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3} = 20\angle 45°\,\Omega$$

$$\mathbf{I}_a = \frac{110\angle 0°}{20\angle 45°} = \underline{\mathbf{5.5\angle -45°\ A}}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120° = \underline{\mathbf{5.5\angle -165°\ A}}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle 120° = \underline{\mathbf{5.5\angle 75°\ A}}$$

Chapter 12, Problem 13.

PS ML In the balanced three-phase Y-Δ system in Fig. 12.46, find the line current I_L and the average power delivered to the load.

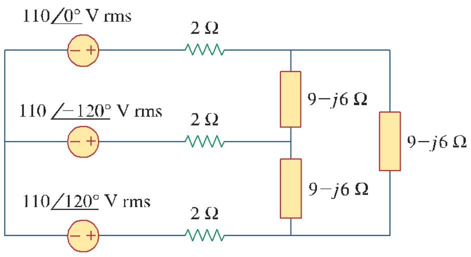
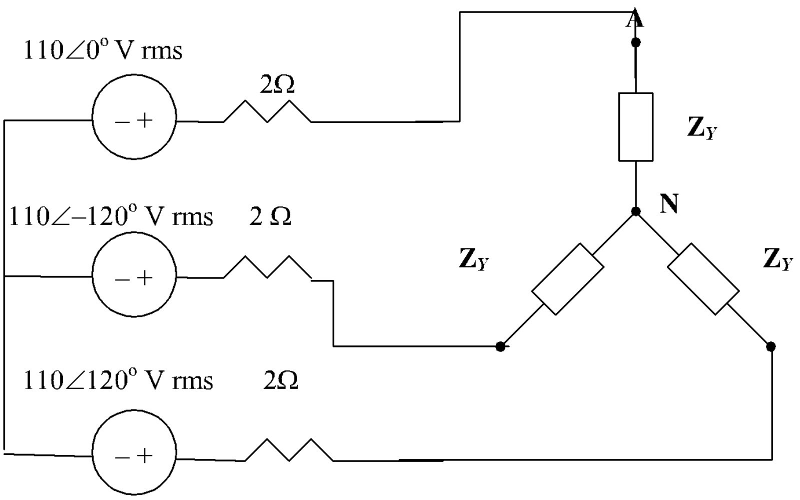


Figure 12.46
For Prob. 12.13.

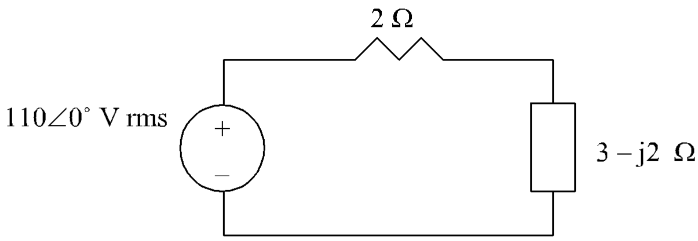
Chapter 12, Solution 13.

Convert the delta load to wye as shown below.



$$Z_Y = \frac{1}{3}Z \; = 3 - j2\;\Omega$$

We consider the single phase equivalent shown below.



$$I_a = \frac{110}{2+3-j2} = 20.4265 < 21.8^{o}$$

$I_L = |I_a| =$ 20.43 A

$S = 3|I_a|^2 Z_Y = 3(20.43)^2(3–j2) = 4514\angle–33.96^\circ = 3744 – j2522$

P = Re(S) = **3744 W**.

Chapter 12, Problem 14.

Obtain the line currents in the three-phase circuit of Fig. 12.47 on the next page.

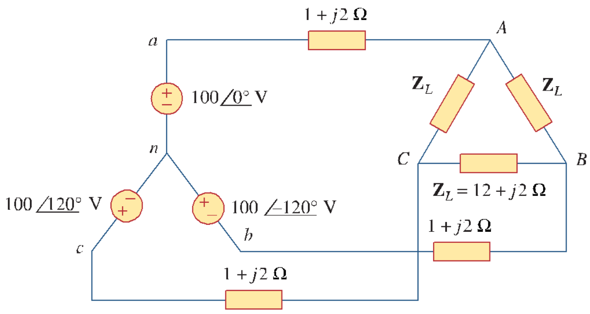
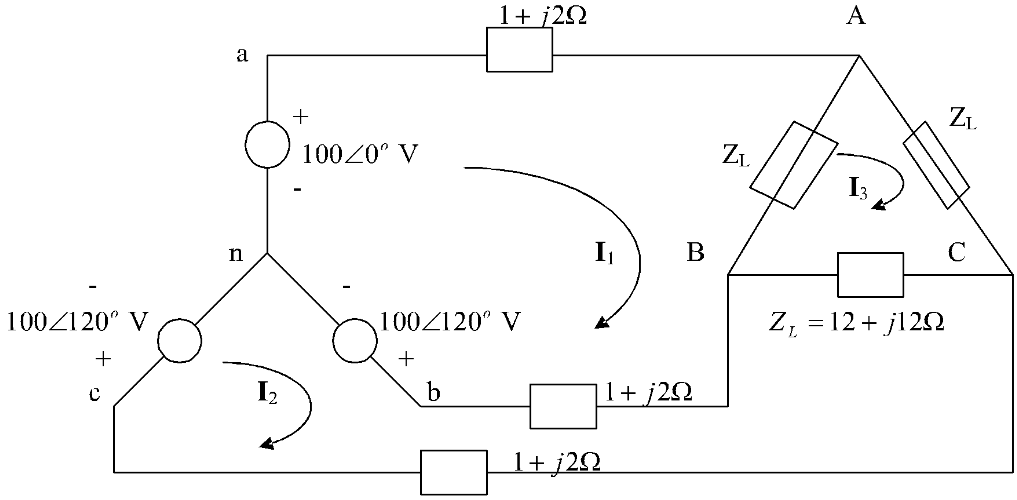


Figure 12.47
For Prob. 12.14.

Chapter 12, Solution 14.

We apply mesh analysis.



For mesh,

$$-100 + 100\angle 120^o + I_1(14 + j16) - (1 + j2)I_2 - (12 + j12)I_3 = 0$$

or

$$(14 + j16)I_1 - (1 + j2)I_2 - (12 + j12)I_3 = 100 + 50 - j86.6 = 150 - j86.6 \qquad (1)$$

For mesh 2,

$$100\angle 120^o - 100\angle -120^o - I_1(1 + j2) - (12 + j12)I_3 + (14 + j16)I_2 = 0$$

or

$$-(1 + j2)I_1 + (14 + j16)I_2 - (12 + j12)I_3 = -50 - j86.6 + 50 - j86.6 = -j173.2 \quad (2)$$

For mesh 3,

$$-(12 + j12)I_1 - (12 + j12)I_2 + (36 + j36)I_3 = 0 \qquad (3)$$

Solving (1) to (3) gives

$$I_1 = -3.161 - j19.3, \qquad I_2 = -10.098 - j16.749, \qquad I_3 = -4.4197 - j12.016$$

$$I_{aA} = I_1 = \underline{19.58\angle -99.3^o \text{ A}}$$

$$I_{bB} = I_2 - I_1 = \underline{7.392\angle 159.8^o \text{ A}}$$

$$I_{cC} = -I_2 = \underline{19.56\angle 58.91^o \text{ A}}$$

Chapter 12, Problem 15.

ps The circuit in Fig. 12.48 is excited by a balanced three-phase source with a line voltage of 210 V. If $\mathbf{Z}_l = 1 + j1\ \Omega$, $\mathbf{Z}_\Delta = 24 - j30\Omega$, and $\mathbf{Z}_Y = 12 + j5\ \Omega$, determine the magnitude of the line current of the combined loads.

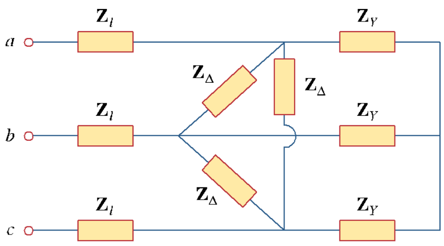


Figure 12.48
For Prob. 12.15.

Chapter 12, Solution 15.

Convert the delta load, $\mathbf{Z}_\Delta$, to its equivalent wye load.

$$\mathbf{Z}_{Ye} = \frac{\mathbf{Z}_\Delta}{3} = 8 - j10$$

$$\mathbf{Z}_p = \mathbf{Z}_Y \parallel \mathbf{Z}_{Ye} = \frac{(12 + j5)(8 - j10)}{20 - j5} = 8.076\angle -14.68°$$

$$\mathbf{Z}_p = 7.812 - j2.047$$

$$\mathbf{Z}_T = \mathbf{Z}_p + \mathbf{Z}_L = 8.812 - j1.047$$

$$\mathbf{Z}_T = 8.874\angle -6.78°$$

We now use the per-phase equivalent circuit.

$$\mathbf{I}_a = \frac{V_p}{\mathbf{Z}_p + \mathbf{Z}_L}, \qquad \text{where} \quad V_p = \frac{210}{\sqrt{3}}$$

$$\mathbf{I}_a = \frac{210}{\sqrt{3}\,(8.874\angle -6.78°)} = 13.66\angle 6.78°$$

$$\mathbf{I}_L = |\mathbf{I}_a| = \underline{\mathbf{13.66\ A}}$$

Chapter 12, Problem 16.

A balanced delta-connected load has a phase current $\mathbf{I}_{AC} = 10\ \angle -30°$ A.

(a) Determine the three line currents assuming that the circuit operates in the positive phase sequence.

(b) Calculate the load impedance if the line voltage is $\mathbf{V}_{AB} = 110\ \angle 0°$ V.

Chapter 12, Solution 16.

(a) $\mathbf{I}_{CA} = -\mathbf{I}_{AC} = 10\angle(-30° + 180°) = 10\angle 150°$

This implies that

$$\mathbf{I}_{AB} = 10\angle 30°$$

$$\mathbf{I}_{BC} = 10\angle -90°$$

$$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30° = \underline{\mathbf{17.32\angle 0°\ A}}$$

$$\mathbf{I}_b = \underline{\mathbf{17.32\angle -120°\ A}}$$

$$\mathbf{I}_c = \underline{\mathbf{17.32\angle 120°\ A}}$$

(b) $$\mathbf{Z}_\Delta = \frac{\mathbf{V}_{AB}}{\mathbf{I}_{AB}} = \frac{110\angle 0°}{10\angle 30°} = \underline{\mathbf{11\angle -30°\ \Omega}}$$

Chapter 12, Problem 17.

A balanced delta-connected load has line current $\mathbf{I}_a = 10\ \angle -25°$ A. Find the phase currents $\mathbf{I}_{AB}$, $\mathbf{I}_{BC}$, and $\mathbf{I}_{CA}$.

Chapter 12, Solution 17.

$$I_a = I_{AB}\sqrt{3} < -30^o \quad \longrightarrow \quad I_{AB} = \frac{I_a}{\sqrt{3} < -30^o} = \frac{10}{\sqrt{3}} < -25^o + 30^o = \underline{5.773 < 5^o\ \text{A}}$$

$$I_{BC} = I_{AB} < -120^o = \underline{5.775 < -115^o\ \text{A}}$$

$$I_{CA} = I_{AB} < +120^o = \underline{5.775 < 125^o\ \text{A}}$$

Chapter 12, Problem 18.

If $\mathbf{V}_{an} = 440\ \angle 60°$ V in the network of Fig. 12.49, find the load phase currents $\mathbf{I}_{AB}$, $\mathbf{I}_{BC}$, and $\mathbf{I}_{CA}$.

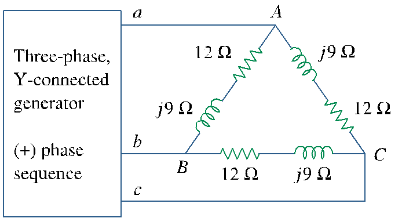


Figure 12.49
For Prob. 12.18.

Chapter 12, Solution 18.

$$\mathbf{V}_{AB} = \mathbf{V}_{an}\sqrt{3}\angle 30° = (440\angle 60°)(\sqrt{3}\angle 30°) = 762.1\angle 90°$$

$$\mathbf{Z}_{\Delta} = 12 + j9 = 15\angle 36.87°$$

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_{\Delta}} = \frac{762.1\angle 90°}{15\angle 36.87°} = \underline{\mathbf{50.81\angle 53.13°\ A}}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\angle -120° = \underline{\mathbf{50.81\angle -66.87°\ A}}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle 120° = \underline{\mathbf{50.81\angle 173.13°\ A}}$$

Chapter 12, Problem 19.

For the Δ-Δ circuit of Fig. 12.50, calculate the phase and line currents.

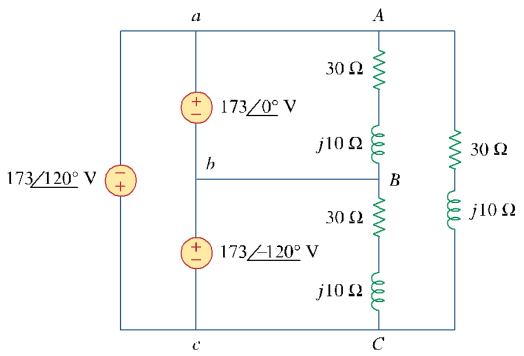


Figure 12.50
For Prob. 12.19.

Chapter 12, Solution 19.

$$\mathbf{Z}_\Delta = 30 + j10 = 31.62\angle 18.43°$$

The phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_\Delta} = \frac{173\angle 0°}{31.62\angle 18.43°} = \underline{\mathbf{5.47\angle -18.43° \ A}}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\angle -120° = \underline{\mathbf{5.47\angle -138.43° \ A}}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle 120° = \underline{\mathbf{5.47\angle 101.57° \ A}}$$

The line currents are

$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA} = \mathbf{I}_{AB}\sqrt{3}\angle -30°$$

$$\mathbf{I}_a = 5.47\sqrt{3}\angle -48.43° = \underline{\mathbf{9.474\angle -48.43° \ A}}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120° = \underline{\mathbf{9.474\angle -168.43° \ A}}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle 120° = \underline{\mathbf{9.474\angle 71.57° \ A}}$$

Chapter 12, Problem 20.

Refer to the Δ-Δ circuit in Fig. 12.51. Find the line and phase currents. Assume that the load impedance is $Z_L = 12 + j9\ \Omega$ per phase.

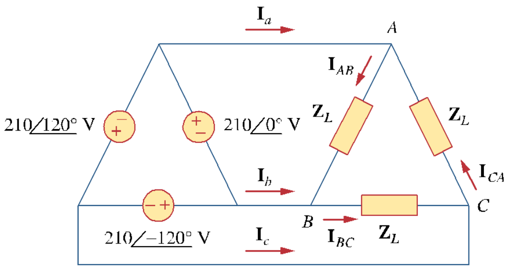


Figure 12.51
For Prob. 12.20.

Chapter 12, Solution 20.

$$\mathbf{Z}_\Delta = 12 + j9 = 15\angle 36.87°$$

The phase currents are

$$\mathbf{I}_{AB} = \frac{210\angle 0°}{15\angle 36.87°} = \underline{\mathbf{14\angle -36.87°\ A}}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\angle -120° = \underline{\mathbf{14\angle -156.87°\ A}}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle 120° = \underline{\mathbf{14\angle 83.13°\ A}}$$

The line currents are

$$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30° = \underline{\mathbf{24.25\angle -66.87°\ A}}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120° = \underline{\mathbf{24.25\angle -186.87°\ A}}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle 120° = \underline{\mathbf{24.25\angle 53.13°\ A}}$$

Chapter 12, Problem 21.

Three 230-V generators form a delta-connected source that is connected to a balanced delta-connected load of $\mathbf{Z}_L = 10 + j8\ \Omega$ per phase as shown in Fig. 12.52.

(a) Determine the value of $\mathbf{I}_{AC}$.
(b) What is the value of $\mathbf{I}_b$?

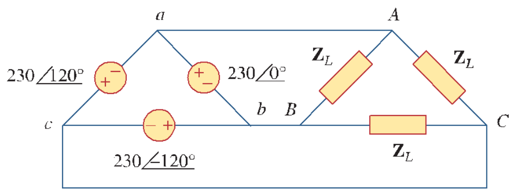


Figure 12.52
For Prob. 12.21.

Chapter 12, Solution 21.

(a) $$I_{AC} = \frac{-230\angle 120°}{10 + j8} = \frac{-230\angle 120°}{12.806\angle 38.66°} = \underline{17.96\angle -98.66°\text{ A(rms)}}$$

17.96∠–98.66˚ A rms

(b) $$I_{bB} = I_{BC} + I_{BA} = I_{BC} - I_{AB} = \frac{230\angle -120}{10 + j8} - \frac{230\angle 0°}{10 + j8}$$
$$= 17.96\angle -158.66° - 17.96\angle -38.66°$$
$$= -16.729 - j6.536 - 14.024 + j11.220 = -30.75 + j4.684$$
$$= \underline{31.10\angle 171.34°\text{ A}}$$

31.1∠171.34˚ A rms

Chapter 12, Problem 22.

Find the line currents $\mathbf{I}_a$, $\mathbf{I}_b$, and $\mathbf{I}_c$ in the three-phase network of Fig. 12.53 below. Take $\mathbf{Z}_\Delta = 12 - j15\Omega$, $\mathbf{Z}_Y = 4 + j6\ \Omega$, and $\mathbf{Z}_l = 2\ \Omega$.

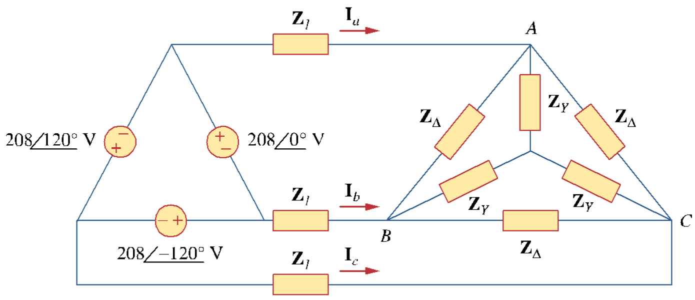


Figure 12.53
For Prob. 12.22.

Chapter 12, Solution 22.

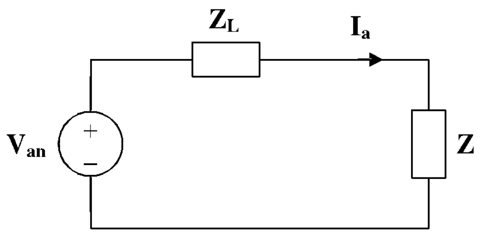
Convert the Δ-connected source to a Y-connected source.

$$\mathbf{V}_{an} = \frac{V_p}{\sqrt{3}}\angle -30° = \frac{208}{\sqrt{3}}\angle -30° = 120\angle -30°$$

Convert the Δ-connected load to a Y-connected load.

$$\mathbf{Z} = \mathbf{Z}_Y \,\|\, \frac{\mathbf{Z}_\Delta}{3} = (4 + j6)\,\|\,(4 - j5) = \frac{(4 + j6)(4 - j5)}{8 + j}$$

$$\mathbf{Z} = 5.723 - j0.2153$$



$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_L + \mathbf{Z}} = \frac{120\angle -30°}{7.723 - j0.2153} = \underline{\mathbf{15.53\angle -28.4°\ A}}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120° = \underline{\mathbf{15.53\angle -148.4°\ A}}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle 120° = \underline{\mathbf{15.53\angle 91.6°\ A}}$$

Chapter 12, Problem 23.

A three-phase balanced system with a line voltage of 202 V rms feeds a delta-connected load with $\mathbf{Z}_p = 25\ \angle 60°\Omega$.

(a) Find the line current.
(b) Determine the total power supplied to the load using two wattmeters connected to the *A* and *C* lines.

Chapter 12, Solution 23.

(a) $$I_{AB} = \frac{V_{AB}}{Z_\Delta} = \frac{202}{25\angle 60^o}$$

$$I_a = I_{AB}\sqrt{3}\angle -30^o = \frac{202\sqrt{3}\angle -30^o}{25\angle 60^o} = 13.995\angle -90^o$$

$$I_L = |\, I_a \,| = \underline{13.995\text{A}}$$

(b)

$$P = P_1 + P_2 = \sqrt{3}V_L I_L \cos\theta = \sqrt{3}(202)\left(\frac{202\sqrt{3}}{25}\right)\cos 60^o = \underline{2.448\ \text{kW}}$$

Chapter 12, Problem 24.

A balanced delta-connected source has phase voltage $\mathbf{V}_{ab} = 416\ \angle 30°$ V and a positive phase sequence. If this is connected to a balanced delta-connected load, find the line and phase currents. Take the load impedance per phase as $60\ \angle 30° \Omega$ and line impedance per phase as $1 + j1\ \Omega$.

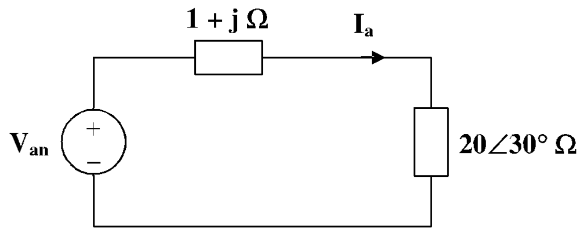
Chapter 12, Solution 24.

Convert both the source and the load to their wye equivalents.

$$\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3} = 20\angle 30° = 17.32 + j10$$

$$\mathbf{V}_{an} = \frac{\mathbf{V}_{ab}}{\sqrt{3}}\angle -30° = 240.2\angle 0°$$

We now use per-phase analysis.



$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{(1 + j) + (17.32 + j10)} = \frac{240.2}{21.37\angle 31°} = \underline{\mathbf{11.24\angle -31°\ A}}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120° = \underline{\mathbf{11.24\angle -151°\ A}}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle 120° = \underline{\mathbf{11.24\angle 89°\ A}}$$

But $\quad \mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30°$

$$\mathbf{I}_{AB} = \frac{11.24\angle -31°}{\sqrt{3}\angle -30°} = \underline{\mathbf{6.489\angle -1°\ A}}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\angle -120° = \underline{\mathbf{6.489\angle -121°\ A}}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle 120° = \underline{\mathbf{6.489\angle 119°\ A}}$$

Chapter 12, Problem 25.

In the circuit of Fig. 12.54, if $\mathbf{V}_{ab} = 440\angle 10°$, $\mathbf{V}_{bc} = 440\angle 250°$, $\mathbf{V}_{ca} = 440\angle 130°$ V, find the line currents.

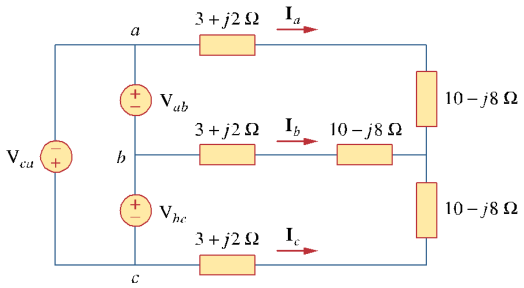


Figure 12.54
For Prob. 12.25.

Chapter 12, Solution 25.

Convert the delta-connected source to an equivalent wye-connected source and consider the single-phase equivalent.

$$\mathbf{I}_a = \frac{440\angle(10° - 30°)}{\sqrt{3}\,\mathbf{Z}_Y}$$

where $\mathbf{Z}_Y = 3 + j2 + 10 - j8 = 13 - j6 = 14.32\angle -24°.78°$

$$\mathbf{I}_a = \frac{440\angle -20°}{\sqrt{3}\,(14.32\angle -24.78°)} = \underline{\mathbf{17.74\angle 4.78°\ A}}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120° = \underline{\mathbf{17.74\angle -115.22°\ A}}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle 120° = \underline{\mathbf{17.74\angle 124.78°\ A}}$$

Chapter 12, Problem 26.

For the balanced circuit in Fig. 12.55, $\mathbf{V}_{ab} = 125\ \angle 0°$ V. Find the line currents $\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$, and $\mathbf{I}_{cC}$.

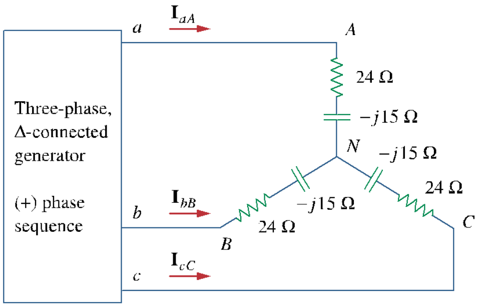


Figure 12.55
For Prob. 12.26.

Chapter 12, Solution 26.

Transform the source to its wye equivalent.

$$\mathbf{V}_{an} = \frac{V_p}{\sqrt{3}}\angle -30° = 72.17\angle -30°$$

Now, use the per-phase equivalent circuit.

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_{an}}{\mathbf{Z}}, \qquad \mathbf{Z} = 24 - j15 = 28.3\angle -32°$$

$$\mathbf{I}_{aA} = \frac{72.17\angle -30°}{28.3\angle -32°} = \underline{\mathbf{2.55\angle 2°\ A}}$$

$$\mathbf{I}_{bB} = \mathbf{I}_{aA}\angle -120° = \underline{\mathbf{2.55\angle -118°\ A}}$$

$$\mathbf{I}_{cC} = \mathbf{I}_{aA}\angle 120° = \underline{\mathbf{2.55\angle 122°\ A}}$$

Chapter 12, Problem 27.

ps A Δ-connected source supplies power to a Y-connected load in a three-phase balanced system. Given that the line impedance is $2 + j1\ \Omega$ per phase while the load impedance is $6 + j4\ \Omega$ per phase, find the magnitude of the line voltage at the load. Assume the source phase voltage $\mathbf{V}_{ab} = 208\ \angle 0°$ V rms.

Chapter 12, Solution 27.

Since Z_L and Z_ℓ are in series, we can lump them together so that

$$Z_Y = 2 + j + 6 + j4 = 8 + j5$$

$$I_a = \frac{\frac{V_P}{\sqrt{3}} < -30^o}{Z_Y} = \frac{208 < -30^o}{\sqrt{3}(8 + j5)}$$

$$V_L = (6 + j4)I_a = \frac{208(0.866 - j0.5)(6 + j4)}{\sqrt{3}(8 + j5)} = 80.81 - j43.54$$

$|V_L|$ = **91.79 V**

Chapter 12, Problem 28.

The line-to-line voltages in a Y-load have a magnitude of 440 V and are in the positive sequence at 60 Hz. If the loads are balanced with $Z_1 = Z_2 = Z_3 = 25\ \angle 30°$, find all line currents and phase voltages.

Chapter 12, Solution 28.

$V_L = |V_{ab}| = 440 = \sqrt{3}V_P$ or $V_P = 440/1.7321 = 254$

For reference, let V_{AN} = **254∠0° V** which leads to V_{BN} = **254∠–120° V** and V_{CN} = **254∠120° V**.

The line currents are found as follows,

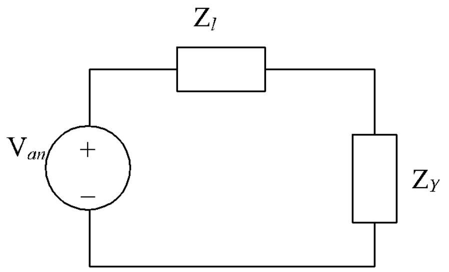
$I_a = V_{AN}/Z_Y = 254/25\angle 30°$ = **10.16∠–30° A**.
This leads to, I_b = **10.16∠–150° A** and I_c = **10.16∠90° A**.

Chapter 12, Problem 29.

ps A balanced three-phase Y-Δ system has $\mathbf{V}_{an} = 120\ \angle 0°$ V rms and $\mathbf{Z}_\Delta = 51 + j45\Omega$. If the line impedance per phase is $0.4 + j1.2\ \Omega$, find the total complex power delivered to the load.

Chapter 12, Solution 29.

We can replace the delta load with a wye load, $Z_Y = Z_\Delta/3 = 17+j15\Omega$.
The per-phase equivalent circuit is shown below.



$$I_a = \frac{V_{an}}{Z_Y + Z_\ell} = \frac{120}{17 + j15 + 0.4 + j1.2} = 5.0475\angle -42.96^\circ$$

$$S = 3S_p = 3|I_a|^2 Z_Y = 3(5.0475)^2(17 + j15) = \underline{1.3 + j1.1465\ \text{kVA}}$$

Chapter 12, Problem 30.

In Fig. 12.56, the rms value of the line voltage is 208 V. Find the average power delivered to the load.

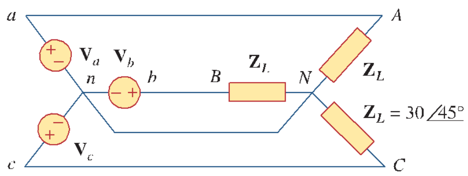
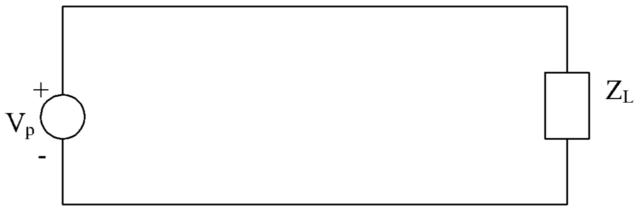


Figure 12.56
For Prob. 12.30.

Chapter 12, Solution 30.

Since this a balanced system, we can replace it by a per-phase equivalent, as shown below.



$$\overline{S} = 3\overline{S}_p = \frac{3V_p^2}{Z_p^*}, \qquad V_p = \frac{V_L}{\sqrt{3}}$$

$$\overline{S} = \frac{V_L^2}{Z_p^*} = \frac{(208)^2}{30\angle -45^o} = 1.4421\angle 45^o \text{ kVA}$$

$$P = S\cos\theta = \underline{1.02 \text{ kW}}$$

Chapter 12, Problem 31.

A balanced delta-connected load is supplied by a 60-Hz three-phase source with a line voltage of 240 V. Each load phase draws 6 kW at a lagging power factor of 0.8. Find:

(a) the load impedance per phase
(b) the line current
(c) the value of capacitance needed to be connected in parallel with each load phase to minimize the current from the source

Chapter 12, Solution 31.

(a)

$$P_p = 6{,}000, \qquad \cos\theta = 0.8, \qquad S_p = \frac{P_P}{\cos\theta} = 6/0.8 = 7.5\,\text{kVA}$$

$$Q_p = S_P \sin\theta = 4.5\,\text{kVAR}$$

$$\overline{S} = 3\overline{S}_p = 3(6 + j4.5) = 18 + j13.5\,\text{kVA}$$

For delta-connected load, $V_p = V_L = 240$ (rms). But

$$\overline{S} = \frac{3V^2{}_p}{Z^*{}_p} \longrightarrow Z^*{}_p = \frac{3V^2{}_p}{S} = \frac{3(240)^2}{(18 + j13.5)x10^3}, \qquad \underline{Z_P = 6.144 + j4.608\Omega}$$

(b) $$P_p = \sqrt{3}V_L I_L \cos\theta \longrightarrow I_L = \frac{6000}{\sqrt{3}x240x0.8} = \underline{18.04\text{ A}}$$

(c) We find C to bring the power factor to unity

$$Q_c = Q_p = 4.5\,\text{kVA} \longrightarrow C = \frac{Q_c}{\omega V^2{}_{rms}} = \frac{4500}{2\pi x60x240^2} = \underline{207.2\,\mu\text{F}}$$

Chapter 12, Problem 32.

A balanced Y-load is connected to a 60-Hz three-phase source with $\mathbf{V}_{ab} = 240\ \angle 0°$ V. The load has pf = 0.5 lagging and each phase draws 5 kW. (a) Determine the load impedance $\mathbf{Z}_Y$. (b) Find $\mathbf{I}_a$, $\mathbf{I}_b$, and $\mathbf{I}_c$.

Chapter 12, Solution 32.

(a) $|V_{ab}| = \sqrt{3}V_p = 240 \quad \longrightarrow \quad V_p = \dfrac{240}{\sqrt{3}} = 138.56$

$$V_{an} = V_p < -30^o$$

$$pf = 0.5 = \cos\theta \quad \longrightarrow \quad \theta = 60^o$$

$$P = S\cos\theta \quad \longrightarrow \quad S = \frac{P}{\cos\theta} = \frac{5}{0.5} = 10 \text{ kVA}$$

$$Q = S\sin\theta = 10\sin 60 = 8.66$$

$$S_p = 5 + j8.66 \text{ kVA}$$

But

$$S_P = \frac{V_p^2}{Z_p^*} \quad \longrightarrow \quad Z_p^* = \frac{V_p^2}{S_p} = \frac{138.56^2}{(5 + j8.66)x10^3} = 0.96 - j1.663$$

$Z_p = \underline{0.96 + j1.663 \quad \Omega}$

(b) $I_a = \dfrac{V_{an}}{Z_Y} = \dfrac{138.56 < -30^o}{0.96 + j1.6627} = \underline{72.17 < -90^o \text{ A}}$

$$I_b = I_a < -120^o = \underline{72.17 < -210^o \text{ A}}$$

$$I_c = I_a < +120^o = \underline{72.17 < 30^o \text{ A}}$$

Chapter 12, Problem 33.

A three-phase source delivers 4800 VA to a wye-connected load with a phase voltage of 208 V and a power factor of 0.9 lagging. Calculate the source line current and the source line voltage.

Chapter 12, Solution 33.

$$\mathbf{S} = \sqrt{3}\, V_L I_L \angle\theta$$

$$S = |\mathbf{S}| = \sqrt{3}\, V_L I_L$$

For a Y-connected load,

$$I_L = I_p, \qquad V_L = \sqrt{3}\, V_p$$

$$S = 3 V_p I_p$$

$$I_L = I_p = \frac{S}{3V_p} = \frac{4800}{(3)(208)} = \underline{\mathbf{7.69\ A}}$$

$$V_L = \sqrt{3}\, V_p = \sqrt{3} \times 208 = \underline{\mathbf{360.3\ V}}$$

Chapter 12, Problem 34.

A balanced wye-connected load with a phase impedance of $10 - j16\ \Omega$ is connected to a balanced three-phase generator with a line voltage of 220 V. Determine the line current and the complex power absorbed by the load.

Chapter 12, Solution 34.

$$V_p = \frac{V_L}{\sqrt{3}} = \frac{220}{\sqrt{3}}$$

$$\mathbf{I}_a = \frac{V_p}{\mathbf{Z}_Y} = \frac{220}{\sqrt{3}(10 - j16)} = \frac{127.02}{18.868\angle -58^\circ} = 6.732\angle 58^\circ$$

$$I_L = I_p = \underline{\mathbf{6.732A}}$$

$$\mathbf{S} = \sqrt{3}\, V_L I_L \angle\theta = \sqrt{3} \times 220 \times 6.732\angle -58^\circ = 2565\angle -58^\circ$$

$$\mathbf{S} = \underline{\mathbf{1359.2 - j2175\ VA}}$$

Chapter 12, Problem 35.

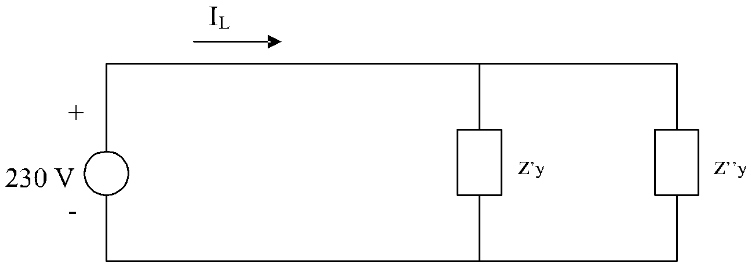
Three equal impedances, $60 + j30\ \Omega$ each, are delta-connected to a 230-V rms, three-phase circuit. Another three equal impedances, $40 + j10\ \Omega$ each, are wye-connected across the same circuit at the same points. Determine:

(a) the line current
(b) the total complex power supplied to the two loads
(c) the power factor of the two loads combined

Chapter 12, Solution 35.

(a) This is a balanced three-phase system and we can use per phase equivalent circuit. The delta-connected load is converted to its wye-connected equivalent

$$Z''_y = \frac{1}{3}Z_\Delta = (60 + j30)/3 = 20 + j10$$



$$Z_y = Z'_y \,//\, Z''_y = (40 + j10)//(20 + j10) = 13.5 + j5.5$$

$$I_L = \frac{230}{13.5 + j5.5} = \underline{14.61 - j5.953\text{ A}}$$

(b) $\overline{S} = V_s I^*_L = \underline{3.361 + j1.368\ \text{kVA}}$

(c) pf = P/S = 0.9261

Chapter 12, Problem 36.

A 4200-V, three-phase transmission line has an impedance of $4 + j10\ \Omega$ per phase. If it supplies a load of 1 MVA at 0.75 power factor (lagging), find:

(a) the complex power
(b) the power loss in the line
(c) the voltage at the sending end

Chapter 12, Solution 36.

(a) S = 1 [0.75 + sin($\cos^{-1}$0.75)] = **0.75 + j0.6614 MVA**

(b) $\overline{S} = 3V_p I^*_p \longrightarrow I^*_p = \frac{S}{3V_p} = \frac{(0.75 + j0.6614)x10^6}{3x4200} = 59.52 + j52.49$

$$P_L = |I_p|^2 R_l = (79.36)^2(4) = \underline{25.19\text{ kW}}$$

(c) $V_s = V_L + I_p(4 + j) = 4.4381 - j0.21\text{ kV} = \underline{4.443\angle -2.709^\circ\text{ kV}}$

Chapter 12, Problem 37.

The total power measured in a three-phase system feeding a balanced wye-connected load is 12 kW at a power factor of 0.6 leading. If the line voltage is 208 V, calculate the line current I_L and the load impedance $\mathbf{Z}_Y$.

Chapter 12, Solution 37.

$$S = \frac{P}{pf} = \frac{12}{0.6} = 20$$

$$\mathbf{S} = S\angle\theta = 20\angle\theta = 12 - j16 \text{ kVA}$$

But $\quad \mathbf{S} = \sqrt{3}\, V_L I_L \angle\theta$

$$I_L = \frac{20\times 10^3}{\sqrt{3}\times 208} = \underline{\mathbf{55.51\ A}}$$

$$\mathbf{S} = 3\left|\mathbf{I}_p\right|^2 \mathbf{Z}_p$$

For a Y-connected load, $I_L = I_p$.

$$\mathbf{Z}_p = \frac{\mathbf{S}}{3\left|I_L\right|^2} = \frac{(12 - j16)\times 10^3}{(3)(55.51)^2}$$

$$\mathbf{Z}_p = \underline{\mathbf{1.298 - j1.731\ \Omega}}$$

Chapter 12, Problem 38.

Given the circuit in Fig. 12.57 below, find the total complex power absorbed by the load.

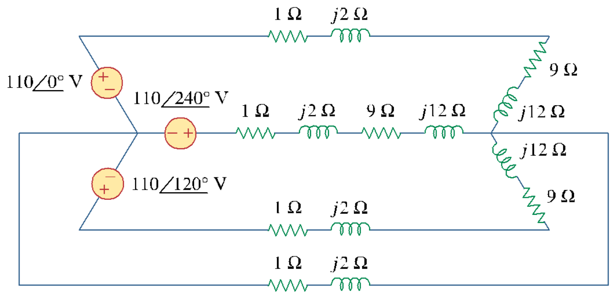


Figure 12.57
For Prob. 12.38.

Chapter 12, Solution 38.

As a balanced three-phase system, we can use the per-phase equivalent shown below.

$$\mathbf{I}_a = \frac{110\angle 0^\circ}{(1+j2)+(9+j12)} = \frac{110\angle 0^\circ}{10+j14}$$

$$\mathbf{S}_p = \frac{1}{2}\left|\mathbf{I}_a\right|^2 \mathbf{Z}_Y = \frac{1}{2}\cdot\frac{(110)^2}{(10^2+14^2)}\cdot(9+j12)$$

The complex power is

$$\mathbf{S} = 3\mathbf{S}_p = \frac{3}{2}\cdot\frac{(110)^2}{296}\cdot(9+j12)$$

$$\mathbf{S} = \underline{\mathbf{551.86 + j735.81\ VA}}$$

Chapter 12, Problem 39.

Find the real power absorbed by the load in Fig. 12.58.

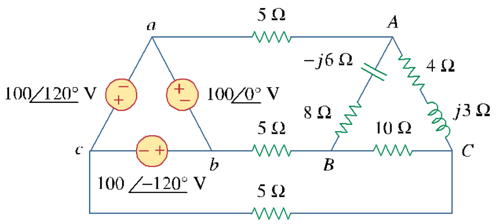
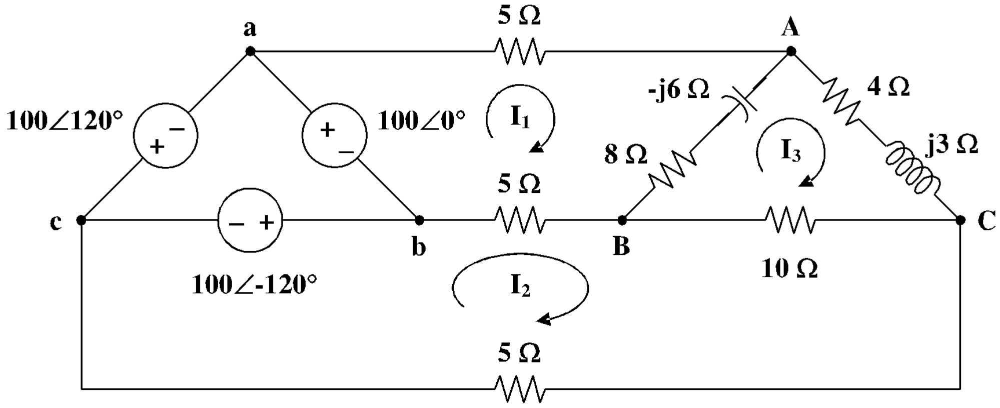


Figure 12.58
For Prob. 12.39.

Chapter 12, Solution 39.

Consider the system shown below.



For mesh 1,

$$100 = (18 - j6)\mathbf{I}_1 - 5\mathbf{I}_2 - (8 - j6)\mathbf{I}_3 \qquad (1)$$

For mesh 2,

$$100\angle\text{-}120° = 20\mathbf{I}_2 - 5\mathbf{I}_1 - 10\mathbf{I}_3$$

$$20\angle\text{-}120° = \text{-}\mathbf{I}_1 + 4\mathbf{I}_2 - 2\mathbf{I}_3 \qquad (2)$$

For mesh 3,

$$0 = \text{-}(8 - j6)\mathbf{I}_1 - 10\mathbf{I}_2 + (22 - j3)\mathbf{I}_3 \qquad (3)$$

To eliminate $\mathbf{I}_2$, start by multiplying (1) by 2,

$$200 = (36 - j12)\mathbf{I}_1 - 10\mathbf{I}_2 - (16 - j12)\mathbf{I}_3 \qquad (4)$$

Subtracting (3) from (4),

$$200 = (44 - j18)\mathbf{I}_1 - (38 - j15)\mathbf{I}_3 \qquad (5)$$

Multiplying (2) by $5/4$,

$$25\angle\text{-}120° = \text{-}1.25\mathbf{I}_1 + 5\mathbf{I}_2 - 2.5\mathbf{I}_3 \qquad (6)$$

Adding (1) and (6),

$$87.5 - j21.65 = (16.75 - j6)\mathbf{I}_1 - (10.5 - j6)\mathbf{I}_3 \qquad (7)$$

In matrix form, (5) and (7) become

$$\begin{bmatrix} 200 \\ 87.5 - j12.65 \end{bmatrix} = \begin{bmatrix} 44 - j18 & \text{-}38 + j15 \\ 16.75 - j6 & \text{-}10.5 + j6 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_3 \end{bmatrix}$$

$$\Delta = 192.5 - j26.25, \quad \Delta_1 = 900.25 - j935.2, \quad \Delta_3 = 110.3 - j1327.6$$

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \frac{1298.1\angle\text{-}46.09°}{194.28\angle\text{-}7.76°} = 6.682\angle\text{-}38.33° = 5.242 - j4.144$$

$$\mathbf{I}_3 = \frac{\Delta_3}{\Delta} = \frac{1332.2\angle\text{-}85.25°}{194.28\angle\text{-}7.76°} = 6.857\angle\text{-}77.49° = 1.485 - j6.694$$

We obtain $\mathbf{I}_2$ from (6),

$$\mathbf{I}_2 = 5\angle\text{-}120° + \frac{1}{4}\mathbf{I}_1 + \frac{1}{2}\mathbf{I}_3$$

$$\mathbf{I}_2 = (\text{-}2.5 - j4.33) + (1.3104 - j1.0359) + (0.7425 - j3.347)$$

$$\mathbf{I}_2 = \text{-}0.4471 - j8.713$$

The average power absorbed by the 8-Ω resistor is

$$P_1 = \left|\mathbf{I}_1 - \mathbf{I}_3\right|^2(8) = \left|3.756 + j2.551\right|^2(8) = 164.89 \text{ W}$$

The average power absorbed by the 4-Ω resistor is

$$P_2 = \left|\mathbf{I}_3\right|^2(4) = (6.8571)^2(4) = 188.1 \text{ W}$$

The average power absorbed by the 10-Ω resistor is

$$P_3 = \left|\mathbf{I}_2 - \mathbf{I}_3\right|^2(10) = \left|\text{-}1.9321 - j2.019\right|^2(10) = 78.12 \text{ W}$$

Thus, the total real power absorbed by the load is

$$P = P_1 + P_2 + P_3 = \underline{\mathbf{431.1\ W}}$$

Chapter 12, Problem 40.

For the three-phase circuit in Fig. 12.59, find the average power absorbed by the delta-connected load with $\mathbf{Z}_\Delta = 21 + j24\Omega$.

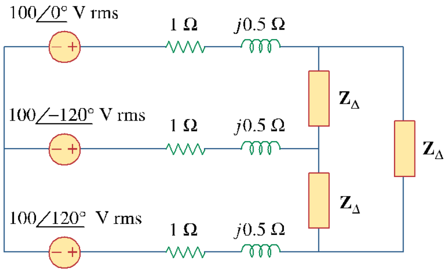


Figure 12.59
For Prob. 12.40.

Chapter 12, Solution 40.

Transform the delta-connected load to its wye equivalent.

$$\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3} = 7 + j8$$

Using the per-phase equivalent circuit above,

$$\mathbf{I}_a = \frac{100\angle 0°}{(1 + j0.5) + (7 + j8)} = 8.567\angle -46.75°$$

For a wye-connected load,

$$I_p = I_a = |\mathbf{I}_a| = 8.567$$

$$\mathbf{S} = 3|\mathbf{I}_p|^2 \mathbf{Z}_p = (3)(8.567)^2(7 + j8)$$

$$P = \text{Re}(\mathbf{S}) = (3)(8.567)^2(7) = \underline{\mathbf{1.541\ kW}}$$

Chapter 12, Problem 41.

A balanced delta-connected load draws 5 kW at a power factor of 0.8 lagging. If the three-phase system has an effective line voltage of 400 V, find the line current.

Chapter 12, Solution 41.

$$S = \frac{P}{pf} = \frac{5\,\text{kW}}{0.8} = 6.25\ \text{kVA}$$

But $\quad S = \sqrt{3}\, V_L I_L$

$$I_L = \frac{S}{\sqrt{3}\, V_L} = \frac{6.25 \times 10^3}{\sqrt{3} \times 400} = \underline{\mathbf{9.021\ A}}$$

Chapter 12, Problem 42.

A balanced three-phase generator delivers 7.2 kW to a wye-connected load with impedance $30 - j40\,\Omega$ per phase. Find the line current I_L and the line voltage V_L.

Chapter 12, Solution 42.

The load determines the power factor.

$$\tan\theta = \frac{40}{30} = 1.333 \longrightarrow \theta = 53.13^\circ$$

$$pf = \cos\theta = 0.6 \quad \text{(leading)}$$

$$\mathbf{S} = 7.2 - j\left(\frac{7.2}{0.6}\right)(0.8) = 7.2 - j9.6\ \text{kVA}$$

But $\quad \mathbf{S} = 3\left|\mathbf{I}_p\right|^2 \mathbf{Z}_p$

$$\left|\mathbf{I}_p\right|^2 = \frac{\mathbf{S}}{3\mathbf{Z}_p} = \frac{(7.2 - j9.6) \times 10^3}{(3)(30 - j40)} = 80$$

$$I_p = 8.944\ \text{A}$$

$$I_L = I_p = \underline{\mathbf{8.944\ A}}$$

$$V_L = \frac{S}{\sqrt{3}\, I_L} = \frac{12 \times 10^3}{\sqrt{3}\,(8.944)} = \underline{\mathbf{774.6\ V}}$$

Chapter 12, Problem 43.

Refer to Fig. 12.48. Obtain the complex power absorbed by the combined loads.

Chapter 12, Solution 43.

$$\mathbf{S} = 3\left|\mathbf{I}_p\right|^2 \mathbf{Z}_p, \qquad I_p = I_L \text{ for Y-connected loads}$$

$$\mathbf{S} = (3)(13.66)^2(7.812 - j2.047)$$

$$\mathbf{S} = \underline{\mathbf{4.373 - j1.145\ kVA}}$$

Chapter 12, Problem 44.

A three-phase line has an impedance of $1 + j3\,\Omega$ per phase. The line feeds a balanced delta-connected load, which absorbs a total complex power of $12 + j5$ k VA. If the line voltage at the load end has a magnitude of 240 V, calculate the magnitude of the line voltage at the source end and the source power factor.

Chapter 12, Solution 44.

For a Δ-connected load,

$$V_p = V_L, \qquad I_L = \sqrt{3}\, I_p$$

$$S = \sqrt{3}\, V_L I_L$$

$$I_L = \frac{S}{\sqrt{3}\, V_L} = \frac{\sqrt{(12^2 + 5^2)} \times 10^3}{\sqrt{3}\,(240)} = 31.273$$

At the source,

$$\mathbf{V}_L' = \mathbf{V}_L + \mathbf{I}_L\, \mathbf{Z}_L$$

$$\mathbf{V}_L' = 240\angle 0° + (31.273)(1 + j3)$$

$$\mathbf{V}_L' = 271.273 + j93.819$$

$$\left|\mathbf{V}_L'\right| = \underline{\mathbf{287.04\ V}}$$

Also, at the source,

$$\mathbf{S}' = \sqrt{3}\mathbf{V}_L'\mathbf{I}_L^*$$

$$\mathbf{S}' = \sqrt{3}\,(271.273 + j93.819)(31.273)$$

$$\theta = \tan^{-1}\left(\frac{93.819}{271.273}\right) = 19.078$$

$$\text{pf} = \cos\theta = \underline{\mathbf{0.9451}}$$

Chapter 12, Problem 45.

A balanced wye-connected load is connected to the generator by a balanced transmission line with an impedance of $0.5 + j2\,\Omega$ per phase. If the load is rated at 450 kW, 0.708 power factor lagging, 440-V line voltage, find the line voltage at the generator.

Chapter 12, Solution 45.

$$\mathbf{S} = \sqrt{3}\, V_L I_L \angle\theta$$

$$I_L = \frac{|\mathbf{S}|\angle -\theta}{\sqrt{3}\, V_L}, \qquad |\mathbf{S}| = \frac{P}{pf} = \frac{450\times10^3}{0.708} = 635.6 \text{ kVA}$$

$$\mathbf{I}_L = \frac{(635.6)\angle -\theta}{\sqrt{3}\times 440} = 834\angle -45° \text{ A}$$

At the source,

$$\mathbf{V}_L = 440\angle 0° + \mathbf{I}_L\,(0.5 + j2)$$
$$\mathbf{V}_L = 440 + (834\angle -45°)(2.062\angle 76°)$$
$$\mathbf{V}_L = 440 + 1719.7\angle 31°$$
$$\mathbf{V}_L = 1914.1 + j885.7$$
$$\mathbf{V}_L = \underline{2.109\angle 24.83° \text{ kV}}$$

Note, this is not normally experienced in practice since transformers are use which can significantly reduce line losses.

Chapter 12, Problem 46.

A three-phase load consists of three 100-Ω resistors that can be wye- or delta-connected. Determine which connection will absorb the most average power from a three-phase source with a line voltage of 110 V. Assume zero line impedance.

Chapter 12, Solution 46.

For the wye-connected load,

$$I_L = I_p, \qquad V_L = \sqrt{3}\,V_p \qquad I_p = V_p / \mathbf{Z}$$

$$\mathbf{S} = 3\mathbf{V}_p\mathbf{I}_p^* = \frac{3|\mathbf{V}_p|^2}{\mathbf{Z}^*} = \frac{3|\mathbf{V}_L/\sqrt{3}|^2}{\mathbf{Z}^*}$$

$$\mathbf{S} = \frac{|\mathbf{V}_L|^2}{\mathbf{Z}^*} = \frac{(110)^2}{100} = 121 \text{ W}$$

For the delta-connected load,

$$V_p = V_L, \qquad I_L = \sqrt{3}\,I_p, \qquad I_p = V_p / \mathbf{Z}$$

$$\mathbf{S} = 3\mathbf{V}_p\mathbf{I}_p^* = \frac{3|\mathbf{V}_p|^2}{\mathbf{Z}^*} = \frac{3|\mathbf{V}_L|^2}{\mathbf{Z}^*}$$

$$\mathbf{S} = \frac{(3)(110)^2}{100} = 363 \text{ W}$$

This shows that the <u>**delta-connected load**</u> will deliver three times more average power than the wye-connected load. This is also evident from $\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3}$.

Chapter 12, Problem 47.

The following three parallel-connected three-phase loads are fed by a balanced three-phase source:

Load 1: 250 kVA, 0.8 pf lagging

Load 2: 300 kVA, 0.95 pf leading

Load 3: 450 kVA, unity pf

If the line voltage is 13.8 kV, calculate the line current and the power factor of the source. Assume that the line impedance is zero.

Chapter 12, Solution 47.

$$\text{pf} = 0.8 \quad (\text{lagging}) \longrightarrow \theta = \cos^{-1}(0.8) = 36.87°$$

$$\mathbf{S}_1 = 250\angle 36.87° = 200 + j150 \text{ kVA}$$

$$\text{pf} = 0.95 \quad (\text{leading}) \longrightarrow \theta = \cos^{-1}(0.95) = -18.19°$$

$$\mathbf{S}_2 = 300\angle -18.19° = 285 - j93.65 \text{ kVA}$$

$$\text{pf} = 1.0 \longrightarrow \theta = \cos^{-1}(1) = 0°$$

$$\mathbf{S}_3 = 450 \text{ kVA}$$

$$\mathbf{S}_T = \mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3 = 935 + j56.35 = 936.7\angle 3.45° \text{ kVA}$$

$$\left|\mathbf{S}_T\right| = \sqrt{3}\, V_L I_L$$

$$I_L = \frac{936.7 \times 10^3}{\sqrt{3}\,(13.8 \times 10^3)} = \underline{\mathbf{39.19\ A\ rms}}$$

$$\text{pf} = \cos\theta = \cos(3.45°) = \underline{\mathbf{0.9982 \quad (lagging)}}$$

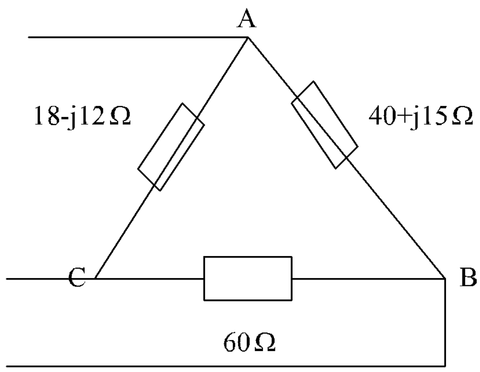
Chapter 12, Problem 48.

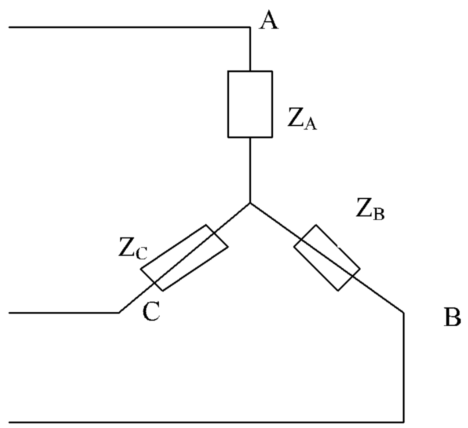
A balanced, positive-sequence wye-connected source has $\mathbf{V}_{an} = 240\ \angle 0°$ V rms and supplies an unbalanced delta-connected load via a transmission line with impedance $2 + j3\,\Omega$ per phase.

(a) Calculate the line currents if $\mathbf{Z}_{AB} = 40 + j15\,\Omega$, $\mathbf{Z}_{BC} = 60\,\Omega$, $\mathbf{Z}_{CA} = 18 - j12\,\Omega$.
(b) Find the complex power supplied by the source.

Chapter 12, Solution 48.

(a) We first convert the delta load to its equivalent wye load, as shown below.



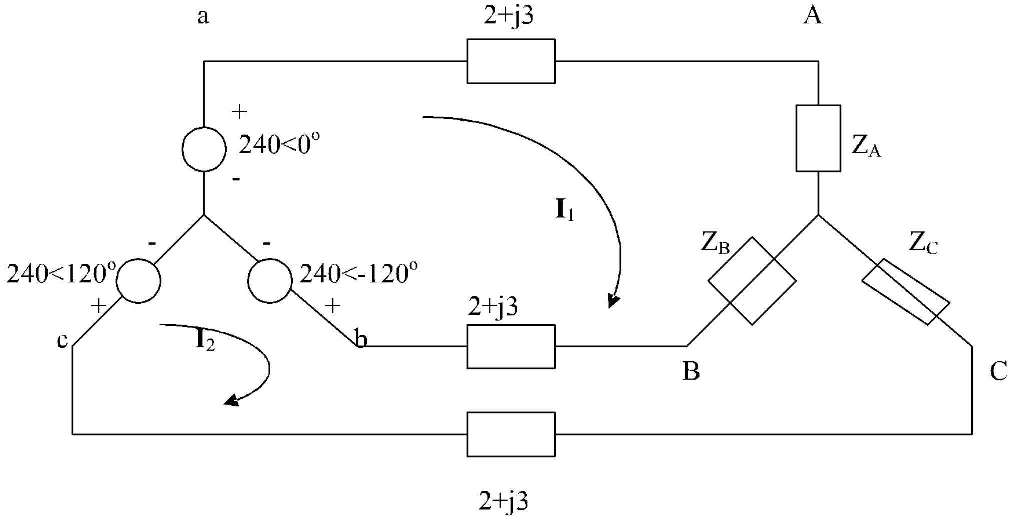


$$Z_A = \frac{(40 + j15)(18 - j12)}{118 + j3} = 7.577 - j1.923$$

$$Z_B = \frac{60(40 + j15).}{118 + j3} = 20.52 + j7.105$$

$$Z_C = \frac{60(18 - j12)}{118 + j3} = 8.992 - j6.3303$$

The system becomes that shown below.



We apply KVL to the loops. For mesh 1,

$$-240 + 240\angle -120^\circ + I_1(2Z_l + Z_A + Z_B) - I_2(Z_B + Z_l) = 0$$

or

$$(32.097 + j11.13)I_1 - (22.52 + j10.105)I_2 = 360 + j207.85 \qquad (1)$$

For mesh 2,

$$240\angle 120^\circ - 240\angle -120^\circ - I_1(Z_B + Z_l) + I_2(2Z_l + Z_B + Z_C) = 0$$

or

$$-(22.52 + j10.105)I_1 + (33.51 + j6.775)I_2 = -j415.69 \qquad (2)$$

Solving (1) and (2) gives

$I_1 = 23.75 - j5.328, \qquad I_2 = 15.165 - j11.89$

$I_{aA} = I_1 = \underline{24.34\angle -12.64^\circ \text{ A}}, \qquad I_{bB} = I_2 - I_1 = \underline{10.81\angle -142.6^\circ \text{ A}}$

$I_{cC} = -I_2 = \underline{19.27\angle 141.9^\circ \text{ A}}$

(b) $\overline{S}_a = (240\angle 0^\circ)(24.34\angle 12.64^\circ) = 5841.6\angle 12.64^\circ$

$\overline{S}_b = (240\angle -120^\circ)(10.81\angle 142.6^\circ) = 2594.4\angle 22.6^\circ$

$\overline{S}_b = (240\angle 120^\circ)(19.27\angle -141.9^\circ) = 4624.8\angle -21.9^\circ$

$\overline{S} = \overline{S}_a + \overline{S}_b + \overline{S}_c = 12.386 + j0.55 \text{ kVA} = \underline{12.4\angle 2.54^\circ \text{ kVA}}$

Chapter 12, Problem 49.

Each phase load consists of a 20-Ω resistor and a 10-Ω inductive reactance. With a line voltage of 220 V rms, calculate the average power taken by the load if:

(a) the three-phase loads are delta-connected
(b) the loads are wye-connected

Chapter 12, Solution 49.

(a) For the delta-connected load, $Z_p = 20 + j10\Omega, \quad V_p = V_L = 220 \text{ (rms)},$

$$S = \frac{3V^2{}_p}{Z^*{}_p} = \frac{3x220^2}{(20 - j10)} = 5808 + j2904 = \underline{6.943\angle 26.56'' \text{ kVA}}$$

or **5.808kW**

(b) For the wye-connected load, $Z_p = 20 + j10\Omega, \quad V_p = V_L / \sqrt{3},$

$$S = \frac{3V^2{}_p}{Z^*{}_p} = \frac{3x220^2}{3(20 - j10)} = \underline{2.164\angle 26.56'' \text{ kVA}} \text{ or } \mathbf{\underline{1.9356 \text{ kW}}}$$

Chapter 12, Problem 50.

A balanced three-phase source with $\mathbf{V}_L$ = 240 V rms is supplying 8 kVA at 0.6 power factor lagging to two wye-connected parallel loads. If one load draws 3 kW at unity power factor, calculate the impedance per phase of the second load.

Chapter 12, Solution 50.

$\overline{S} = \overline{S}_1 + \overline{S}_2 = 8(0.6 + j0.8) = 4.8 + j6.4 \text{kVA}, \qquad \overline{S}_1 = 3 \text{kVA}$

Hence,

$\overline{S}_2 = \overline{S} - \overline{S}_1 = 1.8 + j6.4 \text{kVA}$

But $\overline{S}_2 = \dfrac{3V^2{}_p}{Z^*{}_p}, \quad V_p = \dfrac{V_L}{\sqrt{3}} \longrightarrow \overline{S}_2 = \dfrac{V^2{}_L}{Z^*{}_p}$

$$Z^*{}_p = \frac{V^*{}_L}{\overline{S}_2} = \frac{240^2}{(1.8 + j6.4)x10^3} \longrightarrow \underline{Z_p = 2.346 + j8.34\Omega}$$

Chapter 12, Problem 51.

Consider the Δ-Δ system shown in Fig. 12.60. Take $\mathbf{Z}_1 = 8 + j6\,\Omega$, $\mathbf{Z}_2 = 4.2 - j2.2\,\Omega$, $\mathbf{Z}_3 = 10 + j0\,\Omega$.

(a) Find the phase current $\mathbf{I}_{AB}$, $\mathbf{I}_{BC}$, $\mathbf{I}_{CA}$.
(b) Calculate line currents $\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$, and $\mathbf{I}_{cC}$.

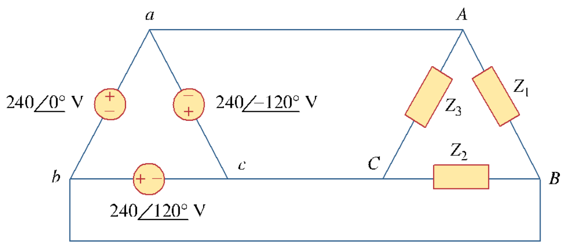


Figure 12.60
For Prob. 12.51.

Chapter 12, Solution 51.

This is an unbalanced system.

$$I_{AB} = \frac{240 < 0^{\circ}}{Z_1} = \frac{240 < 0^{\circ}}{8 + j6} = \underline{19.2\text{-j}14.4 \text{ A}} = \underline{\mathbf{19.2–j14.4\ A}}$$

$$\text{I}_{\text{BC}} = \frac{240\angle 120^{\circ}}{\text{Z}_2} = \frac{240\angle 120^{\circ}}{4.7413\angle -27.65} = 50.62\angle 147.65^{\circ} = \underline{\mathbf{-42.76+j27.09\ A}}$$

$$\text{I}_{\text{CA}} = \frac{240\angle -120^{\circ}}{\text{Z}_3} = \frac{240\angle -120^{\circ}}{10} = \underline{\mathbf{-12–j20.78\ A}}$$

At node A,

$$I_{aA} = I_{AB} - I_{CA} = (19.2 - j14.4) - (-12 - j20.78) = \underline{31.2 + j6.38 \text{ A}} = \underline{\mathbf{31.2+j6.38\ A}}$$

$$\text{I}_{bB} = I_{BC} - I_{AB} = (-42.76 + j27.08) - (19.2 - j14.4) = \underline{-61.96 + j41.48 \text{ A}}$$
$$= \underline{\mathbf{-61.96+j41.48\ A}}$$

$$\text{I}_{cC} = I_{CA} - I_{BC} = (-12 - j20.78) - (-42.76 + j27.08) = \underline{30.76 - j47.86 \text{ A}}$$
$$= \underline{\mathbf{30.76–j47.86\ A}}$$

Chapter 12, Problem 52.

A four-wire wye-wye circuit has

$\mathbf{V}_{an} = 120\angle 120°$, $\mathbf{V}_{bn} = 120\ \angle 0°$

$\mathbf{V}_{cn} = 120\ \angle -120°$ V

If the impedances are

$\mathbf{Z}_{AN} = 20\ \angle 60°$, $\mathbf{Z}_{BN} = 30\ \angle 0°$

$\mathbf{Z}_{cn} = 40\ \angle 30° \Omega$

find the current in the neutral line.

Chapter 12, Solution 52.

Since the neutral line is present, we can solve this problem on a per-phase basis.

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_{AN}} = \frac{120\angle 120°}{20\angle 60°} = 6\angle 60°$$

$$\mathbf{I}_b = \frac{\mathbf{V}_{bn}}{\mathbf{Z}_{BN}} = \frac{120\angle 0°}{30\angle 0°} = 4\angle 0°$$

$$\mathbf{I}_c = \frac{\mathbf{V}_{cn}}{\mathbf{Z}_{CN}} = \frac{120\angle -120°}{40\angle 30°} = 3\angle -150°$$

Thus,

$$-\mathbf{I}_n = \mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c$$

$$-\mathbf{I}_n = 6\angle 60° + 4\angle 0° + 3\angle -150°$$

$$-\mathbf{I}_n = (3 + j5.196) + (4) + (-2.598 - j1.5)$$

$$-\mathbf{I}_n = 4.405 + j3.696 = 5.75\angle 40°$$

$$\mathbf{I}_n = \underline{\mathbf{5.75\angle 220°\ A}}$$

Chapter 12, Problem 53.

In the Y-Y system shown in Fig. 12.61, loads connected to the source are unbalanced. (a) Calculate $\mathbf{I}_a$, $\mathbf{I}_b$, and $\mathbf{I}_c$. (b) Find the total power delivered to the load. Take $\mathbf{V}_p = 240$ V rms.

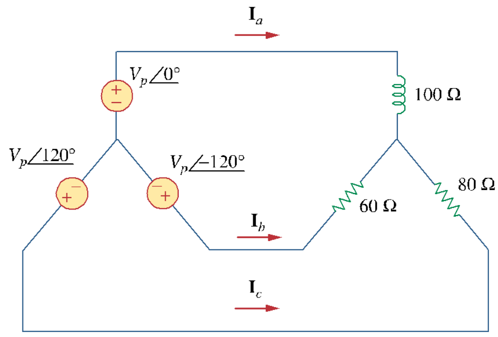
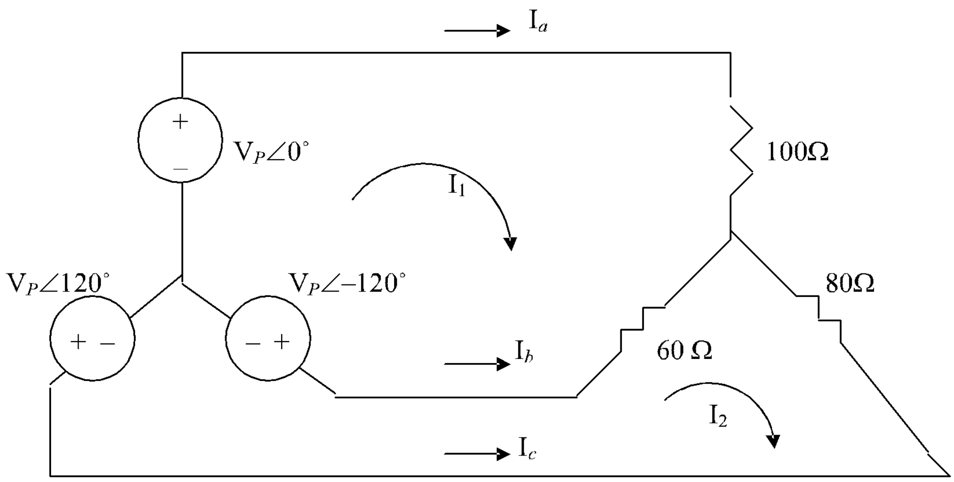


Figure 12.61
For Prob. 12.53.

Chapter 12, Solution 53.

Applying mesh analysis as shown below, we get.



$$240 < -120^{o} - 240 < 0^{o} + I_1 x160 - 60I_2 = 0 \quad \longrightarrow \quad 160I_1 - 60I_2 = 360 + j207.84 \quad (1)$$

$$240 < 120^{o} - 240 < -120^{o} + 140I_2 - 60I_1 = 0 \quad \longrightarrow \quad 140I_2 - 60I_1 = -j415.7 \quad (2)$$

In matrix form, (1) and (2) become

$$\begin{bmatrix} 160 & -60 \\ -60 & 140 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 360 + j207.84 \\ -j415.7 \end{bmatrix}$$

Using MATLAB, we get,

```
>> Z=[160,-60;-60,140]
Z =
  160  -60
  -60  140
>> V=[(360+207.8i);-415.7i]
V =
 1.0e+002 *
  3.6000 + 2.0780i
       0 - 4.1570i
>> I=inv(Z)*V
I =
  2.6809 + 0.2207i
  1.1489 - 2.8747i
```

I_1 = 2.681+j0.2207 and I_2 = 1.1489–j2.875

$I_a = I_1 =$ **$2.69\angle 4.71^\circ$ A**

$I_b = I_2 - I_1 = -1.5321 - j3.096 =$ **$3.454\angle -116.33^\circ$ A**

$I_c = -I_2 =$ **$3.096\angle 111.78^\circ$ A**

$$S_a = |I_a|^2 Z_a = (2.69)^2 x100 = 723.61 \text{ W}$$
$$S_b = |I_b|^2 Z_b = (3.454)^2 x60 = 715.81 \text{ W}$$
$$S_c = |I_c|^2 Z_c = (3.0957)^2 x80 = 766.67 \text{ W}$$

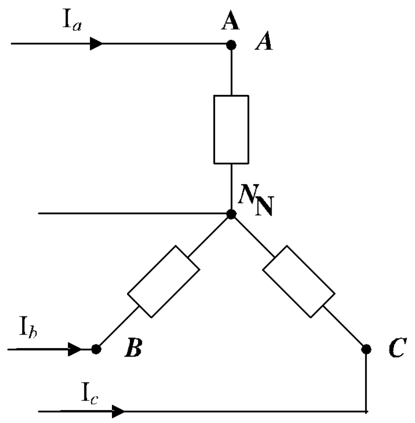
$P = P_a + P_b + P_c =$ **2.205 kW**

Chapter 12, Problem 54.

A balanced three-phase Y-source with $\mathbf{V}_P = 210$ V rms drives a Y-connected three-phase load with phase impedance $\mathbf{Z}_A = 80\,\Omega$, $\mathbf{Z}_B = 60 + j90\,\Omega$, and $\mathbf{Z}_C = j80\,\Omega$. Calculate the line currents and total complex power delivered to the load. Assume that the neutrals are connected.

Chapter 12, Solution 54.

Consider the load as shown below.



$$I_a = \frac{210 < 0^o}{80} = \underline{2.625\text{ A}}$$

$$\mathrm{I_b} = \frac{210\angle 0^\circ}{60 + j90} = \frac{210}{108.17\angle 56.31^\circ} = \underline{\mathbf{1.9414\angle{-56.31}^\circ\ A}}$$

$$I_c = \frac{210 < 0^o}{j80} = \underline{2.625 < -90^o\text{ A}}$$

$$S_a = VI_a^* = 210x2.625 = 551.25$$

$$S_b = VI_b^* = \frac{|V|^2}{Z_b^*} = \frac{210^2}{60 - j90} = 226.15 + j339.2$$

$$S_c = \frac{|V|^2}{Z_c^*} = \frac{210^2}{-j80} = j551.25$$

$$S = S_a + S_b + S_c = \underline{777.4 + j890.45\text{ VA}}$$

Chapter 12, Problem 55.

A three-phase supply, with the line voltage 240 V rms positively phased, has an unbalanced delta-connected load as shown in Fig. 12.62. Find the phase currents and the total complex power.

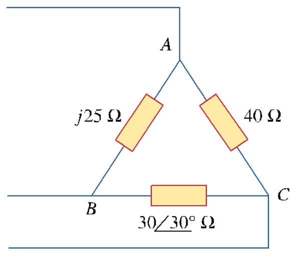


Figure 12.62
For Prob. 12.55.

Chapter 12, Solution 55.

The phase currents are:

$$I_{AB} = 240/j25 = \underline{\mathbf{9.6\angle{-90^\circ}\ A}}$$

$$I_{CA} = 240\angle 120^\circ/40 = \underline{\mathbf{6\angle 120^\circ\ A}}$$

$$I_{BC} = 240\angle{-120^\circ}/30\angle 30^\circ = \underline{\mathbf{8\angle{-150^\circ}\ A}}$$

The complex power in each phase is:

$$S_{AB} = |I_{AB}|^2 Z_{AB} = (9.6)^2 j25 = j2304$$

$$S_{AC} = |I_{AC}|^2 Z_{AC} = (6)^2 40 < 0'' = 1440$$

$$S_{BC} = |I_{BC}|^2 Z_{BC} = (8)^2 30 < 30'' = 1662.77 + j960$$

The total complex power is,

$$S = S_{AB} + S_{AC} + S_{BC} = \underline{3102.77 + j3264 \text{ VA}} = \underline{\mathbf{3.103+j3.264\ kVA}}$$

Chapter 12, Problem 56.

Refer to the unbalanced circuit of Fig. 12.63. Calculate:

(a) the line currents
(b) the real power absorbed by the load
(c) the total complex power supplied by the source

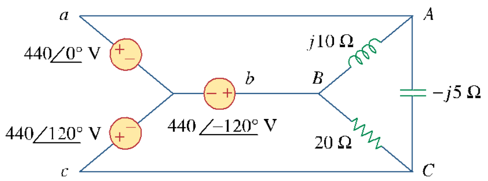
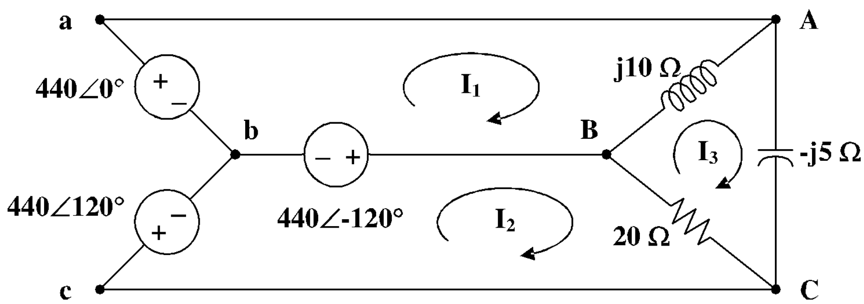


Figure 12.63
For Prob. 12.56.

Chapter 12, Solution 56.

(a) Consider the circuit below.



For mesh 1,

$$440\angle\text{-}120° - 440\angle 0° + j10(\mathbf{I}_1 - \mathbf{I}_3) = 0$$

$$\mathbf{I}_1 - \mathbf{I}_3 = \frac{(440)(1.5 + j0.866)}{j10} = 76.21\angle\text{-}60° \qquad (1)$$

For mesh 2,

$$440\angle 120° - 440\angle\text{-}120° + 20(\mathbf{I}_2 - \mathbf{I}_3) = 0$$

$$\mathbf{I}_3 - \mathbf{I}_2 = \frac{(440)(j1.732)}{20} = j38.1 \qquad (2)$$

For mesh 3,

$$j10(\mathbf{I}_3 - \mathbf{I}_1) + 20(\mathbf{I}_3 - \mathbf{I}_2) - j5\mathbf{I}_3 = 0$$

Substituting (1) and (2) into the equation for mesh 3 gives,

$$\mathbf{I}_3 = \frac{(440)(-1.5 + j0.866)}{j5} = 152.42\angle 60° \qquad (3)$$

From (1),

$$\mathbf{I}_1 = \mathbf{I}_3 + 76.21\angle -60° = 114.315 + j66 = 132\angle 30°$$

From (2),

$$\mathbf{I}_2 = \mathbf{I}_3 - j38.1 = 76.21 + j93.9 = 120.93\angle 50.94°$$

$$\mathbf{I}_a = \mathbf{I}_1 = \underline{\mathbf{132\angle 30° \ A}}$$

$$\mathbf{I}_b = \mathbf{I}_2 - \mathbf{I}_1 = -38.105 + j27.9 = \underline{\mathbf{47.23\angle 143.8° \ A}}$$

$$\mathbf{I}_c = -\mathbf{I}_2 = \underline{\mathbf{120.9\angle 230.9° \ A}}$$

(b)

$$\mathbf{S}_{AB} = \left|\mathbf{I}_1 - \mathbf{I}_3\right|^2 (j10) = j58.08 \text{ kVA}$$

$$\mathbf{S}_{BC} = \left|\mathbf{I}_2 - \mathbf{I}_3\right|^2 (20) = 29.04 \text{ kVA}$$

$$\mathbf{S}_{CA} = \left|\mathbf{I}_3\right|^2 (-j5) = (152.42)^2 (-j5) = -j116.16 \text{ kVA}$$

$$\mathbf{S} = \mathbf{S}_{AB} + \mathbf{S}_{BC} + \mathbf{S}_{CA} = 29.04 - j58.08 \text{ kVA}$$

Real power absorbed = **29.04 kW**

(c) Total complex supplied by the source is

$$\mathbf{S} = \underline{\mathbf{29.04 - j58.08 \ kVA}}$$

Chapter 12, Problem 57.

Determine the line currents for the three-phase circuit of Fig. 12.64. Let $\mathbf{V}_a = 110\angle 0°$, $\mathbf{V}_b = 110\angle -120°$, $\mathbf{V}_c = 110\angle 120°$ V.

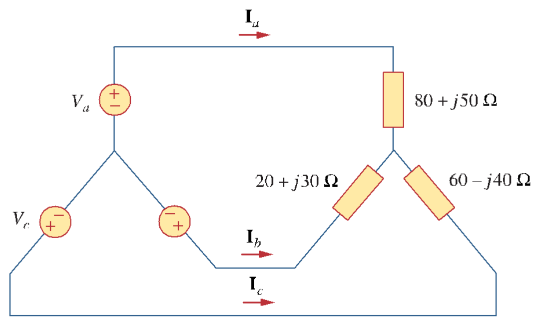
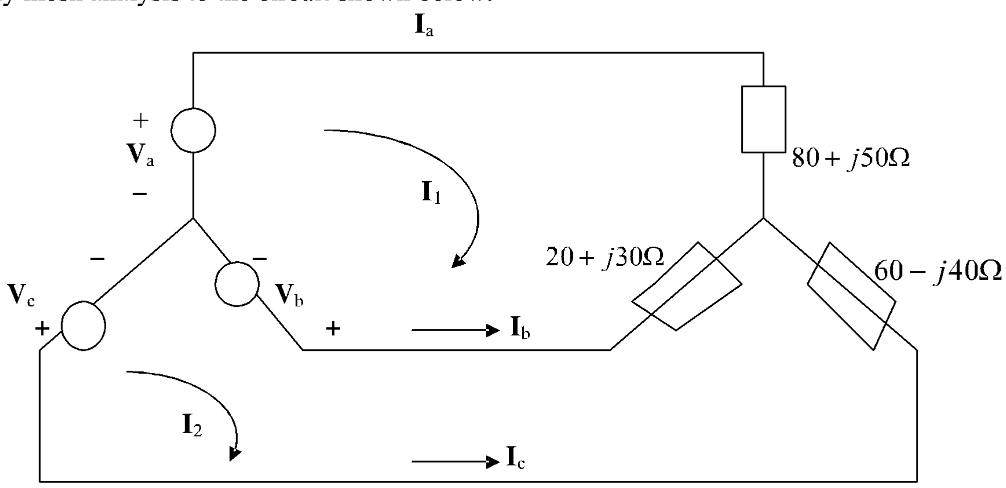


Figure 12.64
For Prob. 12.57.

Chapter 12, Solution 57.
We apply mesh analysis to the circuit shown below.



$$(100 + j80)I_1 - (20 + j30)I_2 = V_a - V_b = 165 + j95.263 \qquad (1)$$

$$-(20 + j30)I_1 + (80 - j10)I_2 = V_b - V_c = -j190.53 \qquad (2)$$

Solving (1) and (2) gives $I_1 = 1.8616 - j0.6084$, $\quad I_2 = 0.9088 - j1.722$.

$I_a = I_1 = \underline{1.9585\angle -18.1^o \text{ A}}$, $\quad I_b = I_2 - I_1 = -0.528 - j1.1136 = \underline{1.4656\angle -130.55^o \text{ A}}$

$I_c = -I_2 = \underline{1.947\angle 117.8^o \text{ A}}$

Chapter 12, Problem 58.

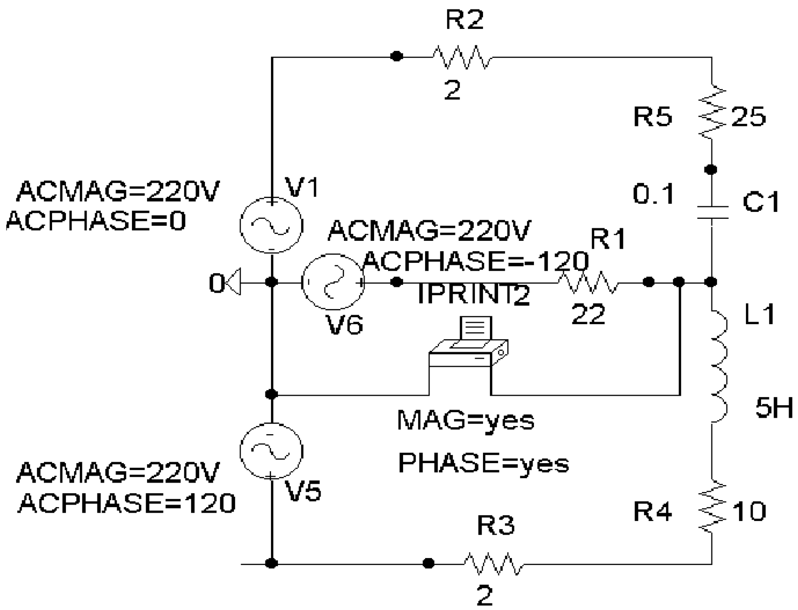
Solve Prob. 12.10 using *PSpice*.

Chapter 12, Solution 58.

The schematic is shown below. IPRINT is inserted in the neutral line to measure the current through the line. In the AC Sweep box, we select Total Ptss = 1, Start Freq. = 0.1592, and End Freq. = 0.1592. After simulation, the output file includes

FREQ	IM(V_PRINT4)	IP(V_PRINT4)
1.592 E–01	1.078 E+01	–8.997 E+01

i.e. I_n = **$\underline{10.78\angle{-89.97°}\ A}$**



Chapter 12, Problem 59.

The source in Fig. 12.65 is balanced and exhibits a positive phase sequence. If f = 60 Hz, use *PSpice* to find $\mathbf{V}_{AN}$, $\mathbf{V}_{BN}$, and $\mathbf{V}_{CN}$.

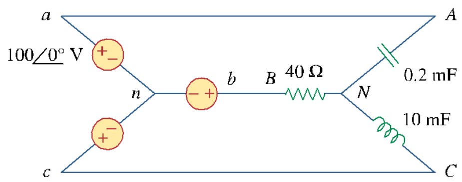


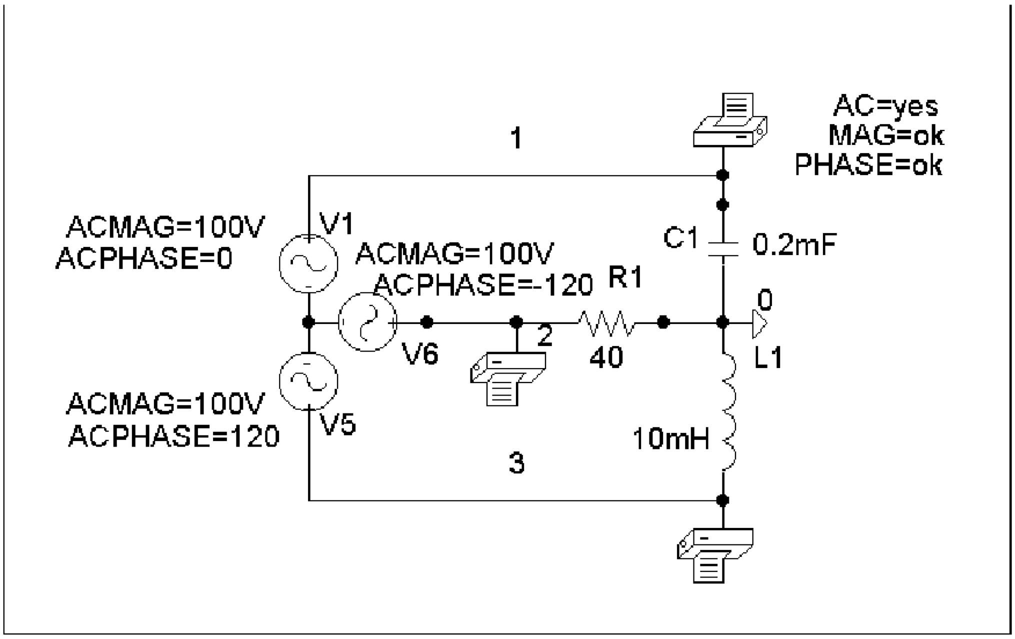
Figure 12.65
For Prob. 12.59.

Chapter 12, Solution 59.

The schematic is shown below. In the AC Sweep box, we set Total Pts = 1, Start Freq = 60, and End Freq = 60. After simulation, we obtain an output file which includes

FREQ	VM(1)	VP(1)
6.000 E+01	2.206 E+02	–3.456 E+01
FREQ	VM(2)	VP(2)
6.000 E+01	2.141 E+02	–8.149 E+01
FREQ	VM(3)	VP(3)
6.000 E+01	4.991 E+01	–5.059 E+01

i.e. V_{AN} = **$220.6\angle -34.56°$**, V_{BN} = **$214.1\angle -81.49°$**, V_{CN} = **$49.91\angle -50.59°$ V**



Chapter 12, Problem 60.

Use *PSpice* to determine $\mathbf{I}_o$ in the single-phase, three-wire circuit of Fig. 12.66. Let $\mathbf{Z}_1 = 15 - j10\,\Omega$, $\mathbf{Z}_2 = 30 + j20\,\Omega$, and $\mathbf{Z}_3 = 12 + j5\,\Omega$.

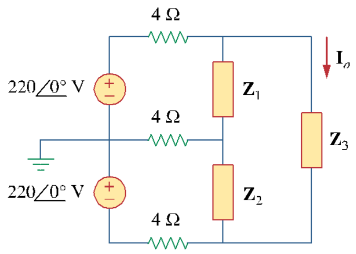


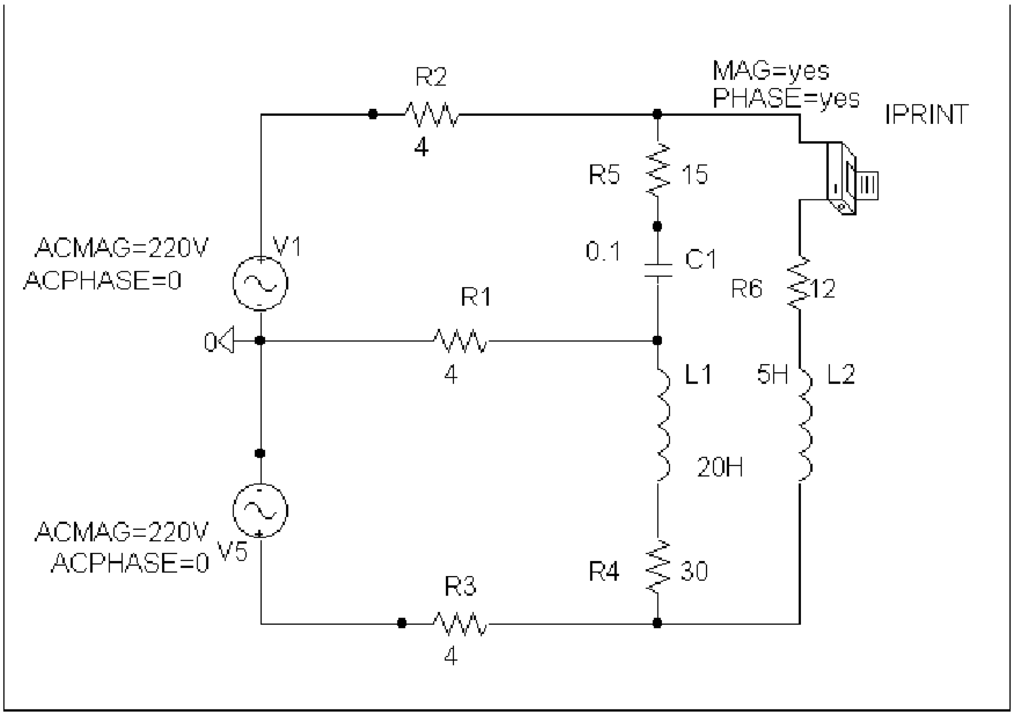
Figure 12.66
For Prob. 12.60.

Chapter 12, Solution 60.

The schematic is shown below. IPRINT is inserted to give I_o. We select Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592 in the AC Sweep box. Upon simulation, the output file includes

FREQ	IM(V_PRINT4)	IP(V_PRINT4)
1.592 E–01	1.421 E+00	–1.355 E+02

from which, I_o = **$1.421\angle{-135.5°}$ A**



Chapter 12, Problem 61.

Given the circuit in Fig. 12.67, use *PSpice* to determine currents $\mathbf{I}_{aA}$ and voltage $\mathbf{V}_{BN}$.

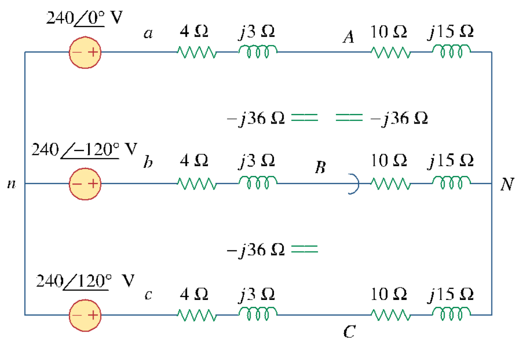


Figure 12.67
For Prob. 12.61.

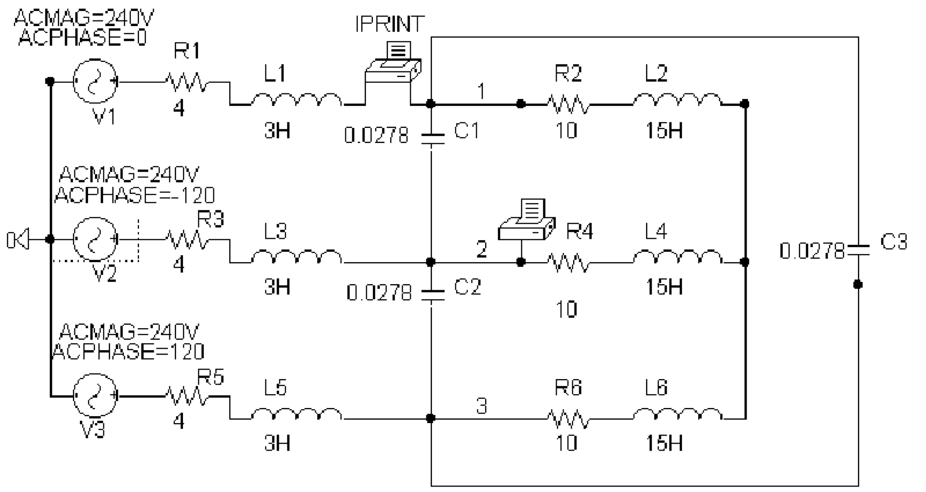
Chapter 12, Solution 61.

The schematic is shown below. Pseudocomponents IPRINT and PRINT are inserted to measure I_{aA} and V_{BN}. In the AC Sweep box, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. Once the circuit is simulated, we get an output file which includes

FREQ	VM(2)	VP(2)
1.592 E–01	2.308 E+02	–1.334 E+02
FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E–01	1.115 E+01	3.699 E+01

from which

$$I_{aA} = \underline{\mathbf{11.15\angle 37^\circ\ A}}, \quad V_{BN} = \underline{\mathbf{230.8\angle -133.4^\circ\ V}}$$



Chapter 12, Problem 62.

The circuit in Fig. 12.68 operates at 60 Hz. Use *PSpice* to find the source current $\mathbf{I}_{ab}$ and the line current $\mathbf{I}_{bB}$.

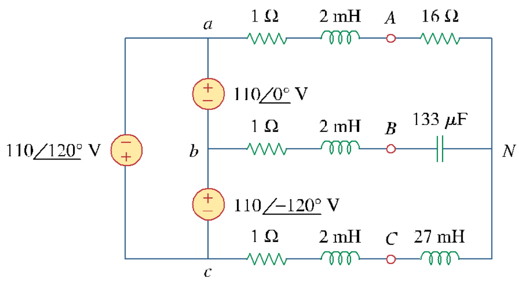


Figure 12.68
For Prob. 12.62.

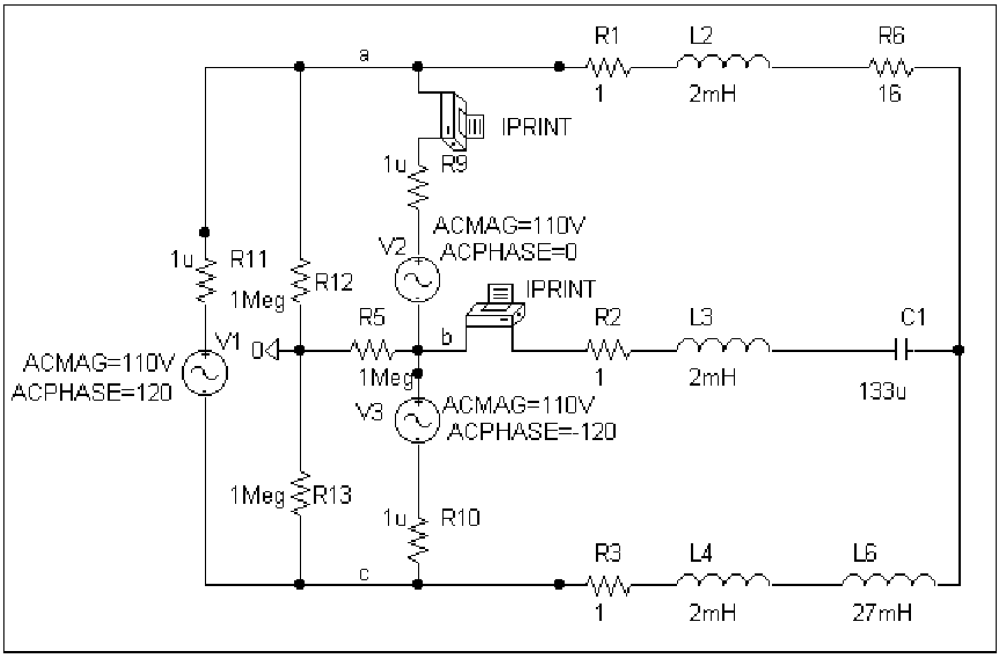
Chapter 12, Solution 62.

Because of the delta-connected source involved, we follow Example 12.12. In the AC Sweep box, we type Total Pts = 1, Start Freq = 60, and End Freq = 60. After simulation, the output file includes

FREQ	IM(V_PRINT2)	IP(V_PRINT2)
6.000 E+01	5.960 E+00	–9.141 E+01
FREQ	IM(V_PRINT1)	IP(V_PRINT1)
6.000 E+01	7.333 E+07	1.200 E+02

From which

$$I_{ab} = \underline{\mathbf{7.333x10^7\angle 120^\circ\ A}},\ I_{bB} = \underline{\mathbf{5.96\angle -91.41^\circ\ A}}$$



Chapter 12, Problem 63.

Use *PSpice* to find currents $\mathbf{I}_{aA}$ and $\mathbf{I}_{AC}$ in the unbalanced three-phase system shown in Fig. 12.69. Let

$\mathbf{Z}_l = 2 + j, \qquad \mathbf{Z}_1 = 40 + j20\,\Omega$,

$\mathbf{Z}_2 = 50 - j30\,\Omega$, $\mathbf{Z}_3 = 25\,\Omega$

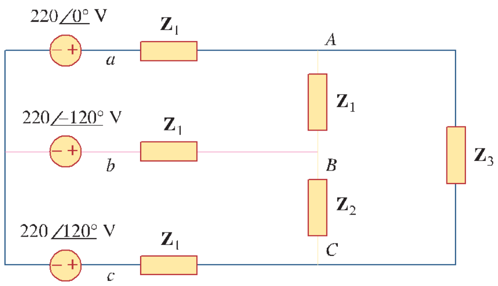
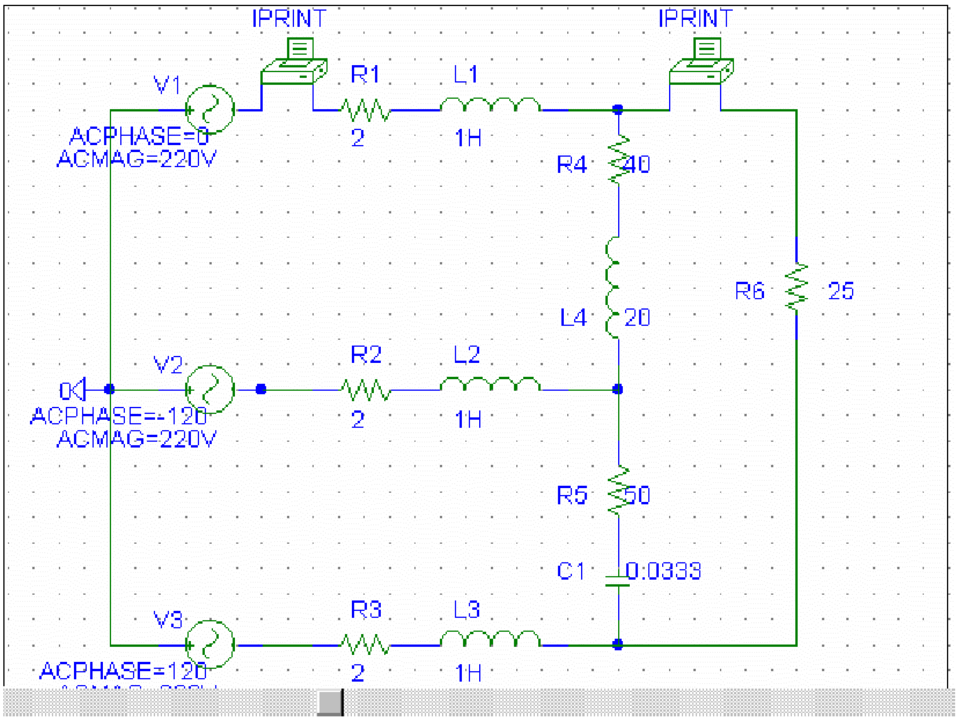


Figure 12.69
For Prob. 12.63.

Chapter 12, Solution 63.

Let $\omega = 1$ so that $L = X/\omega = 20\,H$, and $C = \dfrac{1}{\omega X} = 0.0333\,F$

The schematic is shown below.



When the file is saved and run, we obtain an output file which includes the following:

```
FREQ          IM(V_PRINT1)IP(V_PRINT1)

 1.592E-01    1.867E+01    1.589E+02

FREQ          IM(V_PRINT2)IP(V_PRINT2)

 1.592E-01    1.238E+01    1.441E+02
```

From the output file, the required currents are:

$\underline{I_{aA} = 18.67\angle 158.9^{\circ}\ \text{A},\ \ I_{AC} = 12.38\angle 144.1^{\circ}\ \text{A}}$

Chapter 12, Problem 64.

For the circuit in Fig. 12.58, use *PSpice* to find the line currents and the phase currents.

Chapter 12, Solution 64.

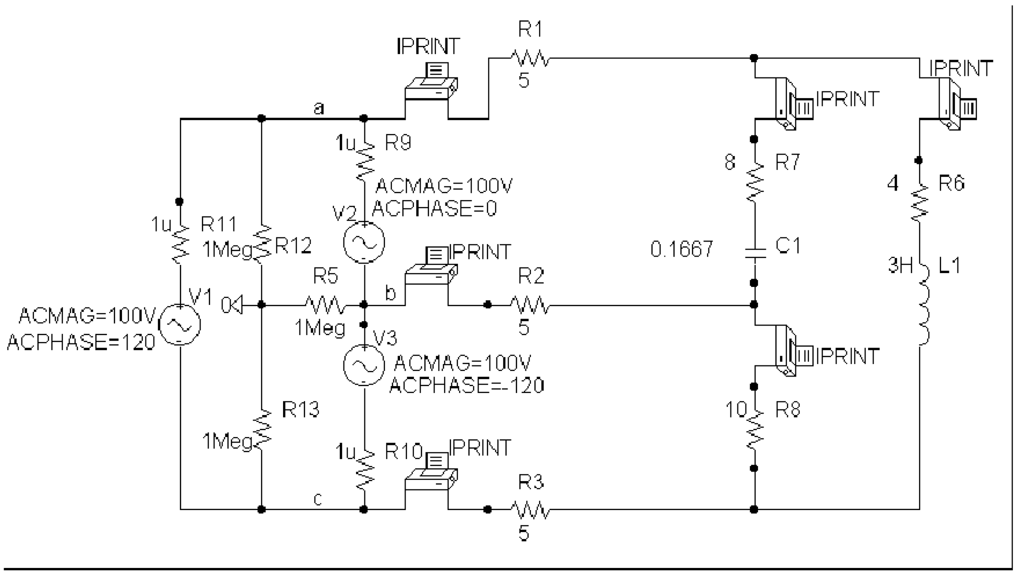
We follow Example 12.12. In the AC Sweep box we type Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation the output file includes

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592 E–01	4.710 E+00	7.138 E+01
FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E–01	6.781 E+07	–1.426 E+02
FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.592 E–01	3.898 E+00	–5.076 E+00
FREQ	IM(V_PRINT4)	IP(V_PRINT4)
1.592 E–01	3.547 E+00	6.157 E+01
FREQ	IM(V_PRINT5)	IP(V_PRINT5)
1.592 E–01	1.357 E+00	9.781 E+01
FREQ	IM(V_PRINT6)	IP(V_PRINT6)
1.592 E–01	3.831 E+00	–1.649 E+02

from this we obtain

I_{aA} = $\underline{\mathbf{4.71\angle 71.38^\circ\ A}}$, I_{bB} = $\underline{\mathbf{6.781\angle -142.6^\circ\ A}}$, I_{cC} = $\underline{\mathbf{3.898\angle -5.08^\circ\ A}}$

I_{AB} = $\underline{\mathbf{3.547\angle 61.57^\circ\ A}}$, I_{AC} = $\underline{\mathbf{1.357\angle 97.81^\circ\ A}}$, I_{BC} = $\underline{\mathbf{3.831\angle -164.9^\circ\ A}}$


IPRINT
R1
5
a
1u
R9
ACMAG=100V
ACPHASE=0
V2
IPRINT
8
R7
4
R6
1u
R11
1Meg
R12
0.1667
C1
3H
L1
R5
R2
b
V1
0
ACMAG=100V
ACPHASE=120
1Meg
5
V3
ACMAG=100V
ACPHASE=-120
10
R8
R13
1Meg
1u
R10
R3
c
5

Chapter 12, Problem 65.

A balanced three-phase circuit is shown in Fig. 12.70 on the next page. Use *PSpice* to find the line currents $\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$, and $\mathbf{I}_{cC}$.

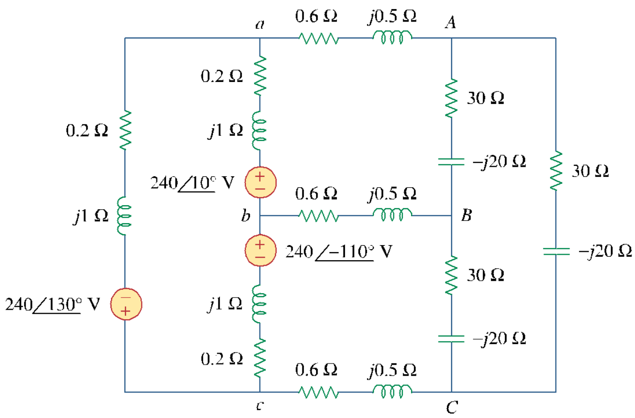


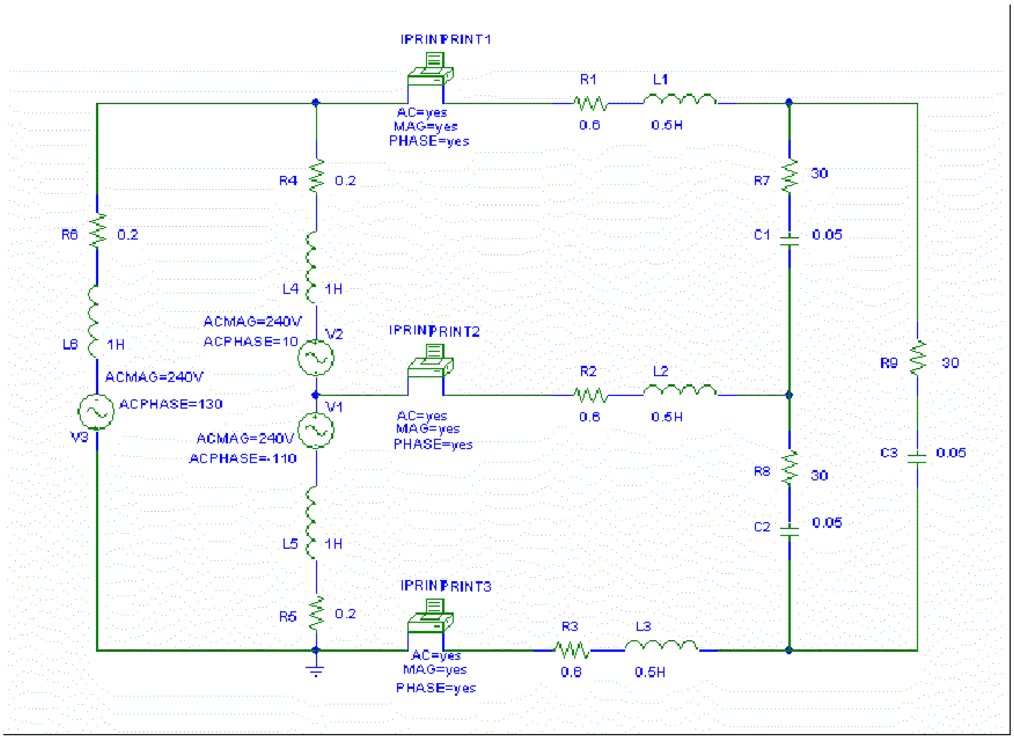
Figure 12.70
For Prob. 12.65.

Chapter 12, Solution 65.

Due to the delta-connected source, we follow Example 12.12. We type Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. The schematic is shown below. After it is saved and simulated, we obtain an output file which includes

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592E-01	1.140E+01	8.664E+00
FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592E-01	1.140E+01	-1.113E+02
FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.592E-01	1.140E+01	1.287E+02

Thus, I_{aA} = **11.02∠12° A**, I_{bB} = **11.02∠–108° A**, I_{cC} = **11.02∠132° A**



Since this is a balanced circuit, we can perform a quick check. The load resistance is large compared to the line and source impedances so we will ignore them (although it would not be difficult to include them).

Converting the sources to a Y configuration we get:

$$V_{an} = 138.56\ \angle{-20^\circ}\ \text{Vrms}$$

and

$$Z_Y = 10 - j6.667 = 12.019\angle{-33.69^\circ}$$

Now we can calculate,

$$I_{aA} = (138.56\ \angle{-20^\circ})/(12.019\angle{-33.69^\circ}) = 11.528\angle 13.69^\circ$$

Clearly, we have a good approximation which is very close to what we really have.

Chapter 12, Problem 66.

A three-phase, four-wire system operating with a 208-V line voltage is shown in Fig. 12.71. The source voltages are balanced. The power absorbed by the resistive wye-connected load is measured by the three-wattmeter method. Calculate:

(a) the voltage to neutral
(b) the currents $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{I}_3$, and $\mathbf{I}_n$
(c) the readings of the wattmeters
(d) the total power absorbed by the load

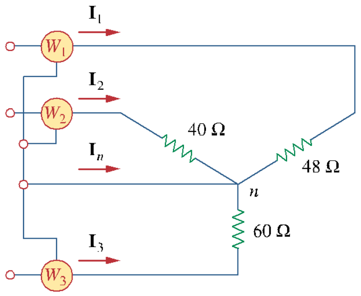


Figure 12.71
For Prob. 12.66.

Chapter 12, Solution 66.

(a) $V_p = \frac{V_L}{\sqrt{3}} = \frac{208}{\sqrt{3}} = \underline{\mathbf{120\ V}}$

(b) Because the load is unbalanced, we have an unbalanced three-phase system. Assuming an abc sequence,

$$\mathbf{I}_1 = \frac{120\angle 0°}{48} = 2.5\angle 0°\ \text{A}$$

$$\mathbf{I}_2 = \frac{120\angle\text{-}120°}{40} = 3\angle\text{-}120°\ \text{A}$$

$$\mathbf{I}_3 = \frac{120\angle 120°}{60} = 2\angle 120°\ \text{A}$$

$$-\mathbf{I}_N = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 = 2.5 + (3)\left(-0.5 - j\frac{\sqrt{3}}{2}\right) + (2)\left(-0.5 + j\frac{\sqrt{3}}{2}\right)$$

$$\mathbf{I}_N = j\frac{\sqrt{3}}{2} = j0.866 = 0.866\angle 90°\ \text{A}$$

Hence,

$I_1 = \underline{\mathbf{2.5\ A}}$, $I_2 = \underline{\mathbf{3\ A}}$, $I_3 = \underline{\mathbf{2\ A}}$, $I_N = \underline{\mathbf{0.866\ A}}$

(c) $P_1 = I_1^2 R_1 = (2.5)^2(48) = \underline{\mathbf{300\ W}}$

$P_2 = I_2^2 R_2 = (3)^2(40) = \underline{\mathbf{360\ W}}$

$P_3 = I_3^2 R_3 = (2)^2(60) = \underline{\mathbf{240\ W}}$

(d) $P_T = P_1 + P_2 + P_3 = \underline{\mathbf{900\ W}}$

Chapter 12, Problem 67.

* As shown in Fig. 12.72, a three-phase four-wire line with a phase voltage of 120 V rms and positive phase sequence supplies a balanced motor load at 260 kVA at 0.85 pf lagging. The motor load is connected to the three main lines marked *a*, *b*, and *c*. In addition, incandescent lamps (unity pf) are connected as follows: 24 kW from line *c* to the neutral, 15 kW from line *b* to the neutral, and 9 kW from line *c* to the neutral.

(a) If three wattmeters are arranged to measure the power in each line, calculate the reading of each meter.
(b) Find the magnitude of the current in the neutral line.

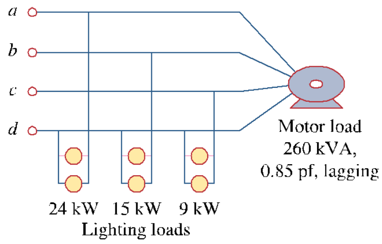


Figure 12.72
For Prob. 12.67.

* An asterisk indicates a challenging problem.

Chapter 12, Solution 67.

(a) The power to the motor is

$$P_T = S\cos\theta = (260)(0.85) = 221\text{ kW}$$

The motor power per phase is

$$P_p = \frac{1}{3}P_T = 73.67\text{ kW}$$

Hence, the wattmeter readings are as follows:

$$W_a = 73.67 + 24 = \underline{\mathbf{97.67\ kW}}$$

$$W_b = 73.67 + 15 = \underline{\mathbf{88.67\ kW}}$$

$$W_c = 73.67 + 9 = \underline{\mathbf{82.67\ kW}}$$

(b) The motor load is balanced so that $I_N = 0$.
For the lighting loads,

$$I_a = \frac{24{,}000}{120} = 200\text{ A}$$

$$I_b = \frac{15{,}000}{120} = 125\text{ A}$$

$$I_c = \frac{9{,}000}{120} = 75\text{ A}$$

If we let

$$\mathbf{I}_a = I_a\angle 0° = 200\angle 0°\text{ A}$$

$$\mathbf{I}_b = 125\angle -120°\text{ A}$$

$$\mathbf{I}_c = 75\angle 120°\text{ A}$$

Then,

$$-\mathbf{I}_N = \mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c$$

$$-\mathbf{I}_N = 200 + (125)\left(-0.5 - j\frac{\sqrt{3}}{2}\right) + (75)\left(-0.5 + j\frac{\sqrt{3}}{2}\right)$$

$$-\mathbf{I}_N = 100 - j43.3\text{ A}$$

$$|\mathbf{I}_N| = \underline{\mathbf{108.97\ A}}$$

Chapter 12, Problem 68.

Meter readings for a three-phase wye-connected alternator supplying power to a motor indicate that the line voltages are 330 V, the line currents are 8.4 A, and the total line power is 4.5 kW. Find:

(a) the load in VA
(b) the load pf
(c) the phase current
(d) the phase voltage

Chapter 12, Solution 68.

(a) $S = \sqrt{3}\, V_L I_L = \sqrt{3}(330)(8.4) = \underline{\textbf{4801 VA}}$

(b) $P = S\cos\theta \longrightarrow \text{pf} = \cos\theta = \dfrac{P}{S}$

$$\text{pf} = \frac{4500}{4801.24} = \underline{\textbf{0.9372}}$$

(c) For a wye-connected load,
$I_p = I_L = \underline{\textbf{8.4 A}}$

(d) $V_p = \dfrac{V_L}{\sqrt{3}} = \dfrac{330}{\sqrt{3}} = \underline{\textbf{190.53 V}}$

Chapter 12, Problem 69.

A certain store contains three balanced three-phase loads. The three loads are:

Load 1: 16 kVA at 0.85 pf lagging

Load 2: 12 kVA at 0.6 pf lagging

Load 3: 8 kW at unity pf

The line voltage at the load is 208 V rms at 60 Hz, and the line impedance is $0.4 + j0.8\,\Omega$. Determine the line current and the complex power delivered to the loads.

Chapter 12, Solution 69.

For load 1,

$$\overline{S}_1 = S_1 \cos\theta_1 + jS_1 \sin\theta_1$$

$$pf = 0.85 = \cos\theta_1 \quad \longrightarrow \quad \theta_1 = 31.79''$$

$$\overline{S}_1 = 13.6 + j8.43 \text{ kVA}$$

For load 2,

$$\overline{S_2} = 12x0.6 + j12x0.8 = 7.2 + j9.6 \text{ kVA}$$

For load 3,

$$\overline{S}_3 = 8 + j0 \text{ kVA}$$

$\overline{S} = \overline{S}_1 + \overline{S_2} + \overline{S_3} = 28.8 + j18.03$ = **<u>28.8+j18.03 kVA</u>**

But $\mathbf{S_P} = \mathbf{V_P I_P^*}$ with $I_P = I_L$

$$I_L^* = \frac{S_P}{V_P} = \frac{(28800 + j18030)}{3x120.08}$$

$I_L = 79.95 - j50.05$ = **<u>94.32∠–32.05° A</u>**. Note, this is relative to 120.08∠0° V. If we assume a positive phase rotation and V_{ab} = 208∠0°, then V_{an} = 120.08∠–30° which yields I_a = **<u>94.32∠–62.05° A</u>**, I_b = **<u>94.32∠177.95° A</u>**, I_c = **<u>94.32∠57.95° A</u>**.

Chapter 12, Problem 70.

The two-wattmeter method gives $P_1 = 1200$ W and $P_2 = -400$ W for a three-phase motor running on a 240-V line. Assume that the motor load is wye-connected and that it draws a line current of 6 A. Calculate the pf of the motor and its phase impedance.

Chapter 12, Solution 70.

$$P_T = P_1 + P_2 = 1200 - 400 = 800$$

$$Q_T = P_2 - P_1 = -400 - 1200 = -1600$$

$$\tan\theta = \frac{Q_T}{P_T} = \frac{-1600}{800} = -2 \longrightarrow \theta = -63.43°$$

$$\text{pf} = \cos\theta = \underline{\mathbf{0.4472 \quad (leading)}}$$

$$Z_p = \frac{V_L}{I_L} = \frac{240}{6} = 40$$

$$\mathbf{Z}_p = \underline{\mathbf{40\angle -63.43° \ \Omega}}$$

Chapter 12, Problem 71.

In Fig. 12.73, two wattmeters are properly connected to the unbalanced load supplied by a balanced source such that $\mathbf{V}_{ab} = 208\angle 0°$ V with positive phase sequence.

(a) Determine the reading of each wattmeter.
(b) Calculate the total apparent power absorbed by the load.

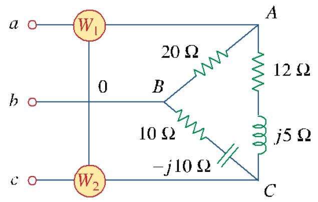


Figure 12.73
For Prob. 12.71.

Chapter 12, Solution 71.

(a) If $\mathbf{V}_{ab} = 208\angle 0°$, $\mathbf{V}_{bc} = 208\angle\text{-}120°$, $\mathbf{V}_{ca} = 208\angle 120°$,

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_{Ab}} = \frac{208\angle 0°}{20} = 10.4\angle 0°$$

$$\mathbf{I}_{BC} = \frac{\mathbf{V}_{bc}}{\mathbf{Z}_{BC}} = \frac{208\angle\text{-}120°}{10\sqrt{2}\angle\text{-}45°} = 14.708\angle\text{-}75°$$

$$\mathbf{I}_{CA} = \frac{\mathbf{V}_{ca}}{\mathbf{Z}_{CA}} = \frac{208\angle 120°}{13\angle 22.62°} = 16\angle 97.38°$$

$$\mathbf{I}_{aA} = \mathbf{I}_{AB} - \mathbf{I}_{CA} = 10.4\angle 0° - 16\angle 97.38°$$

$$\mathbf{I}_{aA} = 10.4 + 2.055 - j15.867$$

$$\mathbf{I}_{aA} = 20.171\angle\text{-}51.87°$$

$$\mathbf{I}_{cC} = \mathbf{I}_{CA} - \mathbf{I}_{BC} = 16\angle 97.83° - 14.708\angle\text{-}75°$$

$$\mathbf{I}_{cC} = 30.64\angle 101.03°$$

$$P_1 = |\mathbf{V}_{ab}||\mathbf{I}_{aA}|\cos(\theta_{V_{ab}} - \theta_{I_{aA}})$$

$$P_1 = (208)(20.171)\cos(0° + 51.87°) = \underline{\mathbf{2590\ W}}$$

$$P_2 = |\mathbf{V}_{cb}||\mathbf{I}_{cC}|\cos(\theta_{V_{cb}} - \theta_{I_{cC}})$$

But $\mathbf{V}_{cb} = \text{-}\mathbf{V}_{bc} = 208\angle 60°$

$$P_2 = (208)(30.64)\cos(60° - 101.03°) = \underline{\mathbf{4808\ W}}$$

(b)

$$P_T = P_1 + P_2 = 7398.17\text{ W}$$

$$Q_T = \sqrt{3}(P_2 - P_1) = 3840.25\text{ VAR}$$

$$\mathbf{S}_T = P_T + jQ_T = 7398.17 + j3840.25\text{ VA}$$

$$S_T = |\mathbf{S}_T| = \underline{\mathbf{8335\ VA}}$$

Chapter 12, Problem 72.

If wattmeters W_1 and W_2 are properly connected respectively between lines *a* and *b* and lines *b* and *c* to measure the power absorbed by the delta-connected load in Fig. 12.44, predict their readings.

Chapter 12, Solution 72.

From Problem 12.11,

$$\mathbf{V}_{AB} = 220\angle 130° \text{ V} \quad \text{and} \quad \mathbf{I}_{aA} = 30\angle 180° \text{ A}$$

$$P_1 = (220)(30)\cos(130° - 180°) = \underline{\mathbf{4242\ W}}$$

$$\mathbf{V}_{CB} = -\mathbf{V}_{BC} = 220\angle 190°$$
$$\mathbf{I}_{cC} = 30\angle -60°$$

$$P_2 = (220)(30)\cos(190° + 60°) = \underline{\mathbf{-\ 2257\ W}}$$

Chapter 12, Problem 73.

For the circuit displayed in Fig. 12.74, find the wattmeter readings.

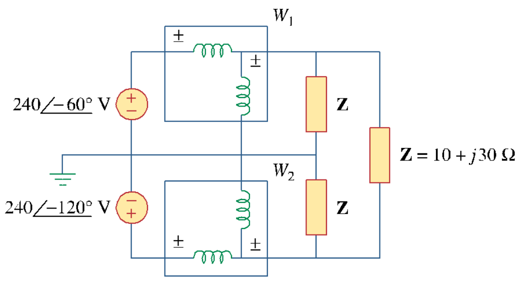
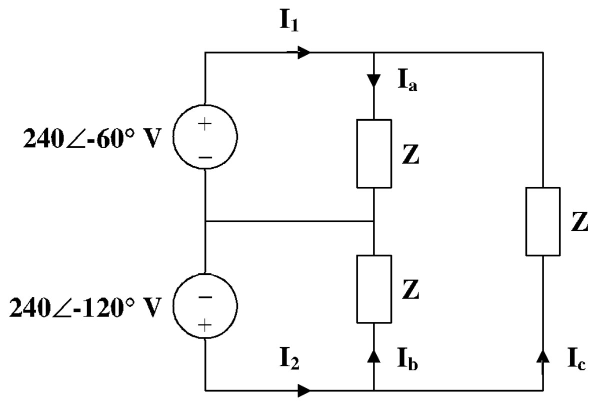


Figure 12.74
For Prob. 12.73.

Chapter 12, Solution 73.

Consider the circuit as shown below.



$$\mathbf{Z} = 10 + j30 = 31.62\angle 71.57°$$

$$\mathbf{I}_a = \frac{240\angle -60°}{31.62\angle 71.57°} = 7.59\angle -131.57°$$

$$\mathbf{I}_b = \frac{240\angle -120°}{31.62\angle 71.57°} = 7.59\angle -191.57°$$

$$\mathbf{I}_c\,\mathbf{Z} + 240\angle -60° - 240\angle -120° = 0$$

$$\mathbf{I}_c = \frac{-240}{31.62\angle 71.57°} = 7.59\angle 108.43°$$

$$\mathbf{I}_1 = \mathbf{I}_a - \mathbf{I}_c = 13.146\angle -101.57°$$

$$\mathbf{I}_2 = \mathbf{I}_b + \mathbf{I}_c = 13.146\angle 138.43°$$

$$P_1 = \text{Re}\left[\mathbf{V}_1\mathbf{I}_1^*\right] = \text{Re}\left[(240\angle -60°)(13.146\angle 101.57°)\right] = \underline{\textbf{2360 W}}$$

$$P_2 = \text{Re}\left[\mathbf{V}_2\mathbf{I}_2^*\right] = \text{Re}\left[(240\angle -120°)(13.146\angle -138.43°)\right] = \underline{\textbf{- 632.8 W}}$$

Chapter 12, Problem 74.

Predict the wattmeter readings for the circuit in Fig. 12.75.

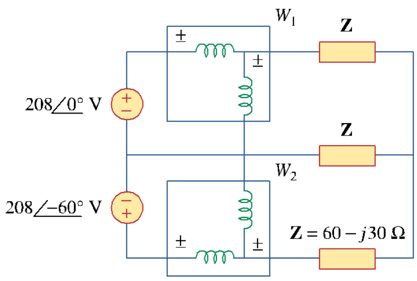
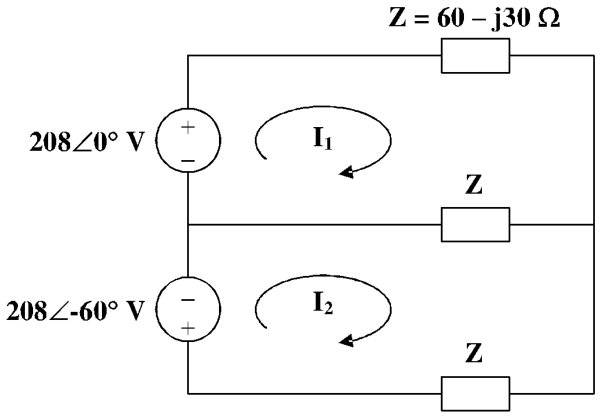


Figure 12.75
For Prob. 12.74.

Chapter 12, Solution 74.

Consider the circuit shown below.



For mesh 1,

$$208 = 2\mathbf{Z}\mathbf{I}_1 - \mathbf{Z}\mathbf{I}_2$$

For mesh 2,

$$-208\angle -60° = -\mathbf{Z}\mathbf{I}_1 + 2\mathbf{Z}\mathbf{I}_2$$

In matrix form,

$$\begin{bmatrix} 208 \\ -208\angle -60° \end{bmatrix} = \begin{bmatrix} 2\mathbf{Z} & -\mathbf{Z} \\ -\mathbf{Z} & 2\mathbf{Z} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

$$\Delta = 3\mathbf{Z}^2, \quad \Delta_1 = (208)(1.5 + j0.866)\mathbf{Z}, \quad \Delta_2 = (208)(j1.732)\mathbf{Z}$$

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \frac{(208)(1.5 + j0.866)}{(3)(60 - j30)} = 1.789\angle 56.56°$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{(208)(j1.732)}{(3)(60 - j30)} = 1.79\angle 116.56°$$

$$P_1 = \text{Re}\left[\mathbf{V}_1\mathbf{I}_1^*\right] = \text{Re}\left[(208)(1.789\angle -56.56°)\right] = \underline{\mathbf{208.98\ W}}$$

$$P_2 = \text{Re}\left[\mathbf{V}_2(-\mathbf{I}_2)^*\right] = \text{Re}\left[(208\angle -60°))(1.79\angle 63.44°)\right] = \underline{\mathbf{371.65\ W}}$$

Chapter 12, Problem 75.

A man has a body resistance of 600Ω. How much current flows through his ungrounded body:

(a) when he touches the terminals of a 12-V autobattery?
(b) when he sticks his finger into a 120-V light socket?

Chapter 12, Solution 75.

(a) $I = \frac{V}{R} = \frac{12}{600} = \underline{\mathbf{20\ mA}}$

(b) $I = \frac{V}{R} = \frac{120}{600} = \underline{\mathbf{200\ mA}}$

Chapter 12, Problem 76.

e2d Show that the I^2R losses will be higher for a 120-V appliance than for a 240-V appliance if both have the same power rating.

Chapter 12, Solution 76.

If both appliances have the same power rating, P,

$$I = \frac{P}{V_s}$$

For the 120-V appliance, $I_1 = \frac{P}{120}$.

For the 240-V appliance, $I_2 = \frac{P}{240}$.

$$\text{Power loss} = I^2 R = \begin{cases} \frac{P^2 R}{120^2} & \text{for the 120-V appliance} \\ \frac{P^2 R}{240^2} & \text{for the 240-V appliance} \end{cases}$$

Since $\frac{1}{120^2} > \frac{1}{240^2}$, **<u>the losses in the 120-V appliance are higher</u>**.

Chapter 12, Problem 77.

A three-phase generator supplied 3.6 kVA at a power factor of 0.85 lagging. If 2500 W are delivered to the load and line losses are 80 W per phase, what are the losses in the generator?

Chapter 12, Solution 77.

$$P_g = P_T - P_{load} - P_{line}, \qquad pf = 0.85$$

But $\quad P_T = 3600\cos\theta = 3600 \times pf = 3060$

$$P_g = 3060 - 2500 - (3)(80) = \underline{\mathbf{320\ W}}$$

Chapter 12, Problem 78.

A three-phase 440-V, 51-kW, 60-kVA inductive load operates at 60 Hz and is wye-connected. It is desired to correct the power factor to 0.95 lagging. What value of capacitor should be placed in parallel with each load impedance?

Chapter 12, Solution 78.

$$\cos\theta_1 = \frac{51}{60} = 0.85 \longrightarrow \theta_1 = 31.79°$$
$$Q_1 = S_1 \sin\theta_1 = (60)(0.5268) = 31.61\text{ kVAR}$$

$$P_2 = P_1 = 51\text{ kW}$$
$$\cos\theta_2 = 0.95 \longrightarrow \theta_2 = 18.19°$$
$$S_2 = \frac{P_2}{\cos\theta_2} = 53.68\text{ kVA}$$
$$Q_2 = S_2 \sin\theta_2 = 16.759\text{ kVAR}$$

$$Q_c = Q_1 - Q_2 = 3.61 - 16.759 = 14.851\text{ kVAR}$$

For each load,

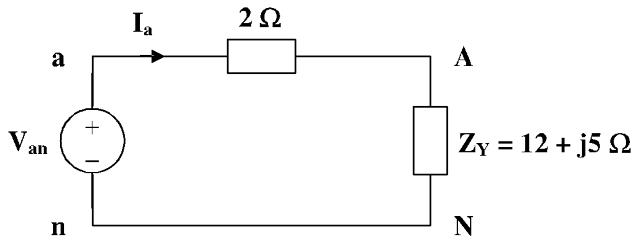
$$Q_{c1} = \frac{Q_c}{3} = 4.95\text{ kVAR}$$
$$C = \frac{Q_{c1}}{\omega V^2} = \frac{4950}{(2\pi)(60)(440)^2} = \underline{\mathbf{67.82\ \mu F}}$$

Chapter 12, Problem 79.

A balanced three-phase generator has an *abc* phase sequence with phase voltage $\mathbf{V}_{an} = 255\angle 0°$ V. The generator feeds an induction motor which may be represented by a balanced Y-connected load with an impedance of $12 + j5\,\Omega$ per phase. Find the line currents and the load voltages. Assume a line impedance of 2 Ω per phase.

Chapter 12, Solution 79.

Consider the per-phase equivalent circuit below.



$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y + 2} = \frac{255\angle 0°}{14 + j5} = \underline{\mathbf{17.15\angle -19.65°\ A}}$$

Thus,

$$\mathbf{I}_b = \mathbf{I}_a\angle -120° = \underline{\mathbf{17.15\angle -139.65°\ A}}$$
$$\mathbf{I}_c = \mathbf{I}_a\angle 120° = \underline{\mathbf{17.15\angle 100.35°\ A}}$$

$$\mathbf{V}_{AN} = \mathbf{I}_a\,\mathbf{Z}_Y = (17.15\angle -19.65°)(13\angle 22.62°) = \underline{\mathbf{223\angle 2.97°\ V}}$$

Thus,

$$\mathbf{V}_{BN} = \mathbf{V}_{AN}\angle -120° = \underline{223\angle -117.03°\ \mathrm{V}}$$
$$\mathbf{V}_{CN} = \mathbf{V}_{AN}\angle 120° = \underline{\mathbf{223\angle 122.97°\ V}}$$

Chapter 12, Problem 80.

A balanced three-phase source furnishes power to the following three loads:

Load 1: 6 kVA at 0.83 pf lagging

Load 2: unknown

Load 3: 8 kW at 0.7071 pf leading

If the line current is 84.6 A rms, the line voltage at the load is 208 V rms, and the combined load has a 0.8 pf lagging, determine the unknown load.

Chapter 12, Solution 80.

$$S = S_1 + S_2 + S_3 = 6[0.83 + j\sin(\cos^{-1} 0.83)] + S_2 + 8(0.7071 - j0.7071)$$

$$S = 10.6368 - j2.31 + S_2 \text{ kVA} \qquad (1)$$

But

$$S = \sqrt{3}V_L I_L \angle\theta = \sqrt{3}(208)(84.6)(0.8 + j0.6) \text{ VA} = 24.383 + \text{j}18.287 \text{ kVA} \qquad (2)$$

From (1) and (2),

$S_2 = 13.746 + j20.6 = 24.76\angle 56.28 \text{ kVA}$

Thus, the unknown load is <u>24.76 kVA at 0.5551 pf lagging</u>.

Chapter 12, Problem 81.

A professional center is supplied by a balanced three-phase source. The center has four balanced three-phase loads as follows:

Load 1: 150 kVA at 0.8 pf leading
Load 2: 100 kW at unity pf
Load 3: 200 kVA at 0.6 pf lagging
Load 4: 80 kW and 95 kVAR (inductive)

If the line impedance is $0.02 + j0.05\ \Omega$ per phase and the line voltage at the loads is 480 V, find the magnitude of the line voltage at the source.

Chapter 12, Solution 81.

$$\text{pf} = 0.8 \quad \text{(leading)} \longrightarrow \theta_1 = -36.87°$$
$$\mathbf{S}_1 = 150\angle -36.87° \text{ kVA}$$

$$\text{pf} = 1.0 \longrightarrow \theta_2 = 0°$$
$$\mathbf{S}_2 = 100\angle 0° \text{ kVA}$$

$$\text{pf} = 0.6 \quad \text{(lagging)} \longrightarrow \theta_3 = 53.13°$$
$$\mathbf{S}_3 = 200\angle 53.13° \text{ kVA}$$

$$\mathbf{S}_4 = 80 + j95 \text{ kVA}$$

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3 + \mathbf{S}_4$$
$$\mathbf{S} = 420 + j165 = 451.2\angle 21.45° \text{ kVA}$$

$$S = \sqrt{3}\, V_L I_L$$
$$I_L = \frac{S}{\sqrt{3}\, V_L} = \frac{451.2\times 10^3}{\sqrt{3}\times 480} = 542.7 \text{ A}$$

For the line,

$$\mathbf{S}_L = 3 I_L^2\, \mathbf{Z}_L = (3)(542.7)^2 (0.02 + j0.05)$$
$$\mathbf{S}_L = 17.67 + j44.18 \text{ kVA}$$

At the source,

$$\mathbf{S}_T = \mathbf{S} + \mathbf{S}_L = 437.7 + j209.2$$
$$\mathbf{S}_T = 485.1\angle 25.55° \text{ kVA}$$

$$V_T = \frac{S_T}{\sqrt{3}\, I_L} = \frac{485.1\times 10^3}{\sqrt{3}\times 542.7} = \underline{\mathbf{516\ V}}$$

Chapter 12, Problem 82.

A balanced three-phase system has a distribution wire with impedance $2 + j6\,\Omega$ per phase. The system supplies two three-phase loads that are connected in parallel. The first is a balanced wye-connected load that absorbs 400 kVA at a power factor of 0.8 lagging. The second load is a balanced delta-connected load with impedance of $10 + j8\,\Omega$ per phase. If the magnitude of the line voltage at the loads is 2400 V rms, calculate the magnitude of the line voltage at the source and the total complex power supplied to the two loads.

Chapter 12, Solution 82.

$$\overline{S}_1 = 400(0.8 + j0.6) = 320 + j240\,\text{kVA}, \qquad \overline{S}_2 = 3\frac{V_p^2}{Z_p^*}$$

For the delta-connected load, $V_L = V_p$

$$\overline{S}_2 = 3x\frac{(2400)^2}{10 - j8} = 1053.7 + j842.93\,\text{kVA}$$

$$\overline{S} = \overline{S}_1 + \overline{S}_2 = 1.3737 + j1.0829\,\text{MVA}$$

Let $I = I_1 + I_2$ be the total line current. For I_1,

$$S_1 = 3V_p I_1^*, \qquad V_p = \frac{V_L}{\sqrt{3}}$$

$$I_1^* = \frac{S_1}{\sqrt{3}V_L} = \frac{(320 + j240)x10^3}{\sqrt{3}(2400)}, \qquad I_1 = 76.98 - j57.735$$

For I_2, convert the load to wye.

$$I_2 = I_p\sqrt{3}\angle -30^o = \frac{2400}{10 + j8}\sqrt{3}\angle -30^o = 273.1 - j289.76$$

$$I = I_1 + I_2 = 350 - j347.5$$

$$V_s = V_L + V_{line} = 2400 + I(3 + j6) = 5.185 + j1.405\,\text{kV} \longrightarrow |V_s| = \underline{5.372\,\text{kV}}$$

Chapter 12, Problem 83.

A commercially available three-phase inductive motor operates at a full load of 120 hp (1 hp = 746 W) at 95 percent efficiency at a lagging power factor of 0.707. The motor is connected in parallel to a 80-kW balanced three-phase heater at unity power factor. If the magnitude of the line voltage is 480 V rms, calculate the line current.

Chapter 12, Solution 83.

$$S_1 = 120x746x0.95(0.707 + j0.707) = 60.135 + j60.135\,\text{kVA}, \qquad \text{S}_2 = 80\,\text{kVA}$$

$$S = S_1 + S_2 = 140.135 + j60.135\,\text{kVA}$$

But $|S| = \sqrt{3}V_L I_L \quad \longrightarrow \quad I_L = \dfrac{|S|}{\sqrt{3}V_L} = \dfrac{152.49x10^3}{\sqrt{3}x480} = \underline{183.42\text{ A}}$

Chapter 12, Problem 84.

* Figure 12.76 displays a three-phase delta-connected motor load which is connected to a line voltage of 440 V and draws 4 kVA at a power factor of 72 percent lagging. In addition, a single 1.8 kVAR capacitor is connected between lines *a* and *b*, while a 800-W lighting load is connected between line *c* and neutral. Assuming the *abc* sequence and taking $\mathbf{V}_{an} = V_p\angle 0°$, find the magnitude and phase angle of currents $\mathbf{I}_a$, $\mathbf{I}_b$, $\mathbf{I}_c$, and $\mathbf{I}_n$.

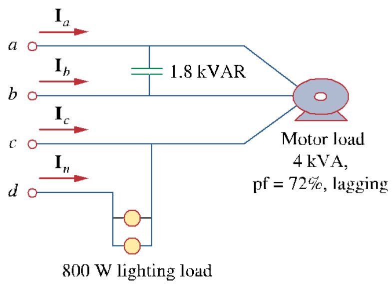


Figure 12.76
For Prob. 12.84.

* An asterisk indicates a challenging problem.

Chapter 12, Solution 84.

We first find the magnitude of the various currents.

For the motor,

$$I_L = \frac{S}{\sqrt{3}\,V_L} = \frac{4000}{440\sqrt{3}} = 5.248 \text{ A}$$

For the capacitor,

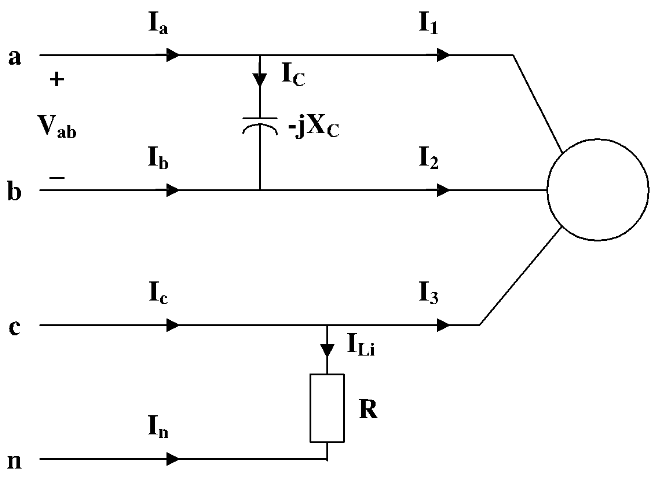
$$I_C = \frac{Q_c}{V_L} = \frac{1800}{440} = 4.091 \text{ A}$$

For the lighting,

$$V_p = \frac{440}{\sqrt{3}} = 254 \text{ V}$$

$$I_{Li} = \frac{P_{Li}}{V_p} = \frac{800}{254} = 3.15 \text{ A}$$

Consider the figure below.



If $\mathbf{V}_{an} = V_p\angle 0°$,

$$\mathbf{V}_{ab} = \sqrt{3}\,V_p\angle 30°$$
$$\mathbf{V}_{cn} = V_p\angle 120°$$

$$\mathbf{I}_C = \frac{\mathbf{V}_{ab}}{-jX_C} = 4.091\angle 120°$$

$$\mathbf{I}_1 = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_\Delta} = 4.091\angle(\theta + 30°)$$

where $\theta = \cos^{-1}(0.72) = 43.95°$

$$\mathbf{I}_1 = 5.249\angle 73.95°$$

$$\mathbf{I}_2 = 5.249\angle -46.05°$$

$$\mathbf{I}_3 = 5.249\angle 193.95°$$

$$\mathbf{I}_{Li} = \frac{\mathbf{V}_{cn}}{R} = 3.15\angle 120°$$

Thus,

$$\mathbf{I}_a = \mathbf{I}_1 + \mathbf{I}_C = 5.249\angle 73.95° + 4.091\angle 120°$$
$$\mathbf{I}_a = \underline{\mathbf{8.608\angle 93.96°\ A}}$$

$$\mathbf{I}_b = \mathbf{I}_2 - \mathbf{I}_C = 5.249\angle -46.05° - 4.091\angle 120°$$
$$\mathbf{I}_b = \underline{\mathbf{9.271\angle -52.16°\ A}}$$

$$\mathbf{I}_c = \mathbf{I}_3 + \mathbf{I}_{Li} = 5.249\angle 193.95° + 3.15\angle 120°$$
$$\mathbf{I}_c = \underline{\mathbf{6.827\angle 167.6°\ A}}$$

$$\mathbf{I}_n = -\mathbf{I}_{Li} = \underline{\mathbf{3.15\angle -60°\ A}}$$

Chapter 12, Problem 85.

e2d Design a three-phase heater with suitable symmetric loads using wye-connected pure resistance. Assume that the heater is supplied by a 240-V line voltage and is to give 27 kW of heat.

Chapter 12, Solution 85.

Let $Z_Y = R$

$$V_p = \frac{V_L}{\sqrt{3}} = \frac{240}{\sqrt{3}} = 138.56 \text{ V}$$

$$P = V_p I_p = \frac{27}{2} = 9 \text{ kW} = \frac{V_p^2}{R}$$

$$R = \frac{V_p^2}{P} = \frac{(138.56)^2}{9000} = 2.133\,\Omega$$

Thus, $Z_Y = \underline{\mathbf{2.133\,\Omega}}$

Chapter 12, Problem 86.

For the single-phase three-wire system in Fig. 12.77, find currents $\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$, and $\mathbf{I}_{nN}$.

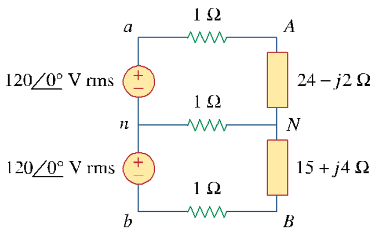
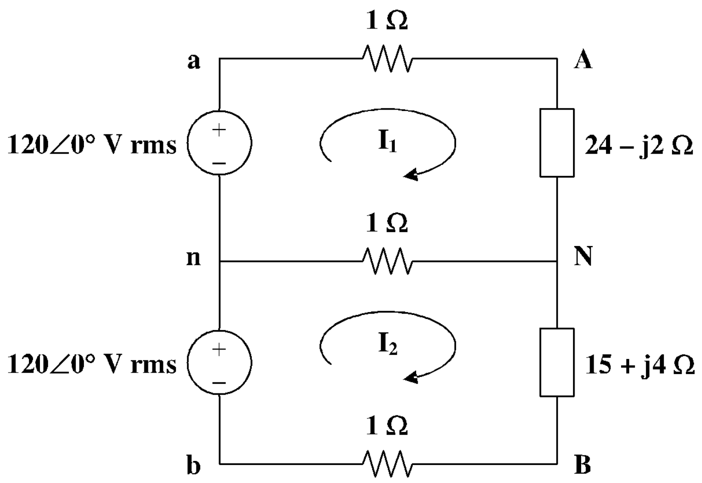


Figure 12.77
For Prob. 12.86.

Chapter 12, Solution 86.

Consider the circuit shown below.



For the two meshes,

$$120 = (26 - j2)\mathbf{I}_1 - \mathbf{I}_2 \qquad (1)$$

$$120 = (17 + j4)\mathbf{I}_2 - \mathbf{I}_1 \qquad (2)$$

In matrix form,

$$\begin{bmatrix} 120 \\ 120 \end{bmatrix} = \begin{bmatrix} 26 - j2 & -1 \\ -1 & 17 + j4 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

$$\Delta = 449 + j70, \quad \Delta_1 = (120)(18 + j4), \quad \Delta_2 = (120)(27 - j2)$$

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \frac{120 \times 18.44\angle 12.53°}{454.42\angle 8.86°} = 4.87\angle 3.67°$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{120 \times 27.07\angle -4.24°}{454.42\angle 8.86°} = 7.15\angle -13.1°$$

$$\mathbf{I}_{aA} = \mathbf{I}_1 = \mathbf{4.87\angle 3.67°\ A}$$

$$\mathbf{I}_{bB} = -\mathbf{I}_2 = \mathbf{7.15\angle 166.9°\ A}$$

$$\mathbf{I}_{nN} = \mathbf{I}_2 - \mathbf{I}_1 = \frac{\Delta_2 - \Delta_1}{\Delta}$$

$$\mathbf{I}_{nN} = \frac{(120)(9 - j6)}{449 + j70} = \mathbf{2.856\angle -42.55°\ A}$$

Chapter 12, Problem 87.

Consider the single-phase three-wire system shown in Fig. 12.78. Find the current in the neutral wire and the complex power supplied by each source. Take $\mathbf{V}_s$ as a $115\angle 0°$-V, 60-Hz source.

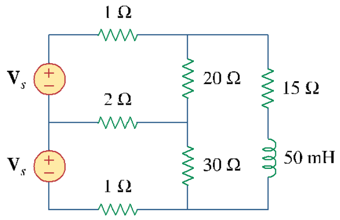
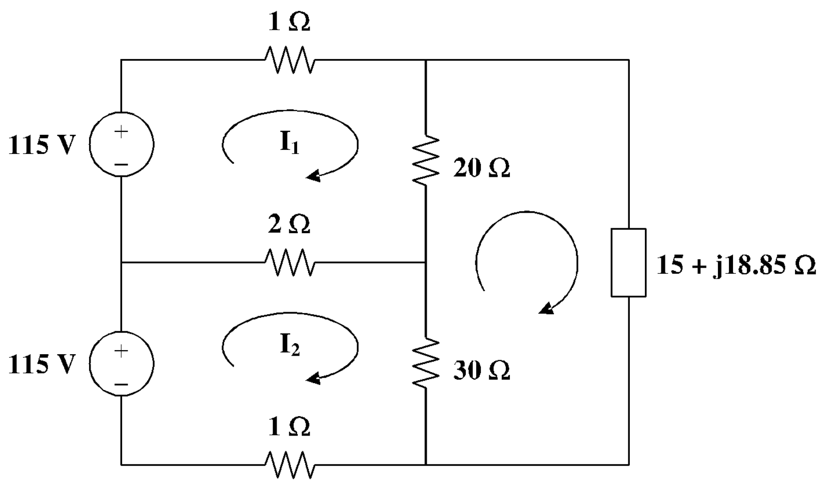


Figure 12.78
For Prob. 12.87.

Chapter 12, Solution 87.

$$L = 50 \text{ mH} \longrightarrow j\omega L = j(2\pi)(60)(5010^{-3}) = j18.85$$

Consider the circuit below.



Applying KVl to the three meshes, we obtain

$$23\mathbf{I}_1 - 2\mathbf{I}_2 - 20\mathbf{I}_3 = 115 \qquad (1)$$

$$-2\mathbf{I}_1 + 33\mathbf{I}_2 - 30\mathbf{I}_3 = 115 \qquad (2)$$

$$-20\mathbf{I}_1 - 30\mathbf{I}_2 + (65 + j18.85)\mathbf{I}_3 = 0 \qquad (3)$$

In matrix form,

$$\begin{bmatrix} 23 & -2 & -20 \\ -2 & 33 & -30 \\ -20 & -30 & 65 + j18.85 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 115 \\ 115 \\ 0 \end{bmatrix}$$

$\Delta = 12{,}775 + j14{,}232$, $\quad \Delta_1 = (115)(1975 + j659.8)$

$\Delta_2 = (115)(1825 + j471.3)$, $\quad \Delta_3 = (115)(1450)$

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \frac{115 \times 2082\angle 18.47°}{19214\angle 48.09°} = 12.52\angle -29.62°$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{115 \times 1884.9\angle 14.48°}{19124\angle 48.09°} = 11.33\angle -33.61°$$

$$\mathbf{I}_n = \mathbf{I}_2 - \mathbf{I}_1 = \frac{\Delta_2 - \Delta_1}{\Delta} = \frac{(115)(-150 - j188.5)}{12{,}775 + j14{,}231.75} = \underline{\mathbf{1.448\angle -176.6°\ A}}$$

$$\mathbf{S}_1 = \mathbf{V}_1\mathbf{I}_1^* = (115)(12.52\angle 29.62°) = \underline{\mathbf{1252 + j711.6\ VA}}$$

$$\mathbf{S}_2 = \mathbf{V}_2\mathbf{I}_2^* = (115)(1.33\angle 33.61°) = \underline{\mathbf{1085 + j721.2\ VA}}$$

Chapter 13, Problem 1.

For the three coupled coils in Fig. 13.72, calculate the total inductance.

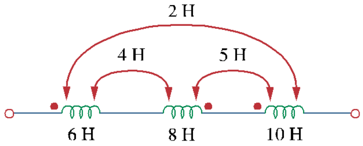


Figure 13.72
For Prob. 13.1.

Chapter 13, Solution 1.

For coil 1, $L_1 - M_{12} + M_{13} = 6 - 4 + 2 = 4$

For coil 2, $L_2 - M_{21} - M_{23} = 8 - 4 - 5 = -1$

For coil 3, $L_3 + M_{31} - M_{32} = 10 + 2 - 5 = 7$

$L_T = 4 - 1 + 7 = 10H$

or $L_T = L_1 + L_2 + L_3 - 2M_{12} - 2M_{23} + 2M_{12}$

$L_T = 6 + 8 + 10 =$ **10H**

Chapter 13, Problem 2.

Determine the inductance of the three series-connected inductors of Fig. 13.73.

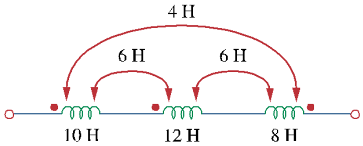


Figure 13.73
For Prob. 13.2.

Chapter 13, Solution 2.

$L = L_1 + L_2 + L_3 + 2M_{12} - 2M_{23} - 2M_{31}$

$= 10 + 12 + 8 + 2x6 - 2x6 - 2x4$

= **22H**

Chapter 13, Problem 3.

Two coils connected in series-aiding fashion have a total inductance of 250 mH. When connected in a series-opposing configuration, the coils have a total inductance of 150 mH. If the inductance of one coil (L_1) is three times the other, find L_1, L_2, and M. What is the coupling coefficient?

Chapter 13, Solution 3.

$L_1 + L_2 + 2M = 250 \text{ mH}$ (1)

$L_1 + L_2 - 2M = 150 \text{ mH}$ (2)

Adding (1) and (2),

$$2L_1 + 2L_2 = 400 \text{ mH}$$

But, $L_1 = 3L_2$, or $8L_2 + 400$, and $L_2 = \underline{\mathbf{50\ mH}}$

$$L_1 = 3L_2 = \underline{\mathbf{150\ mH}}$$

From (2), $150 + 50 - 2M = 150$ leads to $M = \underline{\mathbf{25\ mH}}$

$$k = M/\sqrt{L_1 L_2} = 25/\sqrt{50\text{x}150} = \underline{\mathbf{0.2887}}$$

Chapter 13, Problem 4.

(a) For the coupled coils in Fig. 13.74(a), show that

$$L_{eq} = L_1 + L_2 + 2M$$

(b) For the coupled coils in Fig. 13.74(b), show that

$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

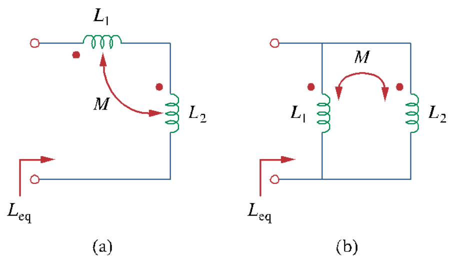
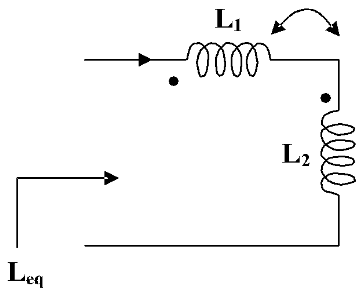


Figure 13.74
For Prob. 13.4.

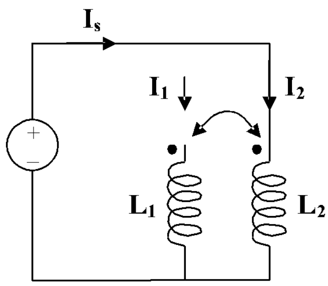
Chapter 13, Solution 4.

(a) For the series connection shown in Figure (a), the current I enters each coil from its dotted terminal. Therefore, the mutually induced voltages have the same sign as the self-induced voltages. Thus,

$$L_{eq} = \underline{\mathbf{L_1 + L_2 + 2M}}$$



(a)



(b)

(b) For the parallel coil, consider Figure (b).

$$I_s = I_1 + I_2 \quad \text{and} \quad Z_{eq} = V_s/I_s$$

Applying KVL to each branch gives,

$$V_s = j\omega L_1 I_1 + j\omega M I_2 \qquad (1)$$

$$V_s = j\omega M I_1 + j\omega L_2 I_2 \qquad (2)$$

or

$$\begin{bmatrix} V_s \\ V_s \end{bmatrix} = \begin{bmatrix} j\omega L_1 & j\omega M \\ j\omega M & j\omega L_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta = -\omega^2 L_1 L_2 + \omega^2 M^2, \quad \Delta_1 = j\omega V_s(L_2 - M), \quad \Delta_2 = j\omega V_s(L_1 - M)$$

$$I_1 = \Delta_1/\Delta, \text{ and } I_2 = \Delta_2/\Delta$$

$$I_s = I_1 + I_2 = (\Delta_1 + \Delta_2)/\Delta = j\omega(L_1 + L_2 - 2M)V_s/(-\omega^2(L_1L_2 - M^2))$$
$$= (L_1 + L_2 - 2M)V_s/(j\omega(L_1L_2 - M^2))$$

$$Z_{eq} = V_s/I_s = j\omega(L_1L_2 - M^2)/(L_1 + L_2 - 2M) = j\omega L_{eq}$$

i.e., $L_{eq} = \underline{\mathbf{(L_1L_2 - M^2)/(L_1 + L_2 - 2M)}}$

Chapter 13, Problem 5.

Two coils are mutually coupled, with L_1 = 25 mH, L_2 = 60 mH, and k = 0.5. Calculate the maximum possible equivalent inductance if:

(a) the two coils are connected in series

(b) the coils are connected in parallel

Chapter 13, Solution 5.

(a) If the coils are connected in series,

$$L = L_1 + L_2 + 2M = 25 + 60 + 2(0.5)\sqrt{25x60} = \underline{\mathbf{123.7\ mH}}$$

(b) If they are connected in parallel,

$$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} = \frac{25x60 - 19.36^2}{25 + 60 - 2x19.36}\,mH = \underline{\mathbf{24.31\ mH}}$$

Chapter 13, Problem 6.

The coils in Fig. 13.75 have $L_1 = 40$ mH, $L_2 = 5$ mH, and coupling coefficient $k = 0.6$. Find $i_1\,(t)$ and $v_2(t)$, given that $v_1(t) = 10 \cos\, \omega t$ and $i_2(t) = 2 \sin\, \omega t$, $\omega\, = 2000$ rad/s.

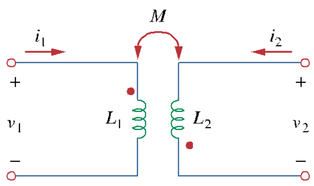


Figure 13.75
For Prob. 13.6.

Chapter 13, Solution 6.

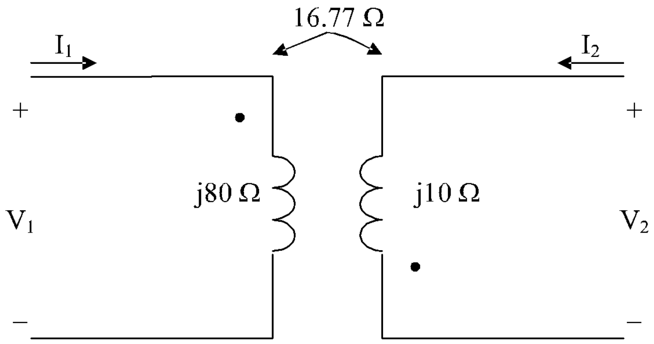
$M = k\sqrt{L_1 L_2} = 0.6\sqrt{40x5} = 8.4853$ mH

$40mH \longrightarrow \quad j\omega L = j2000x40x10^{-3} = j80$

$5mH \longrightarrow \quad j\omega L = j2000x5x10^{-3} = j10$

$8.4853mH \longrightarrow \quad j\omega M = j2000x8.4853x10^{-3} = j16.97$

We analyze the circuit below.



$$V_1 = j80I_1 - j16.97I_2 \qquad (1)$$
$$V_2 = -16.97I_1 + j10I_2 \qquad (2)$$

But $V_1 = 10 < 0^o$ and $I_2 = 2 < -90^o = -j2$. Substituting these in eq.(1) gives

$$I_1 = \frac{V_1 + j16.97I_2}{j80} = \frac{10 + j16.97x(-j2)}{j80} = 0.5493 < -90^o$$

$i_1(t) = \underline{0.5493\sin\omega t \text{ A}}$

From (2),

$$V_2 = -16.97x(-0.j5493) + j10x(-j2) = 20 + j9.3216 = 22.0656 < 24.99^o$$

$v_2(t) = \underline{22.065\cos(\omega t + 25^o) \text{ V}}$

Chapter 13, Problem 7.

For the circuit in Fig. 13.76, find V_o.

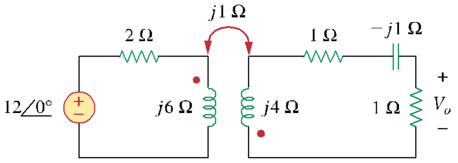
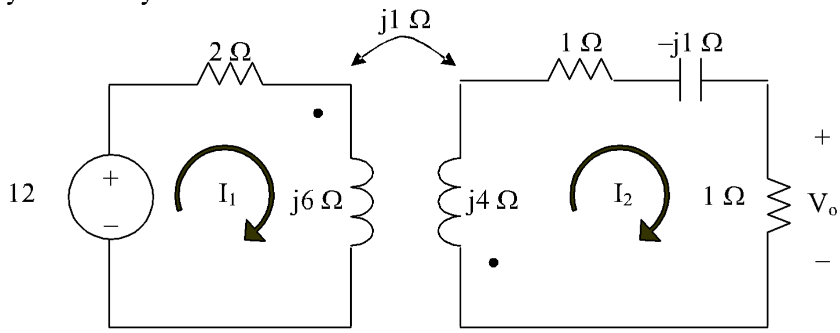


Figure 13.76
For Prob. 13.7.

Chapter 13, Solution 7.

We apply mesh analysis to the circuit as shown below.



For mesh 1,

$$12 = I_1(2 + j6) + jI_2 \qquad (1)$$

For mesh 2,

$$0 = jI_1 + (2 - j1 + j4)I_2$$

or

$$0 = jI_1 + (2 + j3)I_2 \qquad (2)$$

In matrix form,

$$\begin{bmatrix} 12 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 + j6 & j \\ j & 2 + j3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$I_2 = -0.4381 + j0.3164$$

$V_o = I_2 x1 =$ **540.5∠144.16° mV**.

Chapter 13, Problem 8.

PS ML Find $v(t)$ for the circuit in Fig. 13.77.

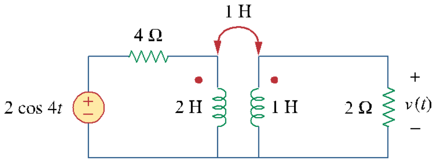


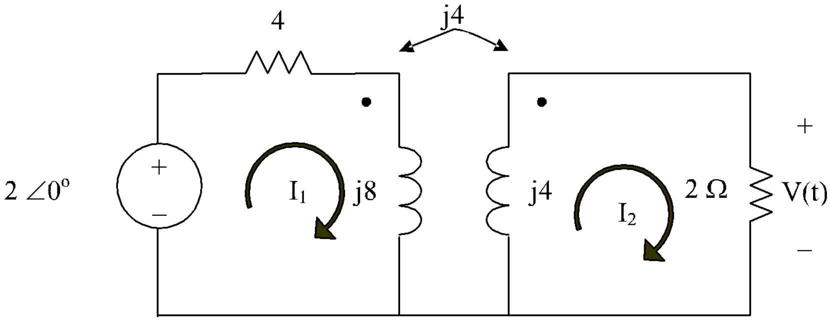
Figure 13.77
For Prob. 13.8.

Chapter 13, Solution 8.

$2H \quad \longrightarrow \quad j\omega L = j4x2 = j8$

$1H \quad \longrightarrow \quad j\omega L = j4x1 = j4$

Consider the circuit below.



$2 = (4 + j8)I_1 - j4I_2 \qquad (1)$

$0 = -j4I_1 + (2 + j4)I_2 \qquad (2)$

In matrix form, these equations become

$$\begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 4+j8 & -j4 \\ -j4 & 2+j4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Solving this leads to

$I_2 = 0.2353 - j0.0588$

$V = 2I_2 = 0.4851 \angle -14.04^\circ$

Thus,

$$\underline{v(t) = 0.4851\cos(4t - 14.04^\circ)\text{ V}}$$

Chapter 13, Problem 9.

Find $\mathbf{V}_x$ in the network shown in Fig. 13.78.

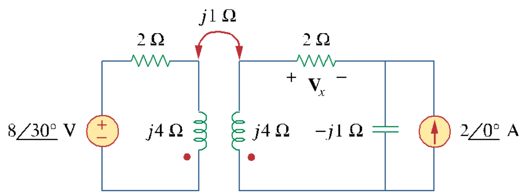
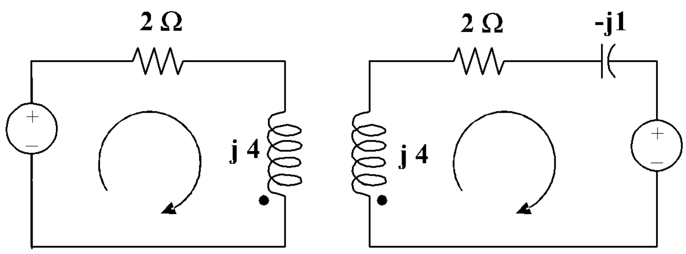


Figure 13.78
For Prob. 13.9.

Chapter 13, Solution 9.

Consider the circuit below.



For loop 1,

$$8\angle 30° = (2 + j4)I_1 - jI_2 \qquad (1)$$

For loop 2,

$$((j4 + 2 - j)I_2 - jI_1 + (-j2) = 0$$

or

$$I_1 = (3 - j2)i_2 - 2 \qquad (2)$$

Substituting (2) into (1),

$$8\angle 30° + (2 + j4)2 = (14 + j7)I_2$$

$$I_2 = (10.928 + j12)/(14 + j7) = 1.037\angle 21.12°$$

$$V_x = 2I_2 = \underline{\mathbf{2.074\angle 21.12°}}$$

Chapter 13, Problem 10.

Find v_o in the circuit of Fig. 13.79.

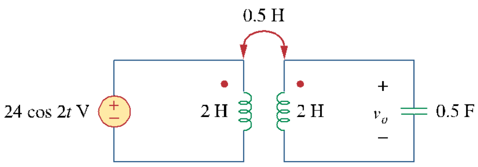


Figure 13.79
For Prob. 13.10.

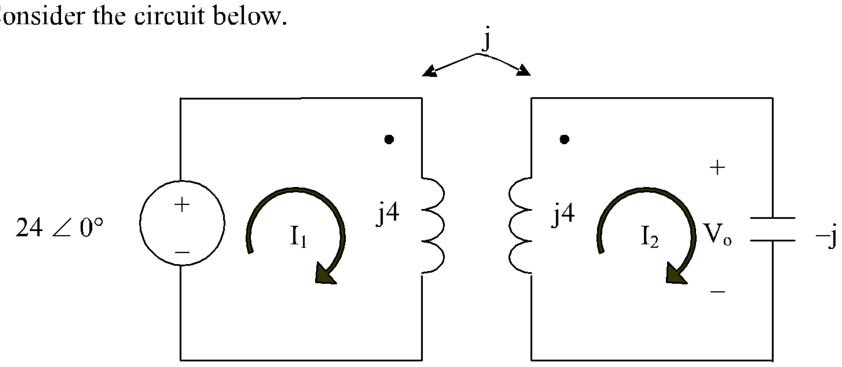
Chapter 13, Solution 10.

$$2H \longrightarrow j\omega L = j2x2 = j4$$

$$0.5H \longrightarrow j\omega L = j2x0.5 = j$$

$$\frac{1}{2}F \longrightarrow 1/j\omega C = \frac{1}{j2x1/2} = -j$$

Consider the circuit below.



$$24 = j4I_1 - jI_2 \qquad (1)$$

$$0 = -jI_1 + (j4 - j)I_2 \longrightarrow 0 = -I_1 + 3I_2 \qquad (2)$$

In matrix form,

$$\begin{bmatrix} 24 \\ 0 \end{bmatrix} = \begin{bmatrix} j4 & -j \\ -1 & 3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Solving this,

$$I_2 = -j2.1818, \qquad V_o = -jI_2 = -2.1818$$

v_o = **−2.1818cos2t V**

Chapter 13, Problem 11.

ML Use mesh analysis to find i_x in Fig. 13.80, where
$i_s = 4\cos(600t)$ A and $v_s = 110\cos(600t + 30^o)$

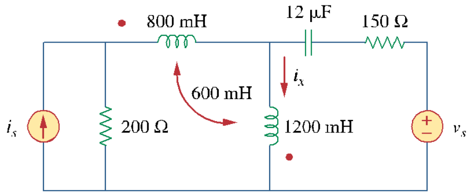


Figure 13.80
For Prob. 13.11.

Chapter 13, Solution 11.

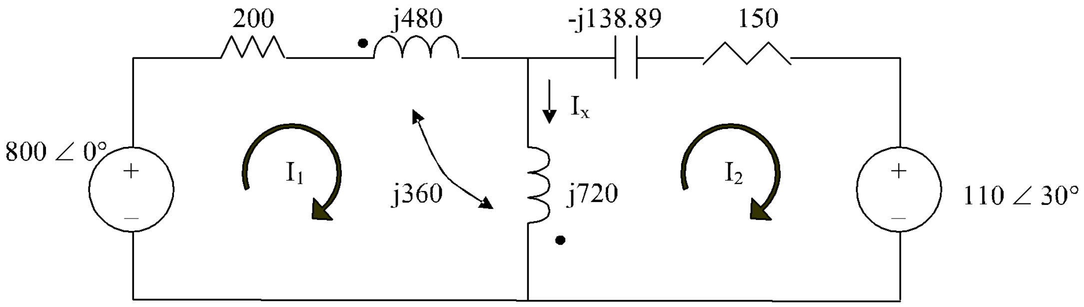
$$800mH \longrightarrow j\omega L = j600x800x10^{-3} = j480$$

$$600mH \longrightarrow j\omega L = j600x600x10^{-3} = j360$$

$$1200mH \longrightarrow j\omega L = j600x1200x10^{-3} = j720$$

$$12\mu F \rightarrow \frac{1}{j\omega C} = \frac{-j}{600x12x10^{-6}} = -j138.89$$

After transforming the current source to a voltage source, we get the circuit shown below.



For mesh 1,

$$800 = (200 + j480 + j720)I_1 + j360I_2 - j720I_2$$

or

$$800 = (200 + j1200)I_1 - j360I_2 \qquad (1)$$

For mesh 2,

$$110\angle 30^\circ + 150 - j138.89 + j720)I_2 + j360I_1 = 0$$

or

$$-95.2628 - j55 = -j360I_1 + (150 + j581.1)I_2 \qquad (2)$$

In matrix form,

$$\begin{bmatrix} 800 \\ -95.2628 - j55 \end{bmatrix} = \begin{bmatrix} 200 + j1200 & -j360 \\ -j360 & 150 + j581.1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Solving this using MATLAB leads to:

```
>> Z = [(200+1200i),-360i;-360i,(150+581.1i)]
Z =
  1.0e+003 *
   0.2000 + 1.2000i       0 - 0.3600i
        0 - 0.3600i   0.1500 + 0.5811i
>> V = [800;(-95.26-55i)]
V =
  1.0e+002 *
   8.0000
  -0.9526 - 0.5500i
>> I = inv(Z)*V
I =
   0.1390 - 0.7242i
   0.0609 - 0.2690i
```

$$I_x = I_1 - I_2 = 0.0781 - j0.4552 = 0.4619\angle -80.26^\circ.$$

Hence, i_x = **461.9cos(600t–80.26°) mA**.

Chapter 13, Problem 12.

Determine the equivalent L_{eq} in the circuit of Fig. 13.81.

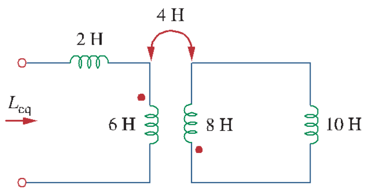
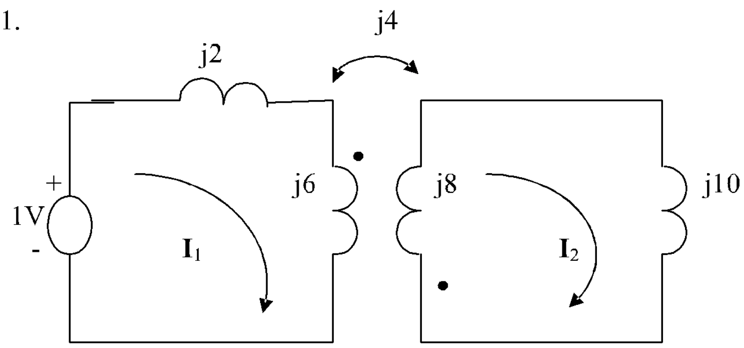


Figure 13.81
For Prob. 13.12.

Chapter 13, Solution 12.

Let $\omega = 1$.



Applying KVL to the loops,

$$1 = j8I_1 + j4I_2 \qquad (1)$$

$$0 = j4I_1 + j18I_2 \qquad (2)$$

Solving (1) and (2) gives $\mathbf{I}_1$ = -j0.1406. Thus

$$Z = \frac{1}{I_1} = jL_{eq} \longrightarrow L_{eq} = \frac{1}{jI_1} = \underline{7.111 \text{ H}}$$

We can also use the equivalent T-section for the transform to find the equivalent inductance.

Chapter 13, Problem 13.

For the circuit in Fig. 13.82, determine the impedance seen by the source.

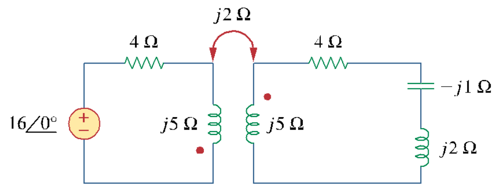


Figure 13.82
For Prob. 13.13.

Chapter 13, Solution 13.

$$Z_{in} = 4 + j(2+5) + \frac{4}{j5 + 4 - j + j2} = 4 + j7 + \frac{4}{4 + j6} = \underline{\mathbf{4.308+j6.538\ \Omega}}.$$

Chapter 13, Problem 14.

Obtain the Thevenin equivalent circuit for the circuit in Fig. 13.83 at terminals *a-b*.

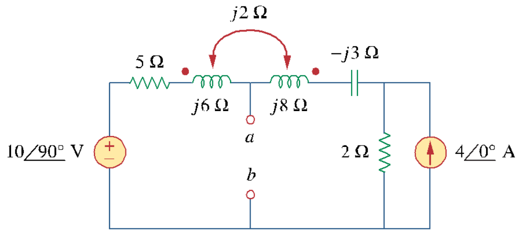
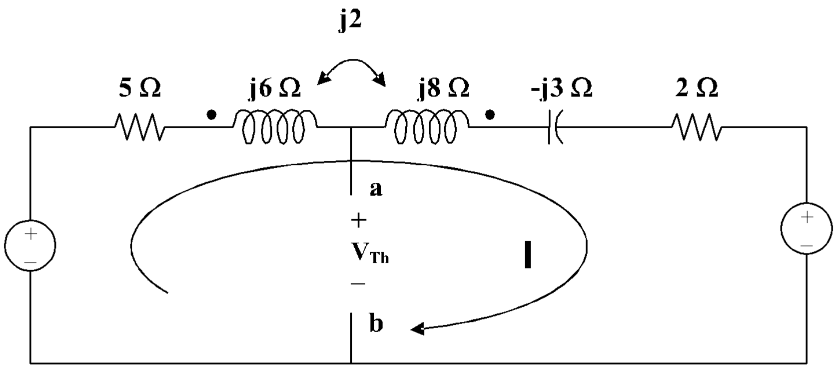


Figure 13.83
For Prob. 13.14.

Chapter 13, Solution 14.

To obtain V_{Th}, convert the current source to a voltage source as shown below.



Note that the two coils are connected series aiding.

$$\omega L = \omega L_1 + \omega L_2 - 2\omega M$$

$$j\omega L = j6 + j8 - j4 = j10$$

Thus,
$$-j10 + (5 + j10 - j3 + 2)I + 8 = 0$$

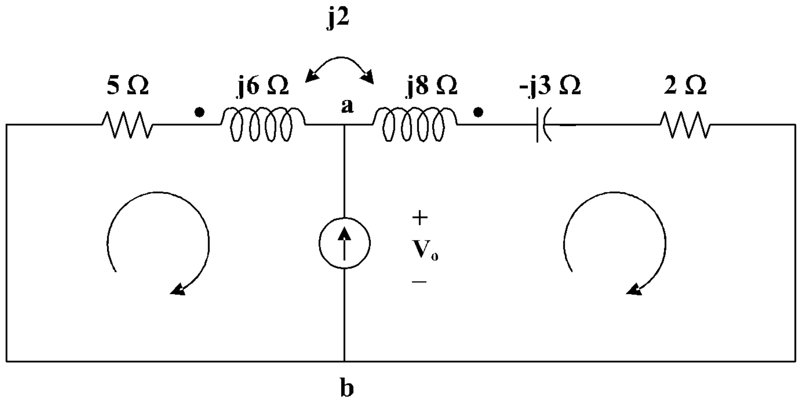
$$I = (-8 + j10)/(7 + j7)$$

But,
$$-j10 + (5 + j6)I - j2I + V_{Th} = 0$$

$$V_{Th} = j10 - (5 + j4)I = j10 - (5 + j4)(-8 + j10)/(7 + j7)$$

$$V_{Th} = \underline{5.349\angle 34.11^\circ}$$

To obtain Z_{Th}, we set all the sources to zero and insert a 1-A current source at the terminals a–b as shown below.



Clearly, we now have only a super mesh to analyze.

$$(5 + j6)I_1 - j2I_2 + (2 + j8 - j3)I_2 - j2I_1 = 0$$

$$(5 + j4)I_1 + (2 + j3)I_2 = 0 \qquad (1)$$

But, $$I_2 - I_1 = 1 \text{ or } I_2 = I_1 - 1 \qquad (2)$$

Substituting (2) into (1), $$(5 + j4)I_1 + (2 + j3)(1 + I_1) = 0$$

$$I_1 = -(2 + j3)/(7 + j7)$$

Now, $$((5 + j6)I_1 - j2I_1 + V_o = 0$$

$$V_o = -(5 + j4)I_1 = (5 + j4)(2 + j3)/(7 + j7) = (-2 + j23)/(7 + j7) = 2.332\angle 50°$$

$$Z_{Th} = V_o/1 = \underline{\mathbf{2.332\angle 50° \text{ ohms}}}$$

Chapter 13, Problem 15.

Find the Norton equivalent for the circuit in Fig. 13.84 at terminals *a*-*b*.

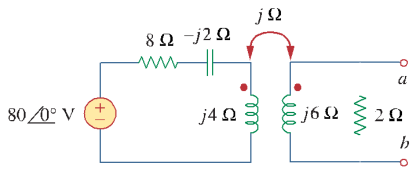
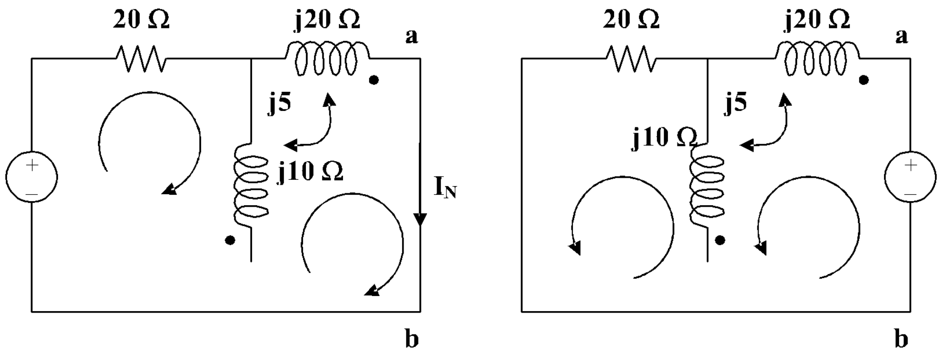


Figure 13.84
For Prob. 13.15.

Chapter 13, Solution 15.

To obtain I_N, short-circuit a–b as shown in Figure (a).



(a) (b)

For mesh 1,

$$60\angle 30° = (20 + j10)I_1 + j5I_2 - j10I_2$$

or $$12\angle 30° = (4 + j2)I_1 - jI_2 \qquad (1)$$

For mesh 2,

$$0 = (j20 + j10)I_2 + j5I_1 - j10I_1$$

or $$I_1 = 6I_2 \qquad (2)$$

Substituting (2) into (1), $$12\angle 30° = (24 + j11)I_2$$

$$I_N = I_2 = 12\angle 30°/(24 + j11) = \underline{\mathbf{1.404\angle 9.44°\ A}}$$

To find Z_N, we set all the sources to zero and insert a 1-volt voltage source at the a–b terminals as shown in Figure (b).

For mesh 1, $$1 = I_1(j10 + j20 - j5x2) + j5I_2 - j10I_2$$

$$1 = j20I_1 - j5I_2 \qquad (3)$$

For mesh 2, $$0 = (20 + j10)I_2 + j5I_1 - j10I_1 \text{ or } (4 + j2)I_2 - jI_1 = 0$$

or $$I_2 = jI_1/(4 + j2) \qquad (4)$$

Substituting (4) into (3), $$1 = j20I_1 - j(j5)I_1/(4 + j2) = (1 + j19.5)I_1$$

$$I_1 = 1/(-1 + j20.5)$$

$$Z_N = 1/I_1 = \underline{\mathbf{(1 + j19.5)\ ohms}}$$

Chapter 13, Problem 16.

Obtain the Norton equivalent at terminals *a*-*b* of the circuit in Fig. 13.85.

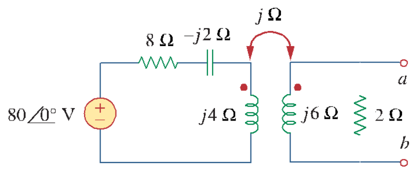
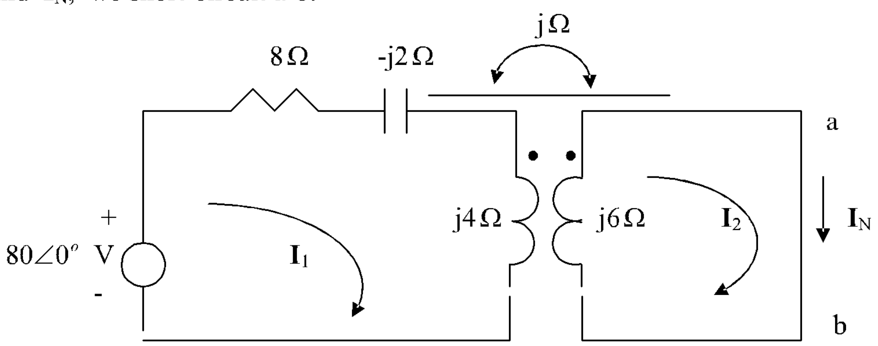


Figure 13.85
For Prob. 13.16.

Chapter 13, Solution 16.

To find $\mathbf{I}_N$, we short-circuit a-b.

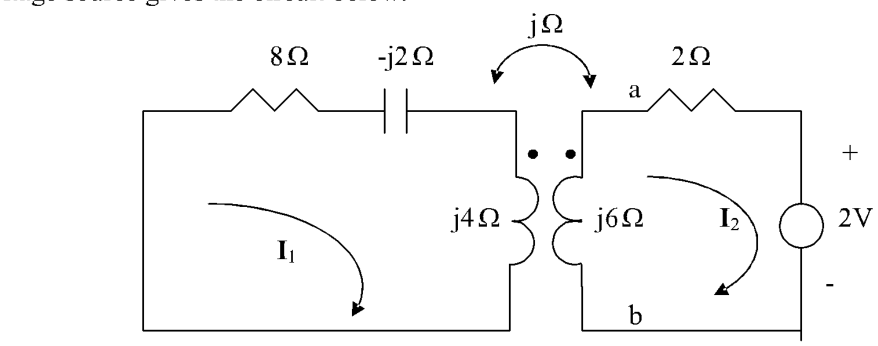


$$-80+(8-j2+j4)I_1-jI_2=0 \longrightarrow (8+j2)I_1-jI_2=80 \qquad (1)$$
$$j6I_2-jI_1=0 \longrightarrow I_1=6I_2 \qquad (2)$$

Solving (1) and (2) leads to

$$I_N=I_2=\frac{80}{48+j11}=1.584-j0.362=\underline{1.6246\angle-12.91^o\ \text{A}}$$

To find $\mathbf{Z}_N$, insert a 1-A current source at terminals a-b. Transforming the current source to voltage source gives the circuit below.



$$0=(8+j2)I_1-jI_2 \longrightarrow I_1=\frac{jI_2}{8+j2} \qquad (3)$$
$$2+(2+j6)I_2-jI_1=0 \qquad (4)$$

Solving (3) and (4) leads to $\mathbf{I}_2$ = -0.1055 +j0.2975, $\mathbf{V}_{ab}$=-j6$\mathbf{I}_2$ = 1.7853 +0.6332

$$Z_N=\frac{V_{ab}}{1}=\underline{1.894\angle19.53^o\,\Omega}$$

Chapter 13, Problem 17.

ML In the circuit of Fig. 13.86, Z_L is a 15-mH inductor having an impedance of $j40\ \Omega$. Determine Z_{in} when $k = 0.6$.

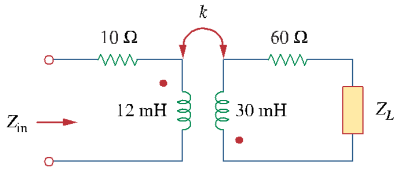


Figure 13.86
For Prob. 13.17.

Chapter 13, Solution 17.

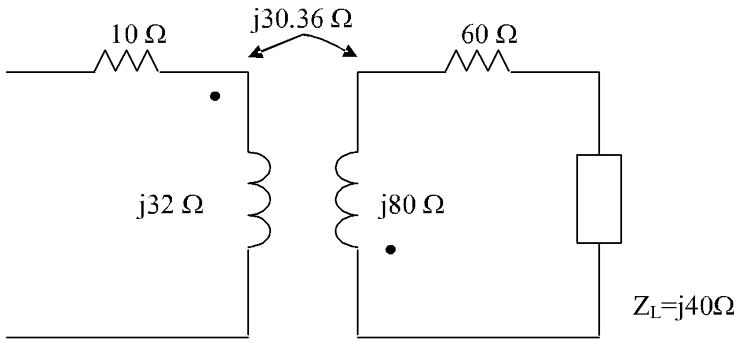
$$j\omega L = j40 \longrightarrow \omega = \frac{40}{L} = \frac{40}{15x10^{-3}} = 2667 \text{ rad/s}$$

$$M = k\sqrt{L_1 L_2} = 0.6\sqrt{12x10^{-3}x30x10^{-3}} = 11.384 \text{ mH}$$

If 15 mH ⟶ 40 Ω

Then 12 mH ⟶ 32 Ω
30 mH ⟶ 80 Ω
11.384 mH ⟶ 30.36 Ω

The circuit becomes that shown below.



$$Z_{in} = 10 + j32 + \frac{\omega^2 M^2}{j80 + 60 + j40} = 10 + j32 + \frac{(30.36)^2}{60 + j120} = \underline{\mathbf{13.073 + j25.86\ \Omega}}.$$

Chapter 13, Problem 18.

PS ML Find the Thevenin equivalent to the left of the load **Z** in the circuit of Fig. 13.87.

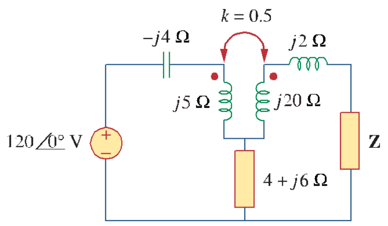


Figure 13.87
For Prob. 13.18.

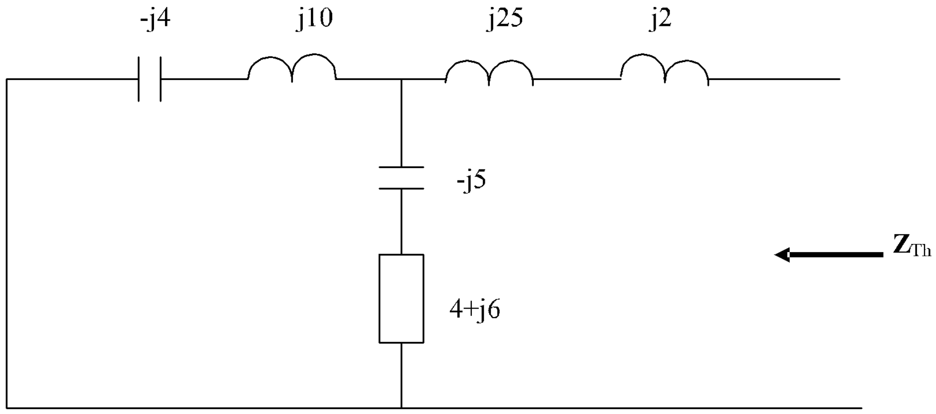
Chapter 13, Solution 18.

Let $\omega = 1$. $L_1 = 5, L_2 = 20, M = k\sqrt{L_1 L_2} = 0.5x10 = 5$

We replace the transformer by its equivalent T-section.

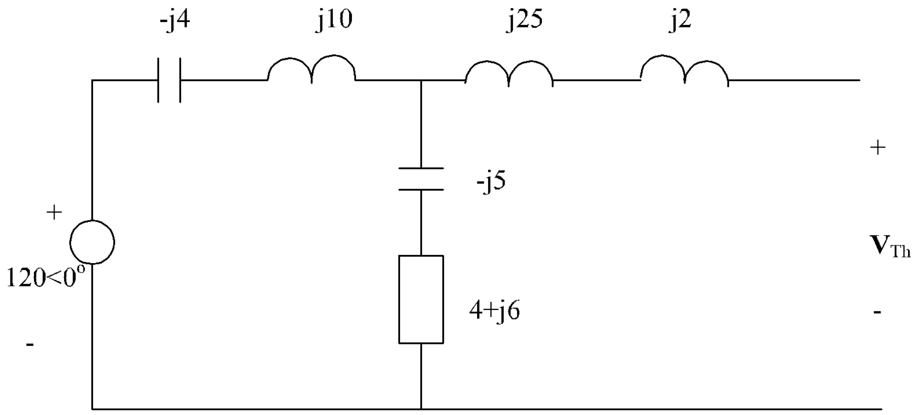
$L_a = L_1 - (-M) = 5 + 5 = 10$, $L_b = L_1 + M = 20 + 5 = 25$, $L_c = -M = -5$

We find $\mathbf{Z}_{Th}$ using the circuit below.



$$Z_{Th} = j27 + (4 + j)//(j6) = j27 + \frac{j6(4+j)}{4+j7} = \underline{2.215 + j29.12\Omega}$$

We find $\mathbf{V}_{Th}$ by looking at the circuit below.



$$V_{Th} = \frac{4+j}{4+j+j6}(120) = \underline{61.37\angle -46.22^o \text{ V}}$$

Chapter 13, Problem 19.

Determine an equivalent T-section that can be used to replace the transformer in Fig. 13.88.

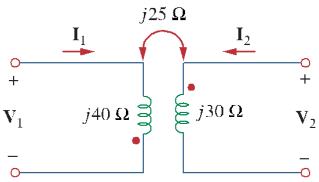


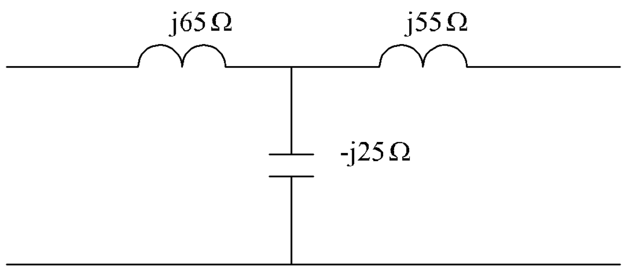
Figure 13.88
For Prob. 13.19.

Chapter 13, Solution 19.

Let $\omega = 1$. $\quad L_a = L_1 - (-M) = 40 + 25 = 65\,\text{H}$

$$L_b = L_2 + M = 30 + 25 = 55\,\text{H}, \qquad L_C = -M = -25$$

Thus, the T-section is as shown below.



Chapter 13, Problem 20.

PS ML Determine currents $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ in the circuit of Fig. 13.89. Find the energy stored in the coupled coils at $t = 2$ ms. Take $\omega = 1{,}000$ rad/s.

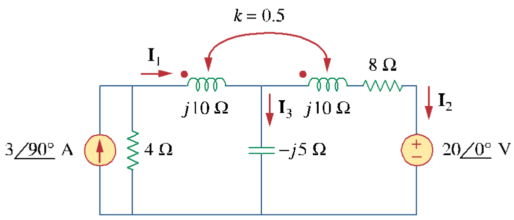
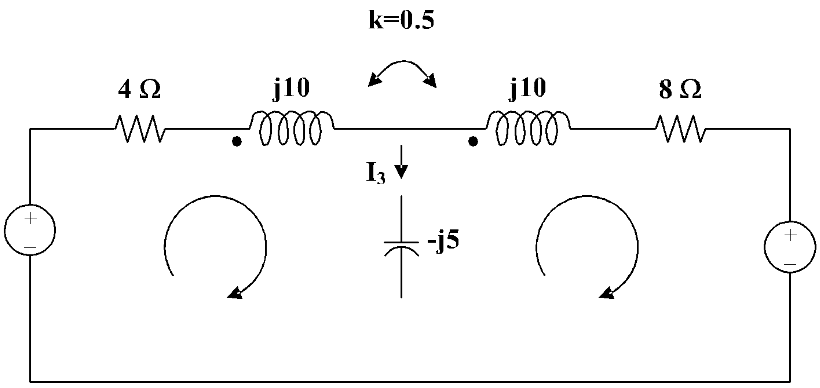


Figure 13.89
For Prob. 13.20.

Chapter 13, Solution 20.

Transform the current source to a voltage source as shown below.



$$k = M/\sqrt{L_1L_2} \quad \text{or} \quad M = k\sqrt{L_1L_2}$$

$$\omega M = k\sqrt{\omega L_1 \omega L_2} = 0.5(10) = 5$$

For mesh 1, $j12 = (4 + j10 - j5)I_1 + j5I_2 + j5I_2 = (4 + j5)I_1 + j10I_2$ (1)

For mesh 2, $0 = 20 + (8 + j10 - j5)I_2 + j5I_1 + j5I_1$

$$-20 = +j10I_1 + (8 + j5)I_2 \qquad (2)$$

From (1) and (2), $\begin{bmatrix} j12 \\ 20 \end{bmatrix} = \begin{bmatrix} 4 + j5 & +j10 \\ +j10 & 8 + j5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

$$\Delta = 107 + j60, \quad \Delta_1 = -60 - j296, \quad \Delta_2 = 40 - j100$$

$$I_1 = \Delta_1/\Delta = \underline{\mathbf{2.462\angle 72.18° \text{ A}}}$$

$$I_2 = \Delta_2/\Delta = \underline{\mathbf{0.878\angle -97.48° \text{ A}}}$$

$$I_3 = I_1 - I_2 = \underline{\mathbf{3.329\angle 74.89° \text{ A}}}$$

$$i_1 = 2.462 \cos(1000t + 72.18°) \text{ A}$$

$$i_2 = 0.878 \cos(1000t - 97.48°) \text{ A}$$

At $t = 2$ ms, $1000t = 2$ rad $= 114.6°$

$$i_1 = 0.9736\cos(114.6° + 143.09°) = -2.445$$

$$i_2 = 2.53\cos(114.6° + 153.61°) = -0.8391$$

The total energy stored in the coupled coils is

$$w = 0.5L_1i_1^2 + 0.5L_2i_2^2 - Mi_1i_2$$

Since $\omega L_1 = 10$ and $\omega = 1000$, $L_1 = L_2 = 10$ mH, $M = 0.5L_1 = 5$mH

$$w = 0.5(10)(-2.445)^2 + 0.5(10)(-0.8391)^2 - 5(-2.445)(-0.8391)$$

w = <u>43.67 mJ</u>

Chapter 13, Problem 21.

PS ML Find $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit of Fig. 13.90. Calculate the power absorbed by the 4-Ω resistor.

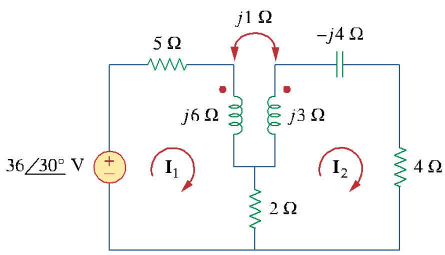


Figure 13.90
For Prob. 13.21.

Chapter 13, Solution 21.

For mesh 1, $36\angle 30° = (7 + j6)I_1 - (2 + j)I_2$ (1)

For mesh 2, $0 = (6 + j3 - j4)I_2 - 2I_1 - jI_1 = -(2 + j)I_1 + (6 - j)I_2$ (2)

Placing (1) and (2) into matrix form, $\begin{bmatrix} 36\angle 30° \\ 0 \end{bmatrix} = \begin{bmatrix} 7 + j6 & -2 - j \\ -2 - j & 6 - j \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

$$\Delta = 45 + j25 = 51.48\angle 29.05°, \quad \Delta_1 = (6 - j)36\angle 30° = 219\angle 20.54°$$

$\Delta_2 = (2 + j)36\angle 30° = 80.5\angle 56.57°$, $I_1 = \Delta_1/\Delta =$ **$4.254\angle -8.51°$ A** , $I_2 = \Delta_2/\Delta =$ **$1.5637\angle 27.52°$ A**

Power absorbed by the 4-ohm resistor,

$$= 0.5(I_2)^2 4 = 2(1.5637)^2 = \mathbf{4.89\ watts}$$

Chapter 13, Problem 22.

* Find current $\mathbf{I}_o$ in the circuit of Fig. 13.91.

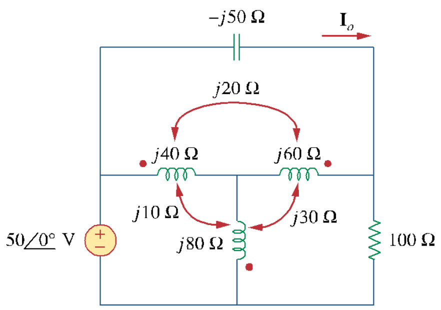
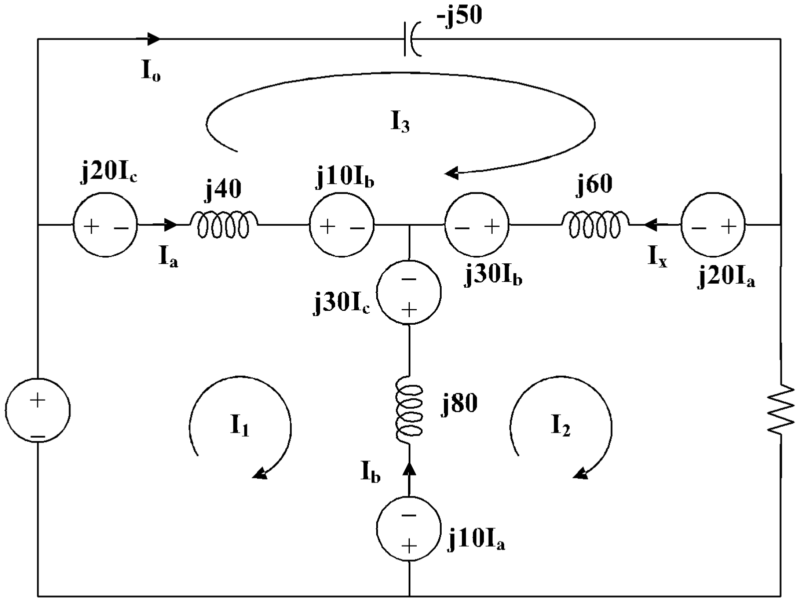


Figure 13.91
For Prob. 13.22.

* An asterisk indicates a challenging problem.

Chapter 13, Solution 22.

With more complex mutually coupled circuits, it may be easier to show the effects of the coupling as sources in terms of currents that enter or leave the dot side of the coil. Figure 13.85 then becomes,



Note the following,

$$I_a = I_1 - I_3$$
$$I_b = I_2 - I_1$$
$$I_c = I_3 - I_2$$

$$\text{and} \quad I_o = I_3$$

Now all we need to do is to write the mesh equations and to solve for I_o.

Loop # 1,

$$-50 + j20(I_3 - I_2)\, j\, 40(I_1 - I_3) + j10(I_2 - I_1) - j30(I_3 - I_2) + j80(I_1 - I_2) - j10(I_1 - I_3) = 0$$

$$j100I_1 - j60I_2 - j40I_3 = 50$$

Multiplying everything by (1/j10) yields $10I_1 - 6I_2 - 4I_3 = -j5$ (1)

Loop # 2,

$j10(I_1 – I_3) + j80(I_2–I_1) + j30(I_3–I_2) – j30(I_2 – I_1) + j60(I_2 – I_3) – j20(I_1 – I_3) + 100I_2 = 0$

$$-j60I_1 + (100 + j80)I_2 – j20I_3 = 0 \qquad (2)$$

Loop # 3,

$-j50I_3 +j20(I_1 –I_3) +j60(I_3 –I_2) +j30(I_2 –I_1) –j10(I_2 –I_1) +j40(I_3 –I_1) –j20(I_3 –I_2) = 0$

$$-j40I_1 – j20I_2 + j10I_3 = 0$$

Multiplying by (1/j10) yields, $-4I_1 – 2I_2 + I_3 = 0$ (3)

Multiplying (2) by (1/j20) yields $-3I_1 + (4 – j5)I_2 – I_3 = 0$ (4)

Multiplying (3) by (1/4) yields $-I_1 – 0.5I_2 – 0.25I_3 = 0$ (5)

Multiplying (4) by (-1/3) yields $I_1 – ((4/3) – j(5/3))I_2 + (1/3)I_3 = -j0.5$ (7)

Multiplying [(6)+(5)] by 12 yields $(-22 + j20)I_2 + 7I_3 = 0$ (8)

Multiplying [(5)+(7)] by 20 yields $-22I_2 – 3I_3 = -j10$ (9)

(8) leads to $I_2 = -7I_3/(-22 + j20) = 0.2355\angle 42.3^o = (0.17418+j0.15849)I_3$ (10)

(9) leads to $I_3 = (j10 – 22I_2)/3$, substituting (1) into this equation produces,

$$I_3 = j3.333 + (-1.2273 – j1.1623)I_3$$

or $I_3 = I_o =$ **$1.3040\angle 63^o$ amp.**

Chapter 13, Problem 23.

PS ML If $M = 0.2$ H and $v_s = 12 \cos 10t$ V in the circuit of Fig. 13.92, find i_1 and i_2 Calculate the energy stored in the coupled coils at $t = 15$ ms.

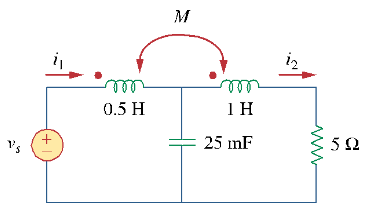


Figure 13.92
For Prob. 13.23.

Chapter 13, Solution 23.

$$\omega = 10$$

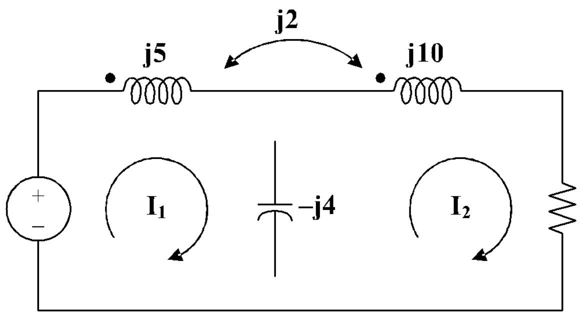
0.5 H converts to $j\omega L_1 = j5$ ohms

1 H converts to $j\omega L_2 = j10$ ohms

0.2 H converts to $j\omega M = j2$ ohms

25 mF converts to $1/(j\omega C) = 1/(10x25x10^{-3}) = -j4$ ohms

The frequency-domain equivalent circuit is shown below.



For mesh 1, $12 = (j5 - j4)I_1 + j2I_2 - (-j4)I_2$

$$-j12 = I_1 + 6I_2 \qquad (1)$$

For mesh 2, $0 = (5 + j10)I_2 + j2I_1 -(-j4)I_1$

$$0 = (5 + j10)I_2 + j6I_1 \qquad (2)$$

From (1), $I_1 = -j12 - 6I_2$

Substituting this into (2) produces,

$$I_2 = 72/(-5 + j26) = 2.7194\angle -100.89°$$

$$I_1 = -j12 - 6\,I_2 = -j12 - 163.17\angle -100.89 = 5.068\angle 52.54°$$

Hence,

i_1 = **<u>5.068cos(10t + 52.54°) A</u>**, i_2 = **<u>2.719cos(10t – 100.89°) A</u>**.

At t = 15 ms, $10t = 10x15x10^{-3}\ 0.15\text{ rad} = 8.59°$

$$i_1 = 5.068\cos(61.13°) = 2.446$$

$$i_2 = 2.719\cos(-92.3°) = -0.1089$$

$w = 0.5(5)(2.446)^2 + 0.5(1)(-0.1089)^2 - (0.2)(2.446)(-0.1089)$ = **<u>15.02 J</u>**

Chapter 13, Problem 24.

ps ML In the circuit of Fig. 13.93,

(a) find the coupling coefficient,
(b) calculate v_o,
(c) determine the energy stored in the coupled inductors at $t = 2$ s.

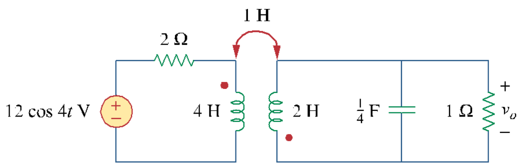


Figure 13.93
For Prob. 13.24.

Chapter 13, Solution 24.

(a) $k = M/\sqrt{L_1 L_2} = 1/\sqrt{4x2} = \underline{\mathbf{0.3535}}$

(b) $\omega = 4$

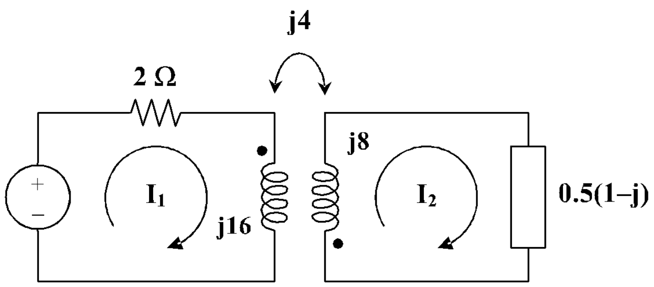
1/4 F leads to $1/(j\omega C) = -j/(4x0.25) = -j$

$1||(-j) = -j/(1-j) = 0.5(1-j)$

1 H produces $j\omega M = j4$

4 H produces j16

2 H becomes j8



$$12 = (2 + j16)I_1 + j4I_2$$

$$\text{or} \qquad 6 = (1 + j8)I_1 + j2I_2 \qquad (1)$$

$$0 = (j8 + 0.5 - j0.5)I_2 + j4I_1 \;\text{ or }\; I_1 = (0.5 + j7.5)I_2/(-j4) \qquad (2)$$

Substituting (2) into (1),

$$24 = (-11.5 - j51.5)I_2 \;\text{ or }\; I_2 = -24/(11.5 + j51.5) = -0.455\angle{-77.41°}$$

$$V_o = I_2(0.5)(1 - j) = 0.3217\angle 57.59°$$

$$v_o = \underline{\mathbf{321.7cos(4t + 57.6°)\ mV}}$$

(c) From (2), $I_1 = (0.5 + j7.5)I_2/(-j4) = 0.855\angle{-81.21°}$

$$i_1 = 0.885\cos(4t - 81.21°)\text{ A},\ i_2 = -0.455\cos(4t - 77.41°)\text{ A}$$

At t = 2s,

$$4t = 8\text{ rad} = 98.37°$$

$$i_1 = 0.885\cos(98.37° - 81.21°) = 0.8169$$

$$i_2 = -0.455\cos(98.37° - 77.41°) = -0.4249$$

$$w = 0.5L_1i_1^2 + 0.5L_2i_2^2 + Mi_1i_2$$

$$= 0.5(4)(0.8169)^2 + 0.5(2)(-.4249)^2 + (1)(0.1869)(-0.4249) = \underline{\mathbf{1.168\ J}}$$

Chapter 13, Problem 25.

PS ML For the network in Fig. 13.94, find $\mathbf{Z}_{ab}$ and $\mathbf{I}_o$.

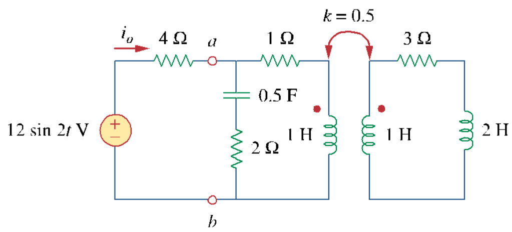


Figure 13.94
For Prob. 13.25.

Chapter 13, Solution 25.

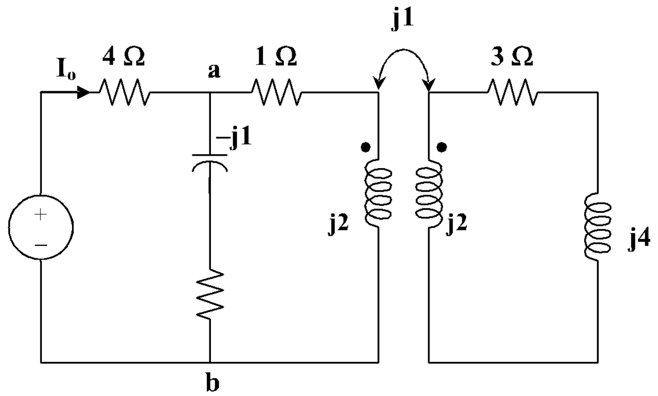
$$m = k\sqrt{L_1 L_2} = 0.5\text{ H}$$

We transform the circuit to frequency domain as shown below.

$$12\sin 2t \text{ converts to } 12\angle 0°,\ \omega = 2$$

$$0.5\text{ F converts to } 1/(j\omega C) = -j$$

$$2\text{ H becomes } j\omega L = j4$$



Applying the concept of reflected impedance,

$$Z_{ab} = (2 - j)||(1 + j2 + (1)^2/(j2 + 3 + j4))$$

$$= (2 - j)||(1 + j2 + (3/45) - j6/45)$$

$$= (2 - j)||(1 + j2 + (3/45) - j6/45)$$

$$= (2 - j)||(1.0667 + j1.8667)$$

$$=(2 - j)(1.0667 + j1.8667)/(3.0667 + j0.8667) = 1.5085\angle 17.9° \text{ ohms}$$

$$I_o = 12\angle 0°/(Z_{ab} + 4) = 12/(5.4355 + j0.4636) = 2.2\angle{-4.88°}$$

$$i_o = \underline{\mathbf{2.2\sin(2t - 4.88°)\ A}}$$

Chapter 13, Problem 26.

PS ML Find $\mathbf{I}_o$ in the circuit of Fig. 13.95. Switch the dot on the winding on the right and calculate $\mathbf{I}_o$ again.

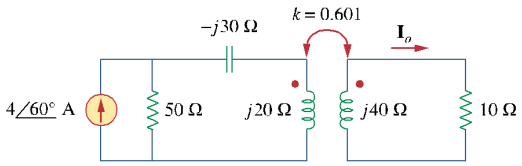


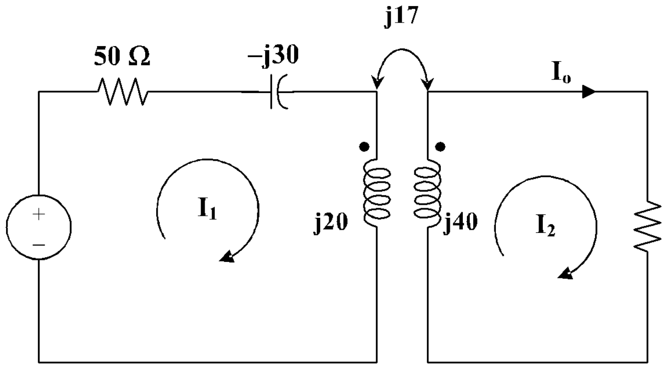
Figure 13.95
For Prob. 13.26.

Chapter 13, Solution 26.

$$M = k\sqrt{L_1 L_2}$$

$$\omega M = k\sqrt{\omega L_1 \omega L_2} = 0.6\sqrt{20x40} = 17$$

The frequency-domain equivalent circuit is shown below.



For mesh 1, $200\angle 60° = (50 - j30 + j20)I_1 + j17I_2 = (50 - j10)I_1 + j17I_2$ (1)

For mesh 2, $0 = (10 + j40)I_2 + j17I_1$ (2)

In matrix form,

$$\begin{bmatrix} 200\angle 60° \\ 0 \end{bmatrix} = \begin{bmatrix} 50 - j10 & j17 \\ j17 & 10 + j40 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta = 900 + j100, \ \Delta_1 = 2000\angle 60°(1 + j4) = 8246.2\angle 136°, \ \Delta_2 = 3400\angle -30°$$

$$I_2 = \Delta_2/\Delta = 3.755\angle -36.34°$$

$$I_o = I_2 = \underline{\mathbf{3.755\angle -36.34° \ A}}$$

Switching the dot on the winding on the right only reverses the direction of I_o. This can be seen by looking at the resulting value of Δ_2 which now becomes $3400\angle 150°$. Thus,

$$I_o = \underline{\mathbf{3.755\angle 143.66° \ A}}$$

Chapter 13, Problem 27.

PS ML Find the average power delivered to the 50-Ω resistor in the circuit of Fig. 13.96.

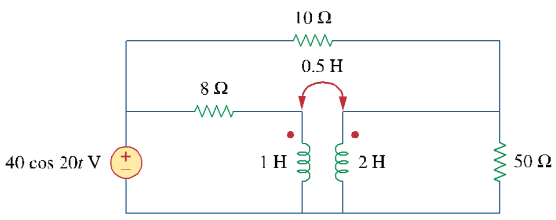
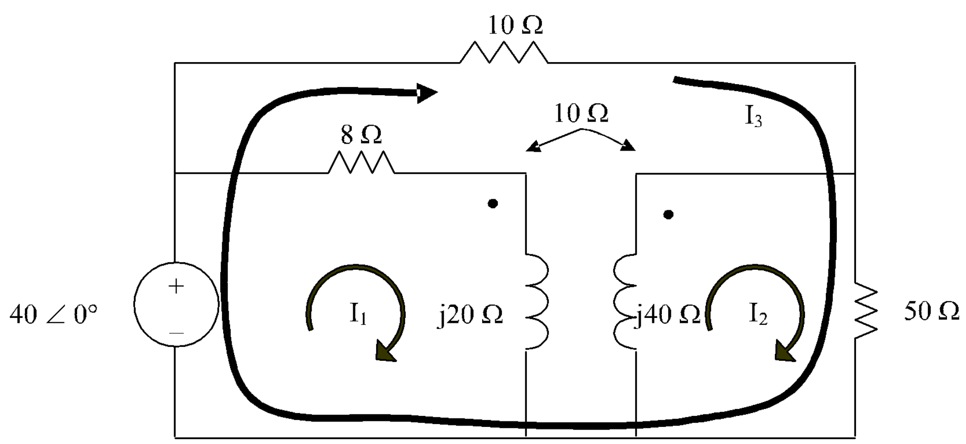


Figure 13.96
For Prob. 13.27.

Chapter 13, Solution 27.

$$1H \longrightarrow j\omega L = j20$$
$$2H \longrightarrow j\omega L = j40$$
$$0.5H \longrightarrow j\omega L = j10$$

We apply mesh analysis to the circuit as shown below.



To make the problem easier to solve, let us have I_3 flow around the outside loop as shown.

For mesh 1,

$$(8+j20)I_1 - j10I_2 = 40 \qquad (1)$$

For mesh 2,

$$-j10I_1 + (50+j40)I_2 + 50I_3 = 0 \qquad (2)$$

For mesh 3,

$$-40 + 50I_2 + 60I_3 = 0 \qquad (3)$$

In matrix form, (1) to (3) become

$$\begin{bmatrix} 8+j20 & -j10 & 0 \\ -j10 & 50+j40 & 50 \\ 0 & 50 & 60 \end{bmatrix} I = \begin{bmatrix} 40 \\ 0 \\ 0 \end{bmatrix}$$

```
>> Z=[(8+20i),-10i,0;-10i,(50+40i),50;0,50,60]
Z =
  8.0000 +20.0000i        0 -10.0000i        0
       0 -10.0000i  50.0000 +40.0000i  50.0000
       0             50.0000            60.0000

>> V=[40;0;0]
V =
   40
    0
    0
>> I=inv(Z)*V
I =
  0.8896 - 1.8427i
  0.3051 - 0.3971i
 -0.2543 + 0.3309i
```

Solving this leads to $I_{50} = I_2 + I_3 = 0.0508 - j0.0662 = 0.08345\angle{-52.5^\circ}$ or $I_{50rms} = 0.08345/1.4142 = 0.059$.

The power delivered to the 50-Ω resistor is

$$P = (I_{50rms})^2R = (0.059)^2 50 = \mathbf{174.05\ mW}.$$

Chapter 13, Problem 28.

ML In the circuit of Fig. 13.97, find the value of X that will give maximum power transfer to the 20-Ω load.

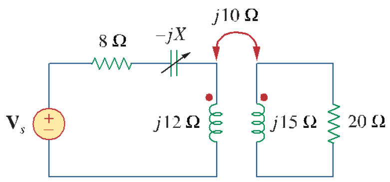
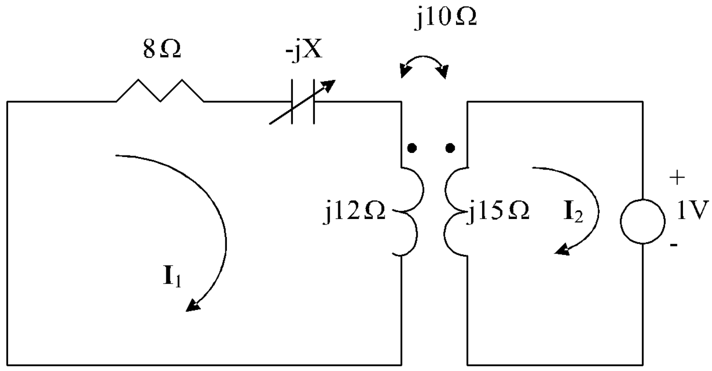


Figure 13.97
For Prob. 13.28.

Chapter 13, Solution 28.

We find Z_{Th} by replacing the 20-ohm load with a unit source as shown below.



For mesh 1, $$0 = (8 - jX + j12)I_1 - j10I_2 \quad (1)$$

For mesh 2,
$$1 + j15I_2 - j10I_1 = 0 \quad \longrightarrow \quad I_1 = 1.5I_2 - 0.1j \quad (2)$$

Substituting (2) into (1) leads to
$$I_2 = \frac{-1.2 + j0.8 + 0.1X}{12 + j8 - j1.5X}$$

$$Z_{Th} = \frac{1}{-I_2} = \frac{12 + j8 - j1.5X}{1.2 - j0.8 - 0.1X}$$

$$|Z_{Th}| = 20 = \frac{\sqrt{12^2 + (8 - 1.5X)^2}}{\sqrt{(1.2 - 0.1X)^2 + 0.8^2}} \quad \longrightarrow \quad 0 = 1.75X^2 + 72X - 624$$

Solving the quadratic equation yields <u>X = 6.425</u>

Chapter 13, Problem 29.

In the circuit of Fig. 13.98, find the value of the coupling coefficient *k* that will make the 10-Ω resistor dissipate 320 W. For this value of *k*, find the energy stored in the coupled coils at $t = 1.5$ s.

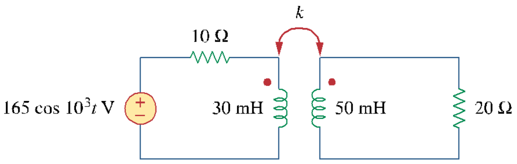


Figure 13.98
For Prob. 13.29.

Chapter 13, Solution 29.

$$30 \text{ mH becomes } j\omega L = j30\text{x}10^{-3}\text{x}10^{3} = j30$$

$$50 \text{ mH becomes } j50$$

$$\text{Let } X = \omega M$$

Using the concept of reflected impedance,

$$Z_{in} = 10 + j30 + X^2/(20 + j50)$$

$$I_1 = V/Z_{in} = 165/(10 + j30 + X^2/(20 + j50))$$

$$p = 0.5|I_1|^2(10) = 320 \text{ leads to } |I_1|^2 = 64 \text{ or } |I_1| = 8$$

$$8 = |165(20 + j50)/(X^2 + (10 + j30)(20 + j50))|$$

$$= |165(20 + j50)/(X^2 - 1300 + j1100)|$$

or $$64 = 27225(400 + 2500)/((X^2 - 1300)^2 + 1{,}210{,}000)$$

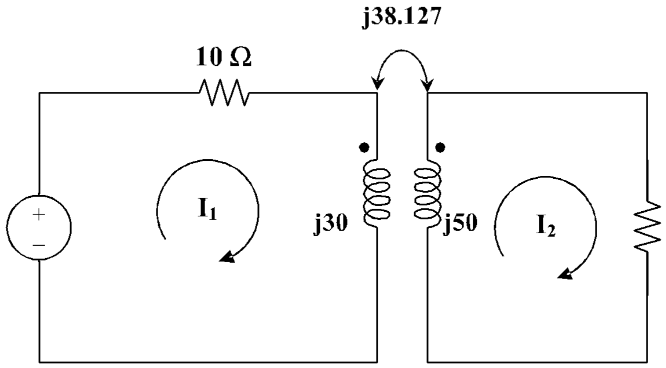
$$(X^2 - 1300)^2 + 1{,}210{,}000 = 1{,}233{,}633$$

$$X = 33.86 \text{ or } 38.13$$

$$\text{If } X = 38.127 = \omega M$$

$$M = 38.127 \text{ mH}$$

$$k = M/\sqrt{L_1 L_2} = 38.127/\sqrt{30\text{x}50} = \underline{\mathbf{0.984}}$$



$$165 = (10 + j30)I_1 - j38.127I_2 \quad (1)$$

$$0 = (20 + j50)I_2 - j38.127I_1 \quad (2)$$

In matrix form,

$$\begin{bmatrix} 165 \\ 0 \end{bmatrix} = \begin{bmatrix} 10 + j30 & -j38.127 \\ -j38.127 & 20 + j50 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta = 154 + j1100 = 1110.73\angle 82.03°, \ \Delta_1 = 888.5\angle 68.2°, \ \Delta_2 = j6291$$

$$I_1 = \Delta_1/\Delta = 8\angle -13.81°, \ I_2 = \Delta_2/\Delta = 5.664\angle 7.97°$$

$$i_1 = 8\cos(1000t - 13.83°), \ i_2 = 5.664\cos(1000t + 7.97°)$$

$$\text{At } t = 1.5 \text{ ms}, \ 1000t = 1.5 \text{ rad} = 85.94°$$

$$i_1 = 8\cos(85.94° - 13.83°) = 2.457$$

$$i_2 = 5.664\cos(85.94° + 7.97°) = -0.3862$$

$$w = 0.5L_1 i_1^2 + 0.5L_2 i_2^2 + Mi_1 i_2$$

$$= 0.5(30)(2.547)^2 + 0.5(50)(-0.3862)^2 - 38.127(2.547)(-0.3862)$$

$$= \underline{\mathbf{130.51\ mJ}}$$

Chapter 13, Problem 30.

(a) Find the input impedance of the circuit in Fig. 13.99 using the concept of reflected impedance.
(b) Obtain the input impedance by replacing the linear transformer by its T equivalent.

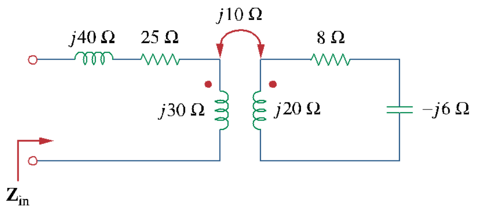


Figure 13.99
For Prob. 13.30.

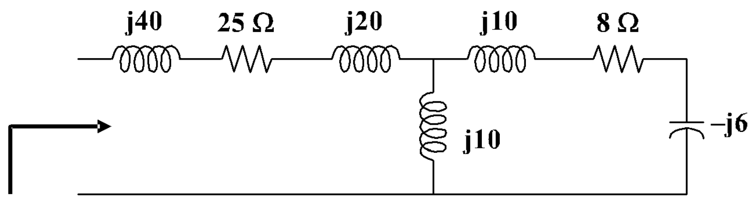
Chapter 13, Solution 30.

(a) $Z_{in} = j40 + 25 + j30 + (10)^2/(8 + j20 - j6)$

$= 25 + j70 + 100/(8 + j14) =$ **(28.08 + j64.62) ohms**

(b) $j\omega L_a = j30 - j10 = j20$, $j\omega L_b = j20 - j10 = j10$, $j\omega L_c = j10$

Thus the Thevenin Equivalent of the linear transformer is shown below.



$Z_{in} = j40 + 25 + j20 + j10||(8 + j4) = 25 + j60 + j10(8 + j4)/(8 + j14)$

$=$ **(28.08 + j64.62) ohms**

Chapter 13, Problem 31.

For the circuit in Fig. 13.100, find:

(a) the *T*-equivalent circuit,
(b) the Π-equivalent circuit.

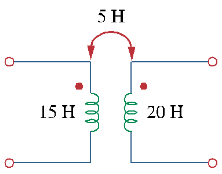


Figure 13.100
For Prob. 13.31.

Chapter 13, Solution 31.

(a)

$$L_a = L_1 - M = \mathbf{\underline{10\ H}}$$

$$L_b = L_2 - M = \mathbf{\underline{15\ H}}$$

$$L_c = M = \mathbf{\underline{5\ H}}$$

(b)

$$L_1L_2 - M^2 = 300 - 25 = 275$$

$$L_A = (L_1L_2 - M^2)/(L_1 - M) = 275/15 = \mathbf{\underline{18.33\ H}}$$

$$L_B = (L_1L_2 - M^2)/(L_1 - M) = 275/10 = \mathbf{\underline{27.5\ H}}$$

$$L_C = (L_1L_2 - M^2)/M = 275/5 = \mathbf{\underline{55\ H}}$$

Chapter 13, Problem 32.

* Two linear transformers are cascaded as shown in Fig. 13.101. Show that

$$\mathbf{Z}_{\text{in}} = \frac{\omega^2 R(L_a^2 + L_a L_b - M_a^2) + j\omega^3(L_a^2 L_b + L_a L_b^2 - L_a M_b^2 - L_b M_a^2)}{\omega^2(L_a L_b + L_b^2 - M_b^2) - j\omega R(L_a + L_b)}$$

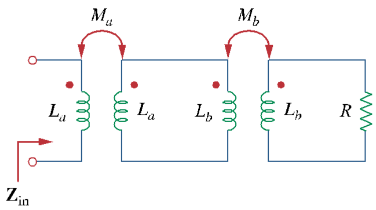
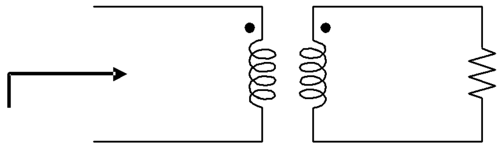


Figure 13.101
For Prob. 13.32.

* An asterisk indicates a challenging problem.

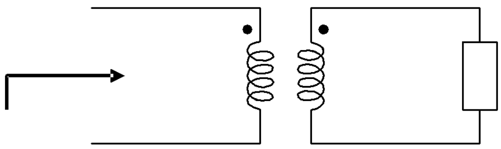
Chapter 13, Solution 32.

We first find Z_{in} for the second stage using the concept of reflected impedance.



$Z_{in}' = j\omega L_b + \omega^2 M_b^2/(R + j\omega L_b) = (j\omega L_b R - \omega^2 L_b^2 + \omega^2 M_b^2)/(R + j\omega L_b)$ (1)

For the first stage, we have the circuit below.



$$Z_{in} = j\omega L_a + \omega^2 M_a^2/(j\omega L_a + Z_{in})$$

$$= (-\omega^2 L_a^2 + \omega^2 M_a^2 + j\omega L_a Z_{in})/(j\omega L_a + Z_{in}) \quad (2)$$

Substituting (1) into (2) gives,

$$= \frac{-\omega^2 L_a^2 + \omega^2 M_a^2 + j\omega L_a \dfrac{(j\omega L_b R - \omega^2 L_b^2 + \omega^2 M_b^2)}{R + j\omega L_b}}{j\omega L_a + \dfrac{j\omega L_b R - \omega^2 L_b^2 + \omega^2 M_b^2}{R + j\omega L_b}}$$

$$= \frac{-R\omega^2 L_a^2 + \omega^2 M_a^2 R - j\omega^3 L_b L_a + j\omega^3 L_b M_a^2 + j\omega L_a(j\omega L_b R - \omega^2 L_b^2 + \omega^2 M_b^2)}{j\omega R La - \omega^2 L_a L_b + j\omega L_b R - \omega^2 L_a^2 + \omega^2 M_b^2}$$

$$Z_{in} = \frac{\boldsymbol{\omega^2 R(L_a^2 + L_a L_b - M_a^2) + j\omega^3(L_a^2 L_b + L_a L_b^2 - L_a M_b^2 - L_b M_a^2)}}{\boldsymbol{\omega^2(L_a L_b + L_b^2 - M_b^2) - j\omega R(L_a + L_b)}}$$

Chapter 13, Problem 33.

ML Determine the input impedance of the air-core transformer circuit of Fig. 13.102.

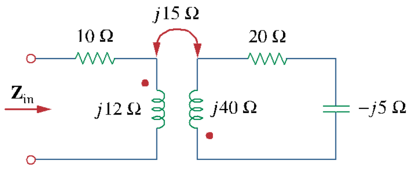


Figure 13.102
For Prob. 13.33.

Chapter 13, Solution 33.

$$Z_{in} = 10 + j12 + (15)^2/(20 + j\,40 - j5) = 10 + j12 + 225/(20 + j35)$$

$$= 10 + j12 + 225(20 - j35)/(400 + 1225)$$

$$= \underline{(12.769 + j7.154)\ \text{ohms}}$$

Chapter 13, Problem 34.

ML Find the input impedance of the circuit in Fig. 13.103.

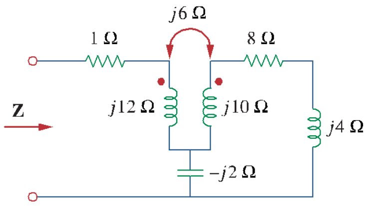
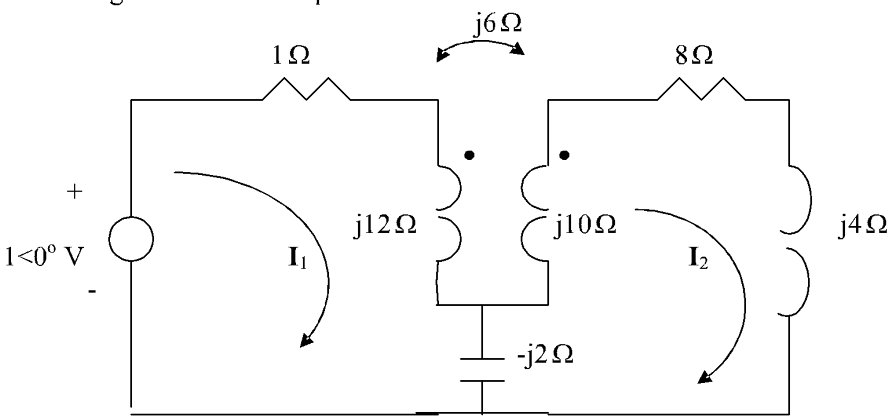


Figure 13.103
For Prob. 13.34.

Chapter 13, Solution 34.

Insert a 1-V voltage source at the input as shown below.



For loop 1,

$$1 = (1 + j10)I_1 - j4I_2 \qquad (1)$$

For loop 2,

$$0 = (8 + j4 + j10 - j2)I_2 + j2I_1 - j6I_1 \longrightarrow 0 = -jI_1 + (2 + j3)I_2 \qquad (2)$$

Solving (1) and (2) leads to $\mathbf{I}_1$=0.019 –j0.1068

$$Z = \frac{1}{I_1} = 1.6154 + j9.077 = \underline{9.219\angle 79.91^o\,\Omega}$$

Alternatively, an easier way to obtain **Z** is to replace the transformer with its equivalent T circuit and use series/parallel impedance combinations. This leads to exactly the same result.

Chapter 13, Problem 35.

PS ML * Find currents $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ in the circuit of Fig. 13.104.

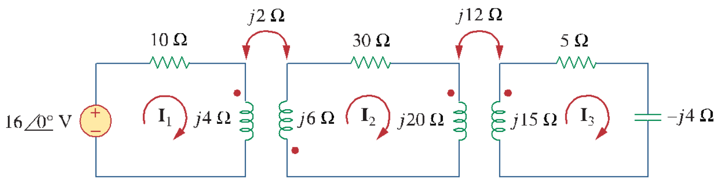


Figure 13.104
For Prob. 13.35.

* An asterisk indicates a challenging problem.

Chapter 13, Solution 35.

For mesh 1,

$$16 = (10 + j4)I_1 + j2I_2 \qquad (1)$$

For mesh 2, $$0 = j2I_1 + (30 + j26)I_2 - j12I_3 \qquad (2)$$

For mesh 3, $$0 = -j12I_2 + (5 + j11)I_3 \qquad (3)$$

We may use MATLAB to solve (1) to (3) and obtain

$$I_1 = 1.3736 - j0.5385 = \underline{1.4754\angle -21.41^o \text{ A}}$$
$$I_2 = -0.0547 - j0.0549 = \underline{0.0775\angle -134.85^o \text{ A}}$$
$$I_3 = -0.0268 - j0.0721 = \underline{0.077\angle -110.41^o \text{ A}}$$

Chapter 13, Problem 36.

As done in Fig. 13.32, obtain the relationships between terminal voltages and currents for each of the ideal transformers in Fig. 13.105.

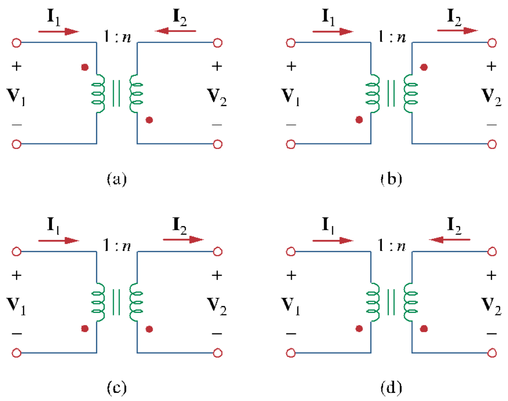


Figure 13.105
For Prob. 13.36.

Chapter 13, Solution 36.

Following the two rules in section 13.5, we obtain the following:

(a) $V_2/V_1 = \underline{\mathbf{-n}}$, $I_2/I_1 = \underline{\mathbf{-1/n}}$ $(n = V_2/V_1)$

(b) $V_2/V_1 = \underline{\mathbf{-n}}$, $I_2/I_1 = \underline{\mathbf{-1/n}}$

(c) $V_2/V_1 = \underline{\mathbf{n}}$, $I_2/I_1 = \underline{\mathbf{1/n}}$

(d) $V_2/V_1 = \underline{\mathbf{n}}$, $I_2/I_1 = \underline{\mathbf{-1/n}}$

Chapter 13, Problem 37.

A 480/2,400-V rms step-up ideal transformer delivers 50 kW to a resistive load. Calculate:

(a) the turns ratio
(b) the primary current
(c) the secondary current

Chapter 13, Solution 37.

(a) $n = \frac{V_2}{V_1} = \frac{2400}{480} = \underline{5}$

(b) $S_1 = I_1 V_1 = S_2 = I_2 V_2 = 50{,}000 \quad \longrightarrow \quad I_1 = \frac{50{,}000}{480} = \underline{104.17\text{ A}}$

(c) $I_2 = \frac{50{,}000}{2400} = \underline{20.83\text{ A}}$

Chapter 13, Problem 38.

A 4-kVA, 2,300/230-V rms transformer has an equivalent impedance of $2\angle 10°\ \Omega$ on the primary side. If the transformer is connected to a load with 0.6 power factor leading, calculate the input impedance.

Chapter 13, Solution 38.

$Z_{in} = Z_p + Z_L/n^2, \quad n = v_2/v_1 = 230/2300 = 0.1$

$v_2 = 230\text{ V}, \quad s_2 = v_2 I_2^*$

$I_2^* = s_2/v_2 = 17.391\angle{-53.13°} \text{ or } I_2 = 17.391\angle 53.13°\text{ A}$

$Z_L = v_2/I_2 = 230\angle 0°/17.391\angle 53.13° = 13.235\angle{-53.13°}$

$Z_{in} = 2\angle 10° + 1323.5\angle{-53.13°}$

$= 1.97 + j0.3473 + 794.1 - j1058.8$

$Z_{in} =$ **$\underline{1.324\angle{-53.05°}\text{ kohms}}$**

Chapter 13, Problem 39.

A 1,200/240-V rms transformer has impedance $60\angle -30°\ \Omega$ on the high-voltage side. If the transformer is connected to a $0.8\angle 10°$ - Ω load on the low-voltage side, determine the primary and secondary currents when the transformer is connected to 1200 V rms.

Chapter 13, Solution 39.

Referred to the high-voltage side,

$Z_L = (1200/240)^2(0.8\angle 10°) = 20\angle 10°$

$Z_{in} = 60\angle -30° + 20\angle 10° = 76.4122\angle -20.31°$

$I_1 = 1200/Z_{in} = 1200/76.4122\angle -20.31° =$ $\underline{\mathbf{15.7\angle 20.31°\ A}}$

Since $S = I_1 v_1 = I_2 v_2,\ I_2 = I_1 v_1/v_2$

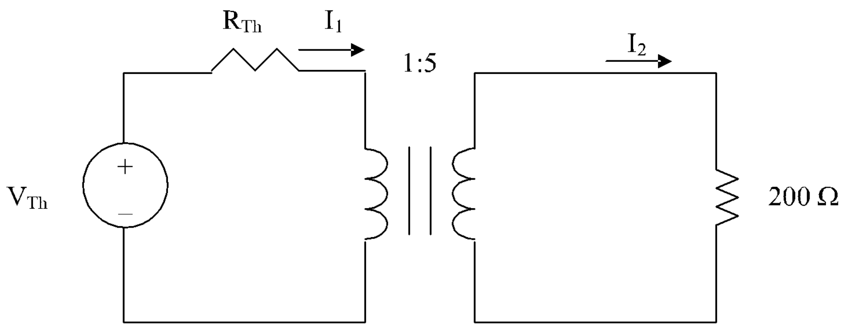
$= (1200/240)(_15.7\angle 20.31°) =$ $\underline{\mathbf{78.5\angle 20.31°\ A}}$

Chapter 13, Problem 40.

The primary of an ideal transformer with a turns ratio of 5 is connected to a voltage source with Thevenin parameters v_{Th} = 10 cos 2000*t* V and R_{Th} = 100Ω Determine the average power delivered to a 200-Ω load connected across the secondary winding.

Chapter 13, Solution 40.

Consider the circuit as shown below.



We reflect the 200-Ω load to the primary side.

$$Z_p = 100 + \frac{200}{5^2} = 108$$

$$I_1 = \frac{10}{108}, \qquad I_2 = \frac{I_1}{n} = 2/108$$

$$P = \frac{1}{2}|I_2|^2 R_L = \frac{1}{2}(\frac{2}{108})^2(200) = \underline{34.3 \text{ mW}}$$

Chapter 13, Problem 41.

PS ML Determine $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit of Fig. 13.106.

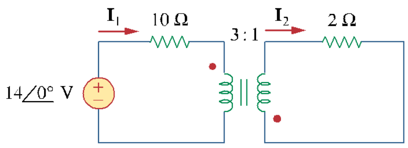


Figure 13.106
For Prob. 13.41.

Chapter 13, Solution 41.

We reflect the 2-ohm resistor to the primary side.

$$Z_{in} = 10 + 2/n^2, \quad n = -1/3$$

Since both I_1 and I_2 enter the dotted terminals, $\quad Z_{in} = 10 + 18 = 28$ ohms

$$I_1 = 14\angle 0°/28 = \underline{\mathbf{0.5\ A}} \text{ and } I_2 = I_1/n = 0.5/(-1/3) = \underline{\mathbf{-1.5\ A}}$$

Chapter 13, Problem 42.

PS ML For the circuit in Fig. 13.107, determine the power absorbed by the 2-Ω resistor. Assume the 80 V is an rms value.

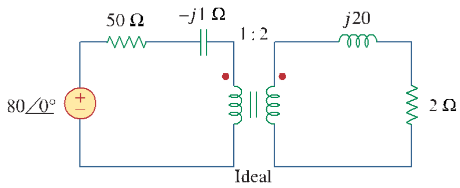
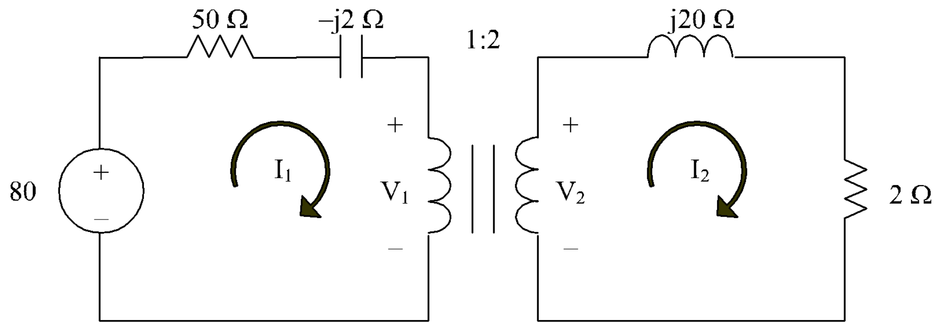


Figure 13.107
For Prob. 13.42.

Chapter 13, Solution 42.

We apply mesh analysis to the circuit as shown below.



For mesh 1,

$$80 = (50 - j2)I_1 + V_1 \qquad (1)$$

For mesh 2,

$$-V_2 + (2 - j20)I_2 = 0 \qquad (2)$$

At the transformer terminals,

$$V_2 = 2V_1 \qquad (3)$$

$$I_1 = 2I_2 \qquad (4)$$

From (1) to (4),

$$\begin{bmatrix} (50-j2) & 0 & 1 & 0 \\ 0 & (2-j20) & 0 & 1 \\ 0 & 0 & 2 & -1 \\ 1 & -2 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 80 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solving this with MATLAB gives

$$I_2 = 0.8051 - j0.0488 = 0.8056\angle -3.47^\circ.$$

The power absorbed by the 2-Ω resistor is

$$P = |I_2|^2 R = (0.8056)^2 2 = \underline{\mathbf{1.3012\ W}}.$$

Chapter 13, Problem 43.

PS ML Obtain $\mathbf{V}_1$ and $\mathbf{V}_2$ in the ideal transformer circuit of Fig. 13.108.

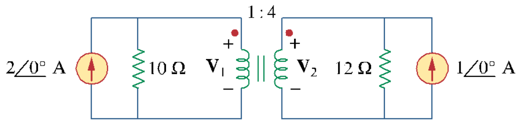
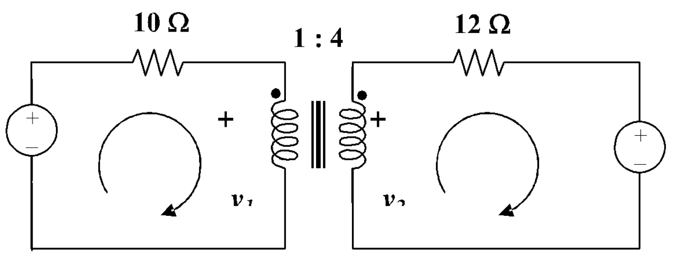


Figure 13.108
For Prob. 13.43.

Chapter 13, Solution 43.

Transform the two current sources to voltage sources, as shown below.



Using mesh analysis, $-20 + 10I_1 + v_1 = 0$

$$20 = v_1 + 10I_1 \quad (1)$$

$$12 + 12I_2 - v_2 = 0 \text{ or } 12 = v_2 - 12I_2 \quad (2)$$

At the transformer terminal, $v_2 = nv_1 = 4v_1 \quad (3)$

$$I_1 = nI_2 = 4I_2 \quad (4)$$

Substituting (3) and (4) into (1) and (2), we get,

$$20 = v_1 + 40I_2 \quad (5)$$

$$12 = 4v_1 - 12I_2 \quad (6)$$

Solving (5) and (6) gives $v_1 = \underline{\mathbf{4.186\ V}}$ and $v_2 = 4v = \underline{\mathbf{16.744\ V}}$

Chapter 13, Problem 44.

*In the ideal transformer circuit of Fig. 13.109, find $i_1(t)$ and $i_2(t)$.

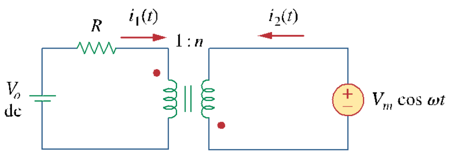


Figure 13.109
For Prob. 13.44.

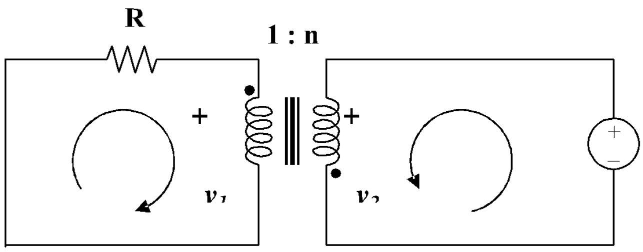
* An asterisk indicates a challenging problem.

Chapter 13, Solution 44.

We can apply the superposition theorem. Let $i_1 = i_1' + i_1''$ and $i_2 = i_2' + i_2''$ where the single prime is due to the DC source and the double prime is due to the AC source. Since we are looking for the steady-state values of i_1 and i_2,

$$i_1' = i_2' = 0.$$

For the AC source, consider the circuit below.



$$V_2/V_1 = -n, \quad I_2''/I_1'' = -1/n$$

But $V_2 = V_m$, $V_1 = -V_m/n$ or $I_1'' = V_m/(Rn)$

$$I_2'' = -I_1''/n = -V_m/(Rn^2)$$

Hence, $i_1(t) =$ **$(V_m/Rn)\cos\omega t$ A**, and $i_2(t) =$ **$(-V_m/(n^2R))\cos\omega t$ A**

Chapter 13, Problem 45.

PS ML For the circuit shown in Fig. 13.110, find the value of the average power absorbed by the 8-Ω resistor.

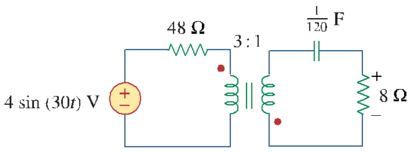
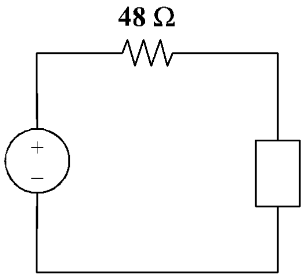


Figure 13.110
For Prob. 13.45.

Chapter 13, Solution 45.



$$Z_L = 8 - \frac{j}{\omega C} = 8 - j4, \quad n = 1/3$$

$$Z = \frac{Z_L}{n^2} = 9Z_L = 72 - j36$$

$$I = \frac{4\angle -90^\circ}{48 + 72 - j36} = \frac{4\angle -90^\circ}{125.28\angle -16.7^\circ} = 0.03193\angle -73.3^\circ$$

We now have some choices, we can go ahead and calculate the current in the second loop and calculate the power delivered to the 8-ohm resistor directly or we can merely say that the power delivered to the equivalent resistor in the primary side must be the same as the power delivered to the 8-ohm resistor. Therefore,

$$P_{8\Omega} = \left|\frac{I^2}{2}\right| 72 = 0.5098\text{x}10^{-3} 72 = \underline{\mathbf{36.71\ mW}}$$

The student is encouraged to calculate the current in the secondary and calculate the power delivered to the 8-ohm resistor to verify that the above is correct.

Chapter 13, Problem 46.

(a) Find $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit of Fig. 13.111 below.
(b) Switch the dot on one of the windings. Find $\mathbf{I}_1$ and $\mathbf{I}_2$ again.

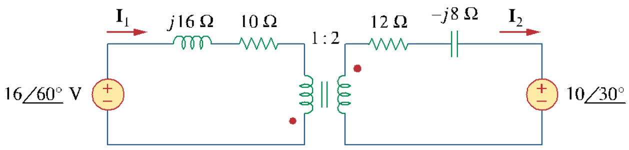
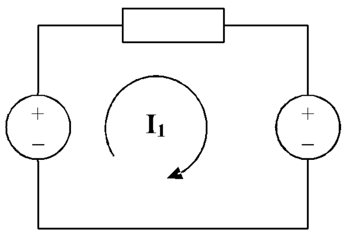


Figure 13.111
For Prob. 13.46.

Chapter 13, Solution 46.

(a) Reflecting the secondary circuit to the primary, we have the circuit shown below.



$Z_{in} = 10 + j16 + (1/4)(12 - j8) = 13 + j14$

$-16\angle 60° + Z_{in}I_1 - 5\angle 30° = 0$ or $I_1 = (16\angle 60° + 5\angle 30°)/(13 + j14)$

Hence, $I_1 =$ **$\underline{1.072\angle 5.88° \text{ A}}$**, and $I_2 = -0.5I_1 =$ **$\underline{0.536\angle 185.88° \text{ A}}$**

(b) Switching a dot will not effect Z_{in} but will effect I_1 and I_2.

$I_1 = (16\angle 60° - 5\angle 30°)/(13 + j14) =$ **$\underline{0.625\ \angle 25 \text{ A}}$**

and $I_2 = 0.5I_1 =$ **$\underline{0.3125\angle 25° \text{ A}}$**

Chapter 13, Problem 47.

PS ML Find $v(t)$ for the circuit in Fig. 13.112.

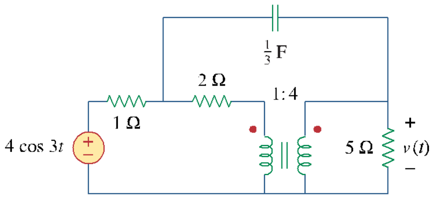
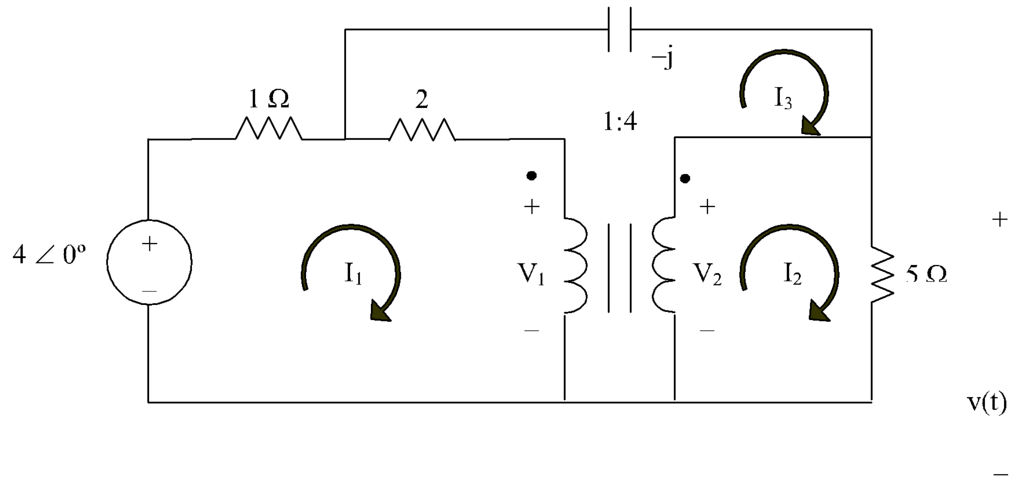


Figure 13.112
For Prob. 13.47.

Chapter 13, Solution 47.

$$1F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j3x1/3} = -j1$$

Consider the circuit shown below.



For mesh 1,

$$3I_1 - 2I_3 + V_1 = 4 \qquad (1)$$

For mesh 2,

$$5I_2 - V_2 = 0 \qquad (2)$$

For mesh 3,

$$-2I_1\ (2-j)I_3 - V_1 + V_2 = 0 \qquad (3)$$

At the terminals of the transformer,

$$V_2 = nV_1 = 4V_1 \qquad (4)$$

$$I_1 = nI_2 = 4I_2 \qquad (5)$$

In matrix form,

$$\begin{bmatrix} 3 & 0 & -2 & 1 & 0 \\ 0 & 5 & 0 & 0 & -1 \\ -2 & 0 & 2-j & -1 & 1 \\ 0 & 0 & 0 & -4 & 1 \\ 1 & -4 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solving this using MATLAB yields

```
A = [3,0,-2,1,0;0,5,0,0,-1;-2,0,(2-i),-1,1;0,0,0,-4,1;1,-4,0,0,0]
U = [4;0;0;0;0]
X = inv(A)*U
>> A = [3,0,-2,1,0;0,5,0,0,-1;-2,0,(2-i),-1,1;0,0,0,-4,1;1,-4,0,0,0]

A =
  Columns 1 through 4

   3.0000           0          -2.0000           1.0000
        0      5.0000                0                0
  -2.0000           0     2.0000 - 1.0000i  -1.0000
        0           0           0          -4.0000
   1.0000     -4.0000                0                0

  Column 5

        0
  -1.0000
   1.0000
   1.0000
        0

>> U = [4;0;0;0;0]

U =
     4
     0
     0
     0
     0

>> X = inv(A)*U

X =
   1.5774 + 0.2722i
   0.3943 + 0.0681i
   0.6125 + 0.4509i
   0.4929 + 0.0851i
   1.9717 + 0.3403i
```

$I_2 = 0.3943+j0.681 = 0.7869\angle 59.93^\circ$ but $V = 5I_2 = 3.934\angle 59.93^\circ$.

$v(t) =$ **<u>3.934cos(3t+59.93°) V</u>**

Chapter 13, Problem 48.

Find $\mathbf{I}_x$ in the ideal transformer circuit of Fig. 13.113.

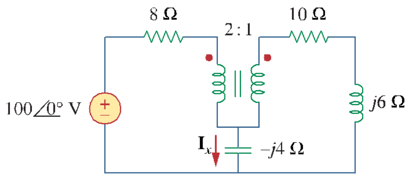
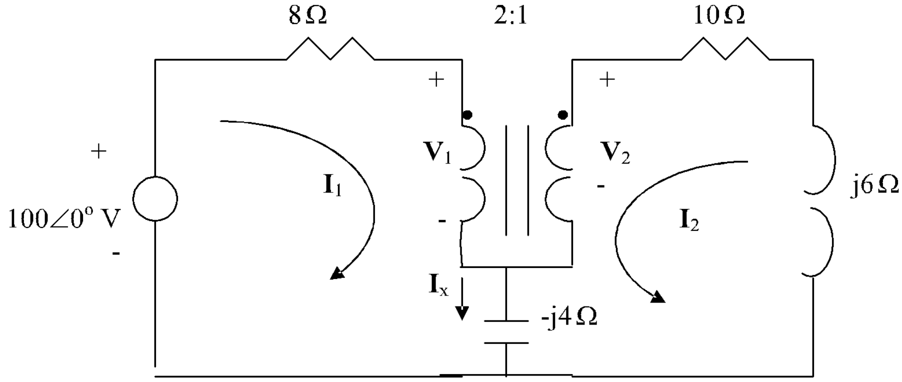


Figure 13.113
For Prob. 13.48.

Chapter 13, Solution 48.

We apply mesh analysis.



$$100 = (8 - j4)I_1 - j4I_2 + V_1 \qquad (1)$$

$$0 = (10 + j2)I_2 - j4I_1 + V_2 \qquad (2)$$

But

$$\frac{V_2}{V_1} = n = \frac{1}{2} \longrightarrow V_1 = 2V_2 \qquad (3)$$

$$\frac{I_2}{I_1} = -\frac{1}{n} = -2 \longrightarrow I_1 = -0.5I_2 \qquad (4)$$

Substituting (3) and (4) into (1) and (2), we obtain

$$100 = (-4 - j2)I_2 + 2V_2 \qquad (1)a$$
$$0 = (10 + j4)I_2 + V_2 \qquad (2)a$$

Solving (1)a and (2)a leads to $\mathbf{I}_2$ = -3.5503 +j1.4793

$$I_x = I_1 + I_2 = 0.5I_2 = \underline{1.923\angle 157.4^o \text{ A}}$$

Chapter 13, Problem 49.

PS ML Find current i_x in the ideal transformer circuit shown in Fig. 13.114.

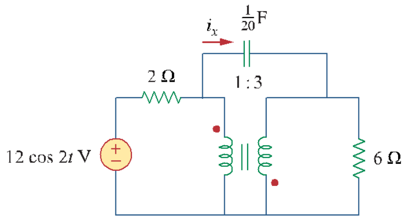
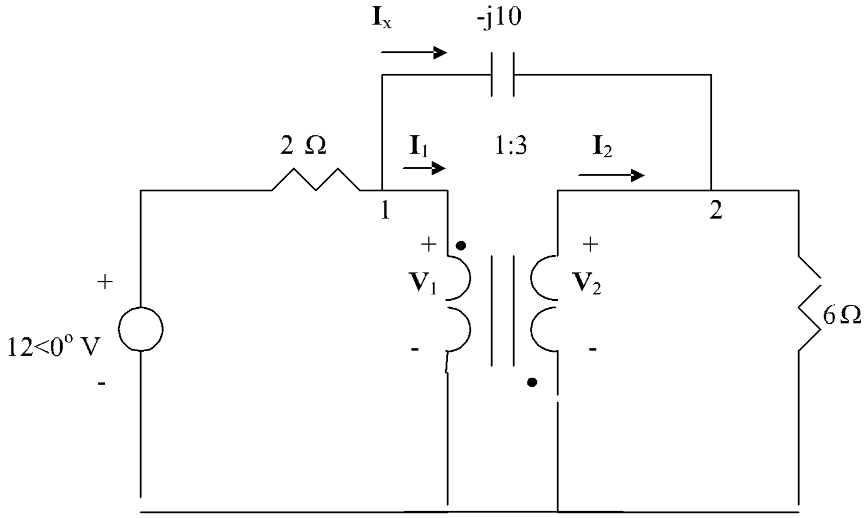


Figure 13.114
For Prob. 13.49.

Chapter 13, Solution 49.

$\omega = 2, \quad \frac{1}{20}\text{F} \longrightarrow \frac{1}{j\omega C} = -j10$



At node 1,

$$\frac{12 - V_1}{2} = \frac{V_1 - V_2}{-j10} + I_1 \longrightarrow 12 = 2I_1 + V_1(1 + j0.2) - j0.2V_2 \qquad (1)$$

At node 2,

$$I_2 + \frac{V_1 - V_2}{-j10} = \frac{V_2}{6} \longrightarrow 0 = 6I_2 + j0.6V_1 - (1 + j0.6)V_2 \qquad (2)$$

At the terminals of the transformer, $V_2 = -3V_1, \quad I_2 = -\frac{1}{3}I_1$

Substituting these in (1) and (2),

$$12 = -6I_2 + V_1(1 + j0.8), \qquad 0 = 6I_2 + V_1(3 + j2.4)$$

Adding these gives $\mathbf{V}_1$=1.829 –j1.463 and

$$I_x = \frac{V_1 - V_2}{-j10} = \frac{4V_1}{-j10} = 0.937\angle 51.34^o$$

$$\underline{i_x = 0.937\cos(2t + 51.34^o)\ \text{A}}$$

Chapter 13, Problem 50.

ML Calculate the input impedance for the network in Fig. 13.115.

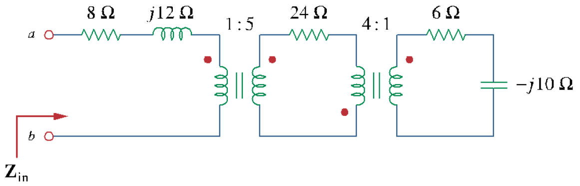


Figure 13.115
For Prob. 13.50.

Chapter 13, Solution 50.

The value of Z_{in} is not effected by the location of the dots since n^2 is involved.

$$Z_{in}' = (6 - j10)/(n')^2, \quad n' = 1/4$$

$$Z_{in}' = 16(6 - j10) = 96 - j160$$

$$Z_{in} = 8 + j12 + (Z_{in}' + 24)/n^2, \quad n = 5$$

$$Z_{in} = 8 + j12 + (120 - j160)/25 = 8 + j12 + 4.8 - j6.4$$

Z_{in} = **(12.8 + j5.6) ohms**

Chapter 13, Problem 51.

ML Use the concept of reflected impedance to find the input impedance and current $\mathbf{I}_1$ in Fig. 13.116.

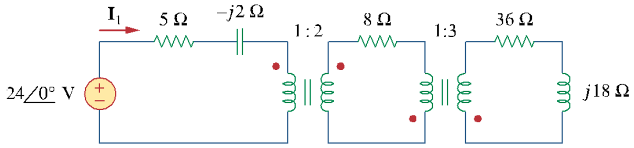


Figure 13.116
For Prob. 13.51.

Chapter 13, Solution 51.

Let $Z_3 = 36 + j18$, where Z_3 is reflected to the middle circuit.

$$Z_R' = Z_L/n^2 = (12 + j2)/4 = 3 + j0.5$$

$$Z_{in} = 5 - j2 + Z_R' = \underline{\mathbf{(8 - j1.5)\ ohms}}$$

$$I_1 = 24\angle 0°/Z_{Th} = 24\angle 0°/(8 - j1.5) = 24\angle 0°/8.14\angle -10.62° = \underline{\mathbf{8.95\angle 10.62°\ A}}$$

Chapter 13, Problem 52.

e2d For the circuit in Fig. 13.117, determine the turns ratio *n* that will cause maximum average power transfer to the load. Calculate that maximum average power.

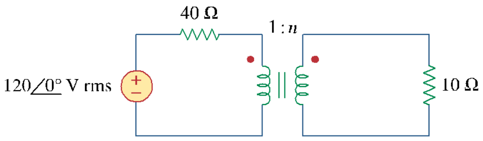


Figure 13.117
For Prob. 13.52.

Chapter 13, Solution 52.

For maximum power transfer,

$$40 = Z_L/n^2 = 10/n^2 \quad \text{or} \quad n^2 = 10/40 \quad \text{which yields } n = 1/2 = 0.5$$

$$I = 120/(40 + 40) = 3/2$$

$$p = I^2R = (9/4)x40 = \underline{\mathbf{90\ watts}}.$$

Chapter 13, Problem 53.

ML Refer to the network in Fig. 13.118.

(a) Find n for maximum power supplied to the 200-Ω load.
(b) Determine the power in the 200-Ω load if $n = 10$.

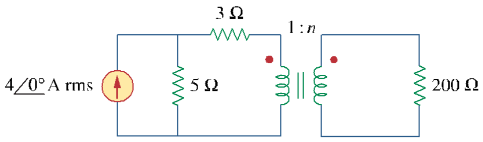
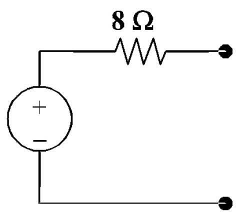


Figure 13.118
For Prob. 13.53.

Chapter 13, Solution 53.

(a) The Thevenin equivalent to the left of the transformer is shown below.



The reflected load impedance is $Z_L' = Z_L/n^2 = 200/n^2$.

For maximum power transfer, $8 = 200/n^2$ produces $n = \underline{\mathbf{5}}$.

(b) If $n = 10$, $Z_L' = 200/10 = 2$ and $I = 20/(8 + 2) = 2$

$p = I^2 Z_L' = (2)^2(2) =$ **8 watts**.

Chapter 13, Problem 54.

e2d A transformer is used to match an amplifier with an 8-Ω load as shown in Fig. 13.119. The Thevenin equivalent of the amplifier is: V_{Th} = 10 V, Z_{Th} = 128 Ω.

(a) Find the required turns ratio for maximum energy power transfer.
(b) Determine the primary and secondary currents.
(c) Calculate the primary and secondary voltages.

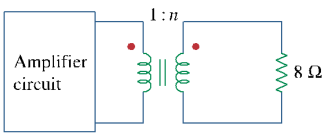
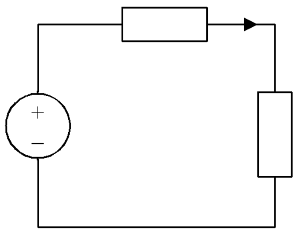


Figure 13.119
For Prob. 13.54.

Chapter 13, Solution 54.

(a)



For maximum power transfer,

$$Z_{Th} = Z_L/n^2, \text{ or } n^2 = Z_L/Z_{Th} = 8/128$$

$$n = \underline{\mathbf{0.25}}$$

(b) $I_1 = V_{Th}/(Z_{Th} + Z_L/n^2) = 10/(128 + 128) = \underline{\mathbf{39.06\ mA}}$

(c) $v_2 = I_2 Z_L = 156.24\text{x}8 \text{ mV} = 1.25 \text{ V}$

But $v_2 = nv_1$ therefore $v_1 = v_2/n = 4(1.25) = \underline{\mathbf{5\ V}}$

Chapter 13, Problem 55.

ML For the circuit in Fig. 13.120, calculate the equivalent resistance.

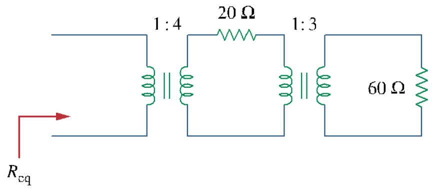


Figure 13.120
For Prob. 13.55.

Chapter 13, Solution 55.

We first reflect the 60-Ω resistance to the middle circuit.

$$Z_L' = 20 + \frac{60}{3^2} = 26.67\Omega$$

We now reflect this to the primary side.

$$Z_L = \frac{Z_L'}{4^2} = \frac{26.67}{16} = \underline{\mathbf{1.6669\ \Omega}}$$

Chapter 13, Problem 56.

Find the power absorbed by the 10-Ω resistor in the ideal transformer circuit of Fig. 13.121.

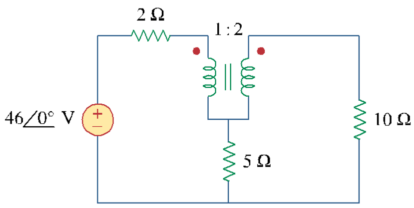
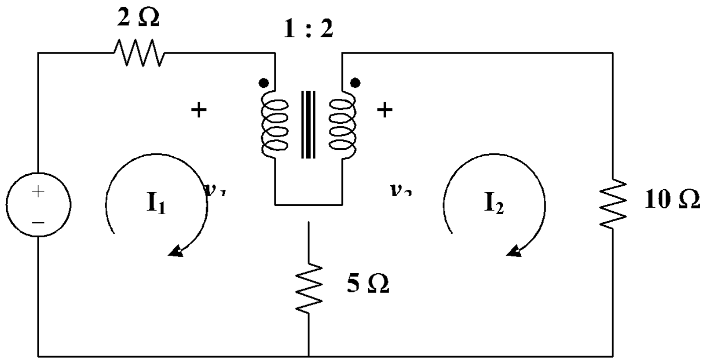


Figure 13.121
For Prob. 13.56.

Chapter 13, Solution 56.

We apply mesh analysis to the circuit as shown below.



For mesh 1, $46 = 7I_1 - 5I_2 + v_1$ (1)

For mesh 2, $v_2 = 15I_2 - 5I_1$ (2)

At the terminals of the transformer,

$$v_2 = nv_1 = 2v_1 \quad (3)$$

$$I_1 = nI_2 = 2I_2 \quad (4)$$

Substituting (3) and (4) into (1) and (2),

$$46 = 9I_2 + v_1 \quad (5)$$

$$v_1 = 2.5I_2 \quad (6)$$

Combining (5) and (6), $46 = 11.5I_2$ or $I_2 = 4$

$P_{10} = 0.5I_2^2(10) =$ **80 watts**.

Chapter 13, Problem 57.

For the ideal transformer circuit of Fig. 13.122 below, find:

(a) $\mathbf{I}_1$ and $\mathbf{I}_2$,
(b) $\mathbf{V}_1$, $\mathbf{V}_2$, and $\mathbf{V}_o$,
(c) the complex power supplied by the source.

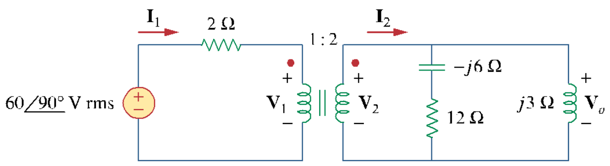


Figure 13.122
For Prob. 13.57.

Chapter 13, Solution 57.

(a) $Z_L = j3||(12 - j6) = j3(12 - j6)/(12 - j3) = (12 + j54)/17$

Reflecting this to the primary side gives

$Z_{in} = 2 + Z_L/n^2 = 2 + (3 + j13.5)/17 = 2.3168\angle 20.04°$

$I_1 = v_s/Z_{in} = 60\angle 90°/2.3168\angle 20.04° =$ **<u>25.9∠69.96° A(rms)</u>**

$I_2 = I_1/n =$ **<u>12.95∠69.96° A(rms)</u>**

(b) $60\angle 90° = 2I_1 + v_1$ or $v_1 = j60 - 2I_1 = j60 - 51.8\angle 69.96°$

$v_1 =$ **<u>21.06∠147.44° V(rms)</u>**

$v_2 = nv_1 =$ **<u>42.12∠147.44° V(rms)</u>**

$v_o = v_2 =$ **<u>42.12∠147.44° V(rms)</u>**

(c) $S = v_s I_1^* = (60\angle 90°)(25.9\angle -69.96°) =$ **<u>1554∠20.04° VA</u>**

Chapter 13, Problem 58.

PS ML Determine the average power absorbed by each resistor in the circuit of Fig. 13.123.

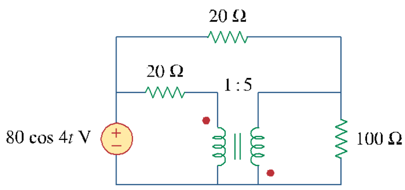
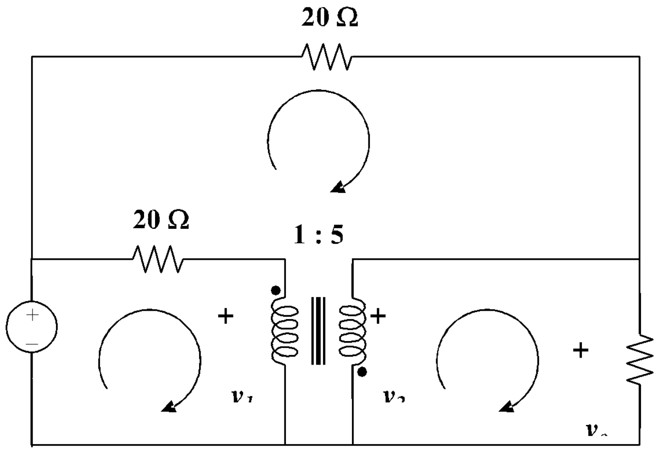


Figure 13.123
For Prob. 13.58.

Chapter 13, Solution 58.

Consider the circuit below.



For mesh1, $80 = 20I_1 - 20I_3 + v_1$ (1)

For mesh 2, $v_2 = 100I_2$ (2)

For mesh 3, $0 = 40I_3 - 20I_1$ which leads to $I_1 = 2I_3$ (3)

At the transformer terminals, $v_2 = -nv_1 = -5v_1$ (4)

$I_1 = -nI_2 = -5I_2$ (5)

From (2) and (4), $-5v_1 = 100I_2$ or $v_1 = -20I_2$ (6)

Substituting (3), (5), and (6) into (1),

$4 = I_1 - I_2 - I_3 = I_1 - (I_1/(-5)) - I_1/2 = (7/10)I_1$

$I_1 = 40/7,\ I_2 = -8/7,\ I_3 = 20/7$

p_{20}(the one between 1 and 3) $= 0.5(20)(I_1 - I_3)^2 = 10(20/7)^2 =$ **81.63 watts**

p_{20}(at the top of the circuit) $= 0.5(20)I_3^2 =$ **81.63 watts**

$p_{100} = 0.5(100)I_2^2 =$ **65.31 watts**

Chapter 13, Problem 59.

PS ML In the circuit of Fig. 13.124, let $v_s = 40 \cos 1000t$. Find the average power delivered to each resistor.

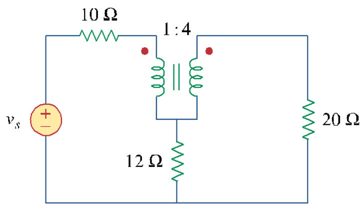
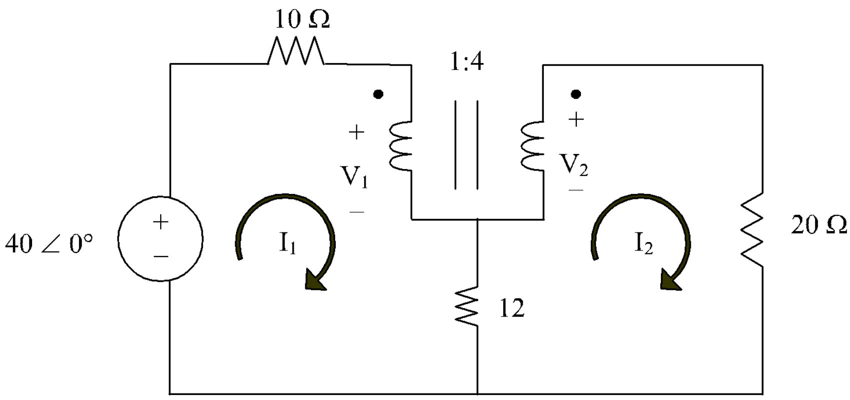


Figure 13.124
For Prob. 13.59.

Chapter 13, Solution 59.

We apply mesh analysis to the circuit as shown below.



For mesh 1,

$$-40 + 22I_1 - 12I_2 + V_1 = 0 \qquad (1)$$

For mesh 2,

$$-12I_1 + 32I_2 - V_2 = 0 \qquad (2)$$

At the transformer terminals,

$$-4V_1 + V_2 = 0 \qquad (3)$$

$$I_1 - 4I_2 = 0 \qquad (4)$$

Putting (1), (2), (3), and (4) in matrix form, we obtain

$$\begin{bmatrix} 22 & -12 & 1 & 0 \\ -12 & 32 & 0 & -1 \\ 0 & 0 & -4 & 1 \\ 1 & -4 & 0 & 0 \end{bmatrix} I = \begin{bmatrix} 40 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

```
>> A=[22,-12,1,0;-12,32,0,-1;0,0,-4,1;1,-4,0,0]
A =
   22  -12    1    0
  -12   32    0   -1
    0    0   -4    1
    1   -4    0    0
>> U=[40;0;0;0]
U =
   40
    0
    0
    0
>> X=inv(A)*U
X =
   2.2222
   0.5556
  -2.2222
  -8.8889
```

For 10-Ω resistor,

$P_{10} = [(2.222)^2/2]10 =$ **24.69 W**

For 12-Ω resistor,

$P_{12} = [(2.222–0.5556)^2/2]12 =$ **16.661 W**

For 20-Ω resistor,

$P_{20} = [(0.5556)^2/2]20 =$ **3.087 W**.

Chapter 13, Problem 60.

Refer to the circuit in Fig. 13.125 on the following page.

(a) Find currents $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$.
(b) Find the power dissipated in the 40-Ω resistor.

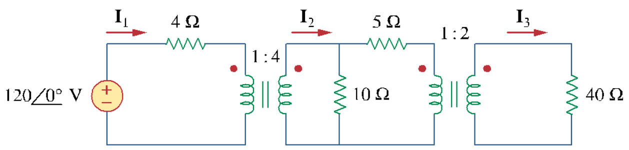


Figure 13.125
For Prob. 13.60.

Chapter 13, Solution 60.

(a) Transferring the 40-ohm load to the middle circuit,

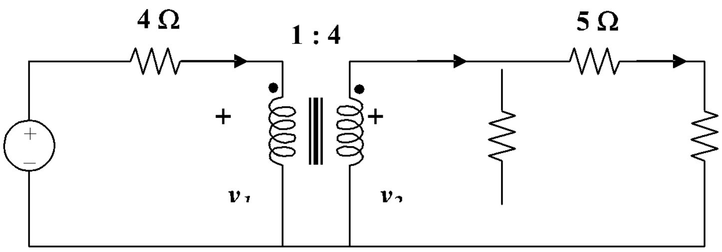
$Z_L' = 40/(n')^2 = 10$ ohms where $n' = 2$

$10||(5 + 10) = 6$ ohms

We transfer this to the primary side.

$Z_{in} = 4 + 6/n^2 = 4 + 96 = 100$ ohms, where $n = 0.25$

$I_1 = 120/100 =$ **1.2 A** and $I_2 = I_1/n =$ **4.8 A**



Using current division, $I_2' = (10/25)I_2 = 1.92$ and $I_3 = I_2'/n' =$ **0.96 A**

(b) $p = 0.5(I_3)^2(40) =$ **18.432 watts**

Chapter 13, Problem 61.

* For the circuit in Fig. 13.126, find $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{V}_o$.

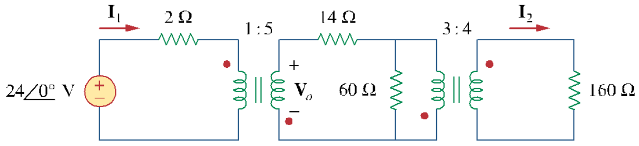


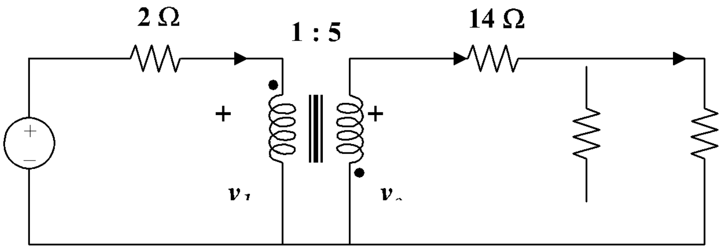
Figure 13.126
For Prob. 13.61.

* An asterisk indicates a challenging problem.

Chapter 13, Solution 61.

We reflect the 160-ohm load to the middle circuit.

$$Z_R = Z_L/n^2 = 160/(4/3)^2 = 90 \text{ ohms, where } n = 4/3$$



$$14 + 60||90 = 14 + 36 = 50 \text{ ohms}$$

We reflect this to the primary side.

$$Z_R' = Z_L'/(n')^2 = 50/5^2 = 2 \text{ ohms when } n' = 5$$

$$I_1 = 24/(2 + 2) = \underline{\mathbf{6A}}$$

$$24 = 2I_1 + v_1 \text{ or } v_1 = 24 - 2I_1 = 12 \text{ V}$$

$$v_o = -nv_1 = \underline{\mathbf{-60\ V}},\ I_o = -I_1/n_1 = -6/5 = -1.2$$

$$I_o' = [60/(60 + 90)]I_o = -0.48\text{A}$$

$$I_2 = -I_o'/n = 0.48/(4/3) = \underline{\mathbf{0.36\ A}}$$

Chapter 13, Problem 62.

For the network in Fig. 13.127, find

(a) the complex power supplied by the source,
(b) the average power delivered to the 18-Ω resistor.

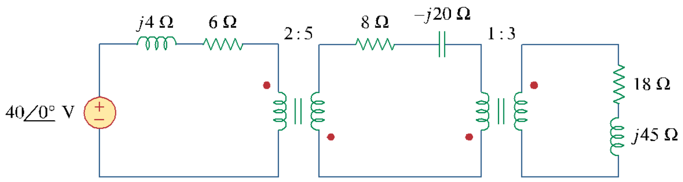


Figure 13.127
For Prob. 13.62.

Chapter 13, Solution 62.

(a) Reflect the load to the middle circuit.

$Z_L' = 8 - j20 + (18 + j45)/3^2 = 10 - j15$

We now reflect this to the primary circuit so that

$Z_{in} = 6 + j4 + (10 - j15)/n^2 = 7.6 + j1.6 = 7.767\angle 11.89°$, where $n = 5/2 = 2.5$

$I_1 = 40/Z_{in} = 40/7.767\angle 11.89° = 5.15\angle -11.89°$

$S = 0.5 v_s I_1^* = (20\angle 0°)(5.15\angle 11.89°) =$ **$\underline{103\angle 11.89° \text{ VA}}$**

(b) $I_2 = -I_1/n, \quad n = 2.5$

$I_3 = -I_2/n', \quad n = 3$

$I_3 = I_1/(nn') = 5.15\angle -11.89°/(2.5x3) = 0.6867\angle -11.89°$

$p = 0.5|I_2|^2(18) = 9(0.6867)^2 =$ **$\underline{4.244 \text{ watts}}$**

Chapter 13, Problem 63.

ML Find the mesh currents in the circuit of Fig. 13.128

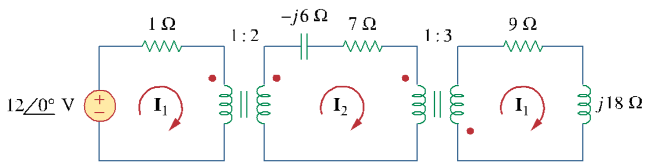


Figure 13.128
For Prob. 13.63.

Chapter 13, Solution 63.

Reflecting the (9 + j18)-ohm load to the middle circuit gives,

$Z_{in}' = 7 - j6 + (9 + j18)/(n')^2 = 7 - j6 + 1 + j2 = 8 - j4$ when $n' = 3$

Reflecting this to the primary side,

$Z_{in} = 1 + Z_{in}'/n^2 = 1 + 2 - j = 3 - j$, where $n = 2$

$I_1 = 12\angle 0°/(3 - j) = 12/3.162\angle -18.43° = $ **$\underline{3.795\angle 18.43 A}$**

$I_2 = I_1/n = $ **$\underline{1.8975\angle 18.43° A}$**

$I_3 = -I_2/n^2 = $ **$\underline{632.5\angle 161.57° mA}$**

Chapter 13, Problem 64.

PS ML For the circuit in Fig. 13.129, find the turns ratio so that the maximum power is delivered to the 30-kΩ resistor.

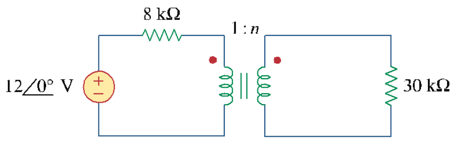
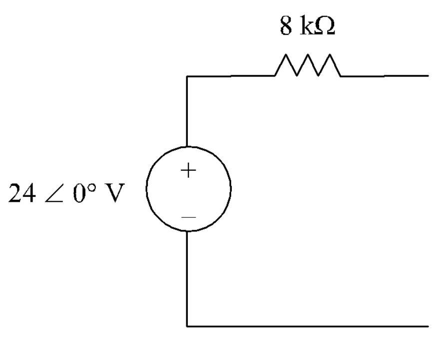


Figure 13.129
For Prob. 13.64.

Chapter 13, Solution 64.

The Thevenin equivalent to the left of the transformer is shown below.



The reflected load impedance is

$$Z_L' = \frac{Z_L}{n^2} = \frac{30k}{n^2}$$

For maximum power transfer,

$$8k\Omega = \frac{30k\Omega}{n^2} \longrightarrow n^2 = 30/8 = 3.75$$

n =**<u>1.9365</u>**

Chapter 13, Problem 65.

* Calculate the average power dissipated by the 20-Ω resistor in Fig. 13.130.

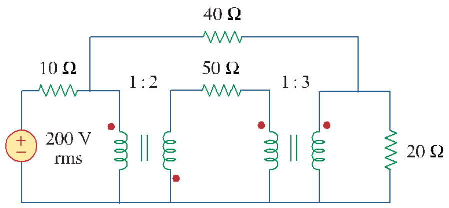
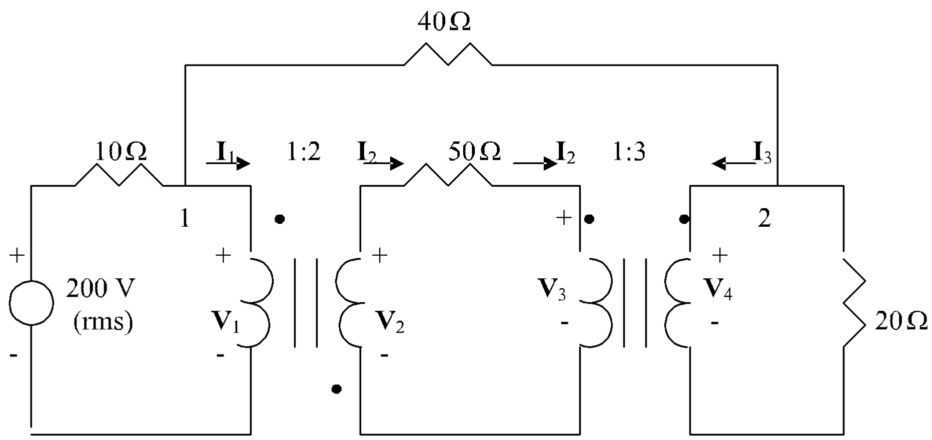


Figure 13.130
For Prob. 13.65.

* An asterisk indicates a challenging problem.

Chapter 13, Solution 65.



At node 1,

$$\frac{200 - V_1}{10} = \frac{V_1 - V_4}{40} + I_1 \quad \longrightarrow \quad 200 = 1.25V_1 - 0.25V_4 + 10I_1 \qquad (1)$$

At node 2,

$$\frac{V_1 - V_4}{40} = \frac{V_4}{20} + I_3 \quad \longrightarrow \quad V_1 = 3V_4 + 40I_3 \qquad (2)$$

At the terminals of the first transformer,

$$\frac{V_2}{V_1} = -2 \quad \longrightarrow \quad V_2 = -2V_1 \qquad (3)$$

$$\frac{I_2}{I_1} = -1/2 \quad \longrightarrow \quad I_1 = -2I_2 \qquad (4)$$

For the middle loop,

$$-V_2 + 50I_2 + V_3 = 0 \quad \longrightarrow \quad V_3 = V_2 - 50I_2 \qquad (5)$$

At the terminals of the second transformer,

$$\frac{V_4}{V_3} = 3 \quad \longrightarrow \quad V_4 = 3V_3 \tag{6}$$

$$\frac{I_3}{I_2} = -1/3 \quad \longrightarrow \quad I_2 = -3I_3 \tag{7}$$

We have seven equations and seven unknowns. Combining (1) and (2) leads to

$$200 = 3.5V_4 + 10I_1 + 50I_3$$

But from (4) and (7), $I_1 = -2I_2 = -2(-3I_3) = 6I_3$. Hence

$$200 = 3.5V_4 + 110I_3 \tag{8}$$

From (5), (6), (3), and (7),

$$V_4 = 3(V_2 - 50I_2) = 3V_2 - 150I_2 = -6V_1 + 450I_3$$

Substituting for V_1 in (2) gives

$$V_4 = -6(3V_4 + 40I_3) + 450I_3 \quad \longrightarrow \quad I_3 = \frac{19}{210}V_4 \tag{9}$$

Substituting (9) into (8) yields

$$200 = 13.452V_4 \quad \longrightarrow \quad V_4 = 14.87$$

$$P = \frac{V^2{}_4}{20} = \underline{11.05\text{ W}}$$

Chapter 13, Problem 66.

An ideal autotransformer with a 1:4 step-up turns ratio has its secondary connected to a 120-Ω load and the primary to a 420-V source. Determine the primary current.

Chapter 13, Solution 66.

$$v_1 = 420 \text{ V} \qquad (1)$$

$$v_2 = 120I_2 \qquad (2)$$

$$v_1/v_2 = 1/4 \text{ or } v_2 = 4v_1 \qquad (3)$$

$$I_1/I_2 = 4 \text{ or } I_1 = 4 I_2 \qquad (4)$$

Combining (2) and (4),

$$v_2 = 120[(1/4)I_1] = 30 I_1$$

$$4v_1 = 30I_1$$

$$4(420) = 1680 = 30I_1 \text{ or } I_1 = \underline{\mathbf{56 A}}$$

Chapter 13, Problem 67.

An autotransformer with a 40 percent tap is supplied by a 400-V, 60-Hz source and is used for step-down operation. A 5-kVA load operating at unity power factor is connected to the secondary terminals. Find:

(a) the secondary voltage
(b) the secondary current
(c) the primary current

Chapter 13, Solution 67.

(a) $$\frac{V_1}{V_2} = \frac{N_1 + N_2}{N_2} = \frac{1}{0.4} \longrightarrow V_2 = 0.4V_1 = 0.4\text{x}400 = \underline{160 \text{ V}}$$

(b) $$S_2 = I_2V_2 = 5{,}000 \longrightarrow I_2 = \frac{5000}{160} = \underline{31.25 \text{ A}}$$

(c) $$S_2 = S_1 = I_1V_1 = 5{,}000 \longrightarrow I_1 = \frac{5000}{400} = \underline{12.5 \text{ A}}$$

Chapter 13, Problem 68.

ML In the ideal autotransformer of Fig. 13.131, calculate $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_o$ Find the average power delivered to the load.

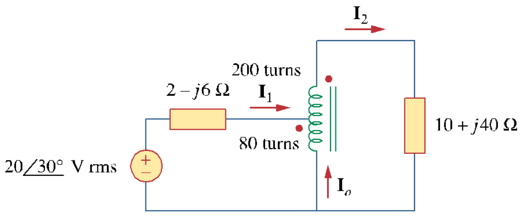
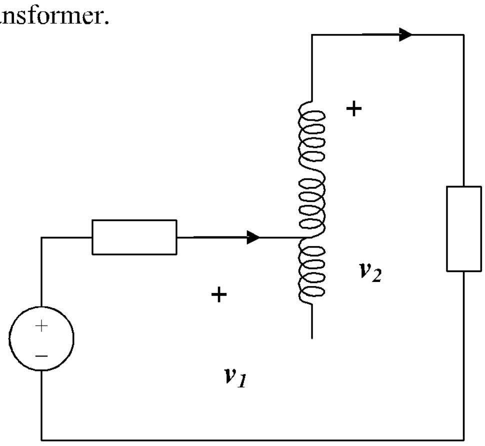


Figure 13.131
For Prob. 13.68.

Chapter 13, Solution 68.

This is a step-up transformer.



For the primary circuit, $20\angle 30° = (2 - j6)I_1 + v_1$ (1)

For the secondary circuit, $v_2 = (10 + j40)I_2$ (2)

At the autotransformer terminals,

$$v_1/v_2 = N_1/(N_1 + N_2) = 200/280 = 5/7,$$

$$\text{thus } v_2 = 7v_1/5 \quad (3)$$

Also, $I_1/I_2 = 7/5$ or $I_2 = 5I_1/7$ (4)

Substituting (3) and (4) into (2), $v_1 = (10 + j40)25I_1/49$

Substituting that into (1) gives $20\angle 30° = (7.102 + j14.408)I_1$

$I_1 = 20\angle 30°/16.063\angle 63.76° =$ <u>**$1.245\angle -33.76°$ A**</u>

$I_2 = 5I_1/7 =$ <u>**$0.8893\angle -33.76°$ A**</u>

$I_o = I_1 - I_2 = [(5/7) - 1]I_1 = -2I_1/7 =$ <u>**$0.3557\angle 146.2°$ A**</u>

$p = |I_2|^2R = (0.8893)^2(10) =$ <u>**7.51 watts**</u>

Chapter 13, Problem 69.

eⒶd * In the circuit of Fig. 13.132, $\mathbf{Z}_L$ is adjusted until maximum average power is delivered to $\mathbf{Z}_L$. Find $\mathbf{Z}_L$ and the maximum average power transferred to it. Take $N_1 = 600$ turns and $N_2 = 200$ turns.

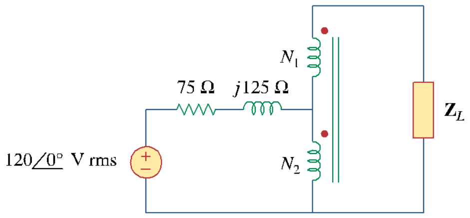
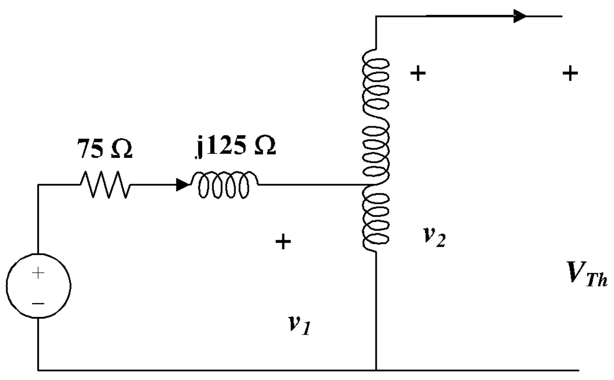


Figure 13.132
For Prob. 13.69.

* An asterisk indicates a challenging problem.

Chapter 13, Solution 69.

We can find the Thevenin equivalent.

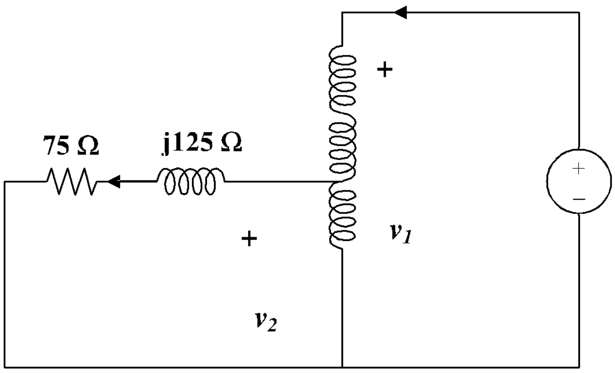


$I_1 = I_2 = 0$

As a step up transformer, $v_1/v_2 = N_1/(N_1 + N_2) = 600/800 = 3/4$

$$v_2 = 4v_1/3 = 4(120)/3 = 160\angle 0° \text{ rms} = V_{Th}.$$

To find Z_{Th}, connect a 1-V source at the secondary terminals. We now have a step-down transformer.



$$v_1 = 1V,\ v_2 = I_2(75 + j125)$$

But $v_1/v_2 = (N_1 + N_2)/N_1 = 800/200$ which leads to $v_1 = 4v_2 = 1$

and $v_2 = 0.25$

$$I_1/I_2 = 200/800 = 1/4 \text{ which leads to } I_2 = 4I_1$$

Hence $0.25 = 4I_1(75 + j125)$ or $I_1 = 1/[16(75 + j125)$

$Z_{Th} = 1/I_1 = 16(75 + j125)$

Therefore, $Z_L = Z_{Th}^* =$ **(1.2 – j2) kΩ**

Since V_{Th} is rms, $p = (|V_{Th}|/2)^2/R_L = (80)^2/1200 =$ **5.333 watts**

Chapter 13, Problem 70.

ML In the ideal transformer circuit shown in Fig. 13.133, determine the average power delivered to the load.

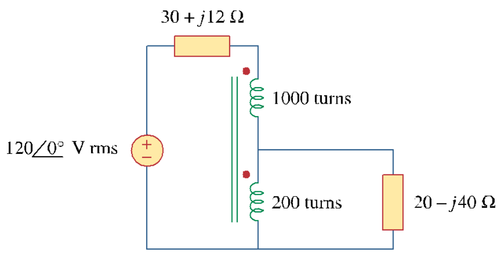
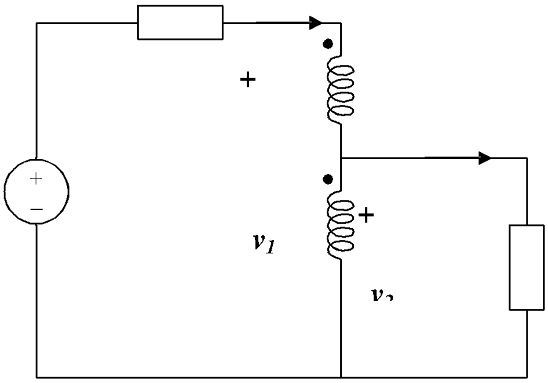


Figure 13.133
For Prob. 13.70.

Chapter 13, Solution 70.

This is a step-down transformer.



$$I_1/I_2 = N_2/(N_1 + N_2) = 200/1200 = 1/6, \text{ or } I_1 = I_2/6 \qquad (1)$$

$$v_1/v_2 = (N_2 + N_2)/N_2 = 6, \text{ or } v_1 = 6v_2 \qquad (2)$$

For the primary loop, $\quad 120 = (30 + j12)I_1 + v_1 \qquad (3)$

For the secondary loop, $\quad v_2 = (20 - j40)I_2 \qquad (4)$

Substituting (1) and (2) into (3),

$$120 = (30 + j12)(I_2/6) + 6v_2$$

and substituting (4) into this yields

$$120 = (49 - j38)I_2 \text{ or } I_2 = 1.935\angle 37.79°$$

$$p = |I_2|^2(20) = \underline{\textbf{74.9 watts}}.$$

Chapter 13, Problem 71.

In the autotransformer circuit in Fig. 13.134, show that

$$\mathbf{Z}_{in} = \left(1 + \frac{N_1}{N_2}\right)^2 \mathbf{Z}_L$$

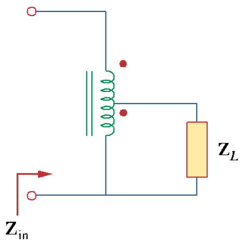


Figure 13.134
For Prob. 13.71.

Chapter 13, Solution 71.

$$Z_{in} = V_1/I_1$$

But $\quad V_1I_1 = V_2I_2$, or $V_2 = I_2Z_L$ and $I_1/I_2 = N_2/(N_1 + N_2)$

Hence $\quad V_1 = V_2I_2/I_1 = Z_L(I_2/I_1)I_2 = Z_L(I_2/I_1)^2I_1$

$$V_1/I_1 = Z_L[(N_1 + N_2)/N_2]^2$$

$$Z_{in} = \underline{\mathbf{[1 + (N_1/N_2)]^2Z_L}}$$

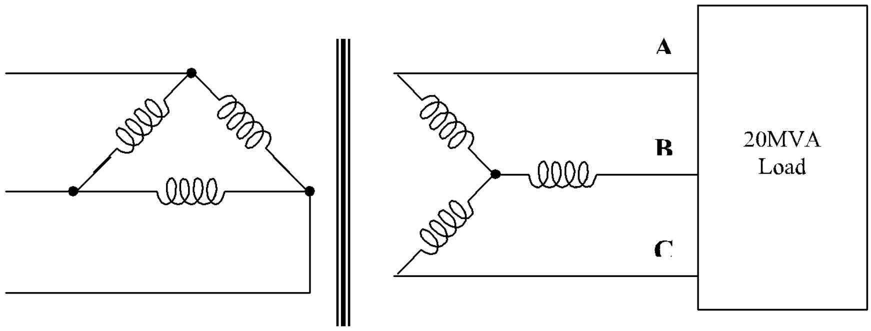
Chapter 13, Problem 72.

e2d In order to meet an emergency, three single-phase transformers with 12,470/7,200 V rms are connected in Δ-Y to form a three-phase transformer which is fed by a 12,470-V transmission line. If the transformer supplies 60 MVA to a load, find:

(a) the turns ratio for each transformer,
(b) the currents in the primary and secondary windings of the transformer,
(c) the incoming and outgoing transmission line currents.

Chapter 13, Solution 72.

(a) Consider just one phase at a time.



$$n = V_L/\sqrt{3}V_{Lp} = 7200/(12470\sqrt{3}) = \underline{\mathbf{1/3}}$$

(b) The load carried by each transformer is 60/3 = 20 MVA.

Hence I_{Lp} = 20 MVA/12.47 k = **<u>1604 A</u>**

I_{Ls} = 20 MVA/7.2 k = **<u>2778 A</u>**

(c) The current in incoming line a, b, c is

$$\sqrt{3}I_{Lp} = \sqrt{3}\text{x}1603.85 = \underline{\mathbf{2778\ A}}$$

Current in each outgoing line A, B, C is

$$2778/(n\sqrt{3}) = \underline{\mathbf{4812\ A}}$$

Chapter 13, Problem 73.

ML Figure 13.135 on the following page shows a three-phase transformer that supplies a Y-connected load.

(a) Identify the transformer connection.
(b) Calculate currents $\mathbf{I}_2$ and $\mathbf{I}_c$.
(c) Find the average power absorbed by the load.

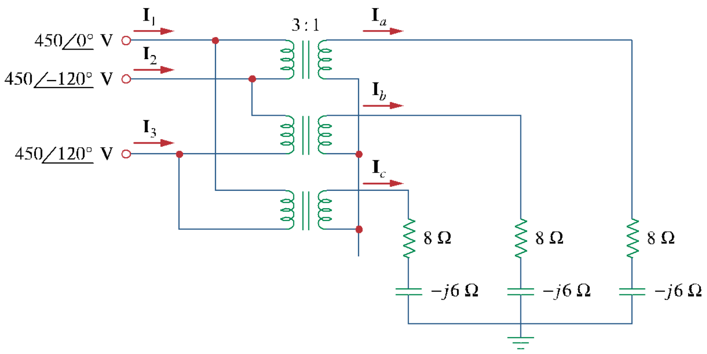


Figure 13.135
For Prob. 13.73.

Chapter 13, Solution 73.

(a) This is a **three-phase Δ-Y transformer**.

(b) $V_{Ls} = nv_{Lp}/\sqrt{3} = 450/(3\sqrt{3}) = 86.6$ V, where $n = 1/3$

As a Y-Y system, we can use per phase equivalent circuit.

$I_a = V_{an}/Z_Y = 86.6\angle 0°/(8 - j6) = 8.66\angle 36.87°$

$I_c = I_a\angle 120° = \underline{\mathbf{8.66\angle 156.87°\ A}}$

$I_{Lp} = n\sqrt{3}\, I_{Ls}$

$I_1 = (1/3)\sqrt{3}\,(8.66\angle 36.87°) = 5\angle 36.87°$

$I_2 = I_1\angle -120° = \underline{\mathbf{5\angle -83.13°\ A}}$

(c) $p = 3|I_a|^2(8) = 3(8.66)^2(8) = \underline{\mathbf{1.8\ kw}}$.

Chapter 13, Problem 74.

Consider the three-phase transformer shown in Fig. 13.136. The primary is fed by a three-phase source with line voltage of 2.4 kV rms, while the secondary supplies a three-phase 120-kW balanced load at pf of 0.8. Determine:

(a) the type of transformer connections,
(b) the values of I_{LS} and I_{PS},
(c) the values of I_{LP} and I_{PP},
(d) the kVA rating of each phase of the transformer.

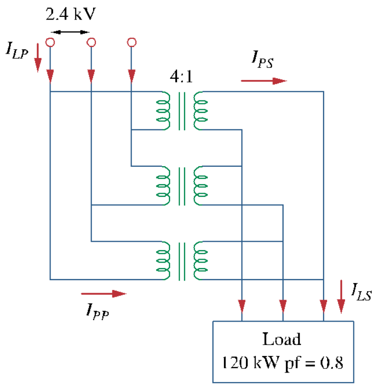


Figure 13.136
For Prob. 13.74.

Chapter 13, Solution 74.

(a) This is a **Δ-Δ connection**.

(b) The easy way is to consider just one phase.

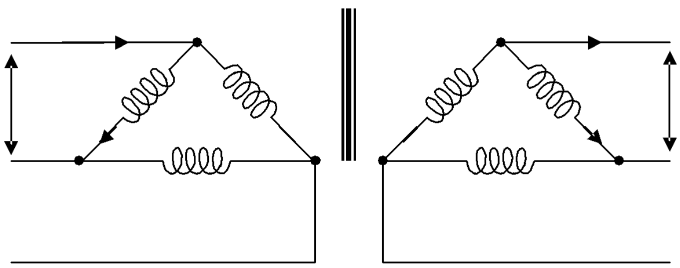
$$1:n = 4:1 \quad \text{or} \quad n = 1/4$$

$n = V_2/V_1$ which leads to $V_2 = nV_1 = 0.25(2400) = 600$

$$\text{i.e. } V_{Lp} = 2400 \text{ V and } V_{Ls} = 600 \text{ V}$$

$S = p/\cos\theta = 120/0.8 \text{ kVA} = 150 \text{ kVA}$

$p_L = p/3 = 120/3 = 40 \text{ kw}$



But $p_{Ls} = V_{ps}I_{ps}$

For the Δ-load, $I_L = \sqrt{3}\,I_p$ and $V_L = V_p$

Hence, $I_{ps} = 40{,}000/600 = 66.67 \text{ A}$

$$I_{Ls} = \sqrt{3}\,I_{ps} = \sqrt{3}\text{x}66.67 = \underline{\mathbf{115.48 \text{ A}}}$$

(c) Similarly, for the primary side

$$p_{pp} = V_{pp}I_{pp} = p_{ps} \text{ or } I_{pp} = 40{,}000/2400 = \underline{\mathbf{16.667 \text{ A}}}$$

$$\text{and} \quad I_{Lp} = \sqrt{3}\,I_p = \underline{\mathbf{28.87 \text{ A}}}$$

(d) Since $S = 150 \text{ kVA}$ therefore $S_p = S/3 = \underline{\mathbf{50 \text{ kVA}}}$

Chapter 13, Problem 75.

A balanced three-phase transformer bank with the Δ-Y connection depicted in Fig. 13.137 is used to step down line voltages from 4,500 V rms to 900 V rms. If the transformer feeds a 120-kVA load, find:

(a) the turns ratio for the transformer,
(b) the line currents at the primary and secondary sides.

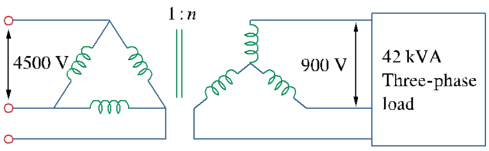


Figure 13.137
For Prob. 13.75.

Chapter 13, Solution 75.

(a) $n = V_{Ls}/(\sqrt{3}\,V_{Lp}) = 900/(4500\sqrt{3}) = \underline{\mathbf{0.11547}}$

(b) $S = \sqrt{3}\,V_{Ls}I_{Ls}$ or $I_{Ls} = 120{,}000/(900\sqrt{3}) = \underline{\mathbf{76.98\ A}}$

$I_{Ls} = I_{Lp}/(n\sqrt{3}) = 76.98/(2.887\sqrt{3}) = \underline{\mathbf{15.395\ A}}$

Chapter 13, Problem 76.

A Y-Δ three-phase transformer is connected to a 60-kVA load with 0.85 power factor (leading) through a feeder whose impedance is $0.05 + j0.1\ \Omega$ per phase, as shown in Fig. 13.138. Find the magnitude of:

(a) the line current at the load,
(b) the line voltage at the secondary side of the transformer,
(c) the line current at the primary side of the transformer.

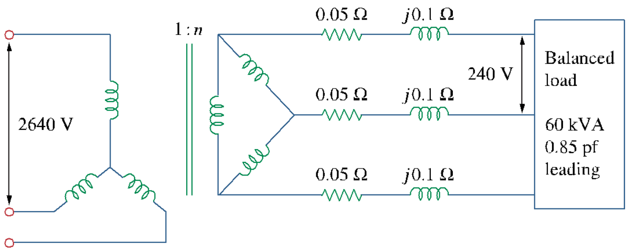


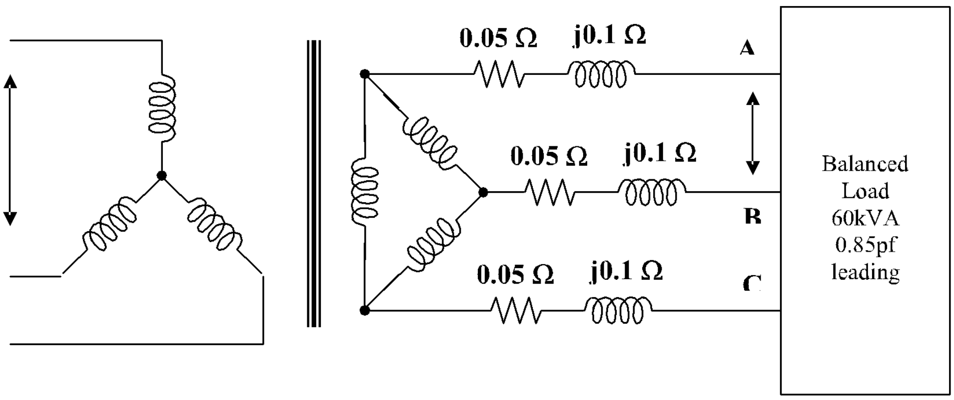
Figure 13.138
For Prob. 13.76.

Chapter 13, Solution 76.

(a) At the load, $V_L = 240\text{ V} = V_{AB}$

$$V_{AN} = V_L/\sqrt{3} = 138.56\text{ V}$$

Since $S = \sqrt{3}V_L I_L$ then $I_L = 60{,}000/(240\sqrt{3}) =$ **<u>144.34 A</u>**



(b) Let $V_{AN} = |V_{AN}|\angle 0° = 138.56\angle 0°$

$\cos\theta = pf = 0.85$ or $\theta = 31.79°$

$I_{AA'} = I_L\angle\theta = 144.34\angle 31.79°$

$V_{A'N'} = ZI_{AA'} + V_{AN}$

$= 138.56\angle 0° + (0.05 + j0.1)(144.34\angle 31.79°)$

$= 138.03\angle 6.69°$

$V_{Ls} = V_{A'N'}\sqrt{3} = 137.8\sqrt{3} =$ **<u>238.7 V</u>**

(c) For Y-Δ connections,

$$n = \sqrt{3}V_{Ls}/V_{ps} = \sqrt{3}\text{x}238.7/2640 = 0.1569$$

$f_{Lp} = nI_{Ls}/\sqrt{3} = 0.1569\text{x}144.34/\sqrt{3} =$ **<u>13.05 A</u>**

Chapter 13, Problem 77.

e2d The three-phase system of a town distributes power with a line voltage of 13.2 kV. A pole transformer connected to single wire and ground steps down the high-voltage wire to 120 V rms and serves a house as shown in Fig. 13.139.

(a) Calculate the turns ratio of the pole transformer to get 120 V.
(b) Determine how much current a 100-W lamp connected to the 120-V hot line draws from the high-voltage line.

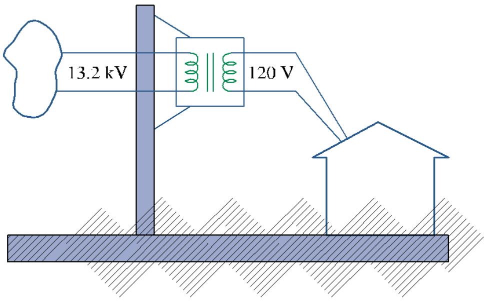


Figure 13.139
For Prob. 13.77.

Chapter 13, Solution 77.

(a) This is a single phase transformer. $V_1 = 13.2$ kV, $V_2 = 120$ V

$n = V_2/V_1 = 120/13{,}200 = 1/110$, therefore $n = \underline{\mathbf{1/110}}$

or 110 turns on the primary to every turn on the secondary.

(b) $P = VI$ or $I = P/V = 100/120 = 0.8333$ A

$I_1 = nI_2 = 0.8333/110 = \underline{\mathbf{7.576\ mA}}$

Chapter 13, Problem 78.

Use *PSpice* to determine the mesh currents in the circuit of Fig. 13.140. Take ω = 1 rad/s.

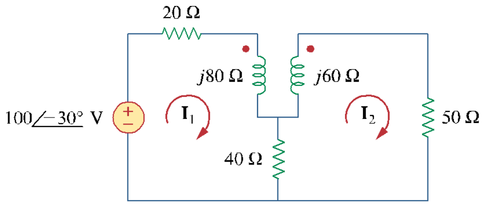


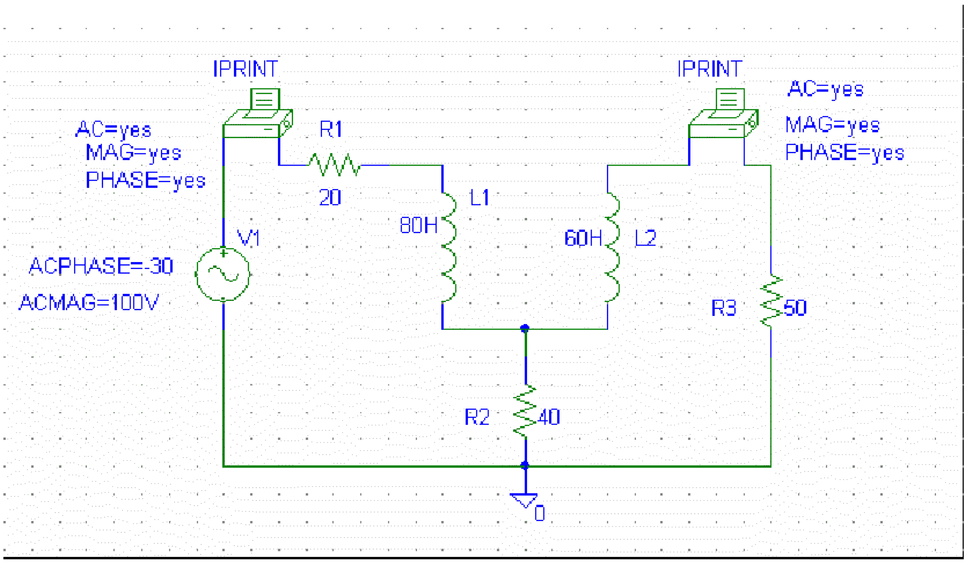
Figure 13.140
For Prob. 13.78.

Chapter 13, Solution 78.

We convert the reactances to their inductive values.

$$X = \omega L \quad \longrightarrow \quad L = \frac{X}{\omega}$$

The schematic is as shown below.



When the circuit is simulated, the output file contains

FREQ IM(V_PRINT1)IP(V_PRINT1)

1.592E-01 9.971E-01 -9.161E+01

FREQ IM(V_PRINT2)IP(V_PRINT2)

1.592E-01 3.687E-01 -1.253E+02

From this, we obtain

I_1 = **<u>997.1∠–91.61° mA</u>**, I_2 = **<u>368.7∠–135.3° mA</u>**.

Chapter 13, Problem 79.

Use *PSpice* to find $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ in the circuit of Fig. 13.141.

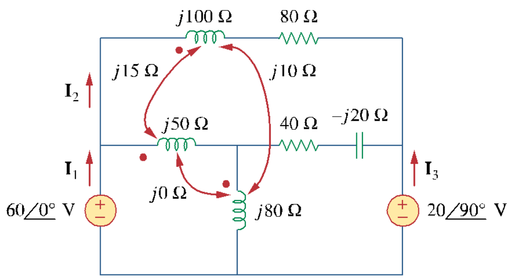


Figure 13.141
For Prob. 13.79.

Chapter 13, Solution 79.

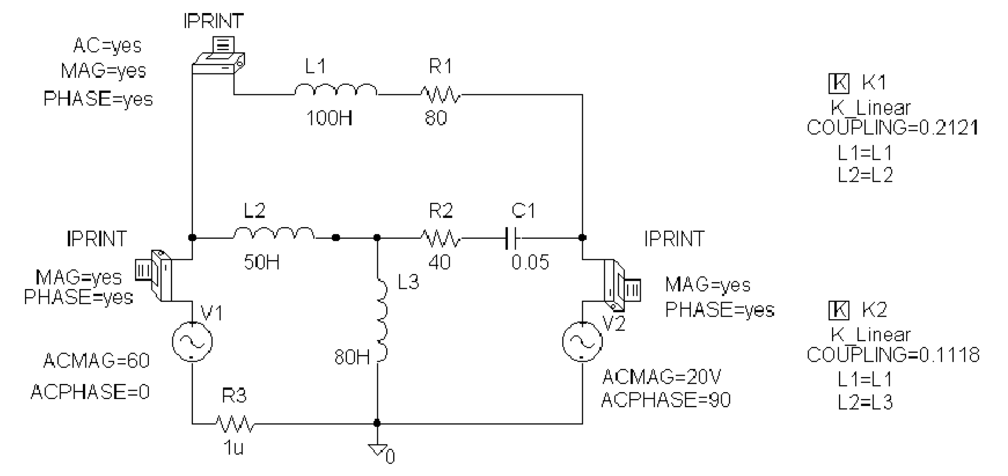
The schematic is shown below.

$$k_1 = 15/\sqrt{5000} = 0.2121, \; k_2 = 10/\sqrt{8000} = 0.1118$$

In the AC Sweep box, we type Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After the circuit is saved and simulated, the output includes

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592 E–01	4.068 E–01	–7.786 E+01
FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E–01	1.306 E+00	–6.801 E+01
FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.592 E–01	1.336 E+00	–5.492 E+01

Thus, I_1 = <u>**1.306∠–68.01° A**</u>, I_2 = <u>**406.8∠–77.86° mA**</u>, I_3 = <u>**1.336∠–54.92° A**</u>



Chapter 13, Problem 80.

Rework Prob. 13.22 using *PSpice*.

Chapter 13, Solution 80.

The schematic is shown below.

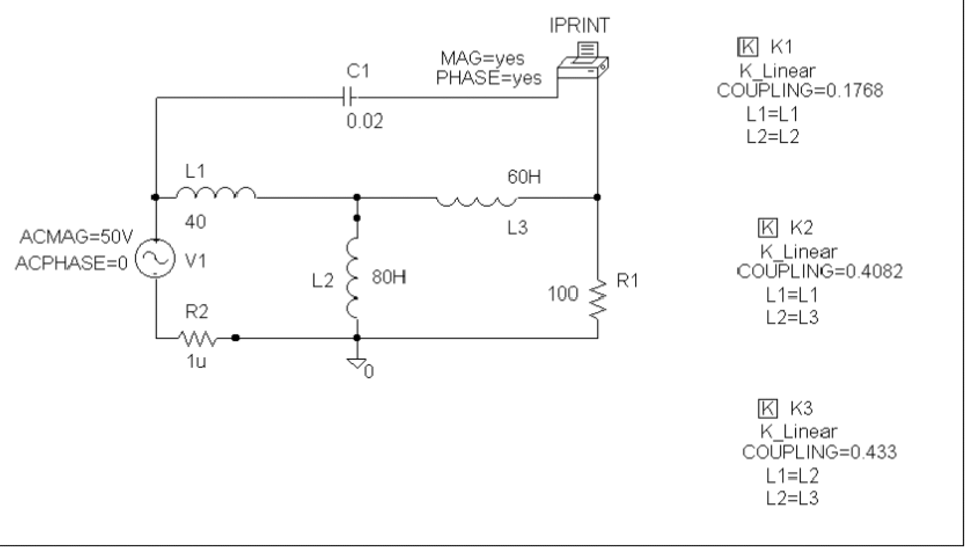
$$k_1 = 10/\sqrt{40x80} = 0.1768, \; k_2 = 20/\sqrt{40x60} = 0.482$$

$$k_3 = 30/\sqrt{80x60} = 0.433$$

In the AC Sweep box, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After the simulation, we obtain the output file which includes

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592 E–01	1.304 E+00	6.292 E+01

i.e. I_o = **<u>1.304∠62.92° A</u>**



Chapter 13, Problem 81.

Use *PSpice* to find $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ in the circuit of Fig. 13.142.

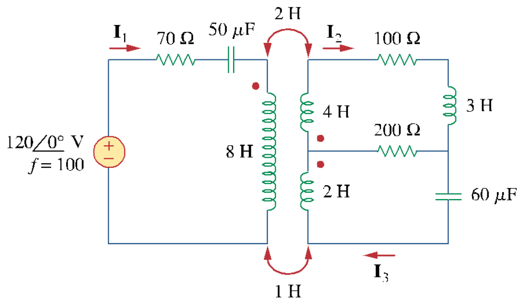


Figure 13.142
For Prob. 13.81.

Chapter 13, Solution 81.

The schematic is shown below.

$$k_1 = 2/\sqrt{4x8} = 0.3535, \; k_2 = 1/\sqrt{2x8} = 0.25$$

In the AC Sweep box, we let Total Pts = 1, Start Freq = 100, and End Freq = 100. After simulation, the output file includes

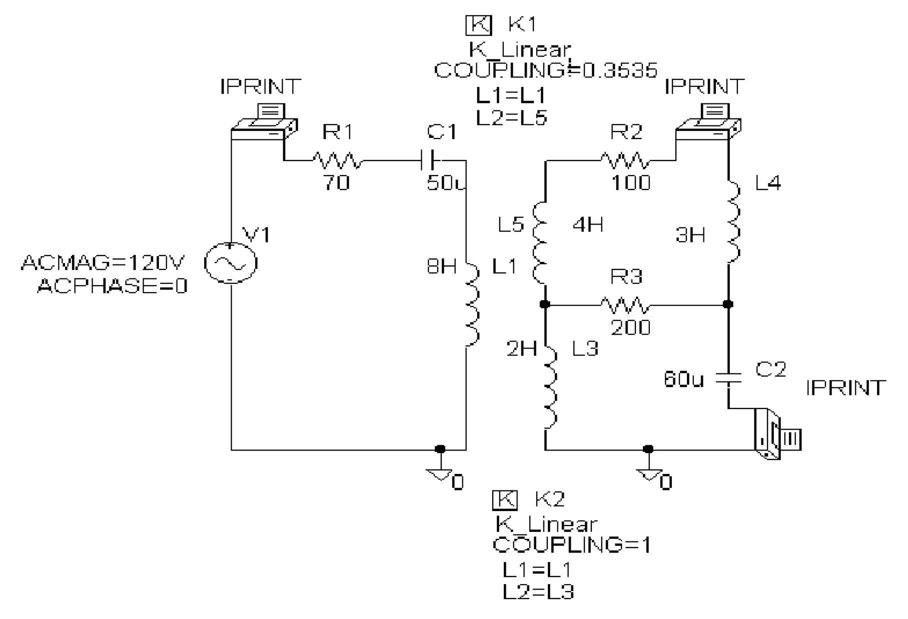
FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.000 E+02	1.0448 E–01	1.396 E+01

FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.000 E+02	2.954 E–02	–1.438 E+02

FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.000 E+02	2.088 E–01	2.440 E+01

i.e. I_1 = **<u>104.5∠13.96° mA</u>**, I_2 = **<u>29.54∠–143.8° mA</u>**,

I_3 = **<u>208.8∠24.4° mA</u>**.



Chapter 13, Problem 82.

Use *PSpice* to find $\mathbf{V}_1$, $\mathbf{V}_2$, and $\mathbf{I}_o$ in the circuit of Fig. 13.143.

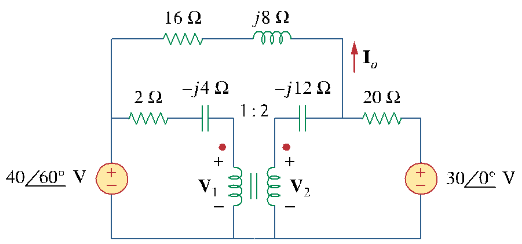


Figure 13.143
For Prob. 13.82.

Chapter 13, Solution 82.

The schematic is shown below. In the AC Sweep box, we type Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, we obtain the output file which includes

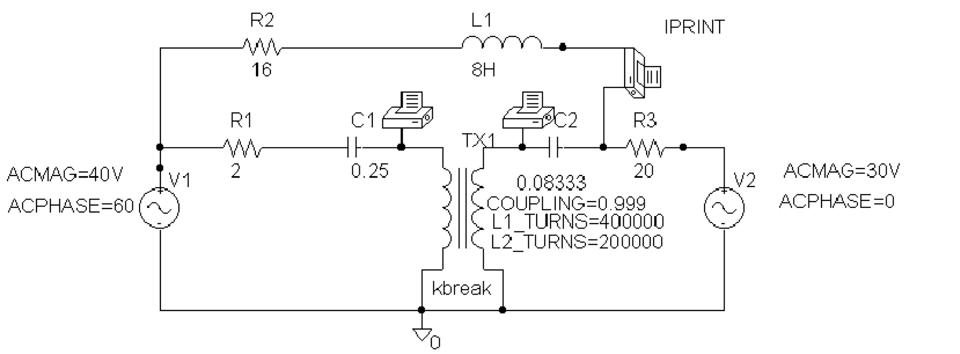
FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592 E–01	1.955 E+01	8.332 E+01

FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E–01	6.847 E+01	4.640 E+01

FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.592 E–01	4.434 E–01	–9.260 E+01

i.e. V_1 = **<u>19.55∠83.32° V</u>**, V_2 = **<u>68.47∠46.4° V</u>**,

I_o = **<u>443.4∠–92.6° mA</u>**.



Chapter 13, Problem 83.

Find $\mathbf{I}_x$ and $\mathbf{V}_x$ in the circuit of Fig. 13.144 using *PSpice*.

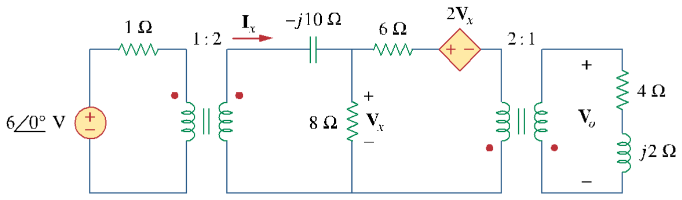


Figure 13.144
For Prob. 13.83.

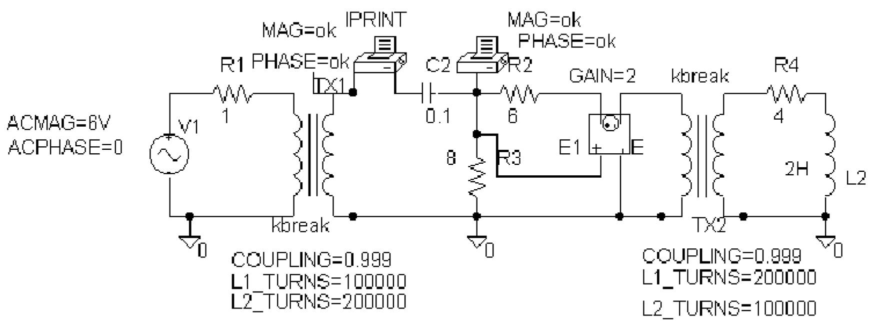
Chapter 13, Solution 83.

The schematic is shown below. In the AC Sweep box, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, the output file includes

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592 E–01	1.080 E+00	3.391 E+01

FREQ	VM($N_0001)	VP($N_0001)
1.592 E–01	1.514 E+01	–3.421 E+01

i.e. $\quad i_X = \underline{\mathbf{1.08\angle 33.91^\circ\ A}}$, $V_x = \underline{\mathbf{15.14\angle{-34.21^\circ}\ V}}$.



Chapter 13, Problem 84.

Determine $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ in the ideal transformer circuit of Fig. 13.145 using *PSpice*.

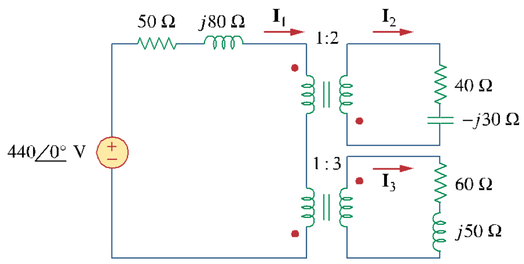


Figure 13.145
For Prob. 13.84.

Chapter 13, Solution 84.

The schematic is shown below. we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, the output file includes

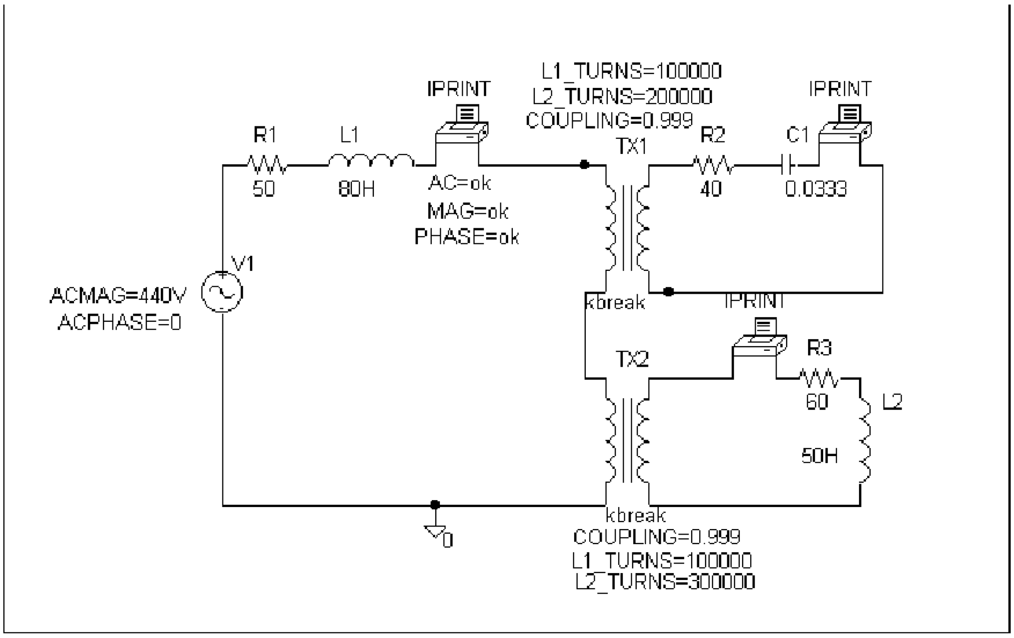
FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592 E–01	4.028 E+00	–5.238 E+01

FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E–01	2.019 E+00	–5.211 E+01

FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.592 E–01	1.338 E+00	–5.220 E+01

i.e. I_1 = $\underline{\mathbf{4.028\angle{-52.38^\circ}\ A}}$, I_2 = $\underline{\mathbf{2.019\angle{-52.11^\circ}\ A}}$,

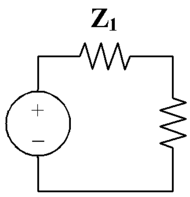
I_3 = $\underline{\mathbf{1.338\angle{-52.2^\circ}\ A}}$.



Chapter 13, Problem 85.

A stereo amplifier circuit with an output impedance of 7.2 kΩ is to be matched to a speaker with an input impedance of 8 Ω by a transformer whose primary side has 3,000 turns. Calculate the number of turns required on the secondary side.

Chapter 13, Solution 85.



For maximum power transfer,

$$Z_1 = Z_L/n^2 \quad \text{or} \quad n^2 = Z_L/Z_1 = 8/7200 = 1/900$$

$$n = 1/30 = N_2/N_1. \text{ Thus } N_2 = N_1/30 = 3000/30 = \underline{\textbf{100 turns}}.$$

Chapter 13, Problem 86.

A transformer having 2,400 turns on the primary and 48 turns on the secondary is used as an impedance-matching device. What is the reflected value of a 3-Ω load connected to the secondary?

Chapter 13, Solution 86.

$$n = N_2/N_1 = 48/2400 = 1/50$$

$$Z_{Th} = Z_L/n^2 = 3/(1/50)^2 = \underline{\textbf{7.5 k}\Omega}$$

Chapter 13, Problem 87.

e2d A radio receiver has an input resistance of 300 Ω. When it is connected directly to an antenna system with a characteristic impedance of 75 Ω, an impedance mismatch occurs. By inserting an impedance-matching transformer ahead of the receiver, maximum power can be realized. Calculate the required turns ratio.

Chapter 13, Solution 87.

$$Z_{Th} = Z_L/n^2 \quad \text{or} \quad n = \sqrt{Z_L / Z_{Th}} = \sqrt{75/300} = \underline{\mathbf{0.5}}$$

Chapter 13, Problem 88.

A step-down power transformer with a turns ratio of $n = 0.1$ supplies 12.6 V rms to a resistive load. If the primary current is 2.5 A rms, how much power is delivered to the load?

Chapter 13, Solution 88.

$$n = V_2/V_1 = I_1/I_2 \quad \text{or} \quad I_2 = I_1/n = 2.5/0.1 = 25\text{ A}$$

$$p = IV = 25\text{x}12.6 = \underline{\mathbf{315\ watts}}$$

Chapter 13, Problem 89.

A 240/120-V rms power transformer is rated at 10 kVA. Determine the turns ratio, the primary current, and the secondary current.

Chapter 13, Solution 89.

$$n = V_2/V_1 = 120/240 = 0.5$$

$$S = I_1V_1 \quad \text{or}\ I_1 = S/V_1 = 10\text{x}10^3/240 = 41.67\text{ A}$$

$$S = I_2V_2 \quad \text{or}\ I_2 = S/V_2 = 10^4/120 = 83.33\text{ A}$$

Chapter 13, Problem 90.

A 4-kVA, 2,400/240-V rms transformer has 250 turns on the primary side. Calculate:

(a) the turns ratio,
(b) the number of turns on the secondary side,
(c) the primary and secondary currents.

Chapter 13, Solution 90.

(a) $n = V_2/V_1 = 240/2400 = \underline{\mathbf{0.1}}$

(b) $n = N_2/N_1$ or $N_2 = nN_1 = 0.1(250) = \underline{\mathbf{25\ turns}}$

(c) $S = I_1V_1$ or $I_1 = S/V_1 = 4x10^3/2400 = \underline{\mathbf{1.6667\ A}}$

$S = I_2V_2$ or $I_2 = S/V_2 = 4x10^4/240 = \underline{\mathbf{16.667\ A}}$

Chapter 13, Problem 91.

A 25,000/240-V rms distribution transformer has a primary current rating of 75 A.

(a) Find the transformer kVA rating.
(b) Calculate the secondary current.

Chapter 13, Solution 91.

(a) The kVA rating is $S = VI = 25{,}000x75 = \underline{\mathbf{1875\ kVA}}$

(b) Since $S_1 = S_2 = V_2I_2$ and $I_2 = 1875x10^3/240 = \underline{\mathbf{7812\ A}}$

Chapter 13, Problem 92.

A 4,800-V rms transmission line feeds a distribution transformer with 1,200 turns on the primary and 28 turns on the secondary. When a 10-Ω load is connected across the secondary, find:

(a) the secondary voltage,
(b) the primary and secondary currents,
(c) the power supplied to the load.

Chapter 13, Solution 92.

(a) $V_2/V_1 = N_2/N_1 = n$, $V_2 = (N_2/N_1)V_1 = (28/1200)4800 = \underline{\mathbf{112\ V}}$

(b) $I_2 = V_2/R = 112/10 = \underline{\mathbf{11.2\ A}}$ and $I_1 = nI_2$, $n = 28/1200$

$I_1 = (28/1200)11.2 = \underline{\mathbf{261.3\ mA}}$

(c) $p = |I_2|^2R = (11.2)^2(10) = \underline{\mathbf{1254\ watts}}$.

Chapter 13, Problem 93.

A four-winding transformer (Fig. 13.146) is often used in equipment (e.g., PCs, VCRs) that may be operated from either 110 V or 220 V. This makes the equipment suitable for both domestic and foreign use. Show which connections are necessary to provide:

(a) an output of 14 V with an input of 110 V,
(b) an output of 50 V with an input of 220 V.

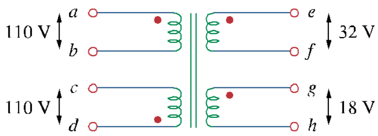
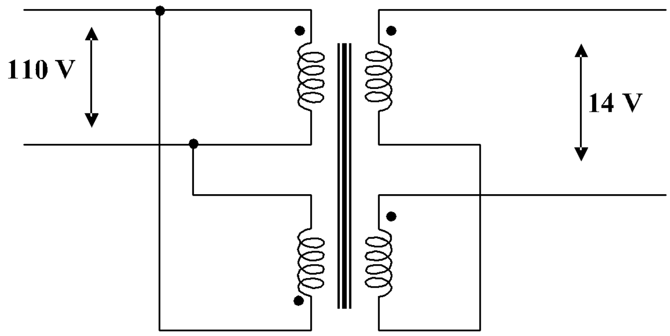


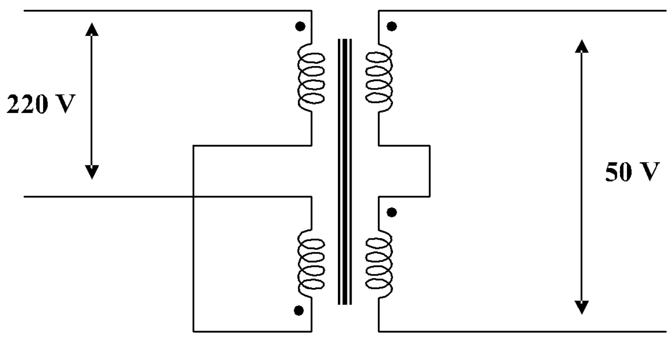
Figure 13.146
For Prob. 13.93.

Chapter 13, Solution 93.

(a) For an input of 110 V, the primary winding must be connected in parallel, with series aiding on the secondary. The coils must be series opposing to give 14 V. Thus, the connections are shown below.



(b) To get 220 V on the primary side, the coils are connected in series, with series aiding on the secondary side. The coils must be connected series aiding to give 50 V. Thus, the connections are shown below.



Chapter 13, Problem 94.

* A 440/110-V ideal transformer can be connected to become a 550/440-V ideal autotransformer. There are four possible connections, two of which are wrong. Find the output voltage of:

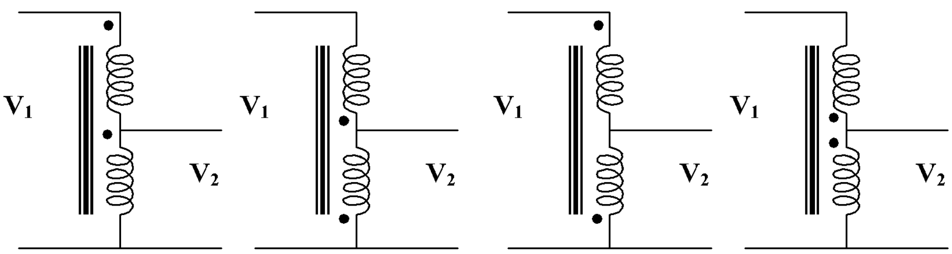
(a) a wrong connection,
(b) the right connection.

* An asterisk indicates a challenging problem.

Chapter 13, Solution 94.

$$V_2/V_1 = 110/440 = 1/4 = I_1/I_2$$

There are four ways of hooking up the transformer as an auto-transformer. However it is clear that there are only two outcomes.



(1) (2) (3) (4)

(1) and (2) produce the same results and (3) and (4) also produce the same results. Therefore, we will only consider Figure (1) and (3).

(a) For Figure (3), $V_1/V_2 = 550/V_2 = (440 - 110)/440 = 330/440$

Thus, $V_2 = 550\text{x}440/330 =$ **733.4 V (not the desired result)**

(b) For Figure (1), $V_1/V_2 = 550/V_2 = (440 + 110)/440 = 550/440$

Thus, $V_2 = 550\text{x}440/550 =$ **440 V (the desired result)**

Chapter 13, Problem 95.

Ten bulbs in parallel are supplied by a 7,200/120-V transformer as shown in Fig. 13.147, where the bulbs are modeled by the 144-Ω resistors. Find:

(a) the turns ratio *n*,
(b) the current through the primary winding.

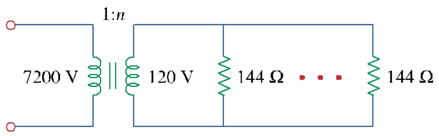


Figure 13.147
For Prob. 13.95.

Chapter 13, Solution 95.

(a) $n = V_s/V_p = 120/7200 = \underline{\mathbf{1/60}}$

(b) $I_s = 10x120/144 = 1200/144$

$$S = V_pI_p = V_sI_s$$

$$I_p = V_sI_s/V_p = (1/60)x1200/144 = \underline{\mathbf{139\ mA}}$$

Chapter 14, Problem 1.

Find the transfer function $\mathbf{V}_o/\mathbf{V}_i$ of the RC circuit in Fig. 14.68. Express it using $\omega_o = 1/RC$.

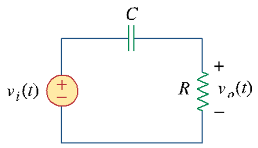


Figure 14.68
For Prob. 14.1.

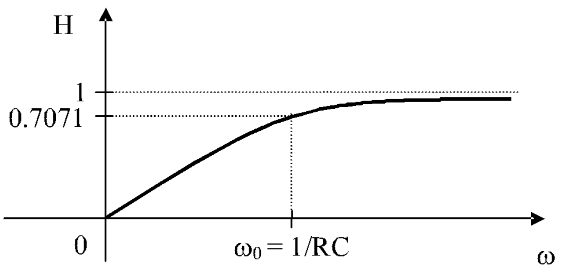
Chapter 14, Solution 1.

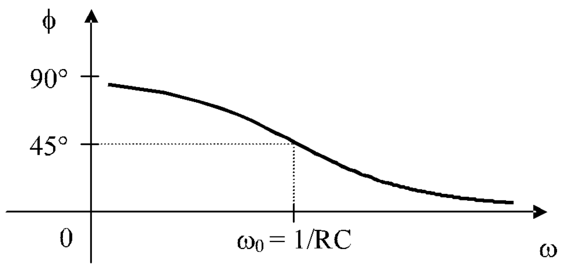
$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{R + 1/j\omega C} = \frac{j\omega RC}{1 + j\omega RC}$$

$$\mathbf{H}(\omega) = \underline{\frac{j\omega/\omega_0}{1 + j\omega/\omega_0}}, \qquad \text{where } \underline{\omega_0 = \frac{1}{RC}}$$

$$H = |\mathbf{H}(\omega)| = \frac{\omega/\omega_0}{\sqrt{1 + (\omega/\omega_0)^2}} \qquad \phi = \angle\mathbf{H}(\omega) = \frac{\pi}{2} - \tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

This is a highpass filter. The frequency response is the same as that for P.P.14.1 except that $\omega_0 = 1/RC$. Thus, the sketches of H and ϕ are shown below.





Chapter 14, Problem 2.

Obtain the transfer function $\mathbf{V}_o(s)/\mathbf{V}_i$ of the circuit in Fig. 14.69.

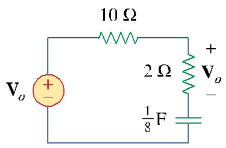


Figure 14.69
For Prob. 14.2.

Chapter 14, Solution 2.

$$H(s) = \frac{V_o}{V_i} = \frac{2 + \dfrac{1}{s/8}}{10 + 20 + \dfrac{1}{s/8}} = \frac{2 + 8/s}{12 + 8/s} = \underline{\frac{1}{6}\frac{s+4}{s+0.6667}}$$

Chapter 14, Problem 3.

For the circuit shown in Fig. 14.70, find $\mathbf{H}(s) = \mathbf{V}_o/\mathbf{V}_i(s)$.

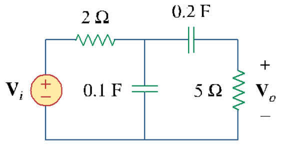


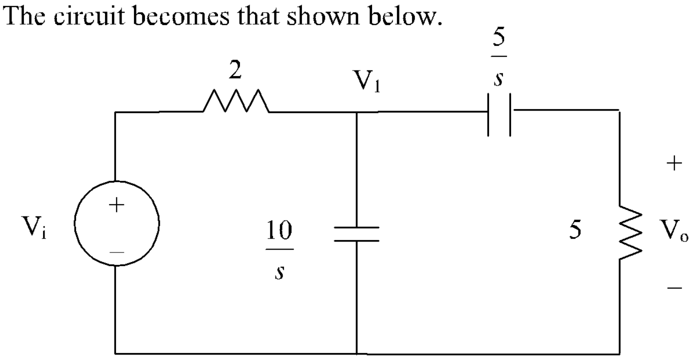
Figure 14.70
For Prob. 14.3.

Chapter 14, Solution 3.

$$0.2F \longrightarrow \frac{1}{j\omega C} = \frac{1}{s(0.2)} = \frac{5}{s}$$

$$0.1F \longrightarrow \frac{1}{s(0.1)} = \frac{10}{s}$$

The circuit becomes that shown below.



$$\text{Let } Z = \frac{10}{s} // (5 + \frac{5}{s}) = \frac{\frac{10}{s}(5+\frac{5}{s})}{5+\frac{15}{s}} = \frac{\frac{10}{s}5(\frac{1+s}{s})}{\frac{5}{s}(3+s)} = \frac{10(s+1)}{s(s+3)}$$

$$V_1 = \frac{Z}{Z+2} V_i$$

$$V_o = \frac{5}{5+5/s} V_1 = \frac{s}{s+1} V_1 = \frac{s}{s+1} \bullet \frac{Z}{Z+2} V_i$$

$$H(s) = V_o / V_i = \frac{s}{s+1} \bullet \frac{\frac{10(s+1)}{s(s+3)}}{2 + \frac{10(s+1)}{s(s+3)}} = \frac{10s}{2s(s+3)+10(s+1)} = \frac{5s}{s^2+8s+5}$$

Chapter 14, Problem 4.

Find the transfer function $\mathbf{H}(\omega) = \mathbf{V}_o/\mathbf{V}_i$ of the circuits shown in Fig. 14.71.

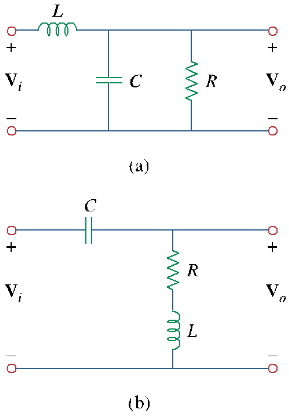


Figure 14.71
For Prob. 14.4.

Chapter 14, Solution 4.

(a) $$R \,\|\, \frac{1}{j\omega C} = \frac{R}{1+j\omega RC}$$

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{\dfrac{R}{1+j\omega RC}}{j\omega L + \dfrac{R}{1+j\omega RC}} = \frac{R}{R + j\omega L(1+j\omega RC)}$$

$$\mathbf{H}(\omega) = \underline{\frac{\mathbf{R}}{\mathbf{-\,\omega^2 RLC + R + j\omega L}}}$$

(b) $$\mathbf{H}(\omega) = \frac{R + j\omega L}{R + j\omega L + 1/j\omega C} = \frac{j\omega C(R + j\omega L)}{1 + j\omega C(R + j\omega L)}$$

$$\mathbf{H}(\omega) = \underline{\frac{\mathbf{-\,\omega^2 LC + j\omega RC}}{\mathbf{1 - \omega^2 LC + j\omega RC}}}$$

Chapter 14, Problem 5.

For each of the circuits shown in Fig. 14.72, find $\mathbf{H}(s) = \mathbf{V}_o(s)/\mathbf{V}_s(s)$.

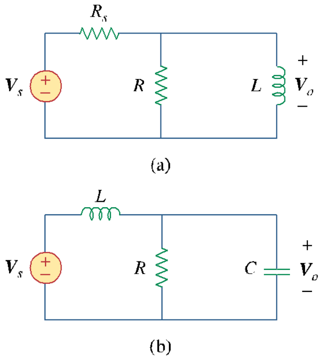


Figure 14.72
For Prob. 14.5.

Chapter 14, Solution 5.

(a) Let $Z = R // sL = \dfrac{sRL}{R + sL}$

$$V_o = \frac{Z}{Z + R_s} V_s$$

$$H(s) = \frac{V_o}{V_s} = \frac{Z}{Z + R_s} = \frac{\frac{sRL}{R + sL}}{R_s + \frac{sRL}{R + sL}} = \underline{\frac{sRL}{RR_s + s(R + R_s)L}}$$

(b) Let $Z = R // \dfrac{1}{sC} = \dfrac{Rx\frac{1}{sC}}{R + \frac{1}{sC}} = \dfrac{R}{1 + sRC}$

$$V_o = \frac{Z}{Z + sL} V_s$$

$$\mathrm{H(s)} = \frac{\mathrm{V_o}}{\mathrm{V_i}} = \frac{\mathrm{Z}}{\mathrm{Z + sL}} = \frac{\frac{\mathrm{R}}{1 + \mathrm{sRC}}}{\mathrm{sL} + \frac{\mathrm{R}}{1 + \mathrm{sRC}}} = \underline{\frac{\mathrm{R}}{\mathrm{s^2 LRC + sL + R}}}$$

Chapter 14, Problem 6.

For the circuit shown in Fig. 14.73, find $\mathbf{H}(s) = \mathbf{I}_o(s)/\mathbf{I}_s(s)$.

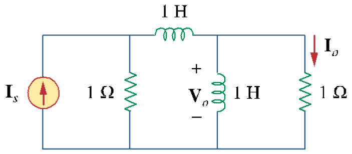


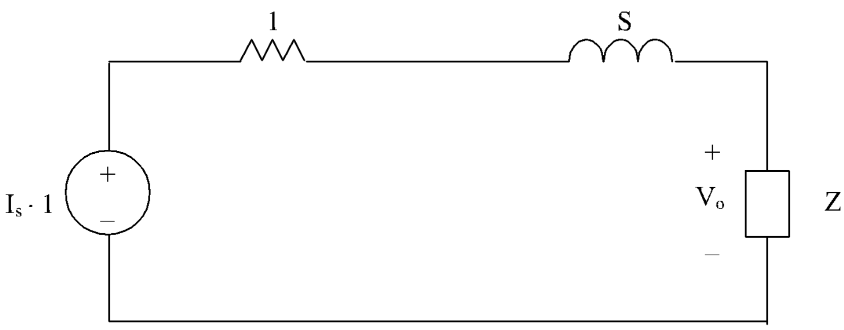
Figure 14.73
For Prob. 14.6.

Chapter 14, Solution 6.

$$1H \longrightarrow j\omega L = sL = s$$

Let $Z = s//1 = \dfrac{s}{s+1}$

We convert the current source to a voltage source as shown below.



$$V_o = \frac{Z}{Z+s+1}(I_s x1) = \frac{\frac{s}{s+1}}{s+1+\frac{s}{s+1}} I_s = \frac{sI_s}{(s+1)^2+s} = \frac{sI_s}{s^2+3s+1}$$

$$I_o = \frac{V_o}{1} = \frac{sI_s}{s^2+3s+1}$$

$$H(s) = \frac{I_o}{I_s} = \frac{s}{s^2+3s+1}$$

Chapter 14, Problem 7.

Calculate $|\mathbf{H}(\omega)|$ if H_{dB} equals

(a) 0.05dB (b) -6.2 dB (c) 104.7 dB

Chapter 14, Solution 7.

(a)
$$0.05 = 20\log_{10} \text{H}$$
$$2.5\times 10^{-3} = \log_{10} \text{H}$$
$$\text{H} = 10^{2.5\times 10^{-3}} = \underline{\mathbf{1.005773}}$$

(b)
$$-6.2 = 20\log_{10}\text{H}$$
$$-0.31 = \log_{10}\text{H}$$
$$\text{H} = 10^{-0.31} = \underline{\mathbf{0.4898}}$$

(c)
$$104.7 = 20\log_{10} \text{H}$$
$$5.235 = \log_{10} \text{H}$$
$$\text{H} = 10^{5.235} = \underline{\mathbf{1.718 \times 10^5}}$$

Chapter 14, Problem 8.

Determine the magnitude (in dB) and the phase (in degrees) of $\mathbf{H}(\omega) =$ at $\omega = 1$ if $\mathbf{H}(\omega)$ equals

(a) 0.05 dB (b) 125 (c) $\dfrac{10j\omega}{2+j\omega}$ (d) $\dfrac{3}{1+j\omega} + \dfrac{6}{2+j\omega}$

Chapter 14, Solution 8.

(a)
$$\text{H} = 0.05$$
$$\text{H}_{\text{dB}} = 20\log_{10} 0.05 = \underline{\mathbf{-26.02}}, \qquad \phi = \underline{\mathbf{0°}}$$

(b)
$$\text{H} = 125$$
$$\text{H}_{\text{dB}} = 20\log_{10} 125 = \underline{\mathbf{41.94}}, \qquad \phi = \underline{\mathbf{0°}}$$

(c)
$$\text{H}(1) = \frac{\text{j}10}{2+\text{j}} = 4.472\angle 63.43°$$
$$\text{H}_{\text{dB}} = 20\log_{10} 4.472 = \underline{\mathbf{13.01}}, \qquad \phi = \underline{\mathbf{63.43°}}$$

(d)
$$\text{H}(1) = \frac{3}{1+\text{j}} + \frac{6}{2+\text{j}} = 3.9 - \text{j}2.7 = 4.743\angle -34.7°$$
$$\text{H}_{\text{dB}} = 20\log_{10} 4.743 = \underline{\mathbf{13.521}}, \qquad \phi = \underline{\mathbf{-34.7°}}$$

Chapter 14, Problem 9.

A ladder network has a voltage gain of

$$\mathbf{H}(\omega) = \frac{10}{(1+j\omega)(10+j\omega)}$$

Sketch the Bode plots for the gain.

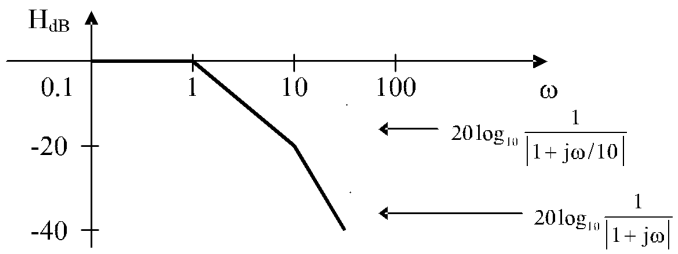
Chapter 14, Solution 9.

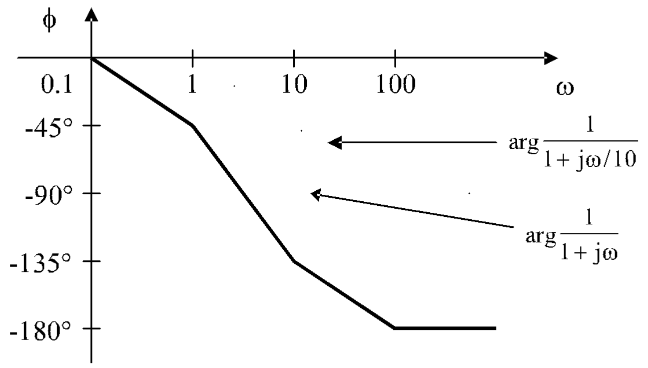
$$\mathbf{H}(\omega) = \frac{1}{(1+j\omega)(1+j\omega/10)}$$

$$H_{dB} = -20\log_{10}|1+j\omega| - 20\log_{10}|1+j\omega/10|$$

$$\phi = -\tan^{-1}(\omega) - \tan^{-1}(\omega/10)$$

The magnitude and phase plots are shown below.





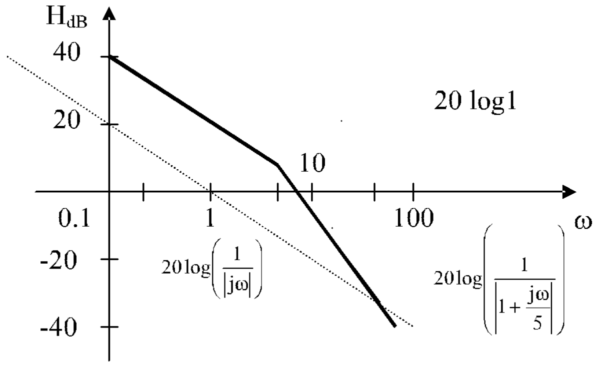
Chapter 14, Problem 10.

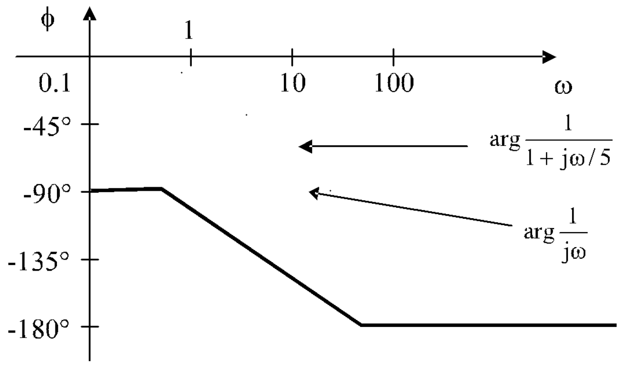
Sketch the Bode magnitude and phase plots of:

$$\mathbf{H}(j\omega) = \frac{50}{j\omega(5 + j\omega)}$$

Chapter 14, Solution 10.

$$H(j\omega) = \frac{50}{j\omega(5 + j\omega)} = \frac{10}{1j\omega\left(1 + \frac{j\omega}{5}\right)}$$





Chapter 14, Problem 11.

Sketch the Bode plots for

$$\mathbf{H}(\omega) = \frac{10 + j\omega}{j\omega(2 + j\omega)}$$

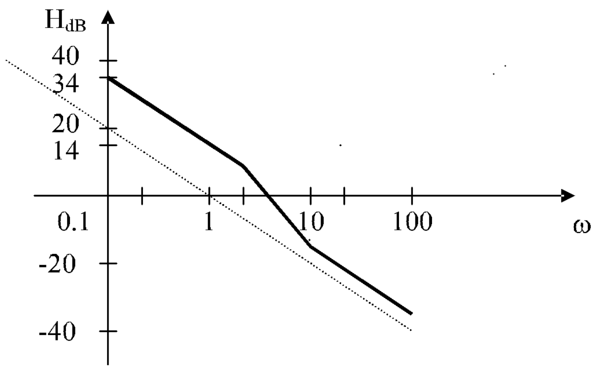
Chapter 14, Solution 11.

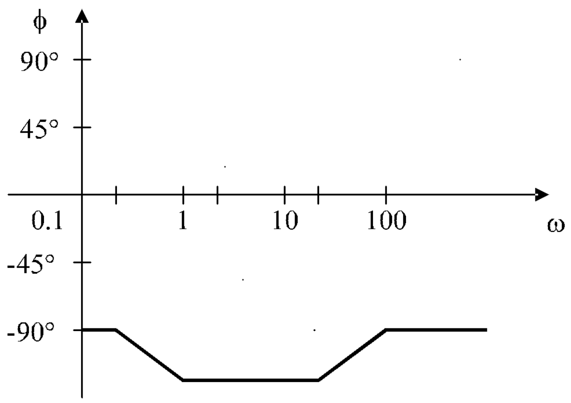
$$\mathbf{H}(\omega) = \frac{5(1 + j\omega/10)}{j\omega(1 + j\omega/2)}$$

$$H_{dB} = 20\log_{10} 5 + 20\log_{10}\left|1 + j\omega/10\right| - 20\log_{10}\left|j\omega\right| - 20\log_{10}\left|1 + j\omega/2\right|$$

$$\phi = -90° + \tan^{-1}\omega/10 - \tan^{-1}\omega/2$$

<u>The magnitude and phase plots are shown below</u>.





Chapter 14, Problem 12.

A transfer function is given by

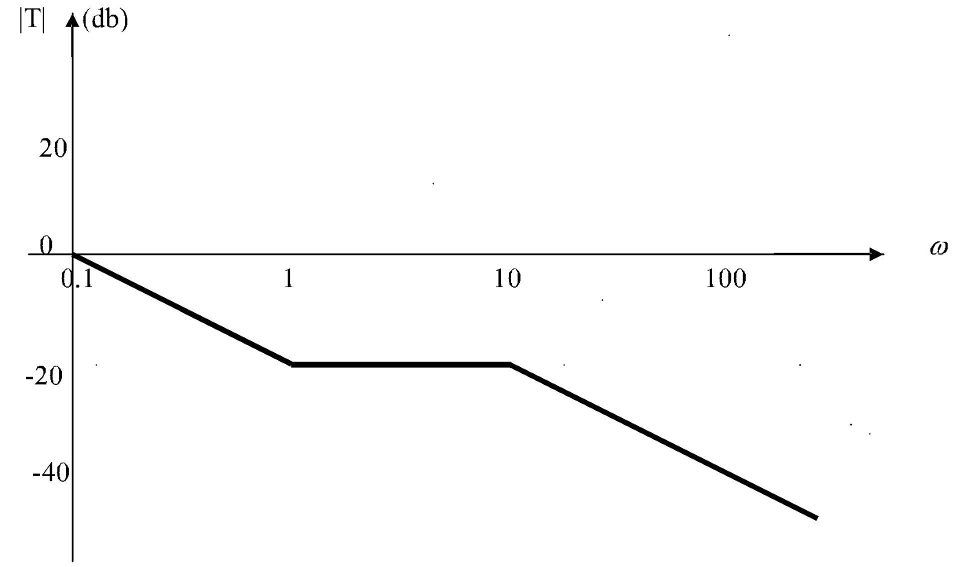
$$T(s) = \frac{s+1}{s(s+10)}$$

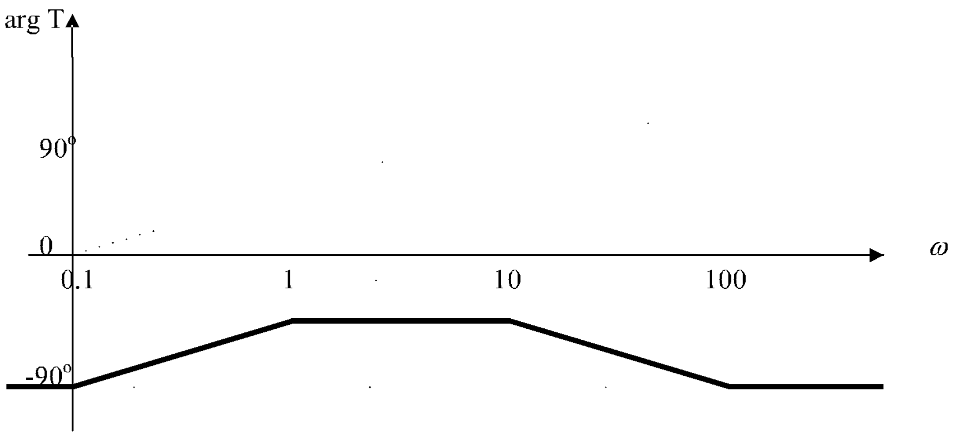
Sketch the magnitude and phase Bode plots.

Chapter 14, Solution 12.

$$T(w) = \frac{0.1(1 + j\omega)}{j\omega(1 + j\omega/10)}, \qquad 20\log 0.1 = -20$$

The plots are shown below.





Chapter 14, Problem 13.

Construct the Bode plots for

$$G(s) = \frac{s+1}{s^2(s+10)}, \qquad s=j\omega$$

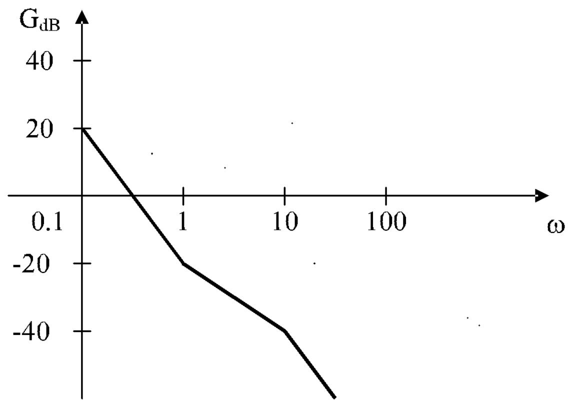
Chapter 14, Solution 13.

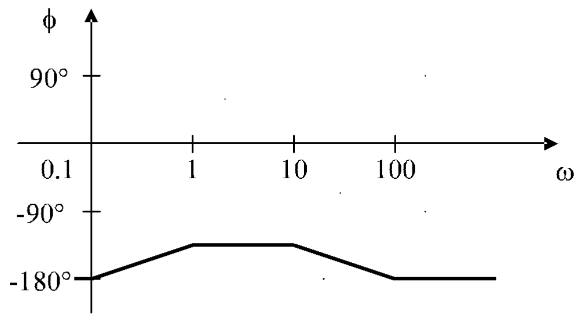
$$\mathbf{G}(\omega) = \frac{1+j\omega}{(j\omega)^2(10+j\omega)} = \frac{(1/10)(1+j\omega)}{(j\omega)^2(1+j\omega/10)}$$

$$G_{dB} = -20 + 20\log_{10}|1+j\omega| - 40\log_{10}|j\omega| - 20\log_{10}|1+j\omega/10|$$

$$\phi = -180° + \tan^{-1}\omega - \tan^{-1}\omega/10$$

__**The magnitude and phase plots are shown below**__.





Chapter 14, Problem 14.

Draw the Bode plots for

$$\mathbf{H}(\omega) = \frac{50(j\omega+1)}{j\omega(-\omega^2+10j\omega+25)}$$

Chapter 14, Solution 14.

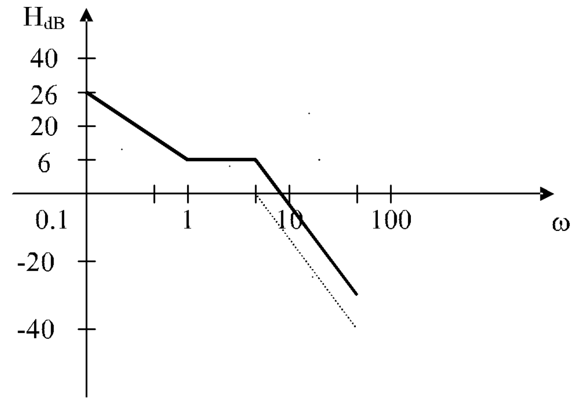
$$\mathbf{H}(\omega) = \frac{50}{25}\frac{1+j\omega}{j\omega\left(1+\frac{j\omega 10}{25}+\left(\frac{j\omega}{5}\right)^2\right)}$$

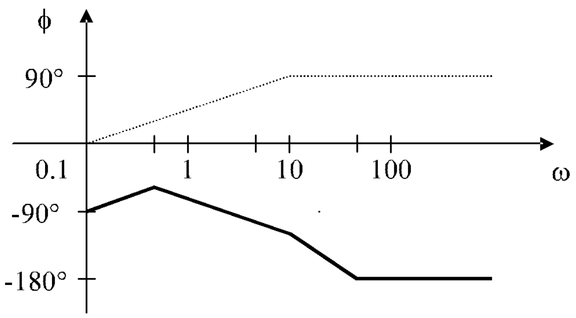
$$H_{dB} = 20\log_{10} 2 + 20\log_{10}\left|1+j\omega\right| - 20\log_{10}\left|j\omega\right|$$

$$-20\log_{10}\left|1+j\omega 2/5+(j\omega/5)^2\right|$$

$$\phi = -90° + \tan^{-1}\omega - \tan^{-1}\left(\frac{\omega 10/25}{1-\omega^2/5}\right)$$

<u>**The magnitude and phase plots are shown below**</u>.





Chapter 14, Problem 15.

Construct the Bode magnitude and phase plots for

$$H(s) = \frac{40(s+1)}{(s+2)(s+10)}, \qquad s=j\omega$$

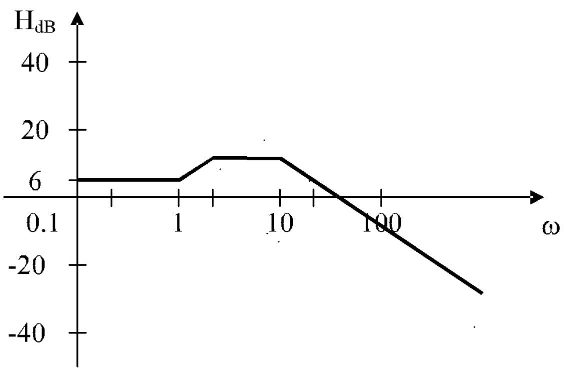
Chapter 14, Solution 15.

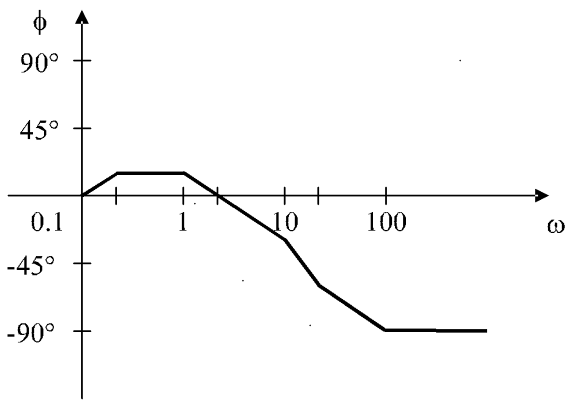
$$\mathbf{H}(\omega) = \frac{40(1+j\omega)}{(2+j\omega)(10+j\omega)} = \frac{2(1+j\omega)}{(1+j\omega/2)(1+j\omega/10)}$$

$$H_{dB} = 20\log_{10} 2 + 20\log_{10}\left|1+j\omega\right| - 20\log_{10}\left|1+j\omega/2\right| - 20\log_{10}\left|1+j\omega/10\right|$$

$$\phi = \tan^{-1}\omega - \tan^{-1}\omega/2 - \tan^{-1}\omega/10$$

<u>The magnitude and phase plots are shown below</u>.





Chapter 14, Problem 16.

Sketch Bode magnitude and phase plots for

$$H(s) = \frac{10}{s(s^2 + s + 16)}, \qquad s = j\omega$$

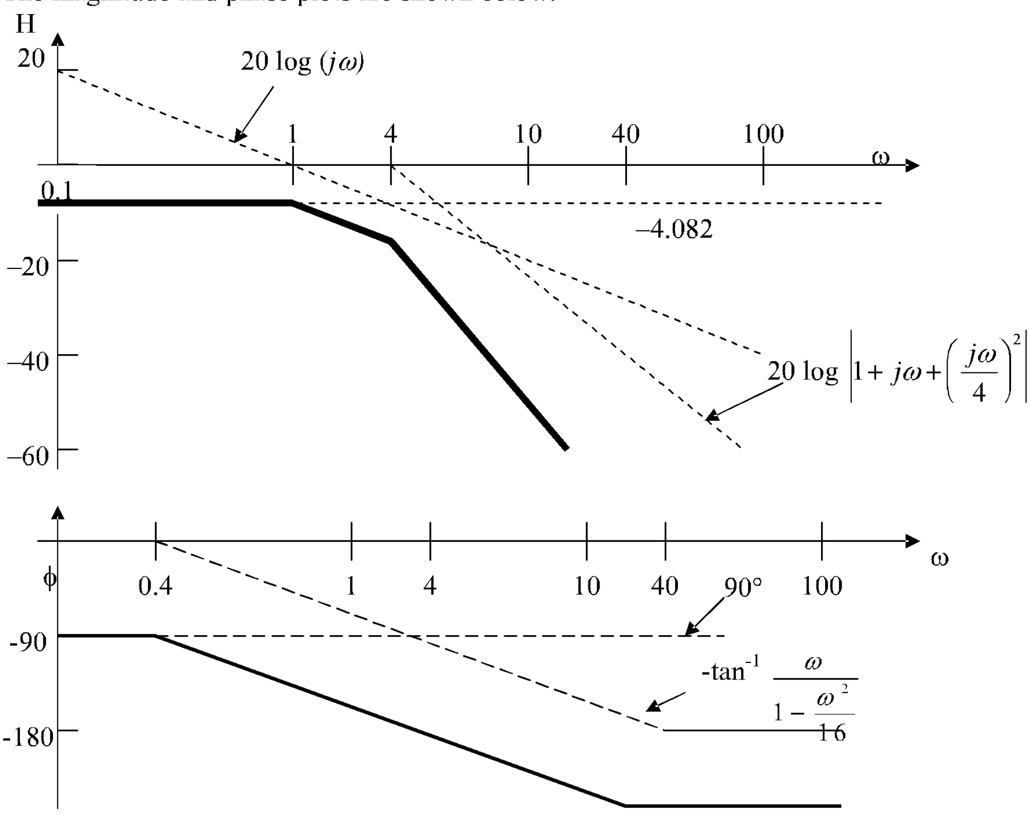
Chapter 14, Solution 16.

$$H(\omega) = \frac{10/16}{j\omega\left[1 + j\omega + \left(\frac{j\omega}{4}\right)^2\right]} = \frac{0.625}{j\omega\left[1 + j\omega + \left(\frac{j\omega}{4}\right)^2\right]}$$

$$H_{dB} = 20\log 0.625 - 20\log|j\omega| - 20\log\left|1 + j\omega + \left(\frac{j\omega}{4}\right)^2\right|$$

(20log0,625= –4.082)

The magnitude and phase plots are shown below.



Chapter 14, Problem 17.

Sketch the Bode plots for

$$G(s) = \frac{s}{(s+2)^2 + (s+1)}, \qquad s = j\omega$$

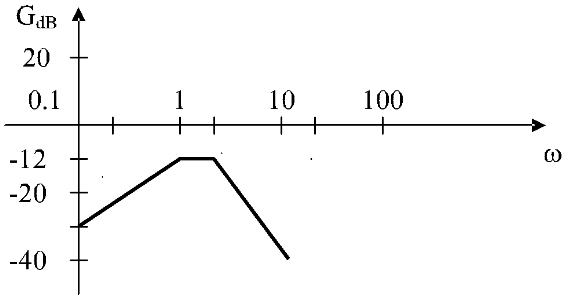
Chapter 14, Solution 17.

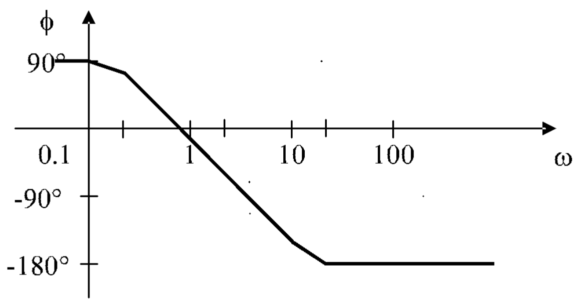
$$\mathbf{G}(\omega) = \frac{(1/4)\, j\omega}{(1+j\omega)(1+j\omega/2)^2}$$

$$G_{dB} = -20\log_{10} 4 + 20\log_{10}\left| j\omega \right| - 20\log_{10}\left| 1+j\omega \right| - 40\log_{10}\left| 1+j\omega/2 \right|$$

$$\phi = -90° - \tan^{-1}\omega - 2\tan^{-1}\omega/2$$

The magnitude and phase plots are shown below.





Chapter 14, Problem 18.

ML

A linear network has this transfer function

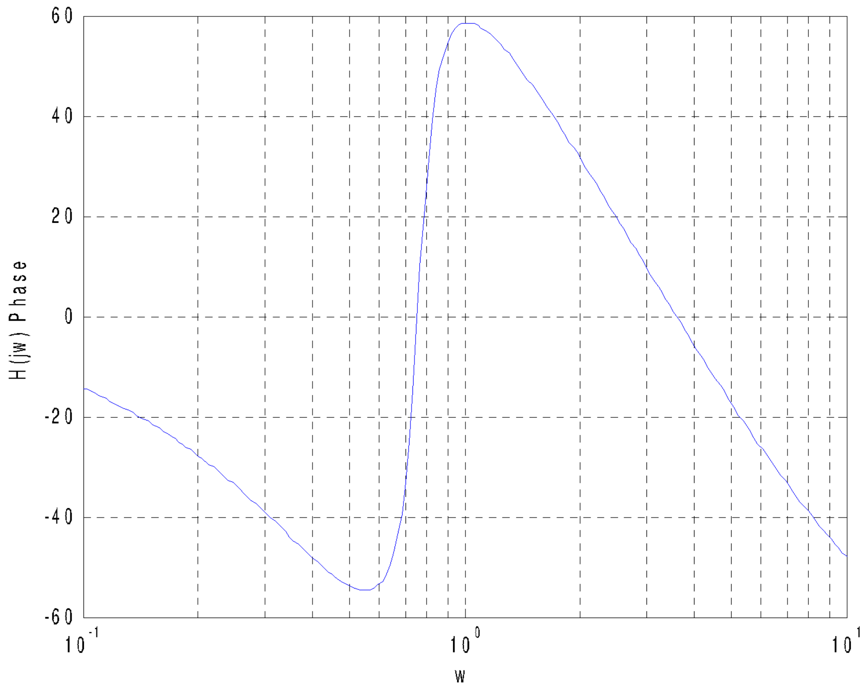
$$H(s) = \frac{7s^2 + s + 4}{(s^3 + 8s^2 + 14s + 5)}, \qquad s = j\omega$$

Use *MATLAB* or equivalent to plot the magnitude and phase (in degrees) of the transfer function. Take $0.1 < \omega < 10$ rads/s.

Chapter 14, Solution 18.

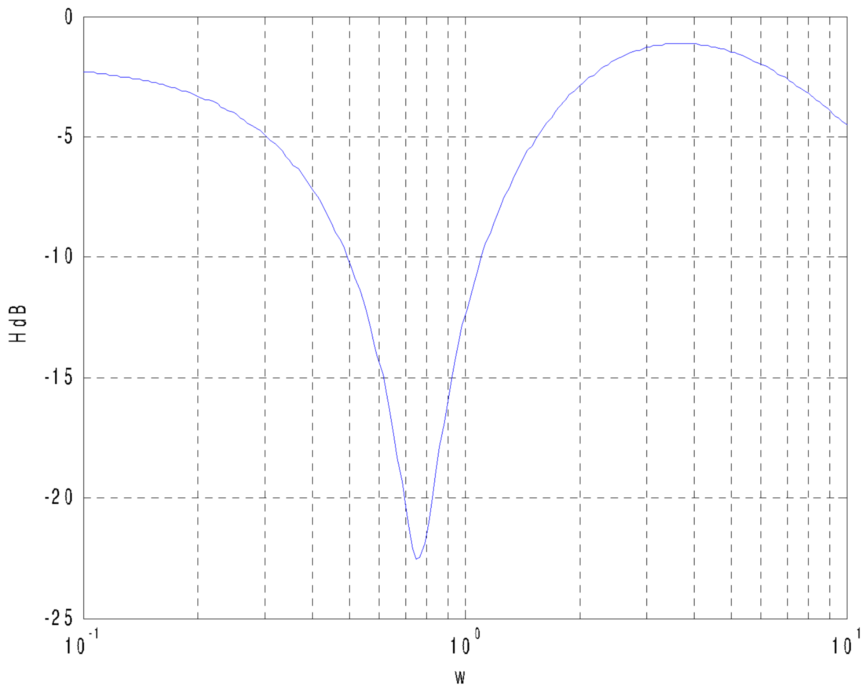
The MATLAB code is shown below.

```
>> w=logspace(-1,1,200);
>> s=i*w;
>> h=(7*s.^2+s+4)./(s.^3+8*s.^2+14*s+5);
>> Phase=unwrap(angle(h))*57.23;
>> semilogx(w,Phase)
>> grid on
```



Now for the magnitude, we need to add the following to the above,

```
>> H=abs(h);
>> HdB=20*log10(H);
>> semilogx(w,HdB);
>> grid on
```



Chapter 14, Problem 19.

Sketch the asymptotic Bode plots of the magnitude and phase for

$$H(s) = \frac{100s}{(s+10)(s+20)(s+40)}, \qquad s=j\omega$$

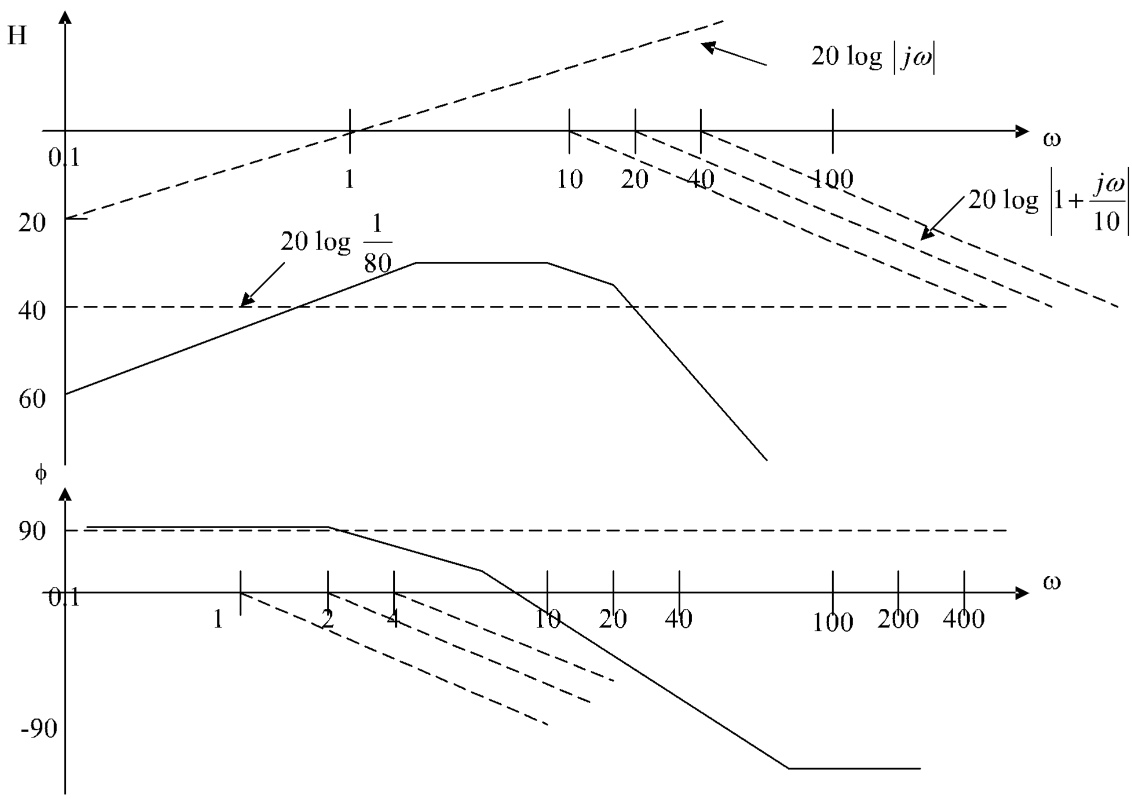
Chapter 14, Solution 19.

$$H(\omega) = \frac{100j\omega}{(j\omega+10)(j\omega+20)(j\omega+40)} = \frac{j\omega/80}{(1+\frac{j\omega}{10})(1+\frac{j\omega}{20})(1+\frac{j\omega}{40})}$$

$$H_{dB} = 20\log(1/80) + 20\log|j\omega/1| - 20\log|1+\frac{j\omega}{10}| - 20\log|1+\frac{j\omega}{20}| - 20\log|1+\frac{j\omega}{40}|$$

(20log(1/80) = -38.06)

The magnitude and phase plots are shown below.



Chapter 14, Problem 20.

Sketch the magnitude Bode plot for the transfer function

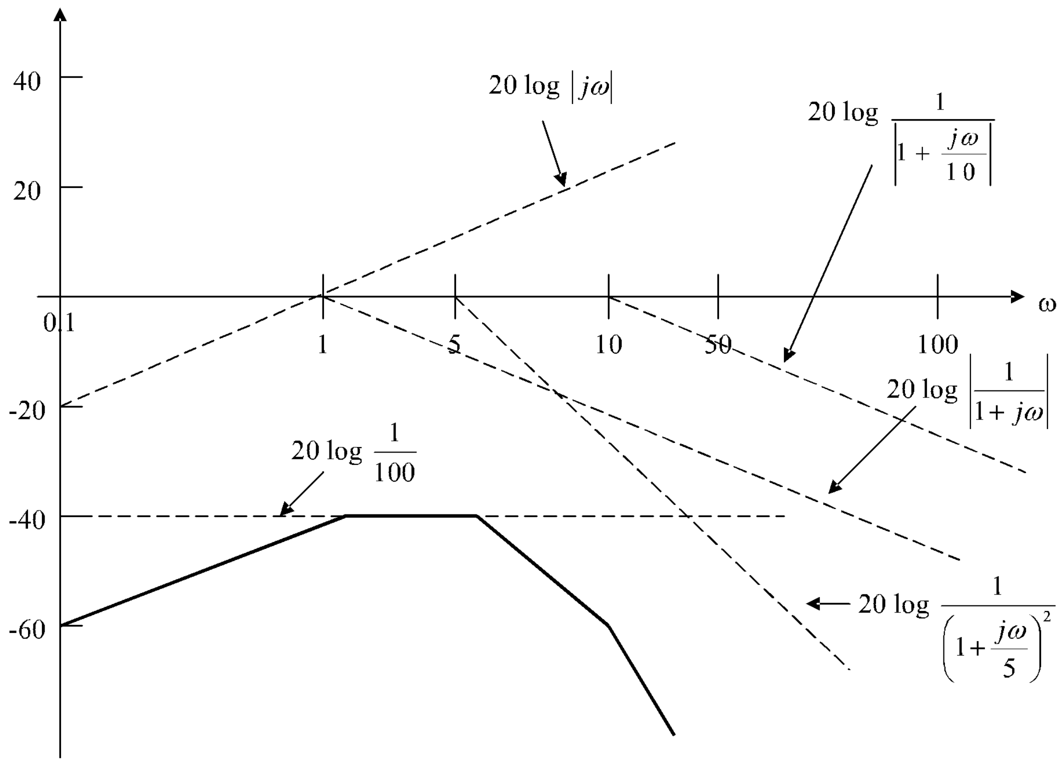
$$H(\omega) = \frac{10j\omega}{(j\omega+1)(j\omega+5)^2(j\omega+40)}$$

Chapter 14, Solution 20.

$$H(\omega) = \frac{10j\omega}{(25)(40)(1+j\omega)(1+j\omega/5)^2(1+j\omega/40)} = \frac{j\omega/100}{(1+j\omega)(1+j\omega/5)^2(1+j\omega/40)}$$

20log(1/100) = -40

The magnitude plot is shown below.



Chapter 14, Problem 21.

Sketch the magnitude Bode plot for

$$H(s) = \frac{s(s+20)}{(s+1)(s^2+60s) = (400)}, \qquad s = j\omega$$

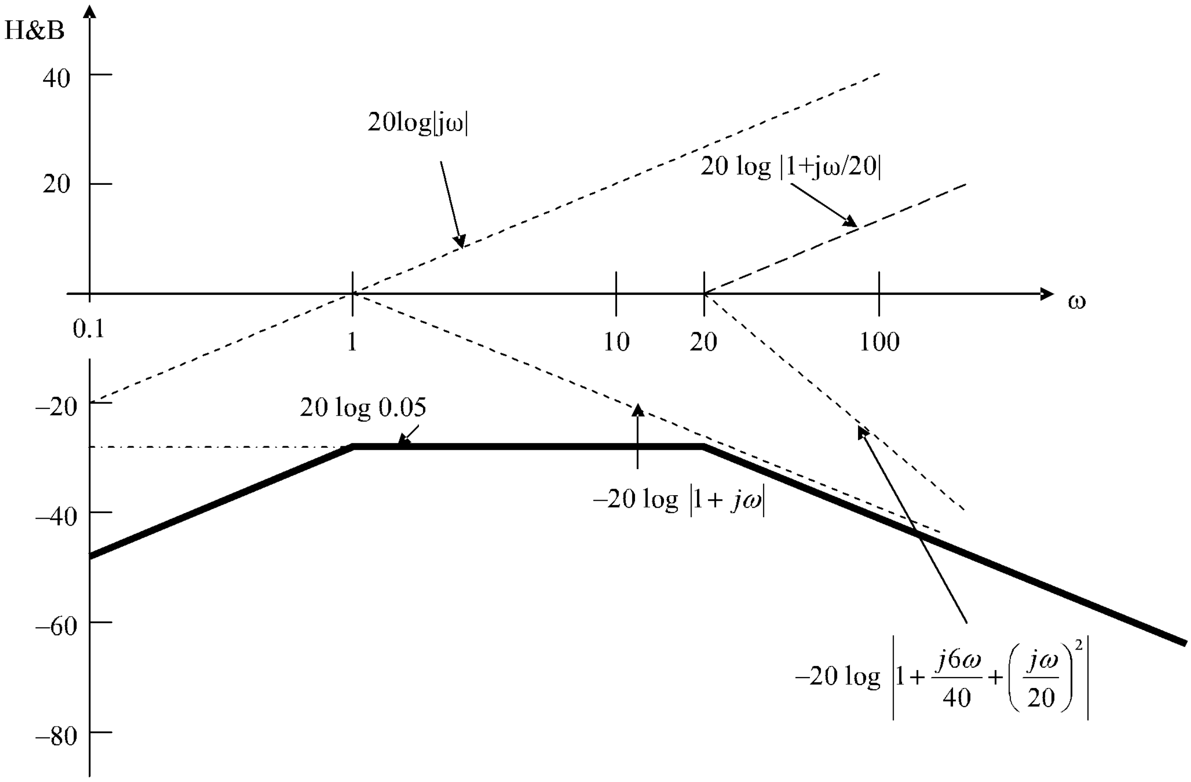
Chapter 14, Solution 21.

$$H(\omega) = \frac{j\omega(j\omega+20)}{(j\omega+1)(-\omega^2+60j\omega+400)} = \frac{20j\omega(1+j\omega/20)}{400(j\omega+1)(1+60j\omega/400+\left(\frac{j\omega}{20}\right)^2)}$$

$$H(\omega) = \frac{0.05j\omega(1+j\omega/20)}{(1+j\omega)\left(1+\frac{6j\omega}{40}+\left(\frac{j\omega}{20}\right)^2\right)}$$

$$H_{dB} = 20\log(0.05) + 20\log|j\omega| + 20\log\left|1+\frac{j\omega}{20}\right| - 20\log|1+j\omega| - 20\log\left|1+\frac{j6\omega}{40}+\left(\frac{j\omega}{20}\right)^2\right|$$

The magnitude plot is as sketched below.



Chapter 14, Problem 22.

Find the transfer function **H**(ω) with the Bode magnitude plot shown in Fig. 14.74.

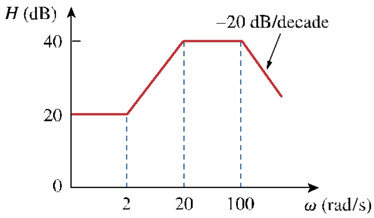


Figure 14.74
For Prob. 14.22.

Chapter 14, Solution 22.

$$20 = 20\log_{10} k \quad \longrightarrow \quad k = 10$$

A zero of slope $+20$ dB/dec at $\omega = 2 \quad \longrightarrow \quad 1 + j\omega/2$

A pole of slope -20 dB/dec at $\omega = 20 \quad \longrightarrow \quad \dfrac{1}{1 + j\omega/20}$

A pole of slope -20 dB/dec at $\omega = 100 \quad \longrightarrow \quad \dfrac{1}{1 + j\omega/100}$

Hence,

$$\mathbf{H}(\omega) = \frac{10(1 + j\omega/2)}{(1 + j\omega/20)(1 + j\omega/100)}$$

$$\mathbf{H}(\omega) = \underline{\mathbf{\frac{10^4\,(2 + j\omega)}{(20 + j\omega)(100 + j\omega)}}}$$

Chapter 14, Problem 23.

The Bode magnitude plot of **H**(ω) is shown in Fig. 14.75. Find **H**(ω).

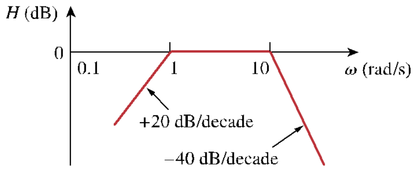


Figure 14.75
For Prob. 14.23.

Chapter 14, Solution 23.

A zero of slope $+20$ dB/dec at the origin $\longrightarrow$ $j\omega$

A pole of slope -20 dB/dec at $\omega = 1$ $\longrightarrow$ $\dfrac{1}{1 + j\omega/1}$

A pole of slope -40 dB/dec at $\omega = 10$ $\longrightarrow$ $\dfrac{1}{(1 + j\omega/10)^2}$

Hence,

$$\mathbf{H}(\omega) = \frac{j\omega}{(1 + j\omega)(1 + j\omega/10)^2}$$

$$\mathbf{H}(\omega) = \frac{\mathbf{100\, j\omega}}{\mathbf{(1 + j\omega)(10 + j\omega)^2}}$$

Chapter 14, Problem 24.

The magnitude plot in Fig. 14.76 represents the transfer function of a preamplifier. Find *H*(*s*).

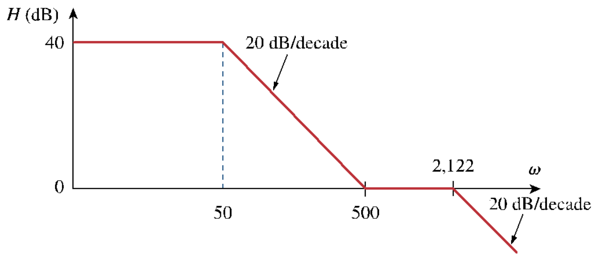


Figure 14.76
For Prob. 14.24.

Chapter 14, Solution 24.

$$40 = 20\log_{10} K \quad \longrightarrow \quad K = 100$$

There is a pole at ω=50 giving 1/(1+jω/50)
There is a zero at ω=500 giving (1 + jω/500).
There is another pole at ω=2122 giving 1/(1 + jω/2122).
Thus,

$$H(\omega) = \frac{40(1 + j\omega/500)}{(1 + j\omega/50)(1 + j\omega/2122)} = \frac{40x\dfrac{1}{500}(s+500)}{\dfrac{1}{50}x\dfrac{1}{2122}(s+50)(s+2122)}$$

or

$$H(s) = \frac{8488(s+500)}{(s+50)(s+2122)}$$

Chapter 14, Problem 25.

A series *RLC* network has $R = 2\ \text{k}\Omega$, $L = 40$ mH, and $C = 1\ \mu\text{F}$. Calculate the impedance at resonance and at one-fourth, one-half, twice, and four times the resonant frequency.

Chapter 14, Solution 25.

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(40\times10^{-3})(1\times10^{-6})}} = 5\ \text{krad/s}$$

$$\mathbf{Z}(\omega_0) = R = \underline{\mathbf{2\ k\Omega}}$$

$$\mathbf{Z}(\omega_0/4) = R + j\left(\frac{\omega_0}{4}L - \frac{4}{\omega_0 C}\right)$$

$$\mathbf{Z}(\omega_0/4) = 2000 + j\left(\frac{5\times10^3}{4}\cdot 40\times10^{-3} - \frac{4}{(5\times10^3)(1\times10^{-6})}\right)$$

$$\mathbf{Z}(\omega_0/4) = 2000 + j(50 - 4000/5)$$

$$\mathbf{Z}(\omega_0/4) = \underline{\mathbf{2 - j0.75\ k\Omega}}$$

$$\mathbf{Z}(\omega_0/2) = R + j\left(\frac{\omega_0}{2}L - \frac{2}{\omega_0 C}\right)$$

$$\mathbf{Z}(\omega_0/2) = 2000 + j\left(\frac{(5\times10^3)}{2}(40\times10^{-3}) - \frac{2}{(5\times10^3)(1\times10^{-6})}\right)$$

$$\mathbf{Z}(\omega_0/2) = 200 + j(100 - 2000/5)$$

$$\mathbf{Z}(\omega_0/2) = \underline{\mathbf{2 - j0.3\ k\Omega}}$$

$$\mathbf{Z}(2\omega_0) = R + j\left(2\omega_0 L - \frac{1}{2\omega_0 C}\right)$$

$$\mathbf{Z}(2\omega_0) = 2000 + j\left((2)(5\times10^3)(40\times10^{-3}) - \frac{1}{(2)(5\times10^3)(1\times10^{-6})}\right)$$

$$\mathbf{Z}(2\omega_0) = \underline{\mathbf{2 + j0.3\ k\Omega}}$$

$$\mathbf{Z}(4\omega_0) = R + j\left(4\omega_0 L - \frac{1}{4\omega_0 C}\right)$$

$$\mathbf{Z}(4\omega_0) = 2000 + j\left((4)(5\times10^3)(40\times10^{-3}) - \frac{1}{(4)(5\times10^3)(1\times10^{-6})}\right)$$

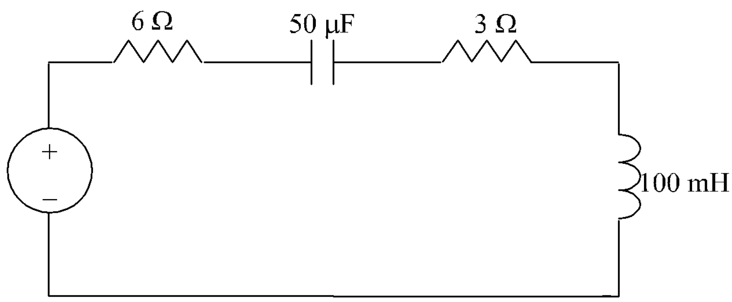
$$\mathbf{Z}(4\omega_0) = \underline{\mathbf{2 + j0.75\ k\Omega}}$$

Chapter 14, Problem 26.

A coil with resistance $3\,\Omega$ and inductance 100 mH is connected in series with a capacitor of 50 pF, a resistor of $6\,\Omega$ and a signal generator that gives 110 V rms at all frequencies. Calculate ω_o, Q, and B at resonance of the resultant series RLC circuit.

Chapter 14, Solution 26.

Consider the circuit as shown below. This is a series RLC resonant circuit.



$$R = 6 + 3 = 9\,\Omega$$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{100x10^{-3}x50x10^{-12}}} = \underline{447.21 \text{ krad/s}}$$

$$Q = \frac{\omega_o L}{R} = \frac{447.21x10^3 x100x10^3}{9} = \underline{4969}$$

$$B = \frac{\omega_o}{Q} = \frac{447.21x10^3}{4969} = \underline{90 \text{ rad/s}}$$

Chapter 14, Problem 27.

Design a series *RLC* resonant circuit with ω_o = 40 rad/s and B = 10 rad/s.

Chapter 14, Solution 27.

$$\omega_o = \frac{1}{\sqrt{LC}} = 40 \quad \longrightarrow \quad LC = \frac{1}{40^2}$$

$$B = \frac{R}{L} = 10 \quad \longrightarrow \quad R = 10L$$

If we select R =**1 Ω**, then L = R/10 = **0.1 H** and

$$C = \frac{1}{40^2 L} = \frac{1}{40^2 x0.1} = \underline{6.25 \text{ mF}}$$

Chapter 14, Problem 28.

Design a series *RLC* circuit with B = 20 rad/s and ω_o = 1,000 rad/s. Find the circuit's Q. Let R = 10 Ω.

Chapter 14, Solution 28.

Let R = 10 Ω.

$$L = \frac{R}{B} = \frac{10}{20} = 0.5 \text{ H}$$

$$C = \frac{1}{\omega_0^2 L} = \frac{1}{(1000)^2 (0.5)} = 2 \ \mu\text{F}$$

$$Q = \frac{\omega_0}{B} = \frac{1000}{20} = 50$$

Therefore, if R = 10 Ω then

L = **0.5 H**, C = **2 μF**, Q = **50**

Chapter 14, Problem 29.

Let $v_s = 20\cos(at)$ V in the circuit of Fig. 14.77. Find ω_o, Q, and B, as seen by the capacitor.

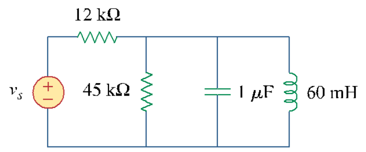
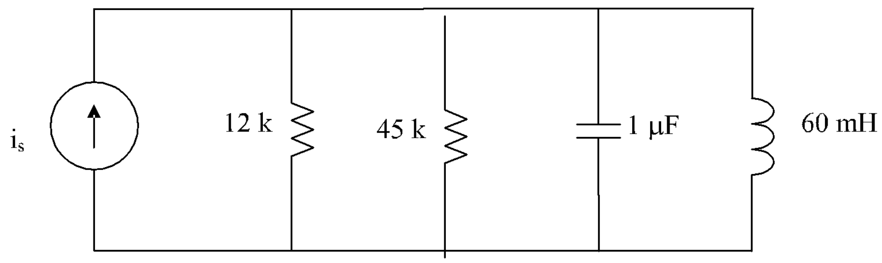


Figure 14.77
For Prob. 14.29.

Chapter 14, Solution 29.

We convert the voltage source to a current source as shown below.



$i_s = \frac{20}{12}\cos\omega t$, R = 12//45= 12x45/57 = 9.4737 kΩ

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{60x10^{-3}x1x10^{-6}}} = \underline{4.082 \text{ krad/s}}$$

$$B = \frac{1}{RC} = \frac{1}{9.4737x10^3 x10^{-6}} = \underline{105.55 \text{ rad/s}}$$

$$Q = \frac{\omega_o}{B} = \frac{4082}{105.55} = \underline{38.674} = \underline{\mathbf{38.67}}$$

Chapter 14, Problem 30.

A circuit consisting of a coil with inductance 10 mH and resistance 20 Ω is connected in series with a capacitor and a generator with an rms voltage of 120 V. Find:

(a) the value of the capacitance that will cause the circuit to be in resonance at 15 kHz
(b) the current through the coil at resonance
(c) the *Q* of the circuit

Chapter 14, Solution 30.

Select $R = 10\,\Omega$.

$$L = \frac{R}{\omega_0 Q} = \frac{10}{(10)(20)} = 0.05\text{ H} = 50\text{ mH}$$

$$C = \frac{1}{\omega_0^2 L} = \frac{1}{(100)(0.05)} = 0.2\text{ F}$$

$$B = \frac{1}{RC} = \frac{1}{(10)(0.2)} = 0.5\text{ rad/s}$$

Therefore, if $R = 10\,\Omega$ then

$L = \underline{\mathbf{50\ mH}}$, $C = \underline{\mathbf{0.2\ F}}$, $B = \underline{\mathbf{0.5\ rad/s}}$

Chapter 14, Problem 31.

e2d

Design a parallel resonant *RLC* circuit with ω_o = 10rad/s and Q = 20. Calculate the bandwidth of the circuit. Let $R = 10\,\Omega$.

Chapter 14, Solution 31.

$$X_L = \omega L \longrightarrow L = \frac{X_L}{\omega}$$

$$B = \frac{R}{L} = \frac{\omega R}{X_L} = \frac{2\pi x10x10^6 x5.6x10^3}{40x10^3} = \underline{8.796x10^6 \text{ rad/s}}$$

Chapter 14, Problem 32.

A parallel *RLC* circuit has the following values:

$R = 60\ \Omega$, $L = 1$ mH, and $C = 50\ \mu$F.

Find the quality factor, the resonant frequency, and the bandwidth of the *RLC* circuit.

Chapter 14, Solution 32.

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-3} x 50 x 10^{-6}}} = \underline{4.472\ \text{krad/s}}$$

$$B = \frac{1}{RC} = \frac{1}{60 x 50 x 10^{-6}} = \underline{333.33\ \text{rad/s}}$$

$$Q = \frac{\omega_o}{B} = \frac{4472}{333.33} = \underline{13.42}$$

Chapter 14, Problem 33.

A parallel resonant circuit with quality factor 120 has a resonant frequency of 6×10^6 rad/s. Calculate the bandwidth and half-power frequencies.

Chapter 14, Solution 33.

$$Q = \omega_o RC \longrightarrow C = \frac{Q}{2\pi f_o R} = \frac{80}{2\pi x 5.6 x 10^6 x 40 x 10^3} = \underline{56.84\ \text{pF}}$$

$$Q = \frac{R}{\omega_o L} \longrightarrow L = \frac{R}{2\pi f_o Q} = \frac{40 x 10^3}{2\pi x 5.6 x 10^6 x 80} = \underline{\mathbf{14.21\ \mu H}}$$

Chapter 14, Problem 34.

A parallel *RLC* circuit is resonant at 5.6 MHz, has a *Q* of 80, and has a resistive branch of 40 kΩ. Determine the values of *L* and *C* in the other two branches.

Chapter 14, Solution 34.

(a) $$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{8 x 10^{-3} x 60 x 10^{-6}}} = \underline{1.443\ \text{krad/s}}$$

(b) $$B = \frac{1}{RC} = \frac{1}{5 x 10^3 x 60 x 10^{-6}} = \underline{3.33\ \text{rad/s}}$$

(c) $$Q = \omega_o RC = 1.443 x 10^3 x 5 x 10^3 x 60 x 10^{-6} = \underline{432.9}$$

Chapter 14, Problem 35.

A parallel *RLC* circuit has $R = 5\text{k}\Omega$, $L = 8$ mH, and $C = \mu\text{F}$. Determine:

(a) the resonant frequency
(b) the bandwidth
(c) the quality factor

Chapter 14, Solution 35.

At resonance,

$$Y = \frac{1}{R} \longrightarrow R = \frac{1}{Y} = \frac{1}{25\times 10^{-3}} = \underline{\mathbf{40\ \Omega}}$$

$$Q = \omega_0 RC \longrightarrow C = \frac{Q}{\omega_0 R} = \frac{80}{(200\times 10^3)(40)} = \underline{\mathbf{10\ \mu F}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \longrightarrow L = \frac{1}{\omega_0^2 C} = \frac{1}{(4\times 10^{10})(10\times 10^{-6})} = \underline{\mathbf{2.5\ \mu H}}$$

$$B = \frac{\omega_0}{Q} = \frac{200\times 10^3}{80} = \underline{\mathbf{2.5\ krad/s}}$$

$$\omega_1 = \omega_0 - \frac{B}{2} = 200 - 1.25 = \underline{\mathbf{198.75\ krad/s}}$$

$$\omega_2 = \omega_0 + \frac{B}{2} = 200 + 1.25 = \underline{\mathbf{201.25\ krad/s}}$$

Chapter 14, Problem 36.

It is expected that a parallel *RLC* resonant circuit has a midband admittance of 25 × 110^{-3} S, quality factor of 80, and a resonant frequency of 200 krad/s. Calculate the values of *R*, *L*, and *C*. Find the bandwidth and the half-power frequencies.

Chapter 14, Solution 36.

$$\omega_0 = \frac{1}{\sqrt{LC}} = 5000\ \text{rad/s}$$

$$\mathbf{Y(\omega_0)} = \frac{1}{R} \longrightarrow \mathbf{Z(\omega_0)} = R = \underline{\mathbf{2\ k\Omega}}$$

$$\mathbf{Y(\omega_0/4)} = \frac{1}{R} + j\left(\frac{\omega_0}{4}C - \frac{4}{\omega_0 L}\right) = 0.5 - j18.75\ \text{mS}$$

$$\mathbf{Z(\omega_0/4)} = \frac{1}{0.0005 - j0.01875} = \underline{\mathbf{1.4212 + j53.3\ \Omega}}$$

$$\mathbf{Y(\omega_0/2)} = \frac{1}{R} + j\left(\frac{\omega_0}{2}C - \frac{2}{\omega_0 L}\right) = 0.5 - j7.5\ \text{mS}$$

$$\mathbf{Z(\omega_0/2)} = \frac{1}{0.0005 - j0.0075} = \underline{\mathbf{8.85 + j132.74\ \Omega}}$$

$$\mathbf{Y(2\omega_0)} = \frac{1}{R} + j\left(2\omega_0 L - \frac{1}{2\omega_0 C}\right) = 0.5 + j7.5\ \text{mS}$$

$$\mathbf{Z(2\omega_0)} = \underline{\mathbf{8.85 - j132.74\ \Omega}}$$

$$\mathbf{Y(4\omega_0)} = \frac{1}{R} + j\left(4\omega_0 L - \frac{1}{4\omega_0 C}\right) = 0.5 + j18.75\ \text{mS}$$

$$\mathbf{Z(4\omega_0)} = \underline{\mathbf{1.4212 - j53.3\ \Omega}}$$

Chapter 14, Problem 37.

Rework Prob. 14.25 if the elements are connected in parallel.

Chapter 14, Solution 37.

$$Z = j\omega L // (R + \frac{1}{j\omega C}) = \frac{j\omega L(R + \frac{1}{j\omega C})}{R + \frac{1}{j\omega C} + j\omega L} = \frac{\left(\frac{L}{C} + j\omega LR\right)\left(R - j(\omega L - \frac{1}{\omega C})\right)}{R^2 + (\omega L - \frac{1}{\omega C})^2}$$

$$\mathrm{Im}(Z) = \frac{\omega LR^2 - \frac{L}{C}\left(\omega L - \frac{1}{\omega C}\right)}{R^2 + (\omega L - \frac{1}{\omega C})^2} = 0 \quad \longrightarrow \quad \omega^2 (LC - R^2C^2) = 1$$

Thus,

$$\underline{\omega = \frac{1}{\sqrt{LC - R^2C^2}}}$$

Chapter 14, Problem 38.

Find the resonant frequency of the circuit in Fig. 14.78.

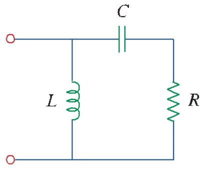


Figure 14.78
For Prob. 14.38.

Chapter 14, Solution 38.

$$Y = \frac{1}{R + j\omega L} + j\omega C = j\omega C + \frac{R - j\omega L}{R^2 + \omega^2 L^2}$$

At resonance, $\text{Im}(\mathbf{Y}) = 0$, i.e.

$$\omega_0 C - \frac{\omega_0 L}{R^2 + \omega_0^2 L^2} = 0$$

$$R^2 + \omega_0^2 L^2 = \frac{L}{C}$$

$$\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} = \sqrt{\frac{1}{(40\times10^{-3})(1\times10^{-6})} - \left(\frac{50}{40\times10^{-3}}\right)^2}$$

$$\omega_0 = \underline{\mathbf{4841\ rad/s}}$$

Chapter 14, Problem 39.

For the "tank" circuit in Fig. 14.79, find the resonant frequency.

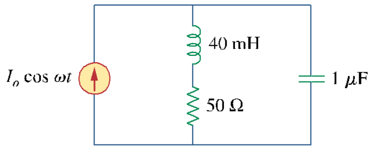


Figure 14.79
For Probs. 14.39 and 14.91.

Chapter 14, Solution 39.

(a) $B = \omega_2 - \omega_1 = 2\pi(f_2 - f_1) = 2\pi(90 - 86)x10^3 = 8\pi krad/s$

$$\omega_o = \frac{1}{2}(\omega_1 + \omega_2) = 2\pi(88)x10^3 = 176\pi X10^3$$

$$B = \frac{1}{RC} \longrightarrow C = \frac{1}{BR} = \frac{1}{8\pi x10^3 x2x10^3} = \underline{19.89nF}$$

(b) $\omega_o = \frac{1}{\sqrt{LC}} \longrightarrow L = \frac{1}{\omega^2{}_o C} = \frac{1}{(176\pi X10^3)^2 x19.89x10^{-9}} = \underline{\mathbf{164.45\ \mu H}}$

(c) $\omega_o = 176\pi = \underline{552.9krad/s}$

(d) $B = 8\pi = \underline{25.13krad/s}$

(e) $Q = \frac{\omega_o}{B} = \frac{176\pi}{8\pi} = \underline{22}$

Chapter 14, Problem 40.

A parallel resonance circuit has a resistance of 2 kΩ and half-power frequencies of 86 kHz and 90 kHz. Determine:

(a) the capacitance
(b) the inductance
(c) the resonant frequency
(d) the bandwidth
(e) the quality factor

Chapter 14, Solution 40.

(a) L = 5 + 10 = 15 mH

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{15x10^{-3}x20x10^{-6}}} = \underline{\textbf{1.8257 k rad/sec}}$$

$$Q = \omega_0 RC = 1.8257x10^3 x25x10^3 x20x10^{-6} = \underline{\textbf{912.8}}$$

$$B = \frac{1}{RC} = \frac{1}{25x10^3 20x10^{-6}} = \underline{\textbf{2 rad/s}}$$

(b) To increase B by 100% means that B' = 4.

$$C' = \frac{1}{RB'} = \frac{1}{25x10^3 x4} = \underline{\textbf{10 µF}}$$

Since $C' = \frac{C_1 C_2}{C_1 + C_2} = 10\mu F$ and $C_1 = 20$ µF, we then obtain $C_2 = 20$ µF.

Therefore, to increase the bandwidth, we merely **add another 20 µF in series with the first one**.

Chapter 14, Problem 41.

For the circuit shown in Fig. 14.80, next page:

(a) Calculate the resonant frequency ω_o, the quality factor Q, and the bandwidth B.

(b) What value of capacitance must be connected in series with the 20-μF capacitor in order to double the bandwidth?

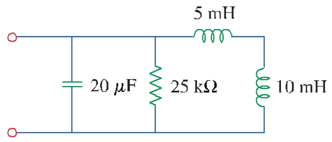


Figure 14.80
For Prob. 14.41.

Chapter 14, Solution 41.

(a) This is a series RLC circuit.

$R = 2 + 6 = 8\,\Omega, \qquad L = 1\,H, \qquad C = 0.4\,F$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.4}} = \underline{\mathbf{1.5811\ rad/s}}$$

$$Q = \frac{\omega_0 L}{R} = \frac{1.5811}{8} = \underline{\mathbf{0.1976}}$$

$$B = \frac{R}{L} = \underline{\mathbf{8\ rad/s}}$$

(b) This is a parallel RLC circuit.

$$3\ \mu F \text{ and } 6\ \mu F \longrightarrow \frac{(3)(6)}{3+6} = 2\ \mu F$$

$C = 2\ \mu F, \qquad R = 2\ k\Omega, \qquad L = 20\ mH$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(2\times10^{-6})(20\times10^{-3})}} = \underline{\mathbf{5\ krad/s}}$$

$$Q = \frac{R}{\omega_0 L} = \frac{2\times10^{3}}{(5\times10^{3})(20\times10^{-3})} = \underline{\mathbf{20}}$$

$$B = \frac{1}{RC} = \frac{1}{(2\times10^{3})(2\times10^{-6})} = \underline{\mathbf{250\ rad/s}}$$

Chapter 14, Problem 42.

For the circuits in Fig. 14.81, find the resonant frequency ω_o, the quality factor Q, and the bandwidth B.

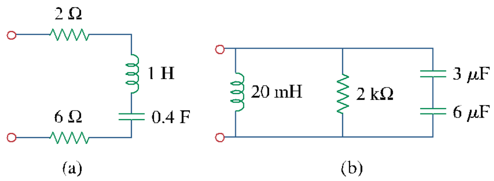


Figure 14.81
For Prob. 14.42.

Chapter 14, Solution 42.

(a) $\mathbf{Z}_{in} = (1/j\omega C) \,||\, (R + j\omega L)$

$$\mathbf{Z}_{in} = \frac{\dfrac{R + j\omega L}{j\omega C}}{R + j\omega L + \dfrac{1}{j\omega C}} = \frac{R + j\omega L}{1 - \omega^2 LC + j\omega RC}$$

$$\mathbf{Z}_{in} = \frac{(R + j\omega L)(1 - \omega^2 LC - j\omega RC)}{(1 - \omega^2 LC)^2 + \omega^2 R^2 C^2}$$

At resonance, $\mathrm{Im}(\mathbf{Z}_{in}) = 0$, i.e.

$$0 = \omega_0 L(1 - \omega_0^2 LC) - \omega_0 R^2 C$$

$$\omega_0^2 L^2 C = L - R^2 C$$

$$\omega_0 = \sqrt{\frac{L - R^2 C}{L^2 C}} = \underline{\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}}$$

(b) $\mathbf{Z}_{in} = R \,||\, (j\omega L + 1/j\omega C)$

$$\mathbf{Z}_{in} = \frac{R\,(j\omega L + 1/j\omega C)}{R + j\omega L + 1/j\omega C} = \frac{R\,(1 - \omega^2 LC)}{(1 - \omega^2 LC) + j\omega RC}$$

$$\mathbf{Z}_{in} = \frac{R\,(1 - \omega^2 LC)[(1 - \omega^2 LC) - j\omega RC]}{(1 - \omega^2 LC)^2 + \omega^2 R^2 C^2}$$

At resonance, $\mathrm{Im}(\mathbf{Z}_{in}) = 0$, i.e.

$$0 = R\,(1 - \omega^2 LC)\,\omega RC$$

$$1 - \omega^2 LC = 0$$

$$\omega_0 = \underline{\frac{\mathbf{1}}{\sqrt{\mathbf{LC}}}}$$

Chapter 14, Problem 43.

Calculate the resonant frequency of each of the circuits in Fig. 14.82.

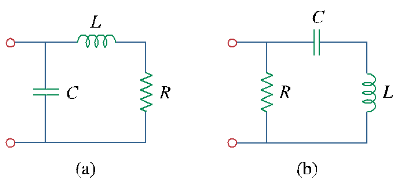
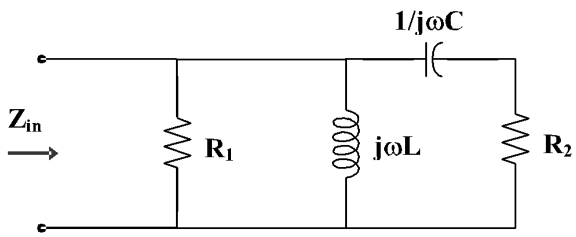


Figure 14.82
For Prob. 14.43.

Chapter 14, Solution 43.

Consider the circuit below.



(a) $\mathbf{Z}_{in} = (R_1 \,||\, j\omega L) \,||\, (R_2 + 1/j\omega C)$

$$\mathbf{Z}_{in} = \left(\frac{R_1 j\omega L}{R_1 + j\omega L}\right) \,||\, \left(R_2 + \frac{1}{j\omega C}\right)$$

$$\mathbf{Z}_{in} = \frac{\dfrac{j\omega R_1 L}{R_1 + j\omega L} \cdot \left(R_2 + \dfrac{1}{j\omega C}\right)}{R_2 + \dfrac{1}{j\omega C} + \dfrac{jR_1 \omega L}{R_1 + j\omega L}}$$

$$\mathbf{Z}_{in} = \frac{j\omega R_1 L(1 + j\omega R_2 C)}{(R_1 + j\omega L)(1 + j\omega R_2 C) - \omega^2 LCR_1}$$

$$\mathbf{Z}_{in} = \frac{-\omega^2 R_1 R_2 LC + j\omega R_1 L}{R_1 - \omega^2 LCR_1 - \omega^2 LCR_2 + j\omega(L + R_1 R_2 C)}$$

$$\mathbf{Z}_{in} = \frac{(-\omega^2 R_1 R_2 LC + j\omega R_1 L)[R_1 - \omega^2 LCR_1 - \omega^2 LCR_2 - j\omega(L + R_1 R_2 C)]}{(R_1 - \omega^2 LCR_1 - \omega^2 LCR_2)^2 + \omega^2 (L + R_1 R_2 C)^2}$$

At resonance, $\text{Im}(\mathbf{Z}_{in}) = 0$, i.e.

$$0 = \omega^3 R_1 R_2 LC(L + R_1 R_2 C) + \omega R_1 L(R_1 - \omega^2 LCR_1 - \omega^2 LCR_2)$$
$$0 = \omega^3 R_1^2 R_2^2 LC^2 + R_1^2 \omega L - \omega^3 R_1^2 L^2 C$$
$$0 = \omega^2 R_2^2 C^2 + 1 - \omega^2 LC$$
$$\omega^2 (LC - R_2^2 C^2) = 1$$

$$\omega_0 = \frac{1}{\sqrt{LC - R_2^2 C^2}}$$

$$\omega_0 = \frac{1}{\sqrt{(0.02)(9\times10^{-6}) - (0.1)^2 (9\times10^{-6})^2}}$$

$$\omega_0 = \underline{\mathbf{2.357\ krad/s}}$$

(b) At $\omega = \omega_0 = 2.357$ krad/s,

$$j\omega L = j(2.357\times10^3)(20\times10^{-3}) = j47.14$$

$$R_1 \,||\, j\omega L = \frac{j47.14}{1 + j47.14} = 0.9996 + j0.0212$$

$$R_2 + \frac{1}{j\omega C} = 0.1 + \frac{1}{j(2.357\times10^3)(9\times10^{-6})} = 0.1 - j47.14$$

$$\mathbf{Z}_{in}(\omega_0) = (R_1 \,||\, j\omega L) \,||\, (R_2 + 1/j\omega C)$$

$$\mathbf{Z}_{in}(\omega_0) = \frac{(0.9996 + j0.0212)(0.1 - j47.14)}{(0.9996 + j0.0212) + (0.1 - j47.14)}$$

$$\mathbf{Z}_{in}(\omega_0) = \underline{\mathbf{1\,\Omega}}$$

Chapter 14, Problem 44.

* For the circuit in Fig. 14.83, find:

(a) the resonant frequency ω_o

(b) $\mathbf{Z}_{in}(\omega_o)$

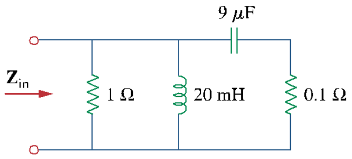
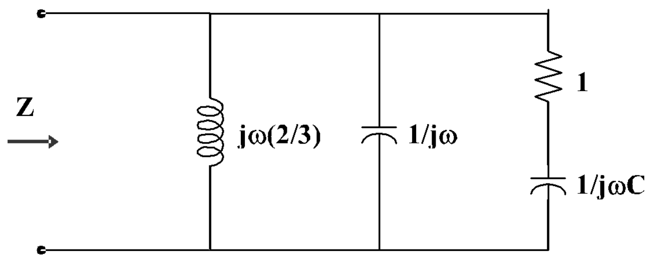


Figure 14.83
For Prob. 14.44.

* An asterisk indicates a challenging problem.

Chapter 14, Solution 44.

We find the input impedance of the circuit shown below.



$$\frac{1}{\mathbf{Z}} = \frac{3}{j\omega 2} + j\omega + \frac{1}{1 + 1/j\omega C}, \qquad \omega = 1$$

$$\frac{1}{\mathbf{Z}} = \text{-}j1.5 + j + \frac{jC}{1 + jC} = \text{-}j0.5 + \frac{C^2 + jC}{1 + C^2}$$

v(t) and i(t) are in phase when **Z** is purely real, i.e.

$$0 = \text{-}0.5 + \frac{C}{1 + C^2} \longrightarrow (C - 1)^2 = 1 \qquad \text{or} \qquad C = \underline{\mathbf{1\,F}}$$

$$\frac{1}{\mathbf{Z}} = \frac{C^2}{1 + C^2} = \frac{1}{2} \longrightarrow \mathbf{Z} = 2\,\Omega$$

$$\mathbf{V} = \mathbf{Z}\mathbf{I} = (2)(10) = 20$$

$$v(t) = 20\sin(t)\ \text{V}, \qquad \text{i.e.} \qquad V_o = \underline{\mathbf{20\ V}}$$

Chapter 14, Problem 45.

For the circuit shown in Fig. 14.84, find ω_o, B, and Q, as seen by the voltage across the inductor.

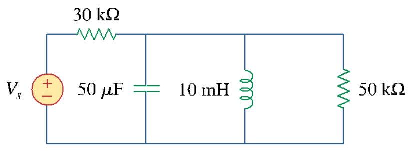
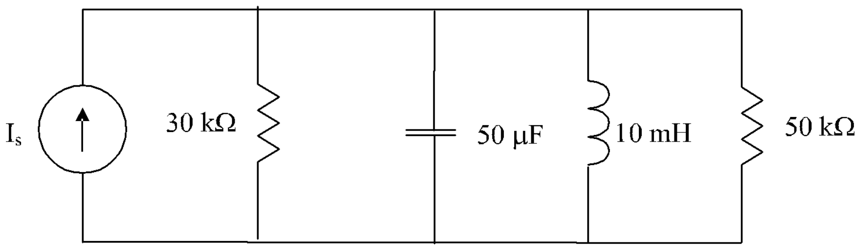


Figure 14.84
For Prob. 14.45.

Chapter 14, Solution 45.

Convert the voltage source to a current source as shown below.



R = 30//50 = 30x50/80 = 18.75 kΩ
This is a parallel resonant circuit.

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10x10^{-3}x50x10^{-6}}} = \underline{447.21 \text{ rad/s}}$$

$$B = \frac{1}{RC} = \frac{1}{18.75x10^{3}x50x10^{-6}} = \underline{1.067 \text{ rad/s}}$$

$$Q = \frac{\omega_o}{B} = \frac{447.21}{1.067} = \underline{419.13}$$

Chapter 14, Problem 46.

For the network illustrated in Fig. 14.85, find

(a) the transfer function $\mathbf{H}(\omega) = \mathbf{V}_o(\omega)/\mathbf{I}(\omega)$,

(b) the magnitude of **H** at $\omega_o = 1$ rad/s.

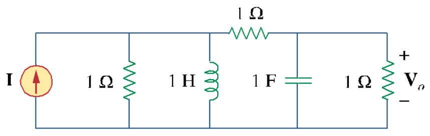


Figure 14.85
For Probs. 14.46, 14.78, and 14.92.

Chapter 14, Solution 46.

(a) This is an RLC series circuit.

$$\omega_o = \frac{1}{\sqrt{LC}} \quad \longrightarrow \quad C = \frac{1}{\omega_o^2 L} = \frac{1}{(2\pi x 15x10^3)^2 x10x10^{-3}} = \underline{11.26\text{nF}}$$

(b) $$Z = R, \;\; I = V/Z = 120/20 = \underline{\mathbf{6\ A}}$$

(c) $$Q = \frac{\omega_o L}{R} = \frac{2\pi x15x10^3 x10x10^{-3}}{20} = 15\pi = \underline{47.12}$$

Chapter 14, Problem 47.

Show that a series *LR* circuit is a lowpass filter if the output is taken across the resistor. Calculate the corner frequency f_c if $L = 2$ mH and $R = 10\ \text{k}\Omega$.

Chapter 14, Solution 47.

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{R + j\omega L} = \frac{1}{1 + j\omega L/R}$$

$H(0) = 1$ and $H(\infty) = 0$ showing that this circuit is a lowpass filter.

At the corner frequency, $\left|H(\omega_c)\right| = \dfrac{1}{\sqrt{2}}$, i.e.

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1 + \left(\dfrac{\omega_c L}{R}\right)^2}} \longrightarrow 1 = \frac{\omega_c L}{R} \qquad \text{or} \qquad \omega_c = \frac{R}{L}$$

Hence,

$$\omega_c = \frac{R}{L} = 2\pi f_c$$

$$f_c = \frac{1}{2\pi} \cdot \frac{R}{L} = \frac{1}{2\pi} \cdot \frac{10 \times 10^3}{2 \times 10^{-3}} = \underline{\mathbf{796\ kHz}}$$

Chapter 14, Problem 48.

Find the transfer function $\mathbf{V}_o/\mathbf{V}_s$ of the circuit in Fig. 14.86. Show that the circuit is a lowpass filter.

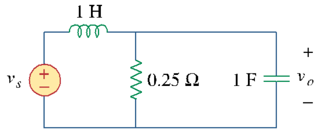


Figure 14.86
For Prob. 14.48.

Chapter 14, Solution 48.

$$\mathbf{H}(\omega) = \frac{R \,||\, \frac{1}{j\omega C}}{j\omega L + R \,||\, \frac{1}{j\omega C}}$$

$$\mathbf{H}(\omega) = \frac{\frac{R/j\omega C}{R + 1/j\omega C}}{j\omega L + \frac{R/j\omega C}{R + 1/j\omega C}}$$

$$\mathbf{H}(\omega) = \underline{\frac{\mathbf{R}}{\mathbf{R + j\omega L - \omega^2 RLC}}}$$

$H(0) = 1$ and $H(\infty) = 0$ showing that **<u>this circuit is a lowpass filter</u>**.

Chapter 14, Problem 49.

Determine the cutoff frequency of the lowpass filter described by

$$\mathbf{H}(\omega) = \frac{4}{2 + j\omega 10}$$

Find the gain in dB and phase of $\mathbf{H}(\omega)$ at ω = 2 rad/s.

Chapter 14, Solution 49.

At dc, $H(0) = \frac{4}{2} = 2$.

Hence, $\left|H(\omega)\right| = \frac{1}{\sqrt{2}} H(0) = \frac{2}{\sqrt{2}}$

$$\frac{2}{\sqrt{2}} = \frac{4}{\sqrt{4 + 100\omega_c^2}}$$

$$4 + 100\omega_c^2 = 8 \longrightarrow \omega_c = 0.2$$

$$H(2) = \frac{4}{2 + j20} = \frac{2}{1 + j10}$$

$$\left|H(2)\right| = \frac{2}{\sqrt{101}} = 0.199$$

In dB, $20\log_{10}\left|H(2)\right| = \underline{\mathbf{-14.023}}$

$\arg H(2) = -\tan^{-1} 10 = \underline{\mathbf{-84.3°}}$

Chapter 14, Problem 50.

Determine what type of filter is in Fig. 14.87. Calculate the corner frequency f_c.

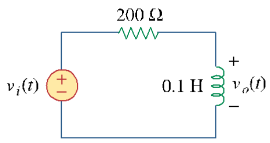


Figure 14.87
For Prob. 14.50.

Chapter 14, Solution 50.

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{j\omega L}{R + j\omega L}$$

$H(0) = 0$ and $H(\infty) = 1$ showing that **<u>this circuit is a highpass filter</u>**.

$$\mathbf{H}(\omega_c) = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1 + \left(\frac{R}{\omega_c L}\right)^2}} \longrightarrow 1 = \frac{R}{\omega_c L}$$

or $\qquad \omega_c = \frac{R}{L} = 2\pi f_c$

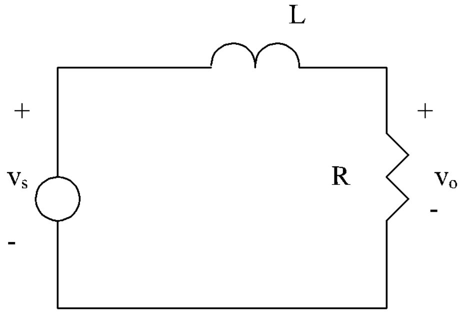
$$f_c = \frac{1}{2\pi} \cdot \frac{R}{L} = \frac{1}{2\pi} \cdot \frac{200}{0.1} = \underline{\mathbf{318.3\ Hz}}$$

Chapter 14, Problem 51.

Design an *RL* lowpass filter that uses a 40-mH coil and has a cutoff frequency of 5 kHz.

Chapter 14, Solution 51.

The lowpass RL filter is shown below.



$$H = \frac{V_o}{V_s} = \frac{R}{R + j\omega L} = \frac{1}{1 + j\omega L/R}$$

$$\omega_c = \frac{R}{L} = 2\pi f_c \quad \longrightarrow \quad R = 2\pi f_c L = 2\pi x 5x10^3 x 40x10^{-3} = \underline{1.256 k\Omega}$$

Chapter 14, Problem 52.

In a highpass *RL* filter with a cutoff frequency of 100 kHz, *L* = 40 mH. Find *R*.

Chapter 14, Solution 52.

$$\omega_c = \frac{R}{L} = 2\pi f_c$$

$$R = 2\pi f_c L = (2\pi)(10^5)(40 \times 10^{-3}) = \underline{\mathbf{25.13\ k\Omega}}$$

Chapter 14, Problem 53.

Design a series *RLC* type bandpass filter with cutoff frequencies of 10 kHz and 11 kHz. Assuming C = 80 pF, find R, L, and Q.

Chapter 14, Solution 53.

$$\omega_1 = 2\pi f_1 = 20\pi \times 10^3$$

$$\omega_2 = 2\pi f_2 = 22\pi \times 10^3$$

$$B = \omega_2 - \omega_1 = 2\pi \times 10^3$$

$$\omega_0 = \frac{\omega_2 + \omega_1}{2} = 21\pi \times 10^3$$

$$Q = \frac{\omega_0}{B} = \frac{21\pi}{2\pi} = \underline{\mathbf{10.5}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \longrightarrow L = \frac{1}{\omega_0^2 C}$$

$$L = \frac{1}{(21\pi \times 10^3)^2 (80 \times 10^{-12})} = \underline{\mathbf{2.872\ H}}$$

$$B = \frac{R}{L} \longrightarrow R = BL$$

$$R = (2\pi \times 10^3)(2.872) = \underline{\mathbf{18.045\ k\Omega}}$$

Chapter 14, Problem 54.

Design a passive bandstop filter with ω_o = 10 rad/s and Q = 20.

Chapter 14, Solution 54.

This is an open-ended problem with several possible solutions. We may choose the bandstop filter in Fig. 14.38.

$$\omega_o = \frac{1}{\sqrt{LC}} = 10 \longrightarrow LC = 0.01$$

$$Q = \frac{\omega_o L}{R} = 10 L/R = 20 \longrightarrow L = 2R$$

If we select L = 1H, then R=0.5 Ω, and C = 0.01/L = 10 mF.

Chapter 14, Problem 55.

Determine the range of frequencies that will be passed by a series *RLC* bandpass filter with $R = 10\Omega$, L = 25mH, and C = 0.4 μF. Find the quality factor.

Chapter 14, Solution 55.

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(25\times 10^{-3})(0.4\times 10^{-6})}} = 10 \text{ krad/s}$$

$$B = \frac{R}{L} = \frac{10}{25\times 10^{-3}} = 0.4 \text{ krad/s}$$

$$Q = \frac{10}{0.4} = \underline{\mathbf{25}}$$

$$\omega_1 = \omega_o - B/2 = 10 - 0.2 = 9.8 \text{ krad/s} \qquad \text{or} \qquad f_1 = \frac{9.8}{2\pi} = 1.56 \text{ kHz}$$

$$\omega_2 = \omega_o + B/2 = 10 + 0.2 = 10.2 \text{ krad/s} \qquad \text{or} \qquad f_2 = \frac{10.2}{2\pi} = 1.62 \text{ kHz}$$

Therefore,

$$\underline{\mathbf{1.56\ kHz < f < 1.62\ kHz}}$$

Chapter 14, Problem 56.

(a) Show that for a bandpass filter,

$$\mathbf{H}(s) = \frac{sB}{s^2 + sB + \omega_0^2}, \quad s = jw$$

where B = bandwidth of the filter and ω_o is the center frequency.

(b) Similarly, show that for a bandstop filter,

$$\mathbf{H}(s) = \frac{s^2 + \omega_0^2}{s^2 + sB + \omega_0^2}, \quad s = jw$$

Chapter 14, Solution 56.

(a) From Eq 14.54,

$$\mathbf{H}(s) = \frac{R}{R + sL + \frac{1}{sC}} = \frac{sRC}{1 + sRC + s^2LC} = \frac{s\frac{R}{L}}{s^2 + s\frac{R}{L} + \frac{1}{LC}}$$

Since $B = \frac{R}{L}$ and $\omega_0 = \frac{1}{\sqrt{LC}}$,

$$\mathbf{H}(s) = \underline{\frac{\mathbf{sB}}{\mathbf{s^2 + sB + \omega_0^2}}}$$

(b) From Eq. 14.56,

$$\mathbf{H}(s) = \frac{sL + \frac{1}{sC}}{R + sL + \frac{1}{sC}} = \frac{s^2 + \frac{1}{LC}}{s^2 + s\frac{R}{L} + \frac{1}{LC}}$$

$$\mathbf{H}(s) = \underline{\frac{\mathbf{s^2 + \omega_0^2}}{\mathbf{s^2 + sB + \omega_0^2}}}$$

Chapter 14, Problem 57.

Determine the center frequency and bandwidth of the bandpass filters in Fig. 14.88.

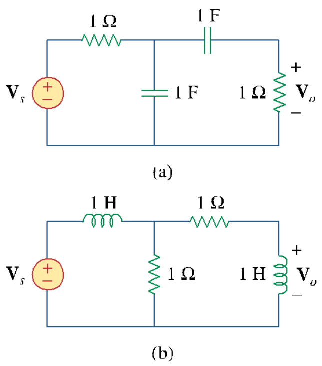
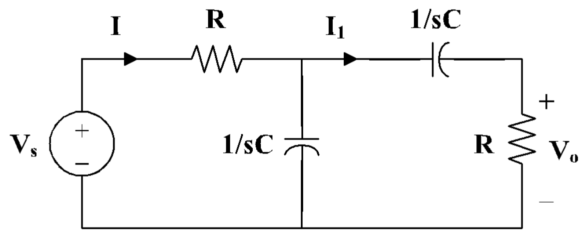


Figure 14.88
For Prob. 14.57.

Chapter 14, Solution 57.

(a) Consider the circuit below.



$$\mathbf{Z}(s) = R + \frac{1}{sC} \,||\, \left(R + \frac{1}{sC}\right) = R + \frac{\frac{1}{sC}\left(R + \frac{1}{sC}\right)}{R + \frac{2}{sC}}$$

$$\mathbf{Z}(s) = R + \frac{1 + sRC}{sC(2 + sRC)}$$

$$\mathbf{Z}(s) = \frac{1 + 3sRC + s^2R^2C^2}{sC(2 + sRC)}$$

$$\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}}$$

$$\mathbf{I}_1 = \frac{1/sC}{2/sC + R}\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}(2 + sRC)}$$

$$\mathbf{V}_o = \mathbf{I}_1 R = \frac{R\mathbf{V}_s}{2 + sRC} \cdot \frac{sC(2 + sRC)}{1 + 3sRC + s^2R^2C^2}$$

$$\mathbf{H}(s) = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{sRC}{1 + 3sRC + s^2R^2C^2}$$

$$\mathbf{H}(s) = \frac{1}{3}\left[\frac{\frac{3}{RC}s}{s^2 + \frac{3}{RC}s + \frac{1}{R^2C^2}}\right]$$

Thus, $\omega_0^2 = \frac{1}{R^2C^2}$ or $\omega_0 = \frac{1}{RC} = \underline{\mathbf{1\ rad/s}}$

$$B = \frac{3}{RC} = \underline{\mathbf{3\ rad/s}}$$

(b) Similarly,

$$\mathbf{Z}(s) = sL + R \,||\, (R + sL) = sL + \frac{R(R + sL)}{2R + sL}$$

$$\mathbf{Z}(s) = \frac{R^2 + 3sRL + s^2L^2}{2R + sL}$$

$$\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}}, \qquad \mathbf{I}_1 = \frac{R}{2R + sL}\mathbf{I} = \frac{R\,\mathbf{V}_s}{\mathbf{Z}(2R + sL)}$$

$$\mathbf{V}_o = \mathbf{I}_1 \cdot sL = \frac{sLR\,\mathbf{V}_s}{2R + sL} \cdot \frac{2R + sL}{R^2 + 3sRL + s^2L^2}$$

$$\mathbf{H}(s) = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{sRL}{R^2 + 3sRL + s^2L^2} = \frac{\frac{1}{3}\left(\frac{3R}{L}s\right)}{s^2 + \frac{3R}{L}s + \frac{R^2}{L^2}}$$

Thus, $\omega_0 = \dfrac{R}{L} = \underline{\mathbf{1\ rad/s}}$

$B = \dfrac{3R}{L} = \underline{\mathbf{3\ rad/s}}$

Chapter 14, Problem 58.

The circuit parameters for a series *RLC* bandstop filter are $R = 2\ \text{k}\Omega$, $L = 0.1$ H, $C = 40$ pF. Calculate:

(a) the center frequency

(b) the half-power frequencies

(c) the quality factor

Chapter 14, Solution 58.

(a) $$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.1)(40\times10^{-12})}} = \underline{\mathbf{0.5\times10^{6}\ rad/s}}$$

(b) $$B = \frac{R}{L} = \frac{2\times10^{3}}{0.1} = 2\times10^{4}$$

$$Q = \frac{\omega_0}{B} = \frac{0.5\times10^{6}}{2\times10^{4}} = 25$$

As a high Q circuit,

$$\omega_1 = \omega_0 - \frac{B}{2} = 10^{4}(50-1) = \underline{\mathbf{490\ krad/s}}$$

$$\omega_2 = \omega_0 + \frac{B}{2} = 10^{4}(50+1) = \underline{\mathbf{510\ krad/s}}$$

(c) As seen in part (b), $Q = \underline{\mathbf{25}}$

Chapter 14, Problem 59.

Find the bandwidth and center frequency of the bandstop filter of Fig. 14.89.

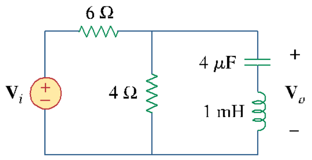
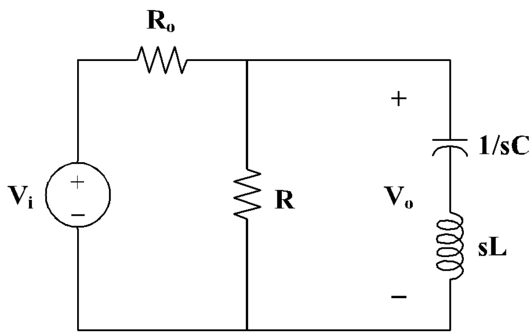


Figure 14.89
For Prob. 14.59.

Chapter 14, Solution 59.

Consider the circuit below.



$$\mathbf{Z}(s) = R \,||\, \left(sL + \frac{1}{sC}\right) = \frac{R\,(sL + 1/sC)}{R + sL + 1/sC}$$

$$\mathbf{Z}(s) = \frac{R\,(1 + s^2LC)}{1 + sRC + s^2LC}$$

$$\mathbf{H} = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{\mathbf{Z}}{\mathbf{Z}+R_o} = \frac{R(1+s^2LC)}{R_o + sRR_oC + s^2LCR_o + R + s^2LCR}$$

$$\mathbf{Z}_{in} = R_o + \mathbf{Z} = R_o + \frac{R(1+s^2LC)}{1+sRC+s^2LC}$$

$$\mathbf{Z}_{in} = \frac{R_o + sRR_oC + s^2LCR_o + R + s^2LCR}{1+sRC+s^2LC}$$

$s = j\omega$

$$\mathbf{Z}_{in} = \frac{R_o + j\omega RR_oC - \omega^2LCR_o + R - \omega^2LCR}{1-\omega^2LC + j\omega RC}$$

$$\mathbf{Z}_{in} = \frac{(R_o + R - \omega^2LCR_o - \omega^2LCR + j\omega RR_oC)(1-\omega^2LC - j\omega RC)}{(1-\omega^2LC)^2 + (\omega RC)^2}$$

$\text{Im}(\mathbf{Z}_{in}) = 0$ implies that

$$-\omega RC[R_o + R - \omega^2LCR_o - \omega^2LCR] + \omega RR_oC(1-\omega^2LC) = 0$$

$$R_o + R - \omega^2LCR_o - \omega^2LCR - R_o + \omega^2LCR_o = 0$$

$$\omega^2LCR = R$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1\times10^{-3})(4\times10^{-6})}} = \underline{\mathbf{15.811\ krad/s}}$$

$$\mathbf{H} = \frac{R(1-\omega^2LC)}{R_o + j\omega RR_oC + R - \omega^2LCR_o - \omega^2LCR}$$

$$H_{max} = H(0) = \frac{R}{R_o + R}$$

or $$H_{max} = H(\infty) = \lim_{\omega\to\infty} \frac{R\left(\frac{1}{\omega^2} - LC\right)}{\frac{R_o+R}{\omega^2} + j\frac{RR_oC}{\omega} - LC(R+R_o)} = \frac{R}{R+R_o}$$

At ω_1 and ω_2, $|\mathbf{H}| = \frac{1}{\sqrt{2}} H_{mzx}$

$$\frac{R}{\sqrt{2}(R_o + R)} = \left| \frac{R(1-\omega^2 LC)}{R_o + R - \omega^2 LC(R_o + R) + j\omega RR_o C} \right|$$

$$\frac{1}{\sqrt{2}} = \frac{(R_o + R)(1-\omega^2 LC)}{\sqrt{(\omega RR_o C)^2 + (R_o + R - \omega^2 LC(R_o + R))^2}}$$

$$\frac{1}{\sqrt{2}} = \frac{10(1-\omega^2 \cdot 4\times 10^{-9})}{\sqrt{(96\times 10^{-6}\omega)^2 + (10-\omega^2 \cdot 4\times 10^{-8})^2}}$$

$$0 = \frac{10(1-\omega^2 \cdot 4\times 10^{-9})}{\sqrt{(96\times 10^{-6}\omega)^2 + (10-\omega^2 \cdot 4\times 10^{-8})^2}} - \frac{1}{\sqrt{2}}$$

$$(10-\omega^2 \cdot 4\times 10^{-8})(\sqrt{2}) - \sqrt{(96\times 10^{-6}\omega)^2 + (10-\omega^2 \cdot 4\times 10^{-8})^2} = 0$$

$$(2)(10-\omega^2 \cdot 4\times 10^{-8})^2 = (96\times 10^{-6}\omega)^2 + (10-\omega^2 \cdot 4\times 10^{-8})^2$$

$$(96\times 10^{-6}\omega)^2 - (10-\omega^2 \cdot 4\times 10^{-8})^2 = 0$$

$$1.6\times 10^{-15}\omega^4 - 8.092\times 10^{-7}\omega^2 + 100 = 0$$

$$\omega^4 - 5.058\times 10^8 + 6.25\times 10^{16} = 0$$

$$\omega^2 = \begin{cases} 2.9109\times 10^8 \\ 2.1471\times 10^8 \end{cases}$$

Hence,

$$\omega_1 = 14.653 \text{ krad/s}$$

$$\omega_2 = 17.061 \text{ krad/s}$$

$$B = \omega_2 - \omega_1 = 17.061 - 14.653 = \underline{\mathbf{2.408\ krad/s}}$$

Chapter 14, Problem 60.

Obtain the transfer function of a highpass filter with a passband gain of 10 and a cutoff frequency of 50 rad/s.

Chapter 14, Solution 60.

$$\mathbf{H}'(\omega) = \frac{j\omega RC}{1 + j\omega RC} = \frac{j\omega}{j\omega + 1/RC} \qquad \text{(from Eq. 14.52)}$$

This has a unity passband gain, i.e. $H(\infty) = 1$.

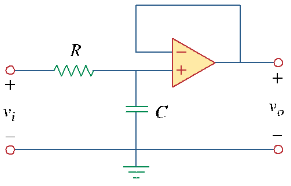
$$\frac{1}{RC} = \omega_c = 50$$

$$\mathbf{H}^{\wedge}(\omega) = 10\,\mathbf{H}'(\omega) = \frac{j10\omega}{50 + j\omega}$$

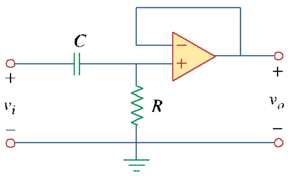
$$\mathbf{H}(\omega) = \underline{\frac{\mathbf{j10\omega}}{\mathbf{50 + j\omega}}}$$

Chapter 14, Problem 61.

Find the transfer function for each of the active filters in Fig. 14.90.



(a)



(b)

Figure 14.90
For Probs. 14.61 and 14.62.

Chapter 14, Solution 61.

(a) $$\mathbf{V}_+ = \frac{1/j\omega C}{R + 1/j\omega C}\mathbf{V}_i, \qquad \mathbf{V}_- = \mathbf{V}_o$$

Since $\mathbf{V}_+ = \mathbf{V}_-$,

$$\frac{1}{1 + j\omega RC}\mathbf{V}_i = \mathbf{V}_o$$

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \underline{\frac{\mathbf{1}}{\mathbf{1 + j\omega RC}}}$$

(b) $$\mathbf{V}_+ = \frac{R}{R + 1/j\omega C}\mathbf{V}_i, \qquad \mathbf{V}_- = \mathbf{V}_o$$

Since $\mathbf{V}_+ = \mathbf{V}_-$,

$$\frac{j\omega RC}{1 + j\omega RC}\mathbf{V}_i = \mathbf{V}_o$$

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \underline{\frac{\mathbf{j\omega RC}}{\mathbf{1 + j\omega RC}}}$$

Chapter 14, Problem 62.

The filter in Fig. 14.90(b) has a 3-dB cutoff frequency at 1 kHz. If its input is connected to a 120-mV variable frequency signal, find the output voltage at:

(a) 200 Hz (b) 2 kHz (c) 10 kHz

Chapter 14, Solution 62.

This is a highpass filter.

$$\mathbf{H}(\omega) = \frac{j\omega RC}{1 + j\omega RC} = \frac{1}{1 - j/\omega RC}$$

$$\mathbf{H}(\omega) = \frac{1}{1 - j\omega_c/\omega}, \qquad \omega_c = \frac{1}{RC} = 2\pi(1000)$$

$$\mathbf{H}(\omega) = \frac{1}{1 - jf_c/f} = \frac{1}{1 - j1000/f}$$

(a) $$\mathbf{H}(f = 200 \text{ Hz}) = \frac{1}{1 - j5} = \frac{\mathbf{V}_o}{\mathbf{V}_i}$$

$$|\mathbf{V}_o| = \frac{120 \text{ mV}}{|1 - j5|} = \underline{\mathbf{23.53 \text{ mV}}}$$

(b) $$\mathbf{H}(f = 2 \text{ kHz}) = \frac{1}{1 - j0.5} = \frac{\mathbf{V}_o}{\mathbf{V}_i}$$

$$|\mathbf{V}_o| = \frac{120 \text{ mV}}{|1 - j0.5|} = \underline{\mathbf{107.3 \text{ mV}}}$$

(c) $$\mathbf{H}(f = 10 \text{ kHz}) = \frac{1}{1 - j0.1} = \frac{\mathbf{V}_o}{\mathbf{V}_i}$$

$$|\mathbf{V}_o| = \frac{120 \text{ mV}}{|1 - j0.1|} = \underline{\mathbf{119.4 \text{ mV}}}$$

Chapter 14, Problem 63.

Design an active first-order highpass filter with

$$\mathbf{H}(s) = -\frac{100s}{s+10}, \qquad s = j\omega$$

Use a 1-μF capacitor.

Chapter 14, Solution 63.

For an active highpass filter,

$$H(s) = -\frac{sC_iR_f}{1+sC_iR_i} \qquad (1)$$

But

$$H(s) = -\frac{10s}{1+s/10} \qquad (2)$$

Comparing (1) and (2) leads to:

$$C_iR_f = 10 \longrightarrow R_f = \frac{10}{C_i} = \underline{10\text{M}\Omega}$$

$$C_iR_i = 0.1 \longrightarrow R_i = \frac{0.1}{C_i} = \underline{100\text{k}\Omega}$$

Chapter 14, Problem 64.

Obtain the transfer function of the active filter in Fig. 14.91 on the next page. What kind of filter is it?

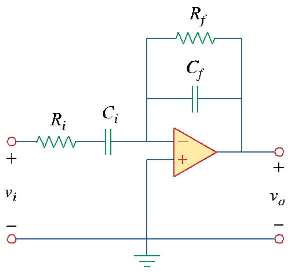


Figure 14.91
For Prob. 14.64.

Chapter 14, Solution 64.

$$Z_f = R_f \parallel \frac{1}{j\omega C_f} = \frac{R_f}{1 + j\omega R_f C_f}$$

$$Z_i = R_i + \frac{1}{j\omega C_i} = \frac{1 + j\omega R_i C_i}{j\omega C_i}$$

Hence,

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{-\mathbf{Z}_f}{\mathbf{Z}_i} = \underline{\frac{-j\omega R_f C_i}{(1 + j\omega R_f C_f)(1 + j\omega R_i C_i)}}$$

This is a bandpass filter. $\mathbf{H}(\omega)$ is similar to the product of the transfer function of a lowpass filter and a highpass filter.

Chapter 14, Problem 65.

A highpass filter is shown in Fig. 14.92. Show that the transfer function is

$$\mathbf{H}(\omega) = \left(1 + \frac{R_f}{R_i}\right) \frac{j\omega RC}{1 + j\omega RC}$$

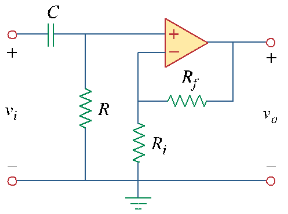


Figure 14.92
For Prob. 14.65.

Chapter 14, Solution 65.

$$\mathbf{V}_+ = \frac{R}{R + 1/j\omega C}\mathbf{V}_i = \frac{j\omega RC}{1 + j\omega RC}\mathbf{V}_i$$

$$\mathbf{V}_- = \frac{R_i}{R_i + R_f}\mathbf{V}_o$$

Since $\mathbf{V}_+ = \mathbf{V}_-$,

$$\frac{R_i}{R_i + R_f}\mathbf{V}_o = \frac{j\omega RC}{1 + j\omega RC}\mathbf{V}_i$$

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \underline{\left(\mathbf{1} + \frac{\mathbf{R_f}}{\mathbf{R_i}}\right)\left(\frac{\mathbf{j\omega RC}}{\mathbf{1 + j\omega RC}}\right)}$$

It is evident that as $\omega \to \infty$, the gain is $\underline{\mathbf{1} + \frac{\mathbf{R_f}}{\mathbf{R_i}}}$ and that the corner frequency is $\underline{\frac{\mathbf{1}}{\mathbf{RC}}}$.

Chapter 14, Problem 66.

A "general" first-order filter is shown in Fig. 14.93.

(a) Show that the transfer function is

$$\mathbf{H}(s) = \frac{R_4}{R_3 + R_4} \times \frac{s + (1/R_1C)[R_1/R_2 - R_3/R_4]}{s + 1/R_2C}$$

$s = j\omega$

(b) What condition must be satisfied for the circuit to operate as a highpass filter?

(c) What condition must be satisfied for the circuit to operate as a lowpass filter?

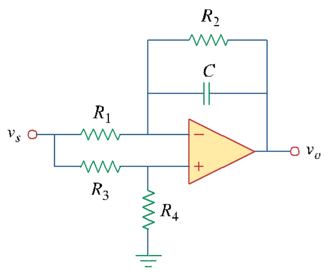


Figure 14.93
For Prob. 14.66.

Chapter 14, Solution 66.

(a) <u>**Proof**</u>

(b) When $\mathbf{R_1 R_4 = R_2 R_3}$,

$$\mathbf{H}(s) = \frac{R_4}{R_3 + R_4} \cdot \frac{s}{s + 1/R_2C}$$

(c) When $\mathbf{R_3 \to \infty}$,

$$\mathbf{H}(s) = \frac{-1/R_1C}{s + 1/R_2C}$$

Chapter 14, Problem 67.

Design an active lowpass filter with dc gain of 0.25 and a corner frequency of 500 Hz.

Chapter 14, Solution 67.

$$\text{DC gain} = \frac{R_f}{R_i} = \frac{1}{4} \longrightarrow R_i = 4R_f$$

$$\text{Corner frequency} = \omega_c = \frac{1}{R_f C_f} = 2\pi(500)\ \text{rad/s}$$

If we select $R_f = 20\ \text{k}\Omega$, then $R_i = 80\ \text{k}\Omega$ and

$$C = \frac{1}{(2\pi)(500)(20\times 10^3)} = 15.915\ \text{nF}$$

Therefore, if $R_f = \underline{\mathbf{20\ k\Omega}}$, then $R_i = \underline{\mathbf{80\ k\Omega}}$ and $C = \underline{\mathbf{15.915\ nF}}$

Chapter 14, Problem 68.

Design an active highpass filter with a high-frequency gain of 5 and a corner frequency of 200 Hz.

Chapter 14, Solution 68.

$$\text{High frequency gain} = 5 = \frac{R_f}{R_i} \longrightarrow R_f = 5R_i$$

$$\text{Corner frequency} = \omega_c = \frac{1}{R_i C_i} = 2\pi(200)\ \text{rad/s}$$

If we select $R_i = 20\ \text{k}\Omega$, then $R_f = 100\ \text{k}\Omega$ and

$$C = \frac{1}{(2\pi)(200)(20\times 10^3)} = 39.8\ \text{nF}$$

Therefore, if $R_i = \underline{\mathbf{20\ k\Omega}}$, then $R_f = \underline{\mathbf{100\ k\Omega}}$ and $C = \underline{\mathbf{39.8\ nF}}$

Chapter 14, Problem 69.

Design the filter in Fig. 14.94 to meet the following requirements:

(a) It must attenuate a signal at 2 kHz by 3 dB compared with its value at 10 MHz.

(b) It must provide a steady-state output of $v_o(t) = 10\sin(2\pi \times 10^8 t + 180^o)$ V for an input $v_s(t) = 4\sin(2\pi \times 10^8 t)$ V.

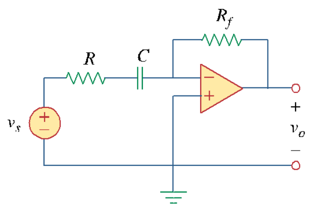


Figure 14.94
For Prob. 14.69.

Chapter 14, Solution 69.

This is a highpass filter with $f_c = 2$ kHz.

$$\omega_c = 2\pi f_c = \frac{1}{RC}$$

$$RC = \frac{1}{2\pi f_c} = \frac{1}{4\pi \times 10^3}$$

10^8 Hz may be regarded as high frequency. Hence the high-frequency gain is

$$\frac{-R_f}{R} = \frac{-10}{4} \qquad \text{or} \qquad R_f = 2.5R$$

If we let $R = \underline{\mathbf{10\ k\Omega}}$, then $R_f = \underline{\mathbf{25\ k\Omega}}$, and $C = \dfrac{1}{4000\pi \times 10^4} = \underline{\mathbf{7.96\ nF}}$.

Chapter 14, Problem 70.

e2d

* A second-order active filter known as a Butterworth filter is shown in Fig. 14.95.

(a) Find the transfer function $\mathbf{V}_o/\mathbf{V}_i$.

(b) Show that it is a lowpass filter.

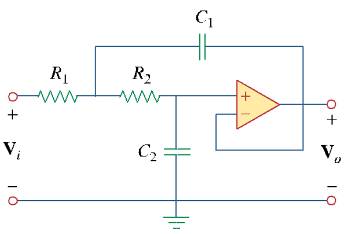


Figure 14.95
For Prob. 14.70.

* an asterisk indicates a challenging problem.

Chapter 14, Solution 70.

(a)
$$\mathbf{H}(s) = \frac{\mathbf{V}_o(s)}{\mathbf{V}_i(s)} = \frac{Y_1Y_2}{Y_1Y_2 + Y_4(Y_1 + Y_2 + Y_3)}$$

where $Y_1 = \dfrac{1}{R_1} = G_1,\ Y_2 = \dfrac{1}{R_2} = G_2,\ Y_3 = sC_1,\ Y_4 = sC_2$.

$$\mathbf{H}(s) = \underline{\frac{\mathbf{G_1G_2}}{\mathbf{G_1G_2 + sC_2(G_1 + G_2 + sC_1)}}}$$

(b)
$$H(0) = \frac{G_1G_2}{G_1G_2} = 1, \qquad H(\infty) = 0$$

showing that **<u>this circuit is a lowpass filter</u>**.

Chapter 14, Problem 71.

Use magnitude and frequency scaling on the circuit of Fig. 14.76 to obtain an equivalent circuit in which the inductor and capacitor have magnitude 1 H and 1 F respectively.

Chapter 14, Solution 71.

$R = 50\,\Omega,\ L = 40\text{ mH},\ C = 1\,\mu\text{F}$

$$L' = \frac{K_m}{K_f} L \longrightarrow 1 = \frac{K_m}{K_f} \cdot (40 \times 10^{-3})$$

$$25K_f = K_m \qquad (1)$$

$$C' = \frac{C}{K_m K_f} \longrightarrow 1 = \frac{10^{-6}}{K_m K_f}$$

$$10^6 K_f = \frac{1}{K_m} \qquad (2)$$

Substituting (1) into (2),

$$10^6 K_f = \frac{1}{25K_f}$$

$$K_f = \underline{\mathbf{0.2 \times 10^{-3}}}$$

$$K_m = 25K_f = \underline{\mathbf{5 \times 10^{-3}}}$$

Chapter 14, Problem 72.

What values of K_m and K_f will scale a 4-mH inductor and a 20-μF capacitor to 1 H and 2 F respectively?

Chapter 14, Solution 72.

$$L'C' = \frac{LC}{K_f^2} \longrightarrow K_f^2 = \frac{LC}{L'C'}$$

$$K_f^2 = \frac{(4\times 10^{-3})(20\times 10^{-6})}{(1)(2)} = 4\times 10^{-8}$$

$$K_f = \underline{\mathbf{2\times 10^{-4}}}$$

$$\frac{L'}{C'} = \frac{L}{C}K_m^2 \longrightarrow K_m^2 = \frac{L'}{C'}\cdot\frac{C}{L}$$

$$K_m^2 = \frac{(1)(20\times 10^{-6})}{(2)(4\times 10^{-3})} = 2.5\times 10^{-3}$$

$$K_m = \underline{\mathbf{5\times 10^{-2}}}$$

Chapter 14, Problem 73.

Calculate the values of R, L, and C that will result in $R = 12\text{k}\Omega$, $L = 40\ \mu$H and $C = 300$ nF respectively when magnitude-scaled by 800 and frequency-scaled by 1000.

Chapter 14, Solution 73.

$$R' = K_m R = (12)(800\times 10^3) = \underline{\mathbf{9.6\ M\Omega}}$$

$$L' = \frac{K_m}{K_f}L = \frac{800}{1000}(40\times 10^{-6}) = \underline{\mathbf{32\ \mu F}}$$

$$C' = \frac{C}{K_m K_f} = \frac{300\times 10^{-9}}{(800)(1000)} = \underline{\mathbf{0.375\ pF}}$$

Chapter 14, Problem 74.

A circuit has $R_1 = 3\ \Omega$, $R_2 = 10\Omega$, $L = 2$H and $C = 1/10$F. After the circuit is magnitude-scaled by 100 and frequency-scaled by 10^6, find the new values of the circuit elements.

Chapter 14, Solution 74.

$$R'_1 = K_m R_1 = 3x100 = \underline{300\Omega}$$

$$R'_2 = K_m R_2 = 10x100 = \underline{1\,k\Omega}$$

$$L' = \frac{K_m}{K_f} L = \frac{10^2}{10^6}(2) = \underline{200\,\mu H}$$

$$C' = \frac{C}{K_m K_f} = \frac{\frac{1}{10}}{10^8} = \underline{1\,nF}$$

Chapter 14, Problem 75.

In an *RLC* circuit, $R = 20\Omega$, $L = 4$ H and $C = 1$ F. The circuit is magnitude-scaled by 10 and frequency-scaled by 10^5. Calculate the new values of the elements.

Chapter 14, Solution 75.

$$R' = K_m R = 20x10 = \underline{200\,\Omega}$$

$$L' = \frac{K_m}{K_f} L = \frac{10}{10^5}(4) = \underline{400\,\mu H}$$

$$C' = \frac{C}{K_m K_f} = \frac{1}{10x10^5} = \underline{1\,\mu F}$$

Chapter 14, Problem 76.

Given a parallel *RLC* circuit with $R = 5\ \text{k}\Omega$, $L = 10$ mH, and $C = 20\ \mu$F, if the circuit is magnitude-scaled by $K_m = 500$ and frequency-scaled by $K_f = 10^5$, find the resulting values of *R*, *L*, and *C*.

Chapter 14, Solution 76.

$$R' = K_m R = 500x5x10^3 = \underline{25\ \text{M}\Omega}$$

$$L' = \frac{K_m}{K_f}L = \frac{500}{10^5}(10mH) = \underline{50\ \mu\text{H}}$$

$$C' = \frac{C}{K_m K_f} = \frac{20x10^{-6}}{500x10^5} = \underline{0.4\ \text{pF}}$$

Chapter 14, Problem 77.

A series *RLC* circuit has $R = 10\Omega$, $\omega_0 = 40$ rad/s, and $B = 5$ rad/s. Find L and C when the circuit is scaled:

(a) in magnitude by a factor of 600,

(b) in frequency by a factor of 1,000,

(c) in magnitude by a factor of 400 and in frequency by a factor of 10^5.

Chapter 14, Solution 77.

L and C are needed before scaling.

$$B = \frac{R}{L} \longrightarrow L = \frac{R}{B} = \frac{10}{5} = 2 \text{ H}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \longrightarrow C = \frac{1}{\omega_0^2 L} = \frac{1}{(1600)(2)} = 312.5\ \mu\text{F}$$

(a) $L' = K_m L = (600)(2) = \underline{\textbf{1200 H}}$

$$C' = \frac{C}{K_m} = \frac{3.125\times10^{-4}}{600} = \underline{\textbf{0.5208}\ \boldsymbol{\mu}\textbf{F}}$$

(b) $L' = \frac{L}{K_f} = \frac{2}{10^3} = \underline{\textbf{2 mH}}$

$$C' = \frac{C}{K_f} = \frac{3.125\times10^{-4}}{10^3} = \underline{\textbf{312.5 nF}}$$

(c) $L' = \frac{K_m}{K_f} L = \frac{(400)(2)}{10^5} = \underline{\textbf{8 mH}}$

$$C' = \frac{C}{K_m K_f} = \frac{3.125\times10^{-4}}{(400)(10^5)} = \underline{\textbf{7.81 pF}}$$

Chapter 14, Problem 78.

Redesign the circuit in Fig. 14.85 so that all resistive elements are scaled by a factor of 1,000 and all frequency-sensitive elements are frequency-scaled by a factor of 10^4.

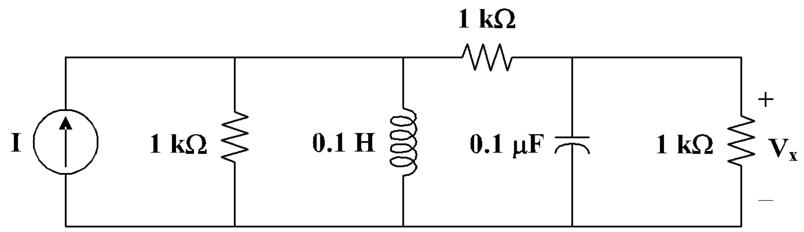
Chapter 14, Solution 78.

$$R' = K_m R = (1000)(1) = 1\ k\Omega$$

$$L' = \frac{K_m}{K_f} L = \frac{10^3}{10^4}(1) = 0.1\ H$$

$$C' = \frac{C}{K_m K_f} = \frac{1}{(10^3)(10^4)} = 0.1\ \mu F$$

<u>**The new circuit is shown below**</u>.



Chapter 14, Problem 79.

* Refer to the network in Fig. 14.96.

(a) Find $\mathbf{Z}_{\text{in}}(s)$.

(b) Scale the elements by $K_m = 10$ and $K_f = 100$. Find $\mathbf{Z}_{\text{in}}(s)$ and ω_0.

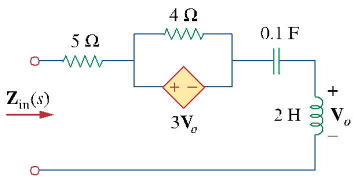
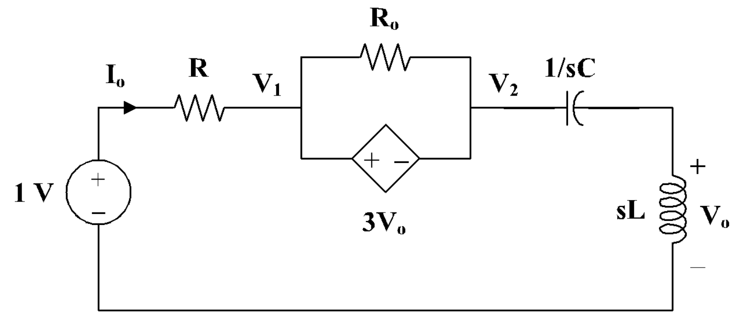


Figure 14.96
For Prob. 14.79.

* An asterisk indicates a challenging problem.

Chapter 14, Solution 79.

(a) Insert a 1-V source at the input terminals.



There is a supernode.

$$\frac{1-\mathbf{V}_1}{R} = \frac{\mathbf{V}_2}{sL + 1/sC} \qquad (1)$$

But $\quad \mathbf{V}_1 = \mathbf{V}_2 + 3\mathbf{V}_o \longrightarrow \mathbf{V}_2 = \mathbf{V}_1 - 3\mathbf{V}_o \qquad (2)$

Also, $\quad \mathbf{V}_o = \frac{sL}{sL + 1/sC}\mathbf{V}_2 \longrightarrow \frac{\mathbf{V}_o}{sL} = \frac{\mathbf{V}_2}{sL + 1/sC} \qquad (3)$

Combining (2) and (3)

$$\mathbf{V}_2 = \mathbf{V}_1 - 3\mathbf{V}_o = \frac{sL + 1/sC}{sL}\mathbf{V}_o$$

$$\mathbf{V}_o = \frac{s^2LC}{1 + 4s^2LC}\mathbf{V}_1 \qquad (4)$$

Substituting (3) and (4) into (1) gives

$$\frac{1-\mathbf{V}_1}{R} = \frac{\mathbf{V}_o}{sL} = \frac{sC}{1 + 4s^2LC}\mathbf{V}_1$$

$$1 = \mathbf{V}_1 + \frac{sRC}{1 + 4s^2LC}\mathbf{V}_1 = \frac{1 + 4s^2LC + sRC}{1 + 4s^2LC}\mathbf{V}_1$$

$$\mathbf{V}_1 = \frac{1 + 4s^2LC}{1 + 4s^2LC + sRC}$$

$$\mathbf{I}_o = \frac{1-\mathbf{V}_1}{R} = \frac{sRC}{R\,(1 + 4s^2LC + sRC)}$$

$$\mathbf{Z}_{in} = \frac{1}{\mathbf{I}_o} = \frac{1 + sRC + 4s^2LC}{sC}$$

$$\mathbf{Z}_{in} = 4sL + R + \frac{1}{sC} \qquad (5)$$

When $R = 5$, $L = 2$, $C = 0.1$,

$$\mathbf{Z}_{in}(s) = \underline{\mathbf{8s + 5 + \frac{10}{s}}}$$

At resonance,

$$\text{Im}(\mathbf{Z}_{in}) = 0 = 4\omega L - \frac{1}{\omega C}$$

or $$\omega_0 = \frac{1}{2\sqrt{LC}} = \frac{1}{2\sqrt{(0.1)(2)}} = \underline{\mathbf{1.118\ rad/s}}$$

(b) After scaling,

$$R' \longrightarrow K_m R$$
$$4\,\Omega \longrightarrow 40\,\Omega$$
$$5\,\Omega \longrightarrow 50\,\Omega$$

$$L' = \frac{K_m}{K_f} L = \frac{10}{100}(2) = 0.2\ \text{H}$$

$$C' = \frac{C}{K_m K_f} = \frac{0.1}{(10)(100)} = 10^{-4}$$

From (5),

$$\mathbf{Z}_{in}(s) = \underline{\mathbf{0.8s + 50 + \frac{10^4}{s}}}$$

$$\omega_0 = \frac{1}{2\sqrt{LC}} = \frac{1}{2\sqrt{(0.2)(10^{-4})}} = \underline{\mathbf{111.8\ rad/s}}$$

Chapter 14, Problem 80.

(a) For the circuit in Fig. 14.97, draw the new circuit after it has been scaled by K_m = 200 and $K_f = 10^4$.

(b) Obtain the Thevenin equivalent impedance at terminals *a*-*b* of the scaled circuit at $\omega = 10^4$ rad/s.

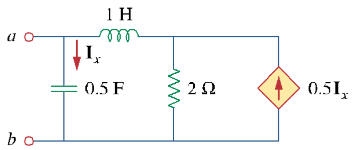


Figure 14.97
For Prob. 14.80.

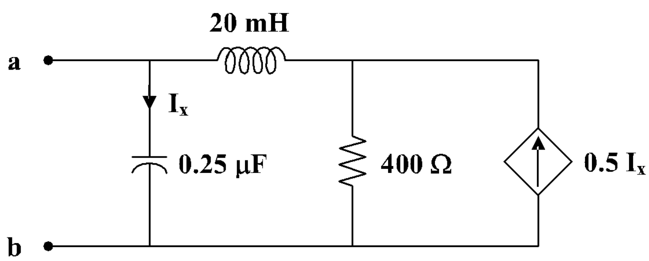
Chapter 14, Solution 80.

(a) $R' = K_m R = (200)(2) = 400\ \Omega$

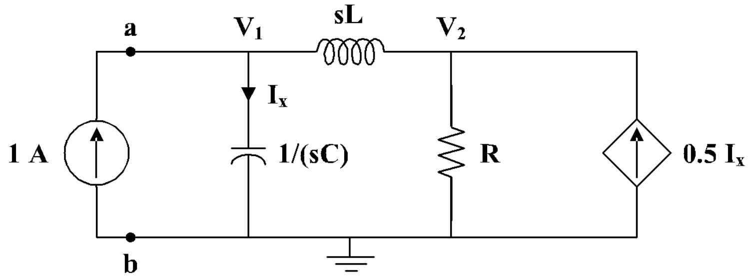
$$L' = \frac{K_m L}{K_f} = \frac{(200)(1)}{10^4} = 20\text{ mH}$$

$$C' = \frac{C}{K_m K_f} = \frac{0.5}{(200)(10^4)} = 0.25\ \mu\text{F}$$

<u>The new circuit is shown below</u>.



(b) Insert a 1-A source at the terminals a-b.



At node 1,

$$1 = sC\mathbf{V}_1 + \frac{\mathbf{V}_1 - \mathbf{V}_2}{sL} \qquad (1)$$

At node 2,

$$\frac{\mathbf{V}_1 - \mathbf{V}_2}{sL} + 0.5\mathbf{I}_x = \frac{\mathbf{V}_2}{R}$$

But, $\mathbf{I}_x = sC\mathbf{V}_1$.

$$\frac{\mathbf{V}_1 - \mathbf{V}_2}{sL} + 0.5sC\mathbf{V}_1 = \frac{\mathbf{V}_2}{R} \qquad (2)$$

Solving (1) and (2),

$$\mathbf{V}_1 = \frac{sL + R}{s^2LC + 0.5sCR + 1}$$

$$\mathbf{Z}_{Th} = \frac{\mathbf{V}_1}{1} = \frac{sL + R}{s^2LC + 0.5sCR + 1}$$

At $\omega = 10^4$,

$$\mathbf{Z}_{Th} = \frac{(j10^4)(20\times10^{-3}) + 400}{(j10^4)^2(20\times10^{-3})(0.25\times10^{-6}) + 0.5(j10^4)(0.25\times10^{-6})(400) + 1}$$

$$\mathbf{Z}_{Th} = \frac{400 + j200}{0.5 + j0.5} = 600 - j200$$

$$\mathbf{Z}_{Th} = \underline{\mathbf{632.5\angle -18.435^\circ\ ohms}}$$

Chapter 14, Problem 81.

The circuit shown in Fig. 14.98 has the impedance

$$Z(s) = \frac{1{,}000(s+1)}{(s+1+j50)(s+1-j50)}, \qquad s = j\omega$$

Find:

(a) the values of R, L, C, and G

(b) the element values that will raise the resonant frequency by a factor of 10^3 by frequency scaling

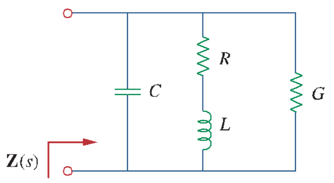


Figure 14.98
For Prob. 14.81.

Chapter 14, Solution 81.

(a)

$$\frac{1}{Z} = G + j\omega C + \frac{1}{R + j\omega L} = \frac{(G + j\omega C)(R + j\omega L) + 1}{R + j\omega L}$$

which leads to $Z = \dfrac{j\omega L + R}{-\omega^2 LC + j\omega(RC + LG) + GR + 1}$

$$Z(\omega) = \frac{j\frac{\omega}{C} + \frac{R}{LC}}{-\omega^2 + j\omega\left(\frac{R}{L} + \frac{G}{C}\right) + \frac{GR + 1}{LC}} \qquad (1)$$

We compare this with the given impedance:

$$Z(\omega) = \frac{1000(j\omega + 1)}{-\omega^2 + 2j\omega + 1 + 2500} \qquad (2)$$

Comparing (1) and (2) shows that

$$\frac{1}{C} = 1000 \longrightarrow C = 1\,\text{mF}, \quad R/L = 1 \longrightarrow R = L$$

$$\frac{R}{L} + \frac{G}{C} = 2 \longrightarrow G = C = 1\ \text{mS}$$

$$2501 = \frac{GR + 1}{LC} = \frac{10^{-3}R + 1}{10^{-3}R} \longrightarrow R = 0.4 = L$$

Thus,

R = **<u>0.4Ω</u>**, L = **<u>0.4 H</u>**, C = **<u>1 mF</u>**, G = **<u>1 mS</u>**

(b) By frequency-scaling, K_f =1000.

R' = **<u>0.4 Ω</u>**, G' = **<u>1 mS</u>**

$$L' = \frac{L}{K_f} = \frac{0.4}{10^3} = \underline{0.4\text{mH}}, \qquad C' = \frac{C}{K_f} = \frac{10^{-3}}{10^{-3}} = \underline{1\mu\text{F}}$$

Chapter 14, Problem 82.

Scale the lowpass active filter in Fig. 14.99 so that its corner frequency increases from 1 rad/s to 200 rad/s. Use a 1-μF capacitor.

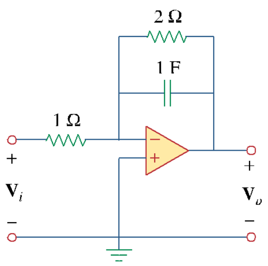


Figure 14.99
For Prob. 14.82.

Chapter 14, Solution 82.

$$C' = \frac{C}{K_m K_f}$$

$$K_f = \frac{\omega'_c}{\omega} = \frac{200}{1} = 200$$

$$K_m = \frac{C}{C'} \cdot \frac{1}{K_f} = \frac{1}{10^{-6}} \cdot \frac{1}{200} = 5000$$

$R' = K_m R = \underline{\mathbf{5\ k\Omega}}$, thus, $R'_f = 2R_i = \underline{\mathbf{10\ k\Omega}}$

Chapter 14, Problem 83.

The op amp circuit in Fig. 14.100 is to be magnitude-scaled by 100 and frequency-scaled by 10^5. Find the resulting element values.

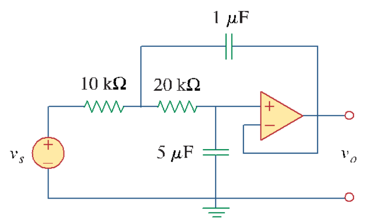


Figure 14.100
For Prob. 14.83.

Chapter 14, Solution 83.

$$1\mu F \longrightarrow C' = \frac{1}{K_m K_f} C = \frac{10^{-6}}{100x10^5} = \underline{0.1\,pF}$$

$$5\mu F \longrightarrow C' = \underline{0.5\,pF}$$

$$10\,k\Omega \longrightarrow R' = K_m R = 100x10\,k\Omega = \underline{1\,M\Omega}$$

$$20\,k\Omega \longrightarrow R' = \underline{2\,M\Omega}$$

Chapter 14, Problem 84.

Using *PSpice*, obtain the frequency response of the circuit in Fig. 14.101 on the next page.

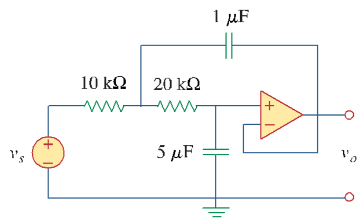
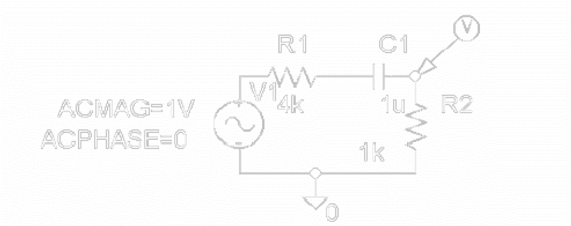
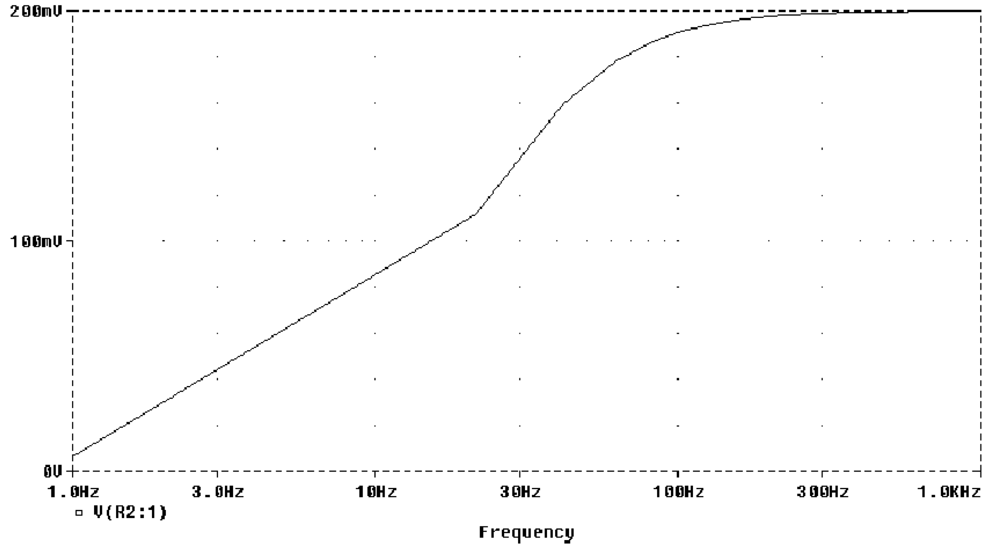


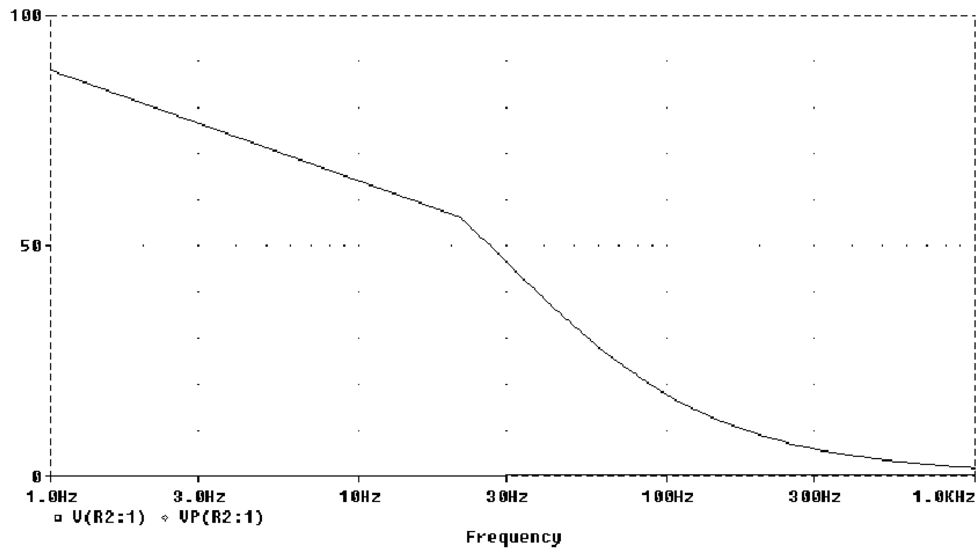
Figure 14.101
For Prob. 14.84.

Chapter 14, Solution 84.

The schematic is shown below. A voltage marker is inserted to measure v_o. In the AC sweep box, we select Total Points = 50, Start Frequency = 1, and End Frequency = 1000. After saving and simulation, we obtain the magnitude and phase plots in the probe menu as shown below.




200mV
100mV
0V
1.0Hz
3.0Hz
10Hz
30Hz
100Hz
300Hz
1.0KHz
▫ V(R2:1)
Frequency


100
50
0
1.0Hz
3.0Hz
10Hz
30Hz
100Hz
300Hz
1.0KHz
▫ V(R2:1) ◇ VP(R2:1)
Frequency

Chapter 14, Problem 85.

Use *PSpice* to obtain the magnitude and phase plots of $\mathbf{V}_0/\mathbf{I}_s$ of the circuit in Fig. 14.102.

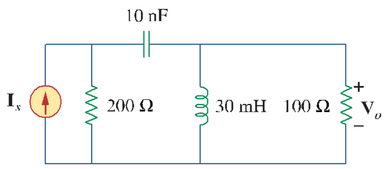
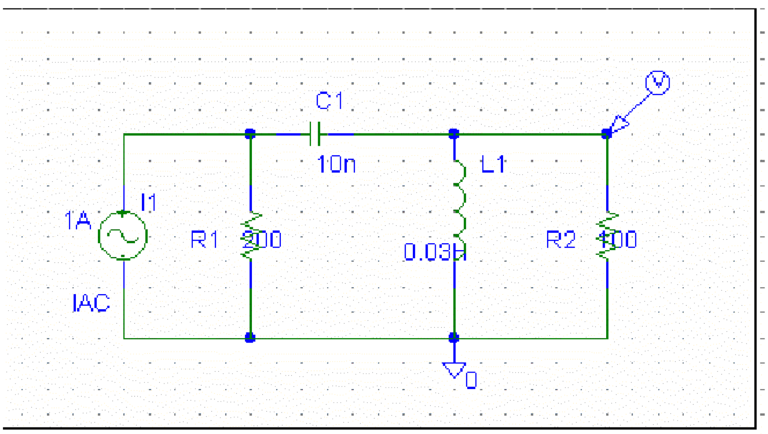
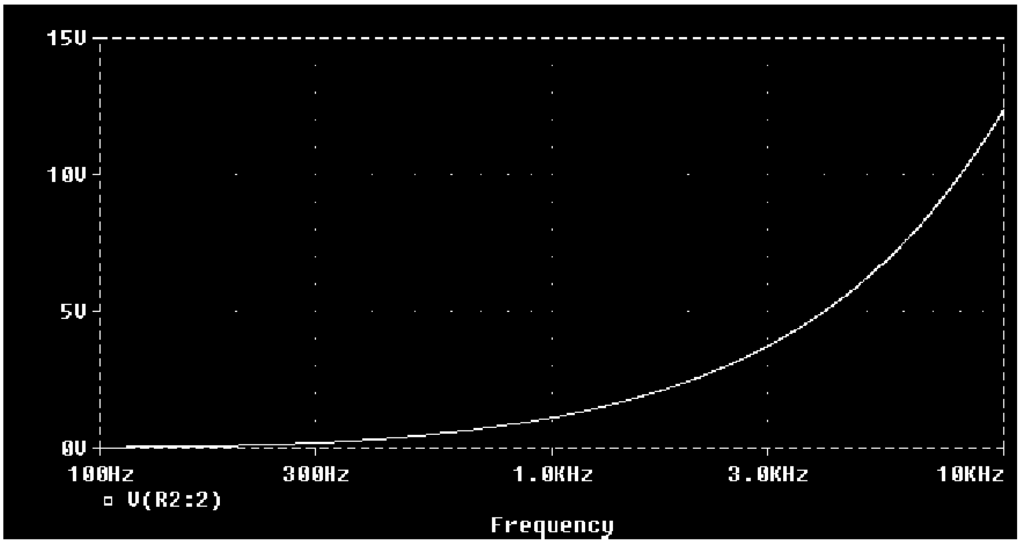


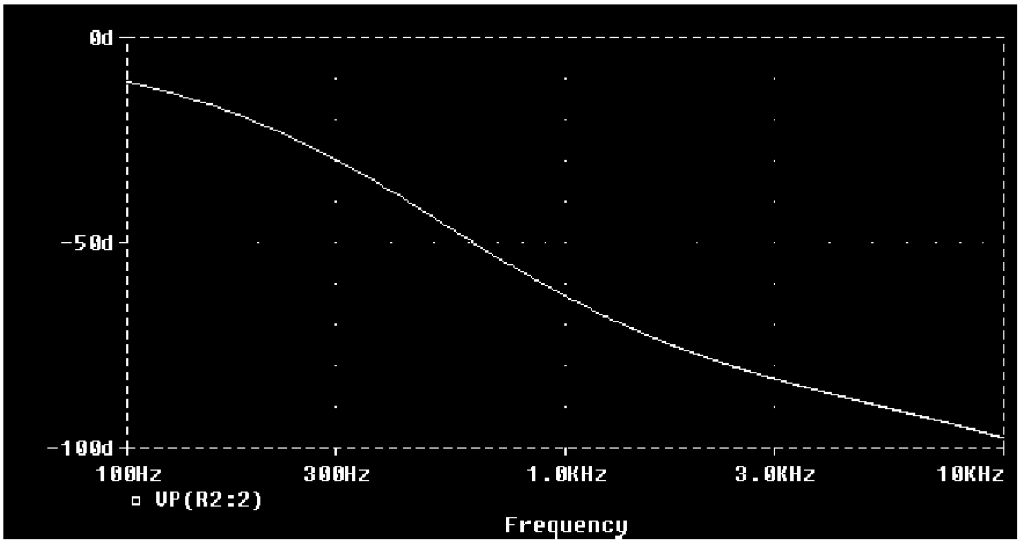
Figure 14.102
For Prob. 14.85.

Chapter 14, Solution 85.

We let $I_s = 1\angle 0^o$ A so that $V_o / I_s = V_o$. The schematic is shown below. The circuit is simulated for $100 < f < 10$ kHz.




15V
10V
5V
0V
100Hz
300Hz
1.0KHz
3.0KHz
10KHz
V(R2:2)
Frequency


0d
-50d
-100d
100Hz
300Hz
1.0KHz
3.0KHz
10KHz
VP(R2:2)
Frequency

Chapter 14, Problem 86.

Use *PSpice* to provide the frequency response (magnitude and phase of *i*) of the circuit in Fig. 14.103. Use linear frequency sweep from 1 to 10,000 Hz.

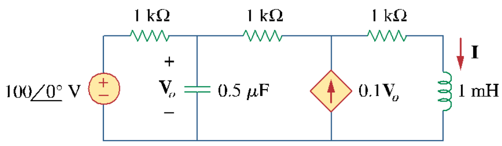
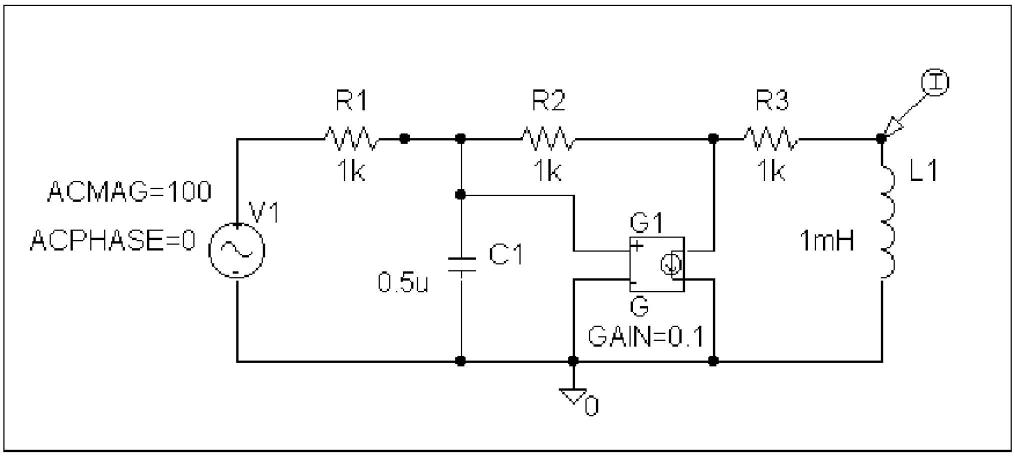
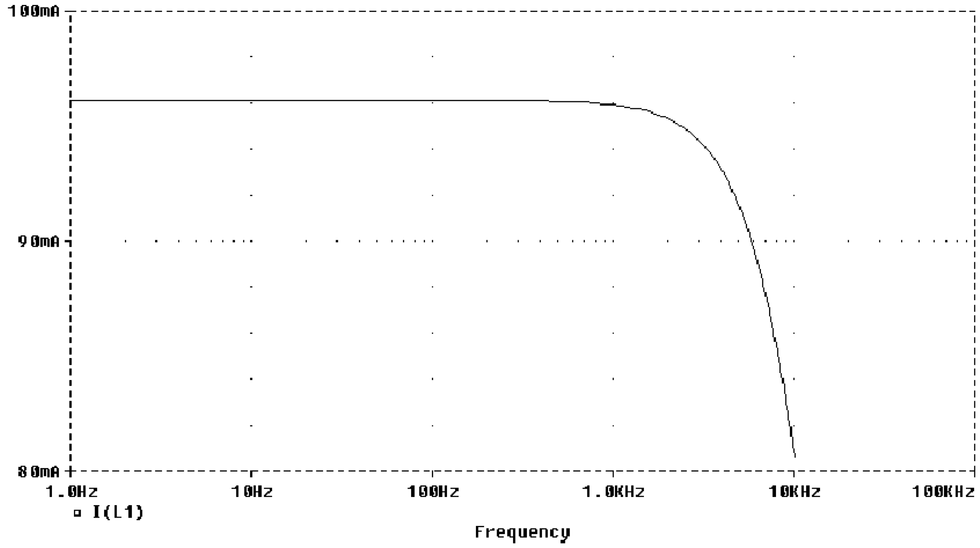


Figure 14.103
For Prob. 14.86.

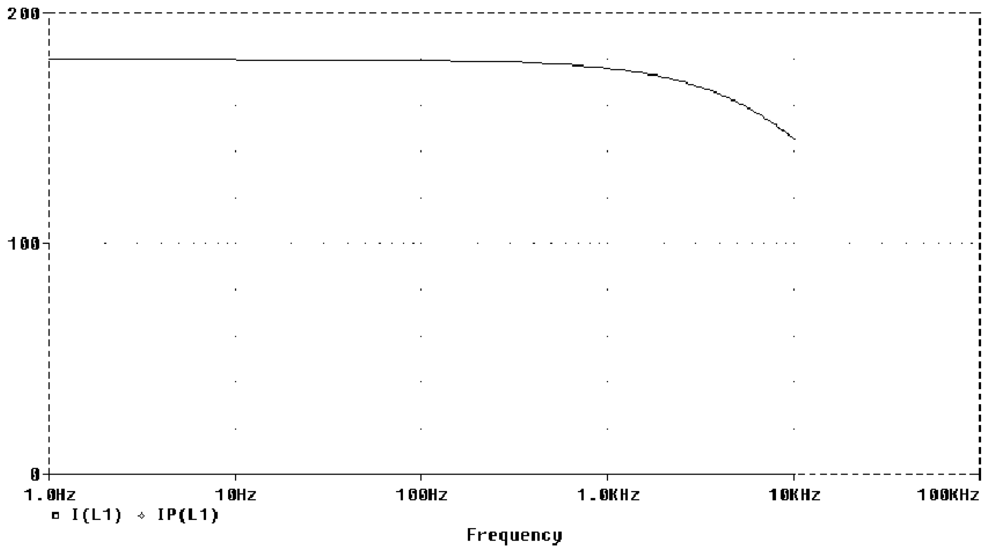
Chapter 14, Solution 86.

The schematic is shown below. A current marker is inserted to measure **I**. We set Total Points = 101, start Frequency = 1, and End Frequency = 10 kHz in the AC sweep box. After simulation, the magnitude and phase plots are obtained in the Probe menu as shown below.





100mA
90mA
80mA
1.0Hz
10Hz
100Hz
1.0KHz
10KHz
100KHz
▫ I(L1)
Frequency



200
100
0
1.0Hz
10Hz
100Hz
1.0KHz
10KHz
100KHz
▫ I(L1) ◇ IP(L1)
Frequency

Chapter 14, Problem 87.

In the interval $0.1 < f < 100$ Hz, plot the response of the network in Fig. 14.104. Classify this filter and obtain ω_0.

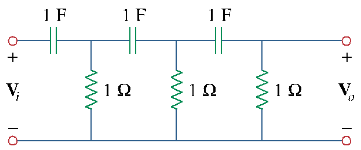
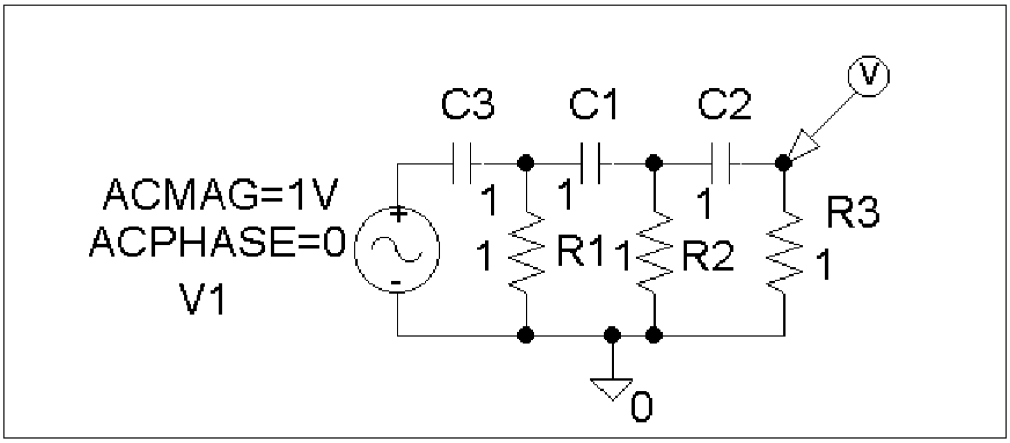
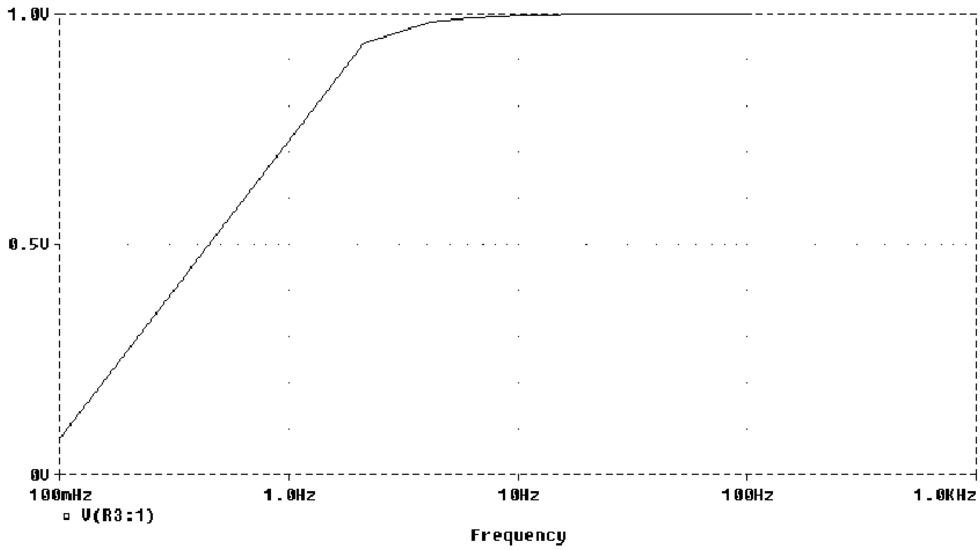


Figure 14.104
For Prob. 14.87.

Chapter 14, Solution 87.

The schematic is shown below. In the AC Sweep box, we set Total Points = 50, Start Frequency = 1, and End Frequency = 100. After simulation, we obtain the magnitude response as shown below. It is evident from the response that the circuit represents a high-pass filter.





Chapter 14, Problem 88.

Use *PSpice* to generate the magnitude and phase Bode plots of $\mathbf{V}_0$ in the circuit of Fig. 14.105.

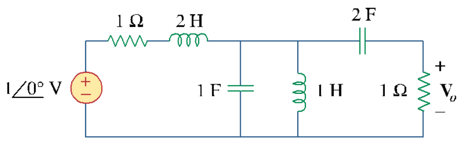
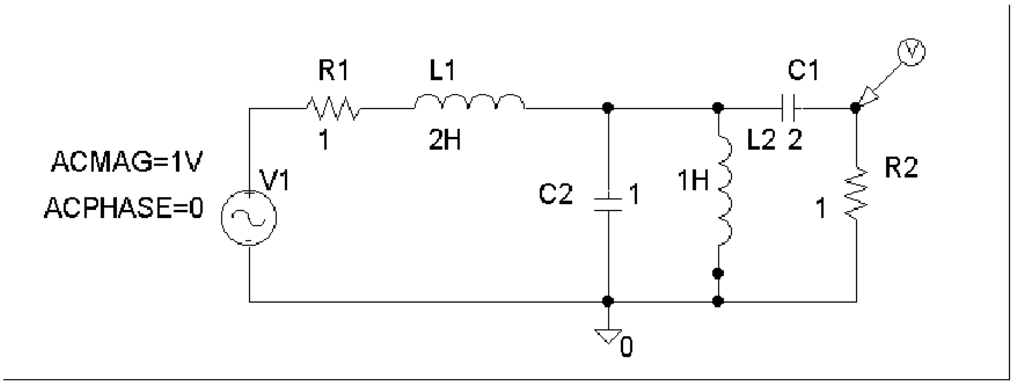
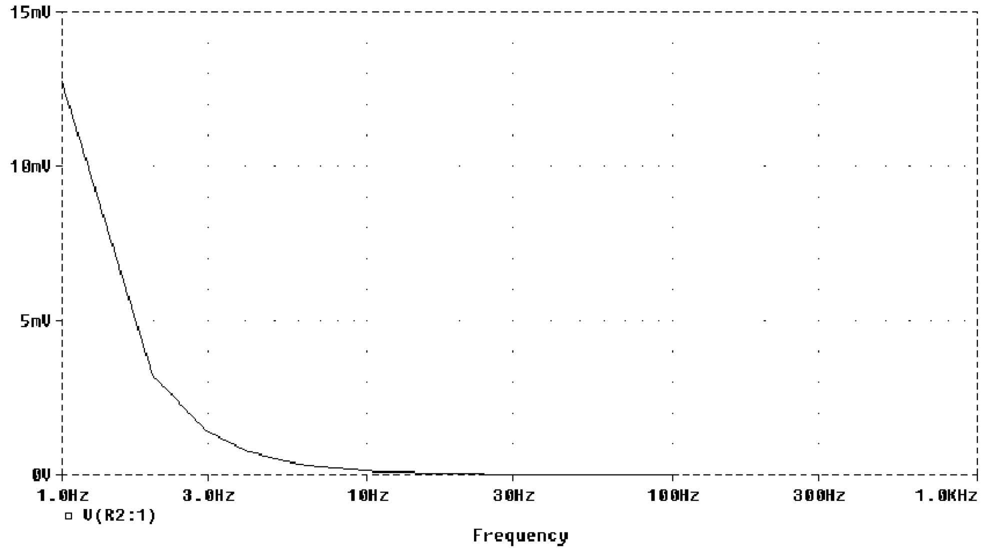


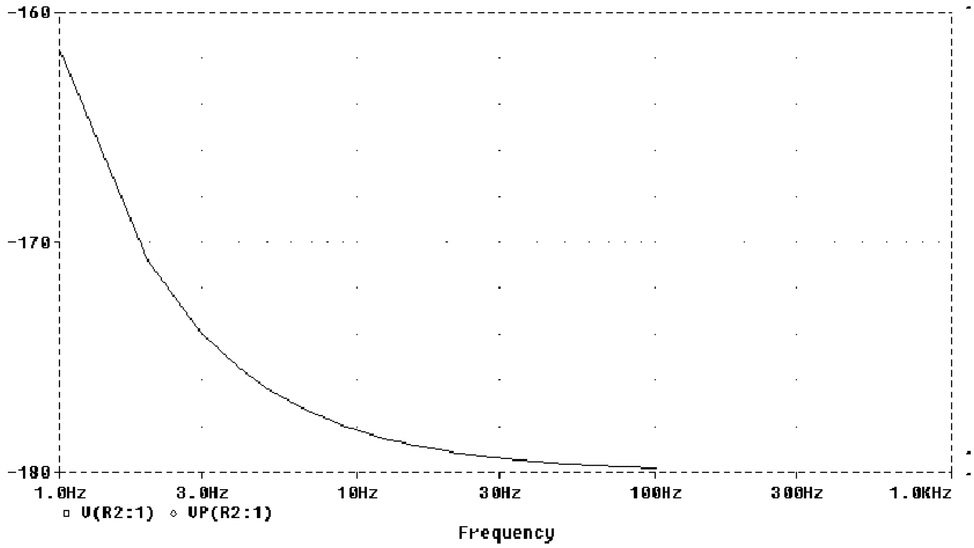
Figure 14.105
For Prob. 14.88.

Chapter 14, Solution 88.

The schematic is shown below. We insert a voltage marker to measure V_o. In the AC Sweep box, we set Total Points = 101, Start Frequency = 1, and End Frequency = 100. After simulation, we obtain the magnitude and phase plots of V_o as shown below.




15mV
10mV
5mV
0V
1.0Hz
3.0Hz
10Hz
30Hz
100Hz
300Hz
1.0KHz
V(R2:1)
Frequency


-160
-170
-180
1.0Hz
3.0Hz
10Hz
30Hz
100Hz
300Hz
1.0KHz
V(R2:1) VP(R2:1)
Frequency

Chapter 14, Problem 89.

Obtain the magnitude plot of the response $\mathbf{V}_0$ in the network of Fig. 14.106 for the frequency interval $100 < f < 1{,}000$ Hz..

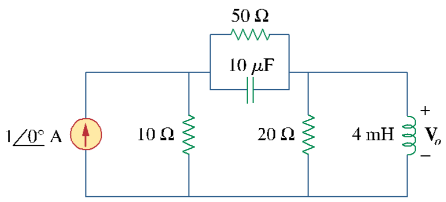
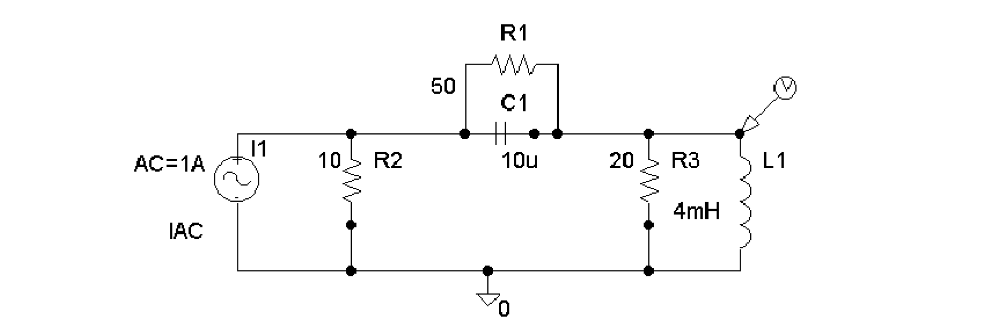
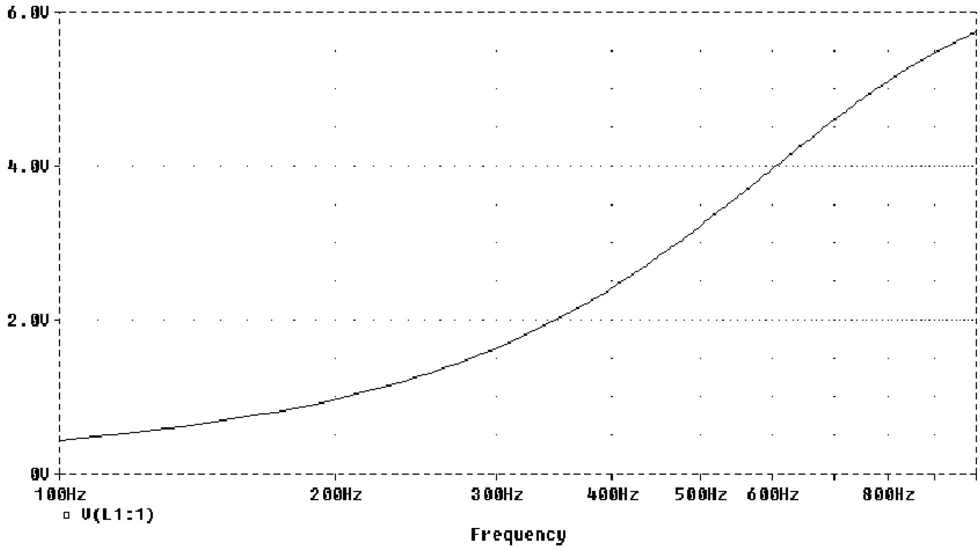


Figure 14.106
For Prob. 14.89.

Chapter 14, Solution 89.

The schematic is shown below. In the AC Sweep box, we type Total Points = 101, Start Frequency = 100, and End Frequency = 1 k. After simulation, the magnitude plot of the response V_o is obtained as shown below.



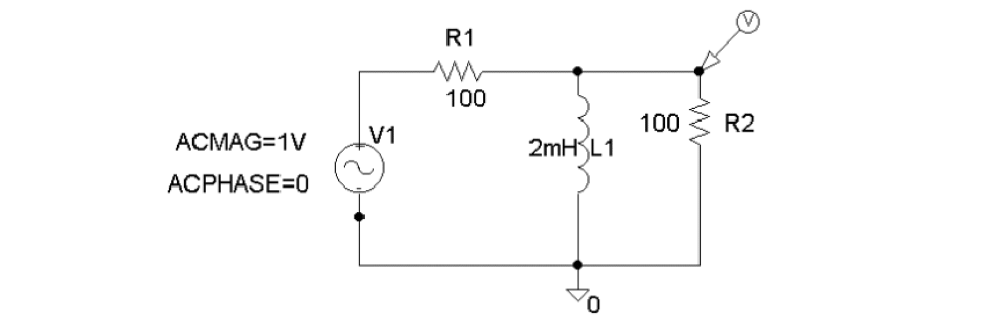


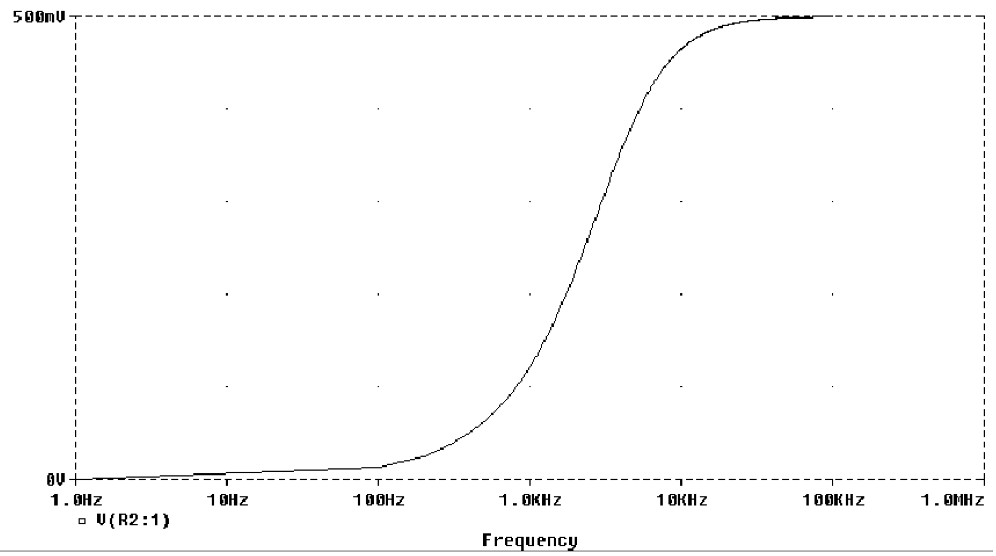
Chapter 14, Problem 90.

Obtain the frequency response of the circuit in Fig. 14.40 (see Practice Problem 14.10). Take $R_1 = R_2 = 100\Omega$, L =2 mH. Use 1 < f < 100,000 Hz.

Chapter 14, Solution 90.

The schematic is shown below. In the AC Sweep box, we set Total Points = 1001, Start Frequency = 1, and End Frequency = 100k. After simulation, we obtain the magnitude plot of the response as shown below. The response shows that the circuit is a high-pass filter.

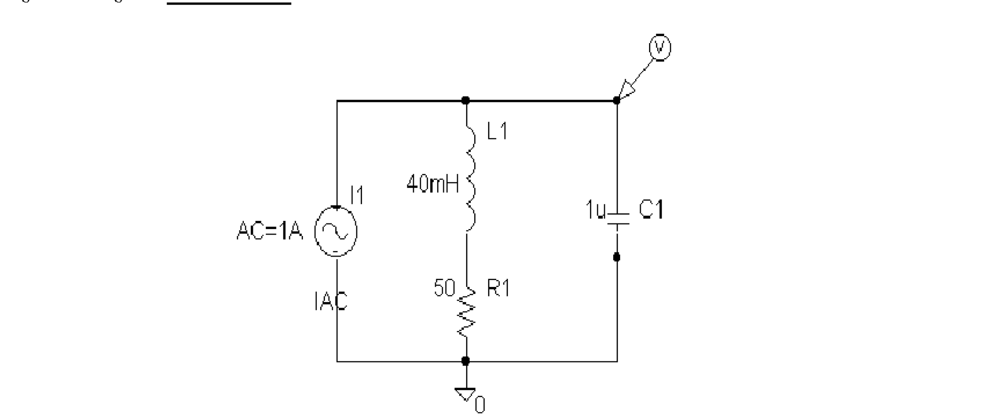


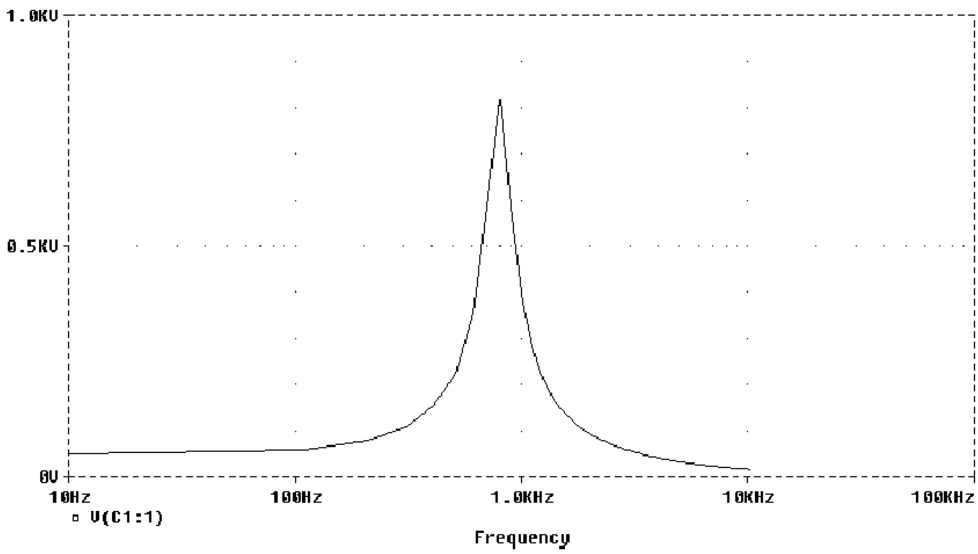


Chapter 14, Problem 91.

For the "tank" circuit of Fig. 14.79, obtain the frequency response (voltage across the capacitor) using *PSpice*. Determine the resonant frequency of the circuit.

Chapter 14, Solution 91.
The schematic is shown below. In the AC Sweep box, we select Total Points = 101, Start Frequency = 10, and End Frequency = 10 k. After simulation, the magnitude plot of the frequency response is obtained. From the plot, we obtain the resonant frequency f_o is approximately equal to 800 Hz so that
$\omega_o = 2\pi f_o =$ **5026 rad/s**.



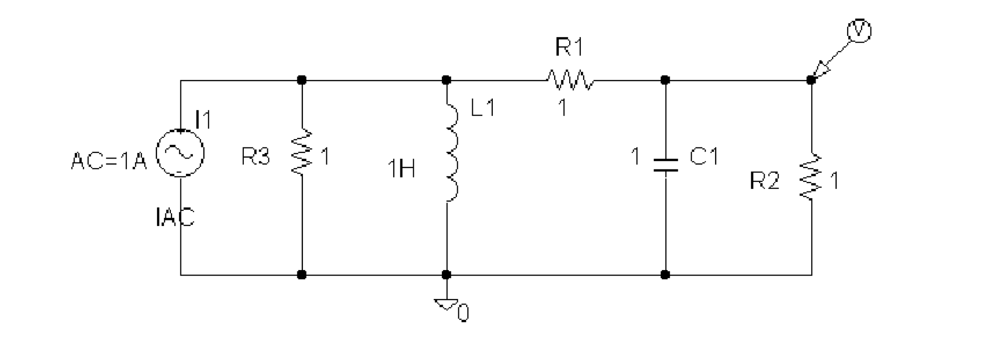


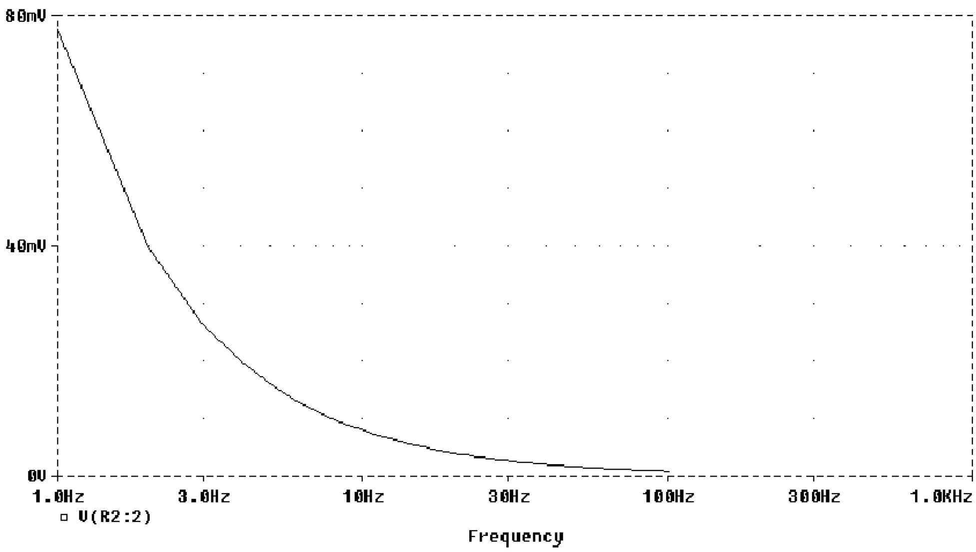
Chapter 14, Problem 92.

Using *PSpice*, plot the magnitude of the frequency response of the circuit in Fig. 14.85.

Chapter 14, Solution 92.

The schematic is shown below. We type Total Points = 101, Start Frequency = 1, and End Frequency = 100 in the AC Sweep box. After simulating the circuit, the magnitude plot of the frequency response is shown below.





Chapter 14, Problem 93.

For the phase shifter circuit shown in Fig. 14.107, find H = V_o/V_s.

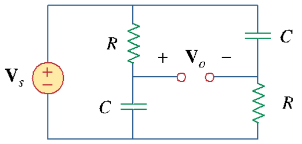
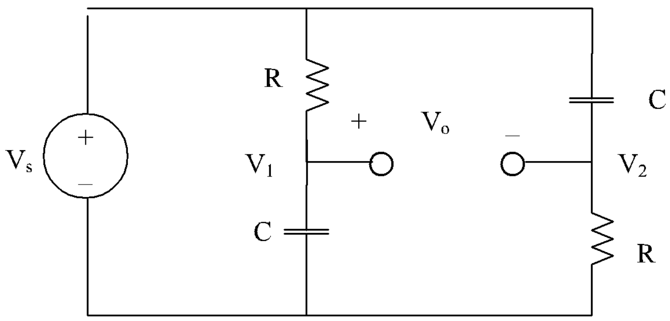


Figure 14.107
For Prob. 14.93.

Chapter 14, Solution 93.

Consider the circuit as shown below.



$$V_1 = \frac{\frac{1}{sC}}{R+\frac{1}{sC}}V_s = \frac{V}{1+sRC}$$

$$V_2 = \frac{R}{R+sC}V_s = \frac{sRC}{1+sRC}V_s$$

$$V_o = V_1 - V_2 = \frac{1-sRC}{1+sRC}V_s$$

Hence,

$$H(s) = \frac{V_o}{V_s} = \frac{1-sRC}{1+sRC}$$

Chapter 14, Problem 94.

e2d

For an emergency situation, an engineer needs to make an *RC* highpass filter. He has one 10-pF capacitor, one 30-pF capacitor, one 1.8-$k\Omega$ resistor, and one 3.3-$k\Omega$ resistor available. Find the greatest cutoff frequency possible using these elements.

Chapter 14, Solution 94.

$$\omega_c = \frac{1}{RC}$$

We make R and C as small as possible. To achieve this, we connect 1.8 kΩ and 3.3 kΩ in parallel so that

$$R = \frac{1.8x3.3}{1.8+3.3} = 1.164\,k\Omega$$

We place the 10-pF and 30-pF capacitors in series so that

C = (10x30)/40 = 7.5 pF

Hence,

$$\omega_c = \frac{1}{RC} = \frac{1}{1.164x10^3 x7.5x10^{-12}} = \underline{114.55x10^6 \text{ rad/s}}$$

Chapter 14, Problem 95.

e2d

A series-tuned antenna circuit consists of a variable capacitor (40 pF to 360 pF) and a 240-μH antenna coil that has a dc resistance of $12\,\Omega$.

(a) Find the frequency range of radio signals to which the radio is tunable.
(b) Determine the value of Q at each end of the frequency range.

Chapter 14, Solution 95.

(a) $$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

When $C = 360\text{ pF}$,

$$f_0 = \frac{1}{2\pi\sqrt{(240\times10^{-6})(360\times10^{-12})}} = 0.541\text{ MHz}$$

When $C = 40\text{ pF}$,

$$f_0 = \frac{1}{2\pi\sqrt{(240\times10^{-6})(40\times10^{-12})}} = 1.624\text{ MHz}$$

Therefore, the frequency range is

$$\underline{\mathbf{0.541\ MHz} < \mathbf{f_0} < \mathbf{1.624\ MHz}}$$

(b) $$Q = \frac{2\pi fL}{R}$$

At $f_0 = 0.541\text{ MHz}$,

$$Q = \frac{(2\pi)(0.541\times10^{6})(240\times10^{-6})}{12} = \underline{\mathbf{67.98}}$$

At $f_0 = 1.624\text{ MHz}$,

$$Q = \frac{(2\pi)(1.624\times10^{6})(240\times10^{-6})}{12} = \underline{\mathbf{204.1}}$$

Chapter 14, Problem 96.

e2d

The crossover circuit in Fig. 14.108 is a lowpass filter that is connected to a woofer. Find the transfer function $\mathbf{H}(\omega) = \mathbf{V}_o(\omega)/\mathbf{V}_i(\omega)$

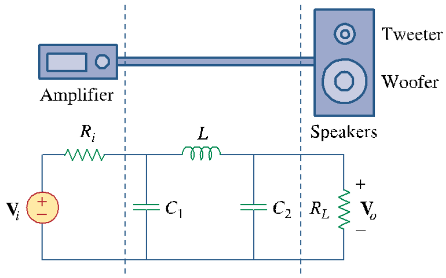
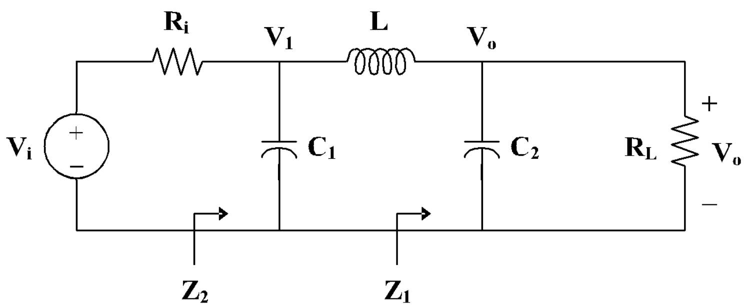


Figure 14.108
For Prob. 14.96.

Chapter 14, Solution 96.



$$\mathbf{Z}_1 = R_L \,||\, \frac{1}{sC_2} = \frac{R_L}{1 + sR_2C_2}$$

$$\mathbf{Z}_2 = \frac{1}{sC_1} \,||\, (sL + \mathbf{Z}_1) = \frac{1}{sC_1} \,||\, \left(\frac{sL + R_L + s^2R_LC_2L}{1 + sR_LC_2} \right)$$

$$\mathbf{Z}_2 = \frac{\dfrac{1}{sC_1} \cdot \dfrac{sL + R_L + s^2R_LC_2L}{1 + sR_LC_2}}{\dfrac{1}{sC_1} + \dfrac{sL + R_L + s^2R_LC_2L}{1 + sR_LC_2}}$$

$$\mathbf{Z}_2 = \frac{sL + R_L + s^2R_LLC_2}{1 + sR_LC_2 + s^2LC_1 + sR_LC_1 + s^3R_LLC_1C_2}$$

$$\mathbf{V}_1 = \frac{\mathbf{Z}_2}{\mathbf{Z}_2 + R_i} \mathbf{V}_i$$

$$\mathbf{V}_o = \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + sL} \mathbf{V}_1 = \frac{\mathbf{Z}_2}{\mathbf{Z}_2 + R_2} \cdot \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + sL} \mathbf{V}_i$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{\mathbf{Z}_2}{\mathbf{Z}_2 + R_2} \cdot \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + sL}$$

where

$$\frac{\mathbf{Z}_2}{\mathbf{Z}_2 + R_2} =$$

$$\frac{sL + R_L + s^2R_LLC_2}{sL + R_L + s^2R_LLC_2 + R_i + sR_iR_LC_2 + s^2R_iLC_1 + sR_iR_LC_1 + s^3R_iR_LLC_1C_2}$$

and $\dfrac{\mathbf{Z}_1}{\mathbf{Z}_1 + sL} = \dfrac{R_L}{R_L + sL + s^2R_LLC_2}$

Therefore,

$$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{\mathbf{R_L(sL + R_L + s^2R_LLC_2)}}{\mathbf{(sL + R_L + s^2R_LLC_2 + R_i + sR_iR_LC_2 + s^2R_iLC_1 + sR_iR_LC_1 + s^3R_iR_LLC_1C_2)(R_L + sL + s^2R_LLC_2)}}$$

where $s = j\omega$.

Chapter 14, Problem 97.

The crossover circuit in Fig. 14.109 is a highpass filter that is connected to a tweeter. Determine the transfer function $\mathbf{H}(\omega) = \mathbf{V}_o(\omega)/\mathbf{V}_i(\omega)$.

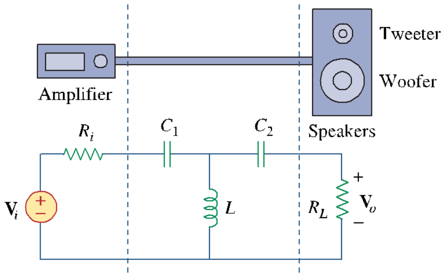
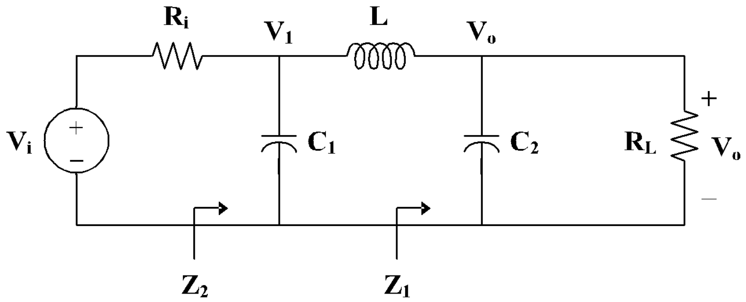


Figure 14.109
For Prob. 14.97.

Chapter 14, Solution 97.



$$\mathbf{Z} = sL \,||\, \left(R_L + \frac{1}{sC_2}\right) = \frac{sL(R_L + 1/sC_2)}{R_L + sL + 1/sC_2}, \qquad s = j\omega$$

$$\mathbf{V}_1 = \frac{\mathbf{Z}}{\mathbf{Z} + R_i + 1/sC_1}\mathbf{V}_i$$

$$\mathbf{V}_o = \frac{R_L}{R_L + 1/sC_2}\mathbf{V}_1 = \frac{R_L}{R_L + 1/sC_2} \cdot \frac{\mathbf{Z}}{\mathbf{Z} + R_i + 1/sC_1}\mathbf{V}_i$$

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R_L}{R_L + 1/sC_2} \cdot \frac{sL(R_L + 1/sC_2)}{sL(R_L + 1/sC_2) + (R_i + 1/sC_1)(R_L + sL + 1/sC_2)}$$

$$\mathbf{H}(\omega) = \underline{\frac{\mathbf{s^3 L R_L C_1 C_2}}{\mathbf{(sR_iC_1 + 1)(s^2LC_2 + sR_LC_2 + 1) + s^2LC_1(sR_LC_2 + 1)}}}$$

where $s = j\omega$.

Chapter 14, Problem 98.

A certain electronic test circuit produced a resonant curve with half-power points at 432 Hz and 454 Hz. If Q = 20, what is the resonant frequency of the circuit?

Chapter 14, Solution 98.

$$B = \omega_2 - \omega_1 = 2\pi(f_2 - f_1) = 2\pi(454 - 432) = 44\pi$$

$$\omega_0 = 2\pi f_0 = QB = (20)(44\pi)$$

$$f_0 = \frac{(20)(44\pi)}{2\pi} = (20)(22) = \underline{\mathbf{440\ Hz}}$$

Chapter 14, Problem 99.

In an electronic device, a series circuit is employed that has a resistance of $100\,\Omega$, a capacitive reactance of $5\ k\Omega$, and an inductive reactance of $300\,\Omega$ when used at 2 MHz. Find the resonant frequency and bandwidth of the circuit.

Chapter 14, Solution 99.

$$X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$C = \frac{1}{2\pi f X_c} = \frac{1}{(2\pi)(2\times10^6)(5\times10^3)} = \frac{10^{-9}}{20\pi}$$

$$X_L = \omega L = 2\pi f L$$

$$L = \frac{X_L}{2\pi f} = \frac{300}{(2\pi)(2\times10^6)} = \frac{3\times10^{-4}}{4\pi}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{\frac{3\times10^{-4}}{4\pi}\cdot\frac{10^{-9}}{20\pi}}} = \underline{8.165\ \text{MHz}}$$

$$B = \frac{R}{L} = (100)\left(\frac{4\pi}{3\times10^{-4}}\right) = \underline{\mathbf{4.188\times10^6\ rad/s}}$$

Chapter 14, Problem 100.

In a certain application, a simple *RC* lowpass filter is designed to reduce high frequency noise. If the desired corner frequency is 20 kHz and $C = 0.5\,\mu F$ find the value of *R*.

Chapter 14, Solution 100.

$$\omega_c = 2\pi f_c = \frac{1}{RC}$$

$$R = \frac{1}{2\pi f_c C} = \frac{1}{(2\pi)(20\times 10^3)(0.5\times 10^{-6})} = \mathbf{15.91\ \Omega}$$

Chapter 14, Problem 101.

In an amplifier circuit, a simple *RC* highpass filter is needed to block the dc component while passing the time-varying component. If the desired rolloff frequency is 15 Hz and $C = 10\,\mu F$ find the value of *R*.

Chapter 14, Solution 101.

$$\omega_c = 2\pi f_c = \frac{1}{RC}$$

$$R = \frac{1}{2\pi f_c C} = \frac{1}{(2\pi)(15)(10\times 10^{-6})} = \mathbf{1.061\ k\Omega}$$

Chapter 14, Problem 102.

Practical *RC* filter design should allow for source and load resistances as shown in Fig. 14.110. Let $R = 4\text{k}\Omega$ and C = 40-nF. Obtain the cutoff frequency when:

(a) $R_s = 0$, $R_L = \infty$,

(b) $R_s = 1k\Omega$, $R_L = 5k\Omega$.

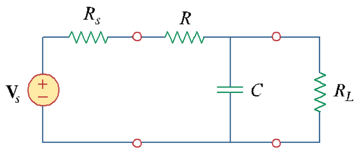


Figure 14.110
For Prob. 14.102.

Chapter 14, Solution 102.

(a) When $R_s = 0$ and $R_L = \infty$, we have a low-pass filter.

$$\omega_c = 2\pi f_c = \frac{1}{RC}$$

$$f_c = \frac{1}{2\pi RC} = \frac{1}{(2\pi)(4\times10^3)(40\times10^{-9})} = \underline{\mathbf{994.7\ Hz}}$$

(b) We obtain R_{Th} across the capacitor.

$$R_{Th} = R_L \parallel (R + R_s)$$

$$R_{Th} = 5 \parallel (4+1) = 2.5\ \text{k}\Omega$$

$$f_c = \frac{1}{2\pi R_{Th} C} = \frac{1}{(2\pi)(2.5\times10^3)(40\times10^{-9})}$$

$$f_c = \underline{\mathbf{1.59\ kHz}}$$

Chapter 14, Problem 103.

The *RC* circuit in Fig. 14.111 is used for a lead compensator in a system design. Obtain the transfer function of the circuit.

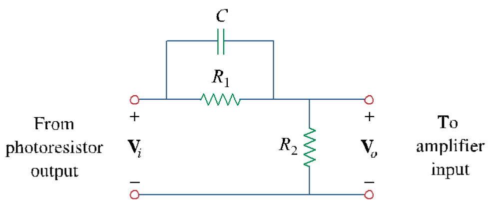


Figure 14.111
For Prob. 14.103.

Chapter 14, Solution 103.

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R_2}{R_2 + R_1 \parallel 1/j\omega C}, \qquad s = j\omega$$

$$\mathbf{H}(s) = \frac{R_2}{R_2 + \dfrac{R_1(1/sC)}{R_1 + 1/sC}} = \frac{R_2(R_1 + 1/sC)}{R_1R_2 + (R_1 + R_2)(1/sC)}$$

$$\mathbf{H}(s) = \underline{\frac{R_2(1 + sCR_1)}{R_1 + R_2 + sCR_1R_2}}$$

Chapter 14, Problem 104.

A low-quality-factor, double-tuned bandpass filter is shown in Fig. 14.112. Use *PSpice* to generate the magnitude plot of $\mathbf{V}_o(\omega)$.

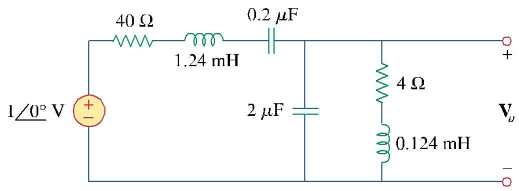
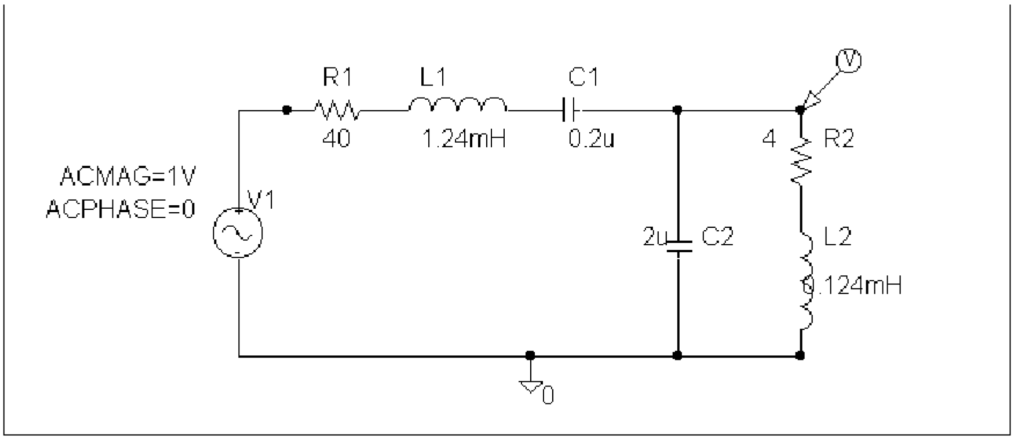
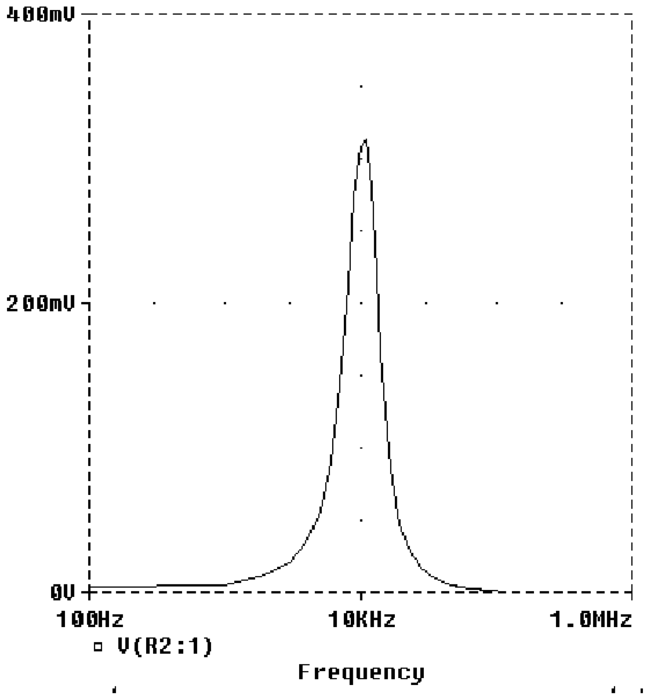


Figure 14.112
For Prob. 14.104.

Chapter 14, Solution 104.

The schematic is shown below. We click Analysis/Setup/AC Sweep and enter Total Points = 1001, Start Frequency = 100, and End Frequency = 100 k. After simulation, we obtain the magnitude plot of the response as shown.





Chapter 15, Problem 1.

Find the Laplace transform of:

(a) cosh *at* (b) sinh *at*

[*Hint:* $\cosh x = \frac{1}{2}\left(e^{x} + e^{-x}\right)$, $\sinh\ x = \frac{1}{2}\left(e^{x} - e^{-x}\right)$.]

Chapter 15, Solution 1.

(a) $$\cosh(at) = \frac{e^{at} + e^{-at}}{2}$$

$$L[\cosh(at)] = \frac{1}{2}\left[\frac{1}{s-a} + \frac{1}{s+a}\right] = \underline{\mathbf{\frac{s}{s^2 - a^2}}}$$

(b) $$\sinh(at) = \frac{e^{at} - e^{-at}}{2}$$

$$L[\sinh(at)] = \frac{1}{2}\left[\frac{1}{s-a} - \frac{1}{s+a}\right] = \underline{\mathbf{\frac{a}{s^2 - a^2}}}$$

Chapter 15, Problem 2.

Determine the Laplace transform of:

(a) $\cos(\omega t + \theta)$ (b) $\sin(\omega t + \theta)$

Chapter 15, Solution 2.

(a) $$f(t) = \cos(\omega t)\cos(\theta) - \sin(\omega t)\sin(\theta)$$

$$F(s) = \cos(\theta)\, L[\cos(\omega t)] - \sin(\theta)\, L[\sin(\omega t)]$$

$$F(s) = \underline{\mathbf{\frac{s\cos(\theta) - \omega\sin(\theta)}{s^2 + \omega^2}}}$$

(b) $$f(t) = \sin(\omega t)\cos(\theta) + \cos(\omega t)\sin(\theta)$$

$$F(s) = \sin(\theta)\, L[\cos(\omega t)] + \cos(\theta)\, L[\sin(\omega t)]$$

$$F(s) = \underline{\mathbf{\frac{s\sin(\theta) - \omega\cos(\theta)}{s^2 + \omega^2}}}$$

Chapter 15, Problem 3.

Obtain the Laplace transform of each of the following functions:

(a) $e^{-2t}\cos 3tu(t)$ (b) $e^{-2t}\sin 4tu(t)$

(c) $e^{-3t}\cosh 2tu(t)$ (d) $e^{-4t}\sinh tu(t)$

(e) $te^{-t}\sin 2tu(t)$

Chapter 15, Solution 3.

(a) $$L\left[e^{-2t}\cos(3t)u(t)\right]=\underline{\mathbf{\frac{s+2}{(s+2)^2+9}}}$$

(b) $$L\left[e^{-2t}\sin(4t)u(t)\right]=\underline{\mathbf{\frac{4}{(s+2)^2+16}}}$$

(c) Since $L\left[\cosh(at)\right]=\dfrac{s}{s^2-a^2}$

$$L\left[e^{-3t}\cosh(2t)u(t)\right]=\underline{\mathbf{\frac{s+3}{(s+3)^2-4}}}$$

(d) Since $L\left[\sinh(at)\right]=\dfrac{a}{s^2-a^2}$

$$L\left[e^{-4t}\sinh(t)u(t)\right]=\underline{\mathbf{\frac{1}{(s+4)^2-1}}}$$

(e) $$L\left[e^{-t}\sin(2t)\right]=\frac{2}{(s+1)^2+4}$$

If $f(t) \longleftrightarrow F(s)$

$tf(t) \longleftrightarrow \dfrac{-d}{ds}F(s)$

Thus, $$L\left[te^{-t}\sin(2t)\right]=\frac{-d}{ds}\left[2\left((s+1)^2+4\right)^{-1}\right]$$

$$=\frac{2}{((s+1)^2+4)^2}\cdot 2(s+1)$$

$$L\left[te^{-t}\sin(2t)\right]=\underline{\mathbf{\frac{4(s+1)}{((s+1)^2+4)^2}}}$$

Chapter 15, Problem 4.

Find the Laplace transforms of the following:

(a) $g(t) = 6\cos(4t - 1)$

(b) $f(t) = 2tu(t) + 5e^{-3(t-2)}u(t-2)$

Chapter 15, Solution 4.

(a) $$G(s) = 6\frac{s}{s^2 + 4^2}e^{-s} = \underline{\frac{6se^{-s}}{s^2 + 16}}$$

(b) $$F(s) = \underline{\frac{2}{s^2} + 5\frac{e^{-2s}}{s + 3}}$$

Chapter 15, Problem 5.

Find the Laplace transform of each of the following functions:

(a) $t^2 \cos(2t + 30°)u(t)$ (b) $3t^4 e^{-2t}u(t)$

(c) $2tu(t) - 4\frac{d}{dt}\delta(t)$ (d) $2e^{-(t-1)}u(t)$

(e) $5u(t/2)$ (f) $6e^{-t/3}u(t)$

(g) $\frac{d^n}{dt^n}\delta(t)$

Chapter 15, Solution 5.

(a)

$$L[\cos(2t + 30°)] = \frac{s\cos(30°) - 2\sin(30°)}{s^2 + 4}$$

$$L[t^2 \cos(2t + 30°)] = \frac{d^2}{ds^2}\left[\frac{s\cos(30°) - 1}{s^2 + 4}\right]$$

$$= \frac{d}{ds}\frac{d}{ds}\left[\left(\frac{\sqrt{3}}{2}s - 1\right)(s^2 + 4)^{-1}\right]$$

$$= \frac{d}{ds}\left[\frac{\sqrt{3}}{2}(s^2 + 4)^{-1} - 2s\left(\frac{\sqrt{3}}{2}s - 1\right)(s^2 + 4)^{-2}\right]$$

$$= \frac{\frac{\sqrt{3}}{2}(-2s)}{(s^2 + 4)^2} - \frac{2\left(\frac{\sqrt{3}}{2}s - 1\right)}{(s^2 + 4)^2} - \frac{2s\left(\frac{\sqrt{3}}{2}\right)}{(s^2 + 4)^2} + \frac{(8s^2)\left(\frac{\sqrt{3}}{2}s - 1\right)}{(s^2 + 4)^3}$$

$$= \frac{-\sqrt{3}s - \sqrt{3}s + 2 - \sqrt{3}s}{(s^2 + 4)^2} + \frac{(8s^2)\left(\frac{\sqrt{3}}{2}s - 1\right)}{(s^2 + 4)^3}$$

$$= \frac{(-3\sqrt{3}s + 2)(s^2 + 4)}{(s^2 + 4)^3} + \frac{4\sqrt{3}s^3 - 8s^2}{(s^2 + 4)^3}$$

$$L[t^2 \cos(2t + 30°)] = \underline{\mathbf{\frac{8 - 12\sqrt{3}s - 6s^2 + \sqrt{3}s^3}{(s^2 + 4)^3}}}$$

(b) $$\mathcal{L}\left[3t^4 e^{-2t}\right] = 3 \cdot \frac{4!}{(s+2)^5} = \underline{\frac{72}{(s+2)^5}}$$

(c) $$\mathcal{L}\left[2t\,u(t) - 4\frac{d}{dt}\delta(t)\right] = \frac{2}{s^2} - 4(s \cdot 1 - 0) = \underline{\frac{\mathbf{2}}{\mathbf{s^2}} - \mathbf{4s}}$$

(d) $$2e^{-(t-1)}\,u(t) = 2e^{-t}\,u(t)$$

$$\mathcal{L}\left[2e^{-(t-1)}\,u(t)\right] = \underline{\frac{\mathbf{2e}}{\mathbf{s+1}}}$$

(e) Using the scaling property,

$$\mathcal{L}\left[5u(t/2)\right] = 5 \cdot \frac{1}{1/2} \cdot \frac{1}{s/(1/2)} = 5 \cdot 2 \cdot \frac{1}{2s} = \underline{\frac{\mathbf{5}}{\mathbf{s}}}$$

(f) $$\mathcal{L}\left[6e^{-t/3}\,u(t)\right] = \frac{6}{s+1/3} = \underline{\frac{\mathbf{18}}{\mathbf{3s+1}}}$$

(g) Let $f(t) = \delta(t)$. Then, $F(s) = 1$.

$$\mathcal{L}\left[\frac{d^n}{dt^n}\delta(t)\right] = \mathcal{L}\left[\frac{d^n}{dt^n}f(t)\right] = s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - \cdots$$

$$\mathcal{L}\left[\frac{d^n}{dt^n}\delta(t)\right] = \mathcal{L}\left[\frac{d^n}{dt^n}f(t)\right] = s^n \cdot 1 - s^{n-1} \cdot 0 - s^{n-2} \cdot 0 - \cdots$$

$$\mathcal{L}\left[\frac{d^n}{dt^n}\delta(t)\right] = \underline{\mathbf{s^n}}$$

Chapter 15, Problem 6.

Find $F(s)$ given that

$$f(t) = \begin{cases} 2t, & 0 < t < 1 \\ t, & 1 < t < 2 \\ 0, & otherwise \end{cases}$$

Chapter 15, Solution 6.

$$F(s) = \int_0^\infty f(t)e^{-st}dt = \int_0^1 2te^{-st}dt + \int_1^2 2e^{-st}dt$$

$$2\frac{e^{-st}}{s^2}(-st-1)\Big|_0^1 + 2\frac{e^{-st}}{-s}\Big|_1^2 = \underline{\frac{2}{s^2}(1-e^{-s}-se^{-2s})}$$

Chapter 15, Problem 7.

Find the Laplace transform of the following signals:

(a) $f(t) = (2t+4)u(t)$

(b) $g(t) = \left(4+3e^{-2t}\right)u(t)$

(c) $h(t) = (6\sin(3t) + 8\cos(3t))u(t)$

(d) $x(t) = \left(e^{-2t}\cosh(4t)\right)u(t)$

Chapter 15, Solution 7.

(a) $\quad \underline{F(s) = \frac{2}{s^2} + \frac{4}{s}}$

(b) $\underline{G(s) = \frac{4}{s} + \frac{3}{s+2}}$

(c) $H(s) = 6\frac{3}{s^2+9} + 8\frac{s}{s^2+9} = \underline{\frac{8s+18}{s^2+9}}$

(d) From Problem 15.1,

$$L\{\cosh at\} = \frac{s}{s^2-a^2}$$

$$X(s) = \frac{s+2}{(s+2)^2-4^2} = \underline{\frac{s+2}{s^2+4s-12}}$$

$$\underline{(a)\ \frac{2}{s^2}+\frac{4}{s},(b)\frac{4}{s}+\frac{3}{s+2},(c)\frac{8s+18}{s^2+9},(d)\frac{s+2}{s^2+4s-12}}$$

Chapter 15, Problem 8.

Find the Laplace transform $F(s)$, given that $f(t)$ is:

(a) $2tu(t-4)$

(b) $5\cos(t)\delta(t-2)$

(c) $e^{-t}u(t-t)$

(d) $\sin(2t)u(t-\tau)$

Chapter 15, Solution 8.

(a) 2t=2(t-4) + 8

f(t) = 2tu(t-4) = 2(t-4)u(t-4) + 8u(t-4)

$$F(s) = \frac{2}{s^2}e^{-4s} + \frac{8}{s}e^{-4s} = \underline{\left(\frac{2}{s^2} + \frac{8}{s}\right)e^{-4s}}$$

(b) $F(s) = \int_0^\infty f(t)e^{-st}dt = \int_0^\infty 5\cos t\delta(t-2)e^{-st}dt = 5\cos te^{-st}\Big|_{t=2} =$ $\underline{\mathbf{5cos(2)e^{-2s}}}$

(c) $e^{-t} = e^{-(t-\tau)}e^{-\tau}$

$$f(t) = e^{-\tau}e^{-(t-\tau)}u(t-\tau)$$

$$F(s) = e^{-\tau}e^{-\tau s}\frac{1}{s+1} = \underline{\frac{e^{-\tau(s+1)}}{s+1}}$$

(d) $\sin 2t = \sin[2(t-\tau)+2\tau] = \sin 2(t-\tau)\cos 2\tau + \cos 2(t-\tau)\sin 2\tau$

$$f(t) = \cos 2\tau \sin 2(t-\tau)u(t-\tau) + \sin 2\tau \cos 2(t-\tau)u(t-\tau)$$

$$F(s) = \underline{\cos 2\tau e^{-\tau s}\frac{2}{s^2+4} + \sin 2\tau e^{-\tau s}\frac{s}{s^2+4}}$$

Chapter 15, Problem 9.

Determine the Laplace transforms of these functions:

(a) $f(t) = (t-4)u(t-2)$

(b) $g(t) = 2e^{-4t}u(t-1)$

(c) $h(t) = 5\cos(2t-1)u(t)$

(d) $p(t) = 6[u(t-2) - u(t-4)]$

Chapter 15, Solution 9.

(a) $f(t) = (t-4)u(t-2) = (t-2)u(t-2) - 2u(t-2)$

$$F(s) = \underline{\frac{e^{-2s}}{s^2} - \frac{2e^{-2s}}{s^2}}$$

(b) $g(t) = 2e^{-4t}u(t-1) = 2e^{-4}e^{-4(t-1)}u(t-1)$

$$G(s) = \underline{\frac{2e^{-s}}{e^4(s+4)}}$$

(c) $h(t) = 5\cos(2t-1)u(t)$

$$\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$$
$$\cos(2t-1) = \cos(2t)\cos(1) + \sin(2t)\sin(1)$$

$$h(t) = 5\cos(1)\cos(2t)u(t) + 5\sin(1)\sin(2t)u(t)$$

$$H(s) = 5\cos(1)\cdot\frac{s}{s^2+4} + 5\sin(1)\cdot\frac{2}{s^2+4}$$

$$H(s) = \underline{\frac{2.702s}{s^2+4} + \frac{8.415}{s^2+4}}$$

(d) $p(t) = 6u(t-2) - 6u(t-4)$

$$P(s) = \underline{\frac{6}{s}e^{-2s} - \frac{6}{s}e^{-4s}}$$

Chapter 15, Problem 10.

In two different ways, find the Laplace transform of

$$g(t) = \frac{d}{dt}\left(te^{-t}\cos t\right)$$

Chapter 15, Solution 10.

(a) By taking the derivative in the time domain,

$$g(t) = (-t e^{-t} + e^{-t})\cos(t) - t e^{-t}\sin(t)$$

$$g(t) = e^{-t}\cos(t) - t e^{-t}\cos(t) - t e^{-t}\sin(t)$$

$$G(s) = \frac{s+1}{(s+1)^2+1} + \frac{d}{ds}\left[\frac{s+1}{(s+1)^2+1}\right] + \frac{d}{ds}\left[\frac{1}{(s+1)^2+1}\right]$$

$$G(s) = \frac{s+1}{s^2+2s+2} - \frac{s^2+2s}{(s^2+2s+2)^2} - \frac{2s+2}{(s^2+2s+2)^2} = \underline{\mathbf{\frac{s^2(s+2)}{(s^2+2s+2)^2}}}$$

(b) By applying the time differentiation property,

$$G(s) = sF(s) - f(0)$$

where $f(t) = t e^{-t}\cos(t)$, $f(0) = 0$

$$G(s) = (s)\cdot\frac{-d}{ds}\left[\frac{s+1}{(s+1)^2+1}\right] = \frac{(s)(s^2+2s)}{(s^2+2s+2)^2} = \underline{\mathbf{\frac{s^2(s+2)}{(s^2+2s+2)^2}}}$$

Chapter 15, Problem 11.

Find $F(s)$ if:

(a) $f(t) = 6e^{-t}\cosh 2t$ (b) $f(t) = 3te^{-2t}\sinh 4t$

(c) $f(t) = 8e^{-3t}\cosh t\,u(t-2)$

Chapter 15, Solution 11.

(a) Since $L[\cosh(at)] = \dfrac{s}{s^2 - a^2}$

$$F(s) = \frac{6(s+1)}{(s+1)^2 - 4} = \underline{\mathbf{\frac{6(s+1)}{s^2 + 2s - 3}}}$$

(b) Since $L[\sinh(at)] = \dfrac{a}{s^2 - a^2}$

$$L[3e^{-2t}\sinh(4t)] = \frac{(3)(4)}{(s+2)^2 - 16} = \frac{12}{s^2 + 4s - 12}$$

$$F(s) = L[t \cdot 3e^{-2t}\sinh(4t)] = \frac{-d}{ds}[12(s^2 + 4s - 12)^{-1}]$$

$$F(s) = (12)(2s+4)(s^2 + 4s - 12)^{-2} = \underline{\mathbf{\frac{24(s+2)}{(s^2 + 4s - 12)^2}}}$$

(c) $\cosh(t) = \dfrac{1}{2}\cdot(e^{t} + e^{-t})$

$$f(t) = 8e^{-3t}\cdot\frac{1}{2}\cdot(e^{t} + e^{-t})u(t-2)$$
$$= 4e^{-2t}u(t-2) + 4e^{-4t}u(t-2)$$
$$= 4e^{-4}e^{-2(t-2)}u(t-2) + 4e^{-8}e^{-4(t-2)}u(t-2)$$

$$L[4e^{-4}e^{-2(t-2)}u(t-2)] = 4e^{-4}e^{-2s}\cdot L[e^{-2}u(t)]$$

$$L[4e^{-4}e^{-2(t-2)}u(t-2)] = \frac{4e^{-(2s+4)}}{s+2}$$

Similarly, $L[4e^{-8}e^{-4(t-2)}u(t-2)] = \dfrac{4e^{-(2s+8)}}{s+4}$

Therefore,

$$F(s) = \frac{4e^{-(2s+4)}}{s+2} + \frac{4e^{-(2s+8)}}{s+4} = \underline{\mathbf{\frac{e^{-(2s+6)}\left[(4e^{2} + 4e^{-2})s + (16e^{2} + 8e^{-2})\right]}{s^2 + 6s + 8}}}$$

Chapter 15, Problem 12.

If $g(t) = e^{-2t}\cos 4t$ find $G(s)$.

Chapter 15, Solution 12.

$$G(s) = \frac{s+2}{(s+2)^2+4^2} = \underline{\frac{s+2}{s^2+4s+20}}$$

Chapter 15, Problem 13.

Find the Laplace transform of the following functions:

(a) $t\cos t\,u(t)$ (b) $e^{-t}t\sin t\,u(t)$ (c) $\dfrac{\sin\beta t}{t}u(t)$

Chapter 15, Solution 13.

(a) $tf(t) \quad \longleftrightarrow \quad -\dfrac{d}{ds}F(s)$

If f(t) = cost, then $F(s) = \dfrac{s}{s^2+1}$ and $-\dfrac{d}{ds}F(s) = -\dfrac{(s^2+1)(1)-s(2s)}{(s^2+1)^2}$

$$\underline{L(t\cos t) = \frac{s^2-1}{(s^2+1)^2}}$$

(b) Let f(t) = e^{-t} sin t.

$$F(s) = \frac{1}{(s+1)^2+1} = \frac{1}{s^2+2s+2}$$

$$\frac{dF}{ds} = \frac{(s^2+2s+2)(0)-(1)(2s+2)}{(s^2+2s+2)^2}$$

$$\underline{L(e^{-t}t\sin t) = -\frac{dF}{ds} = \frac{2(s+1)}{(s^2+2s+2)^2}}$$

(c) $\dfrac{f(t)}{t} \quad \longleftrightarrow \quad \int_s^\infty F(s)ds$

Let $f(t) = \sin\beta t$, then $F(s) = \dfrac{\beta}{s^2+\beta^2}$

$$L\left[\frac{\sin\beta t}{t}\right] = \int_s^\infty \frac{\beta}{s^2+\beta^2}ds = \beta\frac{1}{\beta}\tan^{-1}\frac{s}{\beta}\Big|_s^\infty = \frac{\pi}{2}-\tan^{-1}\frac{s}{\beta} = \underline{\tan^{-1}\frac{\beta}{s}}$$

Chapter 15, Problem 14.

Find the Laplace transform of the signal in Fig. 15.26.

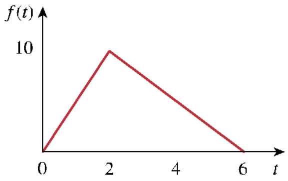
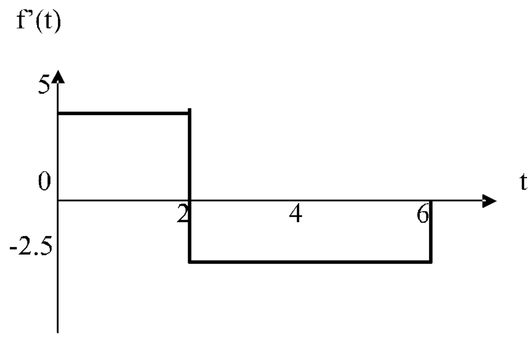


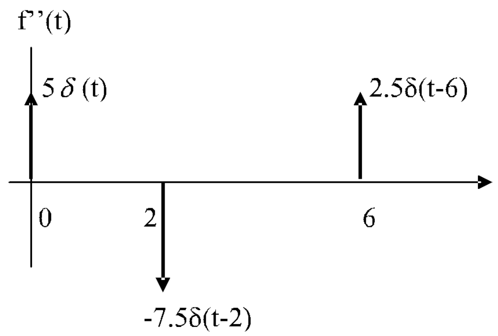
Figure 15.26

For Prob. 15.14.

Chapter 15, Solution 14.

Taking the derivative of f(t) twice, we obtain the figures below.





$$f'' = 5\delta(t) - 7.5\delta(t-2) + 2.5\delta(t-6)$$

Taking the Laplace transform of each term,

$$s^2F(s) = 5 - 7.5e^{-2s} + 2.5e^{-6s} \text{ or } \underline{F(s) = \frac{5}{s} - 7.5\frac{e^{-2s}}{s^2} + 2.5\frac{e^{-6s}}{s^2}}$$

Please note that we can obtain the same answer by representing the function as,

$$f(t) = 5tu(t) - 7.5u(t-2) + 2.5u(t-6).$$

Chapter 15, Problem 15.

Determine the Laplace transform of the function in Fig. 15.27.

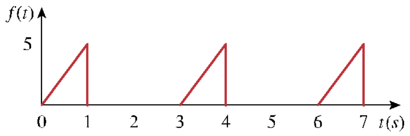


Figure 15.27

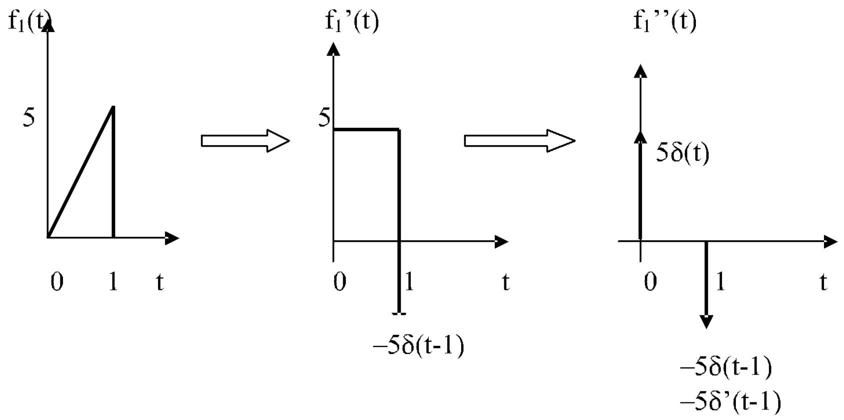
For Prob. 15.15.

Chapter 15, Solution 15.

This is a periodic function with T=3.

$$F(s) = \frac{F_1(s)}{1 - e^{-3s}}$$

To get $F_1(s)$, we consider f(t) over one period.



$f_1'' = 5\delta(t) - 5\delta(t-1) - 5\delta'(t-1)$

Taking the Laplace transform of each term,

$s^2F_1(s) = 5 - 5e^{-s} - 5se^{-s}$ or $F_1(s) = 5(1 - e^{-s} - se^{-s})/s^2$

Hence,

$$F(s) = 5\frac{1 - e^{-s} - se^{-s}}{s^2(1 - e^{-3s})}$$

Alternatively, we can obtain the same answer by noting that $f_1(t) = 5tu(t) - 5tu(t-1) - 5u(t-1)$.

Chapter 15, Problem 16.

Obtain the Laplace transform of *f*(*t*) in Fig. 15.28.

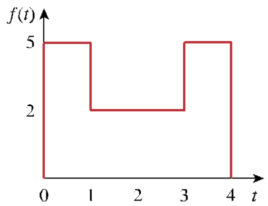


Figure 15.28

For Prob. 15.16.

Chapter 15, Solution 16.

$$f(t) = 5u(t) - 3u(t-1) + 3u(t-3) - 5u(t-4)$$

$$F(s) = \underline{\frac{1}{s}\left[5 - 3e^{-s} + 3e^{-3s} - 5e^{-4s}\right]}$$

Chapter 15, Problem 17.

Find the Laplace transform of $f(t)$ shown in Fig. 15.29.

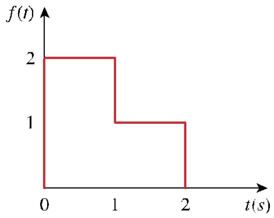
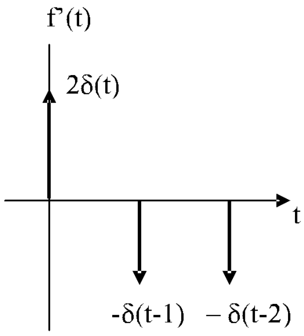


Figure 15.29

For Prob. 15.17.

Chapter 15, Solution 17.

Taking the derivative of f(t) gives f'(t) as shown below.



$$f'(t) = 2\delta(t) - \delta(t-1) - \delta(t-2)$$

Taking the Laplace transform of each term,

$$sF(s) = 2 - e^{-s} - e^{-2s}$$ which leads to

$$F(s) = \underline{\mathbf{[2 - e^{-s} - e^{-2s}]/s}}$$

We can also obtain the same answer noting that $f(t) = 2u(t) - u(t-1) - u(t-2)$.

Chapter 15, Problem 18.

Obtain the Laplace transforms of the functions in Fig. 15.30.

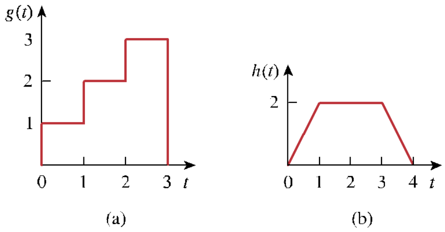


Figure 15.30

For Prob. 15.18.

Chapter 15, Solution 18.

(a) $$g(t) = u(t) - u(t-1) + 2\lfloor u(t-1) - u(t-2) \rfloor + 3\lfloor u(t-2) - u(t-3) \rfloor$$
$$= u(t) + u(t-1) + u(t-2) - 3u(t-3)$$

$$G(s) = \frac{1}{s}(1 + e^{-s} + e^{-2s} - 3e^{-3s})$$

(b) $$h(t) = 2t\lfloor u(t) - u(t-1) \rfloor + 2\lfloor u(t-1) - u(t-3) \rfloor + (8-2t)\lfloor u(t-3) - u(t-4) \rfloor$$

$$= 2tu(t) - 2(t-1)u(t-1) - 2u(t-1) + 2u(t-1) - 2u(t-3) - 2(t-3)u(t-3) + 2u(t-3) + 2(t-4)u(t-4)$$

$$= 2tu(t) - 2(t-1)u(t-1) - 2(t-3)u(t-3) + 2(t-4)u(t-4)$$

$$H(s) = \frac{2}{s^2}(1 - e^{-s}) - \frac{2}{s^2}e^{-3s} + \frac{2}{s^2}e^{-4s} = \frac{2}{s^2}(1 - e^{-s} - e^{-3s} + e^{-4s})$$

Chapter 15, Problem 19.

Calculate the Laplace transform of the train of unit impulses in Fig. 15.31.

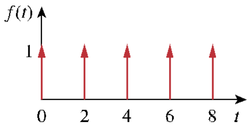


Figure 15.31

For Prob. 15.19.

Chapter 15, Solution 19.

Since $L[\delta(t)] = 1$ and $T = 2$, $F(s) = \dfrac{\mathbf{1}}{\mathbf{1 - e^{-2s}}}$

Chapter 15, Problem 20.

The periodic function shown in Fig. 15.32 is defined over its period as

$$g(t)\begin{cases} \sin \pi t, & 0 < t < 1 \\ 0, & 1 < t < 2 \end{cases}$$

Find $G(s)$

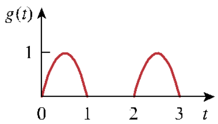


Figure 15.32

For Prob. 15.20.

Chapter 15, Solution 20.

Let $\quad g_1(t) = \sin(\pi t), \quad 0 < t < 1$

$$= \sin(\pi t)[u(t) - u(t-1)]$$

$$= \sin(\pi t)u(t) - \sin(\pi t)u(t-1)$$

Note that $\sin(\pi(t-1)) = \sin(\pi t - \pi) = -\sin(\pi t)$.

So, $\quad g_1(t) = \sin(\pi t)u(t) + \sin(\pi(t-1))u(t-1)$

$$G_1(s) = \frac{\pi}{s^2 + \pi^2}(1 + e^{-s})$$

$$G(s) = \frac{G_1(s)}{1 - e^{-2s}} = \frac{\boldsymbol{\pi(1 + e^{-s})}}{\mathbf{(s^2 + \pi^2)(1 - e^{-2s})}}$$

Chapter 15, Problem 21.

Obtain the Laplace transform of the periodic waveform in Fig. 15.33.

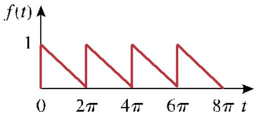


Figure 15.33

For Prob. 15.21.

Chapter 15, Solution 21.

$$T = 2\pi$$

Let $f_1(t) = \left(1 - \dfrac{t}{2\pi}\right)\left[u(t) - u(t - 2\pi)\right]$

$$f_1(t) = u(t) - \frac{t}{2\pi}u(t) + \frac{1}{2\pi}(t - 2\pi)u(t - 2\pi)$$

$$F_1(s) = \frac{1}{s} - \frac{1}{2\pi s^2} + \frac{e^{-2\pi s}}{2\pi s^2} = \frac{2\pi s + \left[-1 + e^{-2\pi s}\right]}{2\pi s^2}$$

$$F(s) = \frac{F_1(s)}{1 - e^{-Ts}} = \underline{\frac{2\pi s - 1 + e^{-2\pi s}}{2\pi s^2 (1 - e^{-2\pi s})}}$$

Chapter 15, Problem 22.

Find the Laplace transforms of the functions in Fig. 15.34.

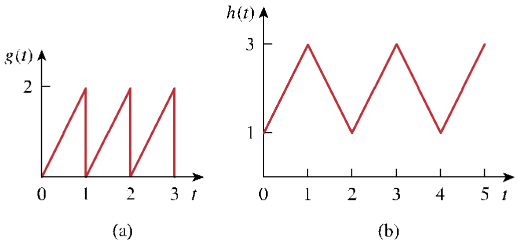


Figure 15.34

For Prob. 15.22.

Chapter 15, Solution 22.

(a) Let $g_1(t) = 2t, \quad 0 < t < 1$

$$= 2t\left[u(t) - u(t-1)\right]$$

$$= 2t\,u(t) - 2(t-1)u(t-1) + 2u(t-1)$$

$$G_1(s) = \frac{2}{s^2} - \frac{2e^{-s}}{s^2} + \frac{2}{s}e^{-s}$$

$$G(s) = \frac{G_1(s)}{1-e^{-sT}}, \quad T = 1$$

$$\mathbf{G(s) = \frac{2(1-e^{-s}+se^{-s})}{s^2(1-e^{-s})}}$$

(b) Let $h = h_0 + u(t)$, where h_0 is the periodic triangular wave.

Let h_1 be h_0 within its first period, i.e.

$$h_1(t) = \begin{cases} 2t & 0 < t < 1 \\ 4-2t & 1 < t < 2 \end{cases}$$

$$h_1(t) = 2t\,u(t) - 2t\,u(t-1) + 4u(t-1) - 2t\,u(t-1) - 2(t-2)u(t-2)$$

$$h_1(t) = 2t\,u(t) - 4(t-1)u(t-1) - 2(t-2)u(t-2)$$

$$H_1(s) = \frac{2}{s^2} - \frac{4}{s^2}e^{-s} - \frac{2e^{-2s}}{s^2} = \frac{2}{s^2}(1-e^{-s})^2$$

$$H_0(s) = \frac{2}{s^2}\frac{(1-e^{-s})^2}{(1-e^{-2s})}$$

$$\mathbf{H(s) = \frac{1}{s} + \frac{2}{s^2}\frac{(1-e^{-s})^2}{(1-e^{-2s})}}$$

Chapter 15, Problem 23.

Determine the Laplace transforms of the periodic functions in Fig. 15.35.

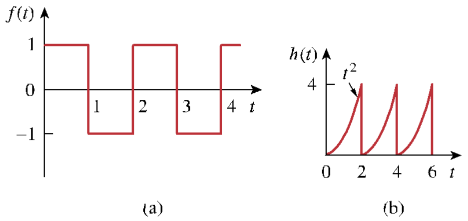


Figure 15.35

For Prob. 15.23.

Chapter 15, Solution 23.

(a) Let $f_1(t) = \begin{cases} 1 & 0 < t < 1 \\ -1 & 1 < t < 2 \end{cases}$

$$f_1(t) = \left[u(t) - u(t-1)\right] - \left[u(t-1) - u(t-2)\right]$$
$$f_1(t) = u(t) - 2u(t-1) + u(t-2)$$

$$F_1(s) = \frac{1}{s}(1 - 2e^{-s} + e^{-2s}) = \frac{1}{s}(1 - e^{-s})^2$$

$$F(s) = \frac{F_1(s)}{(1 - e^{-sT})}, \quad T = 2$$

$$F(s) = \frac{\mathbf{(1 - e^{-s})^2}}{\mathbf{s(1 - e^{-2s})}}$$

(b) Let $h_1(t) = t^2\left[u(t) - u(t-2)\right] = t^2 u(t) - t^2 u(t-2)$

$$h_1(t) = t^2 u(t) - (t-2)^2 u(t-2) - 4(t-2)u(t-2) - 4u(t-2)$$

$$H_1(s) = \frac{2}{s^3}(1 - e^{-2s}) - \frac{4}{s^2}e^{-2s} - \frac{4}{s}e^{-2s}$$

$$H(s) = \frac{H_1(s)}{(1 - e^{-Ts})}, \quad T = 2$$

$$H(s) = \frac{\mathbf{2(1 - e^{-2s}) - 4se^{-2s}(s + s^2)}}{\mathbf{s^3(1 - e^{-2s})}}$$

Chapter 15, Problem 24.

Given that

$$F(s) = \frac{s^2 + 10s + 6}{s(s+1)^2(s+3)}$$

Evaluate $f(0)$ and $f(\infty)$ if they exist.

Chapter 15, Solution 24.

$$f(0) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{s^2 + 10s + 6}{(s+1)^2(s+2)} = \lim_{s\to\infty} \frac{1/s + 10/s^2 + 6/s^3}{(1+1/s)(1+2/s)} = \frac{0}{1} = \underline{0}$$

$$f(\infty) = \lim_{s\to 0} sF(s) = \lim_{s\to 0} \frac{s^2 + 10s + 6}{(s+1)^2(s+2)} = \frac{6}{(1)(2)} = \underline{3}$$

Chapter 15, Problem 25.

Let

$$F(s) = \frac{5(s+1)}{(s+2)(s+3)}$$

(a) Use the initial and final value theorems to find $f(0)$ and $f(\infty)$.
(b) Verify your answer in part (a) by finding $f(t)$, using partial fractions.

Chapter 15, Solution 25.

(a) $$f(0) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{5s(s+1)}{(s+2)(s+3)} = \lim_{s\to\infty} \frac{5(1+1/s)}{(1+2/s)(1+3/s)} = \underline{5}$$

$$f(\infty) = \lim_{s\to 0} sF(s) = \lim_{s\to 0} \frac{5s(s+1)}{(s+2)(s+3)} = \underline{0}$$

(b) $$F(s) = \frac{5(s+1)}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$$

$$A = \frac{5(-1)}{1} = -5, \qquad B = \frac{5(-2)}{-1} = 10$$

$$F(s) = \frac{-5}{s+2} + \frac{10}{s+3} \longrightarrow f(t) = -5e^{-2t} + 10e^{-3t}$$

f(0) = -5 + 10 = **5**
f(∞)= -0 + 0 = **0.**

Chapter 15, Problem 26.

Determine the initial and final values of $f(t)$, if they exist, given that:

(a) $F(s)=\dfrac{s^2+3}{s^3+4s^2+6}$

(b) $F(s)=\dfrac{s^2-2s+1}{(s-2)(s^2+2s+4)}$

Chapter 15, Solution 26.

(a) $$f(0)=\lim_{s\to\infty} sF(s)=\lim_{s\to\infty}\frac{s^3+3s}{s^3+4s^2+6}=\underline{\mathbf{1}}$$

Two poles are not in the left-half plane.
$f(\infty)$ **does not exist**

(b) $$f(0)=\lim_{s\to\infty} sF(s)=\lim_{s\to\infty}\frac{s^3-2s^2+s}{(s-2)(s^2+2s+4)}$$

$$=\lim_{s\to\infty}\frac{1-\frac{2}{s}+\frac{1}{s^2}}{\left(1-\frac{2}{s}\right)\left(1+\frac{2}{s}+\frac{4}{s^2}\right)}=\underline{\mathbf{1}}$$

One pole is not in the left-half plane.
$f(\infty)$ **does not exist**

Chapter 15, Problem 27.

Determine the inverse Laplace transform of each of the following functions:

(a) $F(s) = \frac{1}{s} + \frac{2}{s+1}$

(b) $G(s) = \frac{3s+1}{s+4}$

(c) $H(s) = \frac{4}{(s+1)(s+3)}$

(d) $J(s) = \frac{12}{(s+2)^2(s+4)}$

Chapter 15, Solution 27.

(a) $f(t) = u(t) + 2e^{-t}u(t)$

(b) $G(s) = \dfrac{3(s+4)-11}{s+4} = 3 - \dfrac{11}{s+4}$

$g(t) = \underline{3\delta(t) - 11e^{-4t}u(t)}$

(c) $H(s) = \dfrac{4}{(s+1)(s+3)} = \dfrac{A}{s+1} + \dfrac{B}{s+3}$

$A = 2, \qquad B = -2$

$H(s) = \dfrac{2}{s+1} - \dfrac{2}{s+3}$

$h(t) = \underline{\mathbf{2e^{-t} - 2e^{-3t}}}\ \underline{\mathbf{u(t)}}$

(d) $J(s) = \dfrac{12}{(s+2)^2(s+4)} = \dfrac{A}{s+2} + \dfrac{B}{(s+2)^2} + \dfrac{C}{s+4}$

$B = \dfrac{12}{2} = 6, \quad C = \dfrac{12}{(-2)^2} = 3$

$12 = A(s+2)(s+4) + B(s+4) + C(s+2)^2$

Equating coefficients :

$s^2: \quad 0 = A + C \longrightarrow A = -C = -3$

$s^1: \quad 0 = 6A + B + 4C = 2A + B \longrightarrow B = -2A = 6$

$s^0: \quad 12 = 8A + 4B + 4C = -24 + 24 + 12 = 12$

$J(s) = \dfrac{-3}{s+2} + \dfrac{6}{(s+2)^2} + \dfrac{3}{s+4}$

$j(t) = \underline{\mathbf{3e^{-4t} - 3e^{-2t} + 6te^{-2t}}}\ \underline{\mathbf{u(t)}}$

Chapter 15, Problem 28.

Find the inverse Laplace transform of the following functions:

(a) $F(s)=\dfrac{20(s+2)}{s(s^2+6s+25)}$

(b) $P(s)=\dfrac{6s^2+36s+20}{(s+1)(s+2)(s+3)}$

Chapter 15, Solution 28.

(a) $F(s)=\dfrac{20(s+2)}{s(s^2+6s+25)}=\dfrac{A}{s}+\dfrac{Bs+C}{s^2+6s+25}$

$20(s+2)=A(s^2+6s+25s)+Bs^2+Cs$

Equating components,

s^2 :	0 = A + B or B= - A
s:	20 = 6A + C
constant:	40 – 25 A or A = 8/5, B = -8/5, C= 20 – 6A= 52/5

$$F(s)=\frac{8}{5s}+\frac{-\frac{8}{5}s+\frac{52}{5}}{(s+3)^2+4^2}=\frac{8}{5s}+\frac{-\frac{8}{5}(s+3)+\frac{24}{5}+\frac{52}{5}}{(s+3)^2+4^2}$$

$$f(t)=\underline{\frac{8}{5}u(t)-\frac{8}{5}e^{-3t}\cos 4t+\frac{19}{5}e^{-3t}\sin 4t}$$

(b) $P(s)=\dfrac{6s^2+36s+20}{(s+1)(s+2)(s+3)}=\dfrac{A}{s+1}+\dfrac{B}{s+2}+\dfrac{C}{s+3}$

$$A=\frac{6-36+20}{(-1+2)(-1+3)}=-5$$

$$B=\frac{24-72+20}{(-1)(1)}=28$$

$$C=\frac{54-108+20}{(-2)(-1)}=-17$$

$$P(s)=\frac{-5}{s+1}+\frac{28}{s+2}-\frac{17}{s+3}$$

$$p(t)=\underline{(-5e^{-t}+28e^{-2t}-17e^{-3t})u(t)}$$

Chapter 15, Problem 29.

Find the inverse Laplace transform of:

$$V(s) = \frac{2s + 26}{s\left(s^2 + 4s + 13\right)}$$

Chapter 15, Solution 29.

$$V(s) = \frac{2}{s} + \frac{As + B}{(s+2)^2 + 3^2}; \ 2s^2 + 8s + 26 + As^2 + Bs = 2s + 26 \rightarrow A = -2 \text{ and } B = -6$$

$$V(s) = \frac{2}{s} - \frac{2(s+2)}{(s+2)^2 + 3^2} - \frac{2}{3}\frac{3}{(s+2)^2 + 3^2}$$

$$\underline{v(t) = (2 - 2e^{-2t}\cos 3t - \frac{2}{3}e^{-2t}\sin 3t)u(t), \ t \geq 0}$$

Chapter 15, Problem 30.

Find the inverse Laplace transform of:

(a) $F_1(s)=\dfrac{6s^2+8s+3}{s(s^2+2s+5)}$

(b) $F_2(s)=\dfrac{s^2+5s+6}{(s+1)^2(s+4)}$

(c) $F_3(s)=\dfrac{10}{(s+1)(s^2+4s+8)}$

Chapter 15, Solution 30.

(a) $F_1(s)=\dfrac{6s^2+8s+3}{s(s^2+2s+5)}=\dfrac{A}{s}+\dfrac{Bs+C}{s^2+2s+5}$

$6s^2+8s+3=A(s^2+2s+5)+Bs^2+Cs$

We equate coefficients.

s^2 : $6 = A + B$

s: $8= 2A + C$

constant: $3=5A$ or $A=3/5$

$B=6-A = 27/5$, $C=8-2A = 34/5$

$$F_1(s)=\frac{3/5}{s}+\frac{27s/5+34/5}{s^2+2s+5}=\frac{3/5}{s}+\frac{27(s+1)/5+7/5}{(s+1)^2+2^2}$$

$$\underline{f_1(t)=\left[\frac{3}{5}+\frac{27}{5}e^{-t}\cos 2t+\frac{7}{10}e^{-t}\sin 2t\right]u(t)}$$

(b) $F_2(s) = \frac{s^2+5s+6}{(s+1)^2(s+4)} = \frac{A}{s+1} + \frac{B}{(s+1)^2} + \frac{C}{s+4}$

$s^2+5s+6 = A(s+1)(s+4) + B(s+4) + C(s+1)^2$

Equating coefficients,

s^2 : 1=A+C

s: 5=5A+B+2C

constant: 6=4A+4B+C

Solving these gives

A=7/9, B= 2/3, C=2/9

$$F_2(s) = \frac{7/9}{s+1} + \frac{2/3}{(s+1)^2} + \frac{2/9}{s+4}$$

$$f_2(t) = \underline{\left[\frac{7}{9}e^{-t} + \frac{2}{3}te^{-t} + \frac{2}{9}e^{-4t}\right]u(t)}$$

(c) $F_3(s) = \frac{10}{(s+1)(s^2+4s+8)} = \frac{A}{s+1} + \frac{Bs+C}{s^2+4s+8}$

$10 = A(s^2+4s+8) + B(s^2+s) + C(s+1)$

s^2 : 0 = A + B or B = -A

s: 0=4A+ B + C

constant: 10=8A+C

Solving these yields

A=2, B= -2, C= -6

$$F_3(s) = \frac{2}{s+1} + \frac{-2s-6}{s^2+4s+8} = \frac{2}{s+1} - \frac{2(s+1)}{(s+1)^2+2^2} - \frac{4}{(s+1)^2+2^2}$$

$f_3(t) = \underline{\mathbf{(2e^{-t} - 2e^{-t}\cos(2t) - 2e^{-t}\sin(2t))u(t)}}$.

Chapter 15, Problem 31.

Find $f(t)$ for each $F(s)$:

(a) $\dfrac{10s}{(s+1)(s+2)(s+3)}$

(b) $\dfrac{2s^2+4s+1}{(s+1)(s+2)^3}$

(c) $\dfrac{s+1}{(s+2)(s^2+2s+5)}$

Chapter 15, Solution 31.

(a) $$F(s) = \frac{10s}{(s+1)(s+2)(s+3)} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3}$$

$$A = F(s)\,(s+1)\Big|_{s=-1} = \frac{-10}{2} = -5$$

$$B = F(s)\,(s+2)\Big|_{s=-2} = \frac{-20}{-1} = 20$$

$$C = F(s)\,(s+3)\Big|_{s=-3} = \frac{-30}{2} = -15$$

$$F(s) = \frac{-5}{s+1} + \frac{20}{s+2} - \frac{15}{s+3}$$

$$f(t) = \underline{(-5e^{-t} + 20e^{-2t} - 15e^{-3t})u(t)}$$

(b) $$F(s) = \frac{2s^2+4s+1}{(s+1)(s+2)^3} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{(s+2)^2} + \frac{D}{(s+2)^3}$$

$$A = F(s)(s+1)\big|_{s=-1} = -1$$

$$D = F(s)(s+2)^3\big|_{s=-2} = -1$$

$$2s^2+4s+1 = A(s+2)(s^2+4s+4) + B(s+1)(s^2+4s+4) + C(s+1)(s+2) + D(s+1)$$

Equating coefficients :

s^3: $0 = A + B \longrightarrow B = -A = 1$

s^2: $2 = 6A + 5B + C = A + C \longrightarrow C = 2 - A = 3$

s^1: $4 = 12A + 8B + 3C + D = 4A + 3C + D$

$4 = 6 + A + D \longrightarrow D = -2 - A = -1$

s^0: $1 = 8A + 4B + 2C + D = 4A + 2C + D = -4 + 6 - 1 = 1$

$$F(s) = \frac{-1}{s+1} + \frac{1}{s+2} + \frac{3}{(s+2)^2} - \frac{1}{(s+2)^3}$$

$$f(t) = -e^{-t} + e^{-2t} + 3te^{-2t} - \frac{t^2}{2}e^{-2t}$$

$$\underline{f(t) = \left(-e^{-t} + \left(1 + 3t - \frac{t^2}{2}\right)e^{-2t}\right)u(t)}$$

(c) $$F(s) = \frac{s+1}{(s+2)(s^2+2s+5)} = \frac{A}{s+2} + \frac{Bs+C}{s^2+2s+5}$$

$$A = F(s)(s+2)\big|_{s=-2} = \frac{-1}{5}$$

$$s+1 = A(s^2+2s+5) + B(s^2+2s) + C(s+2)$$

Equating coefficients :

s^2: $0 = A + B \longrightarrow B = -A = \frac{1}{5}$

s^1: $1 = 2A + 2B + C = 0 + C \longrightarrow C = 1$

s^0: $1 = 5A + 2C = -1 + 2 = 1$

$$F(s) = \frac{-1/5}{s+2} + \frac{1/5 \cdot s + 1}{(s+1)^2 + 2^2} = \frac{-1/5}{s+2} + \frac{1/5(s+1)}{(s+1)^2+2^2} + \frac{4/5}{(s+1)^2+2^2}$$

$$\underline{f(t) = (-0.2e^{-2t} + 0.2e^{-t}\cos(2t) + 0.4e^{-t}\sin(2t))u(t)}$$

Chapter 15, Problem 32.

Determine the inverse Laplace transform of each of the following functions:

(a) $\dfrac{8(s+1)(s+3)}{s(s+2)(s+4)}$ (b) $\dfrac{s^2-2s+4}{(s+1)(s+2)^2}$ (c) $\dfrac{s^2+1}{(s+3)(s^2+4s+5)}$

Chapter 15, Solution 32.

(a) $$F(s)=\frac{8(s+1)(s+3)}{s(s+2)(s+4)}=\frac{A}{s}+\frac{B}{s+2}+\frac{C}{s+4}$$

$$A=F(s)s\Big|_{s=0}=\frac{(8)(3)}{(2)(4)}=3$$

$$B=F(s)(s+2)\Big|_{s=-2}=\frac{(8)(-1)}{(-4)}=2$$

$$C=F(s)(s+4)\Big|_{s=-4}=\frac{(8)(-1)(-3)}{(-4)(-2)}=3$$

$$F(s)=\frac{3}{s}+\frac{2}{s+2}+\frac{3}{s+4}$$

$$f(t)=\underline{\mathbf{3u(t)+2e^{-2t}+3e^{-4t}}}$$

(b) $$F(s)=\frac{s^2-2s+4}{(s+1)(s+2)^2}=\frac{A}{s+1}+\frac{B}{s+2}+\frac{C}{(s+2)^2}$$

$$s^2-2s+4=A(s^2+4s+4)+B(s^2+3s+2)+C(s+1)$$

Equating coefficients :

s^2: $1=A+B \longrightarrow B=1-A$

s^1: $-2=4A+3B+C=3+A+C$

s^0: $4=4A+2B+C=-B-2 \longrightarrow B=-6$

$A=1-B=7$ $C=-5-A=-12$

$$F(s)=\frac{7}{s+1}-\frac{6}{s+2}-\frac{12}{(s+2)^2}$$

$$f(t)=\underline{\mathbf{7e^{-t}-6(1+2t)e^{-2t}}}$$

(c) $$F(s) = \frac{s^2+1}{(s+3)(s^2+4s+5)} = \frac{A}{s+3} + \frac{Bs+C}{s^2+4s+5}$$

$$s^2+1 = A(s^2+4s+5) + B(s^2+3s) + C(s+3)$$

Equating coefficients :

s^2: $1 = A + B \longrightarrow B = 1 - A$

s^1: $0 = 4A + 3B + C = 3 + A + C \longrightarrow A + C = -3$

s^0: $1 = 5A + 3C = -9 + 2A \longrightarrow A = 5$

$B = 1 - A = -4$ $C = -A - 3 = -8$

$$F(s) = \frac{5}{s+3} - \frac{4s+8}{(s+2)^2+1} = \frac{5}{s+3} - \frac{4(s+2)}{(s+2)^2+1}$$

$f(t) =$ $\underline{\mathbf{5e^{-3t} - 4e^{-2t}\cos(t)}}$

Chapter 15, Problem 33.

Calculate the inverse Laplace transform of:

(a) $\dfrac{6(s-1)}{s^4-1}$ (b) $\dfrac{se^{-\pi s}}{s^2+1}$ (c) $\dfrac{8}{s(s+1)^3}$

Chapter 15, Solution 33.

(a) $$F(s) = \frac{6(s-1)}{s^4-1} = \frac{6}{(s^2+1)(s+1)} = \frac{As+B}{s^2+1} + \frac{C}{s+1}$$

$$6 = A(s^2+s) + B(s+1) + C(s^2+1)$$

Equating coefficients :

s^2: $0 = A + C \longrightarrow A = \text{-}C$

s^1: $0 = A + B \longrightarrow B = \text{-}A = C$

s^0: $6 = B + C = 2B \longrightarrow B = 3$

$A = \text{-}3,\quad B = 3,\quad C = 3$

$$F(s) = \frac{3}{s+1} + \frac{\text{-}3s+3}{s^2+1} = \frac{3}{s+1} + \frac{\text{-}3s}{s^2+1} + \frac{3}{s^2+1}$$

$$f(t) = \underline{(3e^{-t} + 3\sin(t) - 3\cos(t))u(t)}$$

(b) $$F(s) = \frac{se^{-\pi s}}{s^2+1}$$

$$f(t) = \underline{\mathbf{\cos(t-\pi)u(t-\pi)}}$$

(c) $$F(s) = \frac{8}{s(s+1)^3} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{(s+1)^2} + \frac{D}{(s+1)^3}$$

$A = 8,\quad D = \text{-}8$

$$8 = A(s^3+3s^2+3s+1) + B(s^3+2s^2+s) + C(s^2+s) + Ds$$

Equating coefficients :

s^3: $0 = A + B \longrightarrow B = \text{-}A$

s^2: $0 = 3A + 2B + C = A + C \longrightarrow C = \text{-}A = B$

s^1: $0 = 3A + B + C + D = A + D \longrightarrow D = \text{-}A$

s^0: $A = 8,\quad B = -8,\quad C = -8,\quad D = -8$

$$F(s) = \frac{8}{s} - \frac{8}{s+1} - \frac{8}{(s+1)^2} - \frac{8}{(s+1)^3}$$

$$f(t) = \underline{\mathbf{8\left[1 - e^{-t} - te^{-t} - 0.5t^2e^{-t}\right]u(t)}}$$

Chapter 15, Problem 34.

Find the time functions that have the following Laplace transforms:

(a) $F(s) = 10 + \dfrac{s^2 + 1}{s^2 + 4}$ (b) $G(s) = \dfrac{e^{-s} + 4e^{-2s}}{s^2 + 6s + 8}$ (c) $H(s) = \dfrac{(s+1)e^{-2s}}{s(s+3)(s+4)}$

Chapter 15, Solution 34.

(a) $$F(s) = 10 + \frac{s^2 + 4 - 3}{s^2 + 4} = 11 - \frac{3}{s^2 + 4}$$

$$f(t) = \underline{\mathbf{11\delta(t) - 1.5\sin(2t)}}$$

(b) $$G(s) = \frac{e^{-s} + 4e^{-2s}}{(s+2)(s+4)}$$

Let $$\frac{1}{(s+2)(s+4)} = \frac{A}{s+2} + \frac{B}{s+4}$$

$A = 1/2 \qquad B = 1/2$

$$G(s) = \frac{e^{-s}}{2}\left(\frac{1}{s+2} + \frac{1}{s+4}\right) + 2e^{-2s}\left(\frac{1}{s+2} + \frac{1}{s+4}\right)$$

$$g(t) = \underline{\mathbf{0.5\left[e^{-2(t-1)} - e^{-4(t-1)}\right]u(t-1) + 2\left[e^{-2(t-2)} - e^{-4(t-2)}\right]u(t-2)}}$$

(c) Let $$\frac{s+1}{s(s+3)(s+4)} = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{s+4}$$

$A = 1/12, \qquad B = 2/3, \qquad C = -3/4$

$$H(s) = \left(\frac{1}{12}\cdot\frac{1}{s} + \frac{2/3}{s+3} - \frac{3/4}{s+4}\right)e^{-2s}$$

$$h(t) = \underline{\mathbf{\left(\frac{1}{12} + \frac{2}{3}e^{-3(t-2)} - \frac{3}{4}e^{-4(t-2)}\right)u(t-2)}}$$

Chapter 15, Problem 35.

Obtain $f(t)$ for the following transforms:

(a) $F(s)=\dfrac{(s+3)e^{-6s}}{(s+1)(s+2)}$ (b) $F(s)=\dfrac{4-e^{-2s}}{s^2+5s+4}$ (c) $F(s)=\dfrac{se^{-s}}{(s+3)(s^2+4)}$

Chapter 15, Solution 35.

(a) Let $G(s)=\dfrac{s+3}{(s+1)(s+2)}=\dfrac{A}{s+1}+\dfrac{B}{s+2}$

$A = 2, \qquad B = -1$

$G(s)=\dfrac{2}{s+1}-\dfrac{1}{s+2} \longrightarrow g(t)=2e^{-t}-e^{-2t}$

$F(s)=e^{-6s}G(s) \longrightarrow f(t)=g(t-6)u(t-6)$

$f(t)=\underline{\left[\mathbf{2e^{-(t-6)}-e^{-2(t-6)}}\right]\mathbf{u(t-6)}}$

(b) Let $G(s)=\dfrac{1}{(s+1)(s+4)}=\dfrac{A}{s+1}+\dfrac{B}{s+4}$

$A = 1/3, \qquad B = -1/3$

$G(s)=\dfrac{1}{3(s+1)}-\dfrac{1}{3(s+4)}$

$g(t)=\dfrac{1}{3}\left[e^{-t}-e^{-4t}\right]$

$F(s)=4G(s)-e^{-2t}G(s)$

$f(t)=4g(t)u(t)-g(t-2)u(t-2)$

$f(t)=\underline{\mathbf{\frac{4}{3}\left[e^{-t}-e^{-4t}\right]u(t)-\frac{1}{3}\left[e^{-(t-2)}-e^{-4(t-2)}\right]u(t-2)}}$

(c) Let $G(s) = \frac{s}{(s+3)(s^2+4)} = \frac{A}{s+3} + \frac{Bs+C}{s^2+4}$

$A = -3/13$

$s = A(s^2+4) + B(s^2+3s) + C(s+3)$

Equating coefficients :

$s^2: \quad 0 = A + B \longrightarrow B = -A$

$s^1: \quad 1 = 3B + C$

$s^0: \quad 0 = 4A + 3C$

$A = -3/13, \quad B = 3/13, \quad C = 4/13$

$$13\,G(s) = \frac{-3}{s+3} + \frac{3s+4}{s^2+4}$$

$$13\,g(t) = -3e^{-3t} + 3\cos(2t) + 2\sin(2t)$$

$$F(s) = e^{-s}\,G(s)$$

$$f(t) = g(t-1)\,u(t-1)$$

$$f(t) = \underline{\mathbf{\frac{1}{13}\left[-3e^{-3(t-1)} + 3\cos(2(t-1)) + 2\sin(2(t-1))\right]u(t-1)}}$$

Chapter 15, Problem 36.

Obtain the inverse Laplace transforms of the following functions:

(a) $X(s)=\dfrac{1}{s^2(s+2)(s+3)}$

(b) $Y(s)=\dfrac{1}{s(s+1)^2}$

(c) $Z(s)=\dfrac{1}{s(s+1)(s^2+6s+10)}$

Chapter 15, Solution 36.

(a) $$X(s)=\frac{1}{s^2(s+2)(s+3)}=\frac{A}{s}+\frac{B}{s^2}+\frac{C}{s+2}+\frac{D}{s+3}$$

$$B=1/6, \qquad C=1/4, \qquad D=-1/9$$

$$1=A(s^3+5s^2+6s)+B(s^2+5s+6)+C(s^3+3s^2)+D(s^3+2s^2)$$

Equating coefficients :

s^3: $\quad 0=A+C+D$

s^2: $\quad 0=5A+B+3C+2D=3A+B+C$

s^1: $\quad 0=6A+5B$

s^0: $\quad 1=6B \longrightarrow B=1/6$

$$A=-5/6B=-5/36$$

$$X(s)=\frac{-5/36}{s}+\frac{1/6}{s^2}+\frac{1/4}{s+2}-\frac{1/9}{s+3}$$

$$x(t)=\underline{\mathbf{\frac{-5}{36}u(t)+\frac{1}{6}t+\frac{1}{4}e^{-2t}-\frac{1}{9}e^{-3t}}}$$

(b) $Y(s) = \frac{1}{s(s+1)^2} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{(s+1)^2}$

$A = 1, \qquad C = -1$

$1 = A(s^2 + 2s + 1) + B(s^2 + s) + Cs$

Equating coefficients :

$s^2: \quad 0 = A + B \longrightarrow B = -A$

$s^1: \quad 0 = 2A + B + C = A + C \longrightarrow C = -A$

$s^0: \quad 1 = A, \quad B = -1, \quad C = -1$

$Y(s) = \frac{1}{s} - \frac{1}{s+1} - \frac{1}{(s+1)^2}$

$y(t) = \underline{\mathbf{u(t) - e^{-t} - t\,e^{-t}}}$

(c) $Z(s) = \frac{A}{s} + \frac{B}{s+1} + \frac{Cs + D}{s^2 + 6s + 10}$

$A = 1/10, \qquad B = -1/5$

$1 = A(s^3 + 7s^2 + 16s + 10) + B(s^3 + 6s^2 + 10s) + C(s^3 + s^2) + D(s^2 + s)$

Equating coefficients :

$s^3: \quad 0 = A + B + C$

$s^2: \quad 0 = 7A + 6B + C + D = 6A + 5B + D$

$s^1: \quad 0 = 16A + 10B + D = 10A + 5B \longrightarrow B = -2A$

$s^0: \quad 1 = 10A \longrightarrow A = 1/10$

$A = 1/10, \qquad B = -2A = -1/5, \qquad C = A = 1/10, \qquad D = 4A = \frac{4}{10}$

$10\,Z(s) = \frac{1}{s} - \frac{2}{s+1} + \frac{s+4}{s^2 + 6s + 10}$

$10\,Z(s) = \frac{1}{s} - \frac{2}{s+1} + \frac{s+3}{(s+3)^2 + 1} + \frac{1}{(s+3)^2 + 1}$

$z(t) = \underline{\mathbf{0.1\left[1 - 2e^{-t} + e^{-3t}\cos(t) + e^{-3t}\sin(t)\right]u(t)}}$

Chapter 15, Problem 37.

Find the inverse Laplace transform of:

(a) $H(s)=\dfrac{s+4}{s(s+2)}$

(b) $G(s)=\dfrac{s^2+4s+5}{(s+3)(s^2+2s+2)}$

(c) $F(s)=\dfrac{e^{-4s}}{s+2}$

(d) $D(s)=\dfrac{10s}{(s^2+1)(s^2+4)}$

Chapter 15, Solution 37.

(a) $H(s)=\dfrac{s+4}{s(s+2)}=\dfrac{A}{s}+\dfrac{B}{s+2}$

s+4 =A(s+2) + Bs

Equating coefficients,

s: 1 = A + B

constant: 4= 2 A $\longrightarrow$ A =2, B=1-A = -1

$$H(s)=\frac{2}{s}-\frac{1}{s+2}$$
$$h(t)=2u(t)-e^{-2t}u(t)=\underline{(2-e^{-2t})u(t)}$$

(b) $G(s) = \frac{A}{s+3} + \frac{Bs+C}{s^2+2s+2}$

$$s^2 + 4s + 5 = (Bs + C)(s+3) + A(s^2 + 2s + 2)$$

Equating coefficients,

s^2: 1= B + A (1)

s: 4 = 3B + C + 2A (2)

Constant: 5 =3C + 2A (3)

Solving (1) to (3) gives

$$A = \frac{2}{5}, \quad B = \frac{3}{5}, \quad C = \frac{7}{5}$$

$$G(s) = \frac{0.4}{s+3} + \frac{0.6s+1.4}{s^2+2s+2} = \frac{0.4}{s+3} + \frac{0.6(s+1)+0.8}{(s+1)^2+1}$$

$g(t) = \underline{0.4e^{-3t} + 0.6e^{-t}\cos t + 0.8e^{-t}\sin t}$

(c) $f(t) = \underline{e^{-2(t-4)}u(t-4)}$

(d) $D(s) = \frac{10s}{(s^2+1)(s^2+4)} = \frac{As+B}{s^2+1} + \frac{Cs+D}{s^2+4}$

$$10s = (s^2+4)(As+B) + (s^2+1)(Cs+D)$$

Equating coefficients,

s^3 : 0 = A + C

s^2 : 0 = B + D

s: 10 = 4A + C

constant: 0 = 4B+D

Solving these leads to

A = -10/3, B = 0, C = -10/3, D = 0

$$D(s) == \frac{10s/3}{s^2+1} - \frac{10s/3}{s^2+4}$$

$$d(t) = \underline{\frac{10}{3}\cos t - \frac{10}{3}\cos 2t}$$

Chapter 15, Problem 38.

Find $f(t)$ given that:

(a) $F(s)=\dfrac{s^2+4s}{s^2+10s+26}$

(b) $F(s)=\dfrac{5s^2+7s+29}{s\left(s^2+4s+29\right)}$

Chapter 15, Solution 38.

(a) $$F(s)=\frac{s^2+4s}{s^2+10s+26}=\frac{s^2+10s+26-6s-26}{s^2+10s+26}$$

$$F(s)=1-\frac{6s+26}{s^2+10s+26}$$

$$F(s)=1-\frac{6(s+5)}{(s+5)^2+1^2}+\frac{4}{(s+5)^2+1^2}$$

$$f(t)=\underline{\boldsymbol{\delta(t)-6e^{-t}\cos(5t)+4e^{-t}\sin(5t)}}$$

(b) $$F(s)=\frac{5s^2+7s+29}{s(s^2+4s+29)}=\frac{A}{s}+\frac{Bs+C}{s^2+4s+29}$$

$$5s^2+7s+29=A(s^2+4s+29)+Bs^2+Cs$$

Equating coefficients :

s^0: $29=29A \longrightarrow A=1$

s^1: $7=4A+C \longrightarrow C=7-4A=3$

s^2: $5=A+B \longrightarrow B=5-A=4$

$A=1,\qquad B=4,\qquad C=3$

$$F(s)=\frac{1}{s}+\frac{4s+3}{s^2+4s+29}=\frac{1}{s}+\frac{4(s+2)}{(s+2)^2+5^2}-\frac{5}{(s+2)^2+5^2}$$

$$f(t)=\underline{\boldsymbol{u(t)+4e^{-2t}\cos(5t)-e^{-2t}\sin(5t)}}$$

Chapter 15, Problem 39.

*Determine *f*(*t*) if:

(a) $F(s)=\dfrac{2s^3+4s^2+1}{(s^2+2s+17)(s^2+4s+20)}$

(b) $F(s)=\dfrac{s^2+4}{(s^2+9)(s^2+6s+3)}$

* An asterisk indicates a challenging problem.

Chapter 15, Solution 39.

(a) $$F(s)=\frac{2s^3+4s^2+1}{(s^2+2s+17)(s^2+4s+20)}=\frac{As+B}{s^2+2s+17}+\frac{Cs+D}{s^2+4s+20}$$

$$s^3+4s^2+1=A(s^3+4s^2+20s)+B(s^2+4s+20)$$
$$+C(s^3+2s^2+17s)+D(s^2+2s+17)$$

Equating coefficients :

s^3: $\quad 2=A+C$

s^2: $\quad 4=4A+B+2C+D$

s^1: $\quad 0=20A+4B+17C+2D$

s^0: $\quad 1=20B+17D$

Solving these equations (Matlab works well with 4 unknowns),

$A=-1.6,\qquad B=-17.8,\qquad C=3.6,\qquad D=21$

$$F(s)=\frac{-1.6s-17.8}{s^2+2s+17}+\frac{3.6s+21}{s^2+4s+20}$$

$$F(s)=\frac{(-1.6)(s+1)}{(s+1)^2+4^2}+\frac{(-4.05)(4)}{(s+1)^2+4^2}+\frac{(3.6)(s+2)}{(s+2)^2+4^2}+\frac{(3.45)(4)}{(s+2)^2+4^2}$$

$$f(t)=\underline{\mathbf{-1.6e^{-t}\cos(4t)-4.05e^{-t}\sin(4t)+3.6e^{-2t}\cos(4t)+3.45e^{-2t}\sin(4t)}}$$

(b) $$F(s) = \frac{s^2+4}{(s^2+9)(s^2+6s+3)} = \frac{As+B}{s^2+9} + \frac{Cs+D}{s^2+6s+3}$$

$$s^2+4 = A(s^3+6s^2+3s) + B(s^2+6s+3) + C(s^3+9s) + D(s^2+9)$$

Equating coefficients :

s^3: $\quad 0 = A + C \longrightarrow C = -A$

s^2: $\quad 1 = 6A + B + D$

s^1: $\quad 0 = 3A + 6B + 9C = 6B + 6C \longrightarrow B = -C = A$

s^0: $\quad 4 = 3B + 9D$

Solving these equations,

$A = 1/12, \qquad B = 1/12, \qquad C = -1/12, \qquad D = 5/12$

$$12\,F(s) = \frac{s+1}{s^2+9} + \frac{-s+5}{s^2+6s+3}$$

$$s^2+6s+3 = 0 \longrightarrow \frac{-6 \pm \sqrt{36-12}}{2} = -0.551, -5.449$$

Let $$G(s) = \frac{-s+5}{s^2+6s+3} = \frac{E}{s+0.551} + \frac{F}{s+5.449}$$

$$E = \left.\frac{-s+5}{s+5.449}\right|_{s=-0.551} = 1.133$$

$$F = \left.\frac{-s+5}{s+0.551}\right|_{s=-5.449} = -2.133$$

$$G(s) = \frac{1.133}{s+0.551} - \frac{2.133}{s+5.449}$$

$$12\,F(s) = \frac{s}{s^2+3^2} + \frac{1}{3}\cdot\frac{3}{s^2+3^2} + \frac{1.133}{s+0.551} - \frac{2.133}{s+5.449}$$

$$f(t) = \underline{\mathbf{0.08333\cos(3t) + 0.02778\sin(3t) + 0.0944e^{-0.551t} - 0.1778e^{-5.449t}}}$$

Chapter 15, Problem 40.

Show that

$$L^{-1}\left[\frac{4s^2+7s+13}{(s+2)(s^2+2s+5)}\right]=\left[\sqrt{2}e^{-t}\cos(2t+45°)+3e^{-2t}\right]u(t)$$

Chapter 15, Solution 40.

$$\text{Let } H(s)=\left[\frac{4s^2+7s+13}{(s+2)(s^2+2s+5)}\right]=\frac{A}{s+2}+\frac{Bs+C}{s^2+2s+5}$$

$$4s^2+7s+13=A(s^2+2s+5)+B(s^2+2s)+C(s+2)$$

Equating coefficients gives:

$$s^2: \quad 4=A+B$$

$$s: \quad 7=2A+2B+C \longrightarrow C=-1$$

$$\text{constant}: \quad 13=5A+2C \longrightarrow 5A=15 \text{ or } A=3,\ B=1$$

$$H(s)=\frac{3}{s+2}+\frac{s-1}{s^2+2s+5}=\frac{3}{s+2}+\frac{(s+1)-2}{(s+1)^2+2^2}$$

Hence,

$$h(t)=3e^{-2t}+e^{-t}\cos 2t-e^{-t}\sin 2t=3e^{-2t}+e^{-t}(A\cos\alpha\cos 2t-A\sin\alpha\sin 2t)$$

where $A\cos\alpha=1, \quad A\sin\alpha=1 \longrightarrow A=\sqrt{2}, \quad \alpha=45^{o}$

Thus,

$$h(t)=\underline{\left[\sqrt{2}e^{-t}\cos(2t+45^{o})+3e^{-2t}\right]u(t)}$$

Chapter 15, Problem 41.

* Let $x(t)$ and $y(t)$ be as shown in Fig. 15.36. Find $z(t) = x(t) * y(t)$.

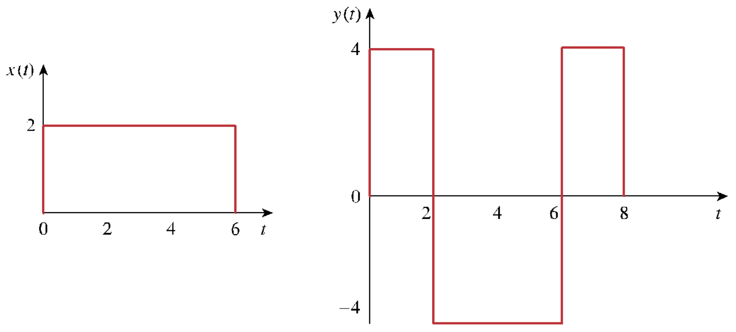


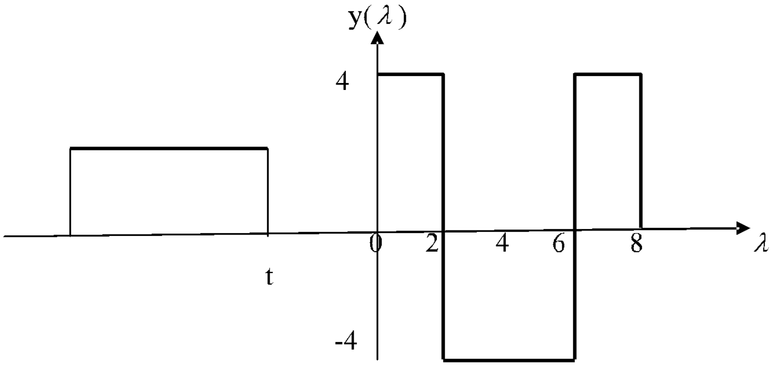
Figure 15.36

For Prob. 15.41.

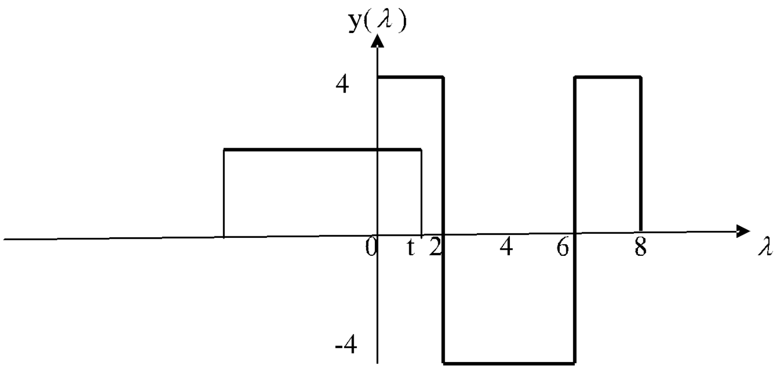
* An asterisk indicates a challenging problem.

Chapter 15, Solution 41.

We fold x(t) and slide on y(t). For t<0, no overlapping as shown below. x(t) =0.

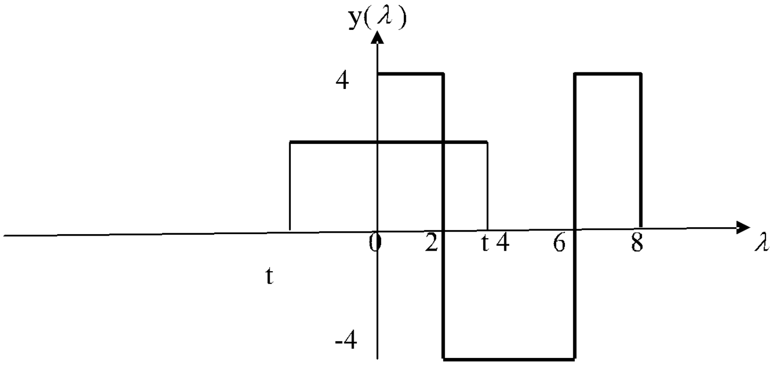


For 0 < t < 2, there is overlapping, as shown below.



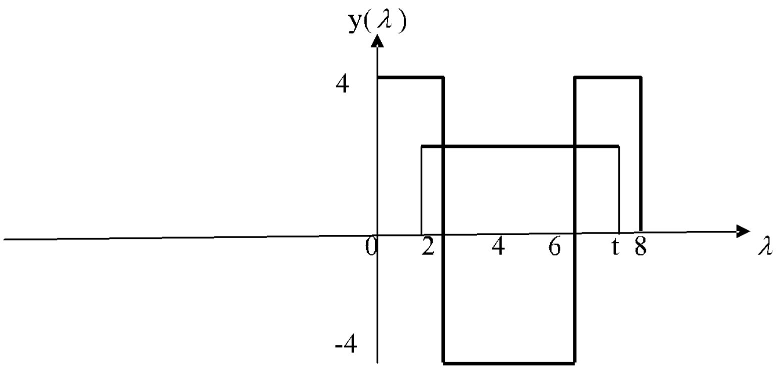
$$z(t) = \int_0^t (2)(4)dt = 8t$$

For 2 < t < 6, the two functions overlap, as shown below.



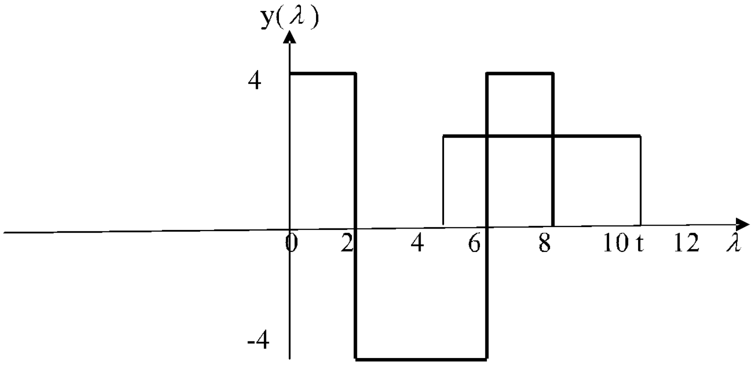
$$z(t) = \int_0^2 (2)(4)d\lambda + \int_0^t (2)(-4)d\lambda = 16 - 8t$$

For 6<t<8, they overlap as shown below.



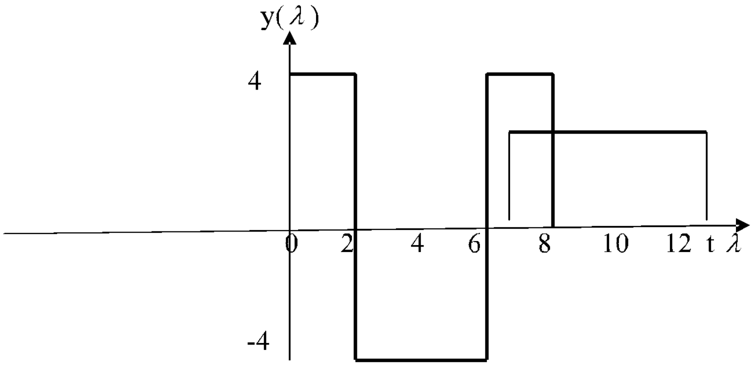
$$z(t) = \int_{t-6}^{2} (2)(4)d\lambda + \int_2^6 (2)(-4)d\lambda + \int_6^t (2)(4)d\lambda = 8\lambda \Big|_{t-6}^{2} - 8\lambda \Big|_2^6 + 8\lambda \Big|_6^t = -16$$

For 8< t <12, they overlap as shown below.



$$z(t) = \int_{t-6}^{6} (2)(-4)d\lambda + \int_{6}^{8}(2)(4)d\lambda = -8\lambda\Big|_{t-6}^{6} + 8\lambda\Big|_{6}^{8} = 8t - 80$$

For 12 < t < 14, they overlap as shown below.



$$z(t) = \int_{t-6}^{8} (2)(4)d\lambda = 8\lambda\Big|_{t-6}^{8} = 112 - 8t$$

Hence,

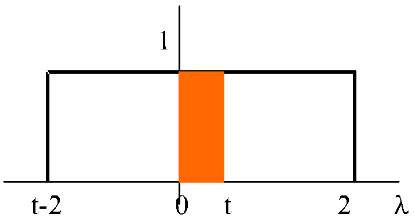
z(t) =		
	8t,	**0<t<2**
	16–8t,	**2<t<6**
	–16,	**6<t<8**
	8t–80,	**8<t<12**
	112–8t,	**12<t<14**
	0,	**otherwise**.

Chapter 15, Problem 42.

Suppose that $f(t) = u(t) - u(t-2)$. Determine $f(t) * f(t)$.

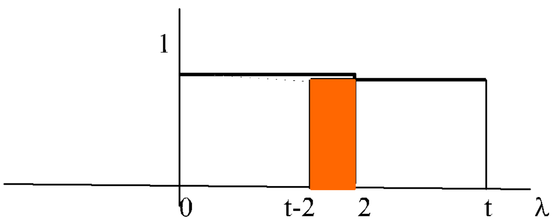
Chapter 15, Solution 42.

For 0<t<2, the signals overlap as shown below.



$$y(t) = f(t) * f(t) = \int_0^t (1)(1) d\lambda = t$$

For 2 < t< 4, they overlap as shown below.



$$y(t) = \int_{t-2}^{2} (1)(1) d\lambda = t \Big|_{t-2}^{2} = 4 - t$$

Thus,

$$y(t) = \begin{cases} t, & 0 < t < 2 \\ 4 - t, & 2 < t < 4 \\ 0, & \text{otherwise} \end{cases}$$

Chapter 15, Problem 43.

Find $y(t) = x(t) * h(t)$ for each paired $x(t)$ and $h(t)$ in Fig. 15.37.

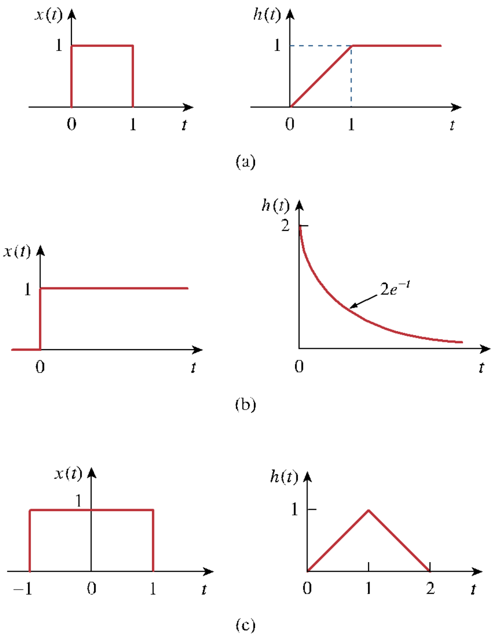


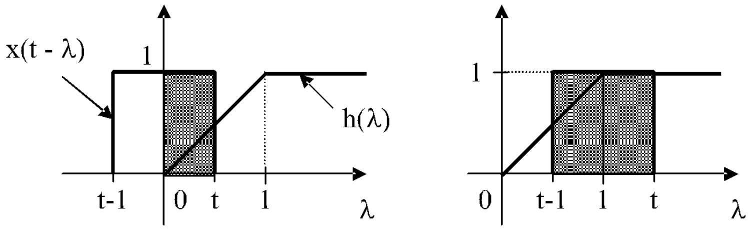
Figure 15.37

For Prob. 15.43.

Chapter 15, Solution 43.

(a) For $0 < t < 1$, $x(t-\lambda)$ and $h(\lambda)$ overlap as shown in Fig. (a).

$$y(t) = x(t) * h(t) = \int_0^t (1)(\lambda)\, d\lambda = \frac{\lambda^2}{2}\Big|_0^t = \frac{t^2}{2}$$



(a) **(b)**

For $1 < t < 2$, $x(t-\lambda)$ and $h(\lambda)$ overlap as shown in Fig. (b).

$$y(t) = \int_{t-1}^{1} (1)(\lambda)\, d\lambda + \int_1^t (1)(1)\, d\lambda = \frac{\lambda^2}{2}\Big|_{t-1}^{1} + \lambda\Big|_1^t = \frac{-1}{2}t^2 + 2t - 1$$

For $t > 2$, there is a complete overlap so that

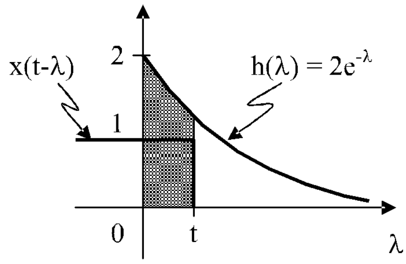
$$y(t) = \int_{t-1}^{t} (1)(1)\, d\lambda = \lambda\Big|_{t-1}^{t} = t - (t-1) = 1$$

Therefore,

$$y(t) = \begin{cases} \mathbf{t^2/2,} & \mathbf{0 < t < 1} \\ \mathbf{-(t^2/2) + 2t - 1,} & \mathbf{1 < t < 2} \\ \mathbf{1,} & \mathbf{t > 2} \\ \mathbf{0,} & \mathbf{otherwise} \end{cases}$$

(b) For $t > 0$, the two functions overlap as shown in Fig. (c).

$$y(t) = x(t) * h(t) = \int_0^t (1)\, 2 e^{-\lambda}\, d\lambda = -2 e^{-\lambda}\Big|_0^t$$



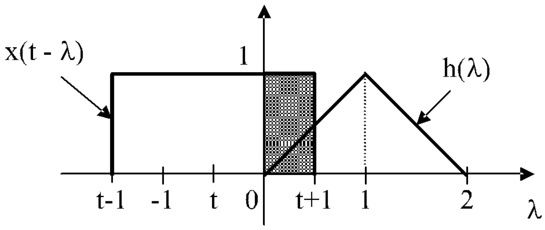
(c)

Therefore,

$$y(t) = \underline{\mathbf{2(1-e^{-t}), \quad t > 0}}$$

(c) For $-1 < t < 0$, $x(t-\lambda)$ and $h(\lambda)$ overlap as shown in Fig. (d).

$$y(t) = x(t) * h(t) = \int_0^{t+1} (1)(\lambda)\, d\lambda = \frac{\lambda^2}{2}\Big|_0^{t+1} = \frac{1}{2}(t+1)^2$$

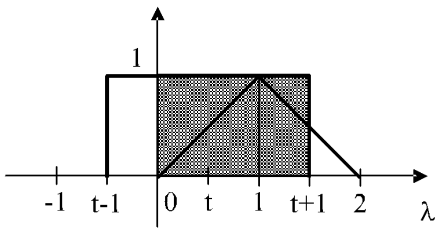


(d)

For $0 < t < 1$, $x(t-\lambda)$ and $h(\lambda)$ overlap as shown in Fig. (e).

$$y(t) = \int_0^1 (1)(\lambda)\, d\lambda + \int_1^{t+1} (1)(2-\lambda)\, d\lambda$$

$$y(t) = \frac{\lambda^2}{2}\Big|_0^1 + \left(2\lambda - \frac{\lambda^2}{2}\right)\Big|_1^{t+1} = \frac{-1}{2}t^2 + t + \frac{1}{2}$$

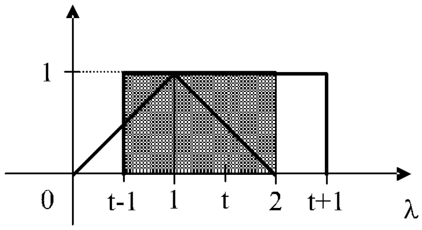


(e)

For $1 < t < 2$, $x(t-\lambda)$ and $h(\lambda)$ overlap as shown in Fig. (f).

$$y(t) = \int_{t-1}^{1} (1)(\lambda)\, d\lambda + \int_{1}^{2} (1)(2-\lambda)\, d\lambda$$

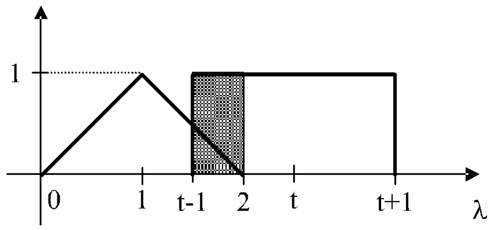
$$y(t) = \frac{\lambda^2}{2}\Big|_{t-1}^{1} + \left(2\lambda - \frac{\lambda^2}{2}\right)\Big|_{1}^{2} = \frac{-1}{2}t^2 + t + \frac{1}{2}$$



(f)

For $2 < t < 3$, $x(t-\lambda)$ and $h(\lambda)$ overlap as shown in Fig. (g).

$$y(t) = \int_{t-1}^{2} (1)(2-\lambda)\, d\lambda = \left(2\lambda - \frac{\lambda^2}{2}\right)\Big|_{t-1}^{2} = \frac{9}{2} - 3t + \frac{1}{2}t^2$$



(g)

Therefore,

$$y(t) = \begin{cases} (t^2/2) + t + 1/2, & -1 < t < 0 \\ -(t^2/2) + t + 1/2, & 0 < t < 2 \\ (t^2/2) - 3t + 9/2, & 2 < t < 3 \\ 0, & \text{otherwise} \end{cases}$$

Chapter 15, Problem 44.

Obtain the convolution of the pairs of signals in Fig. 15.38.

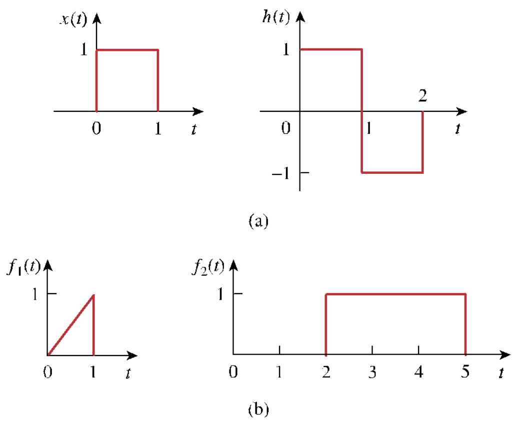


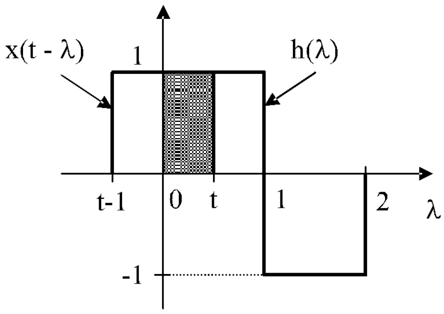
Figure 15.38

For Prob. 15.44.

Chapter 15, Solution 44.

(a) For $0 < t < 1$, $x(t-\lambda)$ and $h(\lambda)$ overlap as shown in Fig. (a).

$$y(t) = x(t) * h(t) = \int_0^t (1)(1)\, d\lambda = t$$



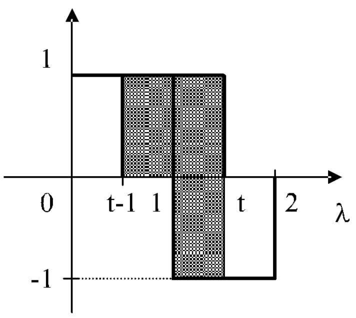
(a)

For $1 < t < 2$, $x(t-\lambda)$ and $h(\lambda)$ overlap as shown in Fig. (b).

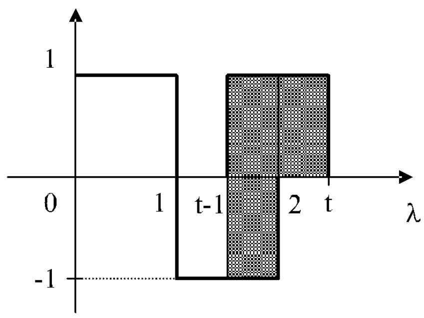
$$y(t) = \int_{t-1}^{1} (1)(1)\, d\lambda + \int_{1}^{t} (-1)(1)\, d\lambda = \lambda\Big|_{t-1}^{1} - \lambda\Big|_{1}^{t} = 3 - 2t$$

For $2 < t < 3$, $x(t-\lambda)$ and $h(\lambda)$ overlap as shown in Fig. (c).

$$y(t) = \int_{t-1}^{2} (1)(-1)\, d\lambda = -\lambda\Big|_{t-1}^{2} = t - 3$$



(b)



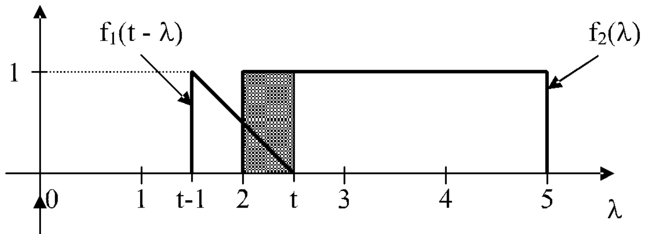
(c)

Therefore,

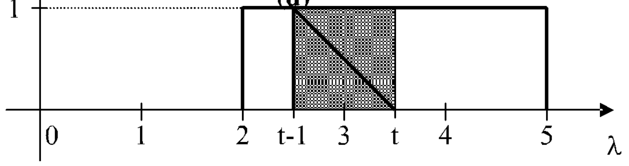
$$y(t) = \begin{cases} \mathbf{t}, & \mathbf{0 < t < 1} \\ \mathbf{3 - 2t}, & \mathbf{1 < t < 2} \\ \mathbf{t - 3}, & \mathbf{2 < t < 3} \\ \mathbf{0}, & \mathbf{otherwise} \end{cases}$$

(b) For $t < 2$, there is no overlap. For $2 < t < 3$, $f_1(t-\lambda)$ and $f_2(\lambda)$ overlap, as shown in Fig. (d).

$$y(t) = f_1(t) * f_2(t) = \int_2^t (1)(t-\lambda)\, d\lambda$$



(d)



(e)

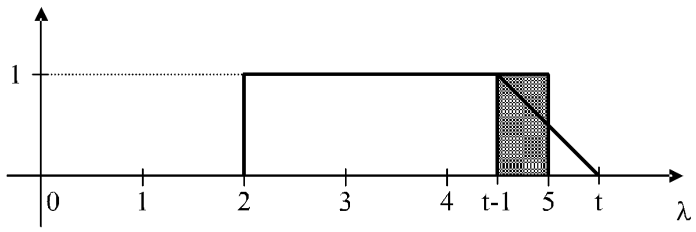
$$= \left(\lambda t - \frac{\lambda^2}{2}\right)\Big|_2^t = \frac{t^2}{2} - 2t + 2$$

For $3 < t < 5$, $f_1(t-\lambda)$ and $f_2(\lambda)$ overlap as shown in Fig. (e).

$$y(t) = \int_{t-1}^t (1)(t-\lambda)\, d\lambda = \left(\lambda t - \frac{\lambda^2}{2}\right)\Big|_{t-1}^t = \frac{1}{2}$$

For $5 < t < 6$, the functions overlap as shown in Fig. (f).

$$y(t) = \int_{t-1}^5 (1)(t-\lambda)\, d\lambda = \left(\lambda t - \frac{\lambda^2}{2}\right)\Big|_{t-1}^5 = \frac{-1}{2}t^2 + 5t - 12$$



(f)

Therefore,
$$y(t) = \begin{cases} \mathbf{(t^2/2) - 2t + 2,} & \mathbf{2 < t < 3} \\ \mathbf{1/2,} & \mathbf{3 < t < 5} \\ \mathbf{-(t^2/2) + 5t - 12,} & \mathbf{5 < t < 6} \\ \mathbf{0,} & \mathbf{otherwise} \end{cases}$$

Chapter 15, Problem 45.

Given $h(t) = 4e^{-2t}u(t)$ and $x(t) = \delta(t) - 2e^{-2t}u(t)$, find $y(t) = x(t) * h(t)$.

Chapter 15, Solution 45.

$$y(t) = h(t) * x(t) = \left[4e^{-2t}u(t)\right] * \left[\delta(t) - 2e^{-2t}u(t)\right]$$

$$= 4e^{-2t}u(t) * \delta(t) - 4e^{-2t}u(t) * 2e^{-2t}u(t) = 4e^{-2t}u(t) - 8e^{-2t}\int_0^t e^o d\lambda$$

$$= \underline{4e^{-2t}u(t) - 8te^{-2t}u(t)}$$

Chapter 15, Problem 46.

Given the following functions

$x(t) = 2\delta(t)$, $y(t) = 4u(t)$, $z(t) = e^{-2t}u(t)$,

evaluate the following convolution operations.

(a) $x(t) * y(t)$

(b) $x(t) * z(t)$

(c) $y(t) * z(t)$

(d) $y(t) * [y(t) + z(t)]$

Chapter 15, Solution 46.

(a) $x(t) * y(t) = 2\delta(t) * 4u(t) = \underline{8u(t)}$

(b) $x(t) * z(t) = 2\delta(t) * e^{-2t}u(t) = \underline{2e^{-2t}u(t)}$

(c) $y(t) * z(t) = 4u(t) * e^{-2t}u(t) = 4\int_0^t e^{-2\lambda}d\lambda = \left.\frac{4e^{-2\lambda}}{-2}\right|_0^t = \underline{2(1-e^{-2t})}$

(d) $y(t) * [y(t) + z(t)] = 4u(t) * [4u(t) + e^{-2t}u(t)] = 4\int [4u(\lambda) + e^{-2\lambda}u(\lambda)]d\lambda$

$$= 4\int_0^t [4 + e^{-2\lambda}]d\lambda = 4[4t + \frac{e^{-2\lambda}}{-2}]\Big|_0^t = \underline{16t - 2e^{-2t} + 2}$$

Chapter 15, Problem 47.

A system has the transfer function

$$H(s)=\frac{s}{(s+1)(s+2)}$$

(a) Find the impulse response of the system.
(b) Determine the output $y(t)$, given that the input is $x(t)=u(t)$

Chapter 15, Solution 47.

(a) $H(s)=\dfrac{s}{(s+1)(s+2)}=\dfrac{A}{s+1}+\dfrac{B}{s+2}$

s=A(s+2) + B(s+1)

We equate the coefficients.

s: 1= A+B
constant: 0 =2A +B

Solving these, A = -1, B= 2.

$$H(s)=\frac{-1}{s+1}+\frac{2}{s+2}$$

$$h(t)=\underline{(-e^{-t}+2e^{-2t})u(t)}$$

(b) $H(s)=\dfrac{Y(s)}{X(s)}\quad\longrightarrow\quad Y(s)=H(s)X(s)=\dfrac{s}{(s+1)(s+2)}\dfrac{1}{s}$

$$Y(s)=\frac{1}{(s+1)(s+2)}=\frac{C}{s+1}+\frac{D}{s+2}$$

C=1 and D=-1 so that

$$Y(s)=\frac{1}{s+1}-\frac{1}{s+2}$$

$$y(t)=\underline{(e^{-t}-e^{-2t})u(t)}$$

Chapter 15, Problem 48.

Find $f(t)$ using convolution given that:

(a) $F(s)=\dfrac{4}{\left(s^2+2s+5\right)^2}$

(b) $F(s)=\dfrac{2s}{(s+1)\left(s^2+4\right)}$

Chapter 15, Solution 48.

(a) Let $G(s) = \frac{2}{s^2 + 2s + 5} = \frac{2}{(s+1)^2 + 2^2}$

$$g(t) = e^{-t}\sin(2t)$$

$$F(s) = G(s)G(s)$$

$$f(t) = L^{-1}\left[G(s)G(s)\right] = \int_0^t g(\lambda)g(t-\lambda)\,d\lambda$$

$$f(t) = \int_0^t e^{-\lambda}\sin(2\lambda)e^{-(t-\lambda)}\sin(2(t-\lambda))\,d\lambda$$

$$\sin(A)\sin(B) = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$f(t) = \frac{1}{2}e^{-t}\int_0^t e^{-\lambda}\left[\cos(2t) - \cos(2(t-2\lambda))\right]d\lambda$$

$$f(t) = \frac{e^{-t}}{2}\cos(2t)\int_0^t e^{-2\lambda}\,d\lambda - \frac{e^{-t}}{2}\int_0^t e^{-2\lambda}\cos(2t-4\lambda)\,d\lambda$$

$$f(t) = \frac{e^{-t}}{2}\cos(2t)\cdot\frac{e^{-2\lambda}}{-2}\Big|_0^t - \frac{e^{-t}}{2}\int_0^t e^{-2\lambda}\left[\cos(2t)\cos(4\lambda) + \sin(2t)\sin(4\lambda)\right]d\lambda$$

$$f(t) = \frac{1}{4}e^{-t}\cos(2t)(-e^{-2t}+1) - \frac{e^{-t}}{2}\cos(2t)\int_0^t e^{-2\lambda}\cos(4\lambda)\,d\lambda$$
$$-\frac{e^{-t}}{2}\sin(2t)\int_0^t e^{-2\lambda}\sin(4\lambda)\,d\lambda$$

$$f(t) = \frac{1}{4}e^{-t}\cos(2t)(1-e^{-2t})$$
$$-\frac{e^{-t}}{2}\cos(2t)\left[\frac{e^{-2\lambda}}{4+16}\left(-2\cos(4\lambda) - 4\sin(4\lambda)\right)\right]_0^t$$
$$-\frac{e^{-t}}{2}\sin(2t)\left[\frac{e^{-2\lambda}}{4+16}\left(-2\sin(4\lambda) + 4\cos(4\lambda)\right)\right]_0^t$$

$$f(t) = \mathbf{\frac{e^{-t}}{2}\cos(2t) - \frac{e^{-3t}}{4}\cos(2t) - \frac{e^{-t}}{20}\cos(2t) + \frac{e^{-3t}}{20}\cos(2t)\cos(4t)}$$
$$\mathbf{+\frac{e^{-3t}}{10}\cos(2t)\sin(4t) + \frac{e^{-t}}{10}\sin(2t)}$$
$$\mathbf{+\frac{e^{-t}}{20}\sin(2t)\sin(4t) - \frac{e^{-t}}{10}\sin(2t)\cos(4t)}$$

(b) Let $X(s) = \dfrac{2}{s+1}$, $Y(s) = \dfrac{s}{s+4}$

$x(t) = 2\,e^{-t}\,u(t)$, $y(t) = \cos(2t)\,u(t)$

$F(s) = X(s)\,Y(s)$

$$f(t) = L^{-1}\left[\,X(s)\,Y(s)\right] = \int_0^{\infty} y(\lambda)\,x(t-\lambda)\,d\lambda$$

$$f(t) = \int_0^{t} \cos(2\lambda)\cdot 2\,e^{-(t-\lambda)}\,d\lambda$$

$$f(t) = 2\,e^{-t}\cdot\frac{e^{\lambda}}{1+4}\left(\cos(2\lambda) + 2\sin(2\lambda)\right)\Big|_0^t$$

$$f(t) = \frac{2}{5}e^{-t}\left[\,e^{t}\left(\,\cos(2t) + 2\sin(2t) - 1\right)\right]$$

$$f(t) = \underline{\mathbf{\frac{2}{5}\cos(2t) + \frac{4}{5}\sin(2t) - \frac{2}{5}e^{-t}}}$$

Chapter 15, Problem 49.

* Use the convolution integral to find:

(a) $t * e^{at}u(t)$

(b) $\cos(t) * \cos(t)u(t)$

* An asterisk indicates a challenging problem.

Chapter 15, Solution 49.

(a) $t*e^{at}u(t) =$

$$\int_0^t e^{a\lambda}(t-\lambda)d\lambda = t\frac{e^{a\lambda}}{a}\Bigg|_0^t - \frac{e^{a\lambda}}{a^2}(a\lambda-1)\Bigg|_0^t = \underline{\frac{t}{a}(e^{at}-1) - \frac{1}{a^2} - \frac{e^{at}}{a^2}(at-1)}$$

(b) $$\cos t * \cos t u(t) = \int_0^t \cos\lambda\cos(t-\lambda)d\lambda = \int_0^t \{\cos t\cos\lambda\cos\lambda + \sin t\sin\lambda\cos\lambda\}d\lambda$$

$$= \left[\cos t\int_0^t \frac{1}{2}[1+\cos 2\lambda]d\lambda + \sin t\int_0^t \cos\lambda d(-\cos\lambda)\right] = \left[\frac{1}{2}\cos t[\lambda + \frac{\sin 2\lambda}{2}]\Bigg|_0^t - \sin t\frac{\cos\lambda}{2}\Bigg|_0^t\right]$$

= **<u>0.5cos(t)(t+0.5sin(2t)) – 0.5sin(t)(cos(t) – 1)</u>**.

Chapter 15, Problem 50.

Use the Laplace transform to solve the differential equation

$$\frac{d^2v(t)}{dt^2}+2\frac{dv(t)}{dt}+10v(t)=3\cos 2t$$

subject to $v(0)=1, dv(0)/dt=-2$.

Chapter 15, Solution 50.

Take the Laplace transform of each term.

$$\left[s^2 V(s)-s\,v(0)-v'(0)\right]+2\left[s\,V(s)-v(0)\right]+10\,V(s)=\frac{3s}{s^2+4}$$

$$s^2 V(s)-s+2+2s\,V(s)-2+10\,V(s)=\frac{3s}{s^2+4}$$

$$(s^2+2s+10)\,V(s)=s+\frac{3s}{s^2+4}=\frac{s^3+7s}{s^2+4}$$

$$V(s)=\frac{s^3+7s}{(s^2+4)(s^2+2s+10)}=\frac{As+B}{s^2+4}+\frac{Cs+D}{s^2+2s+10}$$

$$s^3+7s=A\,(s^3+2s^2+10s)+B\,(s^2+2s+10)+C\,(s^3+4s)+D\,(s^2+4)$$

Equating coefficients :

s^3: $1=A+C \longrightarrow C=1-A$

s^2: $0=2A+B+D$

s^1: $7=10A+2B+4C=6A+2B+4$

s^0: $0=10B+4D \longrightarrow D=-2.5B$

Solving these equations yields

$$A=\frac{9}{26},\qquad B=\frac{12}{26},\qquad C=\frac{17}{26},\qquad D=\frac{-30}{26}$$

$$V(s)=\frac{1}{26}\left[\frac{9s+12}{s^2+4}+\frac{17s-30}{s^2+2s+10}\right]$$

$$V(s)=\frac{1}{26}\left[\frac{9s}{s^2+4}+6\cdot\frac{2}{s^2+4}+17\cdot\frac{s+1}{(s+1)^2+3^2}-\frac{47}{(s+1)^2+3^2}\right]$$

$$\underline{\mathbf{v(t)=\frac{9}{26}\cos(2t)+\frac{6}{26}\sin(2t)+\frac{17}{26}e^{-t}\cos(3t)-\frac{47}{78}e^{-t}\sin(3t)}}$$

Chapter 15, Problem 51.

Given that $v(0) = 2$ and $dv(0)/dt = 4$, solve

$$\frac{d^2v}{dt^2} + 5\frac{dv}{dt} + 6v = 10e^{-t}u(t)$$

Chapter 15, Solution 51.

Taking the Laplace transform of the differential equation yields

$$\left[s^2V(s) - sv(0) - v'(0)\right] + 5[sV(s) - v(0)]] + 6V(s) = \frac{10}{s+1}$$

or $\left(s^2 + 5s + 6\right)V(s) - 2s - 4 - 10 = \dfrac{10}{s+1} \longrightarrow V(s) = \dfrac{2s^2 + 16s + 24}{(s+1)(s+2)(s+3)}$

Let $V(s) = \dfrac{A}{s+1} + \dfrac{B}{s+2} + \dfrac{C}{s+3}$, $A = 5$, $B = 0$, $C = -3$

Hence,

$$v(t) = \underline{(5e^{-t} - 3e^{-3t})u(t)}$$

Chapter 15, Problem 52.

Use the Laplace transform to find $i(t)$ for $t > 0$ if

$$\frac{d^2 i}{dt^2} + 3\frac{di}{dt} + 2i + \delta(t) = 0,$$

$$i(0) = 0, \qquad i'(0) = 3$$

Chapter 15, Solution 52.

Take the Laplace transform of each term.

$$\left[s^2 I(s) - s\,i(0) - i'(0) \right] + 3\left[s\,I(s) - i(0) \right] + 2\,I(s) + 1 = 0$$

$$(s^2 + 3s + 2)\,I(s) - s - 3 - 3 + 1 = 0$$

$$I(s) = \frac{s+5}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$$A = 4, \qquad B = -3$$

$$I(s) = \frac{4}{s+1} - \frac{3}{s+2}$$

$$i(t) = \underline{\mathbf{(4e^{-t} - 3e^{-2t})u(t)}}$$

Chapter 15, Problem 53.

* Use Laplace transforms to solve for $x(t)$ in

$$x(t) = \cos t + \int_0^t e^{\lambda - t} x(\lambda)\, d\lambda$$

* An asterisk indicates a challenging problem.

Chapter 15, Solution 53.

Transform each term.

We begin by noting that the integral term can be rewritten as,

$\int_0^t x(\lambda)e^{-(t-\lambda)}d\lambda$ which is convolution and can be written as $e^{-t}*x(t)$.

Now, transforming each term produces,

$$X(s) = \frac{s}{s^2+1} + \frac{1}{s+1}X(s) \rightarrow \left(\frac{s+1-1}{s+1}\right)X(s) = \frac{s}{s^2+1}$$

$$X(s) = \frac{s+1}{s^2+1} = \frac{s}{s^2+1} + \frac{1}{s^2+1}$$

x(t) = **<u>cos(t) + sin(t)</u>**.

If partial fraction expansion is used we obtain,

x(t) = **<u>1.4141cos(t–45°)</u>**.

This is the same answer and can be proven by using trigonometric identities.

Chapter 15, Problem 54.

Using the Laplace transform, solve the following differential equation for

$$\frac{d^2 i}{dt^2} + 4\frac{di}{dt} + 5i = 2e^{-2t}$$

Subject to $i(0) = 0, i'(0) = 2$.

Chapter 15, Solution 54.

Taking the Laplace transform of each term gives

$$\left[s^2 I(s) - si(0) - i'(0)\right] + 4\left[sI(s) - i(0)\right] + 5I(s) = \frac{2}{s+2}$$

$$\left[s^2 I(s) - 0 - 2\right] + 4\left[sI(s) - 0\right] + 5I(s) = \frac{2}{s+2}$$

$$I(s)(s^2 + 4s + 5) = \frac{2}{s+2} + 2 = \frac{2s+6}{s+2}$$

$$I(s) = \frac{2s+6}{(s+2)(s^2+4s+5)} = \frac{A}{s+2} + \frac{Bs+C}{s^2+4s+5}$$

$$2s + 6 = A(s^2+4s+5) + B(s^2+2s) + C(s+2)$$

We equate the coefficients.

s^2 : $0 = A + B$
s: $2 = 4A + 2B + C$
constant: $6 = 5A + 2C$

Solving these gives
A = 2, B= -2, C = -2

$$I(s) = \frac{2}{s+2} - \frac{2s+2}{s^2+4s+5} = \frac{2}{s+2} - \frac{2(s+2)}{(s+2)^2+1} + \frac{2}{(s+2)^2+1}$$

Taking the inverse Laplace transform leads to:

$$i(t) = \left(2e^{-2t} - 2e^{-2t}\cos t + 2e^{-2t}\sin t\right)u(t) = \underline{2e^{-2t}(1 - \cos t + \sin t)u(t)}$$

Chapter 15, Problem 55.

Solve for $y(t)$ in the following differential equation if the initial conditions are zero.

$$\frac{d^3y}{dt^3}+6\frac{d^2y}{dt^2}+8\frac{dy}{dt}e^{-t}\cos 2t$$

Chapter 15, Solution 55.

Take the Laplace transform of each term.

$$\left[s^3\,Y(s)-s^2\,y(0)-s\,y'(0)-y''(0)\right]+6\left[s^2\,Y(s)-s\,y(0)-y'(0)\right]$$
$$+8\left[s\,Y(s)-y(0)\right]=\frac{s+1}{(s+1)^2+2^2}$$

Setting the initial conditions to zero gives

$$(s^3+6s^2+8s)\,Y(s)=\frac{s+1}{s^2+2s+5}$$

$$Y(s)=\frac{(s+1)}{s(s+2)(s+4)(s^2+2s+5)}=\frac{A}{s}+\frac{B}{s+2}+\frac{C}{s+4}+\frac{Ds+E}{s^2+2s+5}$$

$$A=\frac{1}{40},\qquad B=\frac{1}{20},\qquad C=\frac{-3}{104},\qquad D=\frac{-3}{65},\qquad E=\frac{-7}{65}$$

$$Y(s)=\frac{1}{40}\cdot\frac{1}{s}+\frac{1}{20}\cdot\frac{1}{s+2}-\frac{3}{104}\cdot\frac{1}{s+4}-\frac{1}{65}\cdot\frac{3s+7}{(s+1)^2+2^2}$$

$$Y(s)=\frac{1}{40}\cdot\frac{1}{s}+\frac{1}{20}\cdot\frac{1}{s+2}-\frac{3}{104}\cdot\frac{1}{s+4}-\frac{1}{65}\cdot\frac{3(s+1)}{(s+1)^2+2^2}-\frac{1}{65}\cdot\frac{4}{(s+1)^2+2^2}$$

$$y(t)=\underline{\mathbf{\frac{1}{40}u(t)+\frac{1}{20}e^{-2t}-\frac{3}{104}e^{-4t}-\frac{3}{65}e^{-t}\cos(2t)-\frac{2}{65}e^{-t}\sin(2t)}}$$

Chapter 15, Problem 56.

Solve for $v(t)$ in the integrodifferential equation

$$4\frac{dv}{dt} + 12\int_{-\infty}^{t} v\, dt = 0$$

Given that $v(0) = 2$.

Chapter 15, Solution 56.

Taking the Laplace transform of each term we get:

$$4\left[\mathrm{s\,V(s) - v(0)}\right] + \frac{12}{\mathrm{s}}\mathrm{V(s)} = 0$$

$$\left[4\mathrm{s} + \frac{12}{\mathrm{s}}\right]\mathrm{V(s)} = 8$$

$$\mathrm{V(s)} = \frac{8\mathrm{s}}{4\mathrm{s}^2 + 12} = \frac{2\mathrm{s}}{\mathrm{s}^2 + 3}$$

$$\mathrm{v(t)} = \underline{\mathbf{2\cos\left(\sqrt{3}t\right)}}$$

Chapter 15, Problem 57.

Solve the following integrodifferential equation using the Laplace transform method:

$$\frac{dy(t)}{dt} + 9\int_0^t y(\tau)d\tau = \cos 2t, \qquad y(0) = 1$$

Chapter 15, Solution 57.

Take the Laplace transform of each term.

$$[\,s\,Y(s) - y(0)] + \frac{9}{s}Y(s) = \frac{s}{s^2+4}$$

$$\left(\frac{s^2+9}{s}\right)Y(s) = 1 + \frac{s}{s^2+4} = \frac{s^2+s+4}{s^2+4}$$

$$Y(s) = \frac{s^3+s^2+4s}{(s^2+4)(s^2+9)} = \frac{As+B}{s^2+4} + \frac{Cs+D}{s^2+9}$$

$$s^3+s^2+4s = A(s^3+9s) + B(s^2+9) + C(s^3+4s) + D(s^2+4)$$

Equating coefficients :

s^0: $0 = 9B + 4D$
s^1: $4 = 9A + 4C$
s^2: $1 = B + D$
s^3: $1 = A + C$

Solving these equations gives

$A = 0$, $B = -4/5$, $C = 1$, $D = 9/5$

$$Y(s) = \frac{-4/5}{s^2+4} + \frac{s+9/5}{s^2+9} = \frac{-4/5}{s^2+4} + \frac{s}{s^2+9} + \frac{9/5}{s^2+9}$$

$$y(t) = \underline{\mathbf{-0.4\sin(2t) + \cos(3t) + 0.6\sin(3t)}}$$

Chapter 15, Problem 58.

Given that

$$\frac{dv}{dt} + 2v + 5\int_0^t v(\lambda)d\lambda = 4u(t)$$

with $v(0) = -1$, determine $v(t)$ for $t > 0$.

Chapter 15, Solution 58.

We take the Laplace transform of each term.

$$[sV(s) - v(0)] + 2V(s) + \frac{5}{s}V(s) = \frac{4}{s}$$

$$[sV(s) + 1] + 2V(s) + \frac{5}{s}V(s) = \frac{4}{s} \qquad \longrightarrow \qquad V(s) = \frac{4 - s}{s^2 + 2s + 5}$$

$$V(s) = \frac{-(s+1) + 5}{(s+1)^2 + 2^2} = \frac{-(s+1)}{(s+1)^2 + 2^2} + \frac{5}{2}\frac{2}{(s+1)^2 + 2^2}$$

$$v(t) = \underline{(-e^{-t}\cos 2t + 2.5e^{-t}\sin 2t)u(t)}$$

Chapter 15, Problem 59.

Solve the integrodifferential equation

$$\frac{dy}{dt} + 4y + 3\int_0^t y\,dt = 6e^{-2t}, \qquad y(0) = -1$$

Chapter 15, Solution 59.

Take the Laplace transform of each term of the integrodifferential equation.

$$\left[s\,Y(s) - y(0)\right] + 4\,Y(s) + \frac{3}{s}Y(s) = \frac{6}{s+2}$$

$$(s^2 + 4s + 3)\,Y(s) = s\left(\frac{6}{s+2} - 1\right)$$

$$Y(s) = \frac{s\,(4-s)}{(s+2)(s^2+4s+3)} = \frac{(4-s)\,s}{(s+1)(s+2)(s+3)}$$

$$Y(s) = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3}$$

$A = -2.5, \qquad B = 12, \qquad C = -10.5$

$$Y(s) = \frac{-2.5}{s+1} + \frac{12}{s+2} - \frac{10.5}{s+3}$$

$y(t) =$ **$\underline{-2.5e^{-t} + 12e^{-2t} - 10.5e^{-3t}}$**

Chapter 15, Problem 60.

Solve the following integrodifferential equation

$$2\frac{dx}{dt} + 5x + 3\int_0^t x\,dt + 4 = \sin 4t, \qquad x(0) = 1$$

Chapter 15, Solution 60.

Take the Laplace transform of each term of the integrodifferential equation.

$$2\left[sX(s) - x(0)\right] + 5X(s) + \frac{3}{s}X(s) + \frac{4}{s} = \frac{4}{s^2+16}$$

$$(2s^2+5s+3)X(s) = 2s - 4 + \frac{4s}{s^2+16} = \frac{2s^3-4s^2+36s-64}{s^2+16}$$

$$X(s) = \frac{2s^3-4s^2+36s-64}{(2s^2+5s+3)(s^2+16)} = \frac{s^3-2s^2+18s-32}{(s+1)(s+1.5)(s^2+16)}$$

$$X(s) = \frac{A}{s+1} + \frac{B}{s+1.5} + \frac{Cs+D}{s^2+16}$$

$$A = (s+1)X(s)\Big|_{s=-1} = -6.235$$

$$B = (s+1.5)X(s)\Big|_{s=-1.5} = 7.329$$

When $s = 0$,

$$\frac{-32}{(1.5)(16)} = A + \frac{B}{1.5} + \frac{D}{16} \longrightarrow D = 0.2579$$

$$s^3 - 2s^2 + 18s - 32 = A(s^3 + 1.5s^2 + 16s + 24) + B(s^3 + s^2 + 16s + 16)$$
$$+ C(s^3 + 2.5s^2 + 1.5s) + D(s^2 + 2.5s + 1.5)$$

Equating coefficients of the s^3 terms,

$$1 = A + B + C \longrightarrow C = -0.0935$$

$$X(s) = \frac{-6.235}{s+1} + \frac{7.329}{s+1.5} + \frac{-0.0935s + 0.2579}{s^2+16}$$

$$x(t) = \underline{\mathbf{-6.235e^{-t} + 7.329e^{-1.5t} - 0.0935\cos(4t) + 0.0645\sin(4t)}}$$

Chapter 16, Problem 1.

Determine $i(t)$ in the circuit of Fig. 16.35 by means of the Laplace transform.

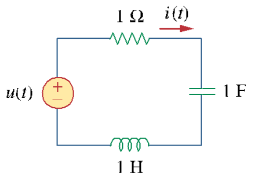
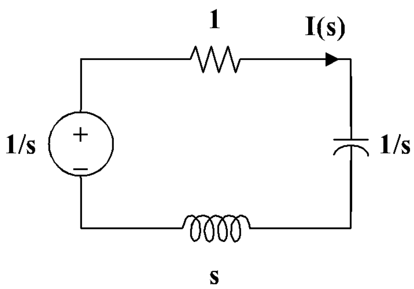


Figure 16.35
For Prob. 16.1.

Chapter 16, Solution 1.

Consider the s-domain form of the circuit which is shown below.



$$I(s) = \frac{1/s}{1+s+1/s} = \frac{1}{s^2+s+1} = \frac{1}{(s+1/2)^2+(\sqrt{3}/2)^2}$$

$$i(t) = \frac{2}{\sqrt{3}} e^{-t/2} \sin\left(\frac{\sqrt{3}}{2} t\right)$$

$$i(t) = \underline{\mathbf{1.155\, e^{-0.5t} \sin(0.866t)\ A}}$$

Chapter 16, Problem 2.

Find v_x in the circuit shown in Fig. 16.36 given v_s .= $4u(t)$V.

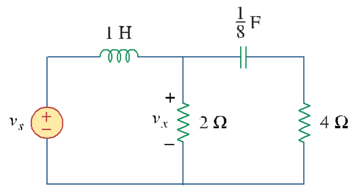
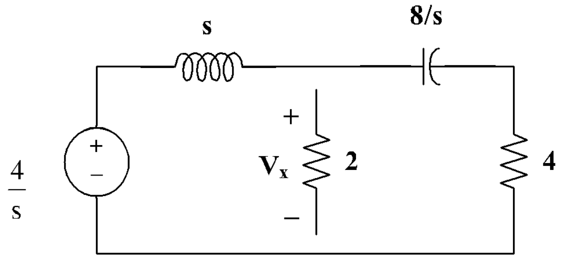


Figure 16.36
For Prob. 16.2.

Chapter 16, Solution 2.



$$\frac{V_x - \frac{4}{s}}{s} + \frac{V_x - 0}{2} + \frac{V_x - 0}{4 + \frac{8}{s}} = 0$$

$$V_x(4s + 8) - \frac{(16s + 32)}{s} + (2s^2 + 4s)V_x + s^2 V_x = 0$$

$$V_x(3s^2 + 8s + 8) = \frac{16s + 32}{s}$$

$$V_x = -16\frac{s + 2}{s(3s^2 + 8s + 8)} = -16\left(\frac{0.25}{s} + \frac{-0.125}{s + \frac{4}{3} + j\frac{\sqrt{8}}{3}} + \frac{-0.125}{s + \frac{4}{3} - j\frac{\sqrt{8}}{3}}\right)$$

$$v_x = \underline{(-4 + 2e^{-(1.3333 + j0.9428)t} + 2e^{-(1.3333 - j0.9428)t})u(t)\,V}$$

$$v_x = \underline{4u(t) - e^{-4t/3}\cos\left(\frac{2\sqrt{2}}{3}t\right) - \frac{6}{\sqrt{2}}e^{-4t/3}\sin\left(\frac{2\sqrt{2}}{3}t\right)\,V}$$

Chapter 16, Problem 3.

Find $i(t)$ for $t > 0$ for the circuit in Fig. 16.37. Assume $i_s = 4u(t) + 2\delta(t)$mA. (Hint: Can we use superposition to help solve this problem?)

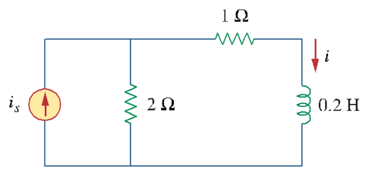
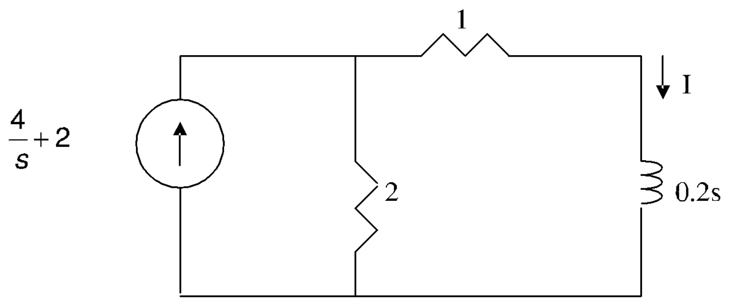


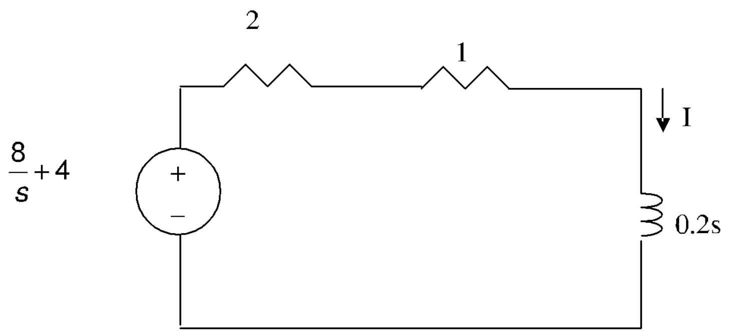
Figure 16.37
For Prob. 16.3.

Chapter 16, Solution 3.

In the s-domain, the circuit becomes that shown below.



We transform the current source to a voltage source and obtain the circuit shown below.



$$I = \frac{\frac{8}{s}+4}{3+0.2s} = \frac{20s+40}{s(s+15)} = \frac{A}{s} + \frac{B}{s+15}$$

$$A = \frac{40}{15} = \frac{8}{3}, \qquad B = \frac{-15x20+40}{-15} = \frac{52}{3}$$

$$I = \frac{8/3}{s} + \frac{52/3}{s+15}$$

$$\underline{i(t) = \left[\frac{8}{3} + \frac{52}{3}e^{-15t}\right]u(t)}$$

Chapter 16, Problem 4.

The capacitor in the circuit of Fig. 16.38 is initially uncharged. Find $v_0(t)$ for $t > 0$.

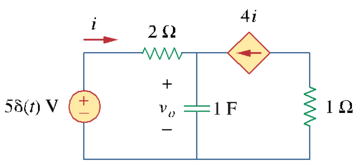
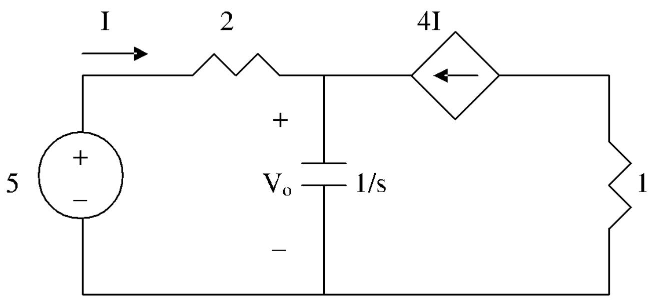


Figure 16.38
For Prob. 16.4.

Chapter 16, Solution 4.

The circuit in the s-domain is shown below.



$$I + 4I = \frac{V_o}{1/s} \quad \longrightarrow \quad 5I = sV_o$$

But $I = \dfrac{5 - V_o}{2}$

$$5\left(\frac{5 - V_o}{2}\right) = sV_o \quad \longrightarrow \quad V_o = \frac{12.5}{s + 5/2}$$

$$v_o(t) = \underline{12.5e^{-2.5t}\ \text{V}}$$

Chapter 16, Problem 5.

If $i_s(t) = e^{-2t}u(t)$ A in the circuit shown in Fig. 16.39, find the value of $i_0(t)$.

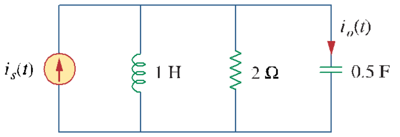
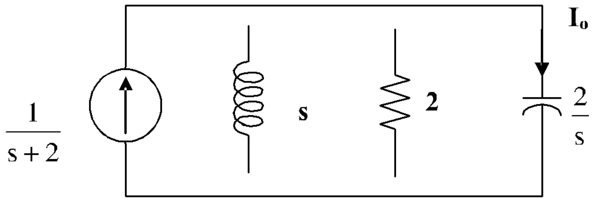


Figure 16.39
For Prob. 16.5.

Chapter 16, Solution 5.



$$V = \frac{1}{s+2}\left(\frac{1}{\frac{1}{s}+\frac{1}{2}+\frac{s}{2}}\right) = \frac{1}{s+2}\left(\frac{2s}{s^2+s+2}\right) = \frac{2s}{(s+2)(s+0.5+j1.3229)(s+0.5-j1.3229)}$$

$$I_o = \frac{Vs}{2} = \frac{s^2}{(s+2)(s+0.5+j1.3229)(s+0.5-j1.3229)}$$

$$= \frac{1}{s+2} + \frac{\frac{(-0.5-j1.3229)^2}{(1.5-j1.3229)(-j2.646)}}{s+0.5+j1.3229} + \frac{\frac{(-0.5+j1.3229)^2}{(1.5+j1.3229)(+j2.646)}}{s+0.5-j1.3229}$$

$$i_o(t) = \underline{\left(e^{-2t} + 0.3779e^{-90^\circ}e^{-t/2}e^{-j1.3229t} + 0.3779e^{90^\circ}e^{-t/2}e^{j1.3229t}\right)u(t)\ A}$$

or

$$\underline{= \left(e^{-2t} - 0.7559\sin 1.3229t\right)u(t)\ A}$$

$$\text{or } i_o(t) = \left(e^{-2t} - \frac{2}{\sqrt{7}}\sin\left(\frac{\sqrt{7}}{2}t\right)\right)u(t)A$$

Chapter 16, Problem 6.

Find $v(t)$, $t > 0$ in the circuit of Fig. 16.40. Let $v_s = 20$ V.

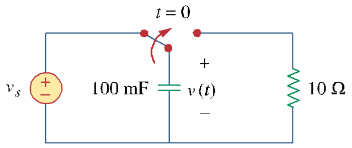
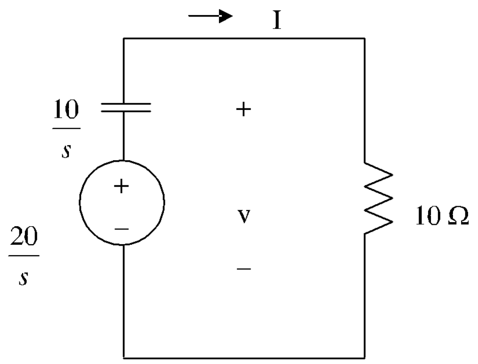


Figure 16.40
For Prob. 16.6.

Chapter 16, Solution 6.

For t<0, $v(0) = v_s = 20$ V
For t>0, the circuit in the s-domain is as shown below.



$$100mF = 0.1F \quad \longrightarrow \quad \frac{1}{sC} = \frac{10}{s}$$

$$I = \frac{20/s}{10 + 10/s} = \frac{2}{s+1}$$

$$V = 10I = \frac{20}{s+1}$$

$$v(t) = \underline{20e^{-t}u(t)}$$

Chapter 16, Problem 7.

Find $v_0(t)$, for all $t > 0$, in the circuit of Fig. 16.41.

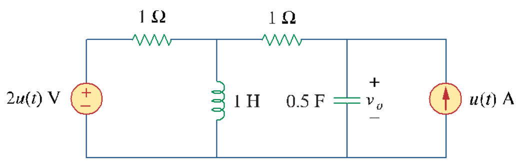
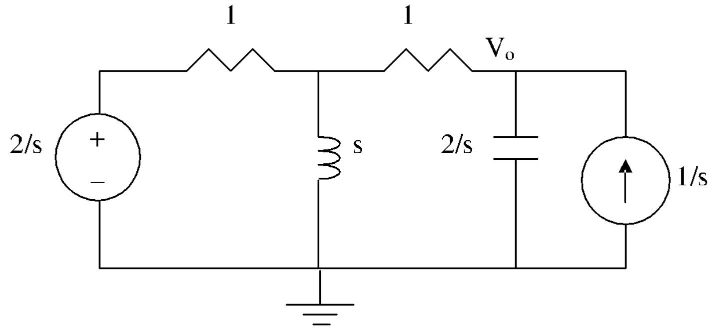


Figure 16.41
For Prob. 16.7.

Chapter 16, Solution 7.

The circuit in the s-domain is shown below. Please note, $i_L(0) = 0$ and $v_o(0) = o$ because both sources were equal to zero for all t<0.



At node 1

$$\frac{2/s - V_1}{1} = \frac{V_1}{s} + \frac{V_1 - V_o}{1} \longrightarrow \frac{2}{s} = V_1(2 + 1/s) - V_o \qquad (1)$$

At node O,

$$\frac{V_1 - V_o}{1} + \frac{1}{s} = \frac{V_o}{2/s} = \frac{s}{2}V_o \longrightarrow V_1 = (1 + s/2)V_o - 1/s \qquad (2)$$

Substituting (2) into (1) gives

$$2/s = (2 + 1/s)(1 + s/2)V_o - \frac{1}{s}(2 + \frac{1}{s}) - V_o$$

$$V_o = \frac{(4s+1)}{s(s^2 + 1.5s + 1)} = \frac{A}{s} + \frac{Bs + C}{s^2 + 1.5s + 1}$$

$$4s + 1 = A(s^2 + 1.5s + 1) + Bs^2 + Cs$$

We equate coefficients.

s^2 : $0 = A + B$ or $B = -A$

s: $4 = 1.5A + C$

constant: $1 = A$, $B = -1$, $C = 4 - 1.5A = 2.5$

$$V_o = \frac{1}{s} + \frac{-s + 2.5}{s^2 + 1.5s + 1} = \frac{1}{s} - \frac{s + 3/4}{(s+3/4)^2 + \left(\frac{\sqrt{7}}{4}\right)^2} + \frac{\frac{3.25}{\frac{\sqrt{7}}{4}} x \frac{\sqrt{7}}{4}}{(s+3/4)^2 + \left(\frac{\sqrt{7}}{4}\right)^2}$$

$$\underline{v(t) = u(t) - e^{-3t/4}\cos\frac{\sqrt{7}}{4}t + 4.9135e^{-3t/4}\sin\frac{\sqrt{7}}{4}t}$$

Chapter 16, Problem 8.

If $v_0(0) = -1$V,obtain $v_0(t)$ in the circuit of Fig. 16.42.

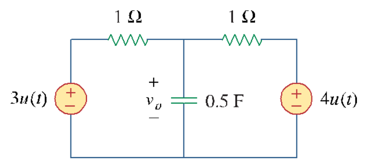
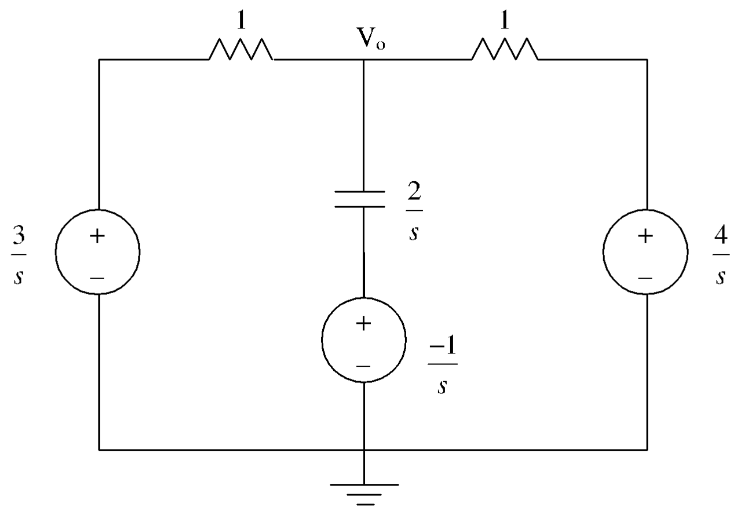


Figure 16.42
For Prob. 16.8.

Chapter 16, Solution 8.

$$\frac{1}{2}F \quad \longrightarrow \quad \frac{1}{sC} = \frac{2}{s}$$

We analyze the circuit in the s-domain as shown below. We apply nodal analysis.



$$\frac{\frac{3}{s} - V_o}{1} + \frac{-\frac{1}{s} - V_o}{\frac{2}{s}} + \frac{\frac{4}{s} - V_o}{1} = 0 \quad \longrightarrow \quad V_0 = \frac{14 - s}{s(s+4)}$$

$$V_o = \frac{A}{s} + \frac{B}{s+4}$$

$$A = \frac{14}{4} = 7/2, \qquad B = \frac{18}{-4} = -9/2$$

$$V_o = \frac{7/2}{s} - \frac{9/2}{s+4}$$

$$\underline{v_o(t) = \left(\frac{7}{2} - \frac{9}{2}e^{-4t}\right)u(t)}$$

Chapter 16, Problem 9.

Find the input impedance $Z_{in}(s)$ of each of the circuits in Fig. 16.43.

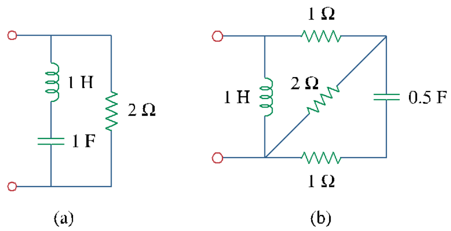
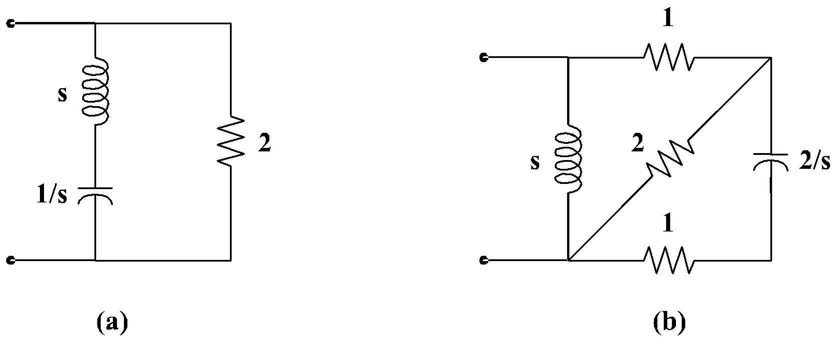


Figure 16.43
For Prob. 16.9.

Chapter 16, Solution 9.

(a) The s-domain form of the circuit is shown in Fig. (a).

$$Z_{in} = 2 \parallel (s + 1/s) = \frac{2(s+1/s)}{2+s+1/s} = \underline{\mathbf{\frac{2(s^2+1)}{s^2+2s+1}}}$$



(b) The s-domain equivalent circuit is shown in Fig. (b).

$$2 \parallel (1+2/s) = \frac{2(1+2/s)}{3+2/s} = \frac{2(s+2)}{3s+2}$$

$$1 + 2 \parallel (1+2/s) = \frac{5s+6}{3s+2}$$

$$Z_{in} = s \parallel \left(\frac{5s+6}{3s+2}\right) = \frac{s\cdot\left(\frac{5s+6}{3s+2}\right)}{s+\left(\frac{5s+6}{3s+2}\right)} = \underline{\mathbf{\frac{s(5s+6)}{3s^2+7s+6}}}$$

Chapter 16, Problem 10.

Use Thevenin's theorem to determine $v_0(t)$, $t > 0$ in the circuit of Fig. 16.44.

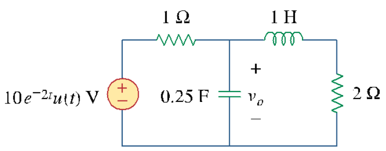


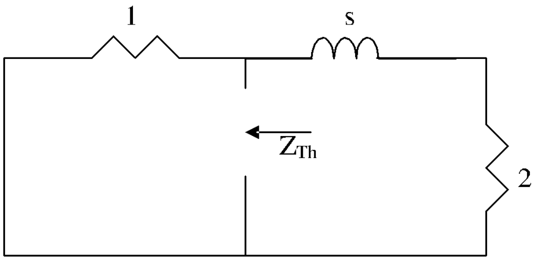
Figure 16.44
For Prob. 16.10.

Chapter 16, Solution 10.

$1H \longrightarrow 1s$ and $i_L(0) = 0$ (the sources is zero for all t<0).

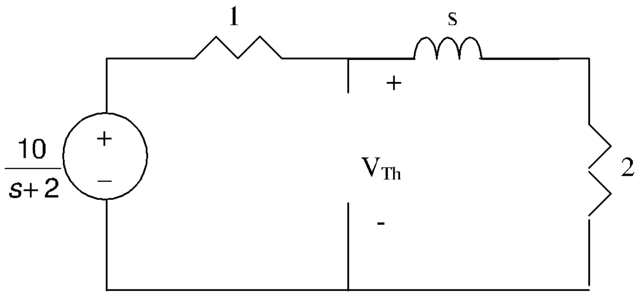
$\frac{1}{4}F \longrightarrow \frac{1}{sC} = \frac{4}{s}$ and $v_C(0) = 0$ (again, there are no source contributions for all t<0).

To find Z_{Th}, consider the circuit below.



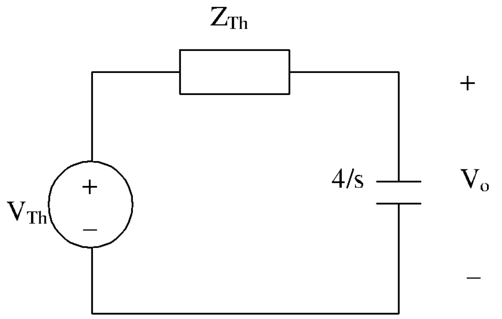
$$Z_{Th} = 1//(s+2) = \frac{s+2}{s+3}$$

To find V_{Th}, consider the circuit below.



$$V_{Th} = \frac{s+2}{s+3}\frac{10}{s+2} = \frac{10}{s+3}$$

The Thevenin equivalent circuit is shown below



$$V_o = \frac{\frac{4}{s}}{\frac{4}{s}+Z_{Th}}V_{Th} = \frac{\frac{4}{s}}{\frac{4}{s}+\frac{s+2}{s+3}}\frac{10}{s+3} = \frac{40}{s^2+6s+12} = \frac{\frac{40}{\sqrt{3}}\sqrt{3}}{(s+3)^3+(\sqrt{3})^2}.$$

$$v_o(t) = \underline{23.094e^{-3t}\sin\sqrt{3}t}$$

Chapter 16, Problem 11.

Solve for the mesh currents in the circuit of Fig. 16.45. You may leave your results in the *s*-domain.

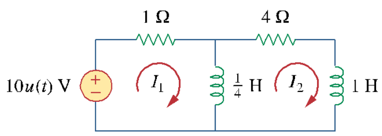
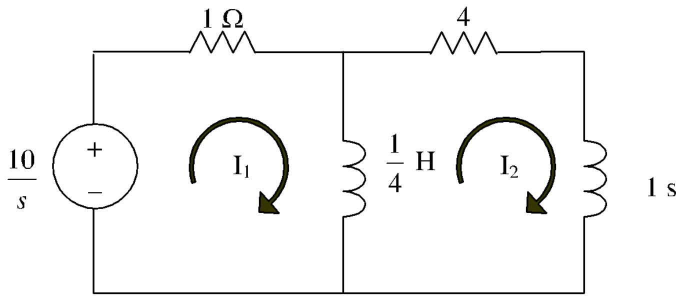


Figure 16.45
For Prob. 16.11.

Chapter 16, Solution 11.

In the s-domain, the circuit is as shown below.



$$\frac{10}{s} = (1+\frac{s}{4})I_1 - \frac{1}{4}sI_2 \qquad (1)$$

$$-\frac{1}{4}sI_1 + I_2(4+\frac{5}{4}s) = 0 \qquad (2)$$

In matrix form,

$$\begin{bmatrix} \frac{10}{s} \\ 0 \end{bmatrix} = \begin{bmatrix} 1+\frac{s}{4} & -\frac{1}{4}s \\ -\frac{1}{4}s & 4+\frac{5}{4}s \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta = \frac{1}{4}s^2 + \frac{9}{4}s + 4$$

$$\Delta_1 = \begin{vmatrix} \frac{10}{s} & -\frac{1}{4}s \\ 0 & 4+\frac{5}{4}s \end{vmatrix} = \frac{40}{s} + \frac{50}{4}$$

$$\Delta_2 = \begin{vmatrix} 1+\frac{s}{4} & \frac{10}{s} \\ -\frac{1}{4}s & 0 \end{vmatrix} = \frac{5}{2}$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{\frac{40}{s} + \frac{25}{2}}{0.25s^2 + 2.25s + 4} = \underline{\frac{50s + 160}{s(s^2 + 9s + 16)}}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{2.5}{0.25s^2 + 2.25s + 4} = \underline{\frac{10}{s^2 + 9s + 16}}$$

Chapter 16, Problem 12.

Find $v_o(t)$ in the circuit of Fig. 16.46.

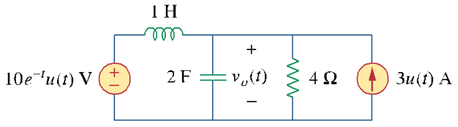
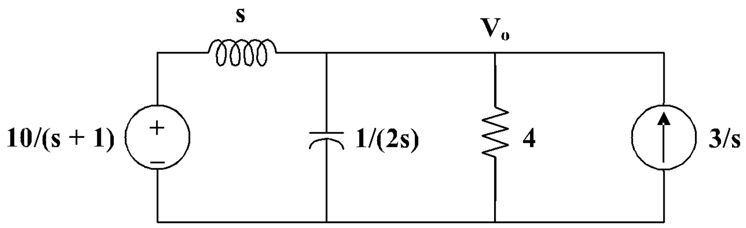


Figure 16.46
For Prob. 16.12.

Chapter 16, Solution 12.
We apply nodal analysis to the s-domain form of the circuit below.



$$\frac{\frac{10}{s+1} - V_o}{s} + \frac{3}{s} = \frac{V_o}{4} + 2sV_o$$

$$(1 + 0.25s + s^2)\,V_o = \frac{10}{s+1} + 15 = \frac{10 + 15s + 15}{s+1}$$

$$V_o = \frac{15s + 25}{(s+1)(s^2 + 0.25s + 1)} = \frac{A}{s+1} + \frac{Bs + C}{s^2 + 0.25s + 1}$$

$$A = (s+1)\,V_o\big|_{s=-1} = \frac{40}{7}$$

$$15s + 25 = A\,(s^2 + 0.25s + 1) + B\,(s^2 + s) + C\,(s+1)$$

Equating coefficients :

s^2: $\quad 0 = A + B \longrightarrow B = \text{-}A$

s^1: $\quad 15 = 0.25A + B + C = \text{-}0.75A + C$

s^0: $\quad 25 = A + C$

$A = 40/7, \quad B = \text{-}\,40/7, \quad C = 135/7$

$$V_o = \frac{\frac{40}{7}}{s+1} + \frac{\frac{-40}{7}s + \frac{135}{7}}{\left(s+\frac{1}{2}\right)^2 + \frac{3}{4}} = \frac{40}{7}\frac{1}{s+1} - \frac{40}{7}\frac{s+\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2 + \frac{3}{4}} + \left(\frac{155}{7}\cdot\frac{2}{\sqrt{3}}\right)\frac{\frac{\sqrt{3}}{2}}{\left(s+\frac{1}{2}\right)^2 + \frac{3}{4}}$$

$$v_o(t) = \frac{40}{7}e^{-t} - \frac{40}{7}e^{-t/2}\cos\!\left(\frac{\sqrt{3}}{2}t\right) + \frac{(155)(2)}{(7)(\sqrt{3})}e^{-t/2}\sin\!\left(\frac{\sqrt{3}}{2}t\right)$$

$$v_o(t) = \underline{\mathbf{5.714\,e^{-t} - 5.714\,e^{-t/2}\cos(0.866t) + 25.57\,e^{-t/2}\sin(0.866t)\ V}}$$

Chapter 16, Problem 13.

Determine $i_0(t)$ in the circuit of Fig. 16.47.

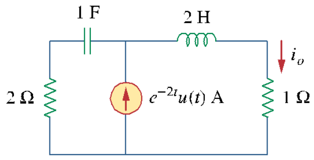
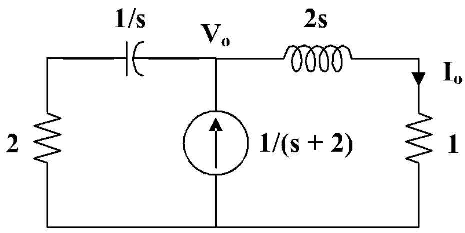


Figure 16.47
For Prob. 16.13.

Chapter 16, Solution 13.

Consider the following circuit.



Applying KCL at node o,

$$\frac{1}{s+2} = \frac{V_o}{2s+1} + \frac{V_o}{2+1/s} = \frac{s+1}{2s+1}V_o$$

$$V_o = \frac{2s+1}{(s+1)(s+2)}$$

$$I_o = \frac{V_o}{2s+1} = \frac{1}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$A = 1, \qquad B = -1$

$$I_o = \frac{1}{s+1} - \frac{1}{s+2}$$

$$i_o(t) = \underline{\left(e^{-t} - e^{-2t}\right)u(t)\ A}$$

Chapter 16, Problem 14.

* Determine $i_0(t)$ in the network shown in Fig. 16.48.

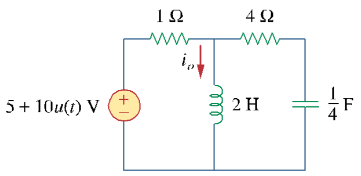
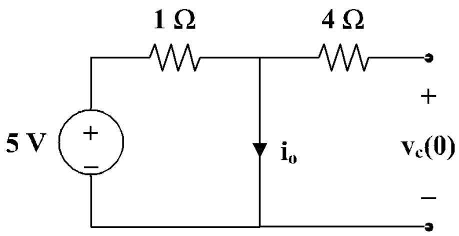


Figure 16.48
For Prob. 16.14.

* An asterisk indicates a challenging problem.

Chapter 16, Solution 14.

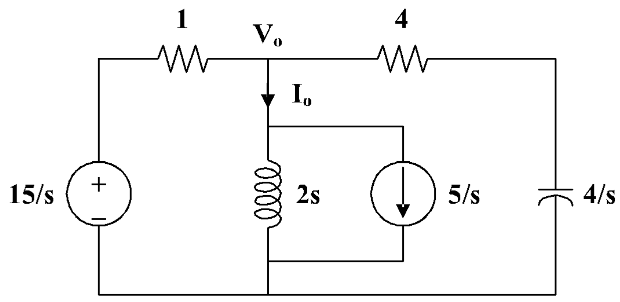
We first find the initial conditions from the circuit in Fig. (a).



(a)

$$i_o(0^-) = 5 \text{ A}, \quad v_c(0^-) = 0 \text{ V}$$

We now incorporate these conditions in the s-domain circuit as shown in Fig.(b).



(b)

At node o,

$$\frac{V_o - 15/s}{1} + \frac{V_o}{2s} + \frac{5}{s} + \frac{V_o - 0}{4 + 4/s} = 0$$

$$\frac{15}{s} - \frac{5}{s} = \left(1 + \frac{1}{2s} + \frac{s}{4(s+1)}\right) V_o$$

$$\frac{10}{s} = \frac{4s^2 + 4s + 2s + 2 + s^2}{4s(s+1)} V_o = \frac{5s^2 + 6s + 2}{4s(s+1)} V_o$$

$$V_o = \frac{40(s+1)}{5s^2 + 6s + 2}$$

$$I_o = \frac{V_o}{2s} + \frac{5}{s} = \frac{4(s+1)}{s(s^2 + 1.2s + 0.4)} + \frac{5}{s}$$

$$I_o = \frac{5}{s} + \frac{A}{s} + \frac{Bs + C}{s^2 + 1.2s + 0.4}$$

$$4(s+1) = A(s^2 + 1.2s + 0.4) + Bs^s + Cs$$

Equating coefficients :

s^0: $4 = 0.4A \longrightarrow A = 10$

s^1: $4 = 1.2A + C \longrightarrow C = -1.2A + 4 = -8$

s^2: $0 = A + B \longrightarrow B = -A = -10$

$$I_o = \frac{5}{s} + \frac{10}{s} - \frac{10s + 8}{s^2 + 1.2s + 0.4}$$

$$I_o = \frac{15}{s} - \frac{10(s+0.6)}{(s+0.6)^2 + 0.2^2} - \frac{10(0.2)}{(s+0.6)^2 + 0.2^2}$$

$$i_o(t) = \underline{\mathbf{\left[15 - 10\,e^{-0.6t}\left(\cos(0.2t) - \sin(0.2t)\right)\right]\ u(t)\ A}}$$

Chapter 16, Problem 15.

Find $V_x(s)$ in the circuit shown in Fig. 16.49.

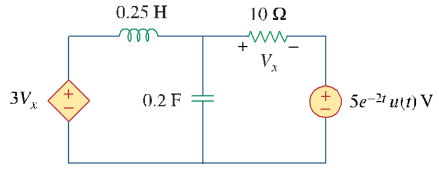
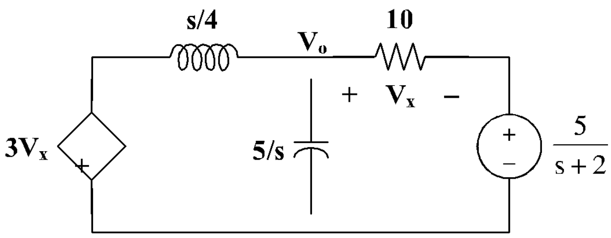


Figure 16.49
For Prob. 16.15.

Chapter 16, Solution 15.
First we need to transform the circuit into the s-domain.



$$\frac{V_o - 3V_x}{s/4} + \frac{V_o - 0}{5/s} + \frac{V_o - \frac{5}{s+2}}{10} = 0$$

$$40V_o - 120V_x + 2s^2V_o + sV_o - \frac{5s}{s+2} = 0 = (2s^2 + s + 40)V_o - 120V_x - \frac{5s}{s+2}$$

But, $V_x = V_o - \frac{5}{s+2} \rightarrow V_o = V_x + \frac{5}{s+2}$

We can now solve for V_x.

$$(2s^2 + s + 40)\left(V_x + \frac{5}{s+2}\right) - 120V_x - \frac{5s}{s+2} = 0$$

$$2(s^2 + 0.5s - 40)V_x = -10\frac{(s^2 + 20)}{s+2}$$

$$V_x = -5\frac{(s^2 + 20)}{(s+2)(s^2 + 0.5s - 40)}$$

Chapter 16, Problem 16.

* Find $i_0(t)$ for $t > 0$ in the circuit of Fig. 16.50.

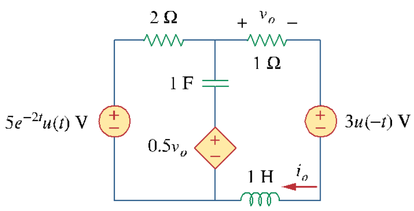
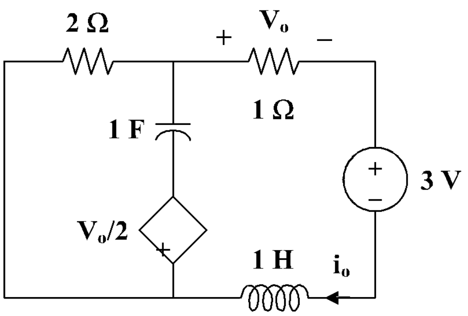


Figure 16.50
For Prob. 16.16.

* An asterisk indicates a challenging problem.

Chapter 16, Solution 16.

We first need to find the initial conditions. For $t < 0$, the circuit is shown in Fig. (a).



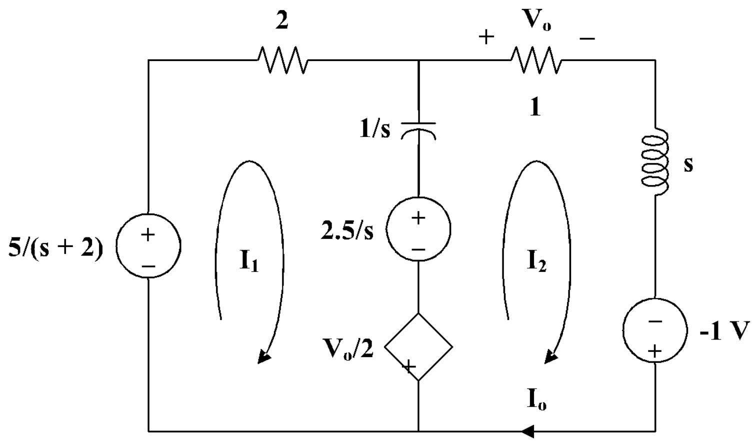
(a)

To dc, the capacitor acts like an open circuit and the inductor acts like a short circuit. Hence,

$$i_L(0) = i_o = \frac{-3}{3} = -1\text{ A}, \qquad v_o = -1\text{ V}$$

$$v_c(0) = -(2)(-1) - \left(\frac{-1}{2}\right) = 2.5\text{ V}$$

We now incorporate the initial conditions for $t > 0$ as shown in Fig. (b).



(b)

For mesh 1,

$$\frac{-5}{s+2} + \left(2 + \frac{1}{s}\right)I_1 - \frac{1}{s}I_2 + \frac{2.5}{s} + \frac{V_o}{2} = 0$$

But, $V_o = I_o = I_2$

$$\left(2+\frac{1}{s}\right)I_1 + \left(\frac{1}{2}-\frac{1}{s}\right)I_2 = \frac{5}{s+2} - \frac{2.5}{s} \qquad (1)$$

For mesh 2,

$$\left(1+s+\frac{1}{s}\right)I_2 - \frac{1}{s}I_1 + 1 - \frac{V_o}{2} - \frac{2.5}{s} = 0$$

$$-\frac{1}{s}I_1 + \left(\frac{1}{2}+s+\frac{1}{s}\right)I_2 = \frac{2.5}{s} - 1 \qquad (2)$$

Put (1) and (2) in matrix form.

$$\begin{bmatrix} 2+\frac{1}{s} & \frac{1}{2}-\frac{1}{s} \\ \frac{-1}{s} & \frac{1}{2}+s+\frac{1}{s} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{5}{s+2} - \frac{2.5}{s} \\ \frac{2.5}{s} - 1 \end{bmatrix}$$

$$\Delta = 2s + 2 + \frac{3}{s}, \qquad \Delta_2 = -2 + \frac{4}{s} + \frac{5}{s(s+2)}$$

$$I_o = I_2 = \frac{\Delta_2}{\Delta} = \frac{-2s^2 + 13}{(s+2)(2s^2+2s+3)} = \frac{A}{s+2} + \frac{Bs+C}{2s^2+2s+3}$$

$$-2s^2 + 13 = A(2s^2+2s+3) + B(s^2+2s) + C(s+2)$$

Equating coefficients :

s^2: $-2 = 2A + B$

s^1: $0 = 2A + 2B + C$

s^0: $13 = 3A + 2C$

Solving these equations leads to

$A = 0.7143$, $B = -3.429$, $C = 5.429$

$$I_o = \frac{0.7143}{s+2} - \frac{3.429s - 5.429}{2s^2+2s+3} = \frac{0.7143}{s+2} - \frac{1.7145s - 2.714}{s^2+s+1.5}$$

$$I_o = \frac{0.7143}{s+2} - \frac{1.7145(s+0.5)}{(s+0.5)^2+1.25} + \frac{(3.194)(\sqrt{1.25})}{(s+0.5)^2+1.25}$$

$$i_o(t) = \underline{\mathbf{\left[0.7143e^{-2t} - 1.7145e^{-0.5t}\cos(1.25t) + 3.194e^{-0.5t}\sin(1.25t)\right]u(t)\ A}}$$

Chapter 16, Problem 17.

Calculate $i_0(t)$ for $t > 0$ in the network of Fig. 16.51.

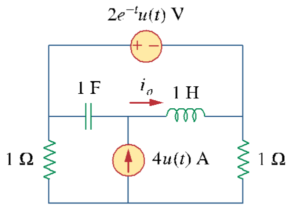
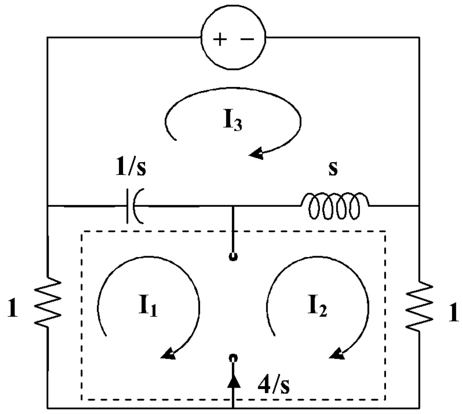


Figure 16.51
For Prob. 16.17.

Chapter 16, Solution 17.

We apply mesh analysis to the s-domain form of the circuit as shown below.



For mesh 3,

$$\frac{2}{s+1} + \left(s + \frac{1}{s}\right)I_3 - \frac{1}{s}I_1 - sI_2 = 0 \qquad (1)$$

For the supermesh,

$$\left(1 + \frac{1}{s}\right)I_1 + (1+s)I_2 - \left(\frac{1}{s} + s\right)I_3 = 0 \qquad (2)$$

Adding (1) and (2) we get, $I_1 + I_2 = -2/(s+1)$ (3)

But $-I_1 + I_2 = 4/s$ (4)

Adding (3) and (4) we get, $I_2 = (2/s) - 1/(s+1)$ (5)

Substituting (5) into (4) yields, $I_1 = -(2/s) - (1/(s+1))$ (6)

Substituting (5) and (6) into (1) we get,

$$\frac{2}{s^2} + \frac{1}{s(s+1)} - 2 + \frac{s}{s+1} + \left(\frac{s^2+1}{s}\right)I_3 = -\frac{2}{s+1}$$

$$I_3 = -\frac{2}{s} + \frac{1.5-0.5j}{s+j} + \frac{1.5+0.5j}{s-j}$$

Substituting (3) into (1) and (2) leads to

$$-\left(s+\frac{1}{s}\right)I_2 + \left(s+\frac{1}{s}\right)I_3 = \frac{2(-s^2+2s+2)}{s^2(s+1)} \quad (4)$$

$$\left(2+s+\frac{1}{s}\right)I_2 - \left(s+\frac{1}{s}\right)I_3 = -\frac{4(s+1)}{s^2} \quad (5)$$

We can now solve for I_o.

$$I_o = I_2 - I_3 = (4/s) - (1/(s+1)) + ((-1.5+0.5j)/(s+j)) + ((-1.5-0.5)/(s-j))$$

or

$$i_o(t) = \underline{\mathbf{[4 - e^{-t} + 1.5811e^{-jt+161.57^\circ} + 1.5811e^{jt-161.57^\circ}]u(t)A}}$$

This is a challenging problem. I did check it with using a Thevenin equivalent circuit and got the same exact answer.

Chapter 16, Problem 18.

(a) Find the Laplace transform of the voltage shown in Fig. 16.52(a). (b) Using that value of $v_s(t)$ in the circuit shown in Fig. 16.52(b), find the value of $v_0(t)$.

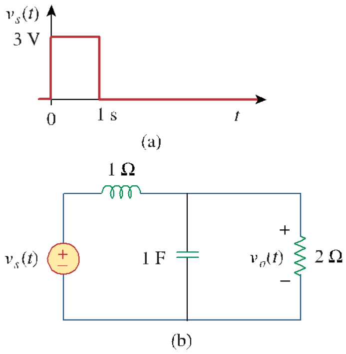
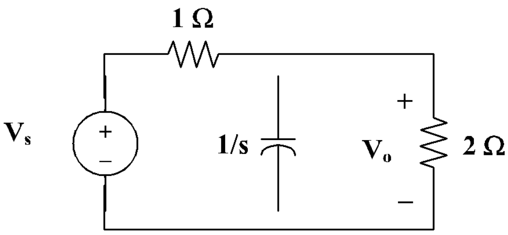


Figure 16.52
For Prob. 16.18.

Chapter 16, Solution 18.

$v_s(t) = 3u(t) – 3u(t–1)$ or $V_s = \frac{3}{s} - \frac{e^{-s}}{s} = \frac{3}{s}(1-e^{-s})$



$$\frac{V_o - V_s}{1} + sV_o + \frac{V_o}{2} = 0 \rightarrow (s+1.5)V_o = V_s$$

$$V_o = \frac{3}{s(s+1.5)}(1-e^{-s}) = \left(\frac{2}{s} - \frac{2}{s+1.5}\right)(1-e^{-s})$$

$$v_o(t) = \underline{[(2-2e^{-1.5t})u(t) - (2-2e^{-1.5(t-1)})u(t-1)]\,V}$$

Chapter 16, Problem 19.

In the circuit of Fig. 16.53, let $i(0) = 1$ A, $v_0(0)$ and $v_s = 4e^{-2t}u(t)$ V. Find $v_0(t)$ for $t > 0$.

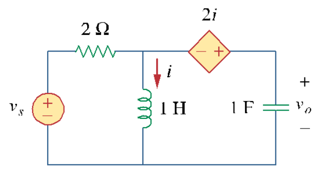
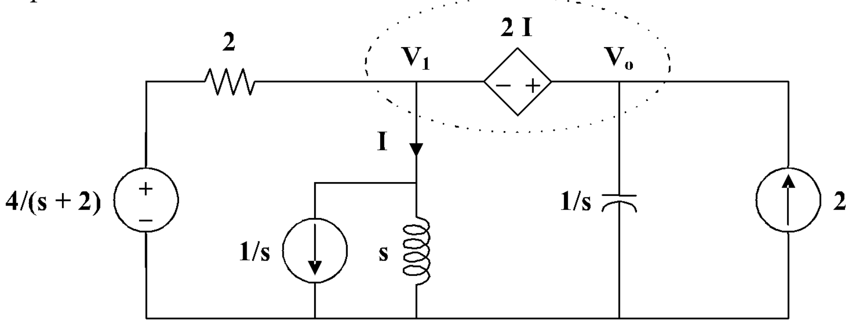


Figure 16.53
For Prob. 16.19.

Chapter 16, Solution 19.

We incorporate the initial conditions in the s-domain circuit as shown below.



At the supernode,

$$\frac{(4/(s+2)) - V_1}{2} + 2 = \frac{V_1}{s} + \frac{1}{s} + sV_o$$

$$\frac{2}{s+2} + 2 = \left(\frac{1}{2} + \frac{1}{s}\right)V_1 + \frac{1}{s} + s\,V_o \qquad (1)$$

But $V_o = V_1 + 2I$ and $I = \frac{V_1 + 1}{s}$

$$V_o = V_1 + \frac{2(V_1+1)}{s} \longrightarrow V_1 = \frac{V_o - 2/s}{(s+2)/s} = \frac{s\,V_o - 2}{s+2} \qquad (2)$$

Substituting (2) into (1)

$$\frac{2}{s+2} + 2 - \frac{1}{s} = \left(\frac{s+2}{2s}\right)\left[\left(\frac{s}{s+2}\right)V_o - \frac{2}{s+2}\right] + s\,V_o$$

$$\frac{2}{s+2} + 2 - \frac{1}{s} + \frac{1}{s} = \left[\left(\frac{1}{2}\right) + s\right]V_o$$

$$\frac{2s+4+2}{(s+2)} = \frac{2s+6}{s+2} = (s+1/2)V_o$$

$$V_o = \frac{2s+6}{(s+2)(s+1/2)} = \frac{A}{s+1/2} + \frac{B}{s+2}$$

$$A = (-1+6)/(-0.5+2) = 3.333, \quad B = (-4+6)/(-2+1/2) = -1.3333$$

$$V_o = \frac{3.333}{s+1/2} - \frac{1.3333}{s+2}$$

Therefore,

$v_o(t) =$ **$(3.333e^{-t/2} - 1.3333e^{-2t})u(t)$ V**

Chapter 16, Problem 20.

Find $v_0(t)$ in the circuit of Fig. 16.54 if $v_x(0) = 2$ V and $i(0) = 1$A.

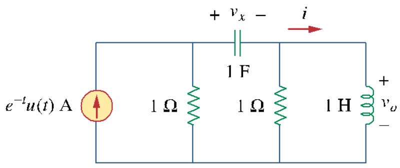
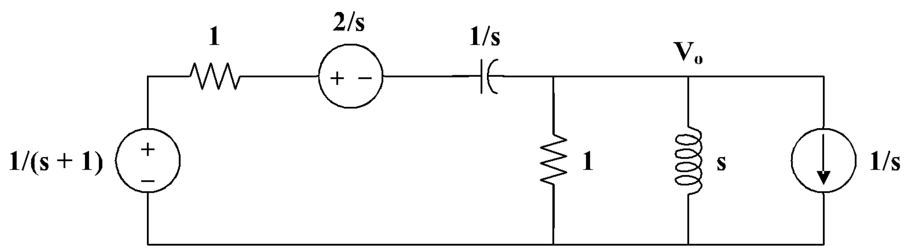


Figure 16.54
For Prob. 16.20.

Chapter 16, Solution 20.

We incorporate the initial conditions and transform the current source to a voltage source as shown.



At the main non-reference node, KCL gives

$$\frac{1/(s+1) - 2/s - V_o}{1 + 1/s} = \frac{V_o}{1} + \frac{V_o}{s} + \frac{1}{s}$$

$$\frac{s}{s+1} - 2 - s\,V_o = (s+1)(1+1/s)\,V_o + \frac{s+1}{s}$$

$$\frac{s}{s+1} - \frac{s+1}{s} - 2 = (2s + 2 + 1/s)\,V_o$$

$$V_o = \frac{-2s^2 - 4s - 1}{(s+1)(2s^2 + 2s + 1)}$$

$$V_o = \frac{-s - 2s - 0.5}{(s+1)(s^2 + s + 0.5)} = \frac{A}{s+1} + \frac{Bs + C}{s^2 + s + 0.5}$$

$$A = (s+1)\,V_o\big|_{s=-1} = 1$$

$$-s^2 - 2s - 0.5 = A\,(s^2 + s + 0.5) + B\,(s^2 + s) + C\,(s+1)$$

Equating coefficients :

$$s^2: \quad -1 = A + B \longrightarrow B = -2$$

$$s^1: \quad -2 = A + B + C \longrightarrow C = -1$$

$$s^0: \quad -0.5 = 0.5A + C = 0.5 - 1 = -0.5$$

$$V_o = \frac{1}{s+1} - \frac{2s+1}{s^2 + s + 0.5} = \frac{1}{s+1} - \frac{2\,(s+0.5)}{(s+0.5)^2 + (0.5)^2}$$

$$v_o(t) = \underline{\left[e^{-t} - 2\,e^{-t/2}\cos(t/2)\right]u(t)\ \text{V}}$$

Chapter 16, Problem 21.

Find the voltage $v_0(t)$ in the circuit of Fig. 16.55 by means of the Laplace transform.

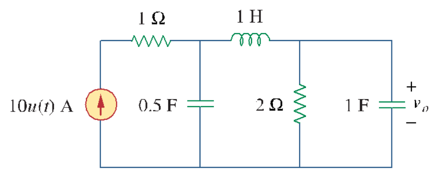
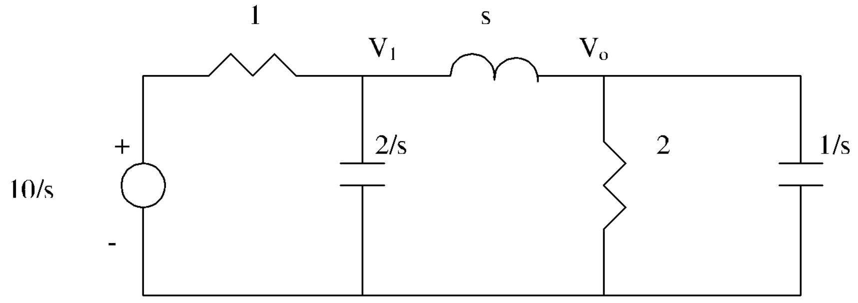


Figure 16.55
For Prob. 16.21.

Chapter 16, Solution 21.

The s-domain version of the circuit is shown below.



At node 1,

$$\frac{\frac{10}{s} - V_1}{1} = \frac{V_1 - V_o}{s} + \frac{s}{2}V_o \quad \longrightarrow \quad 10 = (s+1)V_1 + (\frac{s^2}{2} - 1)V_o \qquad (1)$$

At node 2,

$$\frac{V_1 - V_o}{s} = \frac{V_o}{2} + sV_o \quad \longrightarrow \quad V_1 = V_o(\frac{s}{2} + s^2 + 1) \qquad (2)$$

Substituting (2) into (1) gives

$$10 = (s+1)(s^2 + s/2 + 1)V_o + (\frac{s^2}{2} - 1)V_o = s(s^2 + 2s + 1.5)V_o$$

$$V_o = \frac{10}{s(s^2 + 2s + 1.5)} = \frac{A}{s} + \frac{Bs + C}{s^2 + 2s + 1.5}$$

$$10 = A(s^2 + 2s + 1.5) + Bs^2 + Cs$$

$s^2: \quad 0 = A + B$

$s: \quad 0 = 2A + C$

constant: $\quad 10 = 1.5A \quad \longrightarrow \quad A = 20/3,\ B = -20/3,\ C = -40/3$

$$V_o = \frac{20}{3}\left[\frac{1}{s} - \frac{s+2}{s^2 + 2s + 1.5}\right] = \frac{20}{3}\left[\frac{1}{s} - \frac{s+1}{(s+1)^2 + 0.7071^2} - 1.414\frac{0.7071}{(s+1)^2 + 0.7071^2}\right]$$

Taking the inverse Laplace tranform finally yields

$$\underline{v_o(t) = \frac{20}{3}\left[1 - e^{-t}\cos 0.7071t - 1.414e^{-t}\sin 0.7071t\right]u(t)\ \text{V}}$$

Chapter 16, Problem 22.

Find the node voltages v_1 and v_2 in the circuit of Fig. 16.56 using the Laplace transform technique. Assume that $i_s = 12e^{-t}u(t)$ A and that all initial conditions are zero.

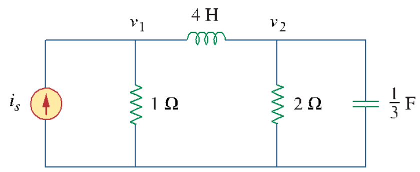
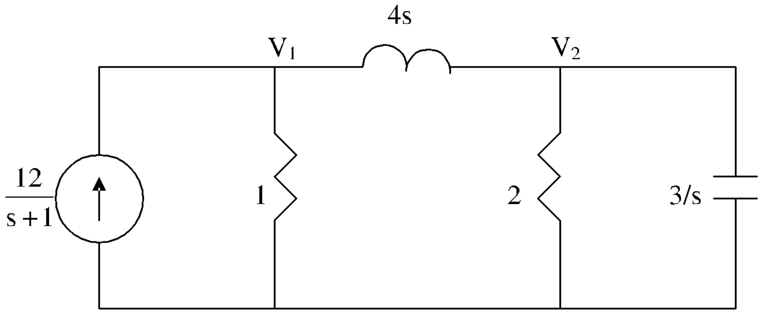


Figure 16.56
For Prob. 16.22.

Chapter 16, Solution 22.
The s-domain version of the circuit is shown below.



At node 1,

$$\frac{12}{s+1} = \frac{V_1}{1} + \frac{V_1 - V_2}{4s} \longrightarrow \frac{12}{s+1} = V_1\left(1 + \frac{1}{4s}\right) - \frac{V_2}{4s} \qquad (1)$$

At node 2,

$$\frac{V_1 - V_2}{4s} = \frac{V_2}{2} + \frac{s}{3}V_2 \longrightarrow V_1 = V_2\left(\frac{4}{3}s^2 + 2s + 1\right) \qquad (2)$$

Substituting (2) into (1),

$$\frac{12}{s+1} = V_2\left[\left(\frac{4}{3}s^2 + 2s + 1\right)\left(1 + \frac{1}{4s}\right) - \frac{1}{4s}\right] = \left(\frac{4}{3}s^2 + \frac{7}{3}s + \frac{3}{2}\right)V_2$$

$$V_2 = \frac{9}{(s+1)(s^2 + \frac{7}{4}s + \frac{9}{8})} = \frac{A}{(s+1)} + \frac{Bs + C}{(s^2 + \frac{7}{4}s + \frac{9}{8})}$$

$$9 = A(s^2 + \frac{7}{4}s + \frac{9}{8}) + B(s^2 + s) + C(s+1)$$

Equating coefficients:

$$s^2: \quad 0 = A + B$$

$$s: \quad 0 = \frac{7}{4}A + B + C = \frac{3}{4}A + C \longrightarrow C = -\frac{3}{4}A$$

$$\text{constant}: \quad 9 = \frac{9}{8}A + C = \frac{3}{8}A \longrightarrow A = 24,\ B = \text{-}24,\ C = \text{-}18$$

$$V_2 = \frac{24}{(s+1)} - \frac{24s+18}{(s^2 + \frac{7}{4}s + \frac{9}{8})} = \frac{24}{(s+1)} - \frac{24(s+7/8)}{(s+\frac{7}{8})^2 + \frac{23}{64}} + \frac{3}{(s+\frac{7}{8})^2 + \frac{23}{64}}$$

Taking the inverse of this produces:

$$\underline{v_2(t) = \left[24e^{-t} - 24e^{-0.875t}\cos(0.5995t) + 5.004e^{-0.875t}\sin(0.5995t)\right]u(t)}$$

Similarly,

$$V_1 = \frac{9\left(\frac{4}{3}s^2 + 2s + 1\right)}{(s+1)(s^2 + \frac{7}{4}s + \frac{9}{8})} = \frac{D}{(s+1)} + \frac{Es+F}{(s^2 + \frac{7}{4}s + \frac{9}{8})}$$

$$9\left(\frac{4}{3}s^2 + 2s + 1\right) = D(s^2 + \frac{7}{4}s + \frac{9}{8}) + E(s^2 + s) + F(s+1)$$

Equating coefficients:

$$s^2: \quad 12 = D + E$$

$$s: \quad 18 = \frac{7}{4}D + E + F \text{ or } 6 = \frac{3}{4}D + F \longrightarrow F = 6 - \frac{3}{4}D$$

$$\text{constant}: \quad 9 = \frac{9}{8}D + F \text{ or } 3 = \frac{3}{8}D \longrightarrow D = 8,\ E = 4,\ F = 0$$

$$V_1 = \frac{8}{(s+1)} + \frac{4s}{(s^2 + \frac{7}{4}s + \frac{9}{8})} = \frac{8}{(s+1)} + \frac{4(s+7/8)}{(s+\frac{7}{8})^2 + \frac{23}{64}} - \frac{7/2}{(s+\frac{7}{8})^2 + \frac{23}{64}}$$

Thus,

$$\underline{v_1(t) = \left[8e^{-t} + 4e^{-0.875t}\cos(0.5995t) - 5.838e^{-0.875t}\sin(0.5995t)\right]u(t)}$$

Chapter 16, Problem 23.

Consider the parallel *RLC* circuit of Fig. 16.57. Find $v(t)$ and $i(t)$ given that $v(0) = 5$ and $i(0) = -2$ A.

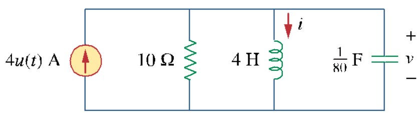
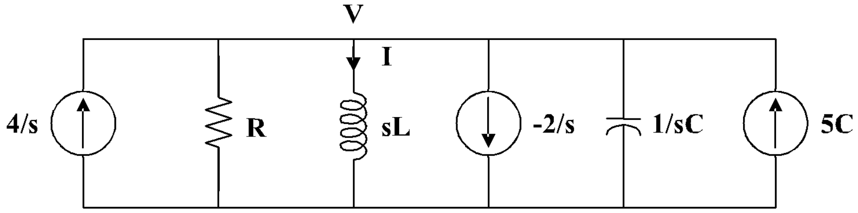


Figure 16.57
For Prob. 16.23.

Chapter 16, Solution 23.

The s-domain form of the circuit with the initial conditions is shown below.



At the non-reference node,

$$\frac{4}{s} + \frac{2}{s} + 5C = \frac{V}{R} + \frac{V}{sL} + sCV$$

$$\frac{6 + 5sC}{s} = \frac{CV}{s}\left(s^2 + \frac{s}{RC} + \frac{1}{LC}\right)$$

$$V = \frac{5s + 6/C}{s^2 + s/RC + 1/LC}$$

But $\quad \frac{1}{RC} = \frac{1}{10/80} = 8, \qquad \frac{1}{LC} = \frac{1}{4/80} = 20$

$$V = \frac{5s + 480}{s^2 + 8s + 20} = \frac{5(s+4)}{(s+4)^2 + 2^2} + \frac{(230)(2)}{(s+4)^2 + 2^2}$$

$$v(t) = \underline{(5e^{-4t}\cos(2t) + 230e^{-4t}\sin(2t))u(t)\ V}$$

$$I = \frac{V}{sL} = \frac{5s + 480}{4s(s^2 + 8s + 20)}$$

$$I = \frac{1.25s + 120}{s(s^2 + 8s + 20)} = \frac{A}{s} + \frac{Bs + C}{s^2 + 8s + 20}$$

$A = 6, \qquad B = -6, \qquad C = -46.75$

$$I = \frac{6}{s} - \frac{6s + 46.75}{s^2 + 8s + 20} = \frac{6}{s} - \frac{6(s+4)}{(s+4)^2 + 2^2} - \frac{(11.375)(2)}{(s+4)^2 + 2^2}$$

$$i(t) = \underline{(6 - 6e^{-4t}\cos(2t) - 11.375e^{-4t}\sin(2t))u(t), \quad t > 0}$$

Chapter 16, Problem 24.

The switch in Fig. 16.58 moves from position 1 to position 2 at t =0. Find $v(t)$, for all t > 0.

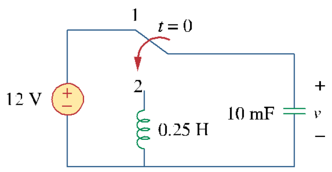
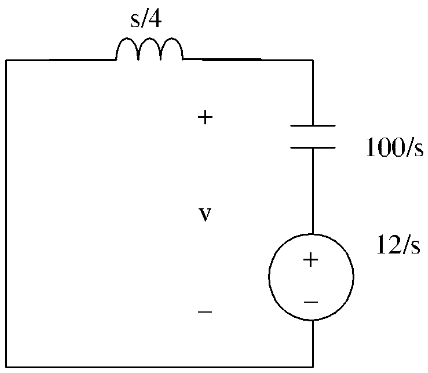


Figure 16.58
For Prob. 16.24.

Chapter 16, Solution 24.

When the switch is position 1, v(0)=12, and $i_L(0) = 0$. When the switch is in position 2, we have the circuit as shown below.



$$10mF = 0.01F \longrightarrow \frac{1}{sC} = \frac{100}{s}$$

$$I = \frac{12/s}{s/4 + 100/s} = \frac{48}{s^2 + 400}, \qquad V = sLI = \frac{s}{4}I = \frac{12s}{s^2 + 400}$$

$$v(t) = \underline{12\cos 20t, \quad t > 0}$$

Chapter 16, Problem 25.

For the *RLC* circuit shown in Fig. 16.59, find the complete response if $v(0) = 2$ V when the switch is closed.

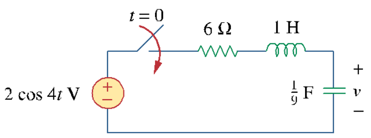
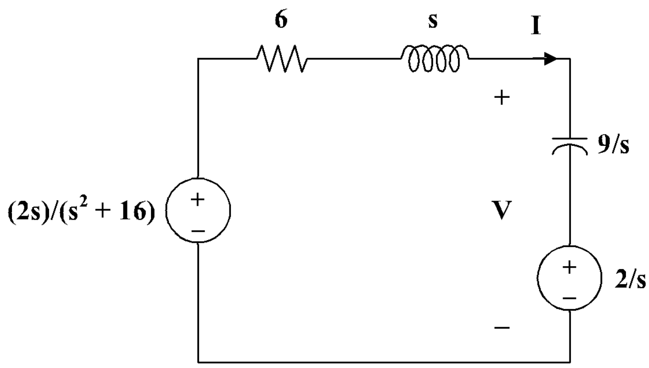


Figure 16.59
For Prob. 16.25.

Chapter 16, Solution 25.

For $t > 0$, the circuit in the s-domain is shown below.



Applying KVL,

$$\frac{-2s}{s^2+16}+\left(6+s+\frac{9}{s}\right)I+\frac{2}{s}=0$$

$$I=\frac{-32}{(s^2+6s+9)(s^2+16)}$$

$$V=\frac{9}{s}I+\frac{2}{s}=\frac{2}{s}+\frac{-288}{s(s+3)^2(s^2+16)}$$

$$=\frac{2}{s}+\frac{A}{s}+\frac{B}{s+3}+\frac{C}{(s+3)^2}+\frac{Ds+E}{s^2+16}$$

$$-288=A(s^4+6s^3+25s^2+96s+144)+B(s^4+3s^3+16s^2+48s)$$
$$+C(s^3+16s)+D(s^4+6s^3+9s^2)+E(s^3+6s^2+9s)$$

Equating coefficients :

s^0:	$-288=144A$	(1)
s^1:	$0=96A+48B+16C+9E$	(2)
s^2:	$0=25A+16B+9D+6E$	(3)
s^3:	$0=6A+3B+C+6D+E$	(4)
s^4:	$0=A+B+D$	(5)

Solving equations (1), (2), (3), (4) and (5) gives

$A=-2,\quad B=2.202,\quad C=3.84,\quad D=-0.202,\quad E=2.766$

$$V(s)=\frac{2.202}{s+3}+\frac{3.84}{(s+3)^2}-\frac{0.202s}{s^2+16}+\frac{(0.6915)(4)}{s^2+16}$$

v(t) = **{$2.202e^{-3t}+3.84te^{-3t}-0.202\cos(4t)+0.6915\sin(4t)$}u(t) V**

Chapter 16, Problem 26.

For the op amp circuit in Fig. 16.60, find $v_0(t)$ for $t > 0$. Take $v_s = 3e^{-5t}\,u(t)$ V.

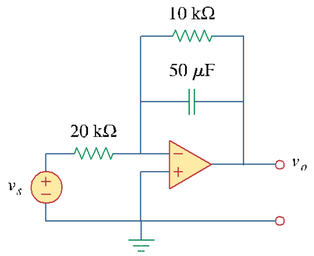
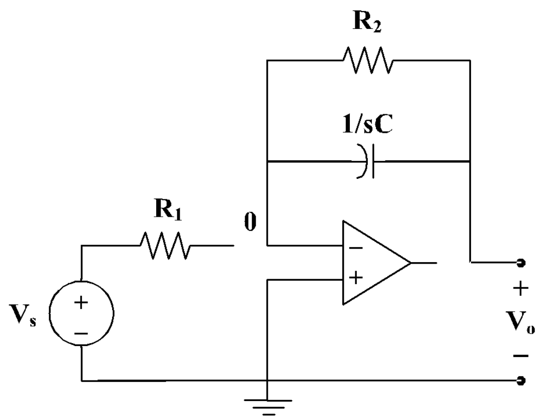


Figure 16.60
For Prob. 16.26.

Chapter 16, Solution 26.

Consider the op-amp circuit below.



At node 0,

$$\frac{V_s - 0}{R_1} = \frac{0 - V_o}{R_2} + (0 - V_o)sC$$

$$V_s = R_1\left(\frac{1}{R_2} + sC\right)(-V_o)$$

$$\frac{V_o}{V_s} = \frac{-1}{sR_1C + R_1/R_2}$$

But $\frac{R_1}{R_2} = \frac{20}{10} = 2$, $R_1C = (20\times10^3)(50\times10^{-6}) = 1$

So, $\frac{V_o}{V_s} = \frac{-1}{s+2}$

$$V_s = 3e^{-5t} \longrightarrow V_s = 3/(s+5)$$

$$V_o = \frac{-3}{(s+2)(s+5)}$$

$$-V_o = \frac{3}{(s+2)(s+5)} = \frac{A}{s+2} + \frac{B}{s+5}$$

$A = 1$, $B = -1$

$$V_o = \frac{1}{s+5} - \frac{1}{s+2}$$

$$v_o(t) = \underline{\left(e^{-5t} - e^{-2t}\right)u(t)}$$

Chapter 16, Problem 27.

Find $I_1(s)$ and $I_2(s)$ in the circuit of Fig. 16.61.

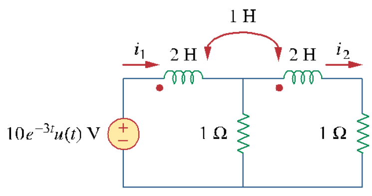
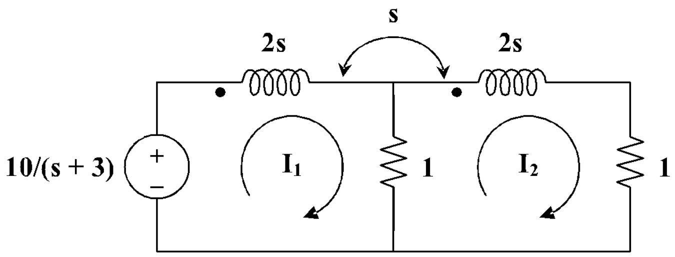


Figure 16.61
For Prob. 16.27.

Chapter 16, Solution 27.

Consider the following circuit.



For mesh 1,

$$\frac{10}{s+3} = (1+2s)\,I_1 - I_2 - s\,I_2$$

$$\frac{10}{s+3} = (1+2s)\,I_1 - (1+s)\,I_2 \qquad (1)$$

For mesh 2,

$$0 = (2+2s)\,I_2 - I_1 - s\,I_1$$

$$0 = -(1+s)\,I_1 + 2\,(s+1)\,I_2 \qquad (2)$$

(1) and (2) in matrix form,

$$\begin{bmatrix} 10/(s+3) \\ 0 \end{bmatrix} = \begin{bmatrix} 2s+1 & -(s+1) \\ -(s+1) & 2\,(s+1) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta = 3s^2 + 4s + 1$$

$$\Delta_1 = \frac{20\,(s+1)}{s+3}$$

$$\Delta_2 = \frac{10\,(s+1)}{s+3}$$

Thus

$$I_1 = \frac{\Delta_1}{\Delta} = \underline{\mathbf{\frac{20\,(s+1)}{(s+3)(3s^2+4s+1)}}}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \underline{\mathbf{\frac{10\,(s+1)}{(s+3)(3s^2+4s+1)}}} = \frac{I_1}{2}$$

Chapter 16, Problem 28.

For the circuit in Fig. 16.62, find $v_0(t)$ for $t > 0$.

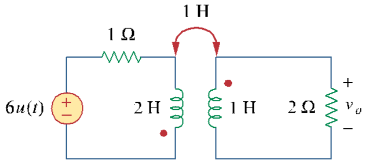
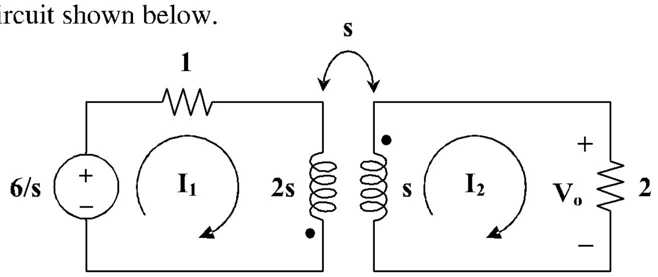


Figure 16.62
For Prob. 16.28.

Chapter 16, Solution 28.

Consider the circuit shown below.



For mesh 1,

$$\frac{6}{s} = (1 + 2s)\, I_1 + s\, I_2 \qquad (1)$$

For mesh 2,

$$0 = s\, I_1 + (2 + s)\, I_2$$

$$I_1 = -\left(1 + \frac{2}{s}\right) I_2 \qquad (2)$$

Substituting (2) into (1) gives

$$\frac{6}{s} = -(1 + 2s)\left(1 + \frac{2}{s}\right) I_2 + s\, I_2 = \frac{-(s^2 + 5s + 2)}{s} I_2$$

or $\quad I_2 = \dfrac{-6}{s^2 + 5s + 2}$

$$V_o = 2\, I_2 = \frac{-12}{s^2 + 5s + 2} = \frac{-12}{(s + 0.438)(s + 4.561)}$$

Since the roots of $s^2 + 5s + 2 = 0$ are -0.438 and -4.561,

$$V_o = \frac{A}{s + 0.438} + \frac{B}{s + 4.561}$$

$$A = \frac{-12}{4.123} = -2.91, \qquad B = \frac{-12}{-4.123} = 2.91$$

$$V_o(s) = \frac{-2.91}{s + 0.438} + \frac{2.91}{s + 4.561}$$

$$v_o(t) = \underline{\mathbf{2.91\left[e^{-4.561t} - e^{0.438t}\right] u(t)\ V}}$$

Chapter 16, Problem 29.

For the ideal transformer circuit in Fig. 16.63, determine $i_0(t)$.

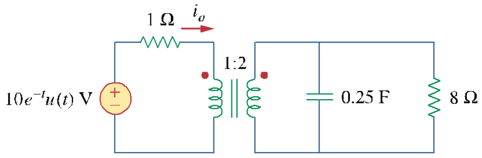
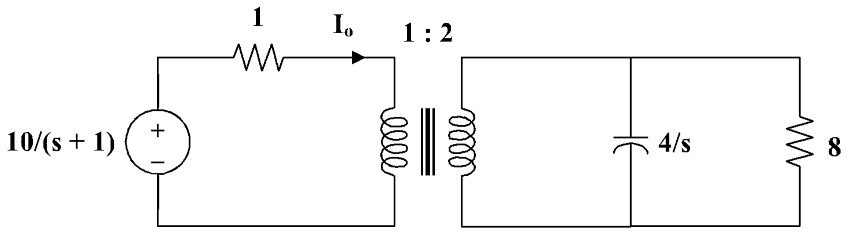


Figure 16.63
For Prob. 16.29.

Chapter 16, Solution 29.
Consider the following circuit.



Let $\quad Z_L = 8 \,||\, \frac{4}{s} = \frac{(8)(4/s)}{8 + 4/s} = \frac{8}{2s+1}$

When this is reflected to the primary side,

$$Z_{in} = 1 + \frac{Z_L}{n^2}, \quad n = 2$$

$$Z_{in} = 1 + \frac{2}{2s+1} = \frac{2s+3}{2s+1}$$

$$I_o = \frac{10}{s+1} \cdot \frac{1}{Z_{in}} = \frac{10}{s+1} \cdot \frac{2s+1}{2s+3}$$

$$I_o = \frac{10s+5}{(s+1)(s+1.5)} = \frac{A}{s+1} + \frac{B}{s+1.5}$$

$$A = -10, \qquad B = 20$$

$$I_o(s) = \frac{-10}{s+1} + \frac{20}{s+1.5}$$

$$i_o(t) = \underline{\mathbf{10\left[2\,e^{-1.5t} - e^{-t}\right]u(t)\ A}}$$

Chapter 16, Problem 30.

The transfer function of a system is

$$H(s) = \frac{s^2}{3s+1}$$

Find the output when the system has an input of $4e^{-t/3}u(t)$.

Chapter 16, Solution 30.

$$Y(s) = H(s)X(s), \qquad X(s) = \frac{4}{s+1/3} = \frac{12}{3s+1}$$

$$Y(s) = \frac{12s^2}{(3s+1)^2} = \frac{4}{3} - \frac{8s+4/3}{(3s+1)^2}$$

$$Y(s) = \frac{4}{3} - \frac{8}{9}\cdot\frac{s}{(s+1/3)^2} - \frac{4}{27}\cdot\frac{1}{(s+1/3)^2}$$

$$\text{Let } G(s) = \frac{-8}{9}\cdot\frac{s}{(s+1/3)^2}$$

Using the time differentiation property,

$$g(t) = \frac{-8}{9}\cdot\frac{d}{dt}(t\,e^{-t/3}) = \frac{-8}{9}\left(\frac{-1}{3}t\,e^{-t/3} + e^{-t/3}\right)$$

$$g(t) = \frac{8}{27}t\,e^{-t/3} - \frac{8}{9}e^{-t/3}$$

Hence,

$$y(t) = \frac{4}{3}u(t) + \frac{8}{27}t\,e^{-t/3} - \frac{8}{9}e^{-t/3} - \frac{4}{27}t\,e^{-t/3}$$

$$\underline{\mathbf{y(t) = \frac{4}{3}u(t) - \frac{8}{9}e^{-t/3} + \frac{4}{27}t\,e^{-t/3}}}$$

Chapter 16, Problem 31.

When the input to a system is a unit step function, the response is 10 cos $2tu(t)$. Obtain the transfer function of the system.

Chapter 16, Solution 31.

$$x(t) = u(t) \longrightarrow X(s) = \frac{1}{s}$$

$$y(t) = 10\cos(2t) \longrightarrow Y(s) = \frac{10s}{s^2 + 4}$$

$$H(s) = \frac{Y(s)}{X(s)} = \underline{\mathbf{\frac{10s^2}{s^2 + 4}}}$$

Chapter 16, Problem 32.

A circuit is known to have its transfer function as

$$H(s) = \frac{s+3}{s^2+4s+5}$$

Find its output when:

(a) the input is a unit step function

(b) the input is $6te^{-2t}\,u(t)$.

Chapter 16, Solution 32.

(a) $\quad Y(s) = H(s)X(s)$

$$= \frac{s+3}{s^2+4s+5}\cdot\frac{1}{s}$$

$$= \frac{s+3}{s(s^2+4s+5)} = \frac{A}{s} + \frac{Bs+C}{s^2+4s+5}$$

$$s+3 = A(s^2+4s+5) + Bs^2 + Cs$$

Equating coefficients :

s^0: $\quad 3 = 5A \longrightarrow A = 3/5$

s^1: $\quad 1 = 4A + C \longrightarrow C = 1 - 4A = -7/5$

s^2: $\quad 0 = A + B \longrightarrow B = -A = -3/5$

$$Y(s) = \frac{3/5}{s} - \frac{1}{5}\cdot\frac{3s+7}{s^2+4s+5}$$

$$Y(s) = \frac{0.6}{s} - \frac{1}{5}\cdot\frac{3(s+2)+1}{(s+2)^2+1}$$

$$y(t) = \underline{\left[\mathbf{0.6 - 0.6\,e^{-2t}\cos(t) - 0.2\,e^{-2t}\sin(t)}\right]\mathbf{u(t)}}$$

(b) $x(t) = 6te^{-2t} \longrightarrow X(s) = \frac{6}{(s+2)^2}$

$$Y(s) = H(s)X(s) = \frac{s+3}{s^2+4s+5} \cdot \frac{6}{(s+2)^2}$$

$$Y(s) = \frac{6(s+3)}{(s+2)^2(s^2+4s+5)} = \frac{A}{s+2} + \frac{B}{(s+2)^2} + \frac{Cs+D}{s^2+4s+5}$$

Equating coefficients :

s^3: $0 = A + C \longrightarrow C = -A$ (1)

s^2: $0 = 6A + B + 4C + D = 2A + B + D$ (2)

s^1: $6 = 13A + 4B + 4C + 4D = 9A + 4B + 4D$ (3)

s^0: $18 = 10A + 5B + 4D = 2A + B$ (4)

Solving (1), (2), (3), and (4) gives

$A = 6$, $B = 6$, $C = -6$, $D = -18$

$$Y(s) = \frac{6}{s+2} + \frac{6}{(s+2)^2} - \frac{6s+18}{(s+2)^2+1}$$

$$Y(s) = \frac{6}{s+2} + \frac{6}{(s+2)^2} - \frac{6(s+2)}{(s+2)^2+1} - \frac{6}{(s+2)^2+1}$$

$$y(t) = \underline{\left[\mathbf{6e^{-2t} + 6te^{-2t} - 6e^{-2t}\cos(t) - 6e^{-2t}\sin(t)}\right]\mathbf{u(t)}}$$

Chapter 16, Problem 33.

When a unit step is applied to a system at $t = 0$ its response is

$$y(t) = \left[4 + \frac{1}{2}e^{-3t} - e^{-2t}(2\cos 4t + 3\sin 4t)\right]u(t)$$

What is the transfer function of the system?

Chapter 16, Solution 33.

$$\mathrm{H(s)} = \frac{\mathrm{Y(s)}}{\mathrm{X(s)}}, \qquad \mathrm{X(s)} = \frac{1}{\mathrm{s}}$$

$$\mathrm{Y(s)} = \frac{4}{\mathrm{s}} + \frac{1}{2(\mathrm{s}+3)} - \frac{2\mathrm{s}}{(\mathrm{s}+2)^2+16} - \frac{(3)(4)}{(\mathrm{s}+2)^2+16}$$

$$\mathrm{H(s)} = \mathrm{s\,Y(s)} = \underline{4 + \frac{\mathrm{s}}{2(\mathrm{s}+3)} - \frac{2\mathrm{s}(\mathrm{s}+2)}{\mathrm{s}^2+4\mathrm{s}+20} - \frac{12\mathrm{s}}{\mathrm{s}^2+4\mathrm{s}+20}}$$

Chapter 16, Problem 34.

For the circuit in Fig. 16.64, find $H(s) = V_0(s)/V_s(s)$. Assume zero initial conditions.

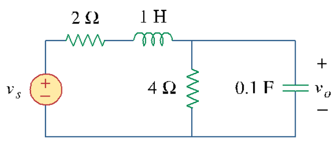
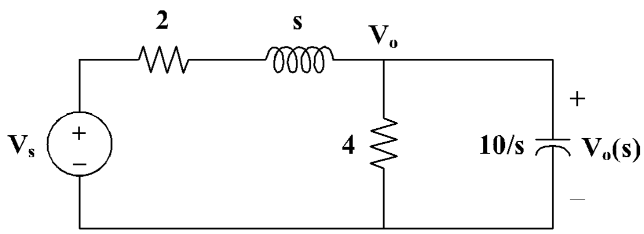


Figure 16.64
For Prob. 16.34.

Chapter 16, Solution 34.

Consider the following circuit.



Using nodal analysis,

$$\frac{V_s - V_o}{s+2} = \frac{V_o}{4} + \frac{V_o}{10/s}$$

$$V_s = (s+2)\left(\frac{1}{s+2} + \frac{1}{4} + \frac{s}{10}\right)V_o = \left(1 + \frac{1}{4}(s+2) + \frac{1}{10}(s^2 + 2s)\right)V_o$$

$$V_s = \frac{1}{20}\left(2s^2 + 9s + 30\right)V_o$$

$$\frac{V_o}{V_s} = \underline{\mathbf{\frac{20}{2s^2 + 9s + 30}}}$$

Chapter 16, Problem 35.

Obtain the transfer function $H(s) = V_0/V_s$ for the circuit of Fig. 16.65.

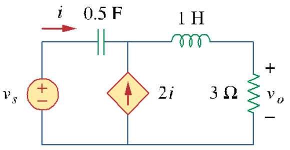
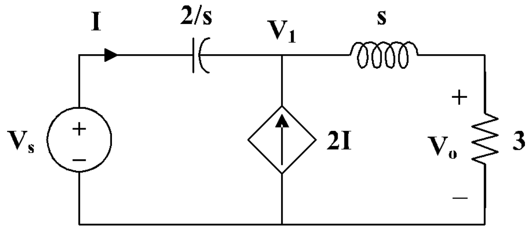


Figure 16.65
For Prob. 16.35.

Chapter 16, Solution 35.
Consider the following circuit.



At node 1,

$$2I + I = \frac{V_1}{s+3}, \qquad \text{where} \quad I = \frac{V_s - V_1}{2/s}$$

$$3 \cdot \frac{V_s - V_1}{2/s} = \frac{V_1}{s+3}$$

$$\frac{V_1}{s+3} = \frac{3s}{2} V_s - \frac{3s}{2} V_1$$

$$\left(\frac{1}{s+3} + \frac{3s}{2}\right) V_1 = \frac{3s}{2} V_s$$

$$V_1 = \frac{3s\,(s+3)}{3s^2 + 9s + 2} V_s$$

$$V_o = \frac{3}{s+3} V_1 = \frac{9s}{3s^2 + 9s + 2} V_s$$

$$H(s) = \frac{V_o}{V_s} = \underline{\mathbf{\frac{9s}{3s^2 + 9s + 2}}}$$

Chapter 16, Problem 36.

The transfer function of a certain circuit is

$$H(s) = \frac{5}{s+1} - \frac{3}{s+2} + \frac{6}{s+4}$$

Find the impulse response of the circuit.

Chapter 16, Solution 36.

Taking the inverse Laplace transform of each term gives

$$h(t) = \underline{\left(5e^{-t} - 3e^{-2t} + 6e^{-4t}\right)u(t)}$$

Chapter 16, Problem 37.

For the circuit in Fig. 16.66, find:

(a) I_1/V_s (b) I_2/V_x

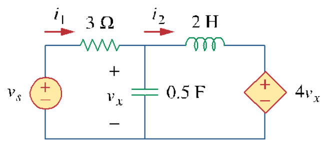
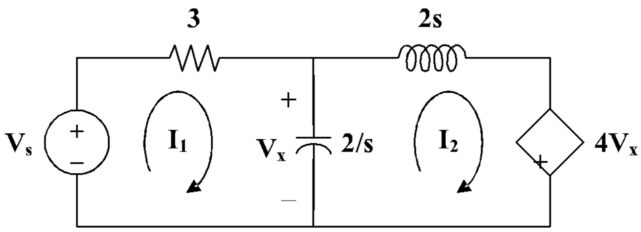


Figure 16.66
For Prob. 16.37.

Chapter 16, Solution 37.

(a) Consider the circuit shown below.



For loop 1,

$$V_s = \left(3 + \frac{2}{s}\right) I_1 - \frac{2}{s} I_2 \qquad (1)$$

For loop 2,

$$4V_x + \left(2s + \frac{2}{s}\right) I_2 - \frac{2}{s} I_1 = 0$$

But, $V_x = (I_1 - I_2)\left(\frac{2}{s}\right)$

So, $$\frac{8}{s}(I_1 - I_2) + \left(2s + \frac{2}{s}\right) I_2 - \frac{2}{s} I_1 = 0$$

$$0 = \frac{-6}{s} I_1 + \left(\frac{6}{s} - 2s\right) I_2 \qquad (2)$$

In matrix form, (1) and (2) become

$$\begin{bmatrix} V_s \\ 0 \end{bmatrix} = \begin{bmatrix} 3+2/s & -2/s \\ -6/s & 6/s-2s \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta = \left(3+\frac{2}{s}\right)\left(\frac{6}{s}-2s\right)-\left(\frac{6}{s}\right)\left(\frac{2}{s}\right)$$

$$\Delta = \frac{18}{s} - 6s - 4$$

$$\Delta_1 = \left(\frac{6}{s} - 2s\right)V_s, \qquad \Delta_2 = \frac{6}{s}V_s$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{(6/s - 2s)}{18/s - 4 - 6s}V_s$$

$$\frac{I_1}{V_s} = \frac{3/s - s}{9/s - 2 - 3} = \underline{\frac{\mathbf{s^2 - 3}}{\mathbf{3s^2 + 2s - 9}}}$$

(b) $I_2 = \dfrac{\Delta_2}{\Delta}$

$$V_x = \frac{2}{s}(I_1 - I_2) = \frac{2}{s}\left(\frac{\Delta_1 - \Delta_2}{\Delta}\right)$$

$$V_x = \frac{2/s\, V_s\,(6/s - 2s - 6/s)}{\Delta} = \frac{-4V_s}{\Delta}$$

$$\frac{I_2}{V_x} = \frac{6/s\, V_s}{-4V_s} = \underline{\frac{\mathbf{-3}}{\mathbf{2s}}}$$

Chapter 16, Problem 38.

Refer to the network in Fig. 16.67. Find the following transfer functions:

(a) $H_1(s) = V_0(s)/V_s(s)$

(b) $H_2(s) = V_0(s)/I_s(s)$

(c) $H_3(s) = I_0(s)/I_s(s)$

(d) $H_4(s) = I_0(s)/V_s(s)$

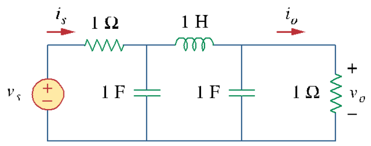
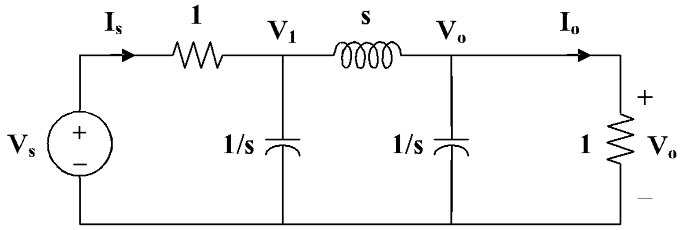


Figure 16.67
For Prob. 16.38.

Chapter 16, Solution 38.

(a) Consider the following circuit.



At node 1,

$$\frac{V_s - V_1}{1} = s\,V_1 + \frac{V_1 - V_o}{s}$$

$$V_s = \left(1 + s + \frac{1}{s}\right)V_1 - \frac{1}{s}V_o \qquad (1)$$

At node o,

$$\frac{V_1 - V_o}{s} = s\,V_o + V_o = (s+1)\,V_o$$

$$V_1 = (s^2 + s + 1)\,V_o \qquad (2)$$

Substituting (2) into (1)

$$V_s = (s + 1 + 1/s)(s^2 + s + 1)V_o - 1/s\,V_o$$

$$V_s = (s^3 + 2s^2 + 3s + 2)V_o$$

$$H_1(s) = \frac{V_o}{V_s} = \underline{\frac{\mathbf{1}}{\mathbf{s^3 + 2s^2 + 3s + 2}}}$$

(b) $I_s = V_s - V_1 = (s^3 + 2s^2 + 3s + 2)V_o - (s^2 + s + 1)V_o$

$I_s = (s^3 + s^2 + 2s + 1)V_o$

$$H_2(s) = \frac{V_o}{I_s} = \underline{\frac{\mathbf{1}}{\mathbf{s^3 + s^2 + 2s + 1}}}$$

(c) $I_o = \dfrac{V_o}{1}$

$$H_3(s) = \frac{I_o}{I_s} = \frac{V_o}{I_s} = H_2(s) = \underline{\frac{\mathbf{1}}{\mathbf{s^3 + s^2 + 2s + 1}}}$$

(d) $$H_4(s) = \frac{I_o}{V_s} = \frac{V_o}{V_s} = H_1(s) = \underline{\frac{\mathbf{1}}{\mathbf{s^3 + 2s^2 + 3s + 2}}}$$

Chapter 16, Problem 39.

Calculate the gain $H(s) = V_0/V_s$ in the op amp circuit of Fig. 16.68.

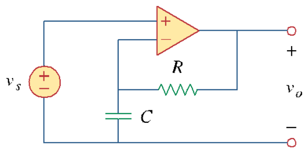
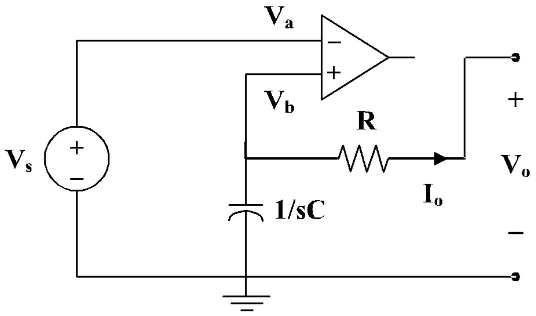


Figure 16.68
For Prob. 16.39.

Chapter 16, Solution 39.

Consider the circuit below.



Since no current enters the op amp, I_o flows through both R and C.

$$V_o = -I_o\left(R + \frac{1}{sC}\right)$$

$$V_a = V_b = V_s = \frac{-I_o}{sC}$$

$$H(s) = \frac{V_o}{V_s} = \frac{R + 1/sC}{1/sC} = \underline{\mathbf{sRC + 1}}$$

Chapter 16, Problem 40.

Refer to the *RL* circuit in Fig. 16.69. Find:

(a) the impulse response *h*(*t*) of the circuit.
(b) the unit step response of the circuit.

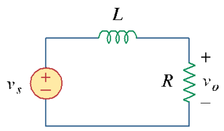


Figure 16.69
For Prob. 16.40.

Chapter 16, Solution 40.

(a) $$H(s) = \frac{V_o}{V_s} = \frac{R}{R + sL} = \frac{R/L}{s + R/L}$$

$$h(t) = \underline{\mathbf{\frac{R}{L} e^{-Rt/L}\ u(t)}}$$

(b) $$v_s(t) = u(t) \longrightarrow V_s(s) = 1/s$$

$$V_o = \frac{R/L}{s + R/L} V_s = \frac{R/L}{s(s + R/L)} = \frac{A}{s} + \frac{B}{s + R/L}$$

$$A = 1, \qquad B = -1$$

$$V_o = \frac{1}{s} - \frac{1}{s + R/L}$$

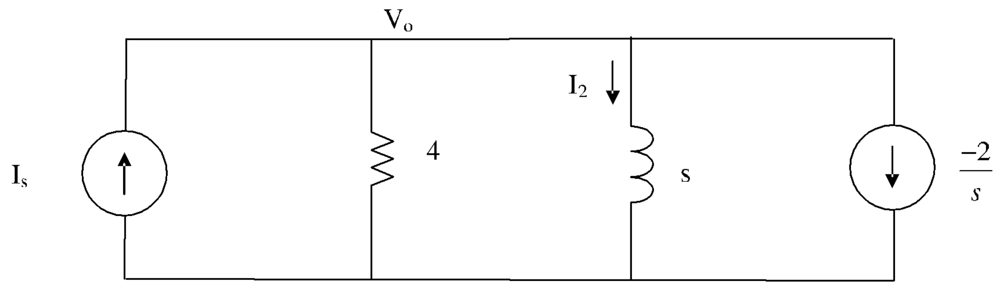
$$v_o(t) = u(t) - e^{-Rt/L}\,u(t) = \underline{\mathbf{(1 - e^{-Rt/L})\,u(t)}}$$

Chapter 16, Problem 41.

A parallel *RL* circuit has $R = 4\,\Omega$ and $L = 1$ H. The input to the circuit is $i_s(t) = 2e^{-t}$ $u(t)$A. Find the inductor current $i_L(t)$ for all $t > 0$ and assume that $i_L(0) = -2$ A.

Chapter 16, Solution 41.

Consider the circuit as shown below.



$$I_s = \frac{V_o}{4} + \frac{V_o}{s} - \frac{2}{s}$$

But $I_S = \dfrac{2}{s+1}$

$$\frac{2}{s+1} = V_o\left(\frac{1}{4} + \frac{1}{s}\right) - \frac{2}{s} \quad \longrightarrow \quad V_o\left(\frac{s+4}{4s}\right) = \frac{2}{s+1} + \frac{2}{s} = \frac{4s+2}{s(s+1)}$$

$$V_o = \frac{8(2s+1)}{(s+1)(s+4)}$$

$$I_L = \frac{V_o}{s} = \frac{8(2s+1)}{s(s+1)(s+4)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+4}$$

$$A = \frac{8(1)}{(1)(4)} = 2, \qquad B = \frac{8(-2+1)}{(-1)(2)} = 8/3, \qquad C = \frac{8(-8+1)}{(-4)(-3)} = -14/3$$

$$I_L = \frac{V_o}{s} = \frac{2}{s} + \frac{8/3}{s+1} + \frac{-14/3}{s+4}$$

$$\underline{i_L(t) = \left(2 + \frac{8}{3}e^{-t} - \frac{14}{3}e^{-4t}\right)u(t)} \; = \; \underline{\left(2 + \frac{8}{3}e^{-t} - \frac{14}{3}e^{-4t}\right)u(t)\text{A}}$$

Chapter 16, Problem 42.

A circuit has a transfer function

$$\text{H}(s) = \frac{s+4}{(s+1)(s+2)^2}$$

Find the impulse response.

Chapter 16, Solution 42.

$$H(s) = \frac{s+4}{(s+1)(s+2)^2} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{(s+2)^2}$$

$$s+4 = A(s+2)^2 + B(s+1)(s+2) + C(s+1) = A(s^2+2s+4) + B(s^2+3s+2) + C(s+1)$$

We equate coefficients.

s^2 : 0=A+B or B=-A

s: 1=4A+3B+C=B+C

constant: 4=4A+2B+C =2A+C

Solving these gives A=3, B=-3, C=-2

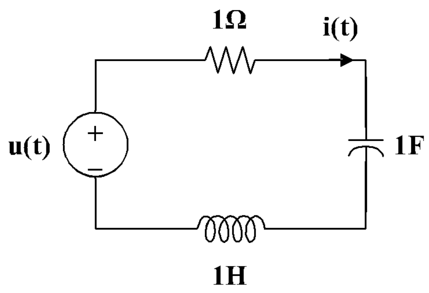
$$H(s) = \frac{3}{s+1} - \frac{3}{s+2} - \frac{2}{(s+2)^2}$$

$$h(t) = \underline{(3e^{-t} - 3e^{-2t} - 2te^{-2t})u(t)}$$

Chapter 16, Problem 43.

Develop the state equations for Prob. 16.1.

Chapter 16, Solution 43.



First select the inductor current i_L and the capacitor voltage v_C to be the state variables.

Applying KVL we get:

$$-u(t) + i + v_C + i' = 0; \; i = v_C'$$

Thus,

$$v_C' = i$$
$$i' = -v_C - i + u(t)$$

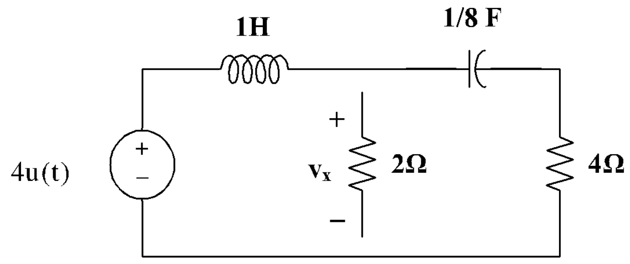
Finally we get,

$$\begin{bmatrix} v_C' \\ i' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} v_C \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t); \; i(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} v_C \\ i \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} u(t)$$

Chapter 16, Problem 44.

Develop the state equations for Prob. 16.2.

Chapter 16, Solution 44.



First select the inductor current i_L and the capacitor voltage v_C to be the state variables.

Applying KCL we get:

$$-i_L + \frac{v_x}{2} + \frac{v_C'}{8} = 0; \quad \text{or} \quad v_C' = 8i_L - 4v_x$$
$$i_L' = 4u(t) - v_x$$
$$v_x = v_C + 4\frac{v_C'}{8} = v_C + \frac{v_C'}{2} = v_C + 4i_L - 2v_x; \text{ or } v_x = 0.3333v_C + 1.3333i_L$$

$$v_C' = 8i_L - 1.3333v_C - 5.333i_L = -1.3333v_C + 2.666i_L$$
$$i_L' = 4u(t) - 0.3333v_C - 1.3333i_L$$

Now we can write the state equations.

$$\underline{\begin{bmatrix} v_C' \\ i_L' \end{bmatrix} = \begin{bmatrix} -1.3333 & 2.666 \\ -0.3333 & -1.3333 \end{bmatrix}\begin{bmatrix} v_C \\ i_L \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \end{bmatrix}u(t)}; \; v_x = \underline{\begin{bmatrix} 0.3333 & 1.3333 \end{bmatrix}\begin{bmatrix} v_C \\ i_L \end{bmatrix}}$$

Chapter 16, Problem 45.

Develop the state equations for the circuit shown in Fig. 16.70.

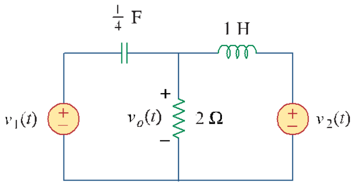
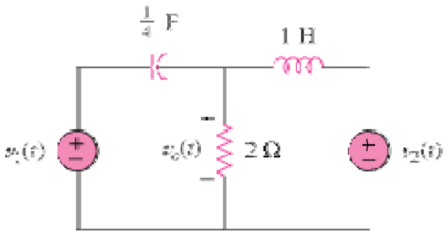


Figure 16.70
For Prob. 16.45.

Chapter 16, Solution 45.



First select the inductor current i_L (current flowing left to right) and the capacitor voltage v_C (voltage positive on the left and negative on the right) to be the state variables.

Applying KCL we get:

$$-\frac{v_C'}{4}+\frac{v_o}{2}+i_L=0 \text{ or } v_C'=4i_L+2v_o$$

$$i_L'=v_o-v_2$$

$$v_o=-v_C+v_1$$

$$v_C'=4i_L-2v_C+2v_1$$

$$i_L'=-v_C+v_1-v_2$$

$$\begin{bmatrix} i_L' \\ v_C' \end{bmatrix}=\begin{bmatrix} 0 & -1 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} i_L \\ v_C \end{bmatrix}+\begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix};\ v_o(t)=\begin{bmatrix} 0 & -1 \end{bmatrix}\begin{bmatrix} i_L \\ v_C \end{bmatrix}+\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix}$$

Chapter 16, Problem 46.

Develop the state equations for the circuit shown in Fig. 16.71.

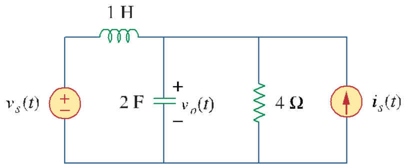
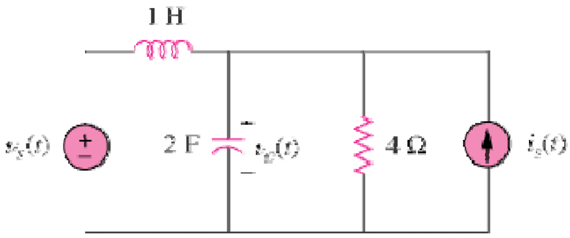


Figure 16.71
For Prob. 16.46.

Chapter 16, Solution 46.



First select the inductor current i_L (left to right) and the capacitor voltage v_C to be the state variables.

Letting $v_o = v_C$ and applying KCL we get:

$$-i_L + v_C' + \frac{v_C}{4} - i_s = 0 \text{ or } v_C' = -0.25v_C + i_L + i_s$$

$$i_L' = -v_C + v_s$$

Thus,

$$\begin{bmatrix} v_C' \\ i_L' \end{bmatrix} = \underline{\begin{bmatrix} -0.25 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} v_C' \\ i_L' \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} v_s \\ i_s \end{bmatrix}}; \; v_o(t) = \underline{\begin{bmatrix} 1 \\ 0 \end{bmatrix}\begin{bmatrix} v_C \\ i_L \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} v_s \\ i_s \end{bmatrix}}$$

Chapter 16, Problem 47.

Develop the state equations for the circuit shown in Fig. 16.72.

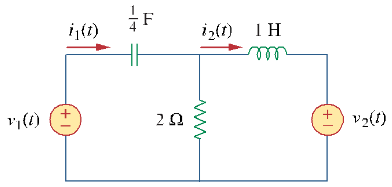
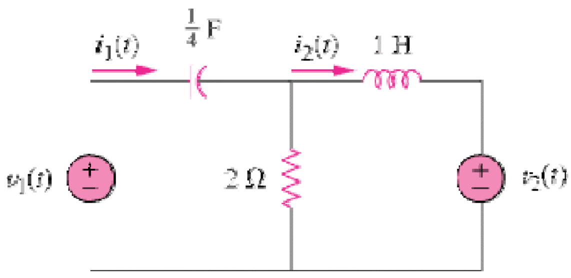


Figure 16.72
For Prob. 16.47.

Chapter 16, Solution 47.



First select the inductor current i_L (left to right) and the capacitor voltage v_C (+ on the left) to be the state variables.

Letting $i_1 = \frac{v_C'}{4}$ and $i_2 = i_L$ and applying KVL we get:

Loop 1:

$$-v_1 + v_C + 2\left(\frac{v_C'}{4} - i_L\right) = 0 \text{ or } v_C' = 4i_L - 2v_C + 2v_1$$

Loop 2:

$$2\left(i_L - \frac{v_C'}{4}\right) + i_L' + v_2 = 0 \text{ or}$$

$$i_L' = -2i_L + \frac{4i_L - 2v_C + 2v_1}{2} - v_2 = -v_C + v_1 - v_2$$

$$i_1 = \frac{4i_L - 2v_C + 2v_1}{4} = i_L - 0.5v_C + 0.5v_1$$

$$\begin{bmatrix} i_L' \\ v_C' \end{bmatrix} = \underline{\begin{bmatrix} 0 & -1 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} i_L \\ v_C \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix}}; \begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix} = \underline{\begin{bmatrix} 1 & -0.5 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} i_L \\ v_C \end{bmatrix} + \begin{bmatrix} 0.5 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix}}$$

Chapter 16, Problem 48.

Develop the state equations for the following differential equation.

$$\frac{d^2y(t)}{dt^2} + \frac{4dy(t)}{dt} + 3y(t) = z(t)$$

Chapter 16, Solution 48.

Let $x_1 = y(t)$. Thus, $x_1' = y' = x_2$ and $x_2' = y'' = -3x_1 - 4x_2 + z(t)$

This gives our state equations.

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \underline{\begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} z(t)}; \quad y(t) = \underline{\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0]z(t)}$$

Chapter 16, Problem 49.

* Develop the state equations for the following differential equation.

$$\frac{d^2y(t)}{dt^2} + \frac{5dy(t)}{dt} + 6y(t) = \frac{dz(t)}{dt}\ z(t)$$

* An asterisk indicates a challenging problem.

Chapter 16, Solution 49.

Let $x_1 = y(t)$ and $x_2 = x_1' - z = y' - z$ or $y' = x_2 + z$

Thus,

$$x_2' = y'' - z' = -6x_1 - 5(x_2 + z) + z' + 2z - z' = -6x_1 - 5x_2 - 3z$$

This now leads to our state equations,

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \underline{\begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ -3 \end{bmatrix} z(t)}; \quad y(t) = \underline{\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0]z(t)}$$

Chapter 16, Problem 50.

* Develop the state equations for the following differential equation.

$$\frac{d^3y(t)}{dt^3} + \frac{6d^2y(t)}{dt^2} + \frac{11dy(t)}{dt} + 6y(t) = z(t)$$

* An asterisk indicates a challenging problem.

Chapter 16, Solution 50.

Let $x_1 = y(t)$, $x_2 = x_1'$, and $x_3 = x_2'$.

Thus,

$$x_3'' = -6x_1 - 11x_2 - 6x_3 + z(t)$$

We can now write our state equations.

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \underline{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} z(t)}; \quad \underline{y(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + [0]z(t)}$$

Chapter 16, Problem 51.

* Given the following state equation, solve for $y(t)$:

$$\dot{\mathbf{x}} = \begin{bmatrix} -4 & 4 \\ -2 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u(t)$$

$$\mathbf{y}(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

* An asterisk indicates a challenging problem.

Chapter 16, Solution 51.

We transform the state equations into the s-domain and solve using Laplace transforms.

$$sX(s) - x(0) = AX(s) + B\left(\frac{1}{s}\right)$$

Assume the initial conditions are zero.

$$(sI - A)X(s) = B\left(\frac{1}{s}\right)$$

$$X(s) = \begin{bmatrix} s+4 & -4 \\ 2 & s \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 2 \end{bmatrix}\left(\frac{1}{s}\right) = \frac{1}{s^2+4s+8}\begin{bmatrix} s & 4 \\ 2 & s+4 \end{bmatrix}\begin{bmatrix} 0 \\ (2/s) \end{bmatrix}$$

$$Y(s) = X_1(s) = \frac{8}{s(s^2+4s+8)} = \frac{1}{s} + \frac{-s-4}{s^2+4s+8}$$

$$= \frac{1}{s} + \frac{-s-4}{(s+2)^2+2^2} = \frac{1}{s} + \frac{-(s+2)}{(s+2)^2+2^2} + \frac{-2}{(s+2)^2+2^2}$$

$$\mathbf{y(t)} = \underline{\left(1 - e^{-2t}(\cos 2t + \sin 2t)\right)u(t)}$$

Chapter 16, Problem 52.

* Given the following state equation, solve for $y_1(t)$.

$$\dot{\mathbf{x}} = \begin{bmatrix} -2 & -1 \\ 2 & -4 \end{bmatrix} x + \begin{bmatrix} 1 & 1 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} u(t) \\ 2u(t) \end{bmatrix}$$

$$\mathbf{y} = \begin{bmatrix} -2 & -2 \\ 1 & -0 \end{bmatrix} x + \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} u(t) \\ 2u(t) \end{bmatrix}$$

* An asterisk indicates a challenging problem.

Chapter 16, Solution 52.

Assume that the initial conditions are zero. Using Laplace transforms we get,

$$X(s) = \begin{bmatrix} s+2 & 1 \\ -2 & s+4 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} 1/s \\ 2/s \end{bmatrix} = \frac{1}{s^2+6s+10} \begin{bmatrix} s+4 & -1 \\ 2 & s+2 \end{bmatrix} \begin{bmatrix} 3/s \\ 4/s \end{bmatrix}$$

$$X_1 = \frac{3s+8}{s((s+3)^2+1^2)} = \frac{0.8}{s} + \frac{-0.8s-1.8}{(s+3)^2+1^2}$$

$$= \frac{0.8}{s} - 0.8\frac{s+3}{(s+3)^2+1^2} + .6\frac{1}{(s+3)^2+1^2}$$

$$x_1(t) = (0.8 - 0.8e^{-3t}\cos t + 0.6e^{-3t}\sin t)u(t)$$

$$X_2 = \frac{4s+14}{s((s+3)^2+1^2} = \frac{1.4}{s} + \frac{-1.4s-4.4}{(s+3)^2+1^2}$$

$$= \frac{1.4}{s} - 1.4\frac{s+3}{(s+3)^2+1^2} - 0.2\frac{1}{(s+3)^2+1^2}$$

$$x_2(t) = (1.4 - 1.4e^{-3t}\cos t - 0.2e^{-3t}\sin t)u(t)$$

$$y_1(t) = -2x_1(t) - 2x_2(t) + 2u(t)$$
$$= \underline{(-2.4 + 4.4e^{-3t}\cos t - 0.8e^{-3t}\sin t)u(t)}$$

$$y_2(t) = x_1(t) - 2u(t) = \underline{(-1.2 - 0.8e^{-3t}\cos t + 0.6e^{-3t}\sin t)u(t)}$$

Chapter 16, Problem 53.

Show that the parallel *RLC* circuit shown in Fig. 16.73 is stable.

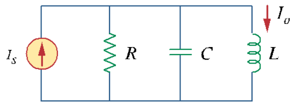


Figure 16.73
For Prob. 16.53.

Chapter 16, Solution 53.

If V_o is the voltage across R, applying KCL at the non-reference node gives

$$I_s = \frac{V_o}{R} + sC\,V_o + \frac{V_o}{sL} = \left(\frac{1}{R} + sC + \frac{1}{sL}\right)V_o$$

$$V_o = \frac{I_s}{\frac{1}{R} + sC + \frac{1}{sL}} = \frac{sRL\,I_s}{sL + R + s^2RLC}$$

$$I_o = \frac{V_o}{R} = \frac{sL\,I_s}{s^2RLC + sL + R}$$

$$H(s) = \frac{I_o}{I_s} = \frac{sL}{s^2RLC + sL + R} = \frac{s/RC}{s^2 + s/RC + 1/LC}$$

The roots

$$s_{1,2} = \frac{-1}{2RC} \pm \sqrt{\frac{1}{(2RC)^2} - \frac{1}{LC}}$$

both lie in the left half plane since R, L, and C are positive quantities.

Thus, **the circuit is stable**.

Chapter 16, Problem 54.

A system is formed by cascading two systems as shown in Fig. 16.74. Given that the impulse response of the systems are

$h_1(t) = 3e^{-t}u(t)$, $\qquad h_2(t) = e^{-4t}u(t)$

(a) Obtain the impulse response of the overall system.
(b) Check if the overall system is stable.



Figure 16.74
For Prob. 16.54.

Chapter 16, Solution 54.

(a) $\quad H_1(s) = \dfrac{3}{s+1}, \qquad H_2(s) = \dfrac{1}{s+4}$

$$H(s) = H_1(s)H_2(s) = \frac{3}{(s+1)(s+4)}$$

$$h(t) = L^{-1}[H(s)] = L^{-1}\left[\frac{A}{s+1} + \frac{B}{s+4}\right]$$

$$A = 1, \qquad B = -1$$

$$h(t) = \underline{\mathbf{(e^{-t} - e^{-4t})u(t)}}$$

(b) Since the poles of H(s) all lie in the left half s-plane, **<u>the system is stable</u>**.

Chapter 16, Problem 55.

Determine whether the op amp circuit in Fig. 16.75 is stable.

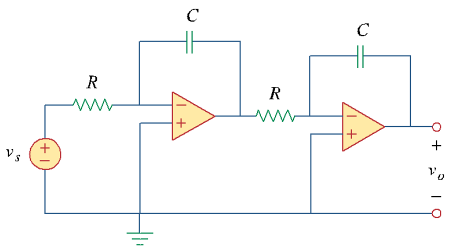


Figure 16.75
For Prob. 16.55.

Chapter 16, Solution 55.

Let V_{o1} be the voltage at the output of the first op amp.

$$\frac{V_{o1}}{V_s} = \frac{-1/sC}{R} = \frac{-1}{sRC}, \qquad \frac{V_o}{V_{o1}} = \frac{-1}{sRC}$$

$$H(s) = \frac{V_o}{V_s} = \frac{1}{s^2R^2C^2}$$

$$h(t) = \frac{t}{R^2C^2}$$

$\lim_{t\to\infty} h(t) = \infty$, i.e. the output is unbounded.

Hence, **<u>the circuit is unstable</u>**.

Chapter 16, Problem 56.

It is desired to realize the transfer function

$$\frac{V_2(s)}{V_1(s)} = \frac{2s}{s^2 + 2s + 6}$$

using the circuit in Fig. 16.76. Choose R = 1 kΩ and find L and C.

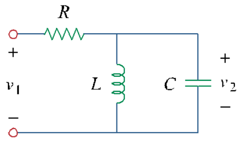


Figure 16.76
For Prob. 16.56.

Chapter 16, Solution 56.

$$sL \parallel \frac{1}{sC} = \frac{sL \cdot \frac{1}{sC}}{sL + \frac{1}{sC}} = \frac{sL}{1 + s^2LC}$$

$$\frac{V_2}{V_1} = \frac{\frac{sL}{1 + s^2LC}}{R + \frac{sL}{1 + s^2LC}} = \frac{sL}{s^2RLC + sL + R}$$

$$\frac{V_2}{V_1} = \frac{s \cdot \frac{1}{RC}}{s^2 + s \cdot \frac{1}{RC} + \frac{1}{LC}}$$

Comparing this with the given transfer function,

$$2 = \frac{1}{RC}, \qquad 6 = \frac{1}{LC}$$

If $R = 1\,k\Omega$, $C = \frac{1}{2R} = \underline{\mathbf{500\ \mu F}}$

$L = \frac{1}{6C} = \underline{\mathbf{333.3\ H}}$

Chapter 16, Problem 57.

Design an op amp circuit, using Fig. 16.77, that will realize the following transfer function:

$$\frac{V_0(s)}{V_i(s)} = -\frac{s+1{,}000}{2(s+4{,}000)}$$

Choose C_1=10 μF; determine R_1, R_2, and C_2

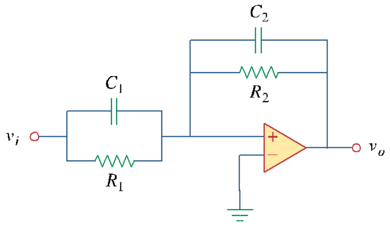
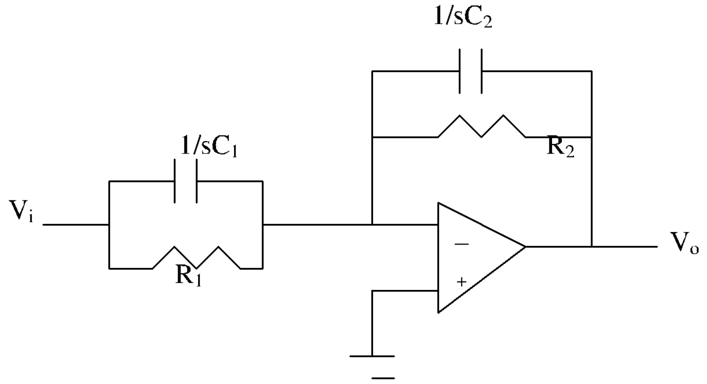


Figure 16.77
For Prob. 16.57.

Chapter 16, Solution 57.

The circuit is transformed in the s-domain as shown below.



Let $Z_1 = R_1 // \frac{1}{sC_1} = \frac{R_1 \frac{1}{sC_1}}{R_1 + \frac{1}{sC_1}} = \frac{R_1}{1 + sR_1C_1}$

$$Z_2 = R_2 // \frac{1}{sC_2} = \frac{R_2 \frac{1}{sC_2}}{R_2 + \frac{1}{sC_2}} = \frac{R_2}{1 + sR_2C_2}$$

This is an inverting amplifier.

$$H(s) = V_o / V_i = -\frac{Z_2}{Z_1} = \frac{-\frac{R_2}{1+sR_2C_2}}{\frac{R_1}{1+sR_1C_1}} = -\frac{R_2}{R_1}\frac{R_1C_1}{R_2C_2}\left[\frac{s+\frac{1}{R_1C_1}}{s+\frac{1}{R_2C_2}}\right] = \frac{-C_1}{C_2}\left[\frac{s+\frac{1}{R_1C_1}}{s+\frac{1}{R_2C_2}}\right]$$

Comparing this with

$$H(s) = -\frac{(s+1000)}{2(s+4000)}$$

we obtain:

$$\frac{C_1}{C_2} = 1/2 \quad \longrightarrow \quad C_2 = 2C_1 = \underline{20\mu F}$$

$$\frac{1}{R_1C_1} = 1000 \quad \longrightarrow \quad R_1 = \frac{1}{1000C_1} = \frac{1}{10^3 x 10 x 10^{-6}} = \underline{100\Omega}$$

$$\frac{1}{R_2C_2} = 4000 \quad \longrightarrow \quad R_2 = \frac{1}{4000C_2} = \frac{1}{4x10^3 x 20 x 10^{-6}} = \underline{\mathbf{12.5\Omega}}$$

Chapter 16, Problem 58.

Realize the transfer function

$$\frac{V_0(s)}{V_s(s)} = \frac{s}{s+10}$$

using the circuit in Fig. 16.78. Let $Y_1 = sC_1$, $Y_2 = 1/R_1$, $Y_3 = sC_2$. Choose $R_1 = 1\text{k}\Omega$ and determine C_1 and C_2.

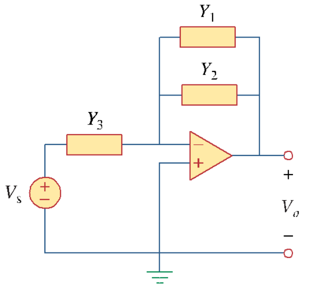


Figure 16.78
For Prob. 16.58.

Chapter 16, Solution 58.

We apply KCL at the noninverting terminal at the op amp.

$$(V_s - 0)\,Y_3 = (0 - V_o)(Y_1 - Y_2)$$

$$Y_3\, V_s = -(Y_1 + Y_2)V_o$$

$$\frac{V_o}{V_s} = \frac{-Y_3}{Y_1 + Y_2}$$

Let $\quad Y_1 = sC_1, \qquad Y_2 = 1/R_1, \qquad Y_3 = sC_2$

$$\frac{V_o}{V_s} = \frac{-sC_2}{sC_1 + 1/R_1} = \frac{-sC_2/C_1}{s + 1/R_1C_1}$$

Comparing this with the given transfer function,

$$\frac{C_2}{C_1} = 1, \qquad \frac{1}{R_1C_1} = 10$$

If $R_1 = 1\,\text{k}\Omega$,

$$C_1 = C_2 = \frac{1}{10^4} = \underline{\mathbf{100\ \mu F}}$$

Chapter 16, Problem 59.

Synthesize the transfer function

$$\frac{V_0(s)}{V_{in}(s)} = \frac{10^6}{s^2 + 100s + 10^6}$$

using the topology of Fig. 16.79. Let $Y_1 = 1/R_1$, $Y_2 = 1/R_2$, $Y_3 = sC_1$, $Y_4 \; sC_2$. Choose $R_1 = 1\text{k}\Omega$ and determine C_1, C_2, and R_2.

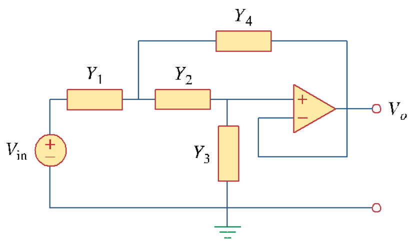
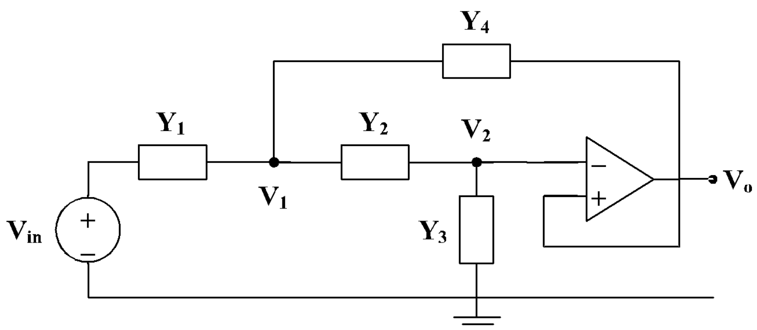


Figure 16.79
For Prob. 16.59.

Chapter 16, Solution 59.

Consider the circuit shown below. We notice that $V_3 = V_o$ and $V_2 = V_3 = V_o$.



At node 1,

$$(V_{in} - V_1)Y_1 = (V_1 - V_o)Y_2 + (V_1 - V_o)Y_4$$

$$V_{in}Y_1 = V_1(Y_1 + Y_2 + Y_4) - V_o(Y_2 + Y_4) \qquad (1)$$

At node 2,

$$(V_1 - V_o)Y_2 = (V_o - 0)Y_3$$

$$V_1 Y_2 = (Y_2 + Y_3)V_o$$

$$V_1 = \frac{Y_2 + Y_3}{Y_2} V_o \qquad (2)$$

Substituting (2) into (1),

$$V_{in}Y_1 = \frac{Y_2 + Y_3}{Y_2} \cdot (Y_1 + Y_2 + Y_4)V_o - V_o(Y_2 + Y_4)$$

$$V_{in}Y_1Y_2 = V_o(Y_1Y_2 + Y_2^2 + Y_2Y_4 + Y_1Y_3 + Y_2Y_3 + Y_3Y_4 - Y_2^2 - Y_2Y_4)$$

$$\frac{V_o}{V_{in}} = \frac{Y_1Y_2}{Y_1Y_2 + Y_1Y_3 + Y_2Y_3 + Y_3Y_4}$$

Y_1 and Y_2 must be resistive, while Y_3 and Y_4 must be capacitive.

Let $\quad Y_1 = \dfrac{1}{R_1}, \qquad Y_2 = \dfrac{1}{R_2}, \qquad Y_3 = sC_1, \qquad Y_4 = sC_2$

$$\frac{V_o}{V_{in}} = \frac{\dfrac{1}{R_1R_2}}{\dfrac{1}{R_1R_2} + \dfrac{sC_1}{R_1} + \dfrac{sC_1}{R_2} + s^2C_1C_2}$$

$$\frac{V_o}{V_{in}} = \frac{\dfrac{1}{R_1R_2C_1C_2}}{s^2 + s \cdot \left(\dfrac{R_1 + R_2}{R_1R_2C_2}\right) + \dfrac{1}{R_1R_2C_1C_2}}$$

Choose $R_1 = 1\ \text{k}\Omega$, then

$$\frac{1}{R_1R_2C_1C_2} = 10^6 \qquad \text{and} \qquad \frac{R_1 + R_2}{R_1R_2C_2} = 100$$

We have three equations and four unknowns. Thus, there is a family of solutions. One such solution is

$$R_2 = \underline{\mathbf{1\ k\Omega}}, \quad C_1 = \underline{\mathbf{50\ nF}}, \quad C_2 = \underline{\mathbf{20\ \mu F}}$$

Chapter 16, Problem 60.

Obtain the transfer function of the op amp circuit in Fig. 16.80 in the form of

$$\frac{V_0(s)}{V_i(s)} = \frac{as}{s^2 + bs + c}$$

where a, b, and c are constants. Determine the constants.

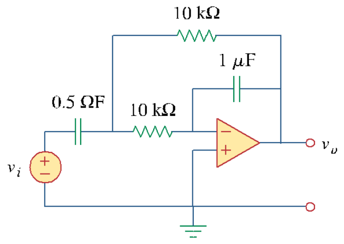
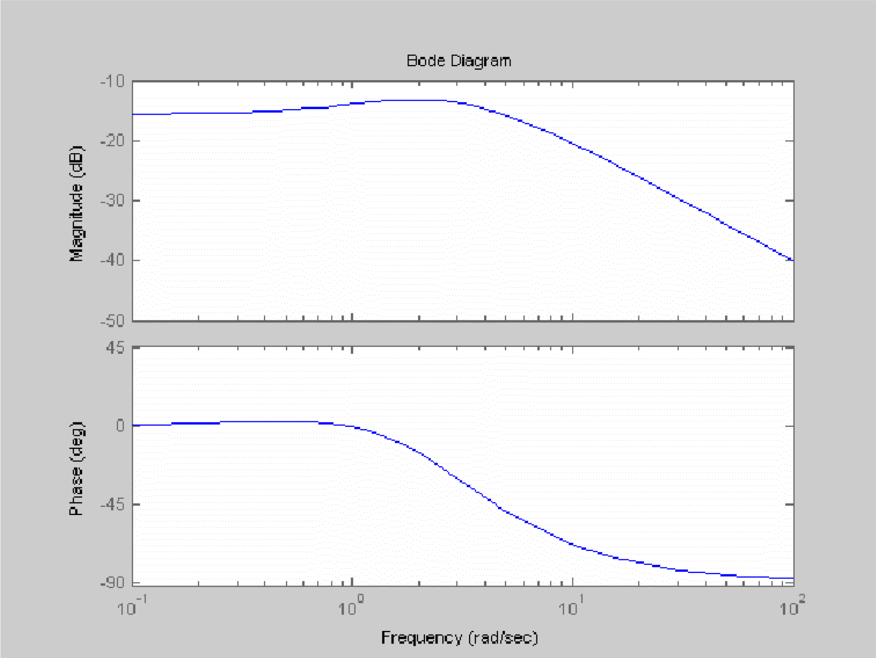


Figure 16.80
For Prob. 16.67.

Chapter 16, Solution 60.

With the following MATLAB codes, the Bode plots are generated as shown below.

```
num=[1 1];
den= [1 5 6];
bode(num,den);
```



Chapter 16, Problem 61.

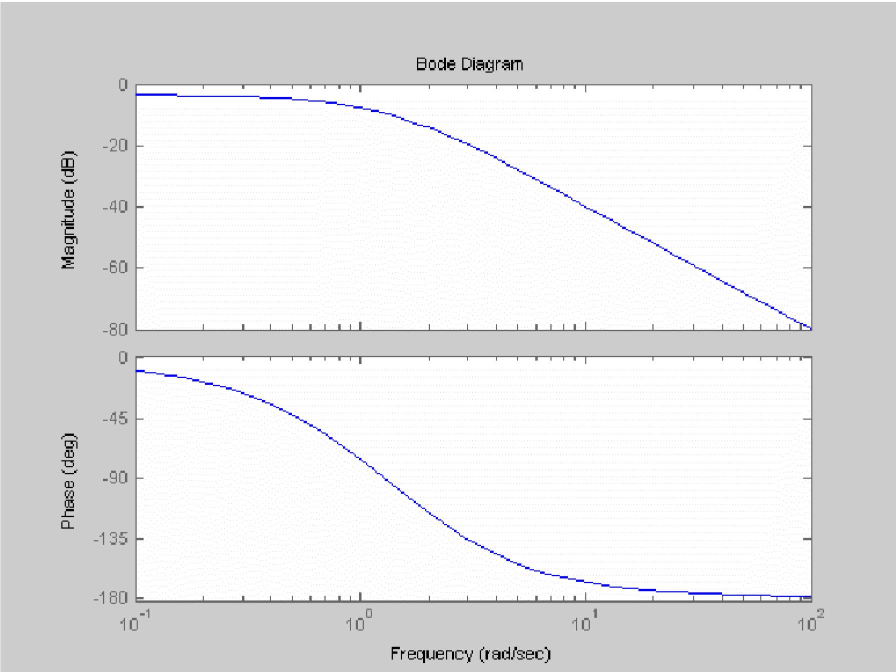
A certain network has an input admittance $Y(s)$. The admittance has a pole at $s = -3$, a zero at $s = -1$, and $Y(\infty) = 0.25$ S.

(a) Find $Y(s)$.
(b) An 8-V battery is connected to the network via a switch. If the switch is closed at $t = 0$, find the current $i(t)$ through $Y(s)$ using the Laplace transform.

Chapter 16, Solution 61.

We use the following codes to obtain the Bode plots below.

num=[1 4];
den= [1 6 11 6];
bode(num,den);



Chapter 16, Problem 62.

A gyrator is a device for simulating an inductor in a network. A basic gyrator circuit is shown in Fig. 16.81. By finding $V_i(s)/I_0(s)$, show that the inductance produced by the gyrator is $L = CR^2$.

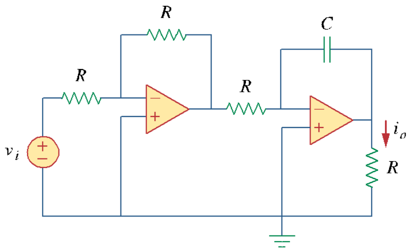
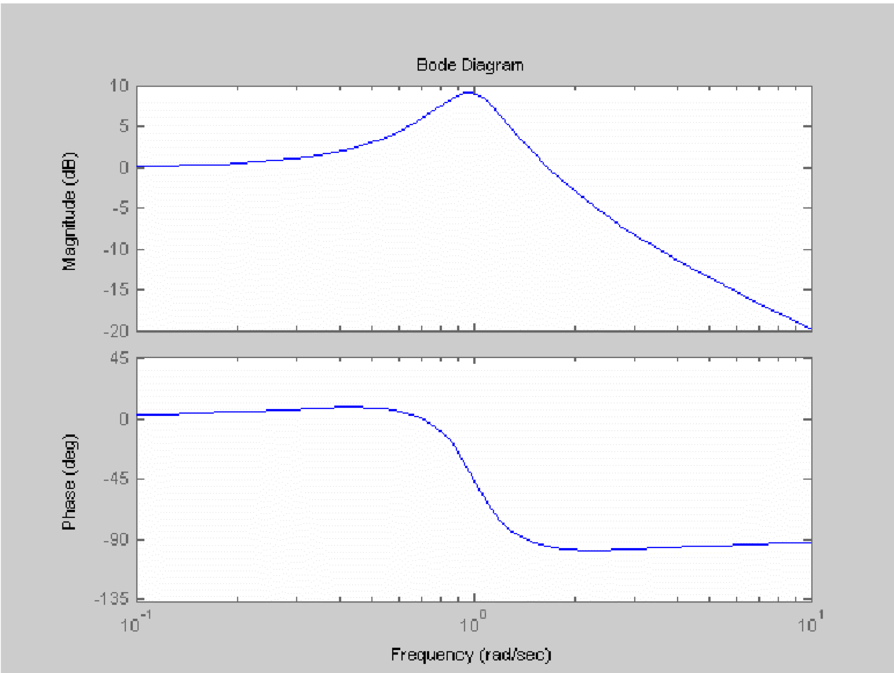


Figure 16.81
For Prob. 16.69.

Chapter 16, Solution 62.
The following codes are used to obtain the Bode plots below.

```
num=[1 1];
den= [1 0.5  1];
bode(num,den);
```



Chapter 17, Problem 1.

Evaluate each of the following functions and see if it is periodic. If periodic, find its period.

(a) $f(t) = \cos \pi t + 2\cos 3\pi t + 3\cos 5\pi t$

(b) $y(t) = \sin t + 4\cos 2\pi t$

(c) $g(t) = \sin 3t \cos 4t$

(d) $h(t) = \cos^2 t$

(e) $z(t) = 4.2\sin(0.4\pi t + 10^\circ)$
$+ 0.8\sin(0.6\pi t + 50^\circ)$

(f) $p(t) = 10$

(g) $q(t) = e^{-\pi t}$

Chapter 17, Solution 1.

(a) This is **periodic** with $\omega = \pi$ which leads to $T = 2\pi/\omega =$ **2**.

(b) y(t) is **not periodic** although sin t and 4 cos 2πt are independently periodic.

(c) Since $\sin A \cos B = 0.5[\sin(A + B) + \sin(A - B)]$,

$$g(t) = \sin 3t \cos 4t = 0.5[\sin 7t + \sin(-t)] = -0.5 \sin t + 0.5 \sin 7t$$

which is harmonic or **periodic** with the fundamental frequency

$$\omega = 1 \text{ or } T = 2\pi/\omega = \mathbf{2\pi}.$$

(d) $h(t) = \cos^2 t = 0.5(1 + \cos 2t)$. Since the sum of a periodic function and a constant is also **periodic**, h(t) is periodic. $\omega = 2$ or $T = 2\pi/\omega = \pi$.

(e) The frequency ratio $0.6|0.4 = 1.5$ makes z(t) **periodic**.

$$\omega = 0.2\pi \text{ or } T = 2\pi/\omega = \mathbf{10}.$$

(f) p(t) = 10 is **not periodic**.

(g) g(t) is **not periodic**.

Chapter 17, Problem 2.

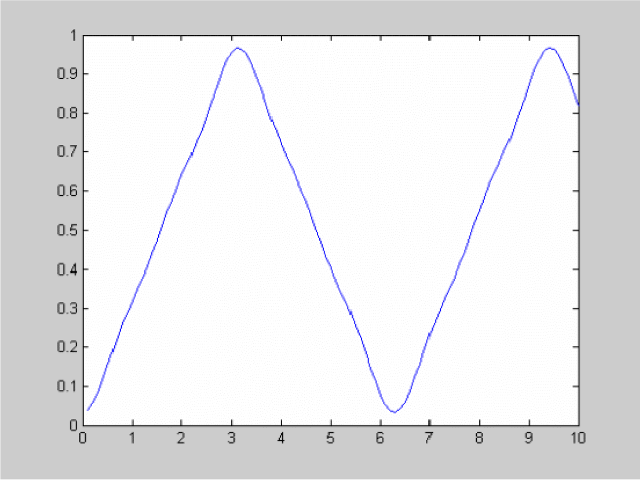
ML Using MATLAB, synthesize the periodic waveform for which the Fourier trigonometric Fourier series is

$$f(t) = \frac{1}{2} - \frac{4}{\pi^2}\left(\cos t + \frac{1}{9}\cos 3t + \frac{1}{25}\cos 5t + \cdots\right)$$

Chapter 17, Solution 2.

The function f(t) has a DC offset and is even. We use the following MATLAB code to plot f(t). The plot is shown below. If more terms are taken, the curve is clearly indicating a triangular wave shape which is easily represented with just the DC component and three, cosinusoidal terms of the expansion.

```
for n=1:100
   tn(n)=n/10;
   t=n/10;
   y1=cos(t);
   y2=(1/9)*cos(3*t);
   y3=(1/25)*cos(5*t);
   factor=4/(pi*pi);
   y(n)=0.5- factor*(y1+y2+y3);
end
plot(tn,y)
```



Chapter 17, Problem 3.

Give the Fourier coefficients a_0, a_n, and b_n of the waveform in Fig. 17.47. Plot the amplitude and phase spectra.

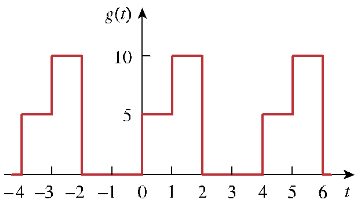


Figure 17.47
For Prob. 17.3.

Chapter 17, Solution 3.

$T = 4, \ \omega_o = 2\pi/T = \pi/2$

$$g(t) = \begin{cases} 5, & 0 < t < 1 \\ 10, & 1 < t < 2 \\ 0, & 2 < t < 4 \end{cases}$$

$$a_o = (1/T)\int_0^T g(t)dt = 0.25[\int_0^1 5dt + \int_1^2 10dt] = \underline{\mathbf{3.75}}$$

$$a_n = (2/T)\int_0^T g(t)\cos(n\omega_o t)dt = (2/4)[\int_0^1 5\cos(\frac{n\pi}{2}t)dt + \int_1^2 10\cos(\frac{n\pi}{2}t)dt]$$

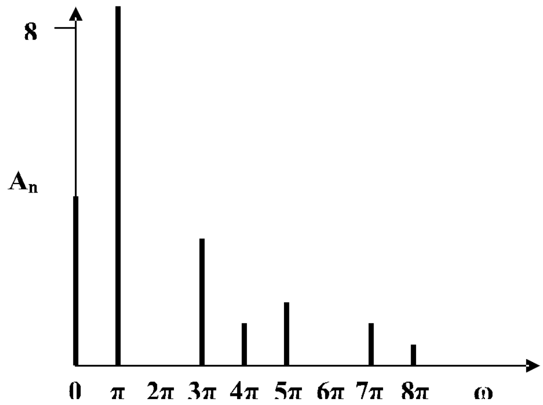
$$= 0.5[5\frac{2}{n\pi}\sin\frac{n\pi}{2}t\Big|_0^1 + 10\frac{2}{n\pi}\sin\frac{n\pi}{2}t\Big|_1^2] = (-1/(n\pi))5\sin(n\pi/2)$$

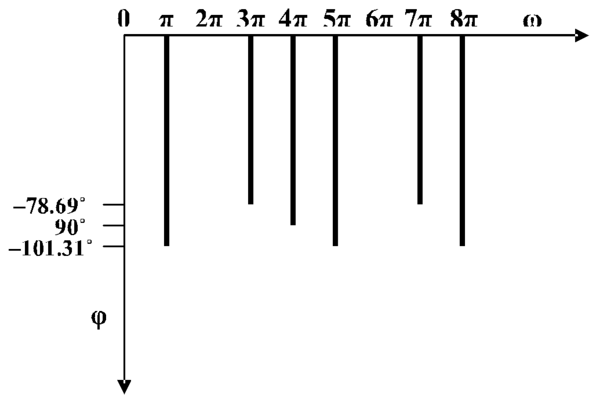
$$a_n = \begin{cases} \mathbf{(5/(n\pi))(-1)^{(n+1)/2}}, & \mathbf{n = odd} \\ \mathbf{0}, & \mathbf{n = even} \end{cases}$$

$$b_n = (2/T)\int_0^T g(t)\sin(n\omega_o t)dt = (2/4)[\int_0^1 5\sin(\frac{n\pi}{2}t)dt + \int_1^2 10\sin(\frac{n\pi}{2}t)dt]$$

$$= 0.5[\frac{-2x5}{n\pi}\cos\frac{n\pi}{2}t\Big|_0^1 - \frac{2x10}{n\pi}\cos\frac{n\pi}{2}t\Big|_1^2] = \underline{\mathbf{(5/(n\pi))[3 - 2\cos n\pi + \cos(n\pi/2)]}}$$

n	a_n	b_n	A_n	phase
1	–1.59	7.95	8.11	–101.31
2	0	0	0	0
3	0.53	2.65	2.70	–78.69
4	0	0.80	0.80	–90
5	–0.32	1.59	1.62	–101.31
6	0	0	0	0
7	0.23	1.15	1.17	–78.69
8	0	0.40	0.40	–90





Chapter 17, Problem 4.

Find the Fourier series expansion of the backward sawtooth waveform of Fig. 17.48. Obtain the amplitude and phase spectra.

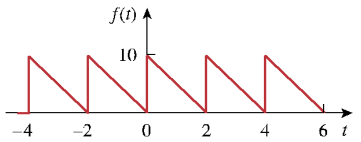


Figure 17.48
For Probs. 17.4 and 17.66.

Chapter 17, Solution 4.

$$f(t) = 10 - 5t, \quad 0 < t < 2, \quad T = 2, \quad \omega_o = 2\pi/T = \pi$$

$$a_o = (1/T)\int_0^T f(t)dt = (1/2)\int_0^2 (10 - 5t)dt = 0.5[10t - (5t^2/2)]\Big|_0^2 = 5$$

$$a_n = (2/T)\int_0^T f(t)\cos(n\omega_o t)dt = (2/2)\int_0^2 (10 - 5t)\cos(n\pi t)dt$$

$$= \int_0^2 (10)\cos(n\pi t)dt - \int_0^2 (5t)\cos(n\pi t)dt$$

$$= \frac{-5}{n^2\pi^2}\cos n\pi t\Big|_0^2 + \frac{5t}{n\pi}\sin n\pi t\Big|_0^2 = [-5/(n^2\pi^2)](\cos 2n\pi - 1) = 0$$

$$b_n = (2/2)\int_0^2 (10 - 5t)\sin(n\pi t)dt$$

$$= \int_0^2 (10)\sin(n\pi t)dt - \int_0^2 (5t)\sin(n\pi t)dt$$

$$= \frac{-5}{n^2\pi^2}\sin n\pi t\Big|_0^2 + \frac{5t}{n\pi}\cos n\pi t\Big|_0^2 = 0 + [10/(n\pi)](\cos 2n\pi) = 10/(n\pi)$$

Hence $\qquad f(t) = \mathbf{5 + \frac{10}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin(n\pi t)}$

Chapter 17, Problem 5.

Obtain the Fourier series expansion for the waveform shown in Fig. 17.49.

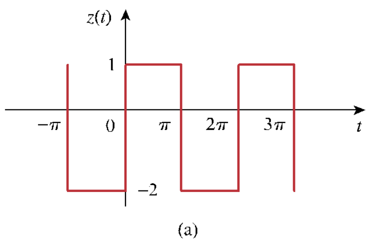


Figure 17.49
For Prob. 17.5.

Chapter 17, Solution 5.

$$T = 2\pi, \qquad \omega = 2\pi / T = 1$$

$$a_o = \frac{1}{T}\int_0^T z(t)dt = \frac{1}{2\pi}[1x\pi - 2x\pi] = -0.5$$

$$a_n = \frac{2}{T}\int_0^T z(t)\cos n\omega_o dt = \frac{1}{\pi}\int_0^{\pi} 1\cos ntdt - \frac{1}{\pi}\int_{\pi}^{2\pi} 2\cos ntdt = \frac{1}{n\pi}\sin ..nt \Big|_0^{\pi} - \frac{2}{n\pi}\sin nt \Big|_{\pi}^{2\pi} = 0$$

$$b_n = \frac{2}{T}\int_0^T z(t)\cos n\omega_o dt = \frac{1}{\pi}\int_0^{\pi} 1\sin ntdt - \frac{1}{\pi}\int_{\pi}^{2\pi} 2\sin ntdt = -\frac{1}{n\pi}\cos nt \Big|_0^{\pi} + \frac{2}{n\pi}\cos nt \Big|_{\pi}^{2\pi} = \begin{cases} \frac{6}{n\pi}, n = odd \\ 0, \quad n = even \end{cases}$$

Thus,

$$z(t) = -0.5 + \sum_{\substack{n=1 \\ n=odd}}^{\infty} \frac{6}{n\pi}\sin nt$$

Chapter 17, Problem 6.

Find the trigonometric Fourier series for

$$f(t) = \begin{cases} 5, & 0 < t < \pi \\ 10, & \pi < t < 2\pi \end{cases} \quad \text{and } f(t+2\pi) = f(t).$$

Chapter 17, Solution 6.

T=2π, ω_o=2π/T = 1

$$a_o = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{2\pi}\left[\int_0^{\pi} 5dt + \int_{\pi}^{2\pi} 10dt\right] = \frac{1}{2\pi}(5\pi + 10\pi) = 7.5$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_o t dt = \frac{2}{2\pi}\left[\int_0^{\pi} 5\cos nt dt + \int_{\pi}^{2\pi} 10\cos nt dt\right] = 0$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega_o t dt = \frac{2}{2\pi}\left[\int_0^{\pi} 5\sin nt dt + \int_{\pi}^{2\pi} 10 \sin nt dt\right] = \frac{1}{\pi}\left[-\frac{1}{n}\cos nt\Big|_0^{\pi} - \frac{1}{n}\cos nt\Big|_{\pi}^{2\pi}\right]$$

$$= \frac{5}{n\pi}\left[\cos \pi n - 1\right] = \begin{cases} -\frac{10}{n\pi}, & n = odd \\ 0, & n = even \end{cases}$$

Thus,

$$\underline{f(t) = 7.5 - \sum_{n=odd}^{\infty} \frac{10}{n\pi}\sin nt}$$

Chapter 17, Problem 7.

ML * Determine the Fourier series of the periodic function in Fig. 17.50.

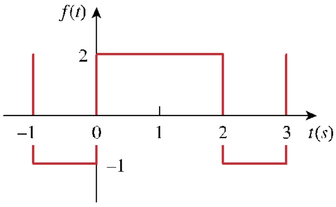


Figure 17.50
For Prob. 17.7.

* An asterisk indicates a challenging problem.

Chapter 17, Solution 7.

$T = 3, \quad \omega_o = 2\pi / T = 2\pi / 3$

$$a_o = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{3}\left[\int_0^2 2dt + \int_2^3 (-1)dt\right] = \frac{1}{3}(4-1) = 1$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos\frac{2n\pi t}{3}dt = \frac{2}{3}\left[\int_0^2 2\cos\frac{2n\pi t}{3}dt + \int_2^3 (-1)\cos\frac{2n\pi t}{3}dt\right]$$

$$= \frac{2}{3}\left[2\frac{3}{2n\pi}\sin\frac{2n\pi t}{3}\Big|_0^2 - 1\frac{3}{2n\pi}\sin\frac{2n\pi t}{3}\Big|_2^3\right] = \frac{3}{n\pi}\sin\frac{4n\pi}{3}$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin\frac{2n\pi t}{3}dt = \frac{2}{3}\left[\int_0^2 2\sin\frac{2n\pi t}{3}dt + \int_2^3 (-1)\sin\frac{2n\pi t}{3}dt\right]$$

$$= \frac{2}{3}\left[-2x\frac{3}{2n\pi}\cos\frac{2n\pi t}{3}\Big|_0^2 + \frac{3}{2n\pi}\cos\frac{2n\pi t}{3}\Big|_2^3\right] = \frac{3}{n\pi}(1 - 2\cos\frac{4n\pi}{3})$$

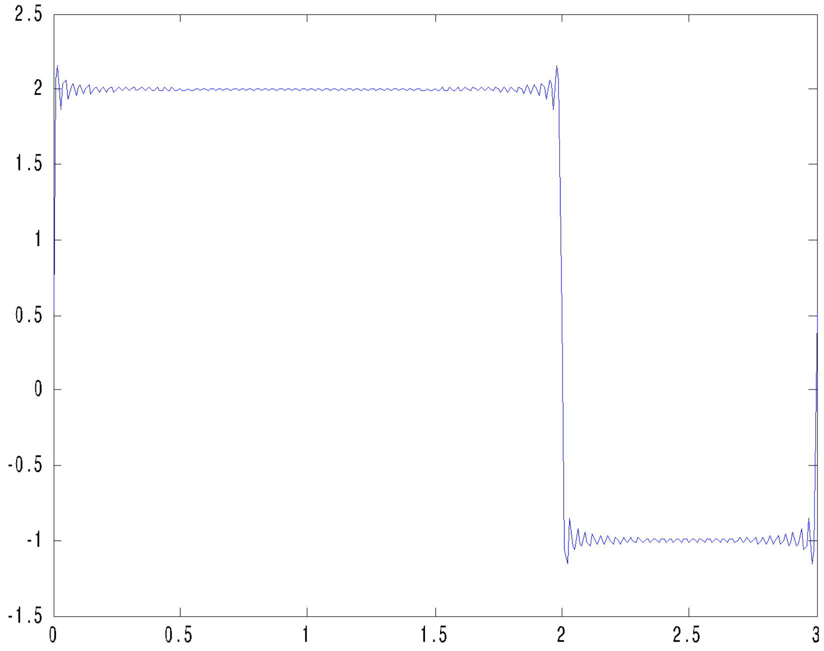
$$= \frac{1}{n\pi}\left(2 - 3\cos\frac{4n\pi}{3} + 1\right) = \frac{3}{n\pi}\left(1 - \cos\frac{4n\pi}{3}\right)$$

Hence,

$$f(t) = 1 + \sum_{n=0}^{\infty}\left[\frac{3}{n\pi}\sin\frac{4n\pi}{3}\cos\frac{2n\pi t}{3} + \frac{3}{n\pi}\left(1-\cos\frac{4n\pi}{3}\right)\sin\frac{2n\pi t}{3}\right]$$

We can now use MATLAB to check our answer,

```
>> t=0:.01:3;
>> f=1*ones(size(t));
>> for n=1:1:99,
f=f+(3/(n*pi))*sin(4*n*pi/3)*cos(2*n*pi*t/3)+(3/(n*pi))*(1-
cos(4*n*pi/3))*sin(2*n*pi*t/3);
end
>> plot(t,f)
```



Clearly we have the same figure we started with!!

Chapter 17, Problem 8.

Obtain the exponential Fourier series of the function in Fig. 17.51.

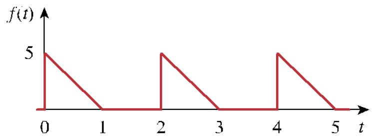


Figure 17.51
For Prob. 17.8.

Chapter 17, Solution 8.

$T = 2, \quad \omega_o = 2\pi / T = \pi$

$$f(t) = \begin{cases} 5(1-t), & 0 < t < 1 \\ 0, & 1 < t < 2 \end{cases}$$

$$c_n = \frac{1}{T}\int_0^T f(t)e^{-jn\omega_o t}dt = \frac{1}{2}\int_0^1 5(1-t)e^{-jn\pi t}dt$$

$$= \frac{5}{2}\int_0^1 e^{-jn\pi t}dt - \frac{5}{2}\int_0^1 te^{-jn\pi t}dt = \frac{5}{2}\frac{e^{-jn\pi t}}{-jn\pi}\Bigg|_0^1 - \frac{5}{2}\frac{e^{-jn\pi t}}{(-jn\pi)^2}(-jn\pi t - 1)\Bigg|_0^1$$

$$= \frac{5}{2}\frac{\left[e^{-jn\pi} - 1\right]}{-jn\pi} - \frac{5}{2}\frac{e^{-jn\pi}}{-n^2\pi^2}(-jn\pi - 1) + \frac{5}{2}\frac{(-1)}{-n^2\pi^2}$$

But $e^{-jn\pi} = \cos\pi n - j\sin n\pi = \cos n\pi + 0 = (-1)^n$

$$c_n = \frac{2.5[1-(-1)^n]}{jn\pi} - \frac{2.5(-1)^n[1+jn\pi]}{n^2\pi^2} + \frac{2.5}{n^2\pi^2}$$

Chapter 17, Problem 9.

Determine the Fourier coefficients a_n and b_n of the first three harmonic terms of the rectified cosine wave in Fig. 17.52.

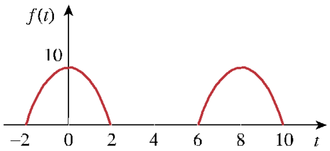


Figure 17.52
For Prob. 17.9.

Chapter 17, Solution 9.

f(t) is an even function, b_n=0.

$$T = 8, \qquad \omega = 2\pi / T = \pi / 4$$

$$a_o = \frac{1}{T}\int_0^T f(t)dt = \frac{2}{8}\left[\int_0^2 10\cos \pi t/4dt + 0\right] = \frac{10}{4}(\frac{4}{\pi})\sin \pi t/4 \Big|_0^2 = \frac{10}{\pi} = 3.183$$

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos n\omega_o dt = \frac{40}{8}[\int_0^2 10\cos \pi t/4\cos n\pi t/4dt + 0] = 5\int_0^2 [\cos \pi t(n+1)/4 + \cos \pi t(n-1)/4]dt$$

For n = 1,

$$a_1 = 5\int_0^2 [\cos \pi t/2 + 1]dt = 5\left[\frac{2}{\pi}\sin \pi t/2dt + t\right]_0^2 = 10$$

For n>1,

$$a_n = \frac{20}{\pi(n+1)}\sin\frac{\pi(n+1)t}{4} + \frac{20}{\pi(n-1)}\sin\frac{\pi(n-1)}{4}\Bigg|_0^2 = \frac{20}{\pi(n+1)}\sin\frac{\pi(n+1)}{2} + \frac{20}{\pi(n-1)}\sin\frac{\pi(n-1)}{2}$$

$$a_2 = \frac{20}{3\pi}\sin 1.5\pi + \frac{20}{\pi}\sin \pi/2 = 4.244, \qquad a_3 = \frac{20}{4\pi}\sin 2\pi + \frac{10}{\pi}\sin \pi = 0$$

Thus,

$$\underline{a_0 = 3.183, \quad a_1 = 10, \quad a_2 = 4.244, \quad a_3 = 0, \quad b_1 = 0 = b_2 = b_3}$$

Chapter 17, Problem 10.

Find the exponential Fourier series for the waveform in Fig. 17.53.

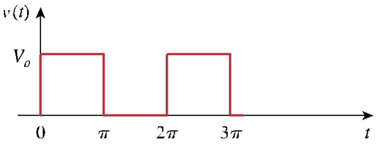


Figure 17.53
For Prob. 17.10.

Chapter 17, Solution 10.

$T = 2\pi, \qquad \omega_o = 2\pi / T = 1$

$$c_n = \frac{1}{T}\int_0^T f(t)e^{-jn\omega_o t}dt = \frac{V_o}{2\pi}\int_0^\pi (1)e^{-jnt}dt = \frac{V_o}{2\pi}\frac{e^{-jnt}}{-jn}\bigg|_0^\pi$$

$$= \frac{V_o}{2n\pi}\left[je^{-jn\pi} - j\right] = \frac{jV_o}{2n\pi}(\cos n\pi - 1)$$

$$f(t) = \sum_{n=-\infty}^{\infty} \frac{jV_o}{2n\pi}(\cos n\pi - 1)e^{jnt}$$

Chapter 17, Problem 11.

Obtain the exponential Fourier series for the signal in Fig. 17.54.

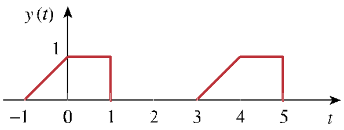


Figure 17.54
For Prob. 17.11.

Chapter 17, Solution 11.

$$T = 4, \qquad \omega_o = 2\pi / T = \pi / 2$$

$$c_n = \frac{1}{T}\int_0^T y(t)e^{-jn\omega_o t}dt = \frac{1}{4}\left[\int_{-1}^{0}(t+1)e^{-jn\pi t/2}dt + \int_0^1 (1)e^{-jn\pi t/2}dt\right]$$

$$c_n = \frac{1}{4}\left[\frac{e^{-jn\pi t/2}}{-n^2\pi^2/4}(-jn\pi t/2 - 1) - \frac{2}{jn\pi}e^{-jn\pi t/2}\,\Big|_{-1}^{0} - \frac{2}{jn\pi}e^{-jn\pi t/2}\,\Big|_0^1\right]$$

$$= \frac{1}{4}\left[\frac{4}{n^2\pi^2} - \frac{2}{jn\pi} + \frac{4}{n^2\pi^2}e^{jn\pi/2}(jn\pi/2 - 1) + \frac{2}{jn\pi}e^{jn\pi/2} - \frac{2}{jn\pi}e^{-jn\pi/2} + \frac{2}{jn\pi}\right]$$

But

$$e^{jn\pi/2} = \cos n\pi/2 + j\sin n\pi/2 = j\sin n\pi/2, \qquad e^{-jn\pi/2} = \cos n\pi/2 - j\sin n\pi/2 = -j\sin n\pi/2$$

$$c_n = \frac{1}{n^2\pi^2}\left[1 + j(jn\pi/2 - 1)\sin n\pi/2 + n\pi\sin n\pi/2\right]$$

$$\underline{y(t) = \sum_{n=-\infty}^{\infty}\frac{1}{n^2\pi^2}\left[1 + j(jn\pi/2 - 1)\sin n\pi/2 + n\pi\sin n\pi/2\right]e^{jn\pi t/2}}$$

Chapter 17, Problem 12.

* A voltage source has a periodic waveform defined over its period as

$v(t) = t(2\pi - t)$ V, $\quad 0 < t < 2\pi$

Find the Fourier series for this voltage.

* An asterisk indicates a challenging problem.

Chapter 17, Solution 12.

A voltage source has a periodic waveform defined over its period as
$v(t) = t(2\pi - t)$ V, $\quad$ for all $0 < t < 2\pi$

Find the Fourier series for this voltage.

$v(t) = 2\pi t - t^2$, $0 < t < 2\pi$, $T = 2\pi$, $\omega_o = 2\pi/T = 1$

$$a_o = (1/T)\int_0^T f(t)dt = \frac{1}{2\pi}\int_0^{2\pi}(2\pi t - t^2)dt = \frac{1}{2\pi}(\pi t^2 - t^3/3)\Big|_0^{2\pi} = \frac{4\pi^3}{2\pi}(1-2/3) = \frac{2\pi^2}{3}$$

$$a_n = \frac{2}{T}\int_0^T (2\pi t - t^2)\cos(nt)dt = \frac{1}{\pi}\left[\frac{2\pi}{n^2}\cos(nt) + \frac{2\pi t}{n}\sin(nt)\right]\Bigg|_0^{2\pi}$$

$$-\frac{1}{\pi n^3}\left[2nt\cos(nt) - 2\sin(nt) + n^2t^2\sin(nt)\right]_0^{2\pi}$$

$$= \frac{2}{n^2}(1-1) - \frac{1}{\pi n^3}4n\pi\cos(2\pi n) = \frac{-4}{n^2}$$

$$b_n = \frac{2}{T}\int_0^T (2nt - t^2)\sin(nt)dt = \frac{1}{\pi}\int (2nt - t^2)\sin(nt)dt$$

$$= \frac{2n}{\pi}\frac{1}{n^2}(\sin(nt) - nt\cos(nt))\Big|_0^{\pi} - \frac{1}{\pi n^3}(2nt\sin(nt) + 2\cos(nt) - n^2t^2\cos(nt))\Big|_0^{2\pi}$$

$$= \frac{-4\pi}{n} + \frac{4\pi}{n} = 0$$

Hence, $\quad f(t) = \underline{\boldsymbol{\frac{2\pi^2}{3} - \sum_{n=1}^{\infty}\frac{4}{n^2}\cos(nt)}}$

Chapter 17, Problem 13.

A periodic function is defined over its period as

$$h(t) = \begin{cases} 10\sin t, & 0 < t < \pi \\ 20\sin(t-\pi), & \pi < t < 2\pi \end{cases}$$

Find the Fourier series of $h(t)$.

Chapter 17, Solution 13.

$T = 2\pi, \ \omega_o = 1$

$$a_o = (1/T)\int_0^T h(t)dt = \frac{1}{2\pi}\left[\int_0^\pi 10\sin t\, dt + \int_\pi^{2\pi} 20\sin(t-\pi)\, dt\right]$$

$$= \frac{1}{2\pi}\left[-10\cos t\Big|_0^\pi - 20\cos(t-\pi)\Big|_\pi^{2\pi}\right] = \frac{30}{\pi}$$

$$a_n = (2/T)\int_0^T h(t)\cos(n\omega_o t)dt$$

$$= [2/(2\pi)]\left[\int_0^\pi 10\sin t\cos(nt)dt + \int_\pi^{2\pi} 20\sin(t-\pi)\cos(nt)dt\right]$$

Since $\sin A\cos B = 0.5[\sin(A+B) + \sin(A-B)]$
$\sin t\cos nt = 0.5[\sin((n+1)t) + \sin((1-n))t]$
$\sin(t-\pi) = \sin t\cos\pi - \cos t\sin\pi = -\sin t$
$\sin(t-\pi)\cos(nt) = -\sin(t)\cos(nt)$

$$a_n = \frac{1}{2\pi}\left[10\int_0^\pi [\sin([1+n]t) + \sin([1-n]t)]dt - 20\int_\pi^{2\pi} [\sin([1+n]t) + \sin([1-n]t)]dt\right]$$

$$= \frac{5}{\pi}\left[\left(-\frac{\cos([1+n]t)}{1+n} - \frac{\cos([1-n]t)}{1-n}\right)\Bigg|_0^\pi + \left(\frac{2\cos([1+n]t)}{1+n} + \frac{2\cos([1-n]t)}{1-n}\right)\Bigg|_\pi^{2\pi}\right]$$

$$a_n = \frac{5}{\pi}\left[\frac{3}{1+n} + \frac{3}{1-n} - \frac{3\cos([1+n]\pi)}{1+n} - \frac{3\cos([1-n]\pi)}{1-n}\right]$$

But, $[1/(1+n)] + [1/(1-n)] = 1/(1-n^2)$

$\cos([n-1]\pi) = \cos([n+1]\pi) = \cos\pi\cos n\pi - \sin\pi\sin n\pi = -\cos n\pi$

$a_n = (5/\pi)[(6/(1-n^2)) + (6\cos(n\pi)/(1-n^2))]$

$= [30/(\pi(1-n^2))](1 + \cos n\pi) = [-60/(\pi(n-1))], \ n = \text{even}$
$= 0, \quad n = \text{odd}$

$$b_n = (2/T)\int_0^T h(t)\sin n\omega_o t\, dt$$

$$= [2/(2\pi)][\int_0^\pi 10\sin t\sin nt\, dt + \int_\pi^{2\pi} 20(-\sin t)\sin nt\, dt$$

But, $\sin A\sin B = 0.5[\cos(A-B) - \cos(A+B)]$

$\sin t\sin nt = 0.5[\cos([1-n]t) - \cos([1+n]t)]$

$$b_n = (5/\pi)\{[(\sin([1-n]t)/(1-n)) - (\sin([1+n]t)/(1+n)]\Big|_0^\pi$$

$$+ [(2\sin([1-n]t)/(1-n)) - (2\sin([1+n]t)/(1+n)]\Big|_\pi^{2\pi}\}$$

$$= \frac{5}{\pi}\left[-\frac{\sin([1-n]\pi)}{1-n} + \frac{\sin([1+n]\pi)}{1+n}\right] = 0$$

Thus, $$h(t) = \frac{30}{\pi} - \frac{60}{\pi}\sum_{k=1}^{\infty}\frac{\cos(2kt)}{(4k^2-1)}$$

Chapter 17, Problem 14.

Find the quadrature (cosine and sine) form of the Fourier series

$$f(t) = 2 + \sum_{n=1}^{\infty} \frac{10}{n^3+1}\cos\left(2nt + \frac{n\pi}{4}\right)$$

Chapter 17, Solution 14.

Since cos(A + B) = cos A cos B – sin A sin B.

$$f(t) = \underline{\mathbf{2 + \sum_{n=1}^{\infty}\left(\frac{10}{n^3+1}\cos(n\pi/4)\cos(2nt) - \frac{10}{n^3+1}\sin(n\pi/4)\sin(2nt)\right)}}$$

Chapter 17, Problem 15.

Express the Fourier series

$$f(t) = 10 + \sum_{n=1}^{\infty} \frac{4}{n^2+1}\cos 10nt + \frac{1}{n^3}\sin 10nt$$

(a) in a cosine and angle form.
(b) in a sine and angle form.

Chapter 17, Solution 15.

(a) $D\cos\omega t + E\sin\omega t = A\cos(\omega t - \theta)$

where $A = \sqrt{D^2 + E^2}$, $\theta = \tan^{-1}(E/D)$

$$A = \sqrt{\frac{16}{(n^2+1)^2} + \frac{1}{n^6}}, \quad \theta = \tan^{-1}((n^2+1)/(4n^3))$$

$$f(t) = \underline{\mathbf{10 + \sum_{n=1}^{\infty}\sqrt{\frac{16}{(n^2+1)^2} + \frac{1}{n^6}}\cos\left(10nt - \tan^{-1}\frac{n^2+1}{4n^3}\right)}}$$

(b) $D\cos\omega t + E\sin\omega t = A\sin(\omega t + \theta)$

where $A = \sqrt{D^2 + E^2}$, $\theta = \tan^{-1}(D/E)$

$$f(t) = \underline{\mathbf{10 + \sum_{n=1}^{\infty}\sqrt{\frac{16}{(n^2+1)^2} + \frac{1}{n^6}}\sin\left(10nt + \tan^{-1}\frac{4n^3}{n^2+1}\right)}}$$

Chapter 17, Problem 16.

The waveform in Fig. 17.55(a) has the following Fourier series:

$$v_1(t) = \frac{1}{2} - \frac{4}{\pi^2}\left(\cos \pi t + \frac{1}{9}\cos 3\pi t + \frac{1}{25}\cos 5\pi t + \cdots\right) \text{V}$$

Obtain the Fourier series of $v_2(t)$ in Fig. 17.55(b).

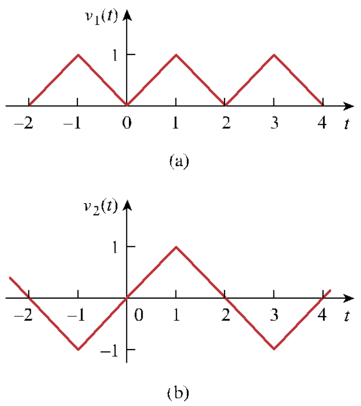
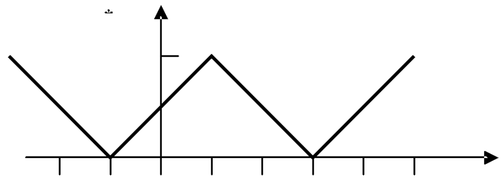


Figure 17.55
For Probs. 17.16 and 17.69.

Chapter 17, Solution 16.

If $v_2(t)$ is shifted by 1 along the vertical axis, we obtain $v_2^*(t)$ shown below, i.e. $v_2^*(t) = v_2(t) + 1$.



Comparing $v_2^*(t)$ with $v_1(t)$ shows that

$$v_2^*(t) = 2v_1((t + t_o)/2)$$

$$\text{where } (t + t_o)/2 = 0 \text{ at } t = -1 \text{ or } t_o = 1$$

Hence $$v_2^*(t) = 2v_1((t + 1)/2)$$

But $$v_2^*(t) = v_2(t) + 1$$

$$v_2(t) + 1 = 2v_1((t+1)/2)$$

$$v_2(t) = -1 + 2v_1((t+1)/2)$$

$$= -1 + 1 - \frac{8}{\pi^2}\left[\cos \pi\left(\frac{t+1}{2}\right) + \frac{1}{9}\cos 3\pi\left(\frac{t+1}{2}\right) + \frac{1}{25}\cos 5\pi\left(\frac{t+1}{2}\right) + \cdots\right]$$

$$v_2(t) = -\frac{8}{\pi^2}\left[\cos\left(\frac{\pi t}{2} + \frac{\pi}{2}\right) + \frac{1}{9}\cos\left(\frac{3\pi t}{2} + \frac{3\pi}{2}\right) + \frac{1}{25}\cos\left(\frac{5\pi t}{2} + \frac{5\pi}{2}\right) + \cdots\right]$$

$$\underline{\mathbf{v_2(t) = -\frac{8}{\pi^2}\left[\sin\left(\frac{\pi t}{2}\right) + \frac{1}{9}\sin\left(\frac{3\pi t}{2}\right) + \frac{1}{25}\sin\left(\frac{5\pi t}{2}\right) + \cdots\right]}}$$

Chapter 17, Problem 17.

Determine if these functions are even, odd, or neither.

(a) $1+t$ (b) t^2-1 (c) $\cos n\pi t \sin n\pi t$
(d) $\sin^2 \pi t$ (e) e^{-t}

Chapter 17, Solution 17.

We replace t by –t in each case and see if the function remains unchanged.

(a) $1 - t$, **neither odd nor even.**

(b) $t^2 - 1$, **even**

(c) $\cos n\pi(-t) \sin n\pi(-t) = -\cos n\pi t \sin n\pi t$, **odd**

(d) $\sin^2 n(-t) = (-\sin \pi t)^2 = \sin^2 \pi t$, **even**

(e) e^t, **neither odd nor even**.

Chapter 17, Problem 18.

Determine the fundamental frequency and specify the type of symmetry present in the functions in Fig. 17.56.

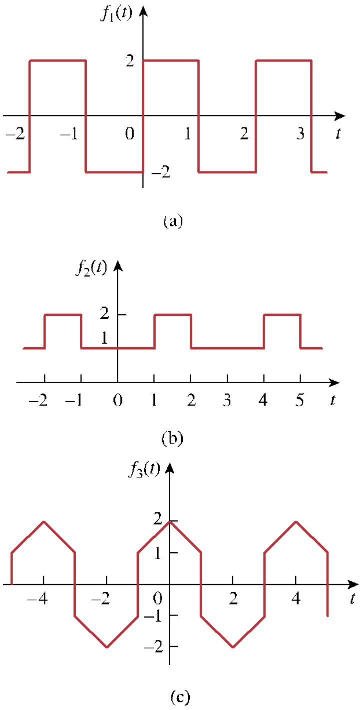


Figure 17.56
For Probs. 17.18 and 17.63.

Chapter 17, Solution 18.

(a) T = 2 leads to $\omega_o = 2\pi/T =$ **π**

$f_1(-t) = -f_1(t)$, showing that $f_1(t)$ is **odd and half-wave symmetric**.

(b) T = 3 leads to $\omega_o = 2\pi/3$

$f_2(t) = f_2(-t)$, showing that $f_2(t)$ is **even**.

(c) T = 4 leads to $\omega_o = \pi/2$

$f_3(t)$ is **even and half-wave symmetric**.

Chapter 17, Problem 19.

Obtain the Fourier series for the periodic waveform in Fig. 17.57.

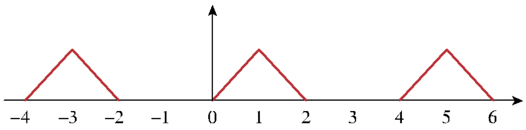


Figure 17.57
For Prob. 17.19.

Chapter 17, Solution 19.

$T = 4, \qquad \omega_o = 2\pi / T = \pi / 2$

$$f(t) = \begin{cases} 10t, & 0 < t < 1 \\ 10(2-t), & 1 < t < 2 \end{cases}$$

$$a_0 = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{4}\int_0^1 10t dt + \frac{1}{4}\int_1^2 10(2-t)dt = \frac{1}{4}5t^2\Big|_0^1 + \frac{10}{4}(2t - \frac{t^2}{2})\Big|_1^2 = 2.5$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_o t dt = \frac{2}{4}\int_0^1 10t\cos n\omega_o t dt + \frac{2}{4}\int_1^2 10(2-t)\cos n\omega_o t dt$$

$$= \frac{20}{n\omega_o}\cos n\omega_o t + \frac{t}{n\omega_o}\sin n\omega_o t\Big|_0^1 + \frac{10}{n\omega_o}\sin n\omega_o t\Big|_1^2 + \frac{5}{n^2\omega_o^2}\cos n\omega_o t + \frac{5t}{n\omega_o}\sin n\omega_o t\Big|_1^2$$

$$= \frac{20}{n\omega_o}(\cos n\pi/2 - 1) + \frac{1}{n\omega_o}\sin n\pi/2 + \frac{10}{n\omega_o}(\sin n\pi - \sin n\pi/2) + \frac{5}{n^2\pi^2/4}\cos n\pi$$

$$- \frac{5}{n^2\pi^2/4}\cos n\pi/2 + \frac{10}{n\omega_o}\sin n\pi - \frac{5}{n\pi/2}\sin n\pi/2$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega_o t dt = \frac{2}{4}\int_0^1 10t\sin n\omega_o t dt + \frac{2}{4}\int_1^2 10(2-t)\sin n\omega_o t dt$$

$$= \frac{5}{n\omega_o}\sin n\omega_o t\Big|_0^1 - \frac{10}{n\omega_o}\cos n\omega_o t\Big|_0^1 - \frac{5}{n^2\omega_o^2}\sin n\omega_o t\Big|_1^2 + \frac{t}{n\omega_o}\cos n\omega_o t\Big|_1^2$$

$$= \frac{5}{n^2\omega_o^2}\sin n\pi/2 - \frac{10}{n\omega_o}(\cos\pi n - \cos n\pi/2) - \frac{5}{n^2\omega_o^2}(\sin\pi n - \sin n\pi/2)$$

$$- \frac{2}{n\omega_o}\cos n\pi - \frac{\cos\pi n/2}{n\omega_o}$$

Chapter 17, Problem 20.

ML Find the Fourier series for the signal in Fig. 17.58. Evaluate $f(t)$ at $t = 2$ using the first three nonzero harmonics.

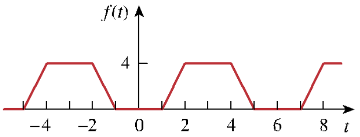


Figure 17.58
For Probs. 17.20 and 17.67.

Chapter 17, Solution 20.

This is an even function.

$$b_n = 0,\ T = 6,\ \omega = 2\pi/6 = \pi/3$$

$$a_o = \frac{2}{T}\int_0^{T/2} f(t)dt = \frac{2}{6}\left[\int_1^2 (4t-4)dt \int_2^3 4\,dt\right]$$

$$= \frac{1}{3}\left[(2t^2 - 4t)\Big|_1^2 + 4(3-2)\right] = 2$$

$$a_n = \frac{4}{T}\int_0^{T/4} f(t)\cos(n\pi t/3)dt$$

$$= (4/6)[\int_1^2 (4t-4)\cos(n\pi t/3)dt + \int_2^3 4\cos(n\pi t/3)dt]$$

$$= \frac{16}{6}\left[\frac{9}{n^2\pi^2}\cos\left(\frac{n\pi t}{3}\right) + \frac{3t}{n\pi}\sin\left(\frac{n\pi t}{3}\right) - \frac{3}{n\pi}\sin\left(\frac{n\pi t}{3}\right)\right]_1^2 + \frac{16}{6}\left[\frac{3}{n\pi}\sin\left(\frac{n\pi t}{3}\right)\right]_2^3$$

$$= [24/(n^2\pi^2)][\cos(2n\pi/3) - \cos(n\pi/3)]$$

Thus $$f(t) = \mathbf{2 + \frac{24}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\left[\cos\left(\frac{2\pi n}{3}\right) - \cos\left(\frac{\pi n}{3}\right)\right]\cos\left(\frac{n\pi t}{3}\right)}$$

At $t = 2$,

$$f(2) = 2 + (24/\pi^2)[(\cos(2\pi/3) - \cos(\pi/3))\cos(2\pi/3)$$

$$+ (1/4)(\cos(4\pi/3) - \cos(2\pi/3))\cos(4\pi/3)$$

$$+ (1/9)(\cos(2\pi) - \cos(\pi))\cos(2\pi) + \text{-----}]$$

$$= 2 + 2.432(0.5 + 0 + 0.2222 + \text{-----})$$

$f(2) =$ **3.756**

Chapter 17, Problem 21.

Determine the trigonometric Fourier series of the signal in Fig. 17.59.

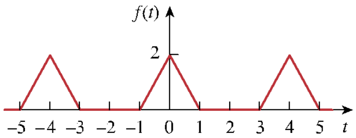


Figure 17.59
For Prob. 17.21.

Chapter 17, Solution 21.

This is an even function.

$b_n = 0,\ T = 4,\ \omega_o = 2\pi/T = \pi/2.$

$$f(t) = \begin{cases} 2 - 2t, & 0 < t < 1 \\ 0, & 1 < t < 2 \end{cases}$$

$$a_o = \frac{2}{4}\int_0^1 2(1 - t)dt = \left[t - \frac{t^2}{2}\right]_0^1 = 0.5$$

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos(n\omega_o t)dt = \frac{4}{4}\int_0^1 2(1 - t)\cos\left(\frac{n\pi t}{2}\right)dt$$

$$= [8/(\pi^2 n^2)][1 - \cos(n\pi/2)]$$

$$\underline{\mathbf{f(t) = \frac{1}{2} + \sum_{n=1}^{\infty}\frac{8}{n^2\pi^2}\left[1 - \cos\left(\frac{n\pi}{2}\right)\right]\cos\left(\frac{n\pi t}{2}\right)}}$$

Chapter 17, Problem 22.

Calculate the Fourier coefficients for the function in Fig. 17.60.

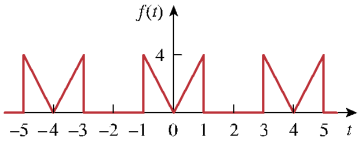


Figure 17.60
For Prob. 17.22.

Chapter 17, Solution 22.

Calculate the Fourier coefficients for the function in Fig. 16.54.

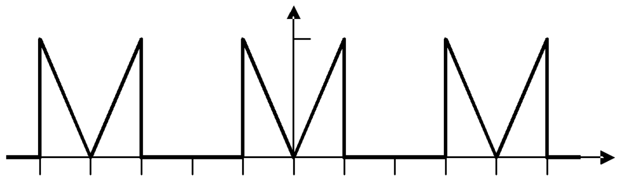


Figure 16.54 For Prob. 16.15

This is an even function, therefore $b_n = 0$. In addition, T=4 and $\omega_o = \pi/2$.

$$a_o = \frac{2}{T}\int_0^{T/2} f(t)dt = \frac{2}{4}\int_0^1 4tdt = t^2\Big|_0^1 = \underline{\mathbf{1}}$$

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos(\omega_o nt)dt = \frac{4}{4}\int_0^1 4t\cos(n\pi t/2)dt$$

$$= 4\left[\frac{4}{n^2\pi^2}\cos(n\pi t/2) + \frac{2t}{n\pi}\sin(n\pi t/2)\right]_0^1$$

$$a_n = \underline{\mathbf{\frac{16}{n^2\pi^2}(\cos(n\pi/2) - 1) + \frac{8}{n\pi}\sin(n\pi/2)}}$$

Chapter 17, Problem 23.

Find the Fourier series of the function shown in Fig. 17.61.

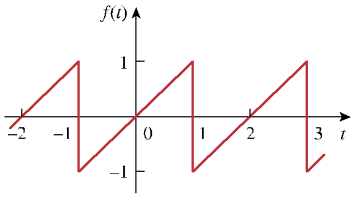


Figure 17.61
For Prob. 17.23.

Chapter 17, Solution 23.

f(t) is an odd function.

$f(t) = t, \quad -1 < t < 1$

$a_o = 0 = a_n, \quad T = 2, \quad \omega_o = 2\pi/T = \pi$

$$b_n = \frac{4}{T}\int_0^{T/2} f(t)\sin(n\omega_o t)dt = \frac{4}{2}\int_0^1 t\sin(n\pi t)dt$$

$$= \frac{2}{n^2\pi^2}\left[\sin(n\pi t) - n\pi t\cos(n\pi t)\right]_0^1$$

$$= -[2/(n\pi)]\cos(n\pi) = 2(-1)^{n+1}/(n\pi)$$

$$f(t) = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin(n\pi t)$$

Chapter 17, Problem 24.

In the periodic function of Fig. 17.62,

(a) find the trigonometric Fourier series coefficients a_2 and b_2,
(b) calculate the magnitude and phase of the component of $f(t)$ that has $\omega_n = 10$ rad/s,
(c) use the first four nonzero terms to estimate $f(\pi/2)$.
(d) show that

$$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{9} + \frac{1}{11} + \cdots$$

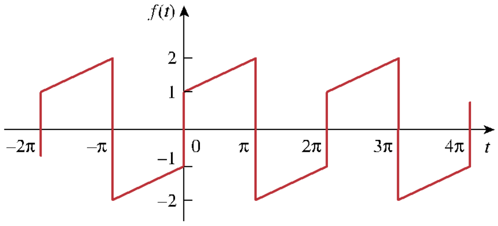


Figure 17.62
For Probs. 17.24 and 17.60.

Chapter 17, Solution 24.

(a) This is an odd function.

$$a_o = 0 = a_n, \quad T = 2\pi, \quad \omega_o = 2\pi/T = 1$$

$$b_n = \frac{4}{T}\int_0^{T/2} f(t)\sin(\omega_o nt)dt$$

$$f(t) = 1 + t/\pi, \qquad 0 < t < \pi$$

$$b_n = \frac{4}{2\pi}\int_0^{\pi} (1 + t/\pi)\sin(nt)dt$$

$$= \frac{2}{\pi}\left[-\frac{1}{n}\cos(nt) + \frac{1}{n^2\pi}\sin(nt) - \frac{t}{n\pi}\cos(nt)\right]_0^{\pi}$$

$$= [2/(n\pi)][1 - 2\cos(n\pi)] = [2/(n\pi)][1 + 2(-1)^{n+1}]$$

a_2 = **0**, $b_2 = [2/(2\pi)][1 + 2(-1)] = -1/\pi$ = **−0.3183**

(b) $\omega n = n\omega_o = 10$ or $n = 10$

$$a_{10} = 0, \; b_{10} = [2/(10\pi)][1 - \cos(10\pi)] = -1/(5\pi)$$

Thus the magnitude is $A_{10} = \sqrt{a_{10}^2 + b_{10}^2} = 1/(5\pi) =$ **0.06366**

and the phase is $\phi_{10} = \tan^{-1}(b_n/a_n) =$ **–90°**

(c) $$f(t) = \sum_{n=1}^{\infty} \frac{2}{n\pi}[1 - 2\cos(n\pi)]\sin(nt)\,\pi$$

$$f(\pi/2) = \sum_{n=1}^{\infty} \frac{2}{n\pi}[1 - 2\cos(n\pi)]\sin(n\pi/2)\,\pi$$

For $n = 1$, $f_1 = (2/\pi)(1 + 2) = 6/\pi$

For $n = 2$, $f_2 = 0$

For $n = 3$, $f_3 = [2/(3\pi)][1 - 2\cos(3\pi)]\sin(3\pi/2) = -6/(3\pi)$

For $n = 4$, $f_4 = 0$

For $n = 5$, $f_5 = 6/(5\pi)$, ----

Thus, $f(\pi/2) = 6/\pi - 6/(3\pi) + 6/(5\pi) - 6/(7\pi)$ ---------

$= (6/\pi)[1 - 1/3 + 1/5 - 1/7 +$ --------$]$

$f(\pi/2) \cong$ **1.3824**

which is within 8% of the exact value of 1.5.

(d) From part (c)

$f(\pi/2) = 1.5 = (6/\pi)[1 - 1/3 + 1/5 - 1/7 +$ - - -$]$

$(3/2)(\pi/6) = [1 - 1/3 + 1/5 - 1/7 +$ - - -$]$

or $\pi/4 =$ **1 – 1/3 + 1/5 – 1/7 + - - -**

Chapter 17, Problem 25.

Determine the Fourier series representation of the function in Fig. 17.63.

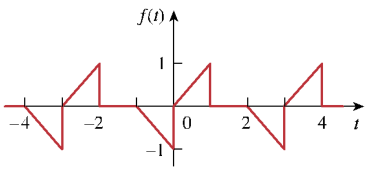


Figure 17.63
For Prob. 17.25.

Chapter 17, Solution 25.

This is a half-wave (odd) function since $f(t-T/2) = -f(t)$.

$a_o = 0$, $a_n = b_n = 0$ for n = even, $T = 3$, $\omega_o = 2\pi/3$.

For n = odd,

$$a_n = \frac{4}{3}\int_0^{1.5} f(t)\cos n\omega_0 t dt = \frac{4}{3}\int_0^1 t\cos n\omega_0 t dt$$

$$= \frac{4}{3}\left[\frac{9}{4\pi^2 n^2}\cos\left(\frac{2\pi nt}{3}\right) + \frac{3t}{2\pi n}\sin\left(\frac{2\pi nt}{3}\right)\right]_0^1$$

$$= \left[\frac{3}{\pi^2 n^2}\left(\cos\left(\frac{2\pi n}{3}\right) - 1\right) + \frac{2}{\pi n}\sin\left(\frac{2\pi n}{3}\right)\right]$$

$$b_n = \frac{4}{3}\int_0^{1.5} f(t)\sin(n\omega_0 t)dt = \frac{4}{3}\int_0^1 t\sin(2\pi nt/3)dt$$

$$= \frac{4}{3}\left[\frac{9}{4\pi^2 n^2}\sin\left(\frac{2\pi nt}{3}\right) - \frac{3t}{2n\pi}\cos\left(\frac{2\pi nt}{3}\right)\right]_0^1$$

$$= \left[\frac{3}{\pi^2 n^2}\sin\left(\frac{2\pi n}{3}\right) - \frac{2}{\pi n}\cos\left(\frac{2\pi n}{3}\right)\right]$$

$$f(t) = \sum_{\substack{n=1 \\ n=odd}}^{\infty} \left\{ \begin{array}{l} \left[\frac{3}{\pi^2 n^2}\left(\cos\left(\frac{2\pi n}{3}\right) - 1\right) + \frac{2}{\pi n}\sin\left(\frac{2\pi n}{3}\right)\right]\cos\left(\frac{2\pi nt}{3}\right) \\ + \left[\frac{3}{\pi^2 n^2}\sin\left(\frac{2\pi n}{3}\right) - \frac{2}{n\pi}\cos\left(\frac{2\pi n}{3}\right)\right]\sin\left(\frac{2\pi nt}{3}\right) \end{array} \right\}$$

Chapter 17, Problem 26.

Find the Fourier series representation of the signal shown in Fig. 17.64.

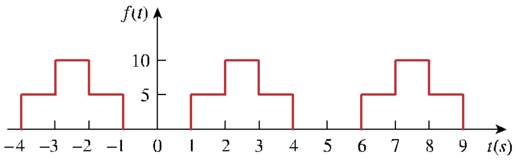


Figure 17.64
For Prob. 17.26.

Chapter 17, Solution 26.

$$T = 4, \quad \omega_o = 2\pi/T = \pi/2$$

$$a_o = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{4}\left[\int_0^1 1\,dt + \int_1^3 2\,dt + \int_3^4 1\,dt\right] = 1$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos(n\omega_o t)dt$$

$$a_n = \frac{2}{4}\left[\int_0^1 1\cos(n\pi t/2)dt + \int_1^3 2\cos(n\pi t/2)dt + \int_3^4 1\cos(n\pi t/2)dt\right]$$

$$= 2\left[\frac{2}{n\pi}\sin\frac{n\pi t}{2}\Big|_0^1 + \frac{4}{n\pi}\sin\frac{n\pi t}{2}\Big|_1^3 + \frac{2}{n\pi}\sin\frac{n\pi t}{2}\Big|_3^4\right]$$

$$= \frac{4}{n\pi}\left[\sin\frac{3n\pi}{2} - \sin\frac{n\pi}{2}\right]$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin(n\omega_o t)dt$$

$$= \frac{2}{4}\left[\int_0^1 1\sin\frac{n\pi t}{2}dt + \int_1^3 2\sin\frac{n\pi t}{2}dt + \int_3^4 1\sin\frac{n\pi t}{2}dt\right]$$

$$= 2\left[-\frac{2}{n\pi}\cos\frac{n\pi t}{2}\Big|_0^1 - \frac{4}{n\pi}\cos\frac{n\pi t}{2}\Big|_1^3 - \frac{2}{n\pi}\cos\frac{n\pi t}{2}\Big|_3^4\right]$$

$$= \frac{4}{n\pi}\left[\cos(n\pi) - 1\right]$$

Hence

$f(t) =$

$$\mathbf{1 + \sum_{n=1}^{\infty}\frac{4}{n\pi}\left[(\sin(3n\pi/2) - \sin(n\pi/2))\cos(n\pi t/2) + (\cos(n\pi) - 1)\sin(n\pi t/2)\right]}$$

Chapter 17, Problem 27.

For the waveform shown in Fig. 17.65 below,

(a) specify the type of symmetry it has,
(b) calculate a_3 and b_3,
(c) find the rms value using the first five nonzero harmonics.

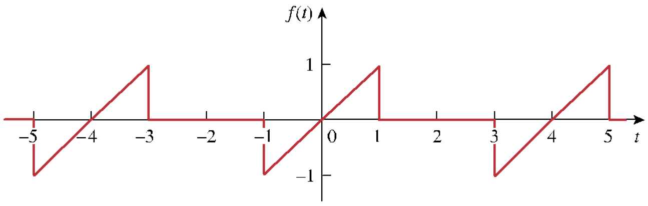


Figure 17.65
For Prob. 17.27.

Chapter 17, Solution 27.

(a) **odd** symmetry.

(b) $a_o = 0 = a_n,\ T = 4,\ \omega_o = 2\pi/T = \pi/2$

$$f(t) = t, \quad 0 < t < 1$$
$$= 0, \quad 1 < t < 2$$

$$b_n = \frac{4}{4}\int_0^1 t\sin\frac{n\pi t}{2}dt = \left[\frac{4}{n^2\pi^2}\sin\frac{n\pi t}{2} - \frac{2t}{n\pi}\cos\frac{n\pi t}{2}\right]_0^1$$

$$= \frac{4}{n^2\pi^2}\sin\frac{n\pi}{2} - \frac{2}{n\pi}\cos\frac{n\pi}{2} - 0$$

$$= 4(-1)^{(n-1)/2}/(n^2\pi^2), \quad n = \text{odd}$$

$$-2(-1)^{n/2}/(n\pi), \quad n = \text{even}$$

$a_3 = \underline{\mathbf{0}}$, $b_3 = 4(-1)/(9\pi^2) = \underline{\mathbf{-0.04503}}$

(c) $b_1 = 4/\pi^2,\ b_2 = 1/\pi,\ b_3 = -4/(9\pi^2),\ b_4 = -1/(2\pi),\ b_5 = 4/(25\pi^2)$

$$F_{rms} = \sqrt{a_o^2 + \frac{1}{2}\sum\left(a_n^2 + b_n^2\right)}$$

$$F_{rms}^2 = 0.5\Sigma b_n^2 = [1/(2\pi^2)][(16/\pi^2) + 1 + (16/(81\pi^2)) + (1/4) + (16/(625\pi^2))]$$

$$= (1/19.729)(2.6211 + 0.27 + 0.00259)$$

$$F_{rms} = \sqrt{0.14667} = \underline{\mathbf{0.383}}$$

Compare this with the exact value of $F_{rms} = \sqrt{\frac{2}{T}\int_0^1 t^2 dt} = \sqrt{1/6} = 0.4082$ or $(0.383/0.4082)\text{x}100 = 93.83\%$, close.

Chapter 17, Problem 28.

ML Obtain the trigonometric Fourier series for the voltage waveform shown in Fig. 17.66.

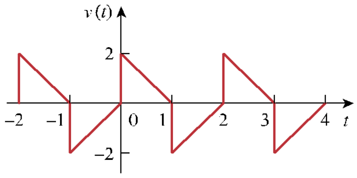


Figure 17.66
For Prob. 17.28.

Chapter 17, Solution 28.

This is half-wave symmetric since f(t – T/2) = –f(t).

$$a_o = 0, \; T = 2, \; \omega_o = 2\pi/2 = \pi$$

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos(n\omega_o t)dt = \frac{4}{2}\int_0^1 (2-2t)\cos(n\pi t)dt$$

$$= 4\left[\frac{1}{n\pi}\sin(n\pi t) - \frac{1}{n^2\pi^2}\cos(n\pi t) - \frac{t}{n\pi}\sin(n\pi t)\right]_0^1$$

$$= [4/(n^2\pi^2)][1-\cos(n\pi)] = \begin{cases} 8/(n^2\pi^2), & n = \text{odd} \\ 0, & n = \text{even}\end{cases}$$

$$b_n = 4\int_0^1 (1-t)\sin(n\pi t)dt$$

$$= 4\left[-\frac{1}{n\pi}\cos(n\pi t) - \frac{1}{n^2\pi^2}\sin(n\pi t) + \frac{t}{n\pi}\cos(n\pi t)\right]_0^1$$

$$= 4/(n\pi), \quad n = \text{odd}$$

$$f(t) = \underline{\sum_{k=1}^{\infty}\left(\frac{8}{n^2\pi^2}\cos(n\pi t) + \frac{4}{n\pi}\sin(n\pi t)\right)}, \; \underline{n = 2k-1}$$

Chapter 17, Problem 29.

Determine the Fourier series expansion of the sawtooth function in Fig. 17.67.

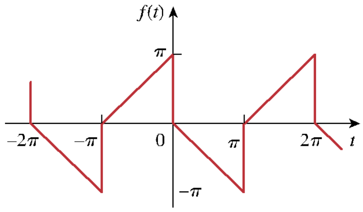


Figure 17.67
For Prob. 17.29.

Chapter 17, Solution 29.

This function is half-wave symmetric.

$$T = 2\pi,\ \omega_o = 2\pi/T = 1,\ f(t) = -t,\ 0 < t < \pi$$

For odd n,
$$a_n = \frac{2}{T}\int_0^{\pi}(-t)\cos(nt)dt = -\frac{2}{n^2\pi}\left[\cos(nt) + nt\sin(nt)\right]_0^{\pi} = 4/(n^2\pi)$$

$$b_n = \frac{2}{\pi}\int_0^{\pi}(-t)\sin(nt)dt = -\frac{2}{n^2\pi}\left[\sin(nt) - nt\cos(nt)\right]_0^{\pi} = -2/n$$

Thus,

$$f(t) = \underline{\mathbf{2\sum_{k=1}^{\infty}\left[\frac{2}{n^2\pi}\cos(nt) - \frac{1}{n}\sin(nt)\right]}}, \qquad \underline{\mathbf{n = 2k-1}}$$

Chapter 17, Problem 30.

(a) If *f*(*t*) is an even function, show that

$$c_n = \frac{2}{T}\int_0^{T/2} f(t)\cos n\omega_o t\, dt$$

(b) If $f(t)$ is an odd function, show that

$$c_n = \frac{j2}{T}\int_0^{T/2} f(t)\sin n\omega_o t\, dt$$

Chapter 17, Solution 30.

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)e^{-jn\omega_o t}dt = \frac{1}{T}\left[\int_{-T/2}^{T/2} f(t)\cos n\omega_o tdt - j\int_{-T/2}^{T/2} f(t)\sin n\omega_o tdt\right] \quad (1)$$

(a) The second term on the right hand side vanishes if f(t) is even. Hence

$$c_n = \frac{2}{T}\int_0^{T/2} f(t)\cos n\omega_o tdt$$

(b) The first term on the right hand side of (1) vanishes if f(t) is odd. Hence,

$$c_n = -\frac{j2}{T}\int_0^{T/2} f(t)\sin n\omega_o tdt$$

Chapter 17, Problem 31.

Let a_n and b_n be the Fourier series coefficients of $f(t)$ and let ω_o be its fundamental frequency. Suppose $f(t)$ is time-scaled to give $h(t) = f(\alpha t)$. Express the $a_n^{'}$ and $b_n^{'}$, and $\omega_o^{'}$, of $h(t)$ in terms of a_n, b_n, and ω_o of $f(t)$.

Chapter 17, Solution 31.

If $h(t) = f(\alpha t)$, $\quad T' = T/\alpha \quad \longrightarrow \quad \omega_o' = \frac{2\pi}{T'} = \frac{2\pi}{T/\alpha} = \underline{\alpha\omega_o}$

$$a_n' = \frac{2}{T'}\int_0^{T'} h(t)\cos n\omega_o' t dt = \frac{2}{T'}\int_0^{T'} f(\alpha t)\cos n\omega_o' t dt$$

Let $\alpha t = \lambda,,\quad dt = d\lambda/\alpha,\quad \alpha T' = T$

$$a_n' = \frac{2\alpha}{T}\int_0^{T} f(\lambda)\cos n\omega_o \lambda d\lambda/\alpha = a_n$$

Similarly, $\quad \underline{b_n' = b_n}$

Chapter 17, Problem 32.

Find $i(t)$ in the circuit of Fig. 17.68 given that

$$i_s(t) = 1 + \sum_{n=1}^{\infty} \frac{1}{n^2} \cos 3nt \text{ A}$$

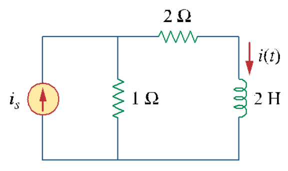


Figure 17.68
For Prob. 17.32.

Chapter 17, Solution 32.

When $i_s = 1$ (DC component)

$$i = 1/(1+2) = 1/3$$

For $n \geq 1$, $\omega_n = 3n, \; I_s = 1/n^2\angle 0°$

$$I = [1/(1+2+j\omega_n{}^2)]I_s = I_s/(3+j6n)$$

$$= \frac{\frac{1}{n^2}\angle 0°}{3\sqrt{1+4n^2}\angle \tan^{-1}(6n/3)} = \frac{1}{3n^2\sqrt{1+4n^2}}\angle -\tan(2n)$$

Thus,

$$\underline{i(t) = \frac{1}{3} + \sum_{n=1}^{\infty} \frac{1}{3n^2\sqrt{1+4n^2}} \cos(3n - \tan^{-1}(2n))}$$

Chapter 17, Problem 33.

In the circuit shown in Fig. 17.69, the Fourier series expansion of $v_s(t)$ is

$$v_s(t) = 3 + \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin(n\pi t)$$

Find $v_o(t)$.

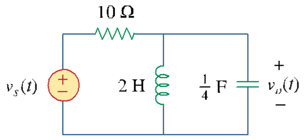


Figure 17.69
For Prob. 17.33.

Chapter 17, Solution 33.

For the DC case, the inductor acts like a short, $V_o = 0$.

For the AC case, we obtain the following:

$$\frac{V_o - V_s}{10} + \frac{V_o}{j2n\pi} + \frac{jn\pi V_o}{4} = 0$$

$$\left(1 + j\left(2.5n\pi - \frac{5}{n\pi}\right)\right)V_o = V_s$$

$$V_o = \frac{V_s}{1 + j\left(2.5n\pi - \frac{5}{n\pi}\right)}$$

$$A_n\angle\Theta_n = \frac{4}{n\pi}\frac{1}{1 + j\left(2.5n\pi - \frac{5}{n\pi}\right)} = \frac{4}{n\pi + j(2.5n^2\pi^2 - 5)}$$

$$A_n = \frac{4}{\sqrt{n^2\pi^2 + (2.5n^2\pi^2 - 5)^2}}; \ \Theta_n = -\tan^{-1}\left(\frac{2.5n^2\pi^2 - 5}{n\pi}\right)$$

$$v_o(t) = \sum_{n=1}^{\infty} A_n \sin(n\pi t + \Theta_n)\,V$$

Chapter 17, Problem 34.

Obtain $v_o(t)$ in the network of Fig. 17.70 if

$$v(t) = \sum_{n=1}^{\infty} \frac{10}{n^2} \cos\left(nt + \frac{n\pi}{4}\right) \text{ V}$$

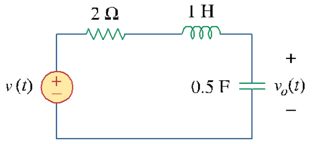
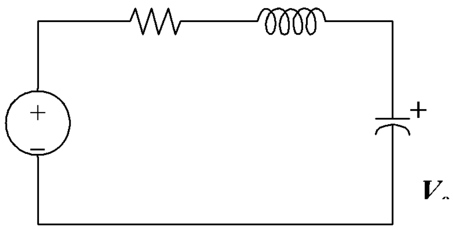


Figure 17.70
For Prob. 17.34.

Chapter 17, Solution 34.

For any n, $V = [10/n^2]\angle(n\pi/4)$, $\omega = n$.

1 H becomes $j\omega_n L = jn$ and 0.5 F becomes $1/(j\omega_n C) = -j2/n$



$V_o = \{-j(2/n)/[2 + jn - j(2/n)]\}V = \{-j2/[2n + j(n^2 - 2)]\}[(10/n^2)\angle(n\pi/4)]$

$$= \frac{20\angle((n\pi/4) - \pi/2)}{n^2\sqrt{4n^2 + (n^2-2)^2}\angle\tan^{-1}((n^2-2)/2n)}$$

$$= \frac{20}{n^2\sqrt{n^2+4}}\angle[(n\pi/4) - (\pi/2) - \tan^{-1}((n^2-2)/2n)]$$

$$v_o(t) = \underline{\boldsymbol{\sum_{n=1}^{\infty} \frac{20}{n^2\sqrt{n^2+4}} \cos\left(nt + \frac{n\pi}{4} - \frac{\pi}{2} - \tan^{-1}\frac{n^2-2}{2n}\right)}}$$

Chapter 17, Problem 35.

If v_s in the circuit of Fig. 17.71 is the same as function $f_2(t)$ in Fig. 17.56(b), determine the dc component and the first three nonzero harmonics of $v_o(t)$.

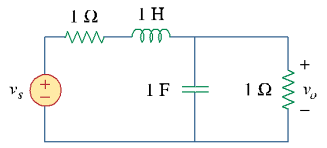


Figure 17.71
For Prob. 17.35.

Chapter 17, Solution 35.

If v_s in the circuit of Fig. 17.72 is the same as function $f_2(t)$ in Fig. 17.57(b), determine the dc component and the first three nonzero harmonics of $v_o(t)$.

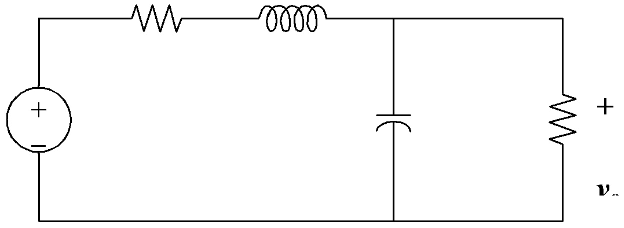


Figure 16.64 For Prob. 16.25

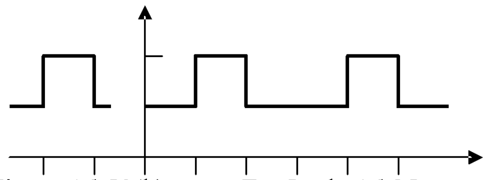


Figure 16.50(b) For Prob. 16.25

The signal is even, hence, $b_n = 0$. In addition, $T = 3$, $\omega_o = 2\pi/3$.

$$v_s(t) = \begin{cases} 1 & \text{for all } 0 < t < 1 \\ 2 & \text{for all } 1 < t < 1.5 \end{cases}$$

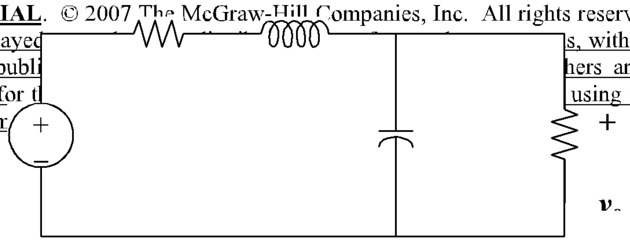
$$a_o = \frac{2}{3}\left[\int_0^1 1dt + \int_1^{1.5} 2dt\right] = \frac{4}{3}$$

$$a_n = \frac{4}{3}\left[\int_0^1 \cos(2n\pi t/3)dt + \int_1^{1.5} 2\cos(2n\pi t/3)dt\right]$$

$$= \frac{4}{3}\left[\frac{3}{2n\pi}\sin(2n\pi t/3)\Big|_0^1 + \frac{6}{2n\pi}\sin(2n\pi t/3)\Big|_1^{1.5}\right] = -\frac{2}{n\pi}\sin(2n\pi/3)$$

$$v_s(t) = \frac{4}{3} - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin(2n\pi/3)\cos(2n\pi t/3)$$

Now consider this circuit,



Let Z = $[-j3/(2n\pi)](1)/(1 - j3/(2n\pi)) = -j3/(2n\pi - j3)$

Therefore, $v_o = Zv_s/(Z + 1 + j2n\pi/3)$. Simplifying, we get

$$v_o = \frac{-j9v_s}{12n\pi + j(4n^2\pi^2 - 18)}$$

For the dc case, n = 0 and $v_s = 3/4$ V and $v_o = v_s/2 = 3/8$ V.

We can now solve for $v_o(t)$

$$v_o(t) = \underline{\left[\frac{3}{8} + \sum_{n=1}^{\infty} A_n \cos\left(\frac{2n\pi t}{3} + \Theta_n\right)\right]\text{volts}}$$

$$\text{where } A_n = \frac{\frac{6}{n\pi}\sin(2n\pi/3)}{\sqrt{16n^2\pi^2 + \left(\frac{4n^2\pi^2}{3} - 6\right)^2}} \text{ and } \Theta_n = 90^\circ - \tan^{-1}\left(\frac{n\pi}{3} - \frac{3}{2n\pi}\right)$$

where we can further simplify A_n to this, $\underline{A_n = \frac{9\sin(2n\pi/3)}{n\pi\sqrt{4n^4\pi^4 + 81}}}$

Chapter 17, Problem 36.

* Find the response i_o for the circuit in Fig. 17.72(a), where $v(t)$ is shown in Fig. 17.72(b).

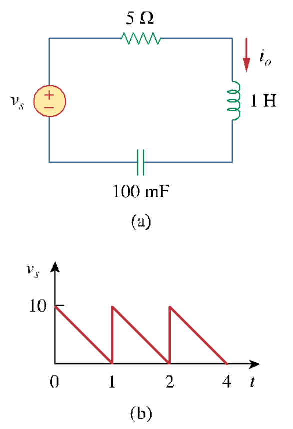


Figure 17.72
For Prob. 17.36.

* An asterisk indicates a challenging problem.

Chapter 17, Solution 36.

We first find the Fourier series expansion of v_s. $T = 1, \quad \omega_o = 2\pi / T = 2\pi$

$$a_0 = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{2}\int_0^1 10(1-t)tdt = 10(t - \frac{t^2}{2})\Big|_1^0 = 5$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_o tdt = 2\int_0^1 10(1-t)\cos 2n\pi tdt$$

$$= 20\left[\frac{1}{2\pi n}\sin 2n\pi t - \frac{1}{4n^2\pi^2}\cos 2n\pi t - \frac{t}{2n\pi}\sin 2n\pi t\right]\Big|_o^1 = 0$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega_o tdt = \frac{2}{2}\int_0^1 10(1-t)t\sin n\omega_o tdt$$

$$= 20\left[-\frac{1}{2n\pi}\cos 2n\pi t - \frac{1}{4n^2\pi^2}\sin 2n\pi t + \frac{1}{2n\pi}\cos 2n\pi t\right]\Big|_0^1 = \frac{10}{n\pi}$$

$$v_s(t) = 5 + \sum_{n=1}^{\infty}\frac{10}{n\pi}\sin 2n\pi t$$

$$1H \longrightarrow j\omega_n L = j\omega_n$$

$$10mF \longrightarrow \frac{1}{j\omega_n C} = \frac{1}{j\omega_n 0.01} = \frac{-j100}{\omega_n}$$

$$I_o = \frac{V_s}{5 + j\omega_n - \frac{j100}{\omega_n}}$$

For dc component, $\omega_0 = 0$ which leads to $I_0 = 0$.
For the nth harmonic,

$$I_n = \frac{\frac{10}{n\pi}\angle 0^\circ}{5 + j2n\pi - \frac{j100}{2n\pi}} = \frac{10}{5n\pi + j(2n^2\pi^2 - 50)} = A_n\angle\phi_n$$

where

$$A_n = \frac{10}{\sqrt{25n^2\pi^2 + (2n^2\pi^2 - 50)^2}}, \qquad \phi_n = -\tan^{-1}\frac{2n^2\pi^2 - 50}{5n\pi}$$

$$i_o(t) = \sum_{n=1}^{\infty} A_n \sin(2n\pi t + \phi_n)$$

Chapter 17, Problem 37.

If the periodic current waveform in Fig. 17.73(a) is applied to the circuit in Fig. 17.73(b), find v_o.

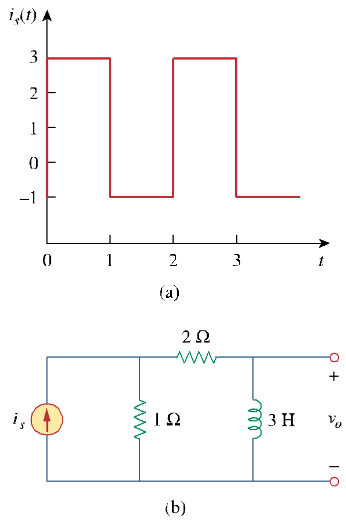


Figure 17.73
For Prob. 17.37.

Chapter 17, Solution 37.

We first need to express i_s in Fourier series. $T = 2, \quad \omega_o = 2\pi / T = \pi$

$$a_o = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{2}\left[\int_0^1 3dt + \int_1^2 1dt\right] = \frac{1}{2}(3+1) = 2$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_o t dt = \frac{2}{2}\left[\int_0^1 3\cos n\pi t dt + \int_1^2 \cos n\pi t dt\right] = \frac{3}{n\pi}\sin n\pi t\Big|_0^1 + \frac{1}{n\pi}\sin n\pi t\Big|_1^2 = 0$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega_o t dt = \frac{2}{2}\left[\int_0^1 3\sin n\pi t dt + \int_1^2 \sin n\pi t dt\right] = \frac{-3}{n\pi}\cos n\pi t\Big|_0^1 + \frac{-1}{n\pi}\cos n\pi t\Big|_1^2 = \frac{2}{n\pi}(1-\cos n\pi)$$

$$i_s(t) = 2 + \sum_{n=1}^{\infty}\frac{2}{n\pi}(1-\cos n\pi)\sin n\pi t$$

By current division,

$$I_o = \frac{1}{1+2+j\omega_n L} I_s = \frac{I_s}{3+j3\omega_n}$$

$$V_o = j\omega_n L I_o = \frac{j\omega_n 3 I_s}{3+j3\omega_n} = \frac{j\omega_n I_s}{1+j\omega_n}$$

For dc component (n=0), $V_o = 0$.
For the nth harmonic,

$$V_o = \frac{jn\pi}{1+jn\pi}\frac{2}{n\pi}(1-\cos n\pi)\angle -90° = \frac{2(1-\cos n\pi)}{\sqrt{1+n^2\pi^2}}\angle(90° - \tan^{-1} n\pi - 90°)$$

$$\underline{v_o(t) = \sum_{n=1}^{\infty}\frac{2(1-\cos \pi n)}{\sqrt{1+n^2\pi^2}}\cos(n\pi t - \tan^{-1} n\pi)}$$

Chapter 17, Problem 38.

If the square wave shown in Fig. 17.74(a) is applied to the circuit in Fig. 17.74(b), find the Fourier series for $v_o(t)$.

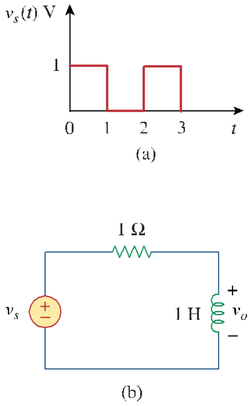


Figure 17.74
For Prob. 17.38.

Chapter 17, Solution 38.

$$v_s(t) = \frac{1}{2} + \frac{2}{\pi}\sum_{k=1}^{\infty}\frac{1}{n}\sin n\pi t, \qquad n = 2k+1$$

$$V_o = \frac{j\omega_n}{1 + j\omega_n}V_s, \qquad \omega_n = n\pi$$

For dc, $\omega_n = 0, \quad V_s = 0.5, \quad V_o = 0$

For nth harmonic, $V_s = \frac{2}{n\pi}\angle -90^o$

$$V_o = \frac{n\pi\angle 90^o}{\sqrt{1+n^2\pi^2}\angle \tan^{-1} n\pi} \bullet \frac{2}{n\pi}\angle 90^o = \frac{2\angle -\tan^{-1} n\pi}{\sqrt{1+n^2\pi^2}}$$

$$v_o(t) = \sum_{k=1}^{\infty}\frac{2}{\sqrt{1+n^2\pi^2}}\cos(n\pi t - \tan^{-1} n\pi), \qquad n = 2k-1$$

Chapter 17, Problem 39.

If the periodic voltage in Fig. 17.75(a) is applied to the circuit in Fig. 17.75(b), find $i_o(t)$.

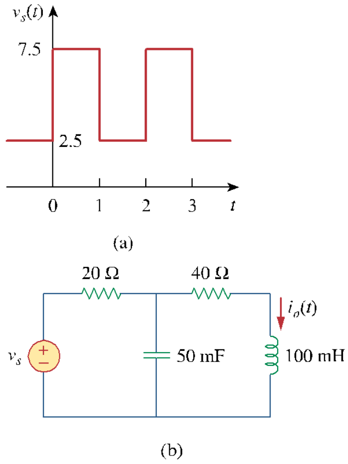


Figure 17.75
For Prob. 17.39.

Chapter 17, Solution 39.

Comparing $v_s(t)$ with f(t) in Figure 15.1, v_s is shifted by 2.5 and the magnitude is 5 times that of f(t).

Hence

$$v_s(t) = 5 + \frac{10}{\pi}\sum_{k=1}^{\infty}\frac{1}{n}\sin(n\pi t), \qquad n = 2k-1$$

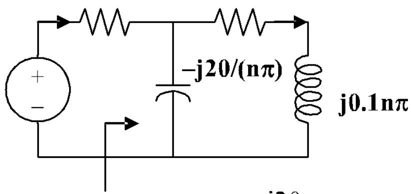
$$T = 2, \quad \omega_o = 2\pi//T = \pi, \quad \omega_n = n\omega_o = n\pi$$

For the DC component, $\quad i_o = 5/(20+40) = 1/12$

For the kth harmonic, $\quad V_s = (10/(n\pi))\angle 0°$

$$100\text{ mH becomes } j\omega_n L = jn\pi x0.1 = j0.1n\pi$$

$$50\text{ mF becomes } 1/(j\omega_n C) = -j20/(n\pi)$$



$$\text{Let } Z = -j20/(n\pi)||(40 + j0.1n\pi) = \frac{-\frac{j20}{n\pi}(40 + j0.1n\pi)}{-\frac{j20}{n\pi} + 40 + j0.1n\pi}$$

$$= \frac{-j20(40 + j0.1n\pi}{-j20 + 40n\pi + j0.1n^2\pi^2} = \frac{2n\pi - j800}{40n\pi + j(0.1n^2\pi^2 - 20)}$$

$$Z_{in} = 20 + Z = \frac{802n\pi + j(2n^2\pi^2 - 1200)}{40n\pi + j(0.1n^2\pi^2 - 20)}$$

$$I = \frac{V_s}{Z_{in}} = \frac{400n\pi + j(n^2\pi^2 - 200)}{n\pi[802n\pi + j(2n^2\pi^2 - 1200)]}$$

$$I_o = \frac{-\frac{j20}{n\pi}I}{-\frac{j20}{n\pi} + (40 + j0.1n\pi)} = \frac{-j20I}{40n\pi + j(0.1n^2\pi^2 - 20)}$$

$$= \frac{-j200}{n\pi[802n\pi + j(2n^2\pi^2 - 1200)]}$$

$$= \frac{200\angle -90° - \tan^{-1}\{(2n^2\pi^2 - 1200)/(802n\pi)\}}{n\pi\sqrt{(802)^2 + (2n^2\pi^2 - 1200)^2}}$$

Thus

$$\underline{\mathbf{i_o(t) = \frac{1}{20} + \frac{200}{\pi}\sum_{k=1}^{\infty} I_n \sin(n\pi t - \theta_n)}}, \qquad \underline{\mathbf{n = 2k - 1}}$$

where $\theta_n = 90° + \tan^{-1}\frac{2n^2\pi^2 - 1200}{802n\pi}$

$$I_n = \frac{1}{n\sqrt{(804n\pi)^2 + (2n^2\pi^2 - 1200)}}$$

Chapter 17, Problem 40.

* The signal in Fig. 17.76(a) is applied to the circuit in Fig. 17.76(b). Find $v_o(t)$.

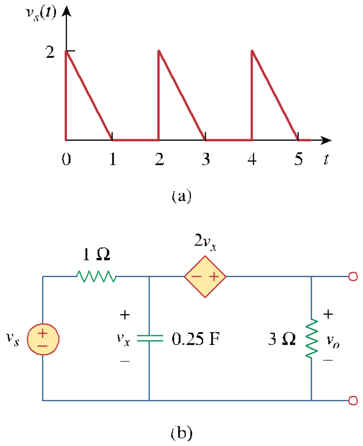


Figure 17.76
For Prob. 17.40.

* An asterisk indicates a challenging problem.

Chapter 17, Solution 40.

$$T = 2, \quad \omega_o = 2\pi/T = \pi$$

$$a_o = \frac{1}{T}\int_0^T v(t)dt = \frac{1}{2}\int_0^1 (2-2t)dt = \left[t - \frac{t^2}{2}\right]_0^1 = 1/2$$

$$a_n = \frac{2}{T}\int_0^T v(t)\cos(n\pi t)dt = \int_0^1 2(1-t)\cos(n\pi t)dt$$

$$= 2\left[\frac{1}{n\pi}\sin(n\pi t) - \frac{1}{n^2\pi^2}\cos(n\pi t) - \frac{t}{n\pi}\sin(n\pi t)\right]_0^1$$

$$= \frac{2}{n^2\pi^2}(1-\cos n\pi) = \begin{cases} 0, & n = \text{even} \\ \dfrac{4}{n^2\pi^2}, & n = \text{odd} \end{cases} = \frac{4}{\pi^2(2n-1)^2}$$

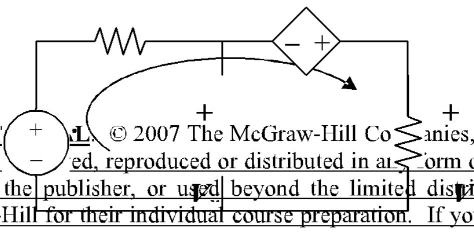
$$b_n = \frac{2}{T}\int_0^T v(t)\sin(n\pi t)dt = 2\int_0^1 (1-t)\sin(n\pi t)dt$$

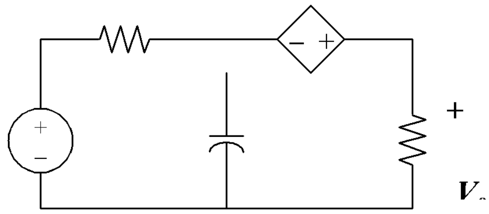
$$= 2\left[-\frac{1}{n\pi}\cos(n\pi t) - \frac{1}{n^2\pi^2}\sin(n\pi t) + \frac{t}{n\pi}\cos(n\pi t)\right]_0^1 = \frac{2}{n\pi}$$

$$v_s(t) = \frac{1}{2} + \sum A_n\cos(n\pi t - \varphi_n)$$

$$\text{where } \phi_n = \tan^{-1}\frac{\pi(2n-1)^2}{2n}, \quad A_n = \sqrt{\frac{4}{n^2\pi^2} + \frac{16}{\pi^4(2n-1)^4}}$$

For the DC component, $v_s = 1/2$. As shown in Figure (a), the capacitor acts like an open circuit.





Applying KVL to the circuit in Figure (a) gives

$$-0.5 - 2V_x + 4i = 0 \qquad (1)$$

But $$-0.5 + i + V_x = 0 \text{ or } -1 + 2V_x + 2i = 0 \qquad (2)$$

Adding (1) and (2), $$-1.5 + 6i = 0 \text{ or } i = 0.25$$

$$V_o = 3i = 0.75$$

For the nth harmonic, we consider the circuit in Figure (b).

$$\omega_n = n\pi,\ V_s = A_n\angle -\phi,\ 1/(j\omega_n C) = -j4/(n\pi)$$

At the supernode,

$$(V_s - V_x)/1 = -[n\pi/(j4)]V_x + V_o/3$$

$$V_s = [1 + jn\pi/4]V_x + V_o/3 \qquad (3)$$

But $-V_x - 2V_x + V_o = 0$ or $V_o = 3V_x$

Substituting this into (3),

$$V_s = [1 + jn\pi/4]V_x + V_x = [2 + jn\pi/4]V_x$$

$$= (1/3)[2 + jn\pi/4]V_o = (1/12)[8 + jn\pi]V_o$$

$$V_o = 12V_s/(8 + jn\pi) = \frac{12A_n\angle -\phi}{\sqrt{64 + n^2\pi^2}\angle \tan^{-1}(n\pi/8)}$$

V_o =

$$\frac{12}{\sqrt{64 + n^2\pi^2}}\sqrt{\frac{4}{n^2\pi^2} + \frac{16}{\pi^4(2n-1)^4}}\angle[\tan^{-1}(n\pi/8) - \tan^{-1}(\pi(2n-1)/(2n))]$$

Thus

$$v_o(t) = \underline{\mathbf{\frac{3}{4} + \sum_{n=1}^{\infty} V_n \cos(n\pi t + \theta_n)}}$$

where

$$V_n = \underline{\mathbf{\frac{12}{\sqrt{64 + n^2\pi^2}}\sqrt{\frac{4}{n^2\pi^2} + \frac{16}{\pi^4(2n-1)^4}}}}$$

$$\theta_n = \underline{\mathbf{\tan^{-1}(n\pi/8) - \tan^{-1}(\pi(2n-1)/(2n))}}$$

Chapter 17, Problem 41.

The full-wave rectified sinusoidal voltage in Fig. 17.77(a) is applied to the lowpass filter in Fig. 17.77(b). Obtain the output voltage $v_o(t)$ of the filter.

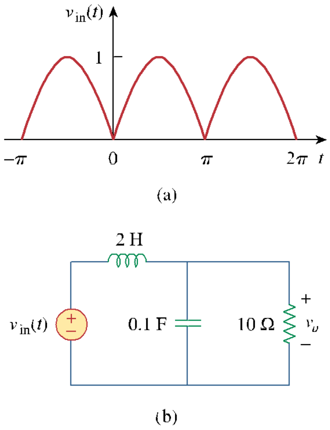


Figure 17.77
For Prob. 17.41.

Chapter 17, Solution 41.

For the full wave rectifier,

$T = \pi,\ \omega_o = 2\pi/T = 2,\ \omega_n = n\omega_o = 2n$

Hence

$$v_{in}(t) = \frac{2}{\pi} - \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{4n^2 - 1}\cos(2nt)$$

For the DC component,

$V_{in} = 2/\pi$

The inductor acts like a short-circuit, while the capacitor acts like an open circuit.

$V_o = V_{in} = 2/\pi$

For the nth harmonic,

$V_{in} = [-4/(\pi(4n^2 - 1))]\angle 0°$

2 H becomes $j\omega_n L = j4n$

0.1 F becomes $1/(j\omega_n C) = -j5/n$

$Z = 10||(-j5/n) = -j10/(2n - j)$

$V_o = [Z/(Z + j4n)]V_{in} = -j10V_{in}/(4 + j(8n - 10))$

$$= -\frac{j10}{4 + j(8n - 10)}\left(-\frac{4\angle 0°}{\pi(4n^2 - 1)}\right)$$

$$= \frac{40\angle\{90° - \tan^{-1}(2n - 2.5)\}}{\pi(4n^2 - 1)\sqrt{16 + (8n - 10)^2}}$$

Hence $\quad v_o(t) = \underline{\mathbf{\frac{2}{\pi} + \sum_{n=1}^{\infty} A_n \cos(2nt + \theta_n)}}$

where

$$A_n = \underline{\mathbf{\frac{20}{\pi(4n^2 - 1)\sqrt{16n^2 - 40n + 29}}}}$$

$\theta_n = \underline{\mathbf{90° - \tan^{-1}(2n - 2.5)}}$

Chapter 17, Problem 42.

The square wave in Fig. 17.78(a) is applied to the circuit in Fig. 17.78(b). Find the Fourier series of $v_o(t)$.

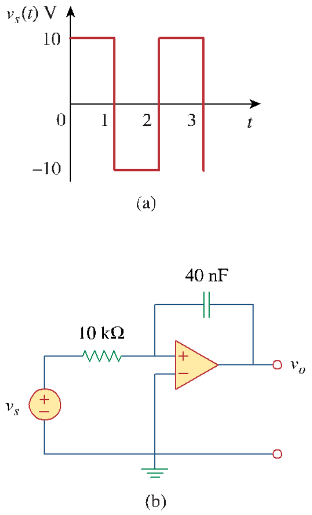


Figure 17.78
For Prob. 17.42.

Chapter 17, Solution 42.

$$v_s = 5 + \frac{20}{\pi}\sum_{k=1}^{\infty}\frac{1}{n}\sin n\pi t, \quad n = 2k - 1$$

$$\frac{V_s - 0}{R} = j\omega_n C(0 - V_o) \qquad \longrightarrow \qquad V_o = \frac{j}{\omega_n RC} V_s, \quad \omega_n = n\omega_o = n\pi$$

For n = 0 (dc component), $V_o = 0$.

For the nth harmonic,

$$V_o = \frac{1\angle 90^o}{n\pi RC} \frac{20}{n\pi} \angle -90^o = \frac{20}{n^2\pi^2 x10^4 x40x10^{-9}} = \frac{10^5}{2n^2\pi^2}$$

Hence,

$$\underline{v_o(t) = \frac{10^5}{2\pi^2} \sum_{k=1}^{\infty} \frac{1}{n^2} \cos n\pi t, \ n = 2k - 1}$$

Alternatively, we notice that this is an integrator so that

$$v_o(t) = -\frac{1}{RC} \int v_s dt = \underline{\frac{10^5}{2\pi^2} \sum_{k=1}^{\infty} \frac{1}{n^2} \cos n\pi t, \ n = 2k - 1}$$

Chapter 17, Problem 43.

The voltage across the terminals of a circuit is

$v(t) = 30 + 20\cos(60\pi t + 45°)$
$+ 10\cos(60\pi t - 45°)$ V

If the current entering the terminal at higher potential is

$i(t) = 6 + 4\cos(60\pi t + 10°)$
$- 2\cos(120\pi t - 60°)$ A

find:

(a) the rms value of the voltage,
(b) the rms value of the current,
(c) the average power absorbed by the circuit.

Chapter 17, Solution 43.

(a) $$V_{rms} = \sqrt{a_0^2 + \frac{1}{2}\sum_{n=1}^{\infty}(a_n^2 + b_n^2)} = \sqrt{30^2 + \frac{1}{2}(20^2 + 10^2)} = \underline{\textbf{33.91 V}}$$

(b) $$I_{rms} = \sqrt{6^2 + \frac{1}{2}(4^2 + 2^2)} = \underline{\textbf{6.782 A}}$$

(c) $$P = V_{dc}I_{dc} + \frac{1}{2}\sum V_n I_n \cos(\Theta_n - \Phi_n)$$

$$= 30x6 + 0.5[20x4\cos(45^o\text{-}10^o) - 10x2\cos(\text{-}45^o\text{+}60^o)]$$

$$= 180 + 32.76 - 9.659 = \underline{\textbf{203.1 W}}$$

Chapter 17, Problem 44.

The voltage and current through an element are, respectively,

$$v(t) = 30\cos(t + 25°) + 10\cos(2t + 35°)$$
$$+ 4\cos(3t - 10°) \text{ V}$$

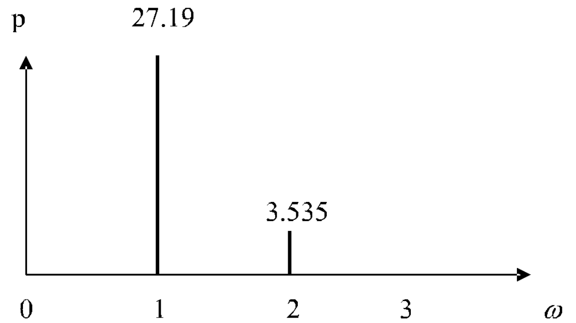
$$i(t) = 2\cos t + \cos(2t + 10°) \text{ A}$$

(a) Find the average power delivered to the element.
(b) Plot the power spectrum.

Chapter 17, Solution 44.

(a) $\quad p = vi = \frac{1}{2}\left[60\cos 25^o + 10\cos 45^o + 0\right] = 27.19 + 3.535 + 0 = \underline{30.73\text{ W}}$

(b) The power spectrum is shown below.



Chapter 17, Problem 45.

A series *RLC* circuit has $R = 10\Omega$, $L = 2$ mH, and $C = 40\ \mu$F. Determine the effective current and average power absorbed when the applied voltage is
$v(t) = 100 \cos 1000t + 50 \cos 2000t$
$+ 25 \cos 3000t$ V

Chapter 17, Solution 45.

$\omega_n = 1000n$

$j\omega_n L = j1000n\text{x}2\text{x}10^{-3} = j2n$

$1/(j\omega_n C) = -j/(1000n\text{x}40\text{x}10^{-6}) = -j25/n$

$Z = R + j\omega_n L + 1/(j\omega_n C) = 10 + j2n - j25/n$

$I = V/Z$

For $n = 1$, $V_1 = 100$, $Z = 10 + j2 - j25 = 10 - j23$

$I_1 = 100/(10 - j23) = 3.987\angle 73.89°$

For $n = 2$, $V_2 = 50$, $Z = 10 + j4 - j12.5 = 10 - j8.5$

$I_2 = 50/(10 - j8.5) = 3.81\angle 40.36°$

For $n = 3$, $V_3 = 25$, $Z = 10 + j6 - j25/3 = 10 - j2.333$

$I_3 = 25/(10 - j2.333) = 2.435\angle 13.13°$

$I_{rms} = \sqrt{0.5(3.987^2 + 3.81^2 + 2.435^2)} = \underline{\mathbf{4.263\ A}}$

$p = R(I_{rms})^2 = \underline{\mathbf{181.7W}}$

Chapter 17, Problem 46.

ML Use *MATLAB* to plot the following sinusoids for $0 < t < 5$:

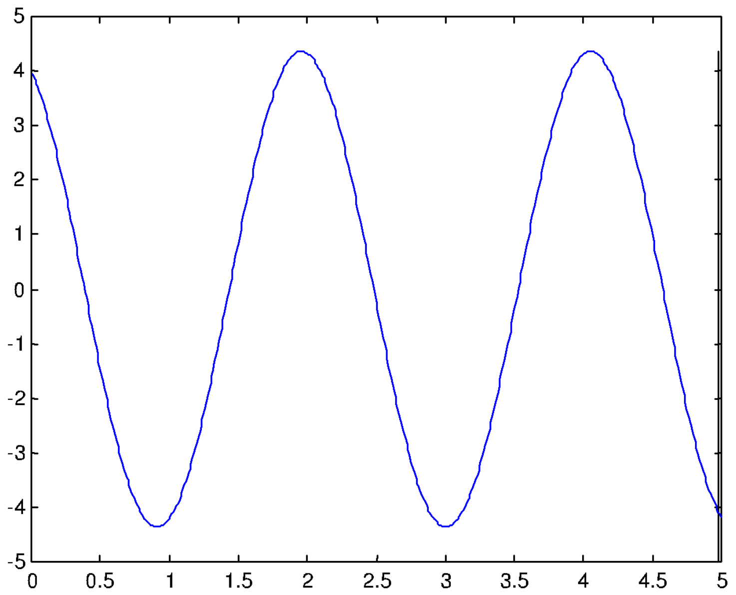
(a) $5 \cos 3t - 2 \cos(3t - \pi/3)$

(b) $8 \sin(\pi t + \pi/4) + 10 \cos(\pi t - \pi/8)$

Chapter 17, Solution 46.

(a)The MATLAB commands are:

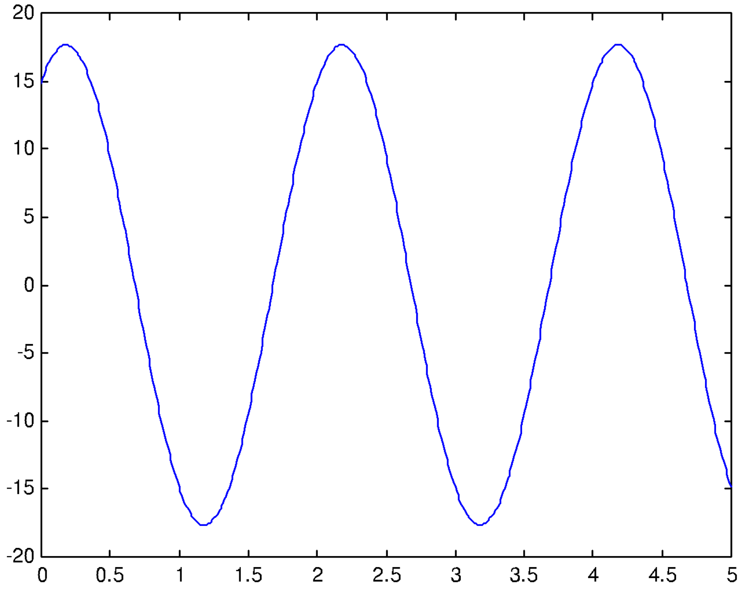
```
t=0:0.01:5;
y=5*cos(3*t) – 2*cos(3*t-pi/3);
plot(t,y)
```



(b) The MATLAB commands are:

```
t=0:0.01:5;
```

```
» x=8*sin(pi*t+pi/4)+10*cos(pi*t-pi/8);
» plot(t,x)
» plot(t,x)
```



Chapter 17, Problem 47.

The periodic current waveform in Fig. 17.79 is applied across a 2-kΩ resistor. Find the percentage of the total average power dissipation caused by the dc component.

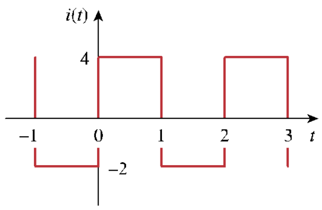


Figure 17.79
For Prob. 17.47.

Chapter 17, Solution 47.

$T = 2, \quad \omega_o = 2\pi / T = \pi$

$$a_o = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{2}\left[\int_0^1 4dt + \int_1^2 (-2)dt\right] = \frac{1}{2}(4-2) = 1$$

$$P = RI_{rms}^2 = \frac{R}{T}\int_0^T f^2(t)dt = \frac{R}{2}\left[\int_0^1 4^2 dt + \int_1^2 (-2)^2 dt\right] = 10R$$

The average power dissipation caused by the dc component is

$$P_0 = Ra_o^2 = R = \underline{10\%} \text{ of P}$$

Chapter 17, Problem 48.

For the circuit in Fig. 17.80,

$$i(t) = 20 + 16\cos(10t + 45°)$$
$$+ 12\cos(20t - 60°)\text{ mA}$$

(a) find $v(t)$, and
(b) calculate the average power dissipated in the resistor.

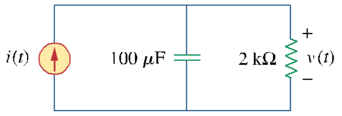


Figure 17.80
For Prob. 17.48.

Chapter 17, Solution 48.

(a) For the DC component, i(t) = 20 mA. The capacitor acts like an open circuit so that

$$v = Ri(t) = 2\text{x}10^3\text{x}20\text{x}10^{-3} = 40$$

For the AC component,

$$\omega_n = 10n, \quad n = 1,2$$

$$1/(j\omega_n C) = -j/(10n\text{x}100\text{x}10^{-6}) = (-j/n)\text{ k}\Omega$$

$$Z = 2||(-j/n) = 2(-j/n)/(2 - j/n) = -j2/(2n - j)$$

$$V = ZI = [-j2/(2n - j)]I$$

For n = 1, $V_1 = [-j2/(2 - j)]16\angle 45° = 14.311\angle -18.43°$ mV

For n = 2, $V_2 = [-j2/(4 - j)]12\angle -60° = 5.821\angle -135.96°$ mV

v(t) = **<u>40 + 0.014311cos(10t – 18.43°) + 0.005821cos(20t – 135.96°) V</u>**

(b) $$p = V_{DC}I_{DC} + \frac{1}{2}\sum_{n=1}^{\infty} V_n I_n \cos(\theta_n - \phi_n)$$

$$= 20\text{x}40 + 0.5\text{x}10\text{x}0.014311\cos(45° + 18.43°)$$
$$+0.5\text{x}12\text{x}0.005821\cos(-60° + 135.96°)$$

= **<u>800.1 mW</u>**

Chapter 17, Problem 49.

(a) For the periodic waveform in Prob. 17.5, find the rms value.

(b) Use the first five harmonic terms of the Fourier series in Prob. 17.5 to determine the effective value of the signal.

(c) Calculate the percentage error in the estimated rms value of $z(t)$ if

$$\%\text{ error} = \left(\frac{\text{estimated value}}{\text{exact value}} - 1\right) \times 100$$

Chapter 17, Solution 49.

(a) $$Z^2_{rms} = \frac{1}{T}\int_0^T z^2(t)dt = \frac{1}{2\pi}\left[\int_0^{\pi} 1dt + \int_{\pi}^{2\pi} 4dt\right] = \frac{1}{2\pi}(5\pi) = 2.5$$

$$\underline{Z_{rms} = 1.581}$$

(b)

$$Z^2_{rms} = a_0^2 + \frac{1}{2}\sum_{n=1}^{\infty}(a_n^2 + b_n^2) = \frac{1}{4} + \frac{1}{2}\sum_{\substack{n=1\\ n=odd}}^{\infty}\frac{36}{n^2\pi^2} = \frac{1}{4} + \frac{18}{\pi^2}\left(1 + 0 + \frac{1}{9} + 0 + \frac{1}{25} + ...\right) = 2.349$$

Z_{rms} = **1.5326**

(c) $$\%\text{error} = \left(1 - \frac{1.5326}{1.581}\right) x100 = \underline{\mathbf{3.061\%}}$$

Chapter 17, Problem 50.

Obtain the exponential Fourier series for $f(t) = t$, $-1 < t < 1$, with $f(t + 2n) = f(t)$ for all integer values of n.

Chapter 17, Solution 50.

$$c_n = \frac{1}{T}\int_0^T f(t)e^{-j\omega_o nt}dt, \qquad \omega_o = \frac{2n}{1} = \pi$$

$$= \frac{1}{2}\int_{-1}^{1} te^{-jn\pi t}dt$$

Using integration by parts,

$$u = t \text{ and } du = dt$$

$$dv = e^{-jn\pi t}dt \text{ which leads to } v = -[1/(2jn\pi)]e^{-jn\pi t}$$

$$c_n = -\frac{t}{2jn\pi}e^{-jn\pi t}\Big|_{-1}^{1} + \frac{1}{2jn\pi}\int_{-1}^{1} e^{-jn\pi t}dt$$

$$= \frac{j}{n\pi}\left[e^{-jn\pi} + e^{jn\pi}\right] + \frac{1}{2n^2\pi^2(-j)^2}e^{-jn\pi t}\Big|_{-1}^{1}$$

$$= [j/(n\pi)]\cos(n\pi) + [1/(2n^2\pi^2)](e^{-jn\pi} - e^{jn\pi})$$

$$c_n = \frac{j(-1)^n}{n\pi} + \frac{2j}{2n^2\pi^2}\sin(n\pi) = \frac{j(-1)^n}{n\pi}$$

Thus

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_o t} = \underline{\sum_{n=-\infty}^{\infty} (-1)^n \frac{j}{n\pi} e^{jn\pi t}}$$

Chapter 17, Problem 51.

Given the periodic function

$f(t) = t^2, \qquad 0 < t < T$

obtain the exponential Fourier series for the special case $T = 2$.

Chapter 17, Solution 51.

$$T = 2, \qquad \omega_o = 2\pi / T = \pi$$

$$c_n = \frac{1}{T}\int_0^T f(t)e^{-jn\omega_o t}dt = \frac{1}{2}\int_0^2 t^2 e^{-jn\pi t}dt = \frac{1}{2}\frac{e^{-jn\pi t}}{(-jn\pi)^3}\left(-n^2\pi^2 t^2 + 2jn\pi t + 2\right)\Big|_0^2$$

$$c_n = \frac{1}{j2n^3\pi^3}(-4n^2\pi^2 + j4n\pi) = \frac{2}{n^2\pi^2}(1 + jn\pi)$$

$$f(t) = \sum_{n=-\infty}^{\infty} \frac{2}{n^2\pi^2}(1 + jn\pi)e^{jn\pi t}$$

Chapter 17, Problem 52.

Calculate the complex Fourier series for $f(t) = e^t$, $-\pi < t < \pi$, with $f(t + 2\pi n) = f(t)$ for all integer values of n.

Chapter 17, Solution 52.

$$c_n = \frac{1}{T}\int_0^T f(t)e^{-j\omega_o nt}dt, \qquad \omega_o = \frac{2n}{1} = \pi$$

$$= \frac{1}{2}\int_{-1}^{1} te^{-jn\pi t}dt$$

Using integration by parts,

$$u = t \text{ and } du = dt$$

$$dv = e^{-jn\pi t}dt \text{ which leads to } v = -[1/(2jn\pi)]e^{-jn\pi t}$$

$$c_n = -\frac{t}{2jn\pi}e^{-jn\pi t}\Big|_{-1}^{1} + \frac{1}{2jn\pi}\int_{-1}^{1} e^{-jn\pi t}dt$$

$$= \frac{j}{n\pi}\left[e^{-jn\pi} + e^{jn\pi t}\right] + \frac{1}{2n^2\pi^2(-j)^2}e^{-jn\pi t}\Big|_{-1}^{1}$$

$$= [j/(n\pi)]\cos(n\pi) + [1/(2n^2\pi^2)](e^{-jn\pi} - e^{jn\pi})$$

$$c_n = \frac{j(-1)^n}{n\pi} + \frac{2j}{2n^2\pi^2}\sin(n\pi) = \frac{j(-1)^n}{n\pi}$$

Thus

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_o t} = \underline{\sum_{n=-\infty}^{\infty} (-1)^n \frac{j}{n\pi} e^{jn\pi t}}$$

Chapter 17, Problem 53.

Find the complex Fourier series for $f(t) = e^{-t}$, $0 < t < 1$, with $f(t + n) = f(t)$ for all integer values of n.

Chapter 17, Solution 53.

$$\omega_o = 2\pi/T = 2\pi$$

$$c_n = \int_0^T e^{-t} e^{-jn\omega_o t} dt = \int_0^1 e^{-(1+jn\omega_o)t} dt$$

$$= \frac{-1}{1 + j2\pi n} e^{-(1+j2n\pi)t}\Big|_0^1 = \frac{-1}{1 + j2n\pi}\left[e^{-(1+j2n\pi)} - 1\right]$$

$$= [1/(j2n\pi)][1 - e^{-1}(\cos(2\pi n) - j\sin(2n\pi))]$$

$$= (1 - e^{-1})/(1 + j2n\pi) = 0.6321/(1 + j2n\pi$$

$$f(t) = \underline{\sum_{n=-\infty}^{\infty} \frac{0.6321 e^{j2n\pi t}}{1 + j2n\pi}}$$

Chapter 17, Problem 54.

Find the exponential Fourier series for the function in Fig. 17.81.

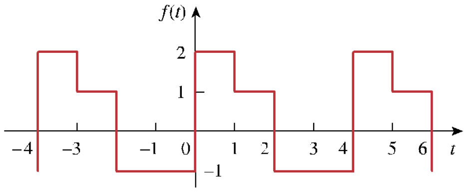


Figure 17.81
For Prob. 17.54.

Chapter 17, Solution 54.

$$T = 4, \ \omega_o = 2\pi/T = \pi/2$$

$$c_n = \frac{1}{T}\int_0^T f(t)e^{-j\omega_o nt}dt$$

$$= \frac{1}{4}\left[\int_0^1 2e^{-jn\pi t/2}dt + \int_1^2 1e^{-jn\pi t/2}dt - \int_2^4 1e^{-jn\pi t/2}dt\right]$$

$$= \frac{j}{2n\pi}\left[2e^{-jn\pi/2} - 2 + e^{-jn\pi} - e^{-jn\pi/2} - e^{-j2n\pi} + e^{-jn\pi}\right]$$

$$= \frac{j}{2n\pi}\left[3e^{-jn\pi/2} - 3 + 2e^{-jn\pi}\right]$$

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_o t}$$

Chapter 17, Problem 55.

Obtain the exponential Fourier series expansion of the half-wave rectified sinusoidal current of Fig. 17.82.

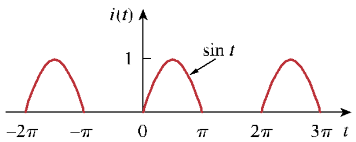


Figure 17.82
For Prob. 17.55.

Chapter 17, Solution 55.

$T = 2\pi, \ \omega_o = 2\pi/T = 1$

$$c_n = \frac{1}{T}\int_0^T i(t)e^{-jn\omega_o t}dt$$

But $$i(t) = \begin{cases} \sin(t), & 0 < t < \pi \\ 0, & \pi < t < 2\pi \end{cases}$$

$$c_n = \frac{1}{2\pi}\int_0^\pi \sin(t)e^{-jnt}dt = \frac{1}{2\pi}\int_0^\pi \frac{1}{2j}(e^{jt} - e^{-jt})e^{-jnt}dt$$

$$= \frac{1}{4\pi j}\left[\frac{e^{jt(1-n)}}{j(1-n)} + \frac{e^{-jt(1+n)}}{j(1+n)}\right]_0^\pi$$

$$= -\frac{1}{4\pi}\left[\frac{e^{j\pi(1-n)} - 1}{1-n} + \frac{e^{-j\pi(n+1)} - 1}{1+n}\right]$$

$$= \frac{1}{4\pi(n^2 - 1)}\left[e^{j\pi(1-n)} - 1 + ne^{j\pi(1-n)} - n + e^{-j\pi(1+n)} - 1 - ne^{-j\pi(1+n)} + n\right]$$

But $e^{j\pi} = \cos(\pi) + j\sin(\pi) = -1 = e^{-j\pi}$

$$c_n = \frac{1}{4\pi(n^2 - 1)}\left[-e^{-jn\pi} - e^{-jn\pi} - ne^{-jn\pi} + ne^{-jn\pi} - 2\right] = \frac{1 + e^{-jn\pi}}{2\pi(1 - n^2)}$$

Thus

$$\underline{i(t) = \sum_{n=-\infty}^{\infty} \frac{1 + e^{-jn\pi}}{2\pi(1 - n^2)}e^{jnt}}$$

Chapter 17, Problem 56.

The Fourier series trigonometric representation of a periodic function is

$$f(t) = 10 + \sum_{n=1}^{\infty}\left(\frac{1}{n^2+1}\cos n\pi t + \frac{n}{n^2+1}\sin n\pi t\right)$$

Find the exponential Fourier series representation of $f(t)$.

Chapter 17, Solution 56.

$c_o = a_o = 10, \ \omega_o = \pi$

$c_o = (a_n - jb_n)/2 = (1 - jn)/[2(n^2 + 1)]$

$$\mathbf{f(t) = 10 + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{(1 - jn)}{2(n^2+1)} e^{jn\pi t}}$$

Chapter 17, Problem 57.

The coefficients of the trigonometric Fourier series representation of a function are:

$b_n = 0, \ a_n = \dfrac{6}{n^3 - 2}, \ n = 0, 1, 2, \cdots$

If $\omega_n = 50n$, find the exponential Fourier series for the function.

Chapter 17, Solution 57.

$a_o = (6/-2) = -3 = c_o$

$c_n = 0.5(a_n - jb_n) = a_n/2 = 3/(n^3 - 2)$

$$f(t) = -3 + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{3}{n^3 - 2} e^{j50nt}$$

Chapter 17, Problem 58.

Find the exponential Fourier series of a function that has the following trigonometric Fourier series coefficients:

$a_0 = \frac{\pi}{4}$, $\quad b_n = \frac{(-1)^n}{n}$, $\quad a_n = \frac{(-1)^n - 1}{\pi n^2}$

Take $T = 2\pi$.

Chapter 17, Solution 58.

$c_n = (a_n - jb_n)/2$, $(-1)^n = \cos(n\pi)$, $\omega_o = 2\pi/T = 1$

$c_n = [(\cos(n\pi) - 1)/(2\pi n^2)] - j\cos(n\pi)/(2n)$

Thus

$$f(t) = \frac{\pi}{4} + \sum \left(\frac{\cos(n\pi) - 1}{2\pi n^2} - j\frac{\cos(n\pi)}{2n} \right) e^{jnt}$$

Chapter 17, Problem 59.

The complex Fourier series of the function in Fig. 17.83(a) is

$$f(t) = \frac{1}{2} - \sum_{n=-\infty}^{\infty} \frac{je^{-j(2n+1)t}}{(2n+1)\pi}$$

Find the complex Fourier series of the function *h*(*t*) in Fig. 17.83(b).

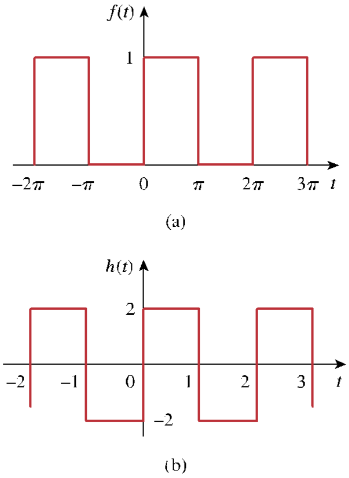


Figure 17.83
For Prob.17.59.

Chapter 17, Solution 59.

For f(t), $T = 2\pi$, $\omega_o = 2\pi/T = 1$.

a_o = DC component = $(1x\pi + 0)/2\pi = 0.5$

For h(t), $T = 2$, $\omega_o = 2\pi/T = \pi$.

$a_o = (2x1 - 2x1)/2 = 0$

Thus by replacing $\omega_o = 1$ with $\omega_o = \pi$ and multiplying the magnitude by four, we obtain

$$h(t) = -\sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{j4e^{-j(2n+1)\pi t}}{(2n+1)\pi}$$

Chapter 17, Problem 60.

Obtain the complex Fourier coefficients of the signal in Fig. 17.62.

Chapter 17, Solution 60.

From Problem 17.24,

$a_o = 0 = a_n,\ b_n = [2/(n\pi)][1 - 2\cos(n\pi)],\ c_o = \underline{\mathbf{0}}$

$c_n = (a_n - jb_n)/2 = \underline{\mathbf{[j/(n\pi)][2\cos(n\pi) - 1],\ n \neq 0}}$.

Chapter 17, Problem 61.

The spectra of the Fourier series of a function are shown in Fig. 17.84. (a) Obtain the trigonometric Fourier series. (b) Calculate the rms value of the function.

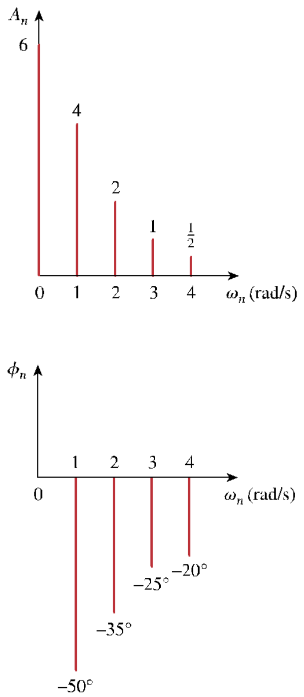


Figure 17.84
For Prob. 17.61.

Chapter 17, Solution 61.

(a) $\omega_o = 1.$

$$f(t) = a_o + \sum A_n \cos(n\omega_o t - \phi_n)$$

$$= 6 + 4\cos(t + 50°) + 2\cos(2t + 35°) + \cos(3t + 25°) + 0.5\cos(4t + 20°)$$

$$= 6 + 4\cos(t)\cos(50°) - 4\sin(t)\sin(50°) + 2\cos(2t)\cos(35°) - 2\sin(2t)\sin(35°) + \cos(3t)\cos(25°) - \sin(3t)\sin(25°) + 0.5\cos(4t)\cos(20°) - 0.5\sin(4t)\sin(20°)$$

$$= \underline{\mathbf{6 + 2.571\cos(t) - 3.73\sin(t) + 1.635\cos(2t) - 1.147\sin(2t) + 0.906\cos(3t) - 0.423\sin(3t) + 0.47\cos(4t) - 0.171\sin(4t)}}$$

(b) $$f_{rms} = \sqrt{a_o^2 + \frac{1}{2}\sum_{n=1}^{\infty} A_n^2}$$

$$f_{rms}^2 = 6^2 + 0.5[4^2 + 2^2 + 1^2 + (0.5)^2] = 46.625$$

$$f_{rms} = \underline{\mathbf{6.828}}$$

Chapter 17, Problem 62.

The amplitude and phase spectra of a truncated Fourier series are shown in Fig. 17.85.

(a) Find an expression for the periodic voltage using the amplitude-phase form. See Eq. (17.10).

(b) Is the voltage an odd or even function of *t*?

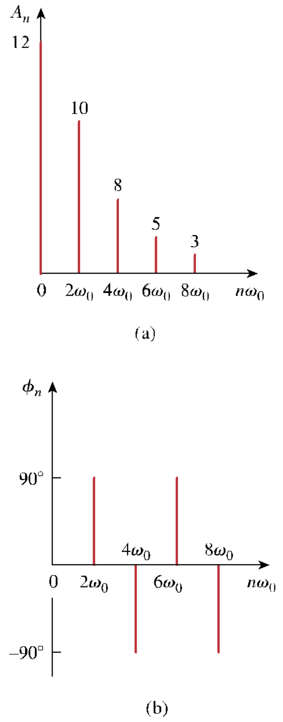


Figure 17.85
For Prob. 17.62.

Chapter 17, Solution 62.

(a)

$$f(t) = 12 + 10\cos(2\omega_o t + 90°) + 8\cos(4\omega_o t - 90°) + 5\cos(6\omega_o t + 90°) + 3\cos(8\omega_o t - 90°)$$

(b) f(t) is an even function of t.

Chapter 17, Problem 63.

Plot the amplitude spectrum for the signal $f_2(t)$ in Fig. 17.56(b). Consider the first five terms.

Chapter 17, Solution 63.

This is an even function.

$T = 3,\ \omega_o = 2\pi/3,\ b_n = 0.$

$$f(t) = \begin{cases} 1, & 0 < t < 1 \\ 2, & 1 < t < 1.5 \end{cases}$$

$$a_o = \frac{2}{T}\int_0^{T/2} f(t)dt = \frac{2}{3}\left[\int_0^1 1dt + \int_1^{1.5} 2dt\right] = (2/3)[1 + 1] = 4/3$$

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos(n\omega_o t)dt = \frac{4}{3}\left[\int_0^1 1\cos(2n\pi t/3)dt + \int_1^{1.5} 2\cos(2n\pi t/3)dt\right]$$

$$= \frac{4}{3}\left[\frac{3}{2n\pi}\sin\left(\frac{2n\pi t}{3}\right)\Bigg|_0^1 + \frac{6}{2n\pi}\sin\left(\frac{2n\pi t}{3}\right)\Bigg|_1^{1.5}\right]$$

$$= [-2/(n\pi)]\sin(2n\pi/3)$$

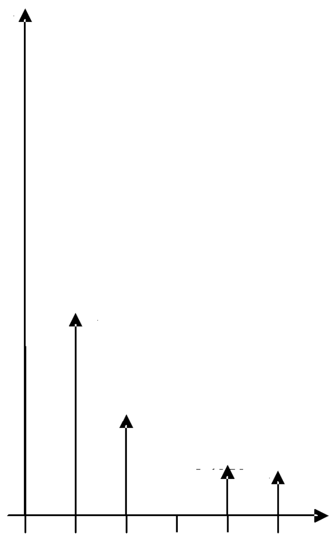
$$f_2(t) = \frac{4}{3} - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{3n\pi}{3}\right)\cos\left(\frac{2n\pi t}{3}\right)$$

$a_o = 4/3 = 1.3333,\ \ \omega_o = 2\pi/3,\ \ a_n = -[2/(n\pi)]\sin(2n\pi t/3)$

$$A_n = \sqrt{a_n^2 + b_n^2} = \left|\frac{2}{n\pi}\sin\left(\frac{2n\pi}{3}\right)\right|$$

$A_1 = 0.5513,\ A_2 = 0.2757,\ A_3 = 0,\ A_4 = 0.1375,\ A_5 = 0.1103$

The amplitude spectra are shown below.



Chapter 17, Problem 64.

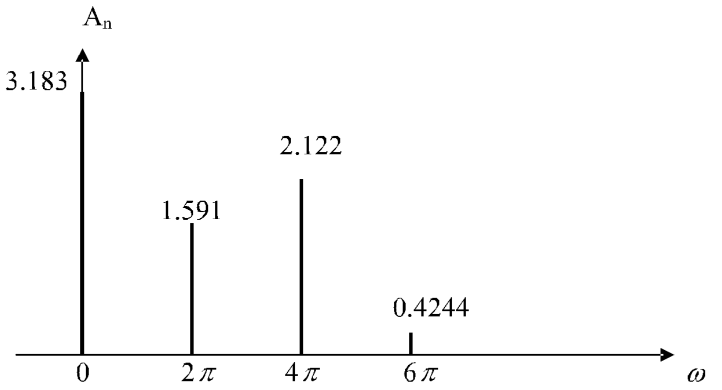
Given that

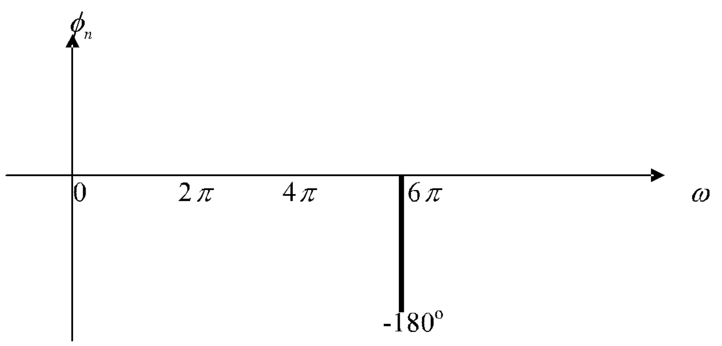
$$v(t) = \frac{10}{\pi}\left[1 + \frac{1}{2}\cos 2\pi t + \frac{2}{3}\cos 4\pi t - \frac{2}{15}\cos 6\pi t + \cdots\right]$$

draw the amplitude and phase spectra for $v(t)$.

Chapter 17, Solution 64.

The amplitude and phase spectra are shown below.





Chapter 17, Problem 65.

Given that

$$f(t) = \sum_{\substack{n=1 \\ n=\text{odd}}}^{\infty} \left(\frac{20}{n^2\pi^2} \cos 2nt - \frac{3}{n\pi} \sin 2nt \right)$$

plot the first five terms of the amplitude and phase spectra for the function.

Chapter 17, Solution 65.

$a_n = 20/(n^2\pi^2)$, $b_n = -3/(n\pi)$, $\omega_n = 2n$

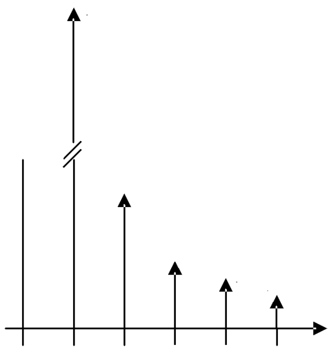
$$A_n = \sqrt{a_n^2 + b_n^2} = \sqrt{\frac{400}{n^4\pi^4} + \frac{9}{n^2\pi^2}}$$

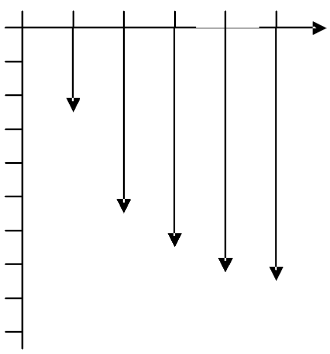
$$= \frac{3}{n\pi}\sqrt{1 + \frac{44.44}{n^2\pi^2}}, \quad n = 1, 3, 5, 7, 9, \text{etc.}$$

n	A_n
1	2.24
3	0.39
5	0.208
7	0.143
9	0.109

$$\phi_n = \tan^{-1}(b_n/a_n) = \tan^{-1}\{[-3/(n\pi)][n^2\pi^2/20]\} = \tan^{-1}(-n \text{x} 0.4712)$$

n	ϕ_n
1	–25.23°
3	–54.73°
5	–67°
7	–73.14°
9	–76.74°
∞	–90°



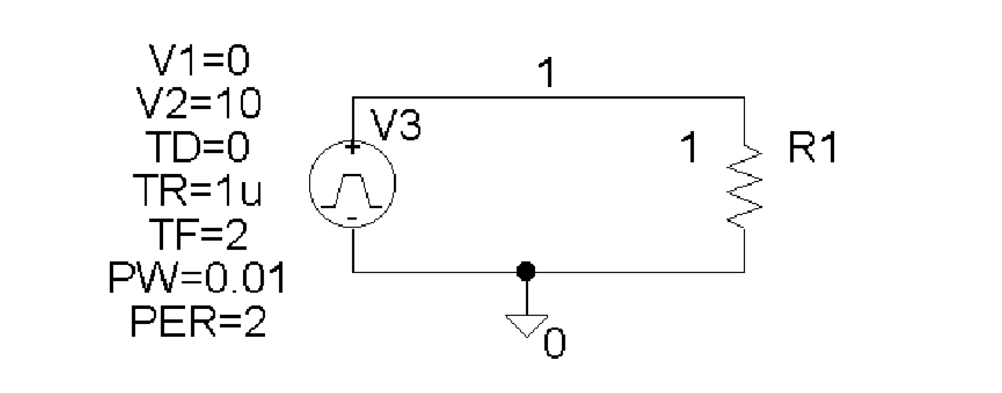


Chapter 17, Problem 66.

Determine the Fourier coefficients for the waveform in Fig. 17.48 using *PSpice*.

Chapter 17, Solution 66.

The schematic is shown below. The waveform is inputted using the attributes of VPULSE. In the Transient dialog box, we enter Print Step = 0.05, Final Time = 12, Center Frequency = 0.5, Output Vars = V(1) and click enable Fourier. After simulation, the output plot is shown below. The output file includes the following Fourier components.



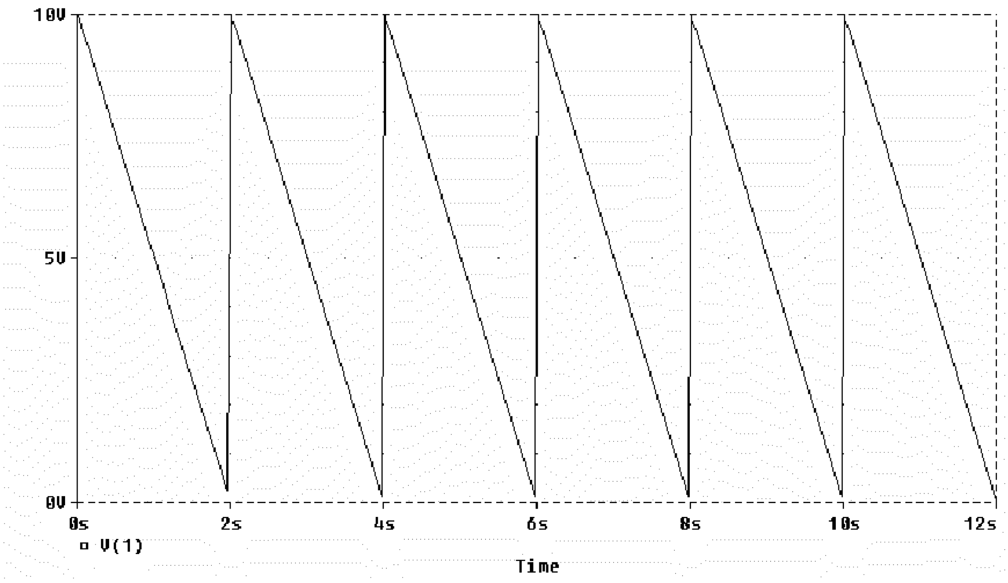
FOURIER COMPONENTS OF TRANSIENT RESPONSE V(1)

DC COMPONENT = 5.099510E+00

HARMONIC FREQUENCY FOURIER NORMALIZED PHASE NORMALIZED

NO	(HZ)	COMPONENT	COMPONENT	(DEG)	PHASE (DEG)
1	5.000E-01	3.184E+00	1.000E+00	1.782E+00	0.000E+00
2	1.000E+00	1.593E+00	5.002E-01	3.564E+00	1.782E+00
3	1.500E+00	1.063E+00	3.338E-01	5.347E+00	3.564E+00
4	2.000E+00	7.978E-01	2.506E-01	7.129E+00	5.347E+00
5	2.500E+00	6.392E-01	2.008E-01	8.911E+00	7.129E+00
6	3.000E+00	5.336E-01	1.676E-01	1.069E+01	8.911E+00
7	3.500E+00	4.583E-01	1.440E-01	1.248E+01	1.069E+01
8	4.000E+00	4.020E-01	1.263E-01	1.426E+01	1.248E+01
9	4.500E+00	3.583E-01	1.126E-01	1.604E+01	1.426E+01

TOTAL HARMONIC DISTORTION = 7.363360E+01 PERCENT

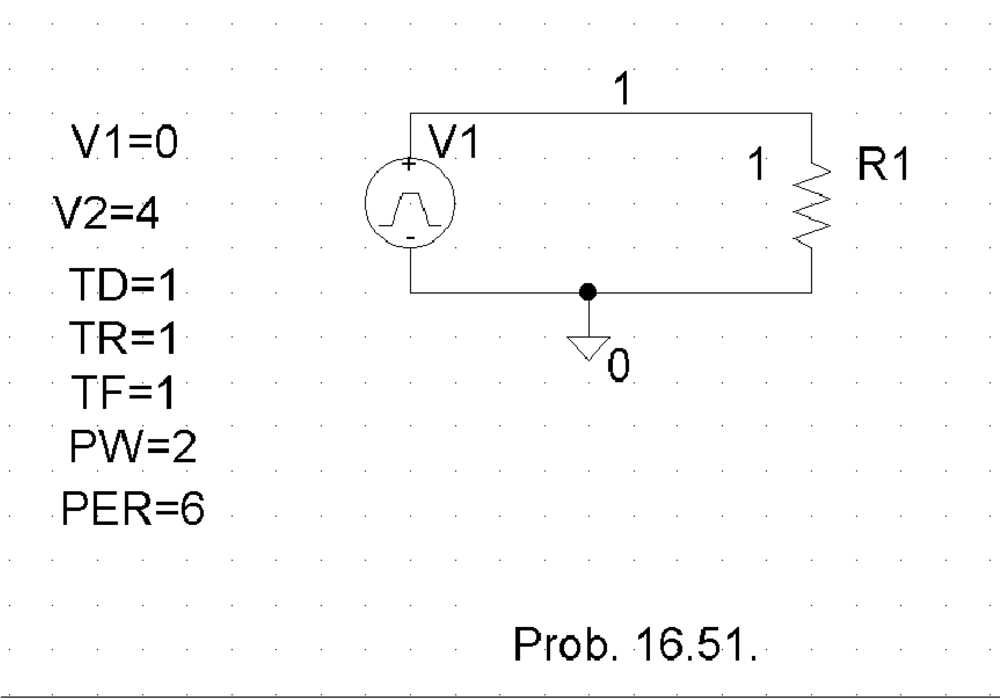


Chapter 17, Problem 67.

Calculate the Fourier coefficients of the signal in Fig. 17.58 using *PSpice*.

Chapter 17, Solution 67.

The Schematic is shown below. In the Transient dialog box, we type "Print step = 0.01s, Final time = 36s, Center frequency = 0.1667, Output vars = v(1)," and click Enable Fourier. After simulation, the output file includes the following Fourier components,



FOURIER COMPONENTS OF TRANSIENT RESPONSE V(1)

DC COMPONENT = 2.000396E+00

HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	1.667E-01	2.432E+00	1.000E+00	-8.996E+01	0.000E+00
2	3.334E-01	6.576E-04	2.705E-04	-8.932E+01	6.467E-01
3	5.001E-01	5.403E-01	2.222E-01	9.011E+01	1.801E+02
4	6.668E-01	3.343E-04	1.375E-04	9.134E+01	1.813E+02
5	8.335E-01	9.716E-02	3.996E-02	-8.982E+01	1.433E-01
6	1.000E+00	7.481E-06	3.076E-06	-9.000E+01	-3.581E-02
7	1.167E+00	4.968E-02	2.043E-02	-8.975E+01	2.173E-01
8	1.334E+00	1.613E-04	6.634E-05	-8.722E+01	2.748E+00
9	1.500E+00	6.002E-02	2.468E-02	9.032E+01	1.803E+02

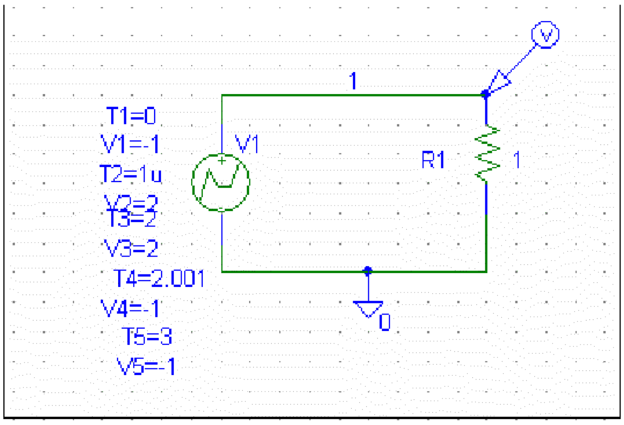
TOTAL HARMONIC DISTORTION = 2.280065E+01 PERCENT

Chapter 17, Problem 68.

Use *PSpice* to find the Fourier components of the signal in Prob. 17.7.

Chapter 17, Solution 68.

Since T=3, f =1/3 = 0.333 Hz. We use the schematic below.



We use VPWL to enter in the signal as shown. In the transient dialog box, we enable Fourier, select 15 for Final Time, 0.01s for Print Step, and 10ms for the Step Ceiling. When the file is saved and run, we obtain the Fourier coefficients as part of the output file as shown below.

FOURIER COMPONENTS OF TRANSIENT RESPONSE V(1)

DC COMPONENT = -1.000000E+00

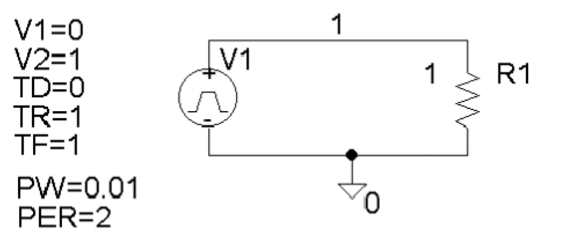
HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	3.330E-01	1.615E-16	1.000E+00	1.762E+02	0.000E+00
2	6.660E-01	5.133E-17	3.179E-01	2.999E+01	-3.224E+02
3	9.990E-01	6.243E-16	3.867E+00	6.687E+01	-4.617E+02
4	1.332E+00	1.869E-16	1.158E+00	7.806E+01	-6.267E+02
5	1.665E+00	6.806E-17	4.215E-01	1.404E+02	-7.406E+02
6	1.998E+00	1.949E-16	1.207E+00	-1.222E+02	-1.179E+03
7	2.331E+00	1.465E-16	9.070E-01	-4.333E+01	-1.277E+03
8	2.664E+00	3.015E-16	1.867E+00	-1.749E+02	-1.584E+03
9	2.997E+00	1.329E-16	8.233E-01	-9.565E+01	-1.681E+03

Chapter 17, Problem 69.

Use *PSpice* to obtain the Fourier coefficients of the waveform in Fig. 17.55(a).

Chapter 17, Solution 69.

The schematic is shown below. In the Transient dialog box, set Print Step = 0.05 s, Final Time = 120, Center Frequency = 0.5, Output Vars = V(1) and click enable Fourier. After simulation, we obtain V(1) as shown below. We also obtain an output file which includes the following Fourier components.

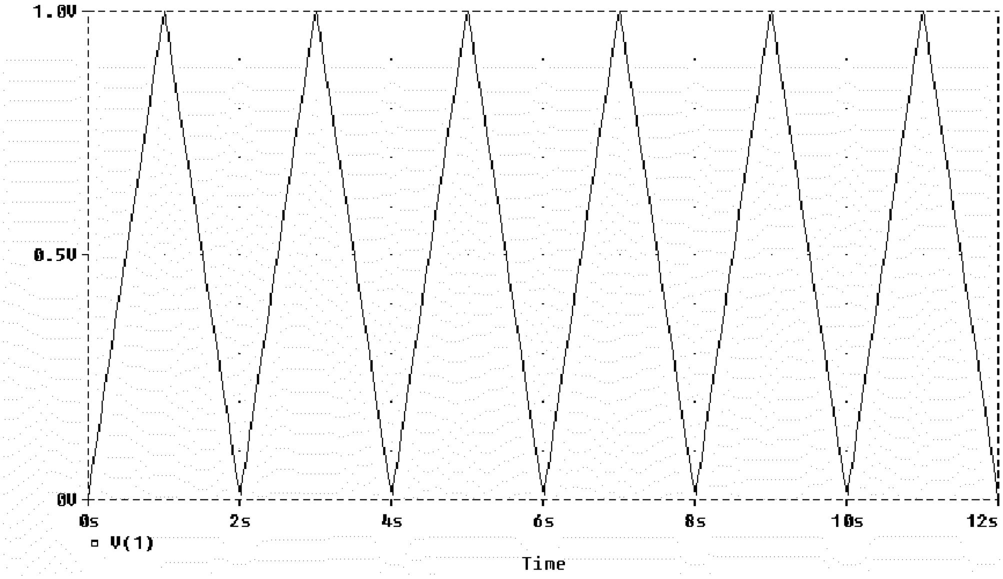


FOURIER COMPONENTS OF TRANSIENT RESPONSE V(1)

DC COMPONENT = 5.048510E-01

HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	5.000E-01	4.056E-01	1.000E+00	-9.090E+01	0.000E+00
2	1.000E+00	2.977E-04	7.341E-04	-8.707E+01	3.833E+00
3	1.500E+00	4.531E-02	1.117E-01	-9.266E+01	-1.761E+00
4	2.000E+00	2.969E-04	7.320E-04	-8.414E+01	6.757E+00
5	2.500E+00	1.648E-02	4.064E-02	-9.432E+01	-3.417E+00
6	3.000E+00	2.955E-04	7.285E-04	-8.124E+01	9.659E+00
7	3.500E+00	8.535E-03	2.104E-02	-9.581E+01	-4.911E+00
8	4.000E+00	2.935E-04	7.238E-04	-7.836E+01	1.254E+01
9	4.500E+00	5.258E-03	1.296E-02	-9.710E+01	-6.197E+00

TOTAL HARMONIC DISTORTION = 1.214285E+01 PERCENT

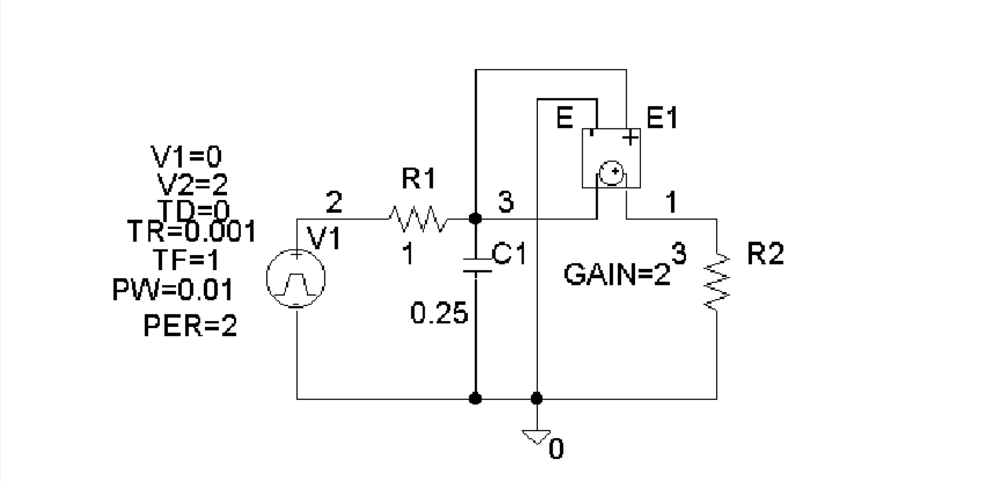


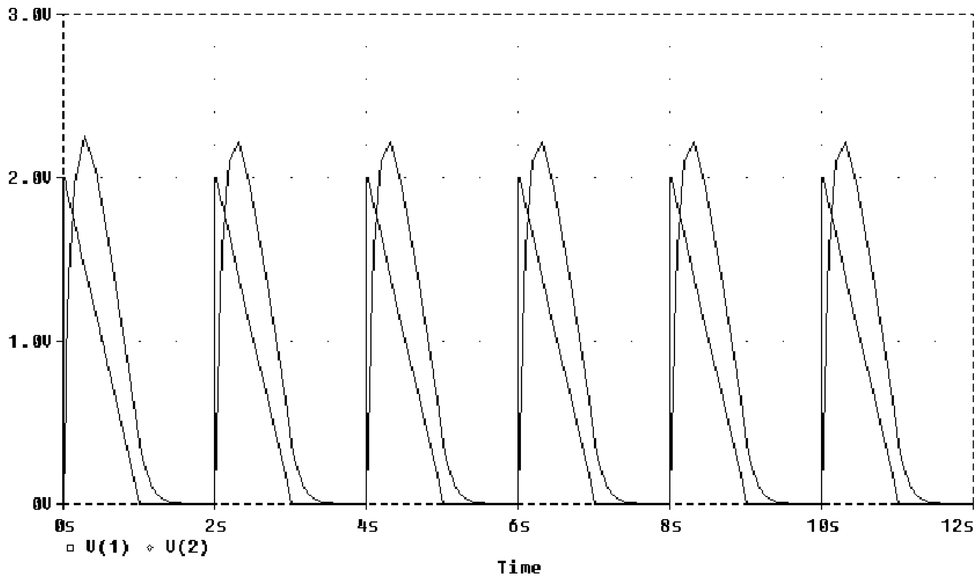
Chapter 17, Problem 70.

Rework Prob. 17.40 using *PSpice*.

Chapter 17, Solution 70.

The schematic is shown below. In the Transient dialog box, we set Print Step = 0.02 s, Final Step = 12 s, Center Frequency = 0.5, Output Vars = V(1) and V(2), and click enable Fourier. After simulation, we compare the output and output waveforms as shown. The output includes the following Fourier components.





FOURIER COMPONENTS OF TRANSIENT RESPONSE V(1)

DC COMPONENT = 7.658051E-01

HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	5.000E-01	1.070E+00	1.000E+00	1.004E+01	0.000E+00
2	1.000E+00	3.758E-01	3.512E-01	-3.924E+01	-4.928E+01
3	1.500E+00	2.111E-01	1.973E-01	-3.985E+01	-4.990E+01
4	2.000E+00	1.247E-01	1.166E-01	-5.870E+01	-6.874E+01
5	2.500E+00	8.538E-02	7.980E-02	-5.680E+01	-6.685E+01
6	3.000E+00	6.139E-02	5.738E-02	-6.563E+01	-7.567E+01
7	3.500E+00	4.743E-02	4.433E-02	-6.520E+01	-7.524E+01
8	4.000E+00	3.711E-02	3.469E-02	-7.222E+01	-8.226E+01
9	4.500E+00	2.997E-02	2.802E-02	-7.088E+01	-8.092E+01

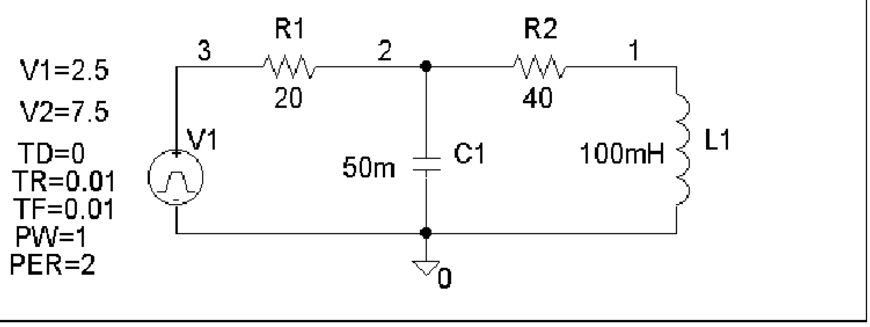
TOTAL HARMONIC DISTORTION = 4.352895E+01 PERCENT

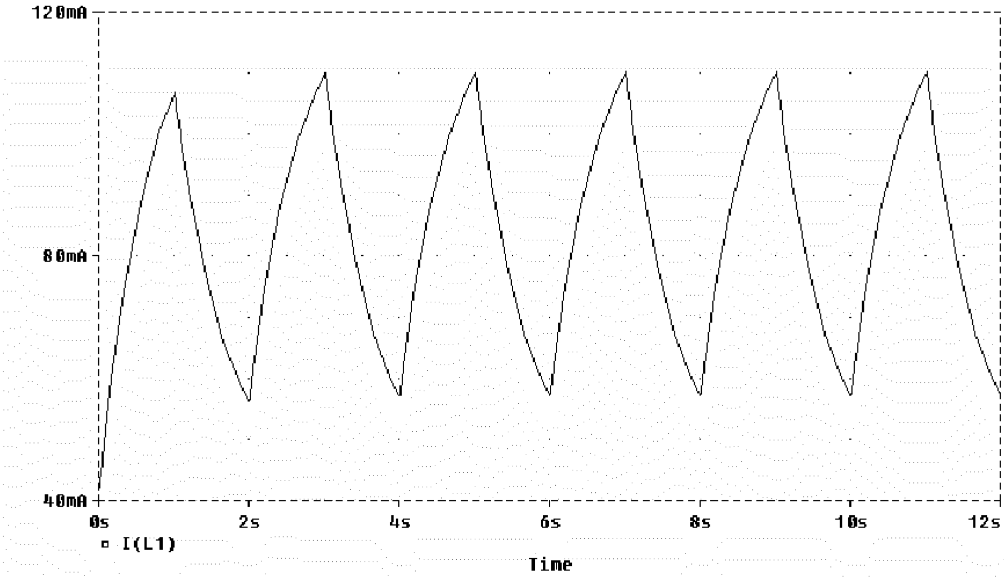
Chapter 17, Problem 71.

Use *PSpice* to solve Prob. 17.39.

Chapter 17, Solution 71.

The schematic is shown below. We set Print Step = 0.05, Final Time = 12 s, Center Frequency = 0.5, Output Vars = I(1), and click enable Fourier in the Transient dialog box. After simulation, the output waveform is as shown. The output file includes the following Fourier components.





FOURIER COMPONENTS OF TRANSIENT RESPONSE I(L_L1)

DC COMPONENT = 8.374999E-02

HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	5.000E-01	2.287E-02	1.000E+00	-6.749E+01	0.000E+00
2	1.000E+00	1.891E-04	8.268E-03	8.174E+00	7.566E+01
3	1.500E+00	2.748E-03	1.201E-01	-8.770E+01	-2.021E+01
4	2.000E+00	9.583E-05	4.190E-03	-1.844E+00	6.565E+01
5	2.500E+00	1.017E-03	4.446E-02	-9.455E+01	-2.706E+01
6	3.000E+00	6.366E-05	2.783E-03	-7.308E+00	6.018E+01
7	3.500E+00	5.937E-04	2.596E-02	-9.572E+01	-2.823E+01
8	4.000E+00	6.059E-05	2.649E-03	-2.808E+01	3.941E+01
9	4.500E+00	2.113E-04	9.240E-03	-1.214E+02	-5.387E+01

TOTAL HARMONIC DISTORTION = 1.314238E+01 PERCENT

Chapter 17, Problem 72.

The signal displayed by a medical device can be approximated by the waveform shown in Fig. 17.86. Find the Fourier series representation of the signal.

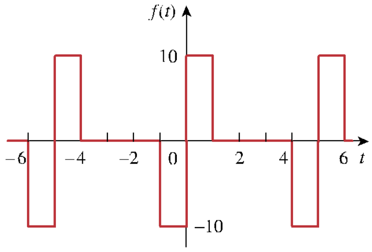


Figure 17.86
For Prob. 17.72.

Chapter 17, Solution 72.

$T = 5, \; \omega_o = 2\pi/T = 2\pi/5$

f(t) is an odd function. $a_o = 0 = a_n$

$$b_n = \frac{4}{T}\int_0^{T/2} f(t)\sin(n\omega_o t)dt = \frac{4}{5}\int_0^{1} 10\sin(0.4n\pi t)dt$$

$$= -\frac{8x5}{2n\pi}\cos(0.4\pi nt)\Big|_0^1 = \frac{20}{n\pi}[1-\cos(0.4n\pi)]$$

$$f(t) = \underline{\mathbf{\frac{20}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}[1-\cos(0.4n\pi)]\sin(0.4n\pi t)}}$$

Chapter 17, Problem 73.

A spectrum analyzer indicates that a signal is made up of three components only: 640 kHz at 2 V, 644 kHz at 1 V, 636 kHz at 1 V. If the signal is applied across a 10-Ω resistor, what is the average power absorbed by the resistor?

Chapter 17, Solution 73.

$$p = \frac{V_{DC}^2}{R} + \frac{1}{2}\sum\frac{V_n^2}{R}$$

$$= 0 + 0.5[(2^2 + 1^2 + 1^2)/10] = \underline{\mathbf{300\ mW}}$$

Chapter 17, Problem 74.

A certain band-limited periodic current has only three frequencies in its Fourier series representation: dc, 50 Hz, and 100 Hz. The current may be represented as

$$i(t) = 4 + 6\sin 100\pi t + 8\cos 100\pi t$$
$$- 3\sin 200\pi t - 4\cos 200\pi t \text{ A}$$

(a) Express $i(t)$ in amplitude-phase form.
(b) If $i(t)$ flows through a 2-Ω resistor, how many watts of average power will be dissipated?

Chapter 17, Solution 74.

(a)

$$A_n = \sqrt{a_n^2 + b_n^2}, \qquad \phi = \tan^{-1}(b_n/a_n)$$

$$A_1 = \sqrt{6^2 + 8^2} = 10, \qquad \phi_1 = \tan^{-1}(6/8) = 36.87°$$

$$A_2 = \sqrt{3^2 + 4^2} = 5, \qquad \phi_2 = \tan^{-1}(3/4) = 36.87°$$

$$i(t) = \underline{\mathbf{\{4 + 10\cos(100\pi t - 36.87°) - 5\cos(200\pi t - 36.87°)\}\ A}}$$

(b)

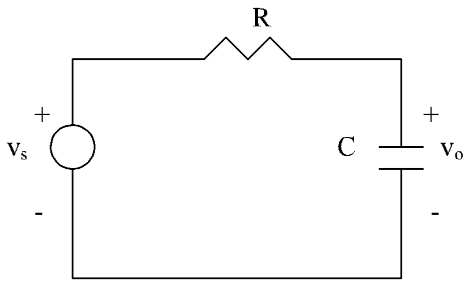
$$p = I_{DC}^2 R + 0.5\sum I_n^2 R$$
$$= 2[4^2 + 0.5(10^2 + 5^2)] = \underline{\mathbf{157\ W}}$$

Chapter 17, Problem 75.

e2d Design a lowpass *RC* filter with a resistance $R = 2\ k\Omega$. The input to the filter is a periodic rectangular pulse train (see Table 17.3) with A = 1 *V*, *T* = 10 ms, and τ = 1 ms. Select *C* such that the dc component of the output is 50 times greater than the fundamental component of the output.

Chapter 17, Solution 75.

The lowpass filter is shown below.



$$v_s = \frac{A\tau}{T} + \frac{2A}{T}\sum_{n=1}^{\infty}\frac{1}{n}\sin\frac{n\pi\tau}{T}\cos n\omega_o t$$

$$V_o = \frac{\frac{1}{j\omega_n C}}{R + \frac{1}{j\omega_n C}}V_s = \frac{1}{1 + j\omega_n RC}V_s, \qquad \omega_n = n\omega_o = 2n\pi / T$$

For n=0, (dc component), $V_o = V_s = \frac{A\tau}{T}$ (1)

For the nth harmonic,

$$V_o = \frac{1}{\sqrt{1 + \omega^2{}_n R^2 C^2}\angle \tan^{-1}\omega_n RC} \bullet \frac{2A}{nT}\sin\frac{n\pi\tau}{T}\angle -90^o$$

When n=1, $|V_o| = \frac{2A}{T}\sin\frac{n\pi\tau}{T} \bullet \frac{1}{\sqrt{1 + \frac{4\pi^2}{T}R^2C^2}}$ (2)

From (1) and (2),

$$\frac{A\tau}{T} = 50\text{x}\frac{2A}{T}\sin\frac{\pi}{10}\frac{1}{\sqrt{1 + \frac{4\pi^2}{T}R^2C^2}} \longrightarrow \sqrt{1 + \frac{4\pi^2}{T}R^2C^2} = \frac{30.9}{\tau} = 3.09\text{x}10^4$$

$$1 + \frac{4\pi^2}{T}R^2C^2 = 10^{10} \longrightarrow C = \frac{T}{2\pi R}10^5 = \frac{10^{-2}\text{x}3.09\text{x}10^4}{4\pi\text{x}10^3} = \underline{24.59\text{ mF}}$$

Chapter 17, Problem 76.

A periodic signal given by $v_s(t) = 10\text{ V}$ for $0 < t < 1$ and 0 V for $1 < t < 2$ is applied to the highpass filter in Fig. 17.87. Determine the value of R such that the output signal $v_o(t)$ has an average power of at least 70 percent of the average power of the input signal.

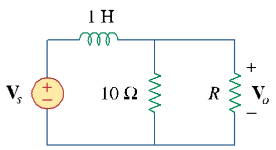


Figure 17.87
For Prob. 17.76.

Chapter 17, Solution 76.

$v_s(t)$ is the same as f(t) in Figure 16.1 except that the magnitude is multiplied by 10. Hence

$$v_o(t) = 5 + \frac{20}{\pi}\sum_{k=1}^{\infty}\frac{1}{n}\sin(n\pi t), \quad n = 2k-1$$

$$T = 2, \ \omega_o = 2\pi/T = 2\pi, \ \omega_n = n\omega_o = 2n\pi$$

$$j\omega_n L = j2n\pi; \ Z = R\|10 = 10R/(10+R)$$

$$V_o = ZV_s/(Z + j2n\pi) = [10R/(10R + j2n\pi(10+R))]V_s$$

$$V_o = \frac{10R\angle -\tan^{-1}\{(n\pi/5R)(10+R)\}}{\sqrt{100R^2 + 4n^2\pi^2(10+R)^2}}V_s$$

$$V_s = [20/(n\pi)]\angle 0°$$

The source current I_s is

$$I_s = \frac{V_s}{Z + j2n\pi} = \frac{V_s}{\frac{10R}{10+R} + j2n\pi} = \frac{(10+R)\frac{20}{n\pi}}{10R + j2n\pi(10+R)}$$

$$= \frac{(10+R)\frac{20}{n\pi}\angle - \tan^{-1}\{(n\pi/3)(10+R)\}}{\sqrt{100R^2 + 4n^2\pi^2(10+R)^2}}$$

$$p_s = V_{DC}I_{DC} + \frac{1}{2}\sum V_{sn}I_{sn}\cos(\theta_n - \phi_n)$$

For the DC case, L acts like a short-circuit.

$$I_s = \frac{5}{\frac{10R}{10+R}} = \frac{5(10+R)}{10R}, \quad V_s = 5 = V_o$$

$$p_s = \frac{25(10+R)}{10R} + \frac{1}{2}\left[\left(\frac{20}{\pi}\right)^2 \frac{(10+R)\cos\left(\tan^{-1}\left(\frac{\pi}{5}(10+R)\right)\right)}{\sqrt{100R^2 + 4\pi^2(10+R)^2}}\right.$$

$$\left. + \left(\frac{10}{\pi}\right)^2 \frac{(10+R)^2\cos\left(\tan^{-1}\left(\frac{2\pi}{5}(10+R)\right)\right)}{\sqrt{100R^2 + 16\pi^2(10+R)^2}} + \cdots\right]$$

$$p_s = \frac{V_{DC}}{R} + \frac{1}{2}\sum_{n=1}^{\infty}\frac{V_{on}}{R}$$

$$= \frac{25}{R} + \frac{1}{2}\left[\frac{100R}{100R^2 + 4\pi^2(10+R)^2} + \frac{100R}{100R^2 + 10\pi^2(10+R)^2} + \cdots\right]$$

We want $p_o = (70/100)\ p_s = 0.7p_s$. Due to the complexity of the terms, we consider only the DC component as an approximation. In fact the DC component has the largest share of the power for both input and output signals.

$$\frac{25}{R} = \frac{7}{10}\text{x}\frac{25(10+R)}{10R}$$

$100 = 70 + 7R$ which leads to $R = 30/7 =$ **4.286 Ω**

Chapter 17, Problem 77.

The voltage across a device is given by

$$v(t) = -2 + 10\cos 4t + 8\cos 6t + 6\cos 8t$$
$$- 5\sin 4t - 3\sin 6t - \sin 8t \text{ V}$$

Find:

(a) the period of $v(t)$,
(b) the average value of $v(t)$,
(c) the effective value of $v(t)$.

Chapter 17, Solution 77.

(a) For the first two AC terms, the frequency ratio is 6/4 = 1.5 so that the highest common factor is 2. Hence $\omega_o = 2$.

$$T = 2\pi/\omega_o = 2\pi/2 = \underline{\pi}$$

(b) The average value is the DC component = $\underline{\mathbf{-2}}$

(c) $$V_{rms} = \sqrt{a_o + \frac{1}{2}\sum_{n=1}^{\infty}(a_n^2 + b_n^2)}$$

$$V_{rms}^2 = (-2)^2 + \frac{1}{2}(10^2 + 8^2 + 6^2 + 3^2 + 1^2) = 121.5$$

$$V_{rms} = \underline{\mathbf{11.02\ V}}$$

Chapter 17, Problem 78.

A certain band-limited periodic voltage has only three harmonics in its Fourier series representation. The harmonics have the following rms values: fundamental 40 V, third harmonic 20 V, fifth harmonic 10 V.

(a) If the voltage is applied across a 5-Ω resistor, find the average power dissipated by the resistor.

(b) If a dc component is added to the periodic voltage and the measured power dissipated increases by 5 percent, determine the value of the dc component added.

Chapter 17, Solution 78.

(a) $$p = \frac{V_{DC}^2}{R} + \frac{1}{2}\sum \frac{V_n^2}{R} = \frac{V_{DC}^2}{R} + \sum \frac{V_{n,rms}^2}{R}$$

$$= 0 + (40^2/5) + (20^2/5) + (10^2/5) = \underline{\mathbf{420\ W}}$$

(b) 5% increase = (5/100)420 = 21

$$p_{DC} = 21\ W = \frac{V_{DC}^2}{R} \quad \text{which leads to} \quad V_{DC}^2 = 21R = 105$$

V_{DC} = **<u>10.25 V</u>**

Chapter 17, Problem 79.

Write a program to compute the Fourier coefficients (up to the 10th harmonic) of the square wave in Table 17.3 with $A = 10$ and $T = 2$.

Chapter 17, Solution 79.

From Table 17.3, it is evident that $a_n = 0$,

$b_n = 4A/[\pi(2n - 1)], \; A = 10.$

A Fortran program to calculate b_n is shown below. The result is also shown.

```
C       FOR PROBLEM 17.79
            DIMENSION B(20)

            A = 10
            PIE = 3.142
            C = 4.*A/PIE
            DO 10 N = 1, 10
            B(N) = C/(2.*FLOAT(N) – 1.)
            PRINT *, N, B(N)
10          CONTINUE
            STOP
            END
```

n	b_n
1	12.731
2	4.243
3	2.546
4	1.8187
5	1.414
6	1.1573
7	0.9793
8	0.8487
9	0.7498
10	0.67

Chapter 17, Problem 80.

Write a computer program to calculate the exponential Fourier series of the half-wave rectified sinusoidal current of Fig. 17.82. Consider terms up to the 10th harmonic.

Chapter 17, Solution 80.

From Problem 17.55,

$$c_n = [1 + e^{-jn\pi}]/[2\pi(1 - n^2)]$$

This is calculated using the Fortran program shown below. The results are also shown.

```
C       FOR PROBLEM 17.80
              COMPLEX X, C(0:20)

              PIE = 3.1415927
              A = 2.0*PIE
              DO 10 N = 0, 10
              IF(N.EQ.1) GO TO 10
              X = CMPLX(0, PIE*FLOAT(N))
              C(N) = (1.0 + CEXP(–X))/(A*(1 – FLOAT(N*N)))
              PRINT *, N, C(N)
10            CONTINUE
              STOP
              END
```

n	c_n
0	$0.3188 + j0$
1	0
2	$-0.1061 + j0$
3	0
4	$-0.2121x10^{-1} + j0$
5	0
6	$-0.9095x10^{-2} + j0$
7	0
8	$-0.5052x10^{-2} + j0$
9	0
10	$-0.3215x10^{-2} + j0$

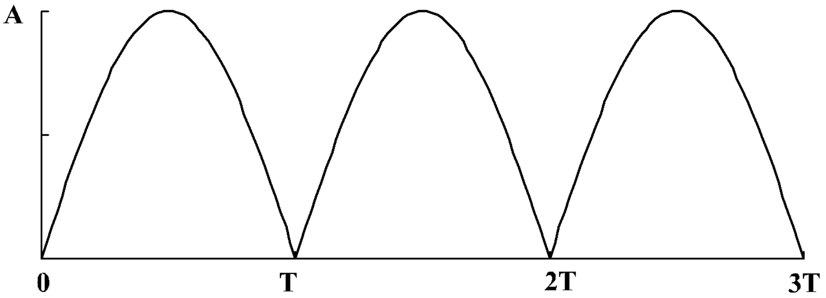
Chapter 17, Problem 81.

Consider the full-wave rectified sinusoidal current in Table 17.3. Assume that the current is passed through a 1-Ω resistor.

(a) Find the average power absorbed by the resistor.
(b) Obtain c_n for n = 1, 2, 3, and 4.
(c) What fraction of the total power is carried by the dc component?
(d) What fraction of the total power is carried by the second harmonic (n = 2)?

Chapter 17, Solution 81.

(a)



$$f(t) = \frac{2A}{\pi} - \frac{4A}{\pi}\sum_{n=1}^{\infty}\frac{1}{4n^2 - 1}\cos(n\omega_o t)$$

The total average power is $p_{avg} = F_{rms}^2 R = F_{rms}^2$ since R = 1 ohm.

$$P_{avg} = F_{rms}^2 = \frac{1}{T}\int_0^T f^2(t)dt = \underline{\mathbf{0.5A^2}}$$

(b) From the Fourier series above

$|c_o| = 2A/\pi$, $|c_n| = |a_n|/2 = 2A/[\pi(4n^2 - 1)]$

n	ω_o	$\lvert c_n \rvert$	$\lvert c_o\rvert^2$ or $2\lvert c_n\rvert^2$	% power
0	0	$2A/\pi$	$4A^2/(\pi^2)$	81.1%
1	$2\omega_o$	$2A/(3\pi)$	$8A^2/(9\pi^2)$	18.01%
2	$4\omega_o$	$2A/(15\pi)$	$8A^2/(225\pi^2)$	0.72%
3	$6\omega_o$	$2A/(35\pi)$	$8A^2/(1225\pi^2)$	0.13%
4	$8\omega_o$	$2A/(63\pi)$	$8A^2/(3969\pi^2)$	0.04%

(c) **81.1%**

(d) **0.72%**

Chapter 17, Problem 82.

A band-limited voltage signal is found to have the complex Fourier coefficients presented in the table below. Calculate the average power that the signal would supply a 4-Ω resistor.

$n\omega_0$	$\lvert c_n \rvert$	θ_n
0	10.0	0°
ω	8.5	15°
2 ω	4.2	30°
3 ω	2.1	45°
4 ω	0.5	60°
5 ω	0.2	75°

Chapter 17, Solution 82.

$$P = \frac{V_{DC}^2}{R} + \frac{1}{2}\sum_{n=1}^{\infty}\frac{V_n^2}{R}$$

Assuming V is an amplitude-phase form of Fourier series. But

$$|A_n| = 2|C_n|, \quad c_o = a_o$$

$$|A_n|^2 = 4|C_n|^2$$

Hence,

$$P = \frac{c_o^2}{R} + 2\sum_{n=1}^{\infty}\frac{c_n^2}{R}$$

Alternatively,

$$P = \frac{V_{rms}^2}{R}$$

where

$$V_{rms}^2 = a_o^2 + \frac{1}{2}\sum_{n=1}^{\infty} A_n^2 = c_o^2 + 2\sum_{n=1}^{\infty} c_n^2 = \sum_{n=-\infty}^{\infty} c_n^2$$

$$= 10^2 + 2(8.5^2 + 4.2^2 + 2.1^2 + 0.5^2 + 0.2^2)$$

$$= 100 + 2x94.57 = 289.14$$

$$P = 289.14/4 = \underline{\mathbf{72.3\ W}}$$

Chapter 18, Problem 1.

Obtain the Fourier transform of the function in Fig. 18.26.

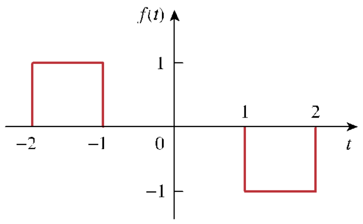


Figure 18.26
For Prob. 18.1.

Chapter 18, Solution 1.

$$f'(t) = \delta(t+2) - \delta(t+1) - \delta(t-1) + \delta(t-2)$$

$$j\omega F(\omega) = e^{j2\omega} - e^{j\omega} - e^{-j\omega} + e^{-j\omega 2}$$

$$= 2\cos 2\omega - 2\cos\omega$$

$$F(\omega) = \underline{\frac{\mathbf{2[\cos 2\omega - \cos\omega]}}{\mathbf{j\omega}}}$$

Chapter 18, Problem 2.

What is the Fourier transform of the triangular pulse in Fig. 18.27?

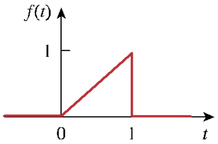
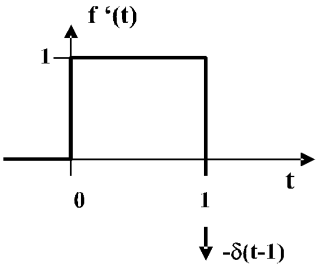
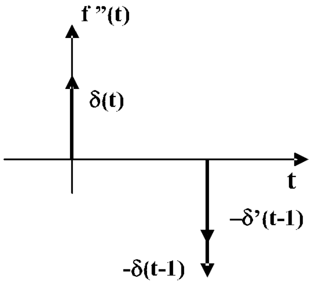


Figure 18.27
For Prob. 18.2.

Chapter 18, Solution 2.

$$f(t) = \begin{cases} t, & 0 < t < 1 \\ 0, & \text{otherwise} \end{cases}$$





$$f''(t) = \delta(t) - \delta(t-1) - \delta'(t-1)$$

Taking the Fourier transform gives

$$-\omega^2 F(\omega) = 1 - e^{-j\omega} - j\omega e^{-j\omega}$$

$$F(\omega) = \frac{(1+j\omega)e^{j\omega} - 1}{\omega^2}$$

or $F(\omega) = \int_0^1 t\, e^{-j\omega t}\, dt$

But $\int x\, e^{ax} dx = \frac{e^{ax}}{a^2}(ax - 1) + c$

$$F(\omega) = \frac{e^{-j\omega}}{(-j\omega)^2}(-j\omega t - 1)\Big|_0^1 = \frac{1}{\omega^2}\left[(1 + j\omega)e^{-j\omega} - 1\right]$$

Chapter 18, Problem 3.

Calculate the Fourier transform of the signal in Fig. 18.28.

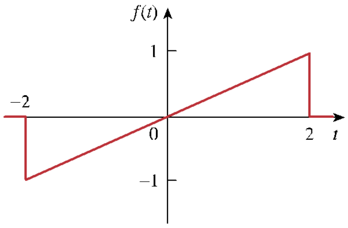


Figure 18.28
For Prob. 18.3.

Chapter 18, Solution 3.

$$f(t) = \frac{1}{2}t, \; -2 < t < 2, \qquad f'(t) = \frac{1}{2}, \; -2 < t < 2$$

$$F(\omega) = \int_{-2}^{2} \frac{1}{2} t\, e^{j\omega t} dt = \frac{e^{-j\omega t}}{2(-j\omega)^2}(-j\omega t - 1)\Big|_{-2}^{2}$$

$$= -\frac{1}{2\omega^2}\left[e^{-j\omega 2}(-j\omega 2 - 1) - e^{j\omega 2}(j\omega 2 - 1)\right]$$

$$= -\frac{1}{2\omega^2}\left[-j\omega 2\left(e^{-j\omega 2} + e^{j\omega 2}\right) + e^{j\omega 2} - e^{-j\omega 2}\right]$$

$$= -\frac{1}{2\omega^2}\left(-j\omega 4\cos 2\omega + j2\sin 2\omega\right)$$

$$F(\omega) = \underline{\frac{j}{\omega^2}(2\omega\cos 2\omega - \sin 2\omega)}$$

Chapter 18, Problem 4.

ML

Find the Fourier transform of the waveform shown in Fig. 18.29.

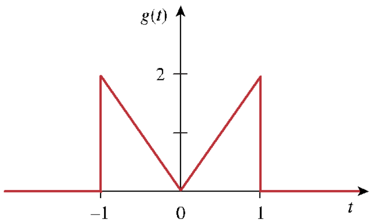
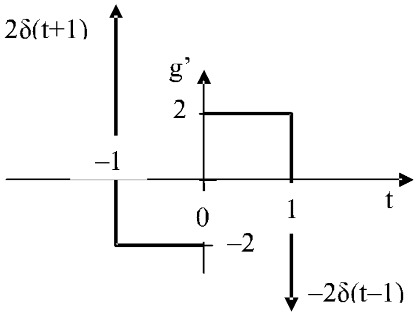
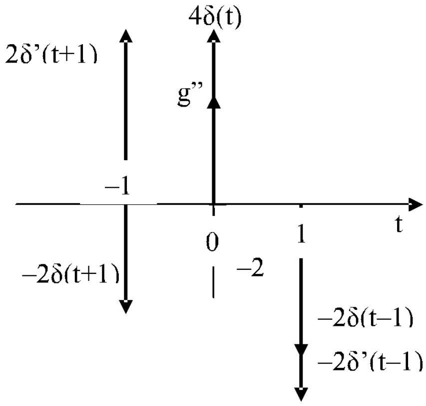


Figure 18.29
For Prob. 18.4.

Chapter 18, Solution 4.





$$g'' = -2\delta(t+1) + 2\delta'(t+1) + 4\delta(t) - 2\delta(t-1) - 2\delta'(t-1)$$

$$(j\omega)^2 G(\omega) = -2e^{j\omega} + 2j\omega e^{j\omega} + 4 - 2e^{-j\omega} - 2j\omega e^{-j\omega}$$
$$= -4\cos\omega - 4\omega\sin\omega + 4$$

$$\underline{G(\omega) = \frac{4}{\omega^2}(\cos\omega + \omega\sin\omega - 1)}$$

Chapter 18, Problem 5.

ML

Obtain the Fourier transform of the signal shown in Fig. 18.30.

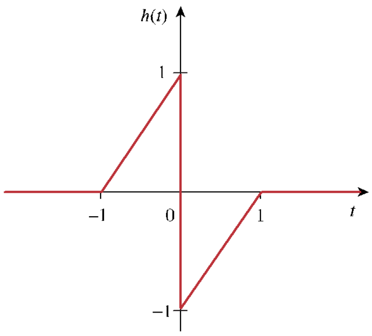
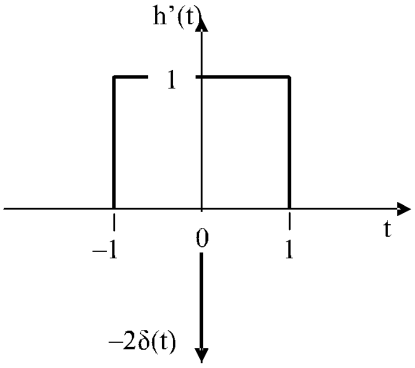
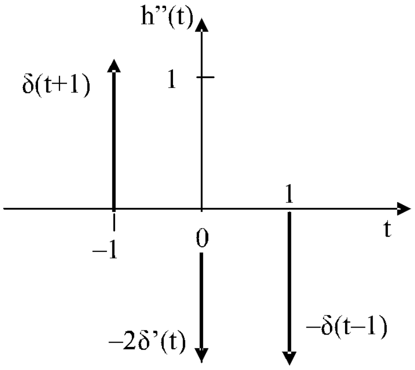


Figure 18.30
For Prob. 18.5.

Chapter 18, Solution 5.





$$h''(t) = \delta(t+1) - \delta(t-1) - 2\delta'(t)$$

$$(j\omega)^2 H(\omega) = e^{j\omega} - e^{-j\omega} - 2j\omega = 2j\sin\omega - 2j\omega$$

$$H(\omega) = \underline{\frac{2j}{\omega} - \frac{2j}{\omega^2}\sin\omega}$$

Chapter 18, Problem 6.

Find the Fourier transforms of both functions in Fig. 18.31 on the following page.

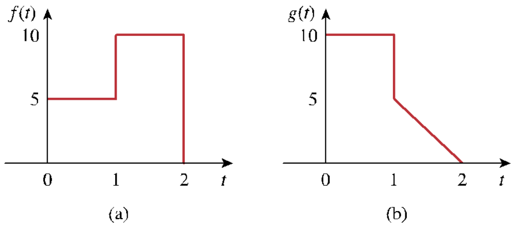
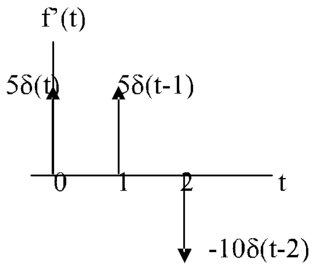


Figure 18.31
For Prob. 18.6.

Chapter 18, Solution 6.

(a) The derivative of f(t) is shown below.



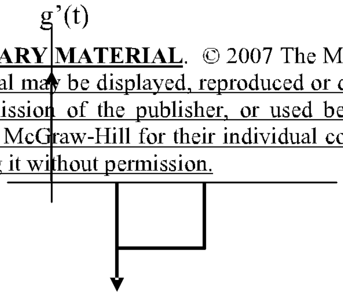
$$f'(t) = 5\delta(t) + 5\delta(t-1) - 10\delta(t-2)$$

Taking the Fourier transform of each term,

$$j\omega F(\omega) = 5 + 5e^{-j\omega} - 10e^{-j2\omega}$$

$$F(\omega) = \frac{5 + 5e^{-j\omega} - 10e^{-j2\omega}}{j\omega}$$

(b) The derivative of g(t) is shown below.



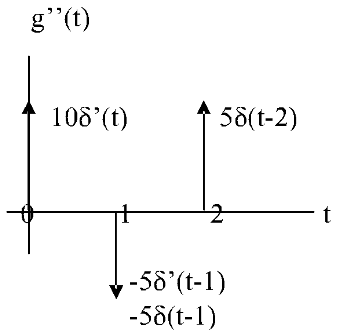
$10\delta(t)$

0 1 2

-5

$-5\delta(t-1)$

The second derivative of g(t) is shown below.



$$g''(t) = 10\delta'(t) - 5\delta'(t-1) - 5\delta(t-1) + 5\delta(t-2)$$

Take the Fourier transform of each term.

$(j\omega)^2 G(j\omega) = 10j\omega - 5j\omega e^{-j\omega} - 5e^{-j\omega} + 5e^{-j2\omega}$ which leads to

$$G(j\omega) = \underline{\mathbf{(-10j\omega + 5j\omega e^{-j\omega} + 5e^{-j\omega} - 5e^{-j2\omega}\quad)/\omega^2}}$$

Chapter 18, Problem 7.

Find the Fourier transforms of the signals in Fig. 18.32.

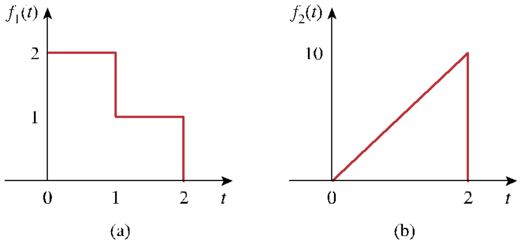
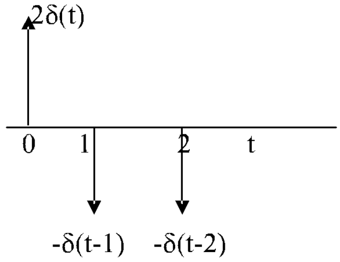


Figure 18.32
For Prob. 18.7.

Chapter 18, Solution 7.

(a) Take the derivative of $f_1(t)$ and obtain $f_1'(t)$ as shown below.



$$f_1'(t) = 2\delta(t) - \delta'(t-1) - \delta(t-2)$$

Take the Fourier transform of each term,

$$j\omega F_1(\omega) = 2 - e^{-j\omega} - e^{-j2\omega}$$

$$F_1(\omega) = \frac{2 - e^{-j\omega} - e^{-j2\omega}}{j\omega}$$

(b) $f_2(t) = 5t$

$$F_2(\omega) = \int_{-\infty}^{\infty} f_2(t)e^{-j\omega}dt = \int_0^2 5te^{-j\omega}dt = \frac{5}{(-j\omega)^2}e^{-j\omega t}(-j\omega - 1)\Big|_0^2$$

$$F_2(\omega) = \frac{5e^{-j2\omega}}{\omega^2}(1 + j\omega 2) - \frac{5}{\omega^2}$$

Chapter 18, Problem 8.

Obtain the Fourier transforms of the signals shown in Fig. 18.33.

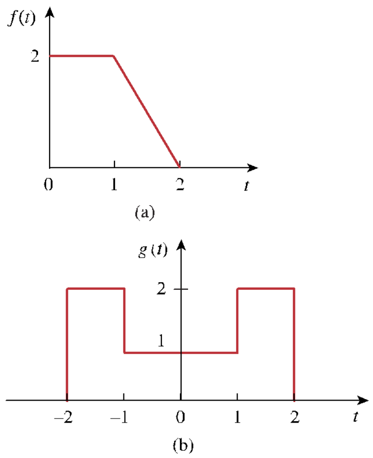


Figure 18.33
For Prob. 18.8.

Chapter 18, Solution 8.

(a)
$$F(\omega) = \int_0^1 2e^{-j\omega t}dt + \int_1^2 (4-2t)e^{-j\omega t}dt$$

$$= \frac{2}{-j\omega}e^{-j\omega t}\Big|_0^1 + \frac{4}{-j\omega}e^{-j\omega t}\Big|_1^2 - \frac{2}{-\omega^2}e^{-j\omega t}(-j\omega t - 1)\Big|_1^2$$

$$F(\omega) = \underline{\frac{2}{\omega^2} + \frac{2}{j\omega}e^{-j\omega} + \frac{2}{j\omega} - \frac{4}{j\omega}e^{-j2\omega} - \frac{2}{\omega^2}(1 + j2\omega)e^{-j2\omega}}$$

(b) g(t) = 2[u(t+2) – u(t-2)] - [u(t+1) – u(t-1)]

$$G(\omega) = \underline{\frac{4\sin 2\omega}{\omega} - \frac{2\sin\omega}{\omega}}$$

Chapter 18, Problem 9.

Determine the Fourier transforms of the signals in Fig. 18.34.

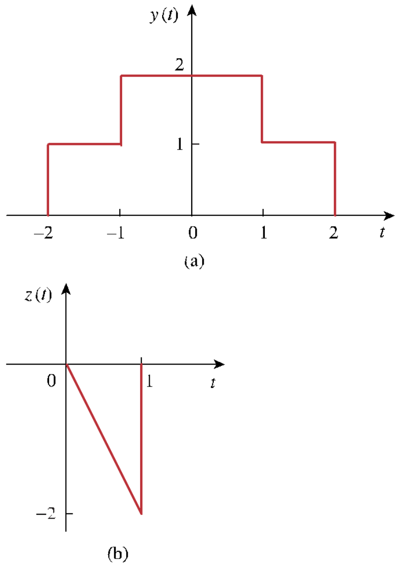


Figure 18.34
For Prob. 18.9.

Chapter 18, Solution 9.

(a) y(t) = u(t+2) – u(t-2) + 2[u(t+1) – u(t-1)]

$$Y(\omega) = \underline{\frac{2}{\omega}\sin 2\omega + \frac{4}{\omega}\sin\omega}$$

(b) $$Z(\omega) = \int_0^1 (-2t)e^{-j\omega t}dt = \frac{-2e^{-j\omega t}}{-\omega^2}(-j\omega t - 1)\Big|_0^1 = \underline{\frac{2}{\omega^2} - \frac{2e^{-j\omega}}{\omega^2}(1 + j\omega)}$$

Chapter 18, Problem 10.

Obtain the Fourier transforms of the signals shown in Fig. 18.35.

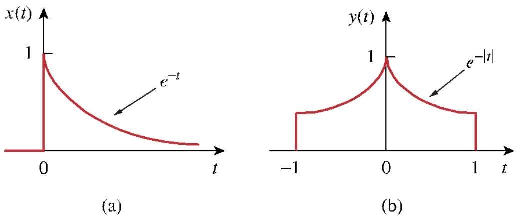


Figure 18.35
For Prob. 18.10.

Chapter 18, Solution 10.

(a) $x(t) = e^{2t}u(t)$

$X(\omega) = \underline{\mathbf{1/(-2 + j\omega)}}$

(b) $$e^{-(t)} = \begin{bmatrix} e^{-t}, & t > 0 \\ e^{t}, & t < 0 \end{bmatrix}$$

$$Y(\omega) = \int_{-1}^{1} y(t)e^{j\omega t}\,dt = \int_{-1}^{0} e^{t}\,e^{j\omega t}\,dt + \int_{0}^{1} e^{-t}\,e^{-j\omega t}\,dt$$

$$= \frac{e^{(1-j\omega)t}}{1-j\omega}\Bigg|_{-1}^{0} + \frac{e^{-(1+j\omega)t}}{-(1+j\omega)}\Bigg|_{0}^{1}$$

$$= \frac{2}{1+\omega^2} - e^{-1}\left[\frac{\cos\omega + j\sin\omega}{1-j\omega} + \frac{\cos\omega - j\sin\omega}{1+j\omega}\right]$$

$$Y(\omega) = \underline{\frac{\mathbf{2}}{\mathbf{1+\omega^2}}\left[\mathbf{1 - e^{-1}(\cos\omega - \omega\sin\omega)}\right]}$$

Chapter 18, Problem 11.

Find the Fourier transform of the “sine-wave pulse” shown in Fig. 18.36.

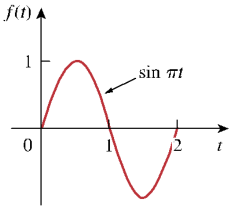


Figure 18.36
For Prob. 18.11.

Chapter 18, Solution 11.

$$f(t) = \sin \pi t\,[u(t) - u(t - 2)]$$

$$F(\omega) = \int_0^2 \sin \pi t\, e^{-j\omega t}\, dt = \frac{1}{2j}\int_0^2 \left(e^{j\pi t} - e^{-j\pi t}\right) e^{-j\omega t}\, dt$$

$$= \frac{1}{2j}\left[\int_0^2 (e^{+j(-\omega+\pi)t} + e^{-j(\omega+\pi)t})dt\right]$$

$$= \frac{1}{2j}\left[\frac{1}{-j(\omega-\pi)} e^{-j(\omega-\pi)t}\Big|_0^2 + \frac{e^{-j(\omega+\pi)t}}{-j(\omega+\pi)}\Big|_0^2\right]$$

$$= \frac{1}{2}\left(\frac{1-e^{-j2\omega}}{\pi-\omega} + \frac{1-e^{-j2\omega}}{\pi+\omega}\right)$$

$$= \frac{1}{2(\pi^2-\omega^2)}\left(2\pi + 2\pi e^{-j2\omega}\right)$$

$$F(\omega) = \underline{\frac{\pi}{\omega^2-\pi^2}\left(e^{-j\omega 2} - 1\right)}$$

Chapter 18, Problem 12.

Find the Fourier transform of the following signals.

(a) $f_1(t) = e^{-3t}\sin(10t)u(t)$

(b) $f_2(t) = e^{-4t}\cos(10t)u(t)$

Chapter 18, Solution 12.

(a) $F_1(\omega) = \dfrac{10}{(3+j\omega)^2+100}$

(b) $F_2(\omega) = \dfrac{4+j\omega}{(4+j\omega)^2+100}$

Chapter 18, Problem 13.

Find the Fourier transform of the following signals:

(a) $f(t) = \cos(at - \pi/3)$, $\quad -\infty < t < \infty$

(b) $g(t) = u(t+1)\sin \pi t$, $\quad -\infty < t < \infty$

(c) $h(t) = (1 + A\sin at)\cos bt$, $\quad -\infty < t < \infty$, where A, a and b are constants

(d) $i(t) = 1 - t$, $\quad 0 < t < 4$

Chapter 18, Solution 13.

(a) We know that $F[\cos at] = \pi[\delta(\omega - a) + \delta(\omega + a)]$.

Using the time shifting property,

$$F[\cos a(t - \pi/3a)] = \pi e^{-j\omega\pi/3a}[\delta(\omega - a) + \delta(\omega + a)] = \underline{\pi e^{-j\pi/3}\delta(\omega - a) + \pi e^{j\pi/3}\delta(\omega + a)}$$

(b) $\sin \pi(t+1) = \sin \pi t \cos \pi + \cos \pi t \sin \pi = -\sin \pi t$

g(t) = -u(t+1) sin (t+1)

Let $x(t) = u(t)\sin t$, then $X(\omega) = \dfrac{1}{(j\omega)^2 + 1} = \dfrac{1}{1 - \omega^2}$

Using the time shifting property,

$$G(\omega) = -\frac{1}{1 - \omega^2}e^{j\omega} = \underline{\frac{e^{j\omega}}{\omega^2 - 1}}$$

(c) Let y(t) = 1 + Asin at, then $Y(\omega) = 2\pi\delta(\omega) + j\pi A[\delta(\omega + a) - \delta(\omega - a)]$

h(t) = y(t) cos bt

Using the modulation property,

$$H(\omega) = \frac{1}{2}[Y(\omega + b) + Y(\omega - b)]$$

$$\underline{H(\omega) = \pi[\delta(\omega + b) + \delta(\omega - b)] + \frac{j\pi A}{2}[\delta(\omega + a + b) - \delta(\omega - a + b) + \delta(\omega + a - b) - \delta(\omega - a - b)]}$$

(d) $$I(\omega) = \int_0^4 (1 - t)e^{-j\omega t}dt = \frac{e^{-j\omega t}}{-j\omega} - \frac{e^{-j\omega t}}{-\omega^2}(-j\omega t - 1)\Big|_0^4 = \underline{\frac{1}{\omega^2} - \frac{e^{-j4\omega}}{j\omega} - \frac{e^{-j4\omega}}{\omega^2}(j4\omega + 1)}$$

Chapter 18, Problem 14.

Find the Fourier transforms of these functions:

(a) $f(t) = e^{-t}\ \cos(3t + \pi)u(t)$

(b) $g(t) = \sin\ \pi t[u(t+1) - u(t-1)]$

(c) $h(t) = e^{-2t} \cos\ \pi tu(t-1)$

(d) $p(t) = e^{-2t} \sin 4tu(-t)$

(e) $q(t) = 4\ \text{sgn}(t-2) + 3\,\delta(t) - 2u(t-2)$

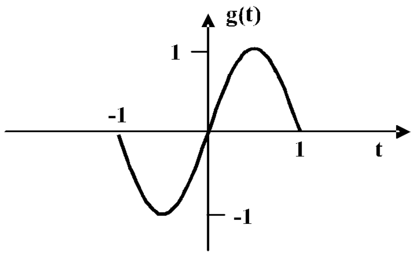
Chapter 18, Solution 14.

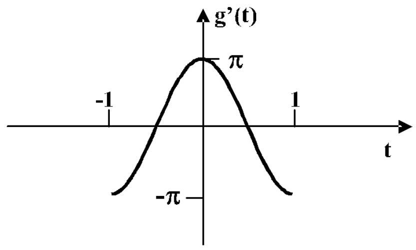
(a) $\cos(3t+\pi) = \cos 3t \cos\pi - \sin 3t \sin\pi = \cos 3t(-1) - \sin 3t(0) = -\cos(3t)$

$$f(t) = -e^{-t}\cos 3t\, u(t)$$

$$F(\omega) = \underline{\frac{-(1+j\omega)}{(1+j\omega)^2+9}}$$

(b)





$$g'(t) = \pi\cos\pi t[u(t-1) - u(t-1)]$$
$$g''(t) = -\pi^2 g(t) - \pi\delta(t+1) + \pi\delta(t-1)$$
$$-\omega^2 G(\omega) = -\pi^2 G(\omega) - \pi e^{j\omega} + \pi e^{-j\omega}$$
$$(\pi^2 - \omega^2)G(\omega) = -\pi(e^{j\omega} - e^{-j\omega}) = -2j\pi\sin\omega$$
$$G(\omega) = \underline{\frac{2j\pi\sin\omega}{\omega^2-\pi^2}}$$

Alternatively, we compare this with Prob. 17.7

f(t) = g(t - 1)

$F(\omega) = G(\omega)e^{-j\omega}$

$$G(\omega) = F(\omega)e^{j\omega} = \frac{\pi}{\omega^2-\pi^2}(e^{-j\omega} - e^{j\omega})$$
$$= \frac{-j2\pi\sin\omega}{\omega^2-\pi^2}$$

$$G(\omega) = \underline{\frac{2j\pi\sin\omega}{\pi^2-\omega^2}}$$

(c) $\cos\pi(t-1) = \cos\pi t\cos\pi + \sin\pi t\sin\pi = \cos\pi t(-1) + \sin\pi t(0) = -\cos\pi t$

Let $x(t) = e^{-2(t-1)}\cos\pi(t-1)u(t-1) = -e^{2}h(t)$

and $y(t) = e^{-2t}\cos(\pi t)u(t)$

$$Y(\omega) = \frac{2+j\omega}{(2+j\omega)^2+\pi^2}$$

$$y(t) = x(t-1)$$

$$Y(\omega) = X(\omega)e^{-j\omega}$$

$$X(\omega) = \frac{(2+j\omega)e^{j\omega}}{(2+j\omega)^2+\pi^2}$$

$$X(\omega) = -e^{2}H(\omega)$$

$$H(\omega) = -e^{-2}X(\omega)$$

$$= \mathbf{\frac{-(2+j\omega)e^{j\omega-2}}{(2+j\omega)^2+\pi^2}}$$

(d) Let $x(t) = e^{-2t}\sin(-4t)u(-t) = y(-t)$

$p(t) = -x(t)$

where $y(t) = e^{2t}\sin 4t\,u(t)$

$$Y(\omega) = \frac{2+j\omega}{(2+j\omega)^2+4^2}$$

$$X(\omega) = Y(-\omega) = \frac{2-j\omega}{(2-j\omega)^2+16}$$

$$p(\omega) = -X(\omega) = \mathbf{\frac{j\omega-2}{(j\omega-2)^2+16}}$$

(e) $$Q(\omega) = \frac{8}{j\omega}e^{-j\omega 2} + 3 - 2\left(\pi\delta(\omega) + \frac{1}{j\omega}\right)e^{-j\omega 2}$$

$$Q(\omega) = \mathbf{\frac{6}{j\omega}e^{j\omega 2} + 3 - 2\pi\delta(\omega)e^{-j\omega 2}}$$

Chapter 18, Problem 15.

Find the Fourier transforms of the following functions:

(a) $f(t) = \delta(t+3) - \delta(t-3)$

(b) $f(t) = \int_{-\infty}^{\infty} 2\delta(t-1)\,dt$

(c) $f(t) = \delta(3t) - \delta'(2t)$

Chapter 18, Solution 15.

(a) $F(\omega) = e^{j3\omega} - e^{-j\omega 3} = \underline{\mathbf{2j\sin 3\omega}}$

(b) Let $g(t) = 2\delta(t-1)$, $G(\omega) = 2e^{-j\omega}$

$$F(\omega) = \mathcal{F}\left(\int_{-\infty}^{t} g(t)\,dt\right)$$
$$= \frac{G(\omega)}{j\omega} + \pi F(0)\delta(\omega)$$
$$= \frac{2e^{-j\omega}}{j\omega} + 2\pi\delta(-1)\delta(\omega)$$
$$= \underline{\frac{\mathbf{2e^{-j\omega}}}{\mathbf{j\omega}}}$$

(c) $\mathcal{F}[\delta(2t)] = \frac{1}{2}\cdot 1$

$$F(\omega) = \frac{1}{3}\cdot 1 - \frac{1}{2}j\omega = \underline{\frac{\mathbf{1}}{\mathbf{3}} - \frac{\mathbf{j\omega}}{\mathbf{2}}}$$

Chapter 18, Problem 16.

* Determine the Fourier transforms of these functions:

(a) $f(t) = 4/t^2$ (b) $g(t) = 8/(4 + t^2)$

* An asterisk indicates a challenging problem.

Chapter 18, Solution 16.

(a) Using duality properly

$$|t| \rightarrow \frac{-2}{\omega^2}$$

$$\frac{-2}{t^2} \rightarrow 2\pi|\omega|$$

or $$\frac{4}{t^2} \rightarrow -4\pi|\omega|$$

$$F(\omega) = F\left(\frac{4}{t^2}\right) = \underline{\mathbf{-4\pi|\omega|}}$$

(b) $$e^{-|a|t} \longrightarrow \frac{2a}{a^2 + \omega^2}$$

$$\frac{2a}{a^2 + t^2} \longrightarrow 2\pi e^{-a|\omega|}$$

$$\frac{8}{a^2 + t^2} \longrightarrow 4\pi e^{-2|\omega|}$$

$$G(\omega) = F\left(\frac{8}{4 + t^2}\right) = \underline{\mathbf{4\pi e^{-2|\omega|}}}$$

Chapter 18, Problem 17.

Find the Fourier transforms of:

(a) $\cos 2tu(t)$ (b) $\sin 10tu(t)$

Chapter 18, Solution 17.

(a) Since $H(\omega) = \mathcal{F}\left(\cos\omega_0 t\ f(t)\right) = \frac{1}{2}\left[F(\omega+\omega_0) + F(\omega-\omega_0)\right]$

where $F(\omega) = \mathcal{F}\left[u(t)\right] = \pi\delta(\omega) + \frac{1}{j\omega}$, $\omega_0 = 2$

$$H(\omega) = \frac{1}{2}\left[\pi\delta(\omega+2) + \frac{1}{j(\omega+2)} + \pi\delta(\omega-2) + \frac{1}{j(\omega-2)}\right]$$

$$= \frac{\pi}{2}\left[\delta(\omega+2) + \delta(\omega-2)\right] - \frac{j}{2}\left[\frac{\omega+2+\omega-2}{(\omega+2)(\omega-2)}\right]$$

$$H(\omega) = \underline{\boldsymbol{\frac{\pi}{2}\left[\delta(\omega+2) + \delta(\omega-2)\right] - \frac{j\omega}{\omega^2-4}}}$$

(b) $G(\omega) = \mathcal{F}\left[\sin\omega_0 t\ f(t)\right] = \frac{j}{2}\left[F(\omega+\omega_0) - F(\omega-\omega_0)\right]$

where $F(\omega) = \mathcal{F}\left[u(t)\right] = \pi\delta(\omega) + \frac{1}{j\omega}$

$$G(\omega) = \frac{j}{2}\left[\pi\delta(\omega+10) + \frac{1}{j(\omega+10)} - \pi\delta(\omega-10) - \frac{1}{j(\omega-10)}\right]$$

$$= \frac{j\pi}{2}\left[\delta(\omega+10) - \delta(\omega-10)\right] + \frac{j}{2}\left[\frac{j}{\omega-10} - \frac{j}{\omega+10}\right]$$

$$= \underline{\boldsymbol{\frac{j\pi}{2}\left[\delta(\omega+10) - \delta(\omega-10)\right] - \frac{10}{\omega^2-100}}}$$

Chapter 18, Problem 18.

Given that $F(\omega)$ = F[$f(t)$], prove the following results, using the definition of Fourier transform:

(a) $F[f(t-t_0)] = e^{-j\omega t_0}F(\omega)$

(b) $F\left[\dfrac{df(t)}{dt}\right] = j\omega F(\omega)$

(c) $F[f(-t)] = F(-\omega)$

(d) $F[tf(t)] = j\dfrac{d}{d\omega}F(\omega)$

Chapter 18, Solution 18.

(a) $F[f(t-t_o)] = \int_{-\infty}^{\infty} f(t-t_o)e^{-j\omega t}dt$

Let $t - t_o = \lambda \longrightarrow t = \lambda + t_o, \quad dt = d\lambda$

$$F[f(t-t_o)] = \int_{-\infty}^{\infty} f(\lambda)e^{-j\omega\lambda}e^{-j\omega t_o}d\lambda = e^{-j\omega t_o}F(\omega)$$

(b) Given that $\quad f(t) = F^{-1}[F(\omega)] = \dfrac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega$

$$f'(t) = \frac{j\omega}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}dt = j\omega F^{-1}[F(\omega)]$$

or

$$F[f'(t)] = j\omega F(\omega)$$

(c) This is a special case of the time scaling property when a = –1. Hence,

$$F[f(-t)] = \frac{1}{|-1|}F(-\omega) = F(-\omega)$$

(d) $F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$

Differentiating both sides respect to ω and multiplying by t yields

$$j\frac{dF(\omega)}{d\omega} = j\int_{-\infty}^{\infty}(-jt)f(t)e^{-j\omega t}dt = \int_{-\infty}^{\infty} tf(t)e^{-j\omega t}dt$$

Hence,

$$j\frac{dF(\omega)}{d\omega} = F[tf(t)]$$

Chapter 18, Problem 19.

Find the Fourier transform of

$f(t) = \cos 2\pi t[u(t) - u(t-1)]$

Chapter 18, Solution 19.

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{j\omega t}\,dt = \frac{1}{2}\int_{0}^{1}\left(e^{j2\pi t} + e^{-j2\pi t}\right)e^{-j\omega t}\,dt$$

$$F(\omega) = \frac{1}{2}\int_{0}^{1}\left[e^{-j(\omega+2\pi)t} + e^{-j(\omega-2\pi)t}\right]dt$$

$$= \frac{1}{2}\left[\frac{1}{-j(\omega+2\pi)}e^{-j(\omega+2\pi)t} + \frac{1}{-j(\omega-2\pi)}e^{-j(\omega-2\pi)t}\right]_0^1$$

$$= -\frac{1}{2}\left[\frac{e^{-j(\omega+2\pi)} - 1}{j(\omega+2\pi)} + \frac{e^{-j(\omega-2\pi)} - 1}{j(\omega-2\pi)}\right]$$

But $\quad e^{j2\pi} = \cos 2\pi + j\sin 2\pi = 1 = e^{-j2\pi}$

$$F(\omega) = -\frac{1}{2}\left(\frac{e^{-j\omega} - 1}{j}\right)\left(\frac{1}{\omega+2\pi} + \frac{1}{\omega-2\pi}\right)$$

$$= \underline{\frac{\mathbf{j\omega}}{\omega^2 - 4\pi^2}\left(e^{-j\omega} - 1\right)}$$

Chapter 18, Problem 20.

(a) Show that a periodic signal with exponential Fourier series

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

has the Fourier transform

$$F(\omega) = \sum_{n=-\infty}^{\infty} c_n \delta(\omega - n\omega_0)$$

where $\omega_0 = 2\pi/T$.

(b) Find the Fourier transform of the signal in Fig. 18.37.

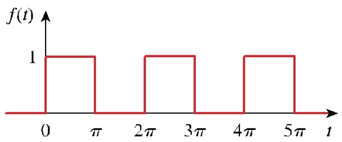


Figure 18.37
For Prob. 18.20(b).

Chapter 18, Solution 20.

(a) $F(c_n) = c_n\delta(\omega)$

$$F\left(c_n e^{jn\omega_o t}\right) = c_n \delta(\omega - n\omega_o)$$

$$F\left(\sum_{n=-\infty}^{\infty} c_n e^{jn\omega_o t}\right) = \underline{\sum_{n=-\infty}^{\infty} \mathbf{c_n \delta(\omega - n\omega_o)}}$$

(b) $T = 2\pi \longrightarrow \omega_o = \dfrac{2\pi}{T} = 1$

$$c_n = \frac{1}{T}\int_0^T f(t) e^{-jn\omega_o t}\,dt = \frac{1}{2\pi}\left(\int_0^\pi 1\cdot e^{-jnt}\,dt + 0\right)$$

$$= \frac{1}{2\pi}\left(-\frac{1}{jn} e^{jnt}\Big|_0^\pi\right) = \frac{j}{2\pi n}\left(e^{-jn\pi} - 1\right)$$

But $e^{-jn\pi} = \cos n\pi + j\sin n\pi = \cos n\pi = (-1)^n$

$$c_n = \frac{j}{2n\pi}\left[(-1)^n - 1\right] = \begin{cases} 0, & n = \text{even} \\ \dfrac{-j}{n\pi}, & n = \text{odd}, n \neq 0 \end{cases}$$

for n = 0

$$c_n = \frac{1}{2\pi}\int_0^\pi 1\,dt = \frac{1}{2}$$

Hence

$$f(t) = \frac{1}{2} - \sum_{\substack{n=-\infty \\ n\neq 0 \\ n=\text{odd}}}^{\infty} \frac{j}{n\pi} e^{jnt}$$

$$F(\omega) = \underline{\frac{1}{2}\delta\omega - \sum_{\substack{n=-\infty \\ n\neq 0 \\ n=\text{odd}}}^{\infty} \frac{j}{n\pi}\delta(\omega - n)}$$

Chapter 18, Problem 21.

Show that

$$\int_{-\infty}^{\infty} \left(\frac{\sin a\omega}{a\omega}\right)^2 d\omega = \frac{\pi}{a}$$

Hint: Use the fact that

$$F[u(t+a) - u(t-a)] = 2a\left(\frac{\sin a\omega}{a\omega}\right).$$

Chapter 18, Solution 21.

Using Parseval's theorem,

$$\int_{-\infty}^{\infty} f^2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(\omega)|^2 d\omega$$

If $f(t) = u(t+a) - u(t+a)$, then

$$\int_{-\infty}^{\infty} f^2(t)dt = \int_{-a}^{a} (1)^2 dt = 2a = \frac{1}{2\pi}\int_{-\infty}^{\infty} 4a^2\left(\frac{\sin a\omega}{a\omega}\right)^2 d\omega$$

or

$$\int_{-\infty}^{\infty} \left(\frac{\sin a\omega}{a\omega}\right)^2 d\omega = \frac{4\pi a}{4a^2} = \frac{\pi}{a} \quad \text{as required.}$$

Chapter 18, Problem 22.

Prove that if $F(\omega)$ is the Fourier transform of $f(t)$,

$$F[f(t)\sin\omega_0 t] = \frac{j}{2}[F(\omega+\omega_0) - F(\omega-\omega_o)]$$

Chapter 18, Solution 22.

$$F[f(t)\sin\omega_o t] = \int_{-\infty}^{\infty} f(t)\frac{\left(e^{j\omega_o t} - e^{-j\omega_o t}\right)}{2j} e^{-j\omega t} dt$$

$$= \frac{1}{2j}\left[\int_{-\infty}^{\infty} f(t)e^{-j(\omega-\omega_o)t} dt - \int_{-\infty}^{\infty} e^{-j(\omega+\omega_o)t} dt\right]$$

$$= \frac{1}{2j}[F(\omega-\omega_o) - F(\omega+\omega_o)]$$

Chapter 18, Problem 23.

If the Fourier transform of $f(t)$ is

$$F(\omega) = \frac{10}{(2+j\omega)(5+j\omega)}$$

determine the transforms of the following:

(a) $f(-3t)$ (b) $f(2t-1)$ (c) $f(t)\cos 2t$

(d) $\frac{d}{dt}f(t)$ (e) $\int_{-\infty}^{t} f(t)dt$

Chapter 18, Solution 23.

(a) f(3t) leads to $\frac{1}{3}\cdot\frac{10}{(2+j\omega/3)(5+j\omega/3)} = \frac{30}{(6+j\omega)(15+j\omega)}$

$$F\,[f(-3t)] = \underline{\frac{\mathbf{30}}{\mathbf{(6-j\omega)(15-j\omega)}}}$$

(b) $f(2t) \longrightarrow \frac{1}{2}\cdot\frac{10}{(2+j\omega/2)(15+j\omega/2)} = \frac{20}{(4+j\omega)(10+j\omega)}$

$$f(2t-1) = f\,[2(t-1/2)] \longrightarrow \underline{\frac{\mathbf{20e^{-j\omega/2}}}{\mathbf{(4+j\omega)(10+j\omega)}}}$$

(c) $f(t)\cos 2t \longrightarrow \frac{1}{2}F(\omega+2)+\frac{1}{2}F(\omega+2)$

$$= \underline{\frac{\mathbf{5}}{\mathbf{[2+j(\omega+2)][5+j(\omega+2)]}} + \frac{\mathbf{5}}{\mathbf{[2+j(\omega-2)[5+j(\omega-2)]]}}}$$

(d) $F\,[f'(t)] = j\omega F(\omega) = \frac{j\omega 10}{(2+j\omega)(5+j\omega)}$

(e) $\int_{-\infty}^{t} f(t)dt \longrightarrow \frac{F(\omega)}{j(\omega)} + \pi F(0)\delta(\omega)$

$$= \frac{10}{j\omega(2+j\omega)(5+j\omega)} + \pi\delta(\omega)\frac{x10}{2x5}$$

$$= \underline{\frac{\mathbf{10}}{\mathbf{j\omega(2+j\omega)(5+j\omega)}} + \pi\delta(\omega)}$$

Chapter 18, Problem 24.

Given that $F[f(t)] = (j\ /\omega)(e^{-j\omega} - 1)$, find the Fourier transforms of:

(a) $x(t) = f(t) + 3$

(b) $y(t) = f(t-2)$

(c) $h(t) = f'(t)$

(d) $g(t) = 4f\left(\frac{2}{3}t\right) + 10f\left(\frac{5}{3}t\right)$

Chapter 18, Solution 24.

(a) $X(\omega) = F(\omega) + F[3]$

$$= \underline{\mathbf{6\pi\delta(\omega) + \frac{j}{\omega}\left(e^{-j\omega} - 1\right)}}$$

(b) $y(t) = f(t-2)$

$$Y(\omega) = e^{-j\omega 2}F(\omega) = \underline{\mathbf{\frac{je^{-j2\omega}}{\omega}\left(e^{-j\omega} - 1\right)}}$$

(c) If $h(t) = f'(t)$

$$H(\omega) = j\omega F(\omega) = j\omega\frac{j}{\omega}\left(e^{-j\omega} - 1\right) = \underline{\mathbf{1 - e^{-j\omega}}}$$

(d) $g(t) = 4f\left(\frac{2}{3}t\right) + 10f\left(\frac{5}{3}t\right)$, $G(\omega) = 4x\frac{3}{2}F\left(\frac{3}{2}\omega\right) + 10x\frac{3}{5}F\left(\frac{3}{5}\omega\right)$

$$= 6\cdot\frac{j}{\frac{3}{2}\omega}\left(e^{-j3\omega/2} - 1\right) + \frac{6j}{\frac{3}{5}\omega}\left(e^{-j3\omega/5} - 1\right)$$

$$= \underline{\mathbf{\frac{j4}{\omega}\left(e^{-j3\omega/2} - 1\right) + \frac{j10}{\omega}\left(e^{-j3\omega/5} - 1\right)}}$$

Chapter 18, Problem 25.

Obtain the inverse Fourier transform of the following signals.

(a) $F(\omega) = \dfrac{5}{j\omega - 2}$

(b) $H(\omega) = \dfrac{12}{\omega^2 + 4}$

(c) $X(\omega) = \dfrac{10}{(j\omega - 1)(j\omega - 2)}$

Chapter 18, Solution 25.

(a) $g(t) = \underline{5e^{2t}u(t)}$

(b) $h(t) = \underline{6e^{-2|t|}}$

(c) $X(\omega) = \dfrac{A}{s-1} + \dfrac{B}{s-2}, \qquad s = j\omega$

$$A = \frac{10}{1-2} = -10, \qquad B = \frac{10}{2-1} = 10$$

$$X(\omega) = \frac{-10}{j\omega - 1} + \frac{10}{j\omega - 2}$$

$x(t) = \underline{-10e^{t}u(t) + 10e^{2t}u(t)}$

Chapter 18, Problem 26.

Determine the inverse Fourier transforms of the following:

(a) $F(\omega) = \dfrac{e^{-j2\omega}}{1 + j\omega}$

(b) $H(\omega) = \dfrac{1}{(j\omega + 4)^2}$

(c) $G(\omega) = 2u(\omega + 1) - 2u(\omega - 1)$

Chapter 18, Solution 26.

(a) $\underline{f(t) = e^{-(t-2)}u(t)}$

(b) $\underline{h(t) = te^{-4t}u(t)}$

(c) If $x(t) = u(t+1) - u(t-1) \longrightarrow X(\omega) = 2\dfrac{\sin\omega}{\omega}$

By using duality property,

$$G(\omega) = 2u(\omega+1) - 2u(\omega-1) \longrightarrow \underline{g(t) = \frac{2\sin t}{\pi t}}$$

Chapter 18, Problem 27.

Find the inverse Fourier transforms of the following functions:

(a) $F(\omega) = \dfrac{100}{j\omega(j\omega+10)}$

(b) $G(\omega) = \dfrac{10j\omega}{(-j\omega+2)(j\omega+3)}$

(c) $H(\omega) = \dfrac{60}{-\omega^2 + j40\omega + 1300}$

(d) $Y(\omega) = \dfrac{\delta(\omega)}{(j\omega+1)(j\omega+2)}$

Chapter 18, Solution 27.

(a) Let $F(s) = \dfrac{100}{s(s+10)} = \dfrac{A}{s} + \dfrac{B}{s+10}, \quad s = j\omega$

$$A = \frac{100}{10} = 10, \quad B = \frac{100}{-10} = -10$$

$$F(\omega) = \frac{10}{j\omega} - \frac{10}{j\omega + 10}$$

$$f(t) = \underline{\mathbf{5sgn(t) - 10e^{-10t}u(t)}}$$

(b) $G(s) = \dfrac{10s}{(2-s)(3+s)} = \dfrac{A}{2-s} + \dfrac{B}{s+3}, \quad s = j\omega$

$$A = \frac{20}{5} = 4, \quad B = \frac{-30}{5} = -6$$

$$G(\omega) = \frac{4}{= -j\omega + 2} - \frac{6}{j\omega + 3}$$

$$g(t) = \underline{\mathbf{4e^{2t}u(-t) - 6e^{-3t}u(t)}}$$

(c) $H(\omega) = \dfrac{60}{(j\omega)^2 + j40\omega + 1300} = \dfrac{60}{(j\omega + 20)^2 + 900}$

$$h(t) = \underline{\mathbf{2e^{-20t}\sin(30t)u(t)}}$$

(d) $y(t) = \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty} \dfrac{\delta(\omega)e^{j\omega t}d\omega}{(2+j\omega)(j\omega+1)} = \dfrac{1}{2}\pi \cdot \dfrac{1}{2} = \underline{\mathbf{\dfrac{1}{4}}}\pi$

Chapter 18, Problem 28.

Find the inverse Fourier transforms of:

(a) $\dfrac{\pi\delta(\omega)}{(5+j\omega)(2+j\omega)}$

(b) $\dfrac{10\delta(\omega+2)}{j\omega(j\omega+1)}$

(c) $\dfrac{20\delta(\omega-1)}{(2+j\omega)(3+j\omega)}$

(d) $\dfrac{5\pi\delta(\omega)}{5+j\omega}+\dfrac{5}{j\omega(5+j\omega)}$

Chapter 18, Solution 28.

(a) $$f(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}F(\omega)e^{j\omega t}\,d\omega=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\pi\delta(\omega)\,e^{j\omega t}}{(5+j\omega)(2+j\omega)}d\omega$$

$$=\frac{1}{2}\frac{1}{(5)(2)}=\frac{1}{20}=\underline{\mathbf{0.05}}$$

(b) $$f(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{10\delta(\omega+2)}{j\omega(j\omega+1)}e^{j\omega t}\,d\omega=\frac{10}{2\pi}\frac{e^{-j2t}}{(-j2)(-j2+1)}$$

$$=\frac{j5}{2\pi}\frac{e^{-j2t}}{1-j2}=\underline{\mathbf{\frac{(-2+j)e^{-j2t}}{2\pi}}}$$

(c) $$f(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{20\delta(\omega-1)e^{j\omega t}}{(2+j\omega)(3+5\omega)}d\omega=\frac{20}{2\pi}\frac{e^{jt}}{(2+j)(3+j)}$$

$$=\frac{20e^{jt}}{2\pi(5+5j)}=\underline{\mathbf{\frac{(1-j)e^{jt}}{\pi}}}$$

(d) Let $$F(\omega)=\frac{5\pi\delta(\omega)}{(5+j\omega)}+\frac{5}{j\omega(5+j\omega)}=F_1(\omega)+F_2(\omega)$$

$$f_1(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{5\pi\delta(\omega)}{5+j\omega}e^{j\omega t}\,d\omega=\frac{5\pi}{2\pi}\cdot\frac{1}{5}=0.5$$

$$F_2(s)=\frac{5}{s(5+s)}=\frac{A}{s}+\frac{B}{s+5}\ ,\quad A=1, B=-1$$

$$F_2(\omega)=\frac{1}{j\omega}-\frac{1}{j\omega+5}$$

$$f_2(t)=\frac{1}{2}\mathrm{sgn}(t)-e^{-5t}=-\frac{1}{2}+u(t)-e^{5t}$$

$$f(t)=f_1(t)+f_2(t)=\ \underline{\mathbf{u(t)-e^{-5t}}}$$

Chapter 18, Problem 29.

* Determine the inverse Fourier transforms of:

(a) $F(\omega) = 4\delta(\omega + 3) + \delta(\omega) + 4\delta(\omega - 3)$
(b) $G(\omega) = 4u(\omega + 2) - 4u(\omega - 2)$
(c) $H(\omega) = 6\cos 2\omega$

* An asterisk indicates a challenging problem.

Chapter 18, Solution 29.

(a) $f(t) = F^{-1}[\delta(\omega)] + F^{-1}[4\delta(\omega+3) + 4\delta(\omega-3)]$

$$= \frac{1}{2\pi} + \frac{4\cos 3t}{\pi} = \underline{\frac{\mathbf{1}}{\mathbf{2\pi}}(\mathbf{1 + 8\cos 3t})}$$

(b) If $h(t) = u(t+2) - u(t-2)$

$$H(\omega) = \frac{2\sin 2\omega}{\omega}$$

$$G(\omega) = 4H(\omega) \longrightarrow g(t) = \frac{1}{2\pi}\cdot\frac{8\sin 2t}{t}$$

$$g(t) = \underline{\frac{\mathbf{4\sin 2t}}{\mathbf{\pi t}}}$$

(c) Since

$$\cos(at) \quad \pi\delta(\omega + a) + \pi\delta(\omega - a)$$

Using the reversal property,

$$2\pi\cos 2\omega \leftrightarrow \pi\delta(t+2) + \pi\delta(t-2)$$

or $F^{-1}[6\cos 2\omega] = \underline{\mathbf{3\delta(t+2) + 3\delta(t-2)}}$

Chapter 18, Problem 30.

For a linear system with input $x(t)$ and output $y(t)$ find the impulse response for the following cases:

(a) $x(t) = e^{-at}\, u(t)$, $\quad y(t) = u(t) - u(-t)$

(b) $x(t) = e^{-t}\, u(t)$, $\quad y(t) = e^{-2t}\, u(t)$

(c) $x(t) = \delta(t)$, $\quad y(t) = e^{-at} \sin btu(t)$

Chapter 18, Solution 30.

(a) $y(t) = \text{sgn}(t) \longrightarrow Y(\omega) = \frac{2}{j\omega}, \qquad X(\omega) = \frac{1}{a + j\omega}$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{2(a + j\omega)}{j\omega} = 2 + \frac{2a}{j\omega} \longrightarrow \underline{h(t) = 2\delta(t) + a[u(t) - u(-t)]}$$

(b) $X(\omega) = \frac{1}{1 + j\omega}, \qquad Y(\omega) = \frac{1}{2 + j\omega}$

$$H(\omega) = \frac{1 + j\omega}{2 + j\omega} = 1 - \frac{1}{2 + j\omega} \longrightarrow \underline{h(t) = \delta(t) - e^{-2t}u(t)}$$

(c) In this case, by definition, $\underline{h(t) = \quad y(t) = e^{-at} \sin bt\, u(t)}$

Chapter 18, Problem 31.

Given a linear system with output $y(t)$ and impulse response $h(t)$, find the corresponding input $x(t)$ for the following cases:

(a) $y(t) = te^{-at}u(t), \qquad h(t) = e^{-at}u(t)$

(b) $y(t) = u(t+1) - u(t-1), \qquad h(t) = \delta(t)$

(c) $y(t) = e^{-at}u(t), \qquad h(t) = \text{sgn}(t)$

Chapter 18, Solution 31.

(a) $$Y(\omega) = \frac{1}{(a+j\omega)^2}, \qquad H(\omega) = \frac{1}{a+j\omega}$$

$$X(\omega) = \frac{Y(\omega)}{H(\omega)} = \frac{1}{a+j\omega} \longrightarrow \underline{x(t) = e^{-at}u(t)}$$

(b) By definition, $\underline{x(t) = y(t) = u(t+1) - u(t-1)}$

(c) $$Y(\omega) = \frac{1}{(a+j\omega)}, \qquad H(\omega) = \frac{2}{j\omega}$$

$$X(\omega) = \frac{Y(\omega)}{H(\omega)} = \frac{j\omega}{2(a+j\omega)} = \frac{1}{2} - \frac{a}{2(a+j\omega)} \longrightarrow \underline{x(t) = \frac{1}{2}\delta(t) - \frac{a}{2}e^{-at}u(t)}$$

Chapter 18, Problem 32.

* Determine the functions corresponding to the following Fourier transforms:

(a) $F_1(\omega) = \dfrac{e^{jw}}{-j\omega+1}$ (b) $F_2(\omega) = 2e^{|\omega|}$

(c) $F_3(\omega) = \dfrac{1}{(1+\omega^2)^2}$ (d) $F_4(\omega) = \dfrac{\delta(\omega)}{1+j2\omega}$

* An asterisk indicates a challenging problem.

Chapter 18, Solution 32.

(a) Since $\dfrac{e^{-j\omega}}{j\omega+1}$ $e^{-(t-1)}u(t-1)$

and $F(-\omega)$ ~~$f(-t)$~~

$F_1(\omega) = \dfrac{e^{j\omega}}{-j\omega+1}$ ~~$f_1(t)$~~ $e^{-(-t-1)}u(-t-1)$

$f_1(t) = \underline{\mathbf{e^{(t+1)}u(-t-1)}}$

(b) From Section 17.3,

$$\frac{2}{t^2+1} \longrightarrow 2\pi e^{-|\omega|}$$

If $F_2(\omega) = 2e^{-|\omega|}$, then

$f_2(t) = \underline{\dfrac{\mathbf{2}}{\boldsymbol{\pi}\mathbf{(t^2+1)}}}$

(d) By partial fractions

$$F_3(\omega) = \frac{1}{(j\omega+1)^2(j\omega-1)^2} = \frac{\frac{1}{4}}{(j\omega+1)^2} + \frac{\frac{1}{4}}{(j\omega+1)} + \frac{\frac{1}{4}}{(j\omega-1)^2} - \frac{\frac{1}{4}}{j\omega-1}$$

Hence $f_3(t) = \dfrac{1}{4}\left(te^{-t} + e^{-t} + te^{t} - e^{t}\right)u(t)$

$= \underline{\mathbf{\frac{1}{4}(t+1)e^{-t}u(t) + \frac{1}{4}(t-1)e^{t}u(t)}}$

(d) $f_4(t) = \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty} F_1(\omega)e^{j\omega t}\,d\omega = \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty} \dfrac{\delta(\omega)e^{j\omega t}}{1+j2\omega} = \underline{\dfrac{\mathbf{1}}{\mathbf{2\pi}}}$

Chapter 18, Problem 33.

* Find *f*(*t*) if:

(a) $F(\omega) = 2\sin \pi\omega[u(\omega+1) - u(\omega-1)]$

(b) $F(\omega) = \dfrac{1}{\omega}(\sin 2\omega - \sin\omega) + \dfrac{j}{\omega}(\cos 2\omega - \cos\omega)$

* An asterisk indicates a challenging problem.

Chapter 18, Solution 33.

(a) Let $x(t) = 2\sin\pi t[u(t+1) - u(t-1)]$

From Problem 17.9(b),

$$X(\omega) = \frac{4j\pi\sin\omega}{\pi^2 - \omega^2}$$

Applying duality property,

$$f(t) = \frac{1}{2\pi}X(-t) = \frac{2j\sin(-t)}{\pi^2 - t^2}$$

$$f(t) = \underline{\frac{2j\sin t}{t^2 - \pi^2}}$$

(b) $F(\omega) = \dfrac{j}{\omega}(\cos 2\omega - j\sin 2\omega) - \dfrac{j}{\omega}(\cos\omega - j\sin\omega)$

$$= \frac{j}{\omega}\left(e^{j2\omega} - e^{-j\omega}\right) = \frac{e^{-j\omega}}{j\omega} - \frac{e^{j2\omega}}{j\omega}$$

$$f(t) = \frac{1}{2}\text{sgn}(t-1) - \frac{1}{2}\text{sgn}(t-2)$$

But $\text{sgn}(t) = 2u(t) - 1$

$$f(t) = u(t-1) - \frac{1}{2} - u(t-2) + \frac{1}{2}$$

$$= \underline{\mathbf{u(t-1) - u(t-2)}}$$

Chapter 18, Problem 34.

Determine the signal $f(t)$ whose Fourier transform is shown in Fig. 18.38. (*Hint:* Use the duality property.)

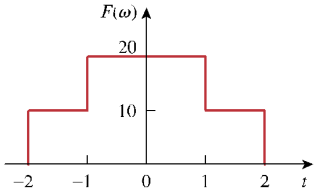


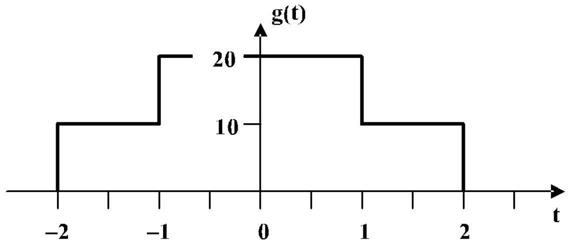
Figure 18.38
For Prob. 18.34.

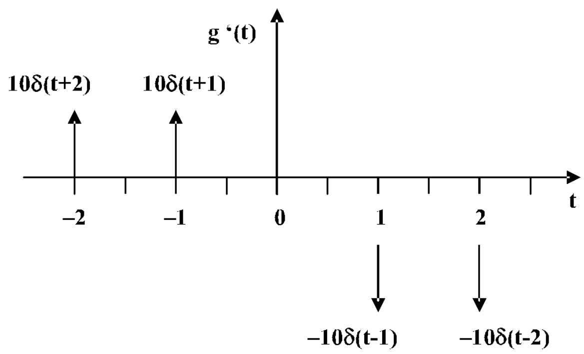
Chapter 18, Solution 34.

First, we find G(ω) for g(t) shown below.

$$g(t) = 10[u(t+2) - u(t-2)] + 10[u(t+1) - u(t-1)]$$
$$g'(t) = 10[\delta(t+2) - \delta(t-2)] + 10[\delta(t+1) - \delta(t-1)]$$

The Fourier transform of each term gives





$$j\omega G(\omega) = 10\left(e^{j\omega 2} - e^{-j\omega 2}\right) + 10\left(e^{j\omega} - e^{-j\omega}\right)$$
$$= 20j\sin 2\omega + 20j\sin\omega$$
$$G(\omega) = \frac{20\sin 2\omega}{\omega} + \frac{20\sin\omega}{\omega} = 40\ \text{sinc}(2\omega) + 20\ \text{sinc}(\omega)$$

Note that G(ω) = G(-ω).

$$F(\omega) = 2\pi G(-\omega)$$
$$f(t) = \frac{1}{2\pi} G(t)$$
$$= \underline{\mathbf{(20/\pi)sinc(2t) + (10/\pi)sinc(t)}}$$

Chapter 18, Problem 35.

A signal $f(t)$ has Fourier transform

$$F(\omega) = \frac{1}{2 + j\omega}$$

Determine the Fourier transform of the following signals:

(a) $x(t) = f(3t - 1)$

(b) $y(t) = f(t) \cos 5t$

(c) $z(t) = \frac{d}{dt} f(t)$

(d) $h(t) = f(t) * f(t)$

(e) $i(t) = tf(t)$

Chapter 18, Solution 35.

(a) x(t) = f[3(t-1/3)]. Using the scaling and time shifting properties,

$$X(\omega) = \frac{1}{3}\frac{1}{2 + j\omega/3}e^{-j\omega/3} = \underline{\frac{e^{-j\omega/3}}{(6 + j\omega)}}$$

(b) Using the modulation property,

$$Y(\omega) = \frac{1}{2}[F(\omega+5) + F(\omega-5)] = \underline{\frac{1}{2}\left[\frac{1}{2 + j(\omega+5)} + \frac{1}{2 + j(\omega-5)}\right]}$$

(c)

$$Z(\omega) = j\omega F(\omega) = \underline{\frac{j\omega}{2 + j\omega}}$$

(d)

$$H(\omega) = F(\omega)F(\omega) = \underline{\frac{1}{(2 + j\omega)^2}}$$

(e)

$$I(\omega) = j\frac{d}{d\omega}F(\omega) = j\frac{(0 - j)}{(2 + j\omega)^2} = \underline{\frac{1}{(2 + j\omega)^2}}$$

Chapter 18, Problem 36.

The transfer function of a circuit is

$$H(\omega) = \frac{2}{j\omega + 2}$$

If the input signal to the circuit is $v_s(t) = e^{-4t}u(t)$ V find the output signal. Assume all initial conditions are zero.

Chapter 18, Solution 36.

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} \longrightarrow Y(\omega) = H(\omega)X(\omega)$$

$$x(t) = v_s(t) = e^{-4t}u(t) \longrightarrow X(\omega) = \frac{1}{4 + j\omega}$$

$$Y(\omega) = \frac{2}{(j\omega + 2)(4 + j\omega)} = \frac{2}{(s+2)(s+4)}, \qquad s = j\omega$$

$$Y(s) = \frac{A}{s+2} + \frac{B}{s+4}$$

$$A = \frac{2}{-2+4} = 1, \qquad B = \frac{2}{-4+2} = -1$$

$$Y(s) = \frac{1}{s+2} - \frac{1}{s+4}$$

$$y(t) = \underline{\left(e^{-2t} - e^{-4t}\right)u(t)}$$

Please note, the units are not known since the transfer function does not give them. If the transfer function was a voltage gain then the units on y(t) would be volts.

Chapter 18, Problem 37.

Find the transfer function $I_o(\omega)/I_s(\omega)$ for the circuit in Fig. 18.39.

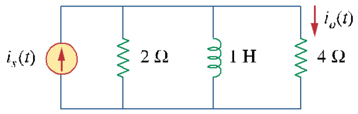


Figure 18.39
For Prob. 18.37.

Chapter 18, Solution 37.

$$2 \| j\omega = \frac{j2\omega}{2 + j\omega}$$

By current division,

$$H(\omega) = \frac{I_o(\omega)}{I_s(\omega)} = \frac{\dfrac{j2\omega}{2 + j\omega}}{4 + \dfrac{j2\omega}{2 + j\omega}} = \frac{j2\omega}{j2\omega + 8 + j4\omega}$$

$$H(\omega) \;=\; \underline{\frac{\mathbf{j\omega}}{\mathbf{4 + j3\omega}}}$$

Chapter 18, Problem 38.

Suppose $v_s(t) = u(t)$ for $t > 0$. Determine $i(t)$ in the circuit of Fig. 18.40, using the Fourier transform.

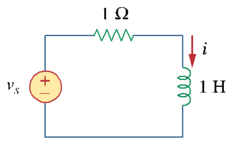


Figure 18.40
For Prob. 18.38.

Chapter 18, Solution 38.

$$V_s = \pi\delta(\omega) + \frac{1}{j\omega}$$

$$I(\omega) = \frac{V_s}{1+j\omega} = \frac{1}{1+j\omega}\left(\pi\delta(\omega) + \frac{1}{j\omega}\right)$$

Let $I(\omega) = I_1(\omega) + I_2(\omega) = \dfrac{\pi\delta(\omega)}{1+j\omega} + \dfrac{1}{j\omega(1+j\omega)}$

$$I_2(\omega) = \frac{1}{j\omega(1+j\omega)} = \frac{A}{s} + \frac{B}{s+1}, \qquad s = j\omega$$

where $A = \dfrac{1}{1} = 1, \qquad B = \dfrac{1}{-1} = -1$

$$I_2(\omega) = \frac{1}{j\omega} + \frac{-1}{j\omega+1} \qquad \longrightarrow \qquad i_2(t) = \frac{1}{2}\text{sgn}(t) - e^{-t}$$

$$I_1(\omega) = \frac{\pi\delta(\omega)}{1+j\omega}$$

$$i_1(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\pi\delta(\omega)}{1+j\omega} e^{j\omega t}\, d\omega = \frac{1}{2}\frac{e^{j\omega t}}{1+j\omega}\bigg|_{\omega=0} = \frac{1}{2}$$

Hence,

$$i(t) = i_1(t) + i_2(t) = \underline{\frac{1}{2} + \frac{1}{2}\text{sgn}(t) - e^{-t}}$$

Chapter 18, Problem 39.

Given the circuit in Fig. 18.41, with its excitation, determine the Fourier transform of *i*(*t*).

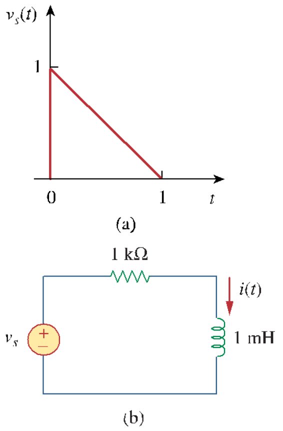


Figure 18.41
For Prob. 18.39.

Chapter 18, Solution 39.

$$V_s(\omega) = \int_{-\infty}^{\infty} (1-t)e^{-j\omega t}dt = \frac{1}{j\omega} + \frac{1}{\omega^2} - \frac{1}{\omega^2}e^{-j\omega}$$

$$I(\omega) = \frac{V_s(\omega)}{10^3 + j\omega x10^{-3}} = \underline{\frac{10^3}{10^6 + j\omega}\left(\frac{1}{j\omega} + \frac{1}{\omega^2} - \frac{1}{\omega^2}e^{-j\omega}\right)}$$

Chapter 18, Problem 40.

Determine the current $i(t)$ in the circuit of Fig. 18.42(b), given the voltage source shown in Fig. 18.42(a).

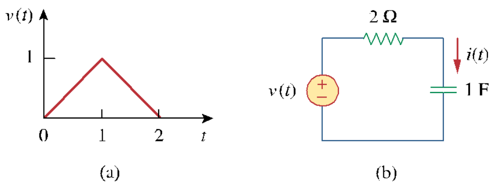


Figure 18.42
For Prob. 18.40.

Chapter 18, Solution 40.

$$\ddot{v}(t) = \delta(t) - 2\delta(t-1) + \delta(t-2)$$

$$-\omega^2 V(\omega) = 1 - 2e^{-j\omega} + e^{j\omega 2}$$

$$V(\omega) = \frac{1 - 2e^{-j\omega} + e^{-j\omega 2}}{-\omega^2}$$

Now $\quad Z(\omega) = 2 + \dfrac{1}{j\omega} = \dfrac{1 + j2\omega}{j\omega}$

$$I = \frac{V(\omega)}{Z(\omega)} = \frac{2e^{j\omega} - e^{j\omega 2} - 1}{\omega^2} \cdot \frac{j\omega}{1 + j2\omega}$$

$$= \frac{1}{j\omega(0.5 + j\omega)}\left(0.5 + 0.5e^{-j\omega 2} - e^{-j\omega}\right)$$

But $\quad \dfrac{1}{s(s+0.5)} = \dfrac{A}{s} + \dfrac{B}{s+0.5} \longrightarrow A = 2, B = -2$

$$I(\omega) = \frac{2}{j\omega}\left(0.5 + 0.5e^{j\omega 2} - e^{-j\omega}\right) - \frac{2}{0.5 + j\omega}\left(0.5 + 0.5e^{-j\omega 2} - e^{-j\omega}\right)$$

$$i(t) = \underline{\mathbf{\frac{1}{2}sgn(t) + \frac{1}{2}sgn(t-2) - sgn(t-1) - e^{-0.5t}u(t) - e^{-0.5(t-2)}u(t-2) - 2e^{-0.5(t-1)}u(t-1)}}$$

Chapter 18, Problem 41.

Determine the Fourier transform of $v(t)$ in the circuit shown in Fig. 18.43.

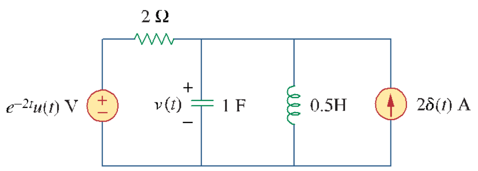
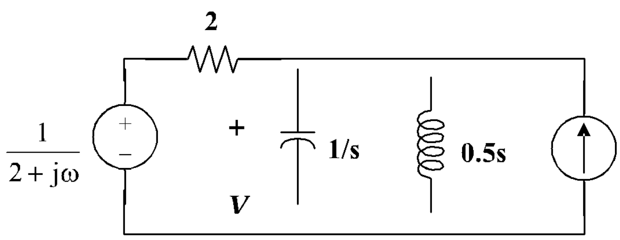


Figure 18.43
For Prob. 18.41.

Chapter 18, Solution 41.



$$\frac{V - \frac{1}{2+j\omega}}{2} + j\omega V + \frac{2V}{j\omega} - 2 = 0$$

$$\left(j\omega - 2\omega^2 + 4\right)V = j4\omega + \frac{j\omega}{2+j\omega} = \frac{-4\omega^2 + j9\omega}{2+j\omega}$$

$$V(\omega) = \frac{2j\omega(4.5 + j2\omega)}{(2+j\omega)(4-2\omega^2+j\omega)}$$

Chapter 18, Problem 42.

Obtain the current $i_o(t)$ in the circuit of Fig. 18.44.

(a) Let $i(t) = \text{sgn}(t)$ A.

(b) Let $i(t) = 4[\text{u(t)} - u(t-1)]$ A.

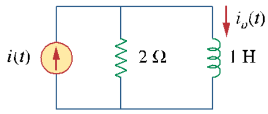


Figure 18.44
For Prob. 18.42.

Chapter 18, Solution 42.

By current division, $I_o = \dfrac{2}{2 + j\omega} \cdot I(\omega)$

(a) For i(t) = 5 sgn (t),

$$I(\omega) = \frac{10}{j\omega}$$

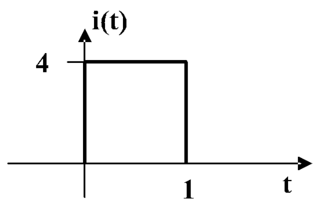
$$I_o = \frac{2}{2 + j\omega} \cdot \frac{10}{j\omega} = \frac{20}{j\omega(2 + j\omega)}$$

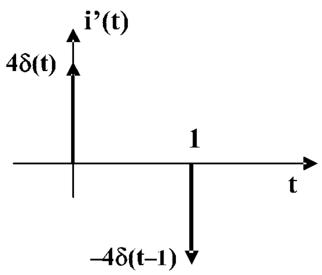
Let $I_o = \dfrac{20}{s(s+2)} = \dfrac{A}{s} + \dfrac{B}{s+2}, \quad A = 10, \; B = -10$

$$I_o(\omega) = \frac{10}{j\omega} - \frac{10}{2 + j\omega}$$

$$i_o(t) = \underline{\mathbf{5\,sgn(t) - 10e^{-2t}u(t)\,A}}$$

(b)





$$i'(t) = 4\delta(t) - 4\delta(t-1)$$

$$j\omega I(\omega) = 4 - 4e^{-j\omega}$$

$$I(\omega) = \frac{4\left(1 - e^{-j\omega}\right)}{j\omega}$$

$$I_o = \frac{8\left(1 - e^{-j\omega}\right)}{j\omega(2 + j\omega)} = 4\left(\frac{1}{j\omega} - \frac{1}{2 + j\omega}\right)\left(1 - e^{-j\omega}\right)$$

$$= \frac{4}{j\omega} - \frac{4}{2 + j\omega} - \frac{4e^{-j\omega}}{j\omega} + \frac{4e^{-j\omega}}{2 + j\omega}$$

$$i_o(t) = \underline{\mathbf{2\,sgn(t) - 2\,sgn(t-1) - 4e^{-2t}u(t) + 4e^{-2(t-1)}u(t-1)\,A}}$$

Chapter 18, Problem 43.

Find $v_o(t)$ in the circuit of Fig. 18.45, where $i_s = 5e^{-t}u(t)$ A.

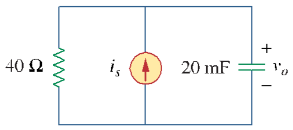


Figure 18.45
For Prob. 18.43.

Chapter 18, Solution 43.

$$20\text{ mF} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j20x10^{-3}\omega} = \frac{50}{j\omega}, \quad i_s = 5e^{-t} \longrightarrow I_s = \frac{5}{1+j\omega}$$

$$V_o = \frac{40}{40+\frac{50}{j\omega}} I_s \bullet \frac{50}{j\omega} = \frac{250}{(s+1)(s+1.25)}, \quad s = j\omega$$

$$V_o = \frac{A}{s+1} + \frac{B}{s+1.25} = 1000\left[\frac{1}{s+1} - \frac{1}{s+1.25}\right]$$

$$v_o(t) = \underline{1000(e^{-1t} - e^{-1.25t})u(t)V}$$

Chapter 18, Problem 44.

If the rectangular pulse in Fig. 18.46(a) is applied to the circuit in Fig. 18.46(b), find v_o at $t = 1$ s.

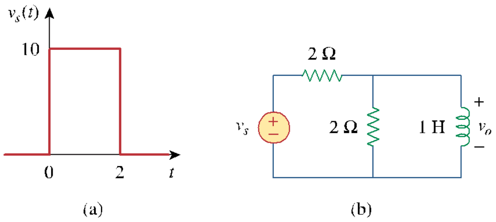
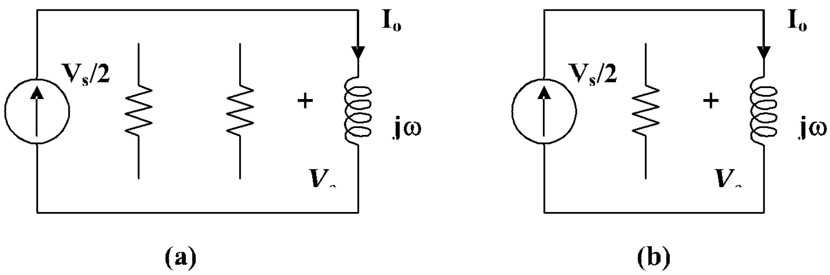


Figure 18.46
For Prob. 18.44.

Chapter 18, Solution 44.

$$1\text{H} \longrightarrow j\omega$$

We transform the voltage source to a current source as shown in Fig. (a) and then combine the two parallel 2Ω resistors, as shown in Fig. (b).



$$2\|2 = 1\Omega,\quad I_o = \frac{1}{1+j\omega}\cdot\frac{V_s}{2}$$

$$V_o = j\omega I_o = \frac{j\omega V_s}{2(1+j\omega)}$$

$$\ddot{v}_s(t) = 10\delta(t) - 10\delta(t-2)$$

$$j\omega V_s(\omega) = 10 - 10e^{-j2\omega}$$

$$V_s(\omega) = \frac{10\left(1-e^{-j2\omega}\right)}{j\omega}$$

$$\text{Hence}\quad V_o = \frac{5\left(1-e^{-j2\omega}\right)}{1+j\omega} = \frac{5}{1+j\omega} - \frac{5}{1+j\omega}e^{-j2\omega}$$

$$v_o(t) = 5e^{-t}u(t) - 5e^{-(t-2)}u(t-2)$$

$$v_o(1) = 5e^{-1} - 0 = \underline{\mathbf{1.839\ V}}$$

Chapter 18, Problem 45.

Use the Fourier transform to find i(t) in the circuit of Fig. 18.47 if $v_s(t) = 10e^{-2t}\,u(t)$.

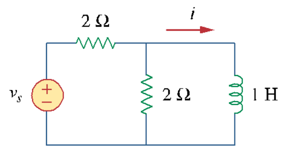
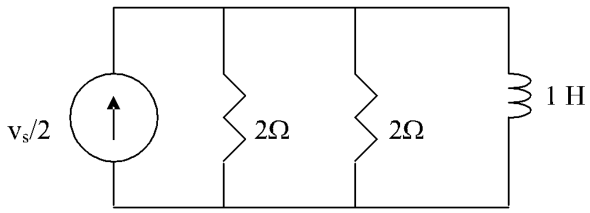


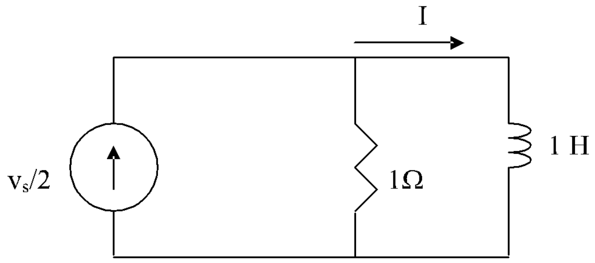
Figure 18.47
For Prob. 18.45.

Chapter 18, Solution 45.

We may convert the voltage source to a current source as shown below.



Combining the two 2-Ω resistors gives 1 Ω. The circuit now becomes that shown below.



$$I = \frac{1}{1+j\omega}\frac{V_s}{2} = \frac{1}{1+j\omega}\frac{5}{2+j\omega} = \frac{5}{(s+1)(s+2)}, \qquad s = j\omega$$

$$= \frac{A}{s+1} + \frac{B}{s+2}$$

where $A = 5/1 = 5, \quad B = 5/-1 = -5$

$$I = \frac{5}{s+1} - \frac{5}{s+2}$$

$$i(t) = \underline{5(e^{-t} - e^{-2t})u(t)}\text{ A}$$

Chapter 18, Problem 46.

Determine the Fourier transform of $i_o(t)$ in the circuit of Fig. 18.48.

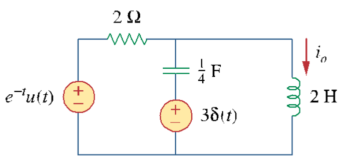


Figure 18.48
For Prob. 18.46.

Chapter 18, Solution 46.

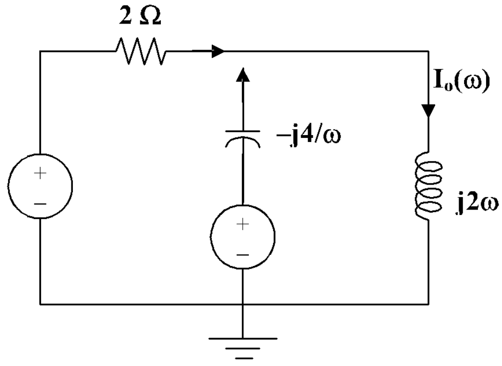
$$\frac{1}{4}F \longrightarrow \frac{1}{j\omega\frac{1}{4}} = \frac{-j4}{\omega}$$

$$2H \longrightarrow j\omega 2$$

$$3\delta(t) \longrightarrow 3$$

$$e^{-t}u(t) \longrightarrow \frac{1}{1+j\omega}$$

The circuit in the frequency domain is shown below:



At node V_o, KCL gives

$$\frac{\frac{1}{1+j\omega} - V_o}{2} + \frac{3 - V_o}{\frac{-j4}{\omega}} = \frac{V_o}{j2\omega}$$

$$\frac{2}{1+j\omega} - 2V_o + j\omega 3 - j\omega V_o = -\frac{j2V_o}{\omega}$$

$$V_o = \frac{\frac{2}{1+j\omega} + j\omega 3}{2 + j\omega - \frac{j2}{\omega}}$$

$$I_o(\omega) = \frac{V_o}{j2\omega} = \frac{\frac{2 + j\omega 3 - 3\omega^2}{1+j\omega}}{j2\omega\left(2 + j\omega - \frac{j2}{\omega}\right)}$$

$$I_o(\omega) = \underline{\mathbf{\frac{2 + j\omega^2 - 3\omega^2}{4 - 6\omega^2 + j(8\omega - 2\omega^3)}}}$$

Chapter 18, Problem 47.

Find the voltage $v_o(t)$ in the circuit of Fig. 18.49. Let $i_s(t) = 8e^{-t}u(t)$ A.

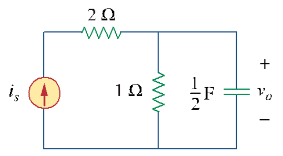


Figure 18.49
For Prob. 18.47.

Chapter 18, Solution 47.

$$\frac{1}{2}F \quad \longrightarrow \quad \frac{1}{j\omega C} = \frac{2}{j\omega}$$

$$I_o = \frac{1}{1+\dfrac{2}{j\omega}} I_s$$

$$V_o = \frac{2}{j\omega} I_o = \frac{\dfrac{2}{j\omega}}{1+\dfrac{2}{j\omega}} I_s = \frac{2}{2+j\omega}\frac{8}{1+j\omega}$$

$$= \frac{16}{(s+1)(s+2)}, \quad s = j\omega$$

$$= \frac{A}{s+1} + \frac{B}{s+2}$$

where $A = 16/1 = 16$, $B = 16/(-1) = -16$

Thus,

$$v_o(t) = \underline{16(e^{-t} - e^{-2t})u(t)\ \text{V}}.$$

Chapter 18, Problem 48.

Find $i_o(t)$ in the op amp circuit of Fig. 18.50.

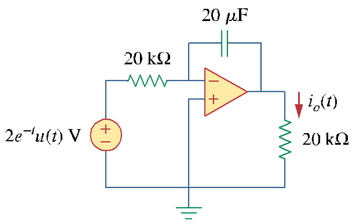


Figure 18.50
For Prob. 18.48.

Chapter 18, Solution 48.

$$0.2\text{F} \longrightarrow \frac{1}{j\omega C} = -\frac{j5}{\omega}$$

As an integrator,

$$RC = 20 \text{x} 10^3 \text{ x } 20 \text{x} 10^{-6} = 0.4$$

$$v_o = -\frac{1}{RC}\int_o^t v_i dt$$

$$V_o = -\frac{1}{RC}\left[\frac{V_i}{j\omega} + \pi V_i(0)\delta(\omega)\right]$$

$$= -\frac{1}{0.4}\left[\frac{2}{j\omega(2+j\omega)} + \pi\delta(\omega)\right]$$

$$I_o = \frac{V_o}{20}\text{mA} = -0.125\left[\frac{2}{j\omega(2+j\omega)} + \pi\delta(\omega)\right]$$

$$= -\frac{0.125}{j\omega} + \frac{0.125}{2+j\omega} - 0.125\pi\delta(\omega)$$

$$i_o(t) = -0.125\,\text{sgn}(t) + 0.125e^{-2t}u(t) - \frac{0.125}{2\pi}\int \pi\delta(\omega)e^{j\omega t}dt$$

$$= 0.125 + 0.25u(t) + 0.125e^{-2t}u(t) - \frac{0.125}{2}$$

$i_o(t)$ = $\underline{\mathbf{0.625 - 0.25u(t) + 0.125e^{-2t}u(t)\,mA}}$

Chapter 18, Problem 49.

Use the Fourier transform method to obtain $v_o(t)$ in the circuit of Fig. 18.51.

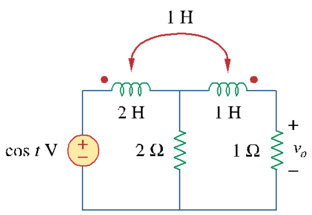
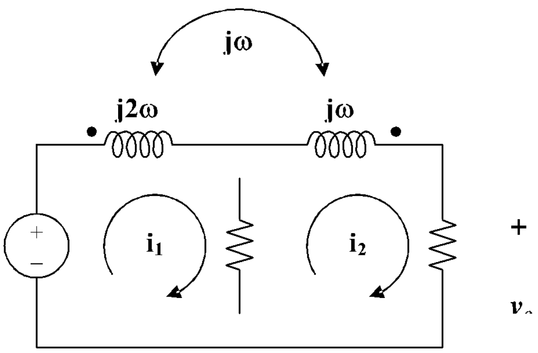


Figure 18.51
For Prob. 18.49.

Chapter 18, Solution 49.

Consider the circuit shown below:



$$V_s = \pi[\delta(\omega+1)+\delta(\omega-2)]$$

For mesh 1, $-V_s + (2 + j2\omega)I_1 - 2I_2 - j\omega I_2 = 0$

$$V_s = 2(1+j\omega)I_1 - (2+j\omega)I_2 \qquad (1)$$

For mesh 2, $\quad 0 = (3 + j\omega)I_2 - 2I_1 - j\omega I_1$

$$I_1 = \frac{(3+\omega)I_2}{(2+\omega)} \qquad (2)$$

Substituting (2) into (1) gives

$$V_s = 2\frac{2(1+j\omega)(3+j\omega)I_2}{2+j\omega} - (2+j\omega)I_2$$

$$V_s(2+\omega) = \left[2\left(3+j4\omega-\omega^2\right) - \left(4+j4\omega-\omega^2\right)\right]I_2$$

$$= I_2\left(2+j4\omega-\omega^2\right)$$

$$I_2 = \frac{(s+2)V_s}{s^2+4s+2},\ s = j\omega$$

$$V_o = I_2 = \frac{(j\omega+2)\pi[\delta(\omega+1)+\delta(\omega-1)]}{(j\omega)^2 + j\omega 4 + 2}$$

$$v_o(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} v_o(\omega)e^{j\omega t}d\omega$$

$$= \int_{-\infty}^{\infty}\frac{\frac{1}{2}(j\omega+2)e^{j\omega t}\delta(\omega+1)d\omega}{(j\omega)^2+j\omega 4+2} + \frac{\frac{1}{2}(j\omega+2)e^{j\omega t}\delta(\omega-1)d\omega}{(j\omega)^2+j\omega 4+2}$$

$$= \frac{\frac{1}{2}(-j+2)e^{jt}}{-1-j4+2} + \frac{\frac{1}{2}(j+2)e^{jt}}{-1+j4+2}$$

$$v_o(t) = \frac{\frac{1}{2}(2-j)(1+j4)}{17}e^{jt} + \frac{\frac{1}{2}(2-j)(1-j4)e^{jt}}{17}$$

$$= \frac{1}{34}(6+j7)e^{jt} + \frac{1}{34}(6-j7)e^{jt}$$

$$= 0.271\,e^{-j(t-13.64^\circ)} + 0.271\,e^{j(t-13.64^\circ)}$$

$v_o(t) =$ <u>**$0.542\cos(t - 13.64^\circ)$V**</u>

Chapter 18, Problem 50.

Determine $v_o(t)$ in the transformer circuit of Fig. 18.52.

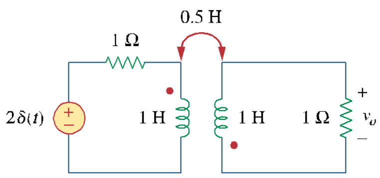
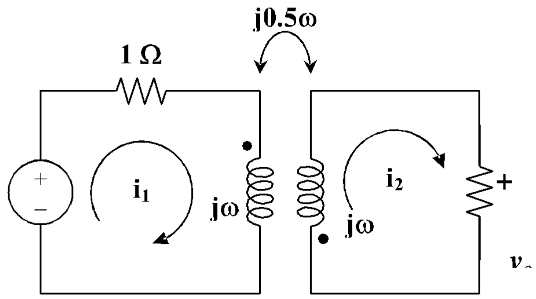


Figure 18.52
For Prob. 18.50.

Chapter 18, Solution 50.

Consider the circuit shown below:



For loop 1,

$$-2+(1+j\omega)I_1+j0.5\omega I_2=0 \qquad (1)$$

For loop 2,

$$(1+j\omega)I_2+j0.5\omega I_1=0 \qquad (2)$$

From (2),

$$I_1=\frac{(1+j\omega)I_2}{-j0.5\omega}=-2\frac{(1+j\omega)I_2}{j\omega}$$

Substituting this into (1),

$$2=\frac{-2(1+j\omega)I_2}{j\omega}+\frac{j\omega}{2}I_2$$

$$2j\omega=-\left(4+j4\omega-\frac{3}{2}\omega^2\right)I_2$$

$$I_2=\frac{2j\omega}{4+j4\omega-1.5\omega^2}$$

$$V_o=I_2=\frac{-2j\omega}{4+j4\omega+1.5(j\omega)^2}$$

$$V_o=\frac{\frac{4}{3}j\omega}{\frac{8}{3}+j\frac{8\omega}{3}+(j\omega)^2}$$

$$=\frac{-4\left(\frac{4}{3}+j\omega\right)}{\left(\frac{4}{3}+j\omega\right)^2+\left(\frac{\sqrt{8}}{3}\right)^2}+\frac{\frac{16}{3}}{\left(\frac{4}{3}+j\omega\right)^2+\left(\frac{\sqrt{8}}{3}\right)^2}$$

$$\mathbf{V_o(t)=-4e^{-4t/3}\cos\left(\frac{\sqrt{8}}{3}t\right)u(t)+5.657e^{-4t/3}\sin\left(\frac{\sqrt{8}}{3}t\right)u(t)\,V}$$

Chapter 18, Problem 51.

Find the energy dissipated by the resistor in the circuit of Fig. 18.53.

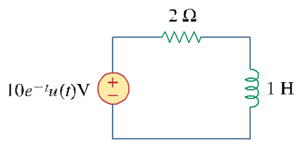


Figure 18.53
For Prob. 18.51.

Chapter 18, Solution 51.

In the frequency domain, the voltage across the 2-Ω resistor is

$$V(\omega) = \frac{2}{2+j\omega}V_s = \frac{2}{2+j\omega}\frac{10}{1+j\omega} = \frac{20}{(s+1)(s+2)}, \qquad s = j\omega$$

$$V(s) = \frac{A}{s+1} + \frac{B}{s+2}$$

$$A = 20/1 = 20, \qquad B = 20/-1 = -20$$

$$V(\omega) = \frac{20}{j\omega+1} - \frac{20}{j\omega+2}$$

$$v(t) = \left(20e^{-t} - 20e^{-2t}\right)u(t)$$

$$W = \frac{1}{2}\int_0^{\infty} v^2(t)dt = 0.5\int 400\left(e^{-2t} + e^{-4t} - 3e^{-3t}\right)dt$$

$$= 200\left(\frac{e^{-2t}}{-2} + \frac{e^{-4t}}{-4} - \frac{2e^{-3t}}{-3}\right)\Bigg|_0^{\infty} = \underline{\mathbf{16.667\ J}}.$$

Chapter 18, Problem 52.

For $F(\omega) = \dfrac{1}{3 + j\omega}$, find $J = \int_{-\infty}^{\infty} f^2(t)dt$.

Chapter 18, Solution 52.

$$J = 2\int_0^\infty f^2(t)\,dt = \frac{1}{\pi}\int_0^\infty |F(\omega)|^2\,d\omega$$

$$= \frac{1}{\pi}\int_0^\infty \frac{1}{9^2 + \omega^2}\,d\omega = \frac{1}{3\pi}\tan^{-1}(\omega/3)\Big|_0^\infty = \frac{1}{3\pi}\frac{\pi}{2} = \underline{\mathbf{(1/6)}}$$

Chapter 18, Problem 53.

If $f(t) = e^{-2|t|}$, find $J = \int_{-\infty}^{\infty} |F(\omega)|^2\, d\omega$.

Chapter 18, Solution 53.

If $f(t) = e^{-2|t|}$, find $J = \int_{-\infty}^{\infty} |F(\omega)|^2\, d\omega$.

$$J = \int_{-\infty}^{\infty} |F(\omega)|^2 d\omega = 2\pi\int_{-\infty}^{\infty} f^2(t)\,dt$$

$$f(t) = \begin{vmatrix} e^{2t}, & t < 0 \\ e^{-2t}, & t > 0 \end{vmatrix}$$

$$J = 2\pi\left[\int_{-\infty}^{0} e^{4t}dt + \int_0^\infty e^{-4t}dt\right] = 2\pi\left[\frac{e^{4t}}{4}\Bigg|_{-\infty}^{0} + \frac{e^{-4t}}{-4}\Bigg|_0^\infty\right] = 2\pi[(1/4) + (1/4)] = \underline{\pi}$$

Chapter 18, Problem 54.

Given the signal $f(t) = 4e^{-t}\,u(t)$ what is the total energy in $f(t)$?

Chapter 18, Solution 54.

$$W_{1\Omega} = \int_{-\infty}^{\infty} f^2(t)\,dt = 16\int_0^\infty e^{-2t}dt = -8e^{-2t}\Big|_0^\infty = \underline{\mathbf{8\ J}}$$

Chapter 18, Problem 55.

Let $f(t) = 5e^{-(t-2)}\, u(t)$ and use it to find the total energy in $f(t)$.

Chapter 18, Solution 55.

$$f(t) = 5e^{2}e^{-t}u(t)$$

$$F(\omega) = 5e^{2}/(1 + j\omega), \quad |F(\omega)|^{2} = 25e^{4}/(1 + \omega^{2})$$

$$W_{1\Omega} = \frac{1}{\pi}\int_{0}^{\infty}|F(\omega)|^{2}\,d\omega = \frac{25e^{4}}{\pi}\int_{0}^{\infty}\frac{1}{1+\omega^{2}}\,d\omega = \frac{25e^{4}}{\pi}\tan^{-1}(\omega)\Big|_{0}^{\infty}$$

$$= 12.5e^{4} = \underline{\mathbf{682.5\ J}}$$

or $$W_{1\Omega} = \int_{-\infty}^{\infty} f^{2}(t)\,dt = 25e^{4}\int_{0}^{\infty} e^{-2t}dt = 12.5e^{4} = \underline{\mathbf{682.5\ J}}$$

Chapter 18, Problem 56.

The voltage across a 1-Ω resistor is $v(t) = te^{-2t}\,u(t)$ V. (a) What is the total energy absorbed by the resistor? (b) What fraction of this energy absorbed is in the frequency band $-2 \le \omega \le 2$?

Chapter 18, Solution 56.

(a) $$W = \int_{-\infty}^{\infty} V^{2}(t)dt = \int_{0}^{\infty} t^{2}e^{-4t}dt = \frac{e^{-4t}}{(-4)^{3}}(16t^{2} + 8t + 2)\Big|_{0}^{\infty} = \frac{2}{64} = \underline{0.0313\ \text{J}}$$

(b) In the frequency domain,

$$V(\omega) = \frac{1}{(2 + j\omega)^{2}}$$

$$|V(\omega)|^{2} = V(\omega)V^{*}(\omega) = \frac{1}{(4 + j\omega)^{2}}$$

$$W_{o} = \frac{1}{2\pi}\int_{-2}^{2}|V(\omega)|^{2}d\omega = \frac{2}{2\pi}\int_{0}^{2}\frac{1}{(4+\omega^{2})^{2}}d\omega$$

$$= \frac{1}{\pi}\frac{1}{2\text{x}4}\left(\frac{\omega}{\omega^{2} + 4} + 0.5\tan^{-1}(0.5\omega)\right)\Bigg|_{0}^{2} = \frac{1}{32\pi} + \frac{1}{64} = 0.0256$$

$$\text{Fraction} = \frac{W_{o}}{W} = \frac{0.0256}{0.0313} = \underline{81.79\%}$$

Chapter 18, Problem 57.

Let $i(t) = 2e^{t}\,u(-t)$A. Find the total energy carried by $i(t)$ and the percentage of the 1-Ω energy in the frequency range of $-5 < \omega < 5$ rad/s.

Chapter 18, Solution 57.

$$W_{1\Omega} = \int_{-\infty}^{\infty} i^2(t)\,dt = \int_{-\infty}^{0} 4e^{2t}dt = 2e^{2t}\Big|_{-\infty}^{0} = \underline{\mathbf{2\ J}}$$

or $\quad I(\omega) = 2/(1 - j\omega), \quad |I(\omega)|^2 = 4/(1 + \omega^2)$

$$W_{1\Omega} = \frac{1}{2\pi}\int_{-\infty}^{\infty} |I(\omega)|^2 d\omega = \frac{4}{2\pi}\int_{-\infty}^{\infty} \frac{1}{(1+\omega^2)} d\omega = \frac{4}{\pi}\tan^{-1}(\omega)\Big|_{0}^{\infty} = \frac{4}{\pi}\frac{\pi}{2} = \underline{\mathbf{2\ J}}$$

In the frequency range, $-5 < \omega < 5$,

$$W = \frac{4}{\pi}\tan^{-1}\omega\Big|_{0}^{5} = \frac{4}{\pi}\tan^{-1}(5) = \frac{4}{\pi}(1.373) = 1.7487$$

$W/\,W_{1\Omega} = 1.7487/2 = 0.8743$ or $\underline{\mathbf{87.43\%}}$

Chapter 18, Problem 58.

e2d

An AM signal is specified by

$f(t) = 10(1 + 4 \cos 200\,\pi t)\cos \pi \times 10^{4}\, t$

Determine the following:

(a) the carrier frequency,
(b) the lower sideband frequency,
(c) the upper sideband frequency.

Chapter 18, Solution 58.

$\omega_m = 200\pi = 2\pi f_m$ which leads to $f_m = 100$ Hz

(a) $\omega_c = \pi x 10^4 = 2\pi f_c$ which leads to $f_c = 10^4/2 = \underline{\mathbf{5\ kHz}}$

(b) lsb $= f_c - f_m = 5{,}000 - 100 = \underline{\mathbf{4{,}900\ Hz}}$

(c) usb $= f_c + f_m = 5{,}000 + 100 = \underline{\mathbf{5{,}100\ Hz}}$

Chapter 18, Problem 59.

For the linear system in Fig. 18.54, when the input voltage is $v_i(t) = 2\delta(t)$ V, the output is $v_0(t) = 10e^{-2t} - 6e^{-4t}$ V. Find the output when the input is $v_i(t) = 4e^{-t}u(t)$ V.

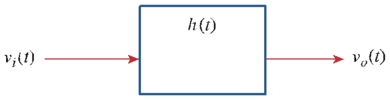


Figure 18.54
For Prob. 18.9.

Chapter 18, Solution 59.

$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)} = \frac{\dfrac{10}{2+j\omega} - \dfrac{6}{4+j\omega}}{2} = \frac{5}{2+j\omega} - \frac{3}{4+j\omega}$$

$$V_o(\omega) = H(\omega)V_i(\omega) = \left(\frac{5}{2+j\omega} - \frac{3}{4+j\omega}\right)\frac{4}{1+j\omega}$$

$$= \frac{20}{(s+1)(s+2)} - \frac{12}{(s+1)(s+4)}, \qquad s = j\omega$$

Using partial fraction,

$$V_o(\omega) = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+1} + \frac{D}{s+4} = \frac{16}{1+j\omega} - \frac{20}{2+j\omega} + \frac{4}{4+j\omega}$$

Thus,

$$v_o(t) = \underline{\left(16e^{-t} - 20e^{-2t} + 4e^{-4t}\right)u(t)\,\text{V}}$$

Chapter 18, Problem 60.

A band-limited signal has the following Fourier series representation:

$i_s(t) = 10 + 8\cos(2\pi t + 30^\circ) + 5\cos(4\pi t - 150^\circ)$mA

If the signal is applied to the circuit in Fig. 18.55, find $v(t)$.

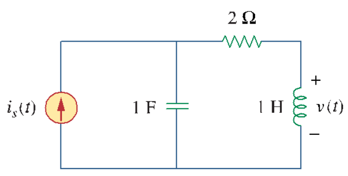
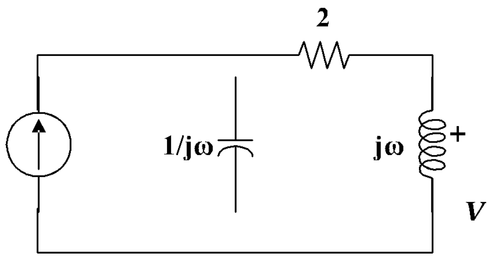


Figure 18.55
For Prob. 18.60.

Chapter 18, Solution 60.



$$V = j\omega I_s \frac{\frac{1}{j\omega}}{\frac{1}{j\omega} + 2 + j\omega} = \frac{j\omega I_s}{1 - \omega^2 + j2\omega}$$

Since the voltage appears across the inductor, there is no DC component.

$$V_1 = \frac{2\pi\angle 90°8}{1 - 4\pi^2 + j4\pi} = \frac{50.27\angle 90°}{-38.48 + j12.566} = 1.2418\angle -71.92°$$

$$V_2 = \frac{4\pi\angle 90°5}{1 - 16\pi^2 + j8\pi} = \frac{62.83\angle 90°}{-156.91 + j25.13} = 0.3954\angle -80.9°$$

$$v(t) = \underline{1.2418\cos(2\pi t - 41.92°) + 0.3954\cos(4\pi t + 129.1°)\,\text{mV}}$$

Chapter 18, Problem 61.

In a system, the input signal $x(t)$ is amplitude-modulated by $m(t) = 2 + \cos\omega_0 t$. The response $y(t) = m(t)x(t)$. Find $Y(\omega)$ in terms of $X(\omega)$.

Chapter 18, Solution 61.

$$y(t) = (2 + \cos\omega_o t)x(t)$$

We apply the Fourier Transform

$$Y(\omega) = \underline{\mathbf{2X(\omega) + 0.5X(\omega+\omega_o) + 0.5X(\omega-\omega_o)}}.$$

Chapter 18, Problem 62.

A voice signal occupying the frequency band of 0.4 to 3.5 kHz is used to amplitude-modulate a 10-MHz carrier. Determine the range of frequencies for the lower and upper sidebands.

Chapter 18, Solution 62.

For the lower sideband, the frequencies range from

10,000,000 – 3,500 Hz = **9,996,500 Hz** to
10,000,000 – 400 Hz = **9,999,600 Hz**

For the upper sideband, the frequencies range from

10,000,000 + 400 Hz = **10,000,400 Hz** to
10,000,000 + 3,500 Hz = **10,003,500 Hz**

Chapter 18, Problem 63.

For a given locality, calculate the number of stations allowable in the AM broadcasting band (540 to 1600 kHz) without interference with one another.

Chapter 18, Solution 63.

Since $f_n = 5$ kHz, $2f_n = 10$ kHz

i.e. the stations must be spaced 10 kHz apart to avoid interference.

$$\Delta f = 1600 - 540 = 1060 \text{ kHz}$$

The number of stations = $\Delta f / 10$ kHz = **106 stations**

Chapter 18, Problem 64.

Repeat the previous problem for the FM broadcasting band (88 to 108 MHz), assuming that the carrier frequencies are spaced 200 kHz apart.

Chapter 18, Solution 64.

$\Delta f = 108 - 88 \text{ MHz} = 20 \text{ MHz}$

The number of stations = 20 MHz/0.2 MHz = **100 stations**

Chapter 18, Problem 65.

The highest-frequency component of a voice signal is 3.4 kHz. What is the Nyquist rate of the sampler of the voice signal?

Chapter 18, Solution 65.

$\omega = 3.4 \text{ kHz}$

$f_s = 2\omega = \mathbf{6.8\ kHz}$

Chapter 18, Problem 66.

A TV signal is band-limited to 4.5 MHz. If samples are to be reconstructed at a distant point, what is the maximum sampling interval allowable?

Chapter 18, Solution 66.

$\omega = 4.5 \text{ MHz}$

$f_c = 2\omega = 9 \text{ MHz}$

$T_s = 1/f_c = 1/(9\text{x}10^6) = 1.11\text{x}10^{-7} = \mathbf{111\ ns}$

Chapter 18, Problem 67.

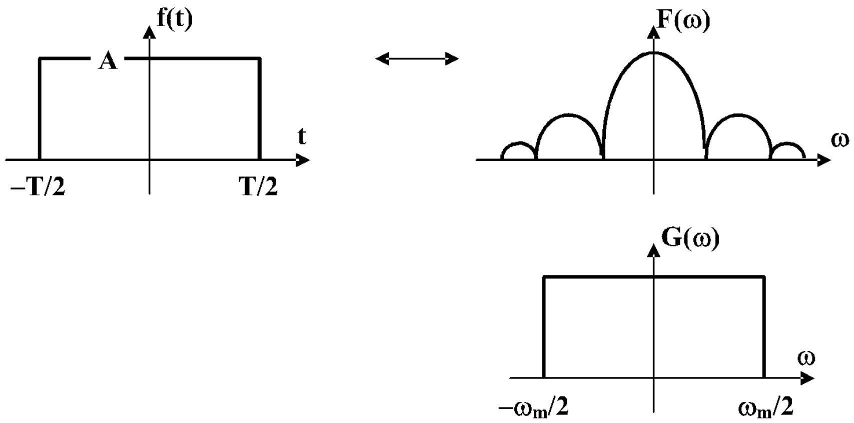
* Given a signal $g(t) = \text{sinc}(200\pi t)$ find the Nyquist rate and the Nyquist interval for the signal.

* An asterisk indicates a challenging problem.

Chapter 18, Solution 67.

We first find the Fourier transform of g(t). We use the results of Example 17.2 in conjunction with the duality property. Let Arect(t) be a rectangular pulse of height A and width T as shown below.

$$\text{Arect}(t) \text{ transforms to } \text{A}\tau\text{sinc}(\omega^2/2)$$



According to the duality property,

$$\text{A}\tau\text{sinc}(\tau t/2) \quad \text{becomes} \quad 2\pi\text{Arect}(\tau)$$

$$g(t) = \text{sinc}(200\pi t) \quad \text{becomes} \quad 2\pi\text{Arect}(\tau)$$

where $A\tau = 1$ and $\tau/2 = 200\pi$ or $T = 400\pi$

i.e. the upper frequency $\omega_u = 400\pi = 2\pi f_u$ or $f_u = 200$ Hz

The Nyquist rate $= f_s =$ **<u>200 Hz</u>**

The Nyquist interval $= 1/f_s = 1/200 =$ **<u>5 ms</u>**

Chapter 18, Problem 68.

The voltage signal at the input of a filter is $v(t) = 50e^{-2|t|}$ V What percentage of the total 1-Ω energy content lies in the frequency range of $1 < \omega < 5$ rad/s?

Chapter 18, Solution 68.

The total energy is

$$W_T = \int_{-\infty}^{\infty} v^2(t)\,dt$$

Since v(t) is an even function,

$$W_T = \int_0^{\infty} 2500e^{-4t}dt = 5000\frac{e^{-4t}}{-4}\Big|_0^{\infty} = 1250 \text{ J}$$

$$V(\omega) = 50x4/(4 + \omega^2)$$

$$W = \frac{1}{2\pi}\int_1^5 |V(\omega)|^2\,d\omega = \frac{1}{2\pi}\int_1^5 \frac{(200)^2}{(4+\omega^2)^2}d\omega$$

But $$\int\frac{1}{(a^2+x^2)^2}dx = \frac{1}{2a^2}\left[\frac{x}{x^2+a^2}+\frac{1}{a}\tan^{-1}(x/a)\right]+C$$

$$W = \frac{2x10^4}{\pi}\frac{1}{8}\left[\frac{\omega}{4+\omega^2}+\frac{1}{2}\tan^{-1}(\omega/2)\right]\Bigg|_1^5$$

$$= (2500/\pi)[(5/29) + 0.5\tan^{-1}(5/2) - (1/5) - 0.5\tan^{-1}(1/2) = 267.19$$

W/W_T = 267.19/1250 = 0.2137 or **21.37%**

Chapter 18, Problem 69.

A signal with Fourier transform

$$F(\omega) = \frac{20}{4 + j\omega}$$

is passed through a filter whose cutoff frequency is 2 rad/s (i.e., $0 < \omega < 2$). What fraction of the energy in the input signal is contained in the output signal?

Chapter 18, Solution 69.

The total energy is

$$W_T = \frac{1}{2\pi}\int_{-\infty}^{\infty}|F(\omega)|^2 d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{400}{4^2+\omega^2}d\omega$$

$$= \frac{400}{\pi}\left[(1/4)\tan^{-1}(\omega/4)\right]_0^{\infty} = \frac{100}{\pi}\frac{\pi}{2} = 50$$

$$W = \frac{1}{2\pi}\int_0^2|F(\omega)|^2 d\omega = \frac{400}{2\pi}\left[(1/4)\tan^{-1}(\omega/4)\right]_0^2$$

$$= [100/(2\pi)]\tan^{-1}(2) = (50/\pi)(1.107) = 17.6187$$

$W/W_T = 17.6187/50 = 0.3524$ or **35.24%**

Chapter 19, Problem 1.

Obtain the z parameters for the network in Fig. 19.65.

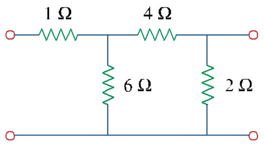
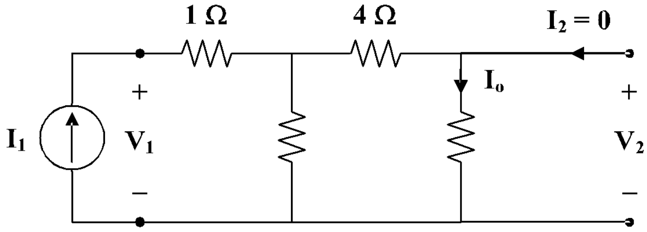


Figure 19.65

For Prob. 19.1 and 19.28.

Chapter 19, Solution 1.

To get $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$, consider the circuit in Fig. (a).



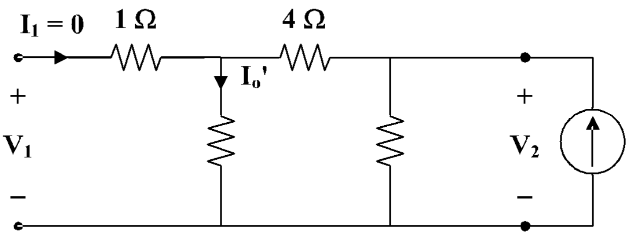
(a)

$$\mathbf{z}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = 1 + 6 \,||\, (4+2) = 4\,\Omega$$

$$\mathbf{I}_o = \frac{1}{2}\mathbf{I}_1, \qquad \mathbf{V}_2 = 2\mathbf{I}_o = \mathbf{I}_1$$

$$\mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1} = 1\,\Omega$$

To get $\mathbf{z}_{22}$ and $\mathbf{z}_{12}$, consider the circuit in Fig. (b).



(b)

$$\mathbf{z}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = 2 \,||\, (4+6) = 1.667\,\Omega$$

$$\mathbf{I}_o' = \frac{2}{2+10}\mathbf{I}_2 = \frac{1}{6}\mathbf{I}_2, \qquad \mathbf{V}_1 = 6\mathbf{I}_o' = \mathbf{I}_2$$

$$\mathbf{z}_{12} = \frac{\mathbf{V}_1}{\mathbf{I}_2} = 1\,\Omega$$

Hence, $$[\mathbf{z}] = \begin{bmatrix} \mathbf{4} & \mathbf{1} \\ \mathbf{1} & \mathbf{1.667} \end{bmatrix}\Omega$$

Chapter 19, Problem 2.

* Find the impedance parameter equivalent of the network in Fig. 19.66.

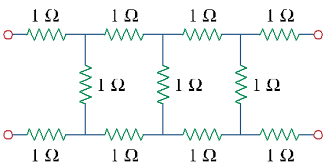


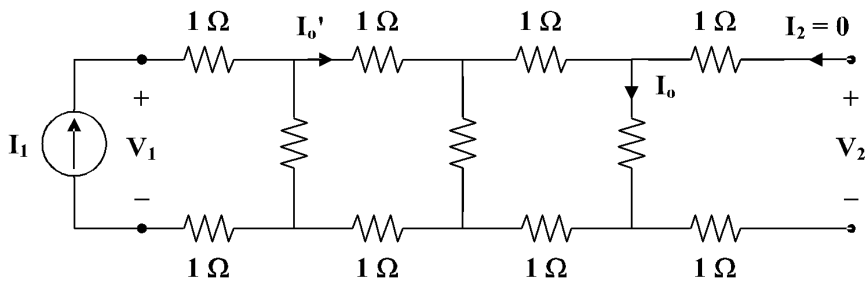
Figure 19.66

For Prob. 19.2.

* An asterisk indicates a challenging problem.

Chapter 19, Solution 2.

Consider the circuit in Fig. (a) to get $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$.



(a)

$$\mathbf{z}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = 2 + 1 \,||\, [2 + 1 \,||\, (2+1)]$$

$$\mathbf{z}_{11} = 2 + 1 \,||\left(2 + \frac{3}{4}\right) = 2 + \frac{(1)(11/4)}{1 + 11/4} = 2 + \frac{11}{15} = 2.733$$

$$\mathbf{I}_o = \frac{1}{1+3}\mathbf{I}_o' = \frac{1}{4}\mathbf{I}_o'$$

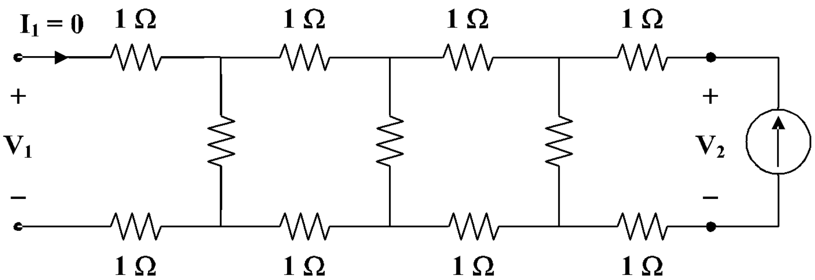
$$\mathbf{I}_o' = \frac{1}{1 + 11/4}\mathbf{I}_1 = \frac{4}{15}\mathbf{I}_1$$

$$\mathbf{I}_o = \frac{1}{4}\cdot\frac{4}{15}\mathbf{I}_1 = \frac{1}{15}\mathbf{I}_1$$

$$\mathbf{V}_2 = \mathbf{I}_o = \frac{1}{15}\mathbf{I}_1$$

$$\mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1} = \frac{1}{15} = \mathbf{z}_{12} = 0.06667$$

To get $\mathbf{z}_{22}$, consider the circuit in Fig. (b).



(b)

$$\mathbf{z}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = 2 + 1 \,||\, (2 + 1 \,||\, 3) = \mathbf{z}_{11} = 2.733$$

Thus,

$$[\mathbf{z}] = \begin{bmatrix} \mathbf{2.733} & \mathbf{0.06667} \\ \mathbf{0.06667} & \mathbf{2.733} \end{bmatrix} \Omega$$

Chapter 19, Problem 3.

Find the z parameters of the circuit in Fig. 19.67.

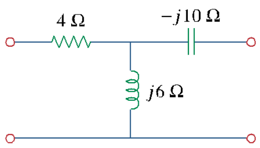


Figure 19.67
For Prob. 19.3.

Chapter 19, Solution 3.

$$z_{12} = j6 = z_{21}$$
$$z_{11} - z_{12} = 4 \quad \longrightarrow \quad z_{11} = z_{12} + 4 = 4 + j6\ \Omega$$
$$z_{22} - z_{12} = -j10 \quad \longrightarrow \quad z_{22} = z_{12} - j10 = -j4\ \Omega$$

$$[z] = \begin{bmatrix} 4+j6 & j6 \\ j6 & -j4 \end{bmatrix} \Omega = \begin{bmatrix} 4+\text{j}6 & \text{j}6 \\ \text{j}6 & -\text{j}4 \end{bmatrix} \Omega$$

Chapter 19, Problem 4.

Calculate the z parameters for the circuit in Fig. 19.68.

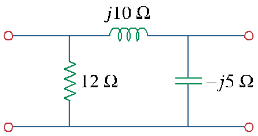
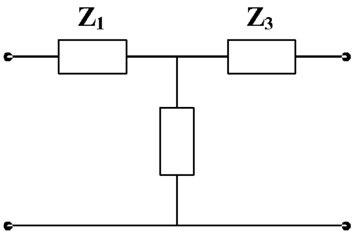


Figure 19.68

For Prob. 19.4.

Chapter 19, Solution 4.

Transform the Π network to a T network.



$$\mathbf{Z}_1 = \frac{(12)(j10)}{12 + j10 - j5} = \frac{j120}{12 + j5}$$

$$\mathbf{Z}_2 = \frac{-j60}{12 + j5}$$

$$\mathbf{Z}_3 = \frac{50}{12 + j5}$$

The z parameters are

$$\mathbf{z}_{12} = \mathbf{z}_{21} = \mathbf{Z}_2 = \frac{(-j60)(12 - j5)}{144 + 25} = -1.775 - j4.26$$

$$\mathbf{z}_{11} = \mathbf{Z}_1 + \mathbf{z}_{12} = \frac{(j120)(12 - j5)}{169} + \mathbf{z}_{12} = 1.775 + j4.26$$

$$\mathbf{z}_{22} = \mathbf{Z}_3 + \mathbf{z}_{21} = \frac{(50)(12 - j5)}{169} + \mathbf{z}_{21} = 1.7758 - j5.739$$

Thus,

$$[\mathbf{z}] = \begin{bmatrix} \mathbf{1.775 + j4.26} & \mathbf{-1.775 - j4.26} \\ \mathbf{-1.775 - j4.26} & \mathbf{1.775 - j5.739} \end{bmatrix} \Omega$$

Chapter 19, Problem 5.

Obtain the *z* parameters for the network in Fig. 19.69 as functions of *s*.

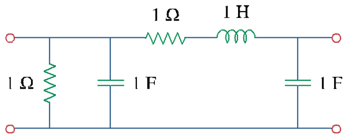
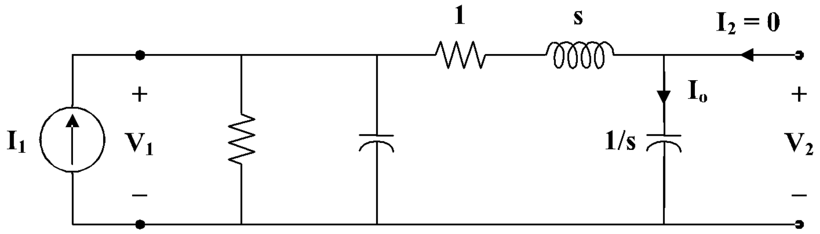


Figure 19.69

For Prob. 19.5.

Chapter 19, Solution 5.

Consider the circuit in Fig. (a).



(a)

$$\mathbf{z}_{11} = 1 \,||\, \frac{1}{s} \,||\, \left(1 + s + \frac{1}{s}\right) = \frac{\frac{1}{s}}{1+\frac{1}{s}} \,||\, \left(1 + s + \frac{1}{s}\right) = \frac{\left(\frac{1}{s+1}\right)\left(1+s+\frac{1}{s}\right)}{\left(\frac{1}{s+1}\right)+1+s+\frac{1}{s}}$$

$$\mathbf{z}_{11} = \frac{s^2+s+1}{s^3+2s^2+3s+1}$$

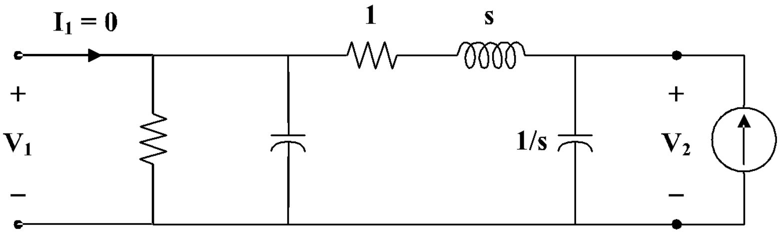
$$\mathbf{I}_o = \frac{1 \,||\, \frac{1}{s}}{1 \,||\, \frac{1}{s} + 1 + s + \frac{1}{s}}\mathbf{I}_1 = \frac{\frac{1}{s+1}}{\frac{1}{s+1}+1+s+\frac{1}{s}}\mathbf{I}_1 = \frac{\frac{s}{s+1}}{\frac{s}{s+1}+s^2+s+1}\mathbf{I}_1$$

$$\mathbf{I}_o = \frac{s}{s^3+2s^2+3s+1}\mathbf{I}_1$$

$$\mathbf{V}_2 = \frac{1}{s}\mathbf{I}_o = \frac{\mathbf{I}_1}{s^3+2s^2+3s+1}$$

$$\mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1} = \frac{1}{s^3+2s^2+3s+1}$$

Consider the circuit in Fig. (b).



(b)

$$\mathbf{z}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{1}{s} \,||\, \left(1 + s + 1 \,||\, \frac{1}{s}\right) = \frac{1}{s} \,||\, \left(1 + s + \frac{1}{s+1}\right)$$

$$\mathbf{z}_{22} = \frac{\left(\frac{1}{s}\right)\left(1 + s + \frac{1}{s+1}\right)}{\frac{1}{s} + 1 + s + \frac{1}{s+1}} = \frac{1 + s + \frac{1}{s+1}}{1 + s + s^2 + \frac{s}{s+1}}$$

$$\mathbf{z}_{22} = \frac{s^2 + 2s + 2}{s^3 + 2s^2 + 3s + 1}$$

$$\mathbf{z}_{12} = \mathbf{z}_{21}$$

Hence,

$$[\mathbf{z}] = \begin{bmatrix} \dfrac{s^2 + s + 1}{s^3 + 2s^2 + 3s + 1} & \dfrac{1}{s^3 + 2s^2 + 3s + 1} \\ \dfrac{1}{s^3 + 2s^2 + 3s + 1} & \dfrac{s^2 + 2s + 2}{s^3 + 2s^2 + 3s + 1} \end{bmatrix}$$

Chapter 19, Problem 6.

Compute the z parameters of the circuit in Fig. 19.70.

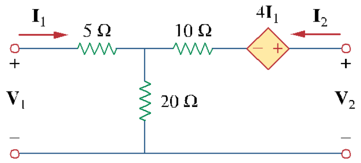
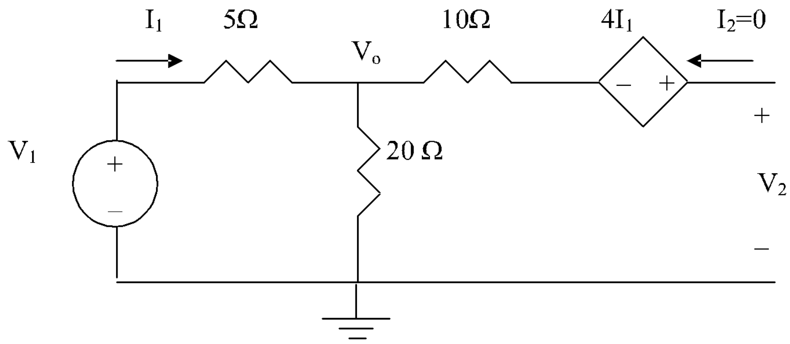


Figure 19.70

For Prob. 19.6 and 19.73.

Chapter 19, Solution 6.

To find z_{11} and z_{21} , consider the circuit below.



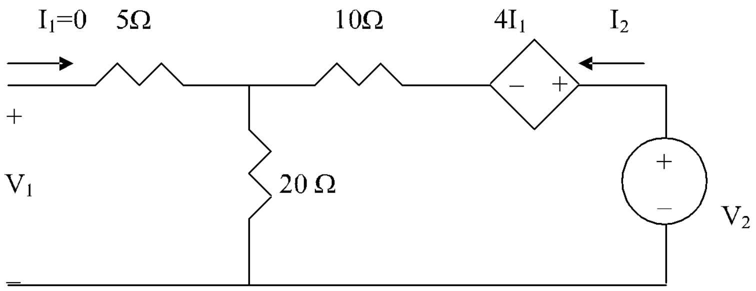
$$z_{11} = \frac{V_1}{I_1} = \frac{(20+5)I_1}{I_1} = 25\ \Omega$$

$$V_o = \frac{20}{25}V_1 = 20I_1$$

$$-V_o - 4I_2 + V_2 = 0 \quad \longrightarrow \quad V_2 = V_o + 4I_1 = 20I_1 + 4I_1 = 24I_1$$

$$z_{21} = \frac{V_2}{I_1} = 24\ \Omega$$

To find z_{12} and z_{22}, consider the circuit below.



$$V_2 = (10+20)I_2 = 30I_2$$

$$z_{22} = \frac{V_2}{I_1} = 30\ \Omega$$

$$V_1 = 20I_2$$

$$z_{12} = \frac{V_1}{I_2} = 20\ \Omega$$

Thus,

$$[z] = \begin{bmatrix} 25 & 20 \\ 24 & 30 \end{bmatrix} \Omega$$

Chapter 19, Problem 7.

Calculate the impedance-parameter equivalent of the circuit in Fig. 19.71.

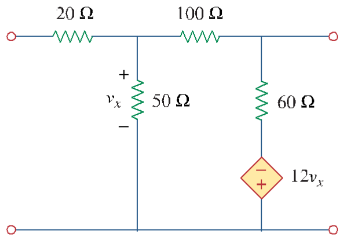
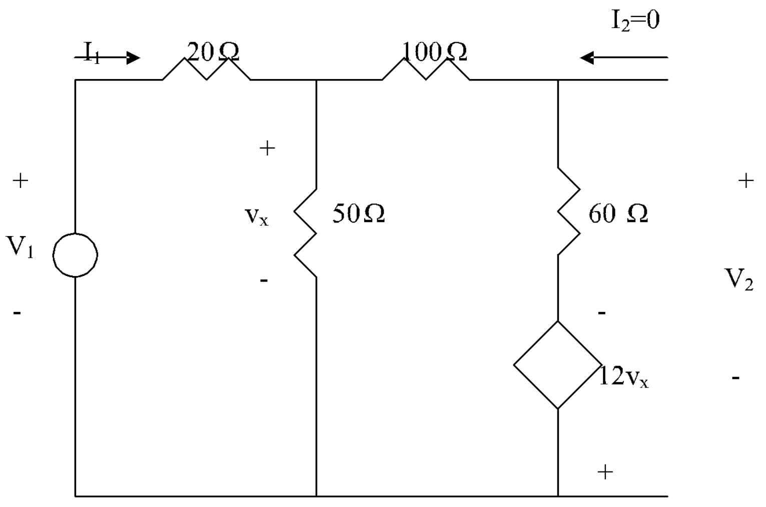


Figure 19.71

For Prob. 19.7 and 19.80.

Chapter 19, Solution 7.

To get z_{11} and z_{21}, we consider the circuit below.



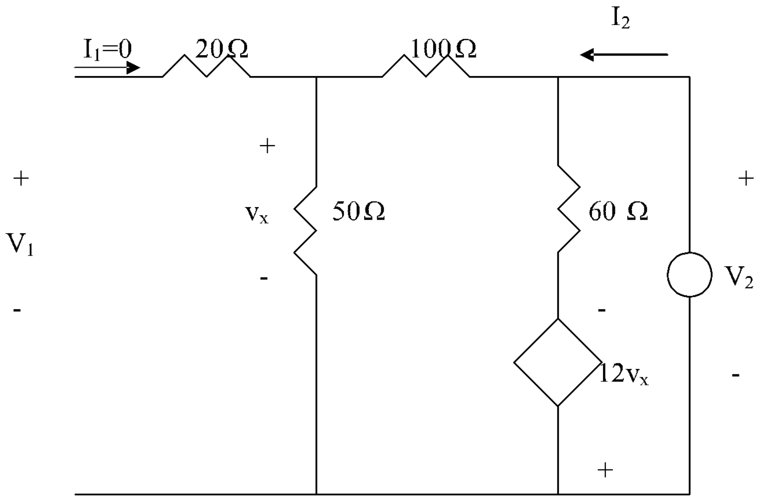
$$\frac{V_1 - V_x}{20} = \frac{V_x}{50} + \frac{V_x + 12V_x}{160} \longrightarrow V_x = \frac{40}{121}V_1$$

$$I_1 = \frac{V_1 - V_x}{20} = \frac{81}{121}(\frac{V_1}{20}) \longrightarrow z_{11} = \frac{V_1}{I_1} = 29.88$$

$$V_2 = 60(\frac{13V_x}{160}) - 12V_x = -\frac{57}{8}V_x = -\frac{57}{8}(\frac{40}{121})V_1 = -\frac{57}{8}(\frac{40}{121})\frac{20x121}{81}I_1$$

$$= -70.37I_1 \longrightarrow z_{21} = \frac{V_2}{I_1} = -70.37$$

To get z_{12} and z_{22}, we consider the circuit below.



$$V_x = \frac{50}{100+50}V_2 = \frac{1}{3}V_2, \quad I_2 = \frac{V_2}{150} + \frac{V_2 + 12V_x}{60} = 0.09V_2$$

$$z_{22} = \frac{V_2}{I_2} = 1/0.09 = 11.11$$

$$V_1 = V_x = \frac{1}{3}V_2 = \frac{11.11}{3}I_2 = 3.704I_2 \longrightarrow z_{12} = \frac{V_1}{I_2} = 3.704$$

Thus,

$$[z] = \begin{bmatrix} 29.88 & 3.704 \\ -70.37 & 11.11 \end{bmatrix}\Omega$$

Chapter 19, Problem 8.

Find the z parameters of the two-port in Fig. 19.72.

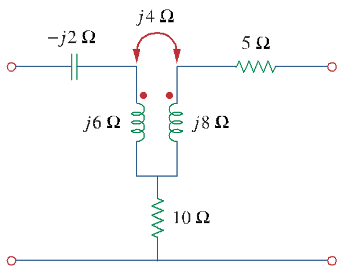
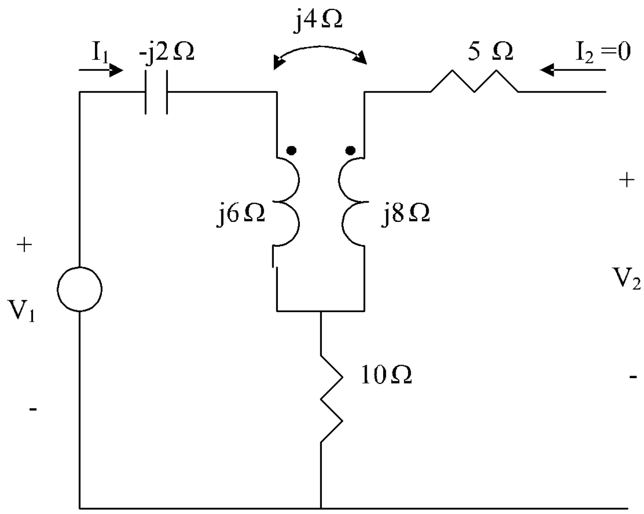


Figure 19.72

For Prob. 19.8.

Chapter 19, Solution 8.

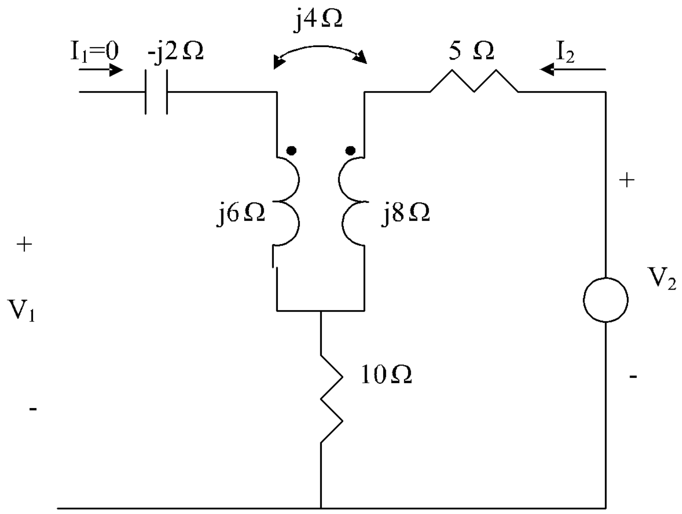
To get z_{11} and z_{21}, consider the circuit below.



$$V_1 = (10 - j2 + j6)I_1 \longrightarrow z_{11} = \frac{V_1}{I_1} = 10 + j4$$

$$V_2 = -10I_1 - j4I_1 \longrightarrow z_{21} = \frac{V_2}{I_1} = -(10 + j4)$$

To get z_{22} and z_{12}, consider the circuit below.



$$V_2 = (5 + 10 + j8)I_2 \longrightarrow z_{22} = \frac{V_2}{I_2} = 15 + j8$$

$$V_1 = -(10 + j4)I_2 \longrightarrow z_{12} = \frac{V_1}{I_2} = -(10 + j4)$$

Thus,

$$[z] = \begin{bmatrix} (10 + j4) & -(10 + j4) \\ -(10 + j4) & (15 + j8) \end{bmatrix} \Omega$$

Chapter 19, Problem 9.

The *y* parameters of a network are:

$$[y] = \begin{bmatrix} 0.5 & -0.2 \\ -0.2 & 0.4 \end{bmatrix}$$

Determine the *z* parameters for the network.

Chapter 19, Solution 9.

$$z_{11} = \frac{y_{22}}{\Delta y} = \frac{0.4}{0.16} = 2.5, \quad \Delta y = y_{11}y_{22} - y_{21}y_{12} = 05x0.4 - 0.2x0.2 = 0.16$$

$$z_{12} = \frac{-y_{12}}{\Delta y} = \frac{0.2}{0.16} = 1.25 = z_{21}$$

$$z_{22} = \frac{y_{11}}{\Delta y} = \frac{0.5}{0.16} = 3.125$$

Thus,

$$[z] = \begin{bmatrix} 2.5 & 1.25 \\ 1.25 & 3.125 \end{bmatrix}\Omega \begin{bmatrix} 2.5 & 1.25 \\ 1.25 & 3.125 \end{bmatrix}\Omega$$

Chapter 19, Problem 10.

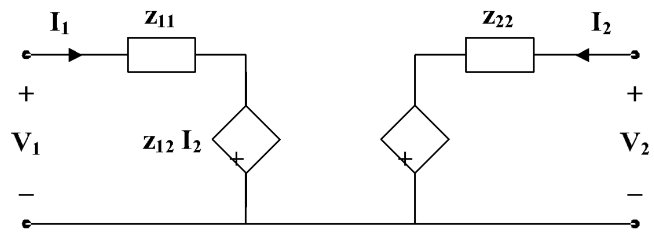
Construct a two-port that realizes each of the following z parameters.

(a) $[\mathbf{z}] = \begin{bmatrix} 25 & 20 \\ 5 & 10 \end{bmatrix} \Omega$

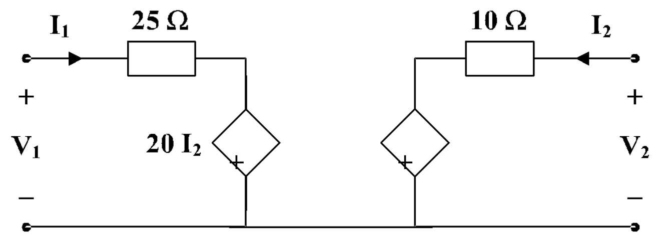
(b) $[\mathbf{z}] = \begin{bmatrix} 1+\frac{3}{s} & \frac{1}{s} \\ \frac{1}{s} & 2s+\frac{1}{s} \end{bmatrix} \Omega$

Chapter 19, Solution 10.

(a) This is a non-reciprocal circuit so that **<u>the two-port looks like the one shown in Figs. (a) and (b)</u>**.

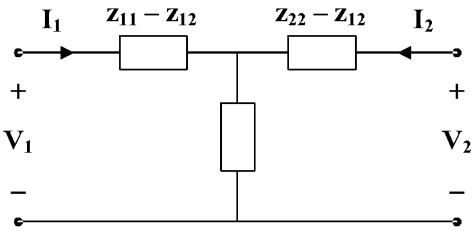


(a)



(b)

(b) This is a reciprocal network and **the two-port look like the one shown in Figs. (c) and (d)**.

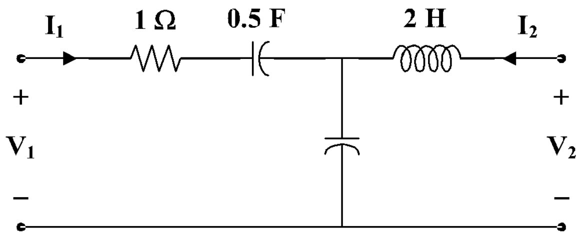


(c)

$$\mathbf{z}_{11} - \mathbf{z}_{12} = 1 + \frac{2}{s} = 1 + \frac{1}{0.5s}$$

$$\mathbf{z}_{22} - \mathbf{z}_{12} = 2s$$

$$\mathbf{z}_{12} = \frac{1}{s}$$



(d)

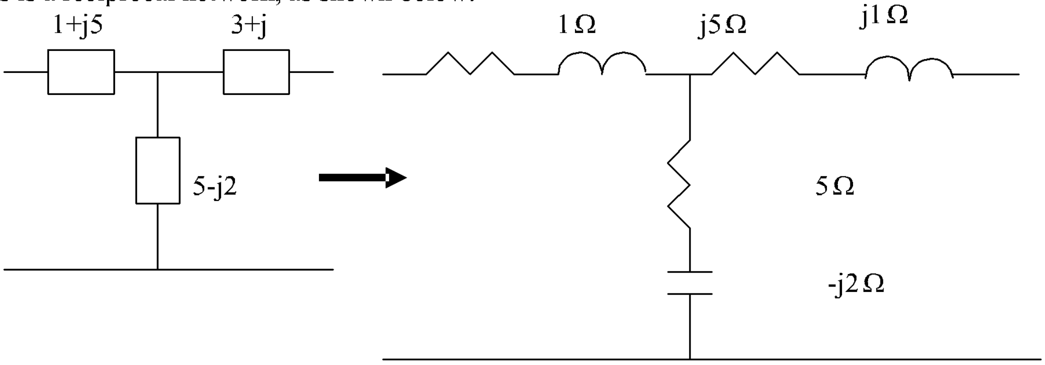
Chapter 19, Problem 11.

Determine a two-port network that is represented by the following *z* parameters:

$$[\mathbf{z}] = \begin{bmatrix} 6+j3 & 5-j2 \\ 5-j2 & 8-j \end{bmatrix} \Omega$$

Chapter 19, Solution 11.

This is a reciprocal network, as shown below.



Chapter 19, Problem 12.

For the circuit shown in Fig. 19.73, let

$$[\mathbf{z}] = \begin{bmatrix} 10 & -6 \\ -4 & 12 \end{bmatrix}$$

Find I_1, I_2, V_1, and V_2.

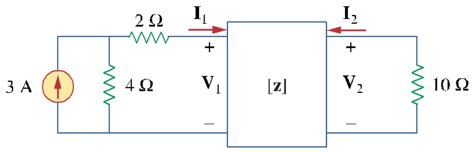


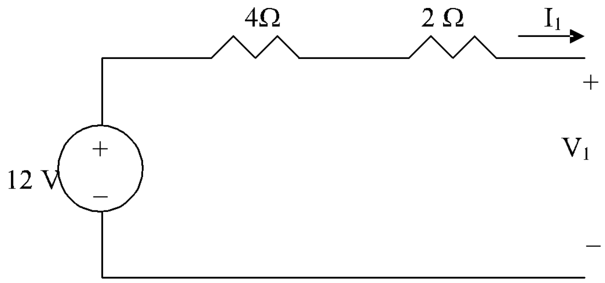
Figure 19.73

For Prob. 19.12.

Chapter 19, Solution 12.

$$V_1 = 10I_1 - 6I_2 \quad (1)$$
$$V_2 = -4I_2 + 12I_2 \quad (2)$$
$$V_2 = -10I_2 \quad (3)$$

If we convert the current source to a voltage source, that portion of the circuit becomes what is shown below.



$$-12 + 6I_1 + V_1 = 0 \longrightarrow V_1 = 12 - 6I_1 \quad (4)$$

Substituting (3) and (4) into (1) and (2), we get

$$12 - 6I_1 = 10I_1 - 6I_2 \longrightarrow 12 = 16I_1 - 6I_2 \quad (5)$$
$$-10I_2 = -4I_1 + 12I_2 \longrightarrow 0 = -4I_1 + 22I_2 \longrightarrow I_1 = 5.5I_2 \quad (6)$$

From (5) and (6),

$$12 = 88I_2 - 6I_2 = 82I_2 \longrightarrow I_2 = \underline{0.1463\ \text{A}}$$
$$I_1 = 5.5I_2 = \underline{0.8049\ \text{A}}$$
$$V_2 = -10I_2 = \underline{-1.463\ \text{V}}$$
$$V_1 = 12 - 6I_1 = \underline{7.1706\ \text{V}}$$

Chapter 19, Problem 13.

Determine the average power delivered to $Z_L = 5 + j4$ in the network of Fig. 19.74. *Note:* The voltage is rms.

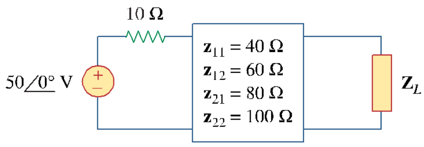
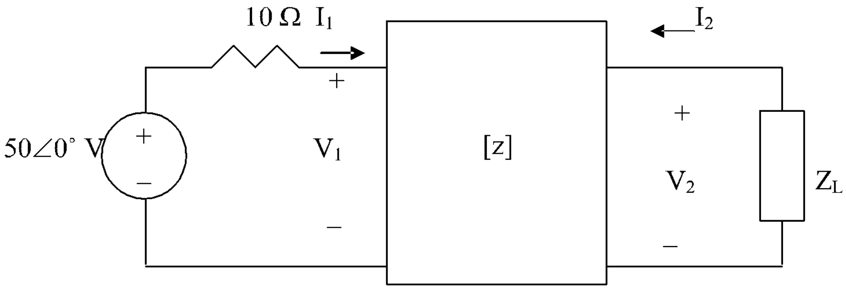


Figure 19.74

For Prob. 19.13.

Chapter 19, Solution 13.

Consider the circuit as shown below.



$V_1 = 40I_1 + 60I_2$ (1)

$V_2 = 80I_1 + 100I_2$ (2)

$V_2 = -I_2 Z_L = -I_2(5 + j4)$ (3)

$50 = V_1 + 10I_1 \longrightarrow V_1 = 50 - 10I_1$ (4)

Substituting (4) in (1)

$50 - 10I_1 = 40I_1 + 60I_2 \longrightarrow 5 = 5I_1 + 6I_2$ (5)

Substituting (3) into (2),

$-I_2(5 + j4) = 80I_1 + 100I_2 \longrightarrow 0 = 80I_1 + (105 + j4)I_2$ (6)

Solving (5) and (6) gives

$I_2 = -7.423 + j3.299$ A

We can check the answer using MATLAB.

First we need to rewrite equations 1-4 as follows,

$$\begin{bmatrix} 1 & 0 & -40 & -60 \\ 0 & 1 & -80 & -100 \\ 0 & 1 & 0 & 5+j4 \\ 1 & 0 & 10 & 0 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ I_1 \\ I_2 \end{bmatrix} = A*X = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 50 \end{bmatrix} = U$$

```
>> A=[1,0,-40,-60;0,1,-80,-100;0,1,0,(5+4i);1,0,10,0]
A =
  1.0e+002 *
   0.0100         0          -0.4000        -0.6000
        0    0.0100          -0.8000        -1.0000
        0    0.0100                0     0.0500 + 0.0400i
   0.0100         0           0.1000              0
>> U=[0;0;0;50]
U =
     0
     0
     0
    50
>> X=inv(A)*U
X =
 -49.0722 +39.5876i
  50.3093 +13.1959i
   9.9072 - 3.9588i
  -7.4227 + 3.2990i
```

$P = |I_2|^2 5 =$ **329.9 W**.

Chapter 19, Problem 14.

For the two-port network shown in Fig. 19.75, show that at the output terminals,

$$\mathbf{Z}_{Th} = \mathbf{z}_{22} - \frac{\mathbf{z}_{12}\mathbf{z}_{21}}{\mathbf{z}_{11} + \mathbf{Z}_s}$$

and

$$\mathbf{V}_{Th} = \frac{\mathbf{z}_{21}}{\mathbf{z}_{11} + \mathbf{Z}_s}\mathbf{V}s$$

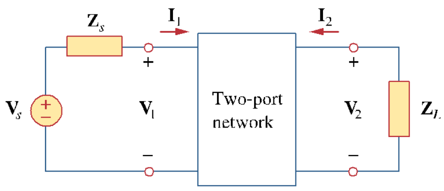
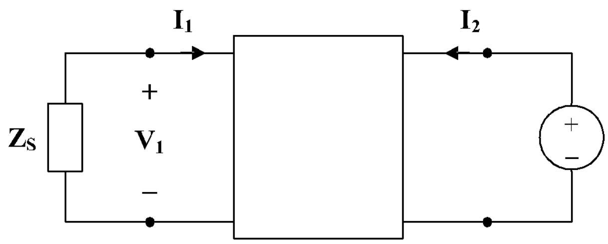


Figure 19.75

For Prob. 19.14 and 19.41.

Chapter 19, Solution 14.

To find $\mathbf{Z}_{Th}$, consider the circuit in Fig. (a).



(a)

$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 \qquad (1)$$

$$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2 \qquad (2)$$

But

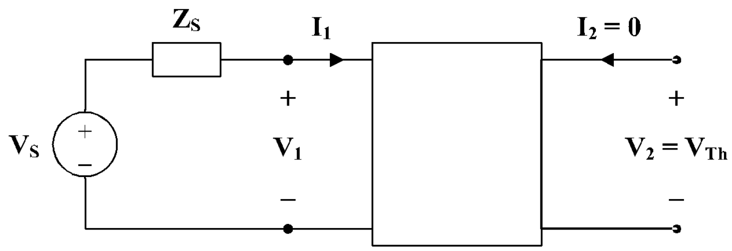
$$\mathbf{V}_2 = 1, \qquad \mathbf{V}_1 = -\mathbf{Z}_s \mathbf{I}_1$$

Hence, $\quad 0 = (\mathbf{z}_{11} + \mathbf{Z}_s)\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 \longrightarrow \mathbf{I}_1 = \frac{-\mathbf{z}_{12}}{\mathbf{z}_{11} + \mathbf{Z}_s}\mathbf{I}_2$

$$1 = \left(\frac{-\mathbf{z}_{21}\mathbf{z}_{12}}{\mathbf{z}_{11} + \mathbf{Z}_s} + \mathbf{z}_{22}\right)\mathbf{I}_2$$

$$\mathbf{Z}_{Th} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{1}{\mathbf{I}_2} = \underline{\mathbf{z}_{22} - \frac{\mathbf{z}_{21}\mathbf{z}_{12}}{\mathbf{z}_{11} + \mathbf{Z}_s}}$$

To find $\mathbf{V}_{Th}$, consider the circuit in Fig. (b).



(b)

$$\mathbf{I}_2 = 0, \qquad \mathbf{V}_1 = \mathbf{V}_s - \mathbf{I}_1\mathbf{Z}_s$$

Substituting these into (1) and (2),

$$\mathbf{V}_s - \mathbf{I}_1\mathbf{Z}_s = \mathbf{z}_{11}\mathbf{I}_1 \longrightarrow \mathbf{I}_1 = \frac{\mathbf{V}_s}{\mathbf{z}_{11} + \mathbf{Z}_s}$$

$$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 = \frac{\mathbf{z}_{21}\mathbf{V}_s}{\mathbf{z}_{11} + \mathbf{Z}_s}$$

$$\mathbf{V}_{Th} = \mathbf{V}_2 = \underline{\frac{\mathbf{z}_{21}\mathbf{V}_s}{\mathbf{z}_{11} + \mathbf{Z}_s}}$$

Chapter 19, Problem 15.

For the two-port circuit in Fig. 19.76,

$$[\mathbf{z}] = \begin{bmatrix} 40 & 60 \\ 80 & 120 \end{bmatrix} \Omega$$

(a) Find $\mathbf{Z}_L$ for maximum power transfer to the load.
(b) Calculate the maximum power delivered to the load.

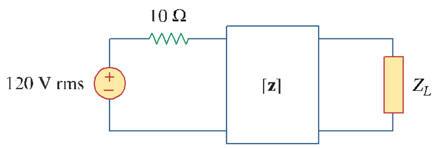


Figure 19.76
For Prob. 19.15.

Chapter 19, Solution 15.

(a) From Prob. 18.12,

$$Z_{Th} = z_{22} - \frac{z_{12}z_{21}}{z_{11} + Z_s} = 120 - \frac{80x60}{40+10} = 24$$

$$\underline{Z_L = Z_{Th} = 24\Omega}$$

(b) $V_{Th} = \frac{z_{21}}{z_{11} + Z_s} V_s = \frac{80}{40+10}(120) = 192$

$$P_{max} = \left(\frac{V_{Th}}{2R_{Th}}\right)^2 R_{Th} = 4^2 x24 = \underline{\mathbf{384W}}$$

Chapter 19, Problem 16.

For the circuit in Fig. 19.77, at $\omega = 2$ rad/s, $\mathbf{z}_{11} = 10\Omega$, $\mathbf{z}_{12} = \mathbf{z}_{21} = j6\Omega$, $\mathbf{z}_{22} = 4\Omega$. Obtain the Thevenin equivalent circuit at terminals *a-b* and calculate v_o.

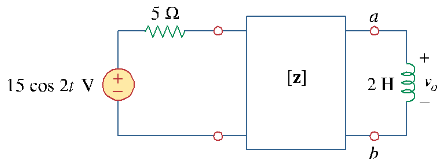
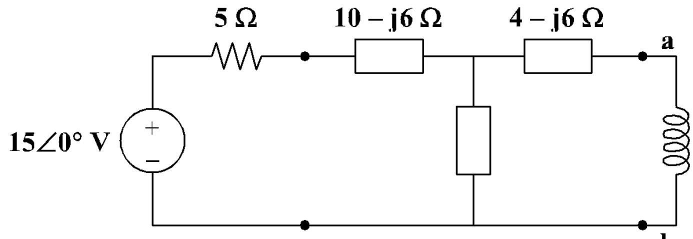


Figure 19.77

For Prob. 19.16.

Chapter 19, Solution 16.

As a reciprocal two-port, the given circuit can be represented as shown in Fig. (a).



(a)

At terminals a-b,

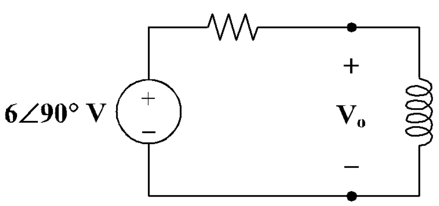
$$\mathbf{Z}_{Th} = (4 - j6) + j6 \,||\, (5 + 10 - j6)$$

$$\mathbf{Z}_{Th} = 4 - j6 + \frac{j6\,(15 - j6)}{15} = 4 - j6 + 2.4 + j6$$

$$\mathbf{Z}_{Th} = \underline{\mathbf{6.4\ \Omega}}$$

$$\mathbf{V}_{Th} = \frac{j6}{j6 + 5 + 10 - j6}(15\angle 0°) = j6 = \underline{\mathbf{6\angle 90°\ V}}$$

The Thevenin equivalent circuit is shown in Fig. (b).



(b)

From this,

$$\mathbf{V}_o = \frac{j4}{6.4 + j4}(j6) = 3.18\angle 148°$$

$$v_o(t) = \underline{\mathbf{3.18\cos(2t + 148°)\ V}}$$

Chapter 19, Problem 17.

* Determine the *z* and *y* parameters for the circuit in Fig. 19.78.

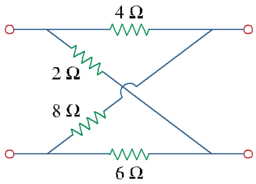


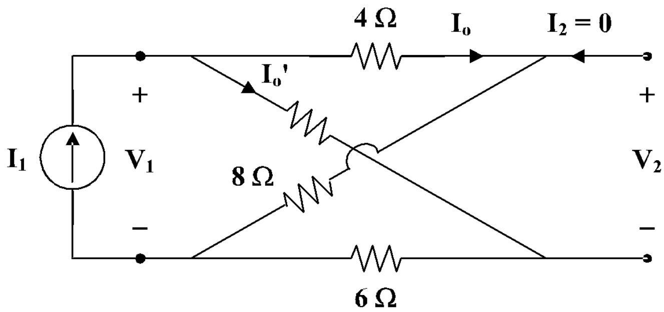
Figure 19.78

For Prob. 19.17.

* An asterisk indicates a challenging problem.

Chapter 19, Solution 17.

To obtain $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$, consider the circuit in Fig. (a).



(a)

In this case, the 4-Ω and 8-Ω resistors are in series, since the same current, $\mathbf{I}_o$, passes through them. Similarly, the 2-Ω and 6-Ω resistors are in series, since the same current, $\mathbf{I}_o^{'}$, passes through them.

$$\mathbf{z}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = (4+8) \,||\, (2+6) = 12 \,||\, 8 = \frac{(12)(8)}{20} = 4.8\,\Omega$$

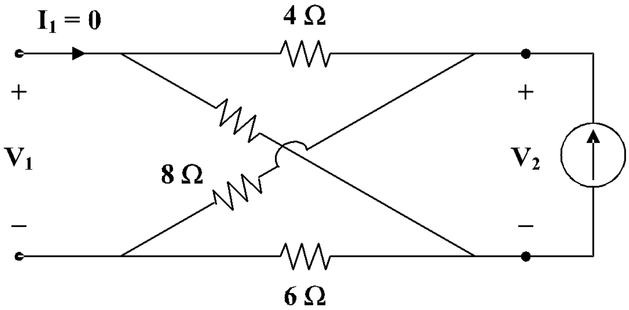
$$\mathbf{I}_o = \frac{8}{8+12}\mathbf{I}_1 = \frac{2}{5}\mathbf{I}_1 \qquad\qquad \mathbf{I}_o^{'} = \frac{3}{5}\mathbf{I}_1$$

But $\quad -\mathbf{V}_2 - 4\mathbf{I}_o + 2\mathbf{I}_o^{'} = 0$

$$\mathbf{V}_2 = -4\mathbf{I}_o + 2\mathbf{I}_o^{'} = \frac{-8}{5}\mathbf{I}_1 + \frac{6}{5}\mathbf{I}_1 = \frac{-2}{5}\mathbf{I}_1$$

$$\mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1} = \frac{-2}{5} = -0.4\,\Omega$$

To get $\mathbf{z}_{22}$ and $\mathbf{z}_{12}$, consider the circuit in Fig. (b).



(b)

$$\mathbf{z}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = (4+2)\,||\,(8+6) = 6\,||\,14 = \frac{(6)(14)}{20} = 4.2\,\Omega$$

$$\mathbf{z}_{12} = \mathbf{z}_{21} = -0.4\,\Omega$$

Thus,

$$[\mathbf{z}] = \begin{bmatrix} \mathbf{4.8} & \mathbf{-0.4} \\ \mathbf{-0.4} & \mathbf{4.2} \end{bmatrix}\Omega$$

We may take advantage of Table 18.1 to get [y] from [z].

$$\Delta_z = (4.8)(4.2) - (0.4)^2 = 20$$

$$\mathbf{y}_{11} = \frac{\mathbf{z}_{22}}{\Delta_z} = \frac{4.2}{20} = 0.21 \qquad \mathbf{y}_{12} = \frac{-\mathbf{z}_{12}}{\Delta_z} = \frac{0.4}{20} = 0.02$$

$$\mathbf{y}_{21} = \frac{-\mathbf{z}_{21}}{\Delta_z} = \frac{0.4}{20} = 0.02 \qquad \mathbf{y}_{22} = \frac{\mathbf{z}_{11}}{\Delta_z} = \frac{4.8}{20} = 0.24$$

Thus,

$$[\mathbf{y}] = \begin{bmatrix} \mathbf{0.21} & \mathbf{0.02} \\ \mathbf{0.02} & \mathbf{0.24} \end{bmatrix}\mathbf{S}$$

Chapter 19, Problem 18.

Calculate the y parameters for the two-port in Fig. 19.79.

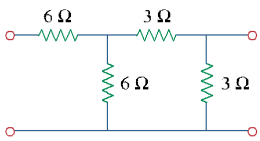
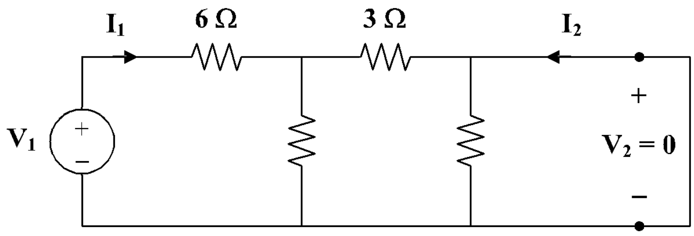


Figure 19.79
For Prob. 19.18 and 19.37.

Chapter 19, Solution 18.

To get $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$, consider the circuit in Fig.(a).



(a)

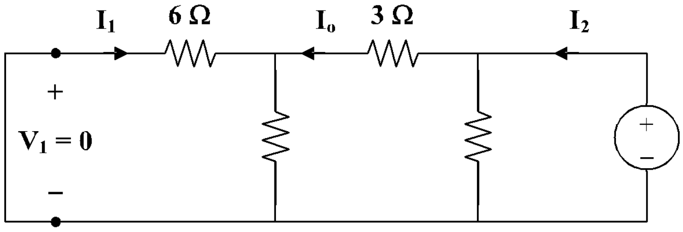
$$\mathbf{V}_1 = (6 + 6 \,||\, 3)\mathbf{I}_1 = 8\mathbf{I}_1$$

$$\mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{1}{8}$$

$$\mathbf{I}_2 = \frac{-6}{6+3}\mathbf{I}_1 = \frac{-2}{3}\frac{\mathbf{V}_1}{8} = \frac{-\mathbf{V}_1}{12}$$

$$\mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \frac{-1}{12}$$

To get $\mathbf{y}_{22}$ and $\mathbf{y}_{12}$, consider the circuit in Fig.(b).



(b)

$$\mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{1}{3 \,||\, (3+6 \,||\, 6)} = \frac{1}{3 \,||\, 6} = \frac{1}{2}$$

$$\mathbf{I}_1 = \frac{-\mathbf{I}_o}{2}, \qquad \mathbf{I}_o = \frac{3}{3+6}\mathbf{I}_2 = \frac{1}{3}\mathbf{I}_2$$

$$\mathbf{I}_1 = \frac{-\mathbf{I}_2}{6} = \left(\frac{-1}{6}\right)\left(\frac{1}{2}\mathbf{V}_2\right) = \frac{-\mathbf{V}_2}{12}$$

$$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{-1}{12} = \mathbf{y}_{21}$$

Thus,

$$[\mathbf{y}] = \begin{bmatrix} \frac{1}{8} & \frac{-1}{12} \\ \frac{-1}{12} & \frac{1}{2} \end{bmatrix} \mathbf{S}$$

Chapter 19, Problem 19.

Find the *y* parameters of the two-port in Fig. 19.80 in terms of *s*.

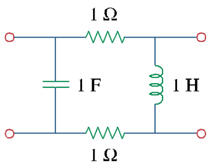
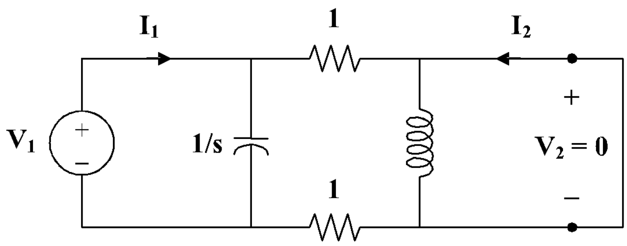


Figure 19.80

For Prob. 19.19.

Chapter 19, Solution 19.

Consider the circuit in Fig.(a) for calculating $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$.



(a)

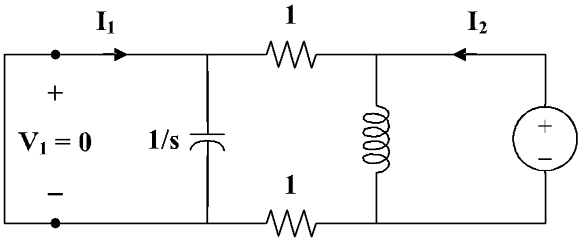
$$\mathbf{V}_1 = \left(\frac{1}{s} \,\|\, 2\right)\mathbf{I}_1 = \frac{2/s}{2 + (1/s)}\mathbf{I}_1 = \frac{2}{2s+1}\mathbf{I}_1$$

$$\mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{2s+1}{2} = s + 0.5$$

$$\mathbf{I}_2 = \frac{(-1/s)}{(1/s)+2}\mathbf{I}_1 = \frac{-\mathbf{I}_1}{2s+1} = \frac{-\mathbf{V}_1}{2}$$

$$\mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = -0.5$$

To get $\mathbf{y}_{22}$ and $\mathbf{y}_{12}$, refer to the circuit in Fig.(b).



(b)

$$\mathbf{V}_2 = (s \,||\, 2)\,\mathbf{I}_2 = \frac{2s}{s+2}\mathbf{I}_2$$

$$\mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{s+2}{2s} = 0.5 + \frac{1}{s}$$

$$\mathbf{I}_1 = \frac{-s}{s+2}\mathbf{I}_2 = \frac{-s}{s+2} \cdot \frac{s+2}{2s}\mathbf{V}_2 = \frac{-\mathbf{V}_2}{2}$$

$$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = -0.5$$

Thus,

$$[\mathbf{y}] = \begin{bmatrix} \mathbf{s+0.5} & \mathbf{-0.5} \\ \mathbf{-0.5} & \mathbf{0.5+1/s} \end{bmatrix} \mathbf{S}$$

Chapter 19, Problem 20.

Find the *y* parameters for the circuit in Fig. 19.81.

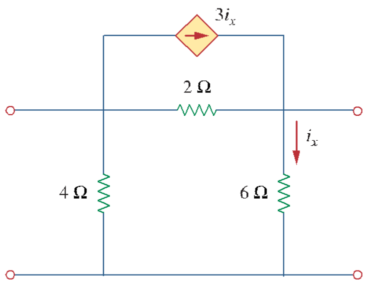
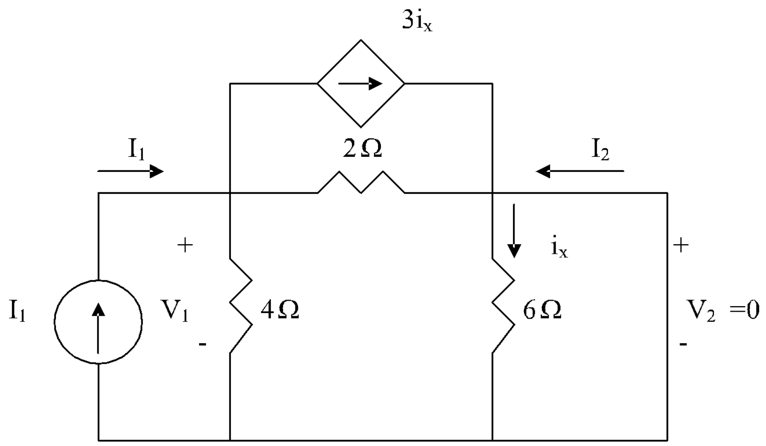


Figure 19.81

For Prob. 19.20.

Chapter 19, Solution 20.

To get y_{11} and y_{21}, consider the circuit below.

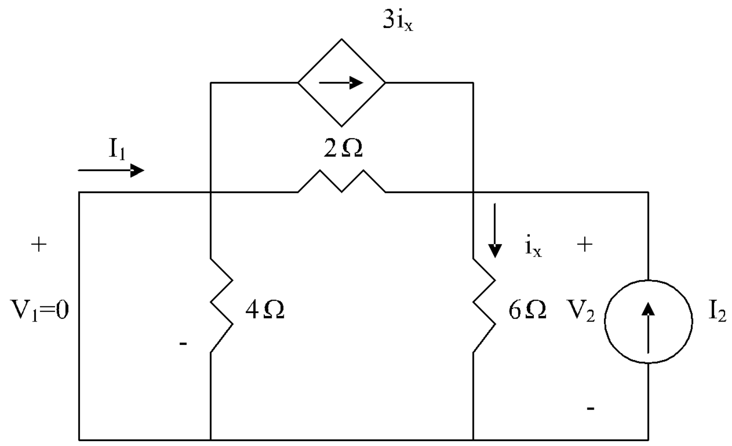


Since 6-ohm resistor is short-circuited, $i_x = 0$

$$V_1 = I_1(4//2) = \frac{8}{6}I_1 \quad \longrightarrow \quad y_{11} = \frac{I_1}{V_1} = 0.75$$

$$I_2 = -\frac{4}{4+2}I_1 = -\frac{2}{3}(\frac{6}{8}V_1) = -\frac{1}{2}V_1 \quad \longrightarrow \quad y_{21} = \frac{I_2}{V_1} = -0.5$$

To get y_{22} and y_{12}, consider the circuit below.



$$i_x = \frac{V_2}{6}, \quad I_2 = i_x - 3i_x + \frac{V_2}{2} = \frac{V_2}{6} \quad \longrightarrow \quad y_{22} = \frac{I_2}{V_2} = \frac{1}{6} = 0.1667$$

$$I_1 = 3i_x - \frac{V_2}{2} = 0 \quad \longrightarrow \quad y_{12} = \frac{I_1}{V_2} = 0$$

Thus,

$$[y] = \begin{bmatrix} 0.75 & 0 \\ -0.5 & 0.1667 \end{bmatrix} S$$

Chapter 19, Problem 21.

Obtain the admittance parameter equivalent circuit of the two-port in Fig. 19.82.

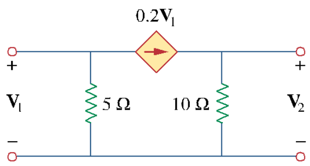
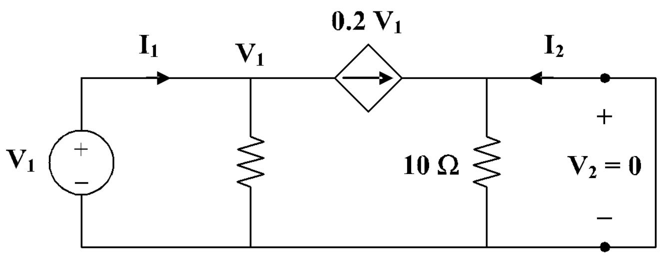


Figure 19.82

For Prob. 19.21.

Chapter 19, Solution 21.

To get $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$, refer to Fig. (a).



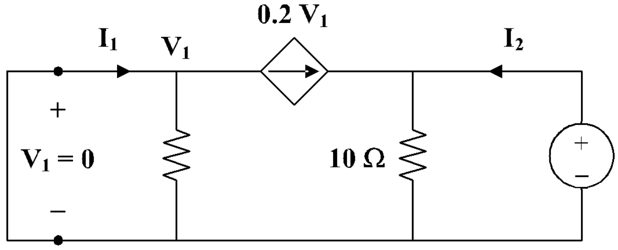
(a)

At node 1,

$$\mathbf{I}_1 = \frac{\mathbf{V}_1}{5} + 0.2\mathbf{V}_1 = 0.4\mathbf{V}_1 \longrightarrow \mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = 0.4$$

$$\mathbf{I}_2 = -0.2\mathbf{V}_1 \longrightarrow \mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = -0.2$$

To get $\mathbf{y}_{22}$ and $\mathbf{y}_{12}$, refer to the circuit in Fig. (b).



(b)

Since $\mathbf{V}_1 = 0$, the dependent current source can be replaced with an open circuit.

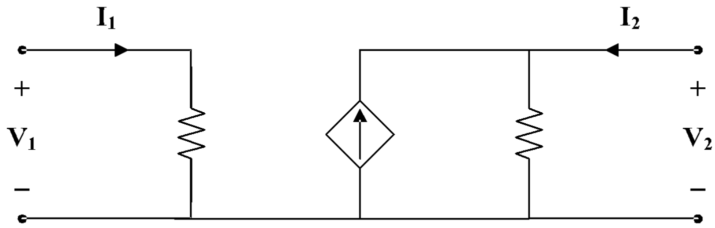
$$\mathbf{V}_2 = 10\mathbf{I}_2 \longrightarrow \mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{1}{10} = 0.1$$

$$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = 0$$

Thus,

$$[\mathbf{y}] = \begin{bmatrix} 0.4 & 0 \\ -0.2 & 0.1 \end{bmatrix} \text{S}$$

Consequently, **<u>the y parameter equivalent circuit is shown in Fig. (c)</u>**.



(c)

Chapter 19, Problem 22.

Obtain the *y* parameters of the two-port network in Fig. 19.83.

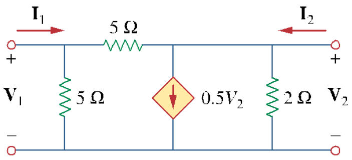
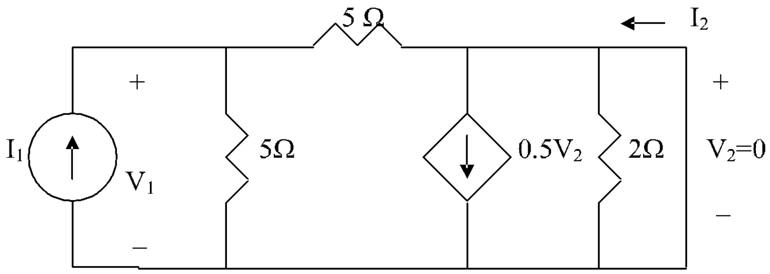


Figure 19.83

For Prob. 19.22.

Chapter 19, Solution 22.

To obtain y_{11} and y_{21}, consider the circuit below.

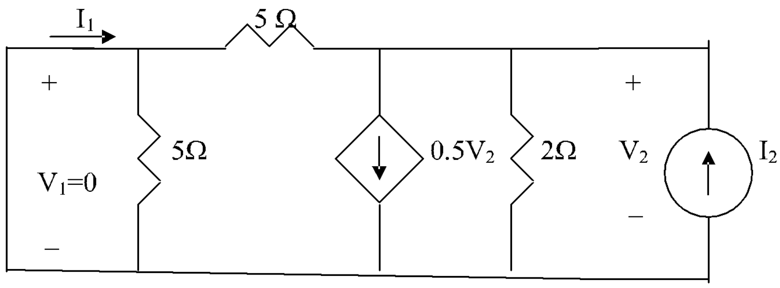


The 2-Ω resistor is short-circuited.

$$V_1 = 5\frac{I_1}{2} \longrightarrow y_{11} = \frac{I_1}{V_1} = \frac{2}{5} = 0.4$$

$$I_2 = \frac{1}{2}I_1 \longrightarrow y_{21} = \frac{I_2}{V_1} = \frac{\frac{1}{2}I_1}{2.5I_1} = 0.2$$

To obtain y_{12} and y_{22}, consider the circuit below.



At the top node, KCL gives

$$I_2 = 0.5V_2 + \frac{V_2}{2} + \frac{V_2}{5} = 1.2V_2 \longrightarrow y_{22} = \frac{I_2}{V_2} = 1.2$$

$$I_1 = -\frac{V_2}{5} = -0.2V_2 \longrightarrow y_{12} = \frac{I_1}{V_2} = -0.2$$

Hence,

$$[y] = \begin{bmatrix} 0.4 & -0.2 \\ 0.2 & 1.2 \end{bmatrix} \text{S}$$

Chapter 19, Problem 23.

(a) Find the *y* parameters of the two-port in Fig. 19.84.
(b) Determine $\mathbf{V}_2(s)$ for $v_s = 2u(t)$V.

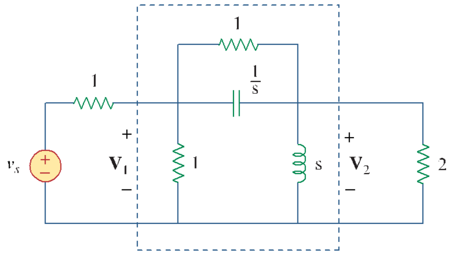


Figure 19.84
For Prob. 19.23.

Chapter 19, Solution 23.

(a)

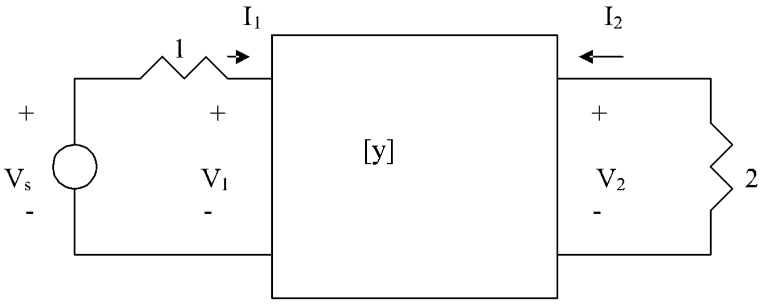
$$-y_{12} = 1/\left(1//\frac{1}{s}\right) = 1 + s \longrightarrow y_{12} = -(s+1)$$

$$y_{11} + y_{12} = 1 \longrightarrow y_{11} = 1 - y_{12} = 1 + s + 1 = s + 2$$

$$y_{22} + y_{12} = s \longrightarrow y_{22} = \frac{1}{s} - y_{12} = \frac{1}{s} + s + 1 = \frac{s^{2s} + s + 1}{s}$$

$$[y] = \begin{bmatrix} s+2 & -(s+1) \\ -(s+1) & \dfrac{s^2+s+1}{s} \end{bmatrix}$$

(b) Consider the network below.



$$V_s = I_1 + V_1 \text{ or } V_s - V_1 = I_1 \qquad (1)$$

$$V_2 = -2I_2 \qquad (2)$$

$$I_1 = y_{11}V_1 + y_{12}V_2 \qquad (3)$$

$$I_2 = y_{21}V_1 + y_{22}V_2 \qquad (4)$$

From (1) and (3)

$$V_s - V_1 = y_{11}V_1 + y_{12}V_2 \longrightarrow V_s = (1 + y_{11})V_1 + y_{12}V_2 \qquad (5)$$

From (2) and (4),

$$-0.5V_2 = y_{21}V_1 + y_{22}V_2 \longrightarrow V_1 = -\frac{1}{y_{21}}(0.5 + y_{22})V_2 \qquad (6)$$

Substituting (6) into (5),

$$V_s = -\frac{(1 + y_{11})(0.5 + y_{22})}{y_{21}}V_2 + y_{12}V_2$$

$$= \frac{2}{s} \longrightarrow V_2 = \frac{2/s}{\left[y_{12} - \frac{1}{y_{21}}(1 + y_{11})(0.5 + y_{22})\right]}$$

$$V_2 = \frac{2/s}{-(s+1) + \frac{1}{s+1}(1 + s + 2)\left(\frac{1}{2} + \frac{s^2 + s + 1}{s}\right)} = \frac{0.8(s+1)}{(s^2 + 1.8s + 1.2)}$$

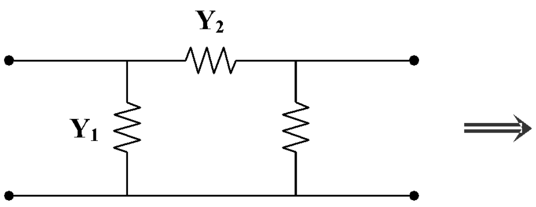
Chapter 19, Problem 24.

Find the resistive circuit that represents these *y* parameters:

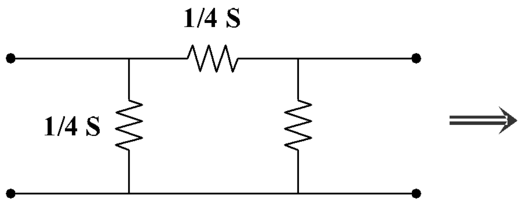
$$[\mathbf{y}] = \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{3}{8} \end{bmatrix}$$

Chapter 19, Solution 24.

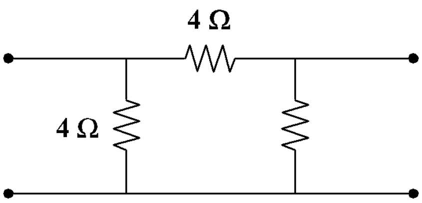
Since this is a reciprocal network, **a Π network is appropriate, as shown below**.



(a)



(b)



(c)

$$\mathbf{Y}_1 = \mathbf{y}_{11} + \mathbf{y}_{12} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}\,\text{S}, \qquad \mathbf{Z}_1 = \underline{\mathbf{4\,\Omega}}$$

$$\mathbf{Y}_2 = -\mathbf{y}_{12} = \frac{1}{4}\,\text{S}, \qquad \mathbf{Z}_2 = \underline{\mathbf{4\,\Omega}}$$

$$\mathbf{Y}_3 = \mathbf{y}_{22} + \mathbf{y}_{21} = \frac{3}{8} - \frac{1}{4} = \frac{1}{8}\,\text{S}, \qquad \mathbf{Z}_3 = \underline{\mathbf{8\,\Omega}}$$

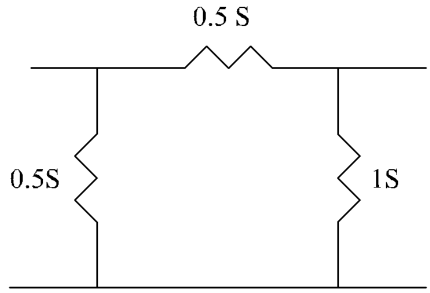
Chapter 19, Problem 25.

Draw the two-port network that has the following *y* parameters:

$$[\mathrm{y}] = \begin{bmatrix} 1 & -0.5 \\ -0.5 & 1.5 \end{bmatrix} \mathrm{S}$$

Chapter 19, Solution 25.

This is a reciprocal network and is shown below.



Chapter 19, Problem 26.

Calculate [y] for the two-port in Fig. 19.85.

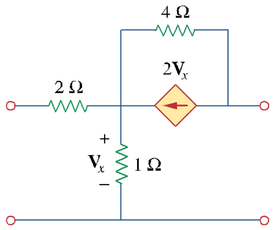
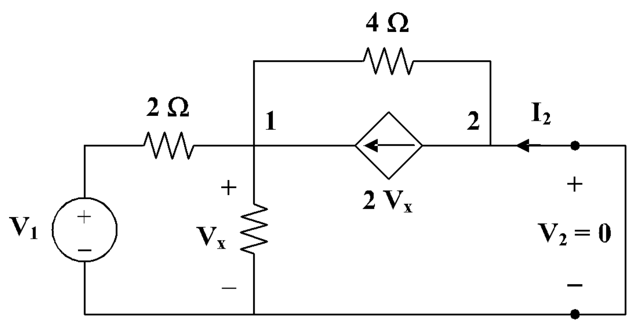


Figure 19.85

For Prob. 19.26.

Chapter 19, Solution 26.

To get $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$, consider the circuit in Fig. (a).



(a)

At node 1,

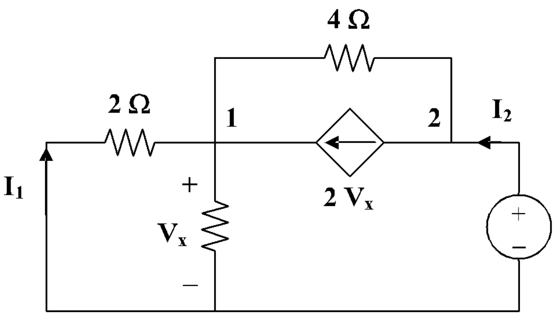
$$\frac{\mathbf{V}_1 - \mathbf{V}_x}{2} + 2\mathbf{V}_x = \frac{\mathbf{V}_x}{1} + \frac{\mathbf{V}_x}{4} \longrightarrow 2\mathbf{V}_1 = \text{-}\mathbf{V}_x \qquad (1)$$

But $$\mathbf{I}_1 = \frac{\mathbf{V}_1 - \mathbf{V}_x}{2} = \frac{\mathbf{V}_1 + 2\mathbf{V}_1}{2} = 1.5\mathbf{V}_1 \longrightarrow \mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = 1.5$$

Also, $$\mathbf{I}_2 + \frac{\mathbf{V}_x}{4} = 2\mathbf{V}_x \longrightarrow \mathbf{I}_2 = 1.75\mathbf{V}_x = \text{-}3.5\mathbf{V}_1$$

$$\mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \text{-}3.5$$

To get $\mathbf{y}_{22}$ and $\mathbf{y}_{12}$, consider the circuit in Fig.(b).



(b)

At node 2,

$$\mathbf{I}_2 = 2\mathbf{V}_x + \frac{\mathbf{V}_2 - \mathbf{V}_x}{4} \qquad (2)$$

At node 1,

$$2\mathbf{V}_x + \frac{\mathbf{V}_2 - \mathbf{V}_x}{4} = \frac{\mathbf{V}_x}{2} + \frac{\mathbf{V}_x}{1} = \frac{3}{2}\mathbf{V}_x \longrightarrow \mathbf{V}_2 = \text{-}\mathbf{V}_x \qquad (3)$$

Substituting (3) into (2) gives

$$\mathbf{I}_2 = 2\mathbf{V}_x - \frac{1}{2}\mathbf{V}_x = 1.5\mathbf{V}_x = \text{-}1.5\mathbf{V}_2$$

$$\mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \text{-}1.5$$

$$\mathbf{I}_1 = \frac{\text{-}\mathbf{V}_x}{2} = \frac{\mathbf{V}_2}{2} \longrightarrow \mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = 0.5$$

Thus,

$$[\mathbf{y}] = \begin{bmatrix} \mathbf{1.5} & \mathbf{0.5} \\ \mathbf{-3.5} & \mathbf{-1.5} \end{bmatrix} \mathbf{S}$$

Chapter 19, Problem 27.

Find the *y* parameters for the circuit in Fig. 19.86.

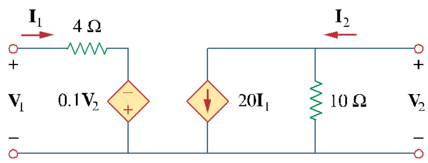
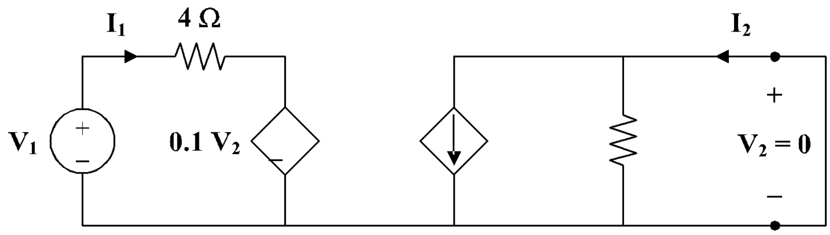


Figure 19.86
For Prob. 19.27.

Chapter 19, Solution 27.

Consider the circuit in Fig. (a).

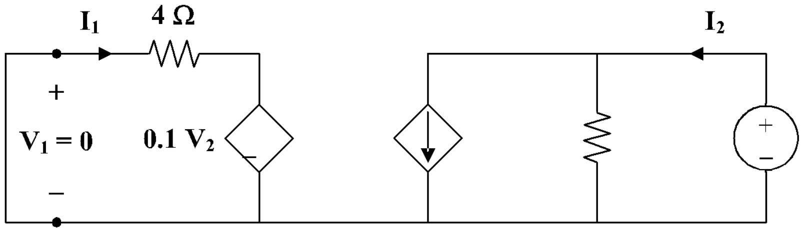


(a)

$$V_1 = 4I_1 \longrightarrow y_{11} = \frac{I_1}{V_1} = \frac{I_1}{4I_1} = 0.25$$

$$I_2 = 20I_1 = 5V_1 \longrightarrow y_{21} = \frac{I_2}{V_1} = 5$$

Consider the circuit in Fig. (b).



(b)

$$4\mathbf{I}_1 = 0.1\mathbf{V}_2 \longrightarrow \mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{0.1}{4} = 0.025$$

$$\mathbf{I}_2 = 20\mathbf{I}_1 + \frac{\mathbf{V}_2}{10} = 0.5\mathbf{V}_2 + 0.1\mathbf{V}_2 = 0.6\mathbf{V}_2 \longrightarrow \mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = 0.6$$

Thus,

$$[\mathbf{y}] = \begin{bmatrix} \mathbf{0.25} & \mathbf{0.025} \\ \mathbf{5} & \mathbf{0.6} \end{bmatrix}\mathbf{S}$$

Alternatively, from the given circuit,

$$\mathbf{V}_1 = 4\mathbf{I}_1 - 0.1\mathbf{V}_2$$
$$\mathbf{I}_2 = 20\mathbf{I}_1 + 0.1\mathbf{V}_2$$

Comparing these with the equations for the h parameters show that

$$\mathbf{h}_{11} = 4, \quad \mathbf{h}_{12} = -0.1, \quad \mathbf{h}_{21} = 20, \quad \mathbf{h}_{22} = 0.1$$

Using Table 18.1,

$$\mathbf{y}_{11} = \frac{1}{\mathbf{h}_{11}} = \frac{1}{4} = 0.25, \qquad \mathbf{y}_{12} = \frac{-\mathbf{h}_{12}}{\mathbf{h}_{11}} = \frac{0.1}{4} = 0.025$$

$$\mathbf{y}_{21} = \frac{\mathbf{h}_{21}}{\mathbf{h}_{11}} = \frac{20}{4} = 5, \qquad \mathbf{y}_{22} = \frac{\Delta_h}{\mathbf{h}_{11}} = \frac{0.4+2}{4} = 0.6$$

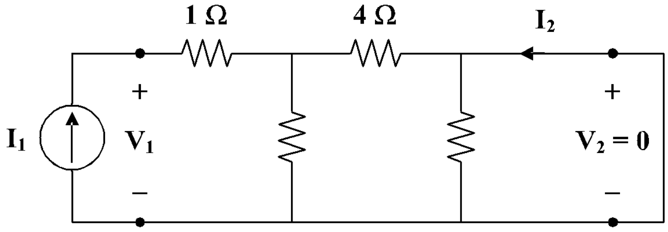
as above.

Chapter 19, Problem 28.

In the circuit of Fig. 19.65, the input port is connected to a 1-A dc current source. Calculate the power dissipated by the $2\text{-}\Omega$ resistor by using the *y* parameters. Confirm your result by direct circuit analysis.

Chapter 19, Solution 28.

We obtain $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$ by considering the circuit in Fig.(a).



(a)

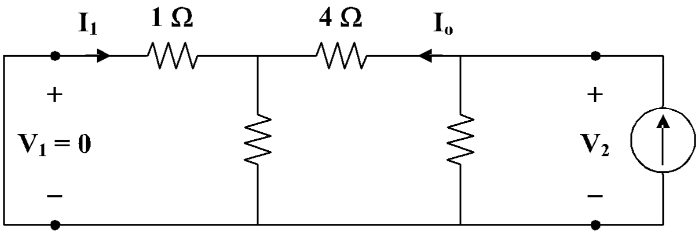
$$\mathbf{Z}_{in} = 1 + 6 \,||\, 4 = 3.4$$

$$\mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{1}{\mathbf{Z}_{in}} = 0.2941$$

$$\mathbf{I}_2 = \frac{-6}{10}\mathbf{I}_1 = \left(\frac{-6}{10}\right)\left(\frac{\mathbf{V}_1}{3.4}\right) = \frac{-6}{34}\mathbf{V}_1$$

$$\mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \frac{-6}{34} = -0.1765$$

To get $\mathbf{y}_{22}$ and $\mathbf{y}_{12}$, consider the circuit in Fig. (b).



(b)

$$\frac{1}{\mathbf{y}_{22}} = 2 \,||\, (4 + 6 \,||\, 1) = 2 \,||\, \left(4 + \frac{6}{7}\right) = \frac{(2)(34/7)}{2 + (34/7)} = \frac{34}{24} = \frac{V_2}{I_2}$$

$$\mathbf{y}_{22} = \frac{24}{34} = 0.7059$$

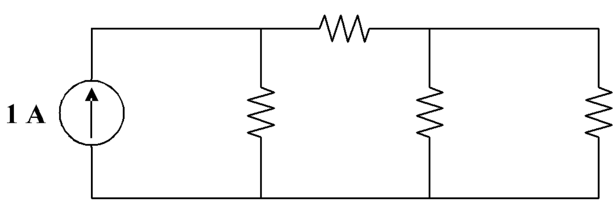
$$\mathbf{I}_1 = \frac{-6}{7}\mathbf{I}_o \qquad \mathbf{I}_o = \frac{2}{2 + (34/7)}\mathbf{I}_2 = \frac{14}{48}\mathbf{I}_2 = \frac{7}{34}\mathbf{V}_2$$

$$\mathbf{I}_1 = \frac{-6}{34}\mathbf{V}_2 \longrightarrow \mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{-6}{34} = -0.1765$$

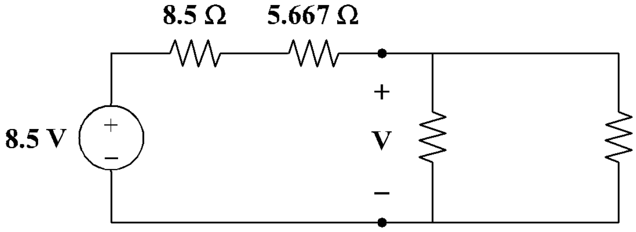
Thus,

$$[\mathbf{y}] = \begin{bmatrix} 0.2941 & -0.1765 \\ -0.1765 & 0.7059 \end{bmatrix} \text{S}$$

The equivalent circuit is shown in Fig. (c). After transforming the current source to a voltage source, we have the circuit in Fig. (d).



(c)



(d)

$$V = \frac{(2 \,||\, 1.889)(8.5)}{2 \,||\, 1.889 + 8.5 + 5.667} = \frac{(0.9714)(8.5)}{0.9714 + 14.167} = 0.5454$$

$$P = \frac{V^2}{R} = \frac{(0.5454)^2}{2} = \underline{\mathbf{0.1487\ W}}$$

Chapter 19, Problem 29.

In the bridge circuit of Fig. 19.87, $I_1 = 10$ A and $I_2 = -4$ A

(a) Find V_1 and V_2 using *y* parameters.
(b) -Confirm the results in part (a) by direct circuit analysis.

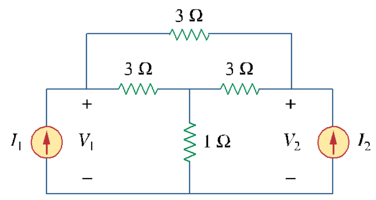
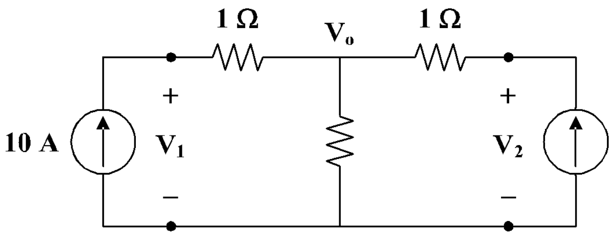


Figure 19.87
For Prob. 19.29.

Chapter 19, Solution 29.

(a) Transforming the Δ subnetwork to Y gives the circuit in Fig. (a).



(a)

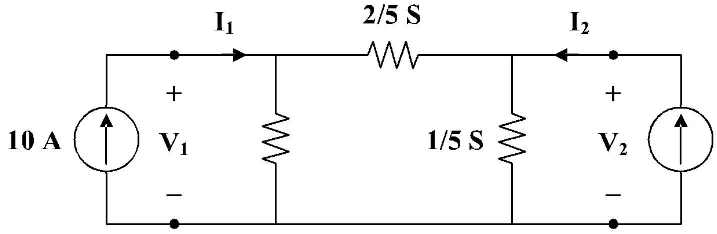
It is easy to get the z parameters

$$\mathbf{z}_{12} = \mathbf{z}_{21} = 2, \quad \mathbf{z}_{11} = 1 + 2 = 3, \quad \mathbf{z}_{22} = 3$$

$$\Delta_z = \mathbf{z}_{11}\mathbf{z}_{22} - \mathbf{z}_{12}\mathbf{z}_{21} = 9 - 4 = 5$$

$$\mathbf{y}_{11} = \frac{\mathbf{z}_{22}}{\Delta_z} = \frac{3}{5} = \mathbf{y}_{22}, \qquad \mathbf{y}_{12} = \mathbf{y}_{21} = \frac{-\mathbf{z}_{12}}{\Delta_z} = \frac{-2}{5}$$

Thus, the equivalent circuit is as shown in Fig. (b).



(b)

$$\mathbf{I}_1 = 10 = \frac{3}{5}\mathbf{V}_1 - \frac{2}{5}\mathbf{V}_2 \longrightarrow 50 = 3\mathbf{V}_1 - 2\mathbf{V}_2 \qquad (1)$$

$$\mathbf{I}_2 = -4 = \frac{-2}{5}\mathbf{V}_1 + \frac{3}{5}\mathbf{V}_2 \longrightarrow -20 = -2\mathbf{V}_1 + 3\mathbf{V}_2$$

$$10 = \mathbf{V}_1 - 1.5\mathbf{V}_2 \longrightarrow \mathbf{V}_1 = 10 + 1.5\mathbf{V}_2 \qquad (2)$$

Substituting (2) into (1),

$$50 = 30 + 4.5\mathbf{V}_2 - 2\mathbf{V}_2 \longrightarrow \mathbf{V}_2 = \underline{\mathbf{8\ V}}$$

$$\mathbf{V}_1 = 10 + 1.5\mathbf{V}_2 = \underline{\mathbf{22\ V}}$$

(b) For direct circuit analysis, consider the circuit in Fig. (a).

For the main non-reference node,

$$10 - 4 = \frac{\mathbf{V}_o}{2} \longrightarrow \mathbf{V}_o = 12$$

$$10 = \frac{\mathbf{V}_1 - \mathbf{V}_o}{1} \longrightarrow \mathbf{V}_1 = 10 + \mathbf{V}_o = \underline{\mathbf{22\ V}}$$

$$-4 = \frac{\mathbf{V}_2 - \mathbf{V}_o}{1} \longrightarrow \mathbf{V}_2 = \mathbf{V}_o - 4 = \underline{\mathbf{8\ V}}$$

Chapter 19, Problem 30.

Find the *h* parameters for the networks in Fig. 19.88.

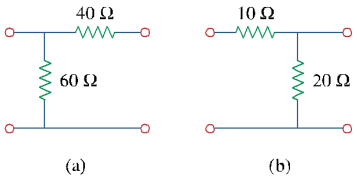


Figure 19.88

For Prob. 19.30.

Chapter 19, Solution 30.

(a) Convert to z parameters; then, convert to h parameters using Table 18.1.

$$\mathbf{z}_{11} = \mathbf{z}_{12} = \mathbf{z}_{21} = 60\,\Omega, \qquad \mathbf{z}_{22} = 100\,\Omega$$

$$\Delta_z = \mathbf{z}_{11}\mathbf{z}_{22} - \mathbf{z}_{12}\mathbf{z}_{21} = 6000 - 3600 = 2400$$

$$\mathbf{h}_{11} = \frac{\Delta_z}{\mathbf{z}_{22}} = \frac{2400}{100} = 24, \qquad \mathbf{h}_{12} = \frac{\mathbf{z}_{12}}{\mathbf{z}_{22}} = \frac{60}{100} = 0.6$$

$$\mathbf{h}_{21} = \frac{-\mathbf{z}_{21}}{\mathbf{z}_{22}} = -0.6, \qquad \mathbf{h}_{22} = \frac{1}{\mathbf{z}_{22}} = 0.01$$

Thus,

$$[\mathbf{h}] = \underline{\begin{bmatrix} \mathbf{24\,\Omega} & \mathbf{0.6} \\ \mathbf{-0.6} & \mathbf{0.01\,S} \end{bmatrix}}$$

(b) Similarly,

$$\mathbf{z}_{11} = 30\,\Omega \qquad \mathbf{z}_{12} = \mathbf{z}_{21} = \mathbf{z}_{22} = 20\,\Omega$$

$$\Delta_z = 600 - 400 = 200$$

$$\mathbf{h}_{11} = \frac{200}{20} = 10 \qquad \mathbf{h}_{12} = \frac{20}{20} = 1$$

$$\mathbf{h}_{21} = -1 \qquad \mathbf{h}_{22} = \frac{1}{20} = 0.05$$

Thus,

$$[\mathbf{h}] = \underline{\begin{bmatrix} \mathbf{10\,\Omega} & \mathbf{1} \\ \mathbf{-1} & \mathbf{0.05\,S} \end{bmatrix}}$$

Chapter 19, Problem 31.

Determine the hybrid parameters for the network in Fig. 19.89.

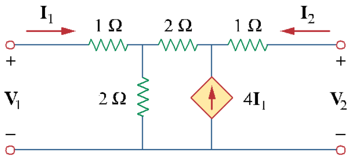
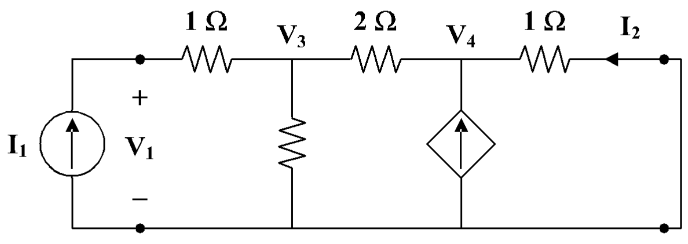


Figure 19.89
For Prob. 19.31.

Chapter 19, Solution 31.

We get $\mathbf{h}_{11}$ and $\mathbf{h}_{21}$ by considering the circuit in Fig. (a).



At node 1,

$$\mathbf{I}_1 = \frac{\mathbf{V}_3}{2} + \frac{\mathbf{V}_3 - \mathbf{V}_4}{2} \longrightarrow 2\mathbf{I}_1 = 2\mathbf{V}_3 - \mathbf{V}_4 \qquad (1)$$

At node 2,

$$\frac{\mathbf{V}_3 - \mathbf{V}_4}{2} + 4\mathbf{I}_1 = \frac{\mathbf{V}_4}{1}$$

$$8\mathbf{I}_1 = -\mathbf{V}_3 + 3\mathbf{V}_4 \longrightarrow 16\mathbf{I}_1 = -2\mathbf{V}_3 + 6\mathbf{V}_4 \qquad (2)$$

Adding (1) and (2),

$$18\mathbf{I}_1 = 5\mathbf{V}_4 \longrightarrow \mathbf{V}_4 = 3.6\mathbf{I}_1$$

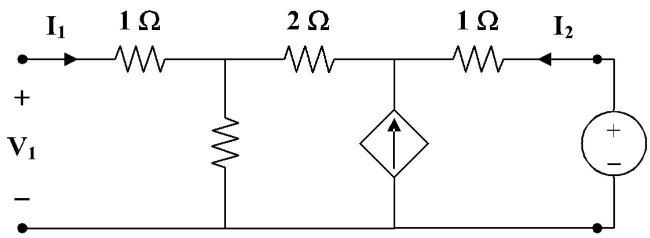
$$\mathbf{V}_3 = 3\mathbf{V}_4 - 8\mathbf{I}_1 = 2.8\mathbf{I}_1$$

$$\mathbf{V}_1 = \mathbf{V}_3 + \mathbf{I}_1 = 3.8\mathbf{I}_1$$

$$\mathbf{h}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = 3.8\ \Omega$$

$$\mathbf{I}_2 = \frac{-\mathbf{V}_4}{1} = -3.6\mathbf{I}_1 \longrightarrow \mathbf{h}_{21} = \frac{\mathbf{I}_2}{\mathbf{I}_1} = -3.6$$

To get $\mathbf{h}_{22}$ and $\mathbf{h}_{12}$, refer to the circuit in Fig. (b). The dependent current source can be replaced by an open circuit since $4\mathbf{I}_1 = 0$.



(b)

$$\mathbf{V}_1 = \frac{2}{2+2+1}\mathbf{V}_2 = \frac{2}{5}\mathbf{V}_2 \longrightarrow \mathbf{h}_{12} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = 0.4$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_2}{2+2+1} = \frac{\mathbf{V}_2}{5} \longrightarrow \mathbf{h}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{1}{5} = 0.2\ \text{S}$$

Thus,

$$[\mathbf{h}] = \begin{bmatrix} \mathbf{38\ \Omega} & \mathbf{0.4} \\ \mathbf{-3.6} & \mathbf{0.2\ S} \end{bmatrix}$$

Chapter 19, Problem 32.

Find the h and g parameters of the two-port network in Fig. 19.90 as functions of s.

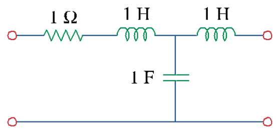
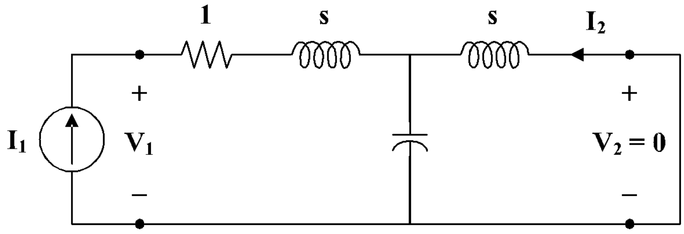


Figure 19.90

For Prob. 19.32.

Chapter 19, Solution 32.

(a) We obtain $\mathbf{h}_{11}$ and $\mathbf{h}_{21}$ by referring to the circuit in Fig. (a).



(a)

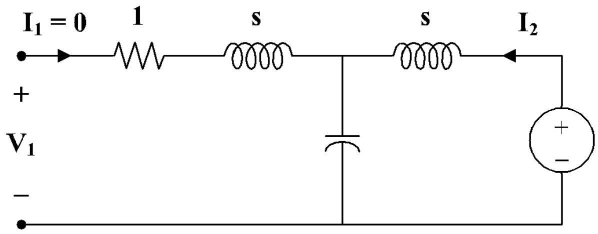
$$\mathbf{V}_1 = \left(1 + s + s \parallel \frac{1}{s}\right)\mathbf{I}_1 = \left(1 + s + \frac{s}{s^2+1}\right)\mathbf{I}_1$$

$$\mathbf{h}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = s + 1 + \frac{s}{s^2+1}$$

By current division,

$$\mathbf{I}_2 = \frac{-1/s}{s + 1/s}\mathbf{I}_1 = \frac{-\mathbf{I}_1}{s+1} \longrightarrow \mathbf{h}_{21} = \frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{-1}{s^2+1}$$

To get $\mathbf{h}_{22}$ and $\mathbf{h}_{12}$, refer to Fig. (b).



(b)

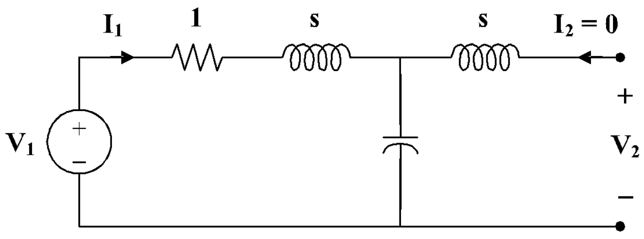
$$\mathbf{V}_1 = \frac{1/s}{s + 1/s}\mathbf{V}_2 = \frac{\mathbf{V}_2}{s^2+1} \longrightarrow \mathbf{h}_{12} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{1}{s^2+1}$$

$$\mathbf{V}_2 = \left(s + \frac{1}{s}\right)\mathbf{I}_2 \longrightarrow \mathbf{h}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{1}{s + 1/s} = \frac{s}{s^2+1}$$

Thus,

$$[\mathbf{h}] = \begin{bmatrix} \mathbf{s} + \mathbf{1} + \dfrac{\mathbf{s}}{\mathbf{s}^2+\mathbf{1}} & \dfrac{\mathbf{1}}{\mathbf{s}^2+\mathbf{1}} \\ \dfrac{\mathbf{-1}}{\mathbf{s}^2+\mathbf{1}} & \dfrac{\mathbf{s}}{\mathbf{s}^2+\mathbf{1}} \end{bmatrix}$$

(b) To get $\mathbf{g}_{11}$ and $\mathbf{g}_{21}$, refer to Fig. (c).

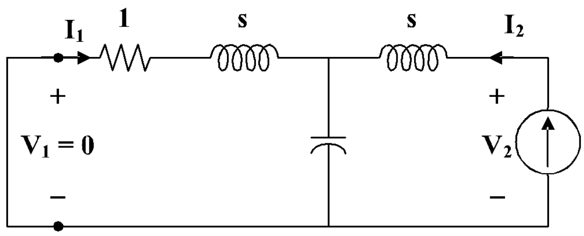


(c)

$$\mathbf{V}_1 = \left(1 + s + \frac{1}{s}\right)\mathbf{I}_1 \longrightarrow \mathbf{g}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{1}{1 + s + 1/s} = \frac{s}{s^2 + s + 1}$$

$$\mathbf{V}_2 = \frac{1/s}{1 + s + 1/s}\mathbf{V}_1 = \frac{\mathbf{V}_1}{s^2 + s + 1} \longrightarrow \mathbf{g}_{21} = \frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{1}{s^2 + s + 1}$$

To get $\mathbf{g}_{22}$ and $\mathbf{g}_{12}$, refer to Fig. (d).



(d)

$$\mathbf{V}_2 = \left(s + \frac{1}{s} \,||\, (s+1)\right)\mathbf{I}_2 = \left(s + \frac{(s+1)/s}{1 + s + 1/s}\right)\mathbf{I}_2$$

$$\mathbf{g}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = s + \frac{s+1}{s^2 + s + 1}$$

$$\mathbf{I}_1 = \frac{-1/s}{1 + s + 1/s}\mathbf{I}_2 = \frac{-\mathbf{I}_2}{s^2 + s + 1} \longrightarrow \mathbf{g}_{12} = \frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{-1}{s^2 + s + 1}$$

Thus,

$$[\mathbf{g}] = \begin{bmatrix} \dfrac{\mathbf{s}}{\mathbf{s}^2 + \mathbf{s} + \mathbf{1}} & \dfrac{\mathbf{-1}}{\mathbf{s}^2 + \mathbf{s} + \mathbf{1}} \\ \dfrac{\mathbf{1}}{\mathbf{s}^2 + \mathbf{s} + \mathbf{1}} & \mathbf{s} + \dfrac{\mathbf{s} + \mathbf{1}}{\mathbf{s}^2 + \mathbf{s} + \mathbf{1}} \end{bmatrix}$$

Chapter 19, Problem 33.

Obtain the *h* parameters for the two-port of Fig. 19.91.

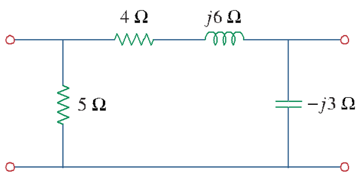
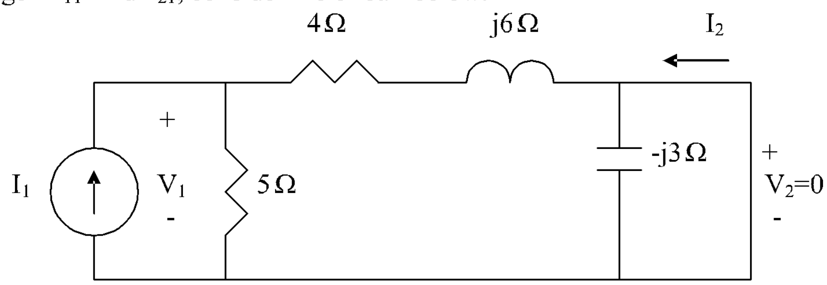


Figure 19.91

For Prob. 19.33.

Chapter 19, Solution 33.

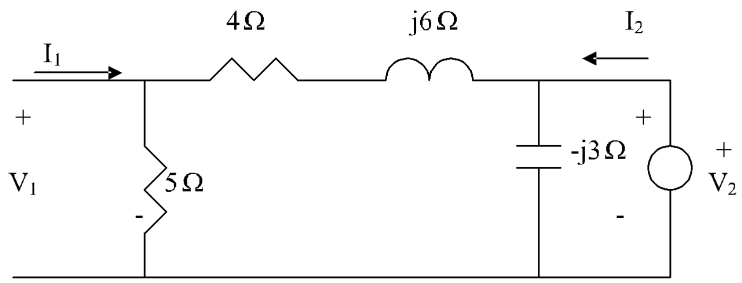
To get h_{11} and h_{21}, consider the circuit below.



$$V_1 = 5//(4+j6)I_1 = \frac{5(4+j6)I_1}{9+j6} \qquad h_{11} = \frac{V_1}{I_1} = 3.0769 + j1.2821$$

Also, $I_2 = -\frac{5}{9+j6}I_1 \longrightarrow h_{21} = \frac{I_2}{I_1} = -0.3846 + j0.2564$

To get h_{22} and h_{12}, consider the circuit below.



$$V_1 = \frac{5}{9+j6}V_2 \longrightarrow h_{12} = \frac{V_1}{V_2} = \frac{5}{9+j6} = 0.3846 - j0.2564$$

$$V_2 = -j3//(9+j6)I_2 \longrightarrow h_{22} = \frac{I_2}{V_2} = \frac{1}{-j3//(9+j6)} = \frac{9+j3}{-j3(9+j6)} = 0.0769 + j0.2821$$

Thus,

$$[h] = \begin{bmatrix} 3.077 + j1.2821 & 0.3846 - j0.2564 \\ -0.3846 + j0.2564 & 0.0769 + j0.2821 \end{bmatrix}$$

Chapter 19, Problem 34.

Obtain the *h* and *g* parameters of the two-port in Fig. 19.92.

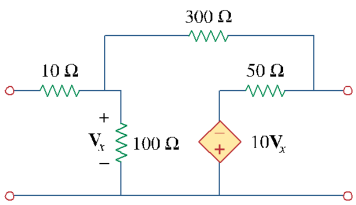
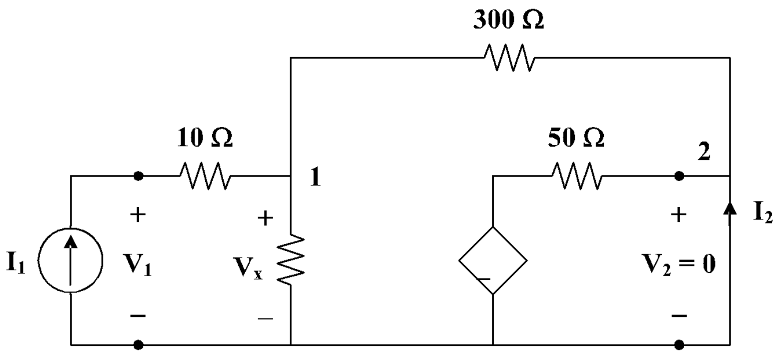


Figure 19.92

For Prob. 19.34.

Chapter 19, Solution 34.

Refer to Fig. (a) to get $\mathbf{h}_{11}$ and $\mathbf{h}_{21}$.



(a)

At node 1,

$$\mathbf{I}_1 = \frac{\mathbf{V}_x}{100} + \frac{\mathbf{V}_x - 0}{300} \longrightarrow 300\mathbf{I}_1 = 4\mathbf{V}_x \qquad (1)$$

$$\mathbf{V}_x = \frac{300}{4}\mathbf{I}_1 = 75\mathbf{I}_1$$

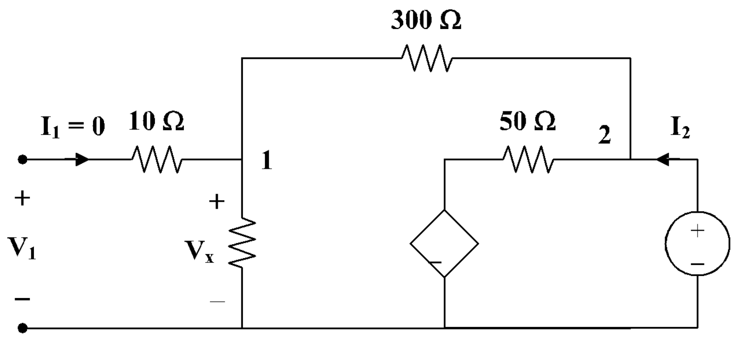
But $\mathbf{V}_1 = 10\mathbf{I}_1 + \mathbf{V}_x = 85\mathbf{I}_1 \longrightarrow \mathbf{h}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = 85\ \Omega$

At node 2,

$$\mathbf{I}_2 = \frac{0+10\mathbf{V}_x}{50} - \frac{\mathbf{V}_x}{300} = \frac{\mathbf{V}_x}{5} - \frac{\mathbf{V}_x}{300} = \frac{75}{5}\mathbf{I}_1 - \frac{75}{300}\mathbf{I}_1 = 14.75\mathbf{I}_1$$

$$\mathbf{h}_{21} = \frac{\mathbf{I}_2}{\mathbf{I}_1} = 14.75$$

To get $\mathbf{h}_{22}$ and $\mathbf{h}_{12}$, refer to Fig. (b).



(b)

At node 2,

$$\mathbf{I}_2 = \frac{\mathbf{V}_2}{400} + \frac{\mathbf{V}_2 + 10\mathbf{V}_x}{50} \longrightarrow 400\mathbf{I}_2 = 9\mathbf{V}_2 + 80\mathbf{V}_x$$

But $$\mathbf{V}_x = \frac{100}{400}\mathbf{V}_2 = \frac{\mathbf{V}_2}{4}$$

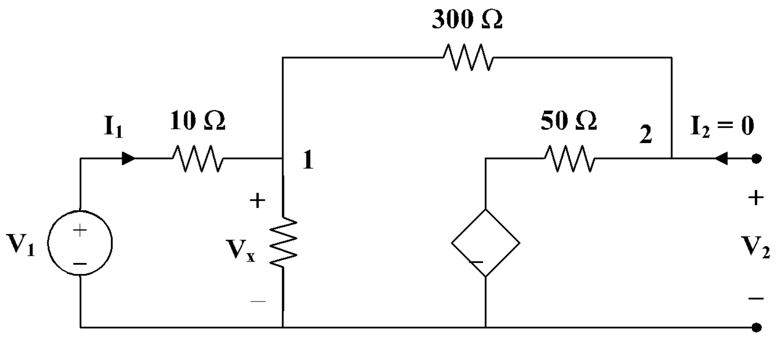
Hence, $$400\mathbf{I}_2 = 9\mathbf{V}_2 + 20\mathbf{V}_2 = 29\mathbf{V}_2$$

$$\mathbf{h}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{29}{400} = 0.0725 \text{ S}$$

$$\mathbf{V}_1 = \mathbf{V}_x = \frac{\mathbf{V}_2}{4} \longrightarrow \mathbf{h}_{12} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{1}{4} = 0.25$$

$$[\mathbf{h}] = \underline{\begin{bmatrix} \mathbf{85\,\Omega} & \mathbf{0.25} \\ \mathbf{14.75} & \mathbf{0.0725\ S} \end{bmatrix}}$$

To get $\mathbf{g}_{11}$ and $\mathbf{g}_{21}$, refer to Fig. (c).



(c)

At node 1,

$$\mathbf{I}_1 = \frac{\mathbf{V}_x}{100} + \frac{\mathbf{V}_x + 10\mathbf{V}_x}{350} \longrightarrow 350\mathbf{I}_1 = 14.5\mathbf{V}_x \qquad (2)$$

But $$\mathbf{I}_1 = \frac{\mathbf{V}_1 - \mathbf{V}_x}{10} \longrightarrow 10\mathbf{I}_1 = \mathbf{V}_1 - \mathbf{V}_x$$

or $$\mathbf{V}_x = \mathbf{V}_1 - 10\mathbf{I}_1 \qquad (3)$$

Substituting (3) into (2) gives

$$350\mathbf{I}_1 = 14.5\mathbf{V}_1 - 145\mathbf{I}_1 \longrightarrow 495\mathbf{I}_1 = 14.5\mathbf{V}_1$$

$$\mathbf{g}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{14.5}{495} = 0.02929 \text{ S}$$

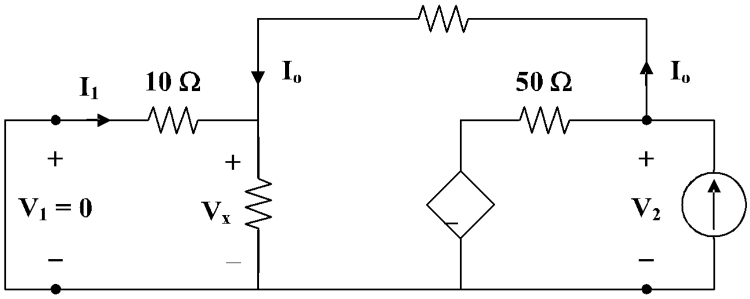
At node 2,

$$\mathbf{V}_2 = (50)\left(\frac{11}{350}\mathbf{V}_x\right) - 10\mathbf{V}_x = \text{-}8.4286\mathbf{V}_x$$

$$= \text{-}8.4286\mathbf{V}_1 + 84.286\mathbf{I}_1 = \text{-}8.4286\mathbf{V}_1 + (84.286)\left(\frac{14.5}{495}\right)\mathbf{V}_1$$

$$\mathbf{V}_2 = \text{-}5.96\mathbf{V}_1 \longrightarrow \mathbf{g}_{21} = \frac{\mathbf{V}_2}{\mathbf{V}_1} = \text{-}5.96$$

To get $\mathbf{g}_{22}$ and $\mathbf{g}_{12}$, refer to Fig. (d).



(d)

$$10 \,||\, 100 = 9.091$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_2 + 10\mathbf{V}_x}{50} + \frac{\mathbf{V}_2}{300 + 9.091}$$

$$309.091\mathbf{I}_2 = 7.1818\mathbf{V}_2 + 61.818\mathbf{V}_x \qquad (4)$$

But $$\mathbf{V}_x = \frac{9.091}{309.091}\mathbf{V}_2 = 0.02941\mathbf{V}_2 \qquad (5)$$

Substituting (5) into (4) gives

$$309.091\mathbf{I}_2 = 9\mathbf{V}_2$$

$$\mathbf{g}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = 34.34\,\Omega$$

$$\mathbf{I}_o = \frac{\mathbf{V}_2}{309.091} = \frac{34.34\mathbf{I}_2}{309.091}$$

$$\mathbf{I}_1 = \frac{-100}{110}\mathbf{I}_o = \frac{-34.34\mathbf{I}_2}{(1.1)(309.091)}$$

$$\mathbf{g}_{12} = \frac{\mathbf{I}_1}{\mathbf{I}_2} = -0.101$$

Thus,

$$[\mathbf{g}] = \begin{bmatrix} \mathbf{0.02929\ S} & \mathbf{-0.101} \\ \mathbf{-5.96} & \mathbf{34.34\,\Omega} \end{bmatrix}$$

Chapter 19, Problem 35.

Determine the h parameters for the network in Fig. 19.93.

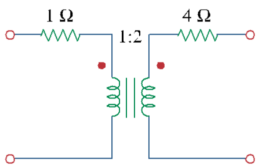
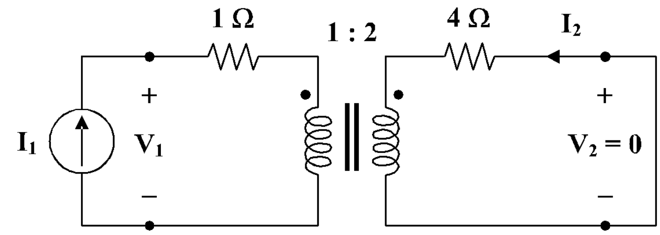


Figure 19.93

For Prob. 19.35.

Chapter 19, Solution 35.

To get $\mathbf{h}_{11}$ and $\mathbf{h}_{21}$ consider the circuit in Fig. (a).



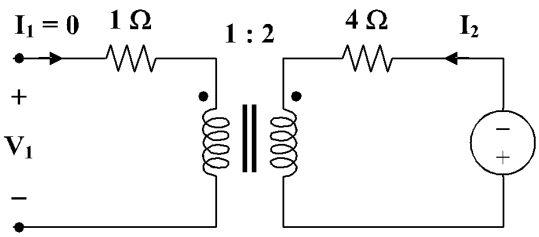
(a)

$$Z_R = \frac{4}{n^2} = \frac{4}{4} = 1$$

$$\mathbf{V}_1 = (1+1)\mathbf{I}_1 = 2\mathbf{I}_1 \longrightarrow \mathbf{h}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = 2\,\Omega$$

$$\frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{-N_2}{N_1} = -2 \longrightarrow \mathbf{h}_{21} = \frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{-1}{2} = -0.5$$

To get $\mathbf{h}_{22}$ and $\mathbf{h}_{12}$, refer to Fig. (b).



(b)

Since $\mathbf{I}_1 = 0$, $\mathbf{I}_2 = 0$.

Hence, $\mathbf{h}_{22} = 0$.

At the terminals of the transformer, we have $\mathbf{V}_1$ and $\mathbf{V}_2$ which are related as

$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{N_2}{N_1} = n = 2 \longrightarrow \mathbf{h}_{12} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{1}{2} = 0.5$$

Thus,

$$[\mathbf{h}] = \begin{bmatrix} \mathbf{2\,\Omega} & \mathbf{0.5} \\ \mathbf{-0.5} & \mathbf{0} \end{bmatrix}$$

Chapter 19, Problem 36.

For the two-port in Fig. 19.94,

$$[\mathbf{h}]\begin{bmatrix} 16\Omega & 3 \\ -2 & 0.01\text{S} \end{bmatrix}$$

Find:

(a) V_2/V_1 (b) I_2/I_1

(c) I_1/V_1 (d) V_2/I_1

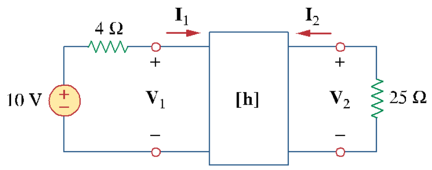
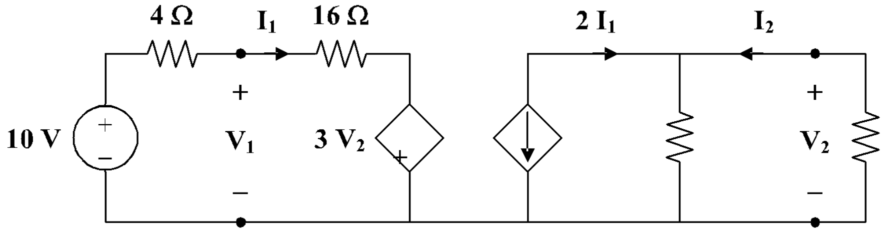


Figure 19.94

For Prob. 19.36.

Chapter 19, Solution 36.

We replace the two-port by its equivalent circuit as shown below.



$$100 \,||\, 25 = 20\ \Omega$$

$$\mathbf{V}_2 = (20)(2\mathbf{I}_1) = 40\mathbf{I}_1 \qquad (1)$$

$$-10 + 20\mathbf{I}_1 + 3\mathbf{V}_2 = 0$$
$$10 = 20\mathbf{I}_1 + (3)(40\mathbf{I}_1) = 140\mathbf{I}_1$$

$$\mathbf{I}_1 = \frac{1}{14}, \qquad \mathbf{V}_2 = \frac{40}{14}$$

$$\mathbf{V}_1 = 16\mathbf{I}_1 + 3\mathbf{V}_2 = \frac{136}{14}$$

$$\mathbf{I}_2 = \left(\frac{100}{125}\right)(2\mathbf{I}_1) = \frac{-8}{70}$$

(a) $$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{40}{136} = \underline{\mathbf{0.2941}}$$

(b) $$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \underline{\mathbf{-1.6}}$$

(c) $$\frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{1}{136} = \underline{\mathbf{7.353 \times 10^{-3}\ S}}$$

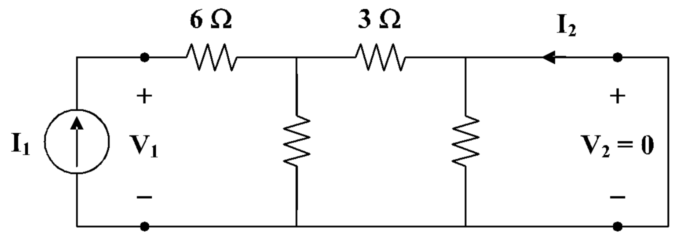
(d) $$\frac{\mathbf{V}_2}{\mathbf{I}_1} = \frac{40}{1} = \underline{\mathbf{40\ \Omega}}$$

Chapter 19, Problem 37.

The input port of the circuit in Fig. 19.79 is connected to a 10-V dc voltage source while the output port is terminated by a $5\text{-}\Omega$ resistor. Find the voltage across the $5\text{-}\Omega$ resistor by using *h* parameters of the circuit. Confirm your result by using direct circuit analysis.

Chapter 19, Solution 37.

(a) We first obtain the h parameters. To get $\mathbf{h}_{11}$ and $\mathbf{h}_{21}$ refer to Fig. (a).



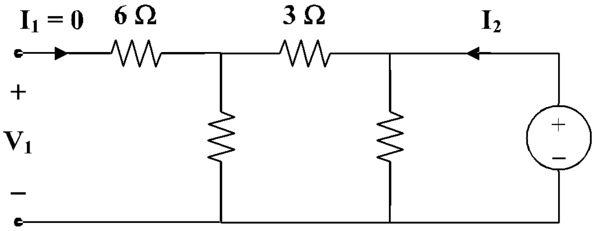
(a)

$3 \parallel 6 = 2$

$$\mathbf{V}_1 = (6+2)\mathbf{I}_1 = 8\mathbf{I}_1 \longrightarrow \mathbf{h}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = 8\,\Omega$$

$$\mathbf{I}_2 = \frac{-6}{3+6}\mathbf{I}_1 = \frac{-2}{3}\mathbf{I}_1 \longrightarrow \mathbf{h}_{21} = \frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{-2}{3}$$

To get $\mathbf{h}_{22}$ and $\mathbf{h}_{12}$, refer to the circuit in Fig. (b).



(b)

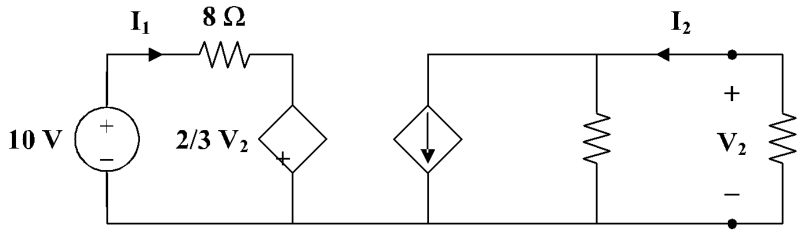
$$3 \parallel 9 = \frac{9}{4}$$

$$\mathbf{V}_2 = \frac{9}{4}\mathbf{I}_2 \longrightarrow \mathbf{h}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{4}{9}$$

$$\mathbf{V}_1 = \frac{6}{6+3}\mathbf{V}_2 = \frac{2}{3}\mathbf{V}_2 \longrightarrow \mathbf{h}_{12} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{2}{3}$$

$$[\mathbf{h}] = \begin{bmatrix} 8\,\Omega & \frac{2}{3} \\ \frac{-2}{3} & \frac{4}{9}\,\text{S} \end{bmatrix}$$

The equivalent circuit of the given circuit is shown in Fig. (c).



(c)

$$8\mathbf{I}_1 + \frac{2}{3}\mathbf{V}_2 = 10 \qquad (1)$$

$$\mathbf{V}_2 = \frac{2}{3}\mathbf{I}_1\left(5 \parallel \frac{9}{4}\right) = \frac{2}{3}\mathbf{I}_1\left(\frac{45}{29}\right) = \frac{30}{29}\mathbf{I}_1$$

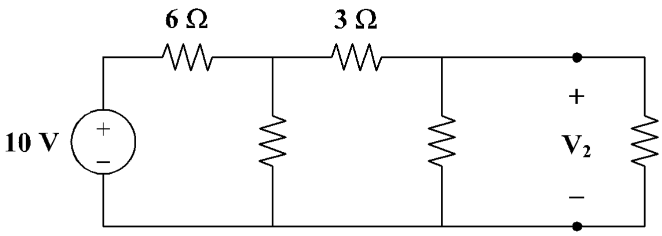
$$\mathbf{I}_1 = \frac{29}{30}\mathbf{V}_2 \qquad (2)$$

Substituting (2) into (1),

$$(8)\left(\frac{29}{30}\right)\mathbf{V}_2 + \frac{2}{3}\mathbf{V}_2 = 10$$

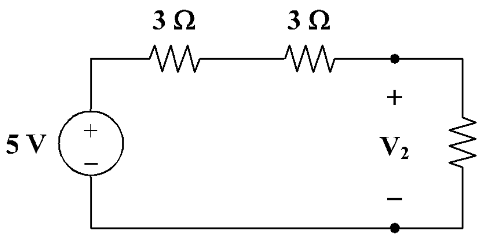
$$\mathbf{V}_2 = \frac{300}{252} = \underline{\mathbf{1.19\ V}}$$

(b) By direct analysis, refer to Fig.(d).



(d)

Transform the 10-V voltage source to a $\frac{10}{6}$-A current source. Since $6 \| 6 = 3\ \Omega$, we combine the two 6-Ω resistors in parallel and transform the current source back to $\frac{10}{6} \times 3 = 5\ \text{V}$ voltage source shown in Fig. (e).



(e)

$$3 \| 5 = \frac{(3)(5)}{8} = \frac{15}{8}$$

$$\mathbf{V}_2 = \frac{15/8}{6 + 15/8}(5) = \frac{75}{63} = \underline{\mathbf{1.1905\ V}}$$

Chapter 19, Problem 38.

The h parameters of the two-port of Fig. 19.95 are:

$$[\mathbf{h}] = \begin{bmatrix} 600\Omega & 0.04 \\ 30 & 2\,\text{mS} \end{bmatrix}$$

Given the $Z_s = 2\,\text{k}\Omega$ and $Z_L = 400\Omega$, find Z_{in} and Z_{out}.

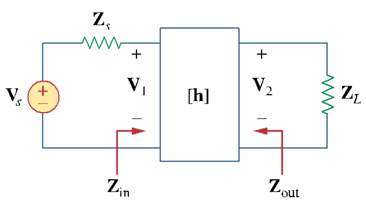


Figure 19.95

For Prob. 19.38.

Chapter 19, Solution 38.

From eq. (19.75),

$$Z_{in} = h_{ie} - \frac{h_{re}h_{fe}R_L}{1+h_{oe}R_L} = h_{11} - \frac{h_{12}h_{21}R_L}{1+h_{22}R_L} = 600 - \frac{0.04x30x400}{1+2x10^{-3}x400} = \underline{333.33\ \Omega}$$

From eq. (19.79),

$$Z_{out} = \frac{R_s + h_{ie}}{(R_s + h_{ie})h_{0e} - h_{re}h_{fe}} = \frac{R_s + h_{11}}{(R_s + h_{11})h_{22} - h_{21}h_{12}} = \frac{2,000+600}{2600x2x10^{-3} - 30x0.04} = \underline{650\ \Omega}$$

Chapter 19, Problem 39.

Obtain the *g* parameters for the wye circuit of Fig. 19.96.

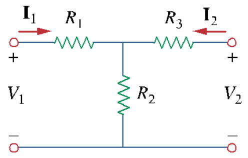
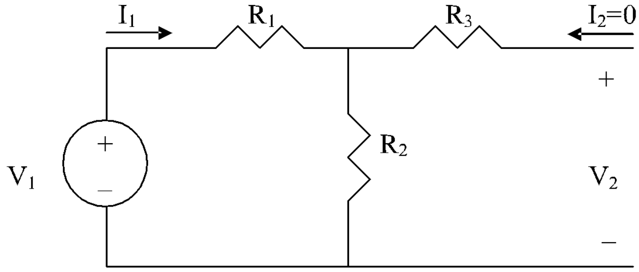


Figure 19.96

For Prob. 19.39.

Chapter 19, Solution 39.

We obtain g_{11} and g_{21} using the circuit below.

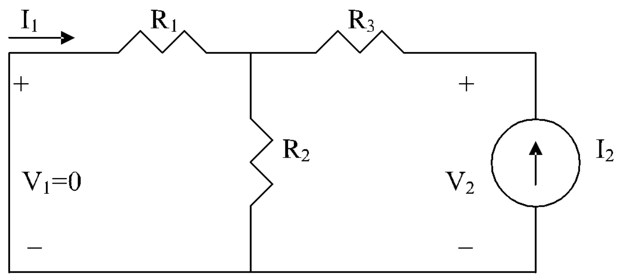


$$I_1 = \frac{V_1}{R_1 + R_2} \longrightarrow g_{11} = \frac{I_1}{V_1} = \underline{\frac{1}{R_1 + R_2}}$$

By voltage division,

$$V_2 = \frac{R_2}{R_1 + R_2} V_1 \longrightarrow g_{21} = \frac{V_2}{V_1} = \underline{\frac{R_2}{R_1 + R_2}}$$

We obtain g_{12} and g_{22} using the circuit below.



By current division,

$$I_1 = -\frac{R_2}{R_1 + R_2} I_2 \longrightarrow g_{12} = \frac{I_1}{I_2} = \underline{-\frac{R_2}{R_1 + R_2}}$$

Also,

$$V_2 = I_2(R_3 + R_1 // R_2) = I_2\left(R_3 + \frac{R_1 R_2}{R_1 + R_2}\right) \quad g_{22} = \frac{V_2}{I_2} = \underline{R_3 + \frac{R_1 R_2}{R_1 + R_2}}$$

$$g_{11} = \frac{1}{R_1 + R_2}, g_{12} = -\frac{R_2}{R_1 + R_2}$$

$$\underline{g_{21} = \frac{R_2}{R_1 + R_2}, g_{22} = R_3 + \frac{R_1 R_2}{R_1 + R_2}}$$

Chapter 19, Problem 40.

Find the *g* parameters for the circuit in Fig. 19.97.

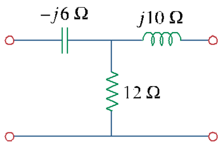
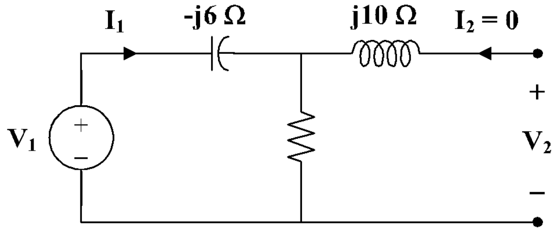


Figure 19.97

For Prob. 19.40.

Chapter 19, Solution 40.

To get $\mathbf{g}_{11}$ and $\mathbf{g}_{21}$, consider the circuit in Fig. (a).

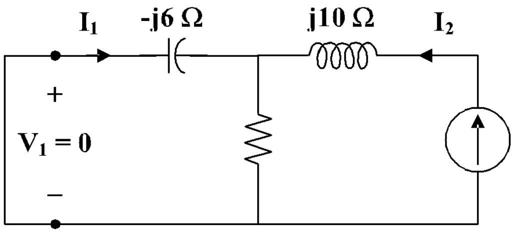


(a)

$$\mathbf{V}_1 = (12 - j6)\mathbf{I}_1 \longrightarrow \mathbf{g}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{1}{12 - j6} = 0.0667 + j0.0333 \text{ S}$$

$$\mathbf{g}_{21} = \frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{12\mathbf{I}_1}{(12 - j6)\mathbf{I}_1} = \frac{2}{2 - j} = 0.8 + j0.4$$

To get $\mathbf{g}_{12}$ and $\mathbf{g}_{22}$, consider the circuit in Fig. (b).



(b)

$$\mathbf{I}_1 = \frac{-12}{12 - j6}\mathbf{I}_2 \longrightarrow \mathbf{g}_{12} = \frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{-12}{12 - j6} = -\mathbf{g}_{21} = -0.8 - j0.4$$

$$\mathbf{V}_2 = (j10 + 12 \,||\, -j6)\mathbf{I}_2$$

$$\mathbf{g}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = j10 + \frac{(12)(-j6)}{12 - j6} = 2.4 + j5.2\,\Omega$$

$$[\mathbf{g}] = \underline{\begin{bmatrix} \mathbf{0.0667 + j0.0333\ S} & \mathbf{-0.8 - j0.4} \\ \mathbf{0.8 + j0.4} & \mathbf{2.4 + j5.2\,\Omega} \end{bmatrix}}$$

Chapter 19, Problem 41.

For the two-port in Fig. 19.75, show that

$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{-\mathbf{g}_{21}}{\mathbf{g}_{11}\mathbf{Z}_L + \Delta_g}$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_s} = \frac{\mathbf{g}_{21}\mathbf{Z}_L}{(1 + \mathbf{g}_{11}\mathbf{Z}_s)(\mathbf{g}_{22} + \mathbf{Z}_L) - \mathbf{g}_{21}\mathbf{g}_{12}\mathbf{Z}_s}$$

where Δ_g is the determinant of [**g**] matrix.

Chapter 19, Solution 41.

For the g parameters

$$\mathbf{I}_1 = \mathbf{g}_{11}\mathbf{V}_1 + \mathbf{g}_{12}\mathbf{I}_2 \qquad (1)$$

$$\mathbf{V}_2 = \mathbf{g}_{21}\mathbf{V}_1 + \mathbf{g}_{22}\mathbf{I}_2 \qquad (2)$$

But $\mathbf{V}_1 = \mathbf{V}_s - \mathbf{I}_1\mathbf{Z}_s$ and

$$\mathbf{V}_2 = -\mathbf{I}_2\mathbf{Z}_L = \mathbf{g}_{21}\mathbf{V}_1 + \mathbf{g}_{22}\mathbf{I}_2$$

$$0 = \mathbf{g}_{21}\mathbf{V}_1 + (\mathbf{g}_{22} + \mathbf{Z}_L)\mathbf{I}_2$$

or

$$\mathbf{V}_1 = \frac{-(\mathbf{g}_{22} + \mathbf{Z}_L)}{\mathbf{g}_{21}}\mathbf{I}_2$$

Substituting this into (1),

$$\mathbf{I}_1 = \frac{(\mathbf{g}_{22}\mathbf{g}_{11} + \mathbf{Z}_L\mathbf{g}_{11} - \mathbf{g}_{21}\mathbf{g}_{12})}{-\mathbf{g}_{21}}\mathbf{I}_2$$

or

$$\underline{\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{-\mathbf{g}_{21}}{\mathbf{g}_{11}\mathbf{Z}_L + \Delta_g}}$$

Also,

$$\mathbf{V}_2 = \mathbf{g}_{21}(\mathbf{V}_s - \mathbf{I}_1\mathbf{Z}_s) + \mathbf{g}_{22}\mathbf{I}_2$$

$$= \mathbf{g}_{21}\mathbf{V}_s - \mathbf{g}_{21}\mathbf{Z}_s\mathbf{I}_1 + \mathbf{g}_{22}\mathbf{I}_2$$

$$= \mathbf{g}_{21}\mathbf{V}_s + \mathbf{Z}_s(\mathbf{g}_{11}\mathbf{Z}_L + \Delta_g)\mathbf{I}_2 + \mathbf{g}_{22}\mathbf{I}_2$$

But

$$\mathbf{I}_2 = \frac{-\mathbf{V}_2}{\mathbf{Z}_L}$$

$$\mathbf{V}_2 = \mathbf{g}_{21}\mathbf{V}_s - [\mathbf{g}_{11}\mathbf{Z}_s\mathbf{Z}_L + \Delta_g\mathbf{Z}_s + \mathbf{g}_{22}]\left[\frac{\mathbf{V}_2}{\mathbf{Z}_L}\right]$$

$$\frac{\mathbf{V}_2[\mathbf{Z}_L + \mathbf{g}_{11}\mathbf{Z}_s\mathbf{Z}_L + \Delta_g\mathbf{Z}_s + \mathbf{g}_{22}]}{\mathbf{Z}_L} = \mathbf{g}_{21}\mathbf{V}_s$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_s} = \frac{\mathbf{g}_{21}\mathbf{Z}_L}{\mathbf{Z}_L + \mathbf{g}_{11}\mathbf{Z}_s\mathbf{Z}_L + \Delta_g\mathbf{Z}_s + \mathbf{g}_{22}}$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_s} = \frac{\mathbf{g}_{21}\mathbf{Z}_L}{\mathbf{Z}_L + \mathbf{g}_{11}\mathbf{Z}_s\mathbf{Z}_L + \mathbf{g}_{11}\mathbf{g}_{22}\mathbf{Z}_s - \mathbf{g}_{21}\mathbf{g}_{12}\mathbf{Z}_s + \mathbf{g}_{22}}$$

$$\underline{\frac{\mathbf{V}_2}{\mathbf{V}_s} = \frac{\mathbf{g}_{21}\mathbf{Z}_L}{(1 + \mathbf{g}_{11}\mathbf{Z}_s)(\mathbf{g}_{22} + \mathbf{Z}_L) - \mathbf{g}_{12}\mathbf{g}_{21}\mathbf{Z}_s}}$$

Chapter 19, Problem 42.

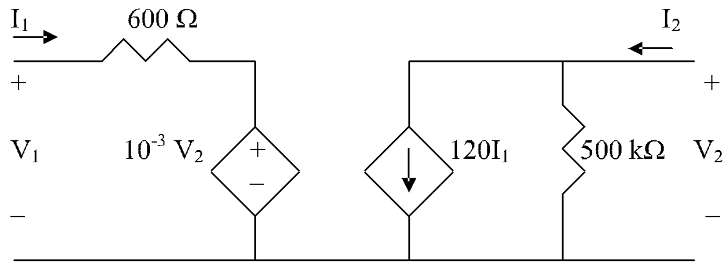
The h parameters of a two-port device are given by

$\mathbf{h}_{11} = 600\Omega$, $\mathbf{h}_{12} = 10^{-3}$, $\mathbf{h}_{21} = 120$, $\mathbf{h}_{22} = 2\times10^{-6}$ S

Draw a circuit model of the device including the value of each element.

Chapter 19, Solution 42.

With the help of Fig. 19.20, we obtain the circuit model below.



Chapter 19, Problem 43.

Find the transmission parameters for the single-element two-port networks in Fig. 19.98.

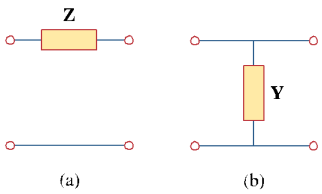
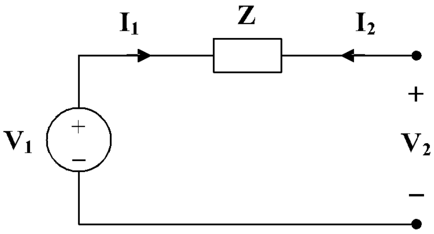


Figure 19.98
For Prob. 19.43.

Chapter 19, Solution 43.

(a) To find $\mathbf{A}$ and $\mathbf{C}$, consider the network in Fig. (a).

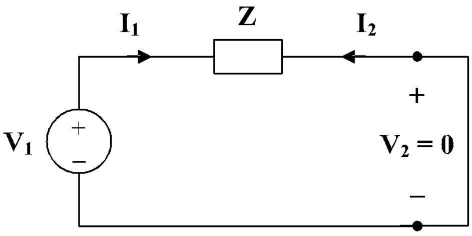


(a)

$$\mathbf{V}_1 = \mathbf{V}_2 \longrightarrow \mathbf{A} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = 1$$

$$\mathbf{I}_1 = 0 \longrightarrow \mathbf{C} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = 0$$

To get $\mathbf{B}$ and $\mathbf{D}$, consider the circuit in Fig. (b).



(b)

$$\mathbf{V}_1 = \mathbf{Z}\mathbf{I}_1, \qquad \mathbf{I}_2 = -\mathbf{I}_1$$

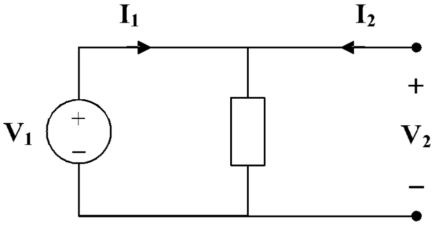
$$\mathbf{B} = \frac{-\mathbf{V}_1}{\mathbf{I}_2} = \frac{-\mathbf{Z}\mathbf{I}_1}{-\mathbf{I}_1} = \mathbf{Z}$$

$$\mathbf{D} = \frac{-\mathbf{I}_1}{\mathbf{I}_2} = 1$$

Hence,

$$[\mathbf{T}] = \begin{bmatrix} \mathbf{1} & \mathbf{Z} \\ \mathbf{0} & \mathbf{1} \end{bmatrix}$$

(b) To find **A** and **C**, consider the circuit in Fig. (c).

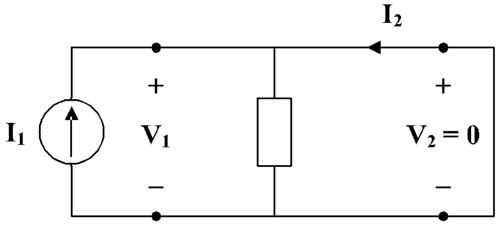


(c)

$$\mathbf{V}_1 = \mathbf{V}_2 \longrightarrow \mathbf{A} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = 1$$

$$\mathbf{V}_1 = \mathbf{Z}\mathbf{I}_1 = \mathbf{V}_2 \longrightarrow \mathbf{C} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{1}{\mathbf{Z}} = \mathbf{Y}$$

To get **B** and **D**, refer to the circuit in Fig.(d).



(d)

$$\mathbf{V}_1 = \mathbf{V}_2 = 0 \qquad \mathbf{I}_2 = -\mathbf{I}_1$$

$$\mathbf{B} = \frac{-\mathbf{V}_1}{\mathbf{I}_2} = 0, \qquad \mathbf{D} = \frac{-\mathbf{I}_1}{\mathbf{I}_2} = 1$$

Thus,

$$[\mathbf{T}] = \underline{\begin{bmatrix} 1 & 0 \\ \mathbf{Y} & 1 \end{bmatrix}}$$

Chapter 19, Problem 44.

Determine the transmission parameters of the circuit in Fig. 19.99.

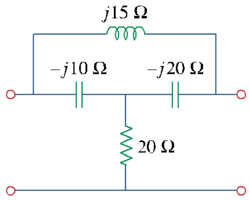
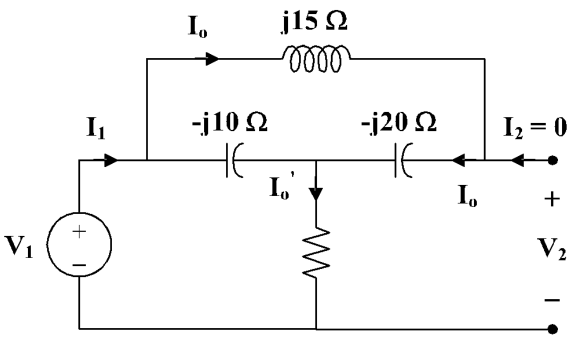


Figure 19.99

For Prob. 19.44.

Chapter 19, Solution 44.

To determine $\mathbf{A}$ and $\mathbf{C}$, consider the circuit in Fig.(a).



(a)

$$\mathbf{V}_1 = [20 + (\text{-j}10) \,||\, (\text{j}15 - \text{j}20)]\mathbf{I}_1$$

$$\mathbf{V}_1 = \left[20 + \frac{(\text{-j}10)(\text{-j}5)}{\text{-j}15}\right]\mathbf{I}_1 = \left[20 - \text{j}\frac{10}{3}\right]\mathbf{I}_1$$

$$\mathbf{I}_o' = \mathbf{I}_1$$

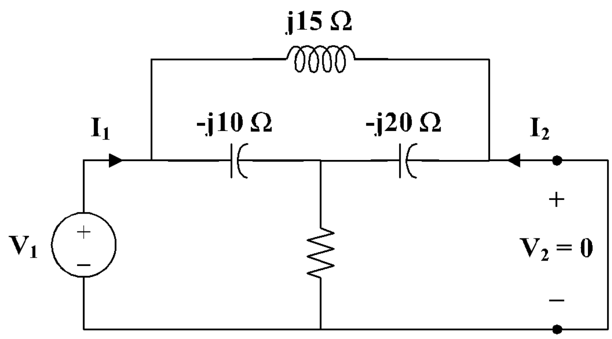
$$\mathbf{I}_o = \left(\frac{\text{-j}10}{\text{-j}10 - \text{j}5}\right)\mathbf{I}_1 = \left(\frac{2}{3}\right)\mathbf{I}_1$$

$$\mathbf{V}_2 = (\text{-j}20)\mathbf{I}_o + 20\mathbf{I}_o' = -\text{j}\frac{40}{3}\text{I}_1 + 20\text{I}_1 = \left(20 - \text{j}\frac{40}{3}\right)\text{I}_1$$

$$\mathbf{A} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{(20 - j10/3)\mathbf{I}_1}{\left(20 - j\frac{40}{3}\right)\mathbf{I}_1} = 0.7692 + j0.3461$$

$$\mathbf{C} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{1}{20 - j\frac{40}{3}} = 0.03461 + j0.023$$

To find **B** and **D**, consider the circuit in Fig. (b).



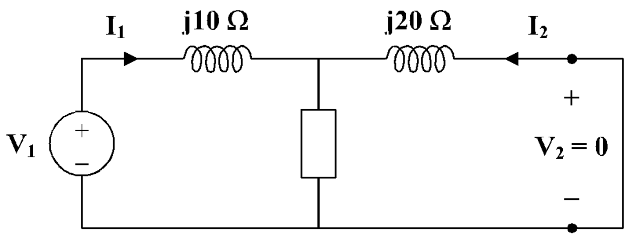
(b)

We may transform the Δ subnetwork to a T as shown in Fig. (c).

$$\mathbf{Z}_1 = \frac{(j15)(-j10)}{j15 - j10 - j20} = j10$$

$$\mathbf{Z}_2 = \frac{(-j10)(-j20)}{-j15} = -j\frac{40}{3}$$

$$\mathbf{Z}_3 = \frac{(j15)(-j20)}{-j15} = j20$$



(c)

$$-\mathbf{I}_2 = \frac{20 - j40/3}{20 - j40/3 + j20}\mathbf{I}_1 = \frac{3 - j2}{3 + j}\mathbf{I}_1$$

$$\mathbf{D} = \frac{-\mathbf{I}_1}{\mathbf{I}_2} = \frac{3 + j}{3 - j2} = 0.5385 + j0.6923$$

$$\mathbf{V}_1 = \left[j10 + \frac{(j20)(20 - j40/3)}{20 - j40/3 + j20} \right]\mathbf{I}_1$$

$$\mathbf{V}_1 = [\, j10 + 2\,(9 + j7)\,]\mathbf{I}_1 = j\mathbf{I}_1\,(24 - j18)$$

$$\mathbf{B} = \frac{-\mathbf{V}_1}{\mathbf{I}_2} = \frac{-\,j\mathbf{I}_1\,(24 - j18)}{\frac{-(3-j2)}{3+j}\mathbf{I}_1} = \frac{6}{13}(-15 + j55)$$

$$\mathbf{B} = -6.923 + j25.385\,\Omega$$

$$[\mathbf{T}] = \underline{\begin{bmatrix} \mathbf{0.7692 + j0.3461} & \mathbf{-6.923 + j25.385\,\Omega} \\ \mathbf{0.03461 + j0.023\,S} & \mathbf{0.5385 + j0.6923} \end{bmatrix}}$$

Chapter 19, Problem 45.

Find the **ABCD** parameters for the circuit in Fig. 19.100.

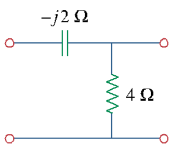
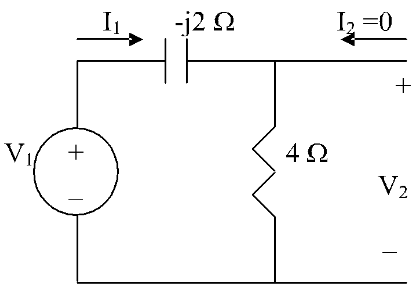


Figure 19.100
For Prob. 19.45.

Chapter 19, Solution 45.

To determine A and C, consider the circuit below.

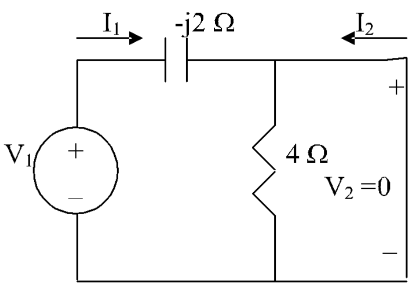


$$V_1 = (4 - j2)I_1, \qquad V_2 = 4I_1$$

$$A = \frac{V_1}{V_2} = \frac{4 - j2}{4} = 1 - j0.5$$

$$C = \frac{I_1}{V_2} = \frac{I_1}{4I_1} = 0.25$$

To determine B and D, consider the circuit below.



The 4-Ω resistor is short-circuited. Hence,

$$I_2 = -I_1, \qquad D = -\frac{I_1}{I_2} = 1$$

$$V_1 = -j2I_1 = j2I_2 \qquad B = -\frac{V_1}{I_2} = -\frac{j2I_2}{I_2} = -2j\Omega$$

Hence,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 - j0.5 & -j2\Omega \\ 0.25S & 1 \end{bmatrix} = \begin{bmatrix} 1 - \text{j}0.5 & -\text{j}2\Omega \\ 0.25\text{S} & 1 \end{bmatrix}$$

Chapter 19, Problem 46.

Find the transmission parameters for the circuit in Fig. 19.101.

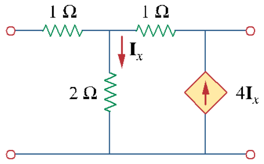
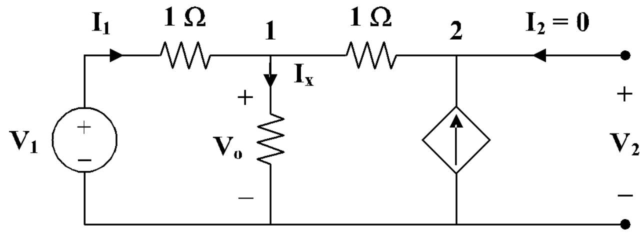


Figure 19.101

For Prob. 19.46.

Chapter 19, Solution 46.

To get $\mathbf{A}$ and $\mathbf{C}$, refer to the circuit in Fig.(a).



(a)

At node 1,

$$\mathbf{I}_1 = \frac{\mathbf{V}_o}{2} + \frac{\mathbf{V}_o - \mathbf{V}_2}{1} \longrightarrow 2\mathbf{I}_1 = 3\mathbf{V}_o - 2\mathbf{V}_2 \qquad (1)$$

At node 2,

$$\frac{\mathbf{V}_o - \mathbf{V}_2}{1} = 4\mathbf{I}_x = \frac{4\mathbf{V}_o}{2} = 2\mathbf{V}_o \longrightarrow \mathbf{V}_o = -\mathbf{V}_2 \qquad (2)$$

From (1) and (2),

$$2\mathbf{I}_1 = -5\mathbf{V}_2 \longrightarrow \mathbf{C} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{-5}{2} = -2.5\text{ S}$$

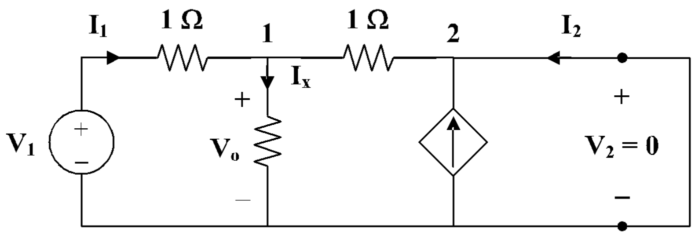
But

$$\mathbf{I}_1 = \frac{\mathbf{V}_1 - \mathbf{V}_o}{1} = \mathbf{V}_1 + \mathbf{V}_2$$

$$-2.5\mathbf{V}_2 = \mathbf{V}_1 + \mathbf{V}_2 \longrightarrow \mathbf{V}_1 = -3.5\mathbf{V}_2$$

$$\mathbf{A} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = -3.5$$

To get **B** and **D**, consider the circuit in Fig. (b).



(b)

At node 1,

$$\mathbf{I}_1 = \frac{\mathbf{V}_o}{2} + \frac{\mathbf{V}_o}{1} \longrightarrow 2\mathbf{I}_1 = 3\mathbf{V}_o \qquad (3)$$

At node 2,

$$\mathbf{I}_2 + \frac{\mathbf{V}_o}{1} + 4\mathbf{I}_x = 0$$

$$-\mathbf{I}_2 = \mathbf{V}_o + 2\mathbf{V}_o = 0 \longrightarrow \mathbf{I}_2 = \text{-}3\mathbf{V}_o \qquad (4)$$

Adding (3) and (4),

$$2\mathbf{I}_1 + \mathbf{I}_2 = 0 \longrightarrow \mathbf{I}_1 = \text{-}0.5\mathbf{I}_2 \qquad (5)$$

$$\mathbf{D} = \frac{\text{-}\mathbf{I}_1}{\mathbf{I}_2} = 0.5$$

But $$\mathbf{I}_1 = \frac{\mathbf{V}_1 - \mathbf{V}_o}{1} \longrightarrow \mathbf{V}_1 = \mathbf{I}_1 + \mathbf{V}_o \qquad (6)$$

Substituting (5) and (4) into (6),

$$\mathbf{V}_1 = \frac{\text{-}1}{2}\mathbf{I}_2 + \frac{\text{-}1}{3}\mathbf{I}_2 = \frac{\text{-}5}{6}\mathbf{I}_2$$

$$\mathbf{B} = \frac{\text{-}\mathbf{V}_1}{\mathbf{I}_2} = \frac{5}{6} = 0.8333\,\Omega$$

Thus,

$$[\mathbf{T}] = \begin{bmatrix} \mathbf{-3.5} & \mathbf{0.8333\,\Omega} \\ \mathbf{-2.5\,S} & \mathbf{-0.5} \end{bmatrix}$$

Chapter 19, Problem 47.

Obtain the **ABCD** parameters for the network in Fig. 19.102.

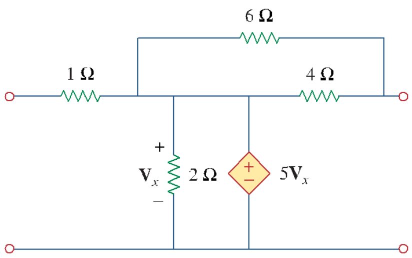
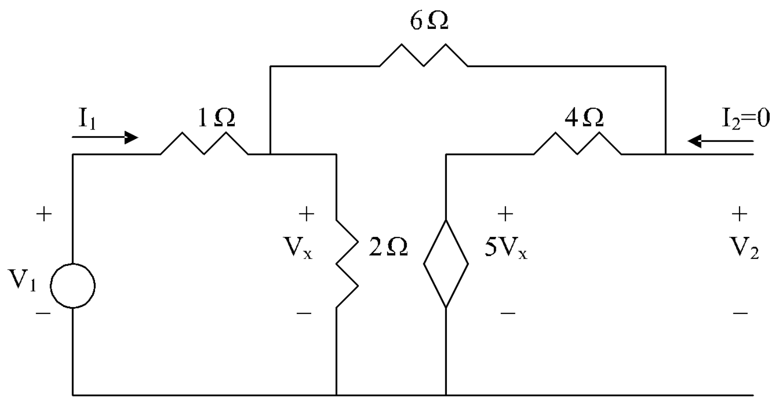


Figure 19.102

For Prob. 19.47.

Chapter 19, Solution 47.

To get A and C, consider the circuit below.

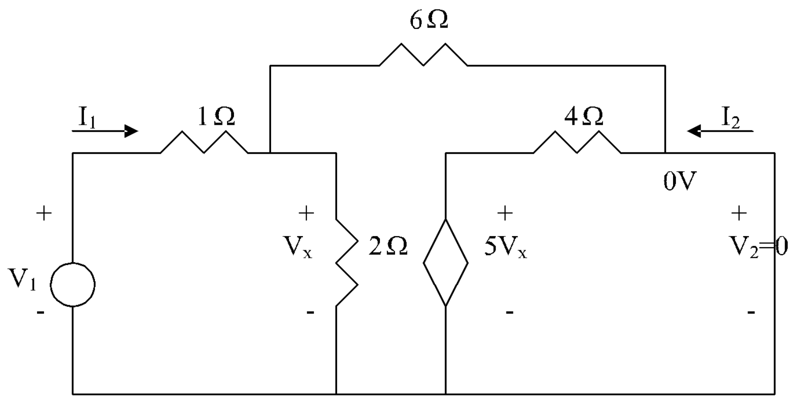


$$\frac{V_1 - V_x}{1} = \frac{V_x}{2} + \frac{V_x - 5V_x}{10} \longrightarrow V_1 = 1.1V_x$$

$$V_2 = 4(-0.4V_x) + 5V_x = 3.4V_x \longrightarrow A = \frac{V_1}{V_2} = 1.1/3.4 = 0.3235$$

$$I_1 = \frac{V_1 - V_x}{1} = 1.1V_x - V_x = 0.1V_x \longrightarrow C = \frac{I_1}{V_2} = 0.1/3.4 = 0.02941$$

To get B and D, consider the circuit below.



$$\frac{V_1 - V_x}{1} = \frac{V_x}{6} + \frac{V_x}{2} \longrightarrow V_1 = \frac{10}{6}V_x \qquad (1)$$

$$I_2 = -\frac{5V_x}{4} - \frac{V_x}{6} = -\frac{17}{12}V_x \qquad (2)$$

$$V_1 = I_1 + V_x \qquad (3)$$

From (1) and (3)

$$I_1 = V_1 - V_x = \frac{4}{6}V_x \longrightarrow D = -\frac{I_1}{I_2} = \frac{4}{6}(\frac{12}{17}) = 0.4706$$

$$B = -\frac{V_1}{I_2} = \frac{10}{6}(\frac{12}{17}) = 1.176$$

$$[T] = \begin{bmatrix} 0.3235 & 1.176 \\ 0.02941 & 0.4706 \end{bmatrix}$$

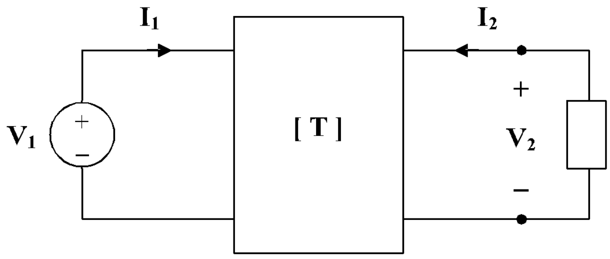
Chapter 19, Problem 48.

For a two-port, let **A** = 4, **B** = 30Ω, **C** = 0.1 S, and **D** = 1.5. Calculate the input impedance, $\mathbf{Z}_{in} = \mathbf{V}_1 / \mathbf{I}_1$ when:

(a) the output terminals are short-circuited,
(b) the output port is open-circuited,
(c) the output port is terminated by a $10\text{-}\Omega$ load.

Chapter 19, Solution 48.

(a) Refer to the circuit below.



$$\mathbf{V}_1 = 4\mathbf{V}_2 - 30\mathbf{I}_2 \qquad (1)$$
$$\mathbf{I}_1 = 0.1\mathbf{V}_2 - \mathbf{I}_2 \qquad (2)$$

When the output terminals are shorted, $\mathbf{V}_2 = 0$.
So, (1) and (2) become

$\mathbf{V}_1 = \text{-}30\mathbf{I}_2$ and $\mathbf{I}_1 = \text{-}\mathbf{I}_2$

Hence,

$$\mathbf{Z}_{in} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \underline{\mathbf{30\,\Omega}}$$

(b) When the output terminals are open-circuited, $\mathbf{I}_2 = 0$.
So, (1) and (2) become

$\mathbf{V}_1 = 4\mathbf{V}_2$

$\mathbf{I}_1 = 0.1\mathbf{V}_2$ or $\mathbf{V}_2 = 10\mathbf{I}_1$

$\mathbf{V}_1 = 40\mathbf{I}_1$

$$\mathbf{Z}_{in} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \underline{\mathbf{40\,\Omega}}$$

(c) When the output port is terminated by a 10-Ω load, $\mathrm{V}_2 = \text{-}10\mathrm{I}_2$.
So, (1) and (2) become

$\mathbf{V}_1 = \text{-}40\mathbf{I}_2 - 30\mathbf{I}_2 = \text{-}70\mathbf{I}_2$

$\mathbf{I}_1 = \text{-}\mathbf{I}_2 - \mathbf{I}_2 = \text{-}2\mathbf{I}_2$

$\mathbf{V}_1 = 35\mathbf{I}_1$

$$\mathbf{Z}_{in} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \underline{\mathbf{35\,\Omega}}$$

Alternatively, we may use $\mathbf{Z}_{in} = \dfrac{\mathbf{A}\mathbf{Z}_L + \mathbf{B}}{\mathbf{C}\mathbf{Z}_L + \mathbf{D}}$

Chapter 19, Problem 49.

Using impedances in the *s* domain, obtain the transmission parameters for the circuit in Fig. 19.103.

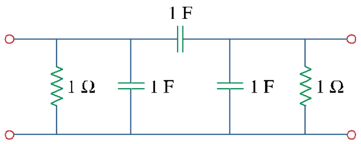
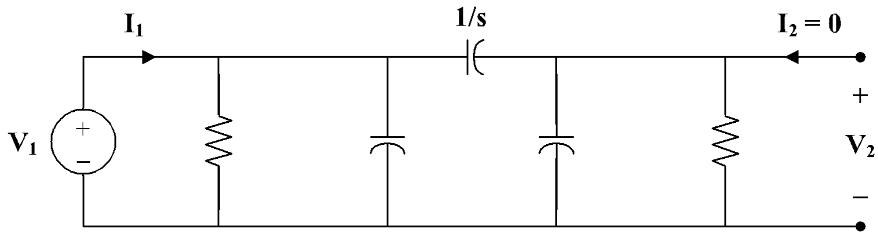


Figure 19.103

For Prob. 19.49.

Chapter 19, Solution 49.

To get $\mathbf{A}$ and $\mathbf{C}$, refer to the circuit in Fig.(a).



(a)

$$1\,||\,\frac{1}{s}=\frac{1/s}{1+1/s}=\frac{1}{s+1}$$

$$\mathbf{V}_2=\frac{1\,||\,1/s}{1/s+1\,||\,1/s}\mathbf{V}_1$$

$$\mathbf{A}=\frac{\mathbf{V}_1}{\mathbf{V}_2}=\frac{\frac{1}{s}+\frac{1}{s+1}}{\frac{1}{s+1}}=\frac{2s+1}{s}$$

$$\mathbf{V}_1=\mathbf{I}_1\left(\frac{1}{s+1}\right)||\left(\frac{1}{s}+\frac{1}{s+1}\right)=\mathbf{I}_1\left(\frac{1}{s+1}\right)||\left(\frac{2s+1}{s(s+1)}\right)$$

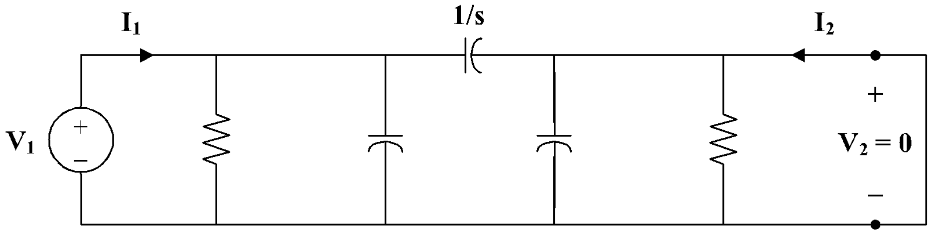
$$\frac{\mathbf{V}_1}{\mathbf{I}_1}=\frac{\left(\frac{1}{s+1}\right)\cdot\left(\frac{2s+1}{s(s+1)}\right)}{\frac{1}{s+1}+\frac{2s+1}{s(s+1)}}=\frac{2s+1}{(s+1)(3s+1)}$$

But $\mathbf{V}_1 = \mathbf{V}_2 \cdot \frac{2s+1}{s}$

Hence, $\frac{\mathbf{V}_2}{\mathbf{I}_1} \cdot \frac{2s+1}{s} = \frac{2s+1}{(s+1)(3s+1)}$

$$\mathbf{C} = \frac{\mathbf{V}_2}{\mathbf{I}_1} = \frac{(s+1)(3s+1)}{s}$$

To get $\mathbf{B}$ and $\mathbf{D}$, consider the circuit in Fig. (b).



(b)

$$\mathbf{V}_1 = \mathbf{I}_1\left(1 \,||\, \frac{1}{s} \,||\, \frac{1}{s}\right) = \mathbf{I}_1\left(1 \,||\, \frac{1}{2s}\right) = \frac{\mathbf{I}_1}{2s+1}$$

$$\mathbf{I}_2 = \frac{\frac{-1}{s+1}\mathbf{I}_1}{\frac{1}{s+1} + \frac{1}{s}} = \frac{-s}{2s+1}\mathbf{I}_1$$

$$\mathbf{D} = \frac{-\mathbf{I}_1}{\mathbf{I}_2} = \frac{2s+1}{s} = 2 + \frac{1}{s}$$

$$\mathbf{V}_1 = \left(\frac{1}{2s+1}\right)\left(\frac{2s+1}{-s}\right)\mathbf{I}_2 = \frac{\mathbf{I}_2}{-s} \longrightarrow \mathbf{B} = \frac{-\mathbf{V}_1}{\mathbf{I}_2} = \frac{1}{s}$$

Thus,

$$[\mathbf{T}] = \begin{bmatrix} \frac{2s+1}{s} & \frac{1}{s} \\ \frac{(s+1)(3s+1)}{s} & 2 + \frac{1}{s} \end{bmatrix}$$

Chapter 19, Problem 50.

Derive the s-domain expression for the t parameters of the circuit in Fig. 19.104.

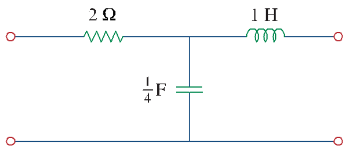
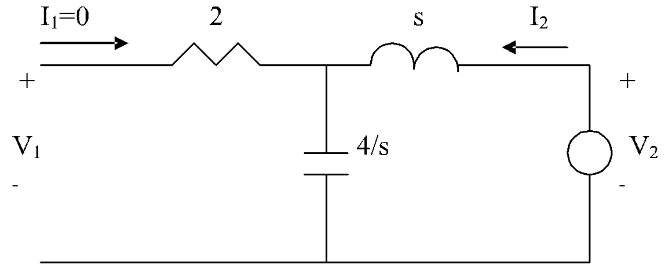


Figure 19.104
For Prob. 19.50.

Chapter 19, Solution 50.

To get a and c, consider the circuit below.

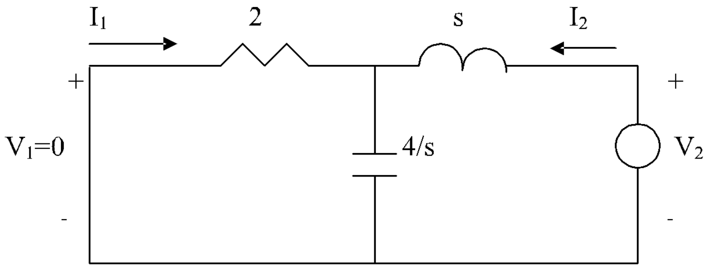


$$V_1 = \frac{4/s}{s+4/s}V_2 = \frac{4}{s^2+4}V_2 \quad \longrightarrow \quad a = V_2/V_1 = 1+0.25s^2$$

$$V_2 = (s+4/s)I_2 \text{ or}$$

$$I_2 = \frac{V_2}{s+4/s} = \frac{(1+0.25s^2)V_1}{s+4/s} \quad \longrightarrow \quad c = \frac{I_2}{V_1} = \frac{s+0.25s^3}{s^2+4}$$

To get b and d, consider the circuit below.



$$I_1 = \frac{-4/s}{2+4/s}I_2 = -\frac{2I_2}{s+2} \quad \longrightarrow \quad d = -\frac{I_2}{I_1} = 1+0.5s$$

$$V_2 = (s+2//\frac{4}{s})I_2 = \frac{(s^2+2s+4)}{s+2}I_2$$

$$= -\frac{(s^2+2s+4)}{s+2}\frac{(s+2)}{2}I_1 \quad \longrightarrow \quad b = -\frac{V_2}{I_1} = 0.5s^2+s+2$$

$$[t] = \begin{bmatrix} 0.25s^2+1 & 0.5s^2+s+2 \\ \dfrac{0.25s^2+s}{s^2+4} & 0.5s+1 \end{bmatrix}$$

Chapter 19, Problem 51.

Obtain the *t* parameters for the network in Fig. 19.105.

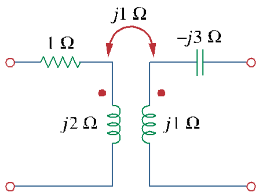
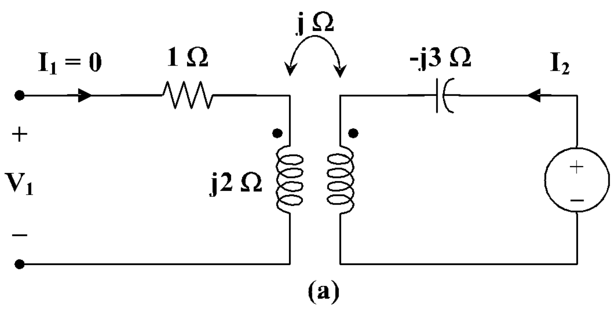


Figure 19.105
For Prob. 19.51.

Chapter 19, Solution 51.

To get $\mathbf{a}$ and $\mathbf{c}$, consider the circuit in Fig. (a).



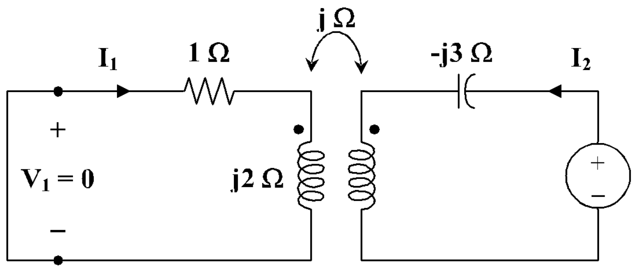
$$\mathbf{V}_2 = \mathbf{I}_2(\mathrm{j} - \mathrm{j}3) = \text{-j}2\,\mathbf{I}_2$$

$$\mathbf{V}_1 = \text{-j}\mathbf{I}_2$$

$$\mathbf{a} = \frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{-\,\mathrm{j}2\,\mathbf{I}_2}{-\,\mathrm{j}\mathbf{I}_2} = 2$$

$$\mathbf{c} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \frac{1}{-\,\mathrm{j}} = \mathrm{j}$$

To get **b** and **d**, consider the circuit in Fig. (b).



(b)

For mesh 1,

$$0 = (1 + j2)\mathbf{I}_1 - j\mathbf{I}_2$$

or
$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{1 + j2}{j} = 2 - j$$

$$\mathbf{d} = \frac{-\mathbf{I}_2}{\mathbf{I}_1} = -2 + j$$

For mesh 2,

$$\mathbf{V}_2 = \mathbf{I}_2\,(j - j3) - j\mathbf{I}_1$$
$$\mathbf{V}_2 = \mathbf{I}_1\,(2 - j)(-j2) - j\mathbf{I}_1 = (-2 - j5)\mathbf{I}_1$$

$$\mathbf{b} = \frac{-\mathbf{V}_2}{\mathbf{I}_1} = 2 + j5$$

Thus,

$$[\mathbf{t}] = \underline{\begin{bmatrix} \mathbf{2} & \mathbf{2 + j5} \\ \mathbf{j} & \mathbf{-2 + j} \end{bmatrix}}$$

Chapter 19, Problem 52.

(a) For the *T* network in Fig. 19.106, show that the *h* parameters are:

$\mathbf{h}_{11} = R_1 + \dfrac{R_2 R_3}{R_1 + R3}$, $\mathbf{h}_{12} = \dfrac{R_2}{R_2 + R_3}$

$\mathbf{h}_{21} = -\dfrac{R_2}{R_2 + R_3}$, $\mathbf{h}_{22} = \dfrac{1}{R_2 + R_3}$

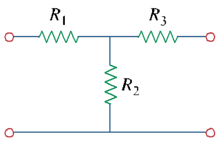


Figure 19.106

For Prob. 19.52.

(b) For the same network, show that the transmission parameters are:

$\mathbf{A} = 1 + \dfrac{R_1}{R_2}$, $\mathbf{B} = R_3 + \dfrac{R_1}{R_2}(R_2 + R_3)$

$\mathbf{C} = \dfrac{1}{R_2}$, $\mathbf{D} = 1 + \dfrac{R_3}{R_2}$

Chapter 19, Solution 52.

It is easy to find the z parameters and then transform these to h parameters and T parameters.

$$[\mathbf{z}] = \begin{bmatrix} R_1 + R_2 & R_2 \\ R_2 & R_2 + R_3 \end{bmatrix}$$

$$\Delta_z = (R_1 + R_2)(R_2 + R_3) - R_2^2$$
$$= R_1R_2 + R_2R_3 + R_3R_1$$

(a) $$[\mathbf{h}] = \begin{bmatrix} \dfrac{\Delta_z}{\mathbf{z}_{22}} & \dfrac{\mathbf{z}_{12}}{\mathbf{z}_{22}} \\ \dfrac{-\mathbf{z}_{21}}{\mathbf{z}_{22}} & \dfrac{1}{\mathbf{z}_{22}} \end{bmatrix} = \begin{bmatrix} \dfrac{R_1R_2 + R_2R_3 + R_3R_1}{R_2 + R_3} & \dfrac{R_2}{R_2 + R_3} \\ \dfrac{-R_2}{R_2 + R_3} & \dfrac{1}{R_2 + R_3} \end{bmatrix}$$

Thus,

$$\underline{\mathbf{h}_{11} = \mathbf{R}_1 + \frac{\mathbf{R}_2\mathbf{R}_3}{\mathbf{R}_2 + \mathbf{R}_3}}, \quad \underline{\mathbf{h}_{12} = \frac{\mathbf{R}_2}{\mathbf{R}_2 + \mathbf{R}_3} = -\mathbf{h}_{21}}, \quad \underline{\mathbf{h}_{22} = \frac{1}{\mathbf{R}_2 + \mathbf{R}_3}}$$

as required.

(b) $$[\mathbf{T}] = \begin{bmatrix} \dfrac{\mathbf{z}_{11}}{\mathbf{z}_{21}} & \dfrac{\Delta_z}{\mathbf{z}_{21}} \\ \dfrac{1}{\mathbf{z}_{21}} & \dfrac{\mathbf{z}_{22}}{\mathbf{z}_{21}} \end{bmatrix} = \begin{bmatrix} \dfrac{R_1 + R_2}{R_2} & \dfrac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} \\ \dfrac{1}{R_2} & \dfrac{R_2 + R_3}{R_2} \end{bmatrix}$$

Hence,

$$\underline{\mathbf{A} = 1 + \frac{\mathbf{R}_1}{\mathbf{R}_2}}, \quad \underline{\mathbf{B} = \mathbf{R}_3 + \frac{\mathbf{R}_1}{\mathbf{R}_2}(\mathbf{R}_2 + \mathbf{R}_3)}, \quad \underline{\mathbf{C} = \frac{1}{\mathbf{R}_2}}, \quad \underline{\mathbf{D} = 1 + \frac{\mathbf{R}_3}{\mathbf{R}_2}}$$

as required.

Chapter 19, Problem 53.

Through derivation, express the *z* parameters in terms of the **ABCD** parameters.

Chapter 19, Solution 53.

For the z parameters,

$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 \qquad (1)$$

$$\mathbf{V}_2 = \mathbf{z}_{12}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2 \qquad (2)$$

For **ABCD** parameters,

$$\mathbf{V}_1 = \mathbf{A}\,\mathbf{V}_2 - \mathbf{B}\,\mathbf{I}_2 \qquad (3)$$

$$\mathbf{I}_1 = \mathbf{C}\,\mathbf{V}_2 - \mathbf{D}\,\mathbf{I}_2 \qquad (4)$$

From (4),

$$\mathbf{V}_2 = \frac{\mathbf{I}_1}{\mathbf{C}} + \frac{\mathbf{D}}{\mathbf{C}}\mathbf{I}_2 \qquad (5)$$

Comparing (2) and (5),

$$\mathbf{z}_{21} = \frac{1}{\mathbf{C}}, \qquad \mathbf{z}_{22} = \frac{\mathbf{D}}{\mathbf{C}}$$

Substituting (5) into (3),

$$\mathbf{V}_1 = \frac{\mathbf{A}}{\mathbf{C}}\mathbf{I}_1 + \left(\frac{\mathbf{AD}}{\mathbf{C}} - \mathbf{B}\right)\mathbf{I}_2$$

$$= \frac{\mathbf{A}}{\mathbf{C}}\mathbf{I}_1 + \frac{\mathbf{AD} - \mathbf{BC}}{\mathbf{C}}\mathbf{I}_2 \qquad (6)$$

Comparing (6) and (1),

$$\mathbf{z}_{11} = \frac{\mathbf{A}}{\mathbf{C}} \qquad \mathbf{z}_{12} = \frac{\mathbf{AD} - \mathbf{BC}}{\mathbf{C}} = \frac{\Delta_T}{\mathbf{C}}$$

Thus,

$$[Z] = \begin{bmatrix} \frac{\mathbf{A}}{\mathbf{C}} & \frac{\Delta_T}{\mathbf{C}} \\ \frac{1}{\mathbf{C}} & \frac{\mathbf{D}}{\mathbf{C}} \end{bmatrix}$$

Chapter 19, Problem 54.

Show that the transmission parameters of a two-port may be obtained from the *y* parameters as:

$$A = -\frac{y_{22}}{y_{21}}, \qquad B = -\frac{1}{y_{21}}$$

$$C = -\frac{\Delta_y}{y_{21}}, \qquad D = -\frac{y_{11}}{y_{21}}$$

Chapter 19, Solution 54.

For the y parameters

$$\mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2 \qquad (1)$$

$$\mathbf{I}_2 = \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2 \qquad (2)$$

From (2),

$$\mathbf{V}_1 = \frac{\mathbf{I}_2}{\mathbf{y}_{21}} - \frac{\mathbf{y}_{22}}{\mathbf{y}_{21}}\mathbf{V}_2$$

or

$$\mathbf{V}_1 = \frac{-\mathbf{y}_{22}}{\mathbf{y}_{12}}\mathbf{V}_2 + \frac{1}{\mathbf{y}_{21}}\mathbf{I}_2 \qquad (3)$$

Substituting (3) into (1) gives

$$\mathbf{I}_1 = \frac{-\mathbf{y}_{11}\mathbf{y}_{22}}{\mathbf{y}_{21}}\mathbf{V}_2 + \mathbf{y}_{12}\mathbf{V}_2 + \frac{\mathbf{y}_{11}}{\mathbf{y}_{21}}\mathbf{I}_2$$

or

$$\mathbf{I}_1 = \frac{-\Delta_y}{\mathbf{y}_{21}}\mathbf{V}_2 + \frac{\mathbf{y}_{11}}{\mathbf{y}_{21}}\mathbf{I}_2 \qquad (4)$$

Comparing (3) and (4) with the following equations

$$\mathbf{V}_1 = \mathbf{A}\mathbf{V}_2 - \mathbf{B}\mathbf{I}_2$$

$$\mathbf{I}_1 = \mathbf{C}\mathbf{V}_2 - \mathbf{D}\mathbf{I}_2$$

clearly shows that

$$\underline{\mathbf{A} = \frac{-\mathbf{y}_{22}}{\mathbf{y}_{21}}}, \quad \underline{\mathbf{B} = \frac{-1}{\mathbf{y}_{21}}}, \quad \underline{\mathbf{C} = \frac{-\Delta_y}{\mathbf{y}_{21}}}, \quad \underline{\mathbf{D} = \frac{-\mathbf{y}_{11}}{\mathbf{y}_{21}}}$$

as required.

Chapter 19, Problem 55.

Prove that the *g* parameters can be obtained from the *z* parameters as

$$\mathbf{g}_{11} = \frac{1}{\mathbf{z}_{11}}, \qquad \mathbf{g}_{12} = -\frac{\mathbf{z}_{12}}{\mathbf{z}_{11}}$$

$$\mathbf{g}_{21} = \frac{\mathbf{z}_{21}}{\mathbf{z}_{11}}, \qquad \mathbf{g}_{22} = \frac{\Delta_z}{\mathbf{z}_{11}}$$

Chapter 19, Solution 55.

For the z parameters

$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 \qquad (1)$$

$$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2 \qquad (2)$$

From (1),

$$\mathbf{I}_1 = \frac{1}{\mathbf{z}_{11}}\mathbf{V}_1 - \frac{\mathbf{z}_{12}}{\mathbf{z}_{11}}\mathbf{I}_2 \qquad (3)$$

Substituting (3) into (2) gives

$$\mathbf{V}_2 = \frac{\mathbf{z}_{21}}{\mathbf{z}_{11}}\mathbf{V}_1 + \left(\mathbf{z}_{22} - \frac{\mathbf{z}_{21}\mathbf{z}_{12}}{\mathbf{z}_{11}}\right)\mathbf{I}_2$$

or

$$\mathbf{V}_2 = \frac{\mathbf{z}_{21}}{\mathbf{z}_{11}}\mathbf{V}_1 + \frac{\Delta_z}{\mathbf{z}_{11}}\mathbf{I}_2 \qquad (4)$$

Comparing (3) and (4) with the following equations

$$\mathbf{I}_1 = \mathbf{g}_{11}\mathbf{V}_1 + \mathbf{g}_{12}\mathbf{I}_2$$

$$\mathbf{V}_2 = \mathbf{g}_{21}\mathbf{V}_1 + \mathbf{g}_{22}\mathbf{I}_2$$

indicates that

$$\underline{\mathbf{g}_{11} = \frac{1}{\mathbf{z}_{11}}}, \quad \underline{\mathbf{g}_{12} = \frac{-\mathbf{z}_{12}}{\mathbf{z}_{11}}}, \quad \underline{\mathbf{g}_{21} = \frac{\mathbf{z}_{21}}{\mathbf{z}_{11}}}, \quad \underline{\mathbf{g}_{22} = \frac{\Delta_z}{\mathbf{z}_{11}}}$$

as required.

Chapter 19, Problem 56.

For the network of Fig. 19.107, obtain $\mathbf{V}_o/\mathbf{V}_s$.

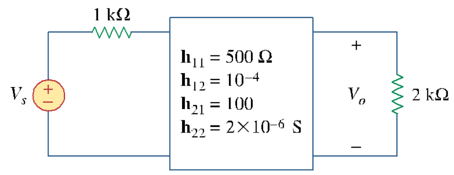
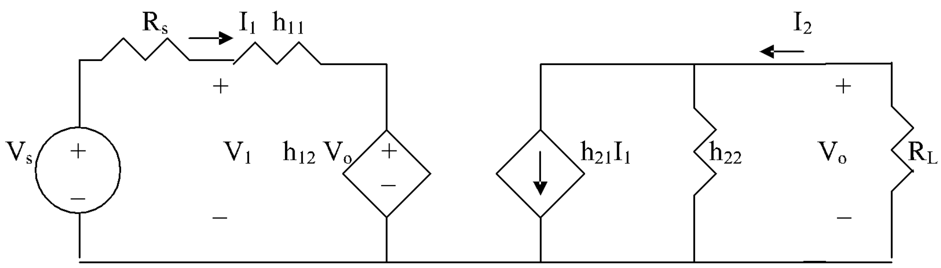


Figure 19.107

For Prob. 19.56.

Chapter 19, Solution 56.

Using Fig. 19.20, we obtain the equivalent circuit as shown below.



We can solve this using MATLAB. First, we generate 4 equations from the given circuit. It may help to let $V_s = 10$ V.

$-10 + R_sI_1 + V_1 = 0$ or $V_1 + 1000I_1 = 10$
$-10 + R_sI_1 + h_{11}I_1 + h_{12}V_o = 0$ or $0.0001V_s + 1500 = 10$
$I_2 = -V_o/R_L$ or $V_o + 2000I_2 = 0$
$h_{21}I_1 + h_{22}V_o - I_2 = 0$ or $2x10^{-6}V_o + 100I_1 - I_2 = 0$

```
>> A=[1,0,1000,0;0,0.0001,1500,0;0,1,0,2000;0,(2*10^-6),100,-1]
A =
  1.0e+003 *
    0.0010         0    1.0000         0
         0    0.0000    1.5000         0
         0    0.0010         0    2.0000
         0    0.0000    0.1000   -0.0010
>> U=[10;10;0;0]
U =
    10
    10
     0
     0
>> X=inv(A)*U
X =
  1.0e+003 *
    0.0032
   -1.3459
    0.0000
    0.0007
```

Gain = $V_o/V_s = -1,345.9/10 =$ **–134.59**.

There is a second approach we can take to check this problem. First, the resistive value of h_{22} is quite large, 500 kΩ versus R_L so can be ignored. Working on the right side of the circuit we obtain the following,

$I_2 = 100I_1$ which leads to $V_o = -I_2x2k = -2x10^5I_1$.

Now the left hand loop equation becomes,

$-V_s + (1000 + 500 + 10^{-4}(-2x10^5))I_1 = 1480I_1$.

Solving for V_o/V_s we get,

$V_o/V_s = -200,000/1480 =$ **–134.14**.

Our answer checks!

Chapter 19, Problem 57.

Given the transmission parameters

$$[\mathbf{T}] = \begin{bmatrix} 3 & 20 \\ 1 & 7 \end{bmatrix}$$

obtain the other five two-port parameters.

Chapter 19, Solution 57.

$$\Delta_T = (3)(7) - (20)(1) = 1$$

$$[\mathbf{z}] = \begin{bmatrix} \dfrac{\mathbf{A}}{\mathbf{C}} & \dfrac{\Delta_T}{\mathbf{C}} \\ \dfrac{1}{\mathbf{C}} & \dfrac{\mathbf{D}}{\mathbf{C}} \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{3} & \mathbf{1} \\ \mathbf{1} & \mathbf{7} \end{bmatrix} \Omega}$$

$$[\mathbf{y}] = \begin{bmatrix} \dfrac{\mathbf{D}}{\mathbf{B}} & \dfrac{-\Delta_T}{\mathbf{B}} \\ \dfrac{-1}{\mathbf{B}} & \dfrac{\mathbf{A}}{\mathbf{B}} \end{bmatrix} = \underline{\begin{bmatrix} \dfrac{\mathbf{7}}{\mathbf{20}} & \dfrac{\mathbf{-1}}{\mathbf{20}} \\ \dfrac{\mathbf{-1}}{\mathbf{20}} & \dfrac{\mathbf{3}}{\mathbf{20}} \end{bmatrix} \mathbf{S}}$$

$$[\mathbf{h}] = \begin{bmatrix} \dfrac{\mathbf{B}}{\mathbf{D}} & \dfrac{\Delta_T}{\mathbf{D}} \\ \dfrac{-1}{\mathbf{D}} & \dfrac{\mathbf{C}}{\mathbf{D}} \end{bmatrix} = \underline{\begin{bmatrix} \dfrac{\mathbf{20}}{\mathbf{7}}\,\Omega & \dfrac{\mathbf{1}}{\mathbf{7}} \\ \dfrac{\mathbf{-1}}{\mathbf{7}} & \dfrac{\mathbf{1}}{\mathbf{7}}\,\mathbf{S} \end{bmatrix}}$$

$$[\mathbf{g}] = \begin{bmatrix} \dfrac{\mathbf{C}}{\mathbf{A}} & \dfrac{-\Delta_T}{\mathbf{A}} \\ \dfrac{1}{\mathbf{A}} & \dfrac{\mathbf{B}}{\mathbf{A}} \end{bmatrix} = \underline{\begin{bmatrix} \dfrac{\mathbf{1}}{\mathbf{3}}\,\mathbf{S} & \dfrac{\mathbf{-1}}{\mathbf{3}} \\ \dfrac{\mathbf{1}}{\mathbf{3}} & \dfrac{\mathbf{20}}{\mathbf{3}}\,\Omega \end{bmatrix}}$$

$$[\mathbf{t}] = \begin{bmatrix} \dfrac{\mathbf{D}}{\Delta_T} & \dfrac{\mathbf{B}}{\Delta_T} \\ \dfrac{\mathbf{C}}{\Delta_T} & \dfrac{\mathbf{A}}{\Delta_T} \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{7} & \mathbf{20}\,\Omega \\ \mathbf{1\,S} & \mathbf{3} \end{bmatrix}}$$

Chapter 19, Problem 58.

A two-port is described by

$\mathbf{V}_1 = \mathbf{I}_1 + 2\mathbf{V}_2$, $\qquad \mathbf{I}_2 = -2\mathbf{I}_1 + 0.4\mathbf{V}_2$

Find: (a) the *y* parameters, (b) the transmission parameters.

Chapter 19, Solution 58.

The given set of equations is for the h parameters.

$$[\mathbf{h}] = \begin{bmatrix} 1\,\Omega & 2 \\ -2 & 0.4\,\text{S} \end{bmatrix} \qquad \Delta_h = (1)(0.4) - (2)(-2) = 4.4$$

(a) $$[\mathbf{y}] = \begin{bmatrix} \dfrac{1}{\mathbf{h}_{11}} & \dfrac{-\mathbf{h}_{12}}{\mathbf{h}_{11}} \\ \dfrac{\mathbf{h}_{21}}{\mathbf{h}_{11}} & \dfrac{\Delta_h}{\mathbf{h}_{11}} \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{1} & \mathbf{-2} \\ \mathbf{-2} & \mathbf{4.4} \end{bmatrix}\mathbf{S}}$$

(b) $$[\mathbf{T}] = \begin{bmatrix} \dfrac{-\Delta_h}{\mathbf{h}_{21}} & \dfrac{-\mathbf{h}_{11}}{\mathbf{h}_{21}} \\ \dfrac{-\mathbf{h}_{22}}{\mathbf{h}_{21}} & \dfrac{-1}{\mathbf{h}_{21}} \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{2.2} & \mathbf{0.5\,\Omega} \\ \mathbf{0.2\,S} & \mathbf{0.5} \end{bmatrix}}$$

Chapter 19, Problem 59.

Given that

$$[\mathbf{g}] = \begin{bmatrix} 0.06\,\text{S} & -0.4 \\ 0.2 & 2\Omega \end{bmatrix}$$

determine:

(a) [**z**] (b) [**y**] (c) [**h**] (d) [**T**]

Chapter 19, Solution 59.

$$\Delta_g = (0.06)(2) - (-0.4)(0.2) = 0.12 + 0.08 = 0.2$$

(a) $$[\mathbf{z}] = \begin{bmatrix} \dfrac{1}{g_{11}} & \dfrac{-g_{12}}{g_{11}} \\ \dfrac{g_{21}}{g_{11}} & \dfrac{\Delta_g}{g_{11}} \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{16.667} & \mathbf{6.667} \\ \mathbf{3.333} & \mathbf{3.333} \end{bmatrix}\Omega}$$

(b) $$[\mathbf{y}] = \begin{bmatrix} \dfrac{\Delta_g}{g_{22}} & \dfrac{g_{12}}{g_{22}} \\ \dfrac{-g_{21}}{g_{22}} & \dfrac{1}{g_{22}} \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{0.1} & \mathbf{-0.2} \\ \mathbf{-0.1} & \mathbf{0.5} \end{bmatrix}\mathbf{S}}$$

(c) $$[\mathbf{h}] = \begin{bmatrix} \dfrac{g_{22}}{\Delta_g} & \dfrac{-g_{12}}{\Delta_g} \\ \dfrac{-g_{21}}{\Delta_g} & \dfrac{g_{11}}{\Delta_g} \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{10\,\Omega} & \mathbf{2} \\ \mathbf{-1} & \mathbf{0.3\,S} \end{bmatrix}}$$

(d) $$[\mathbf{T}] = \begin{bmatrix} \dfrac{1}{g_{21}} & \dfrac{g_{22}}{g_{21}} \\ \dfrac{g_{11}}{g_{21}} & \dfrac{\Delta_g}{g_{21}} \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{5} & \mathbf{10\,\Omega} \\ \mathbf{0.3\,S} & \mathbf{1} \end{bmatrix}}$$

Chapter 19, Problem 60.

Design a **T** network necessary to realize the following z parameters at $\omega = 10^6$ rad/s

$$[\mathbf{z}] = \begin{bmatrix} 4+j3 & 2 \\ 2 & 5-j \end{bmatrix} \text{k}\Omega$$

Chapter 19, Solution 60.

Comparing this with Fig. 19.5,

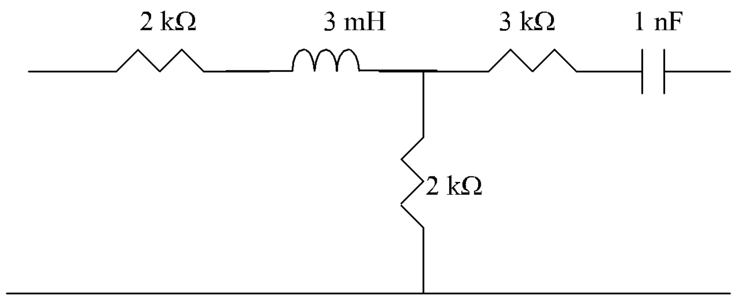
$$z_{11} - z_{12} = 4 + j3 - 2 = 2 + j3k\Omega$$

$z_{22} - z_{12} = 5 - j - 2 = 3 - j$ kΩ

$$X_L = 3x10^3 = \omega L \quad \longrightarrow \quad L = \frac{3x10^3}{10^6} = 3mH$$

$X_C = 1x10^3 = 1/(\omega C)$ or $C = 1/(10^3 x10^6) = 1$ nF

Hence, the resulting T network is shown below.



Chapter 19, Problem 61.

For the bridge circuit in Fig. 19.108, obtain:

(a) the *z* parameters
(b) the *h* parameters
(c) the transmission parameters

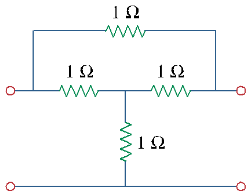
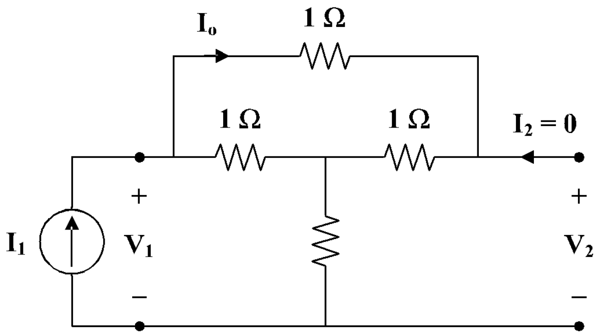


Figure 19.108

For Prob. 19.61.

Chapter 19, Solution 61.

(a) To obtain $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$, consider the circuit in Fig. (a).



(a)

$$\mathbf{V}_1 = \mathbf{I}_1[1+1\,||\,(1+1)] = \mathbf{I}_1\left(1+\frac{2}{3}\right) = \frac{5}{3}\mathbf{I}_1$$

$$\mathbf{z}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{5}{3}$$

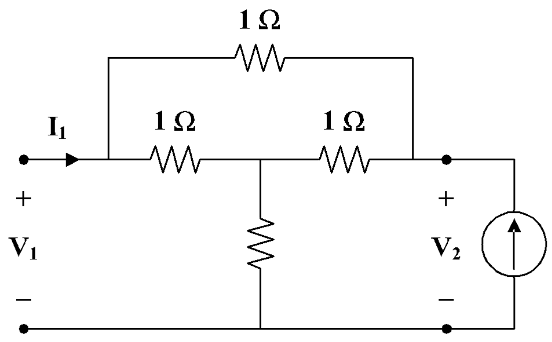
$$\mathbf{I}_o = \frac{1}{1+2}\mathbf{I}_1 = \frac{1}{3}\mathbf{I}_1$$

$$-\mathbf{V}_2 + \mathbf{I}_o + \mathbf{I}_1 = 0$$

$$\mathbf{V}_2 = \frac{1}{3}\mathbf{I}_1 + \mathbf{I}_1 = \frac{4}{3}\mathbf{I}_1$$

$$\mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1} = \frac{4}{3}$$

To obtain $\mathbf{z}_{22}$ and $\mathbf{z}_{12}$, consider the circuit in Fig. (b).



(b)

Due to symmetry, this is similar to the circuit in Fig. (a).

$$\mathbf{z}_{22} = \mathbf{z}_{11} = \frac{5}{3}, \qquad \mathbf{z}_{21} = \mathbf{z}_{12} = \frac{4}{3}$$

$$[\mathbf{z}] = \begin{bmatrix} \frac{5}{3} & \frac{4}{3} \\ \frac{4}{3} & \frac{5}{3} \end{bmatrix} \Omega$$

(b) $$[\mathbf{h}] = \begin{bmatrix} \frac{\Delta_z}{\mathbf{z}_{22}} & \frac{\mathbf{z}_{12}}{\mathbf{z}_{22}} \\ \frac{-\mathbf{z}_{21}}{\mathbf{z}_{22}} & \frac{1}{\mathbf{z}_{22}} \end{bmatrix} = \begin{bmatrix} \frac{3}{5}\Omega & \frac{4}{5} \\ \frac{-4}{5} & \frac{3}{5}\text{S} \end{bmatrix}$$

(c) $$[\mathbf{T}] = \begin{bmatrix} \frac{\mathbf{z}_{11}}{\mathbf{z}_{21}} & \frac{\Delta_z}{\mathbf{z}_{21}} \\ \frac{1}{\mathbf{z}_{21}} & \frac{\mathbf{z}_{22}}{\mathbf{z}_{21}} \end{bmatrix} = \begin{bmatrix} \frac{5}{4} & \frac{3}{4}\Omega \\ \frac{3}{4}\text{S} & \frac{5}{4} \end{bmatrix}$$

Chapter 19, Problem 62.

Find the z parameters of the op amp circuit in Fig. 19.109. Obtain the transmission parameters.

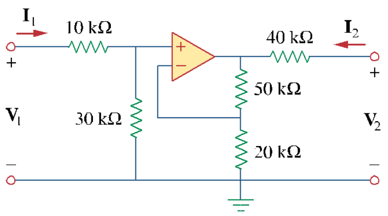
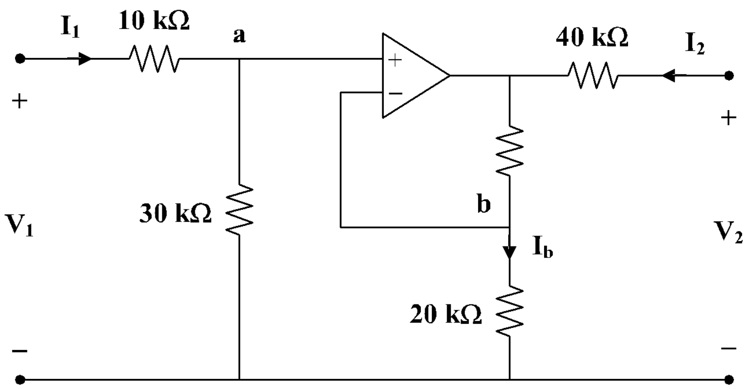


Figure 19.109

For Prob. 19.62.

Chapter 19, Solution 62.

Consider the circuit shown below.



Since no current enters the input terminals of the op amp,

$$\mathbf{V}_1 = (10+30)\times 10^3\, \mathbf{I}_1 \qquad (1)$$

But $$\mathbf{V}_a = \mathbf{V}_b = \frac{30}{40}\mathbf{V}_1 = \frac{3}{4}\mathbf{V}_1$$

$$\mathbf{I}_b = \frac{\mathbf{V}_b}{20\times 10^3} = \frac{3}{80\times 10^3}\mathbf{V}_1$$

which is the same current that flows through the 50-kΩ resistor.

Thus, $$\mathbf{V}_2 = 40\times 10^3\, \mathbf{I}_2 + (50+20)\times 10^3\, \mathbf{I}_b$$

$$\mathbf{V}_2 = 40\times 10^3\, \mathbf{I}_2 + 70\times 10^3 \cdot \frac{3}{80\times 10^3}\mathbf{V}_1$$

$$\mathbf{V}_2 = \frac{21}{8}\mathbf{V}_1 + 40\times 10^3\, \mathbf{I}_2$$

$$\mathbf{V}_2 = 105\times 10^3\, \mathbf{I}_1 + 40\times 10^3\, \mathbf{I}_2 \qquad (2)$$

From (1) and (2),

$$[\mathbf{z}] = \begin{bmatrix} \mathbf{40} & \mathbf{0} \\ \mathbf{105} & \mathbf{40} \end{bmatrix} \mathbf{k\Omega}$$

$$\Delta_z = \mathbf{z}_{11}\mathbf{z}_{22} - \mathbf{z}_{12}\mathbf{z}_{21} = 16\times 10^8$$

$$[\mathbf{T}] = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} = \begin{bmatrix} \frac{\mathbf{z}_{11}}{\mathbf{z}_{21}} & \frac{\Delta_z}{\mathbf{z}_{21}} \\ \frac{1}{\mathbf{z}_{21}} & \frac{\mathbf{z}_{22}}{\mathbf{z}_{21}} \end{bmatrix} = \begin{bmatrix} \mathbf{0.381} & \mathbf{15.24\ k\Omega} \\ \mathbf{9.52\ \mu S} & \mathbf{0.381} \end{bmatrix}$$

Chapter 19, Problem 63.

Determine the *z* parameters of the two-port in Fig. 19.110.

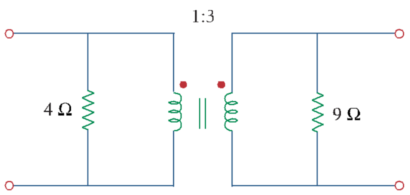
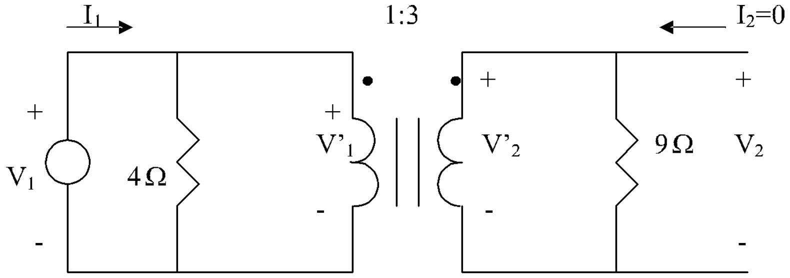


Figure 19.110

For Prob. 19.63.

Chapter 19, Solution 63.

To get z_{11} and z_{21}, consider the circuit below.

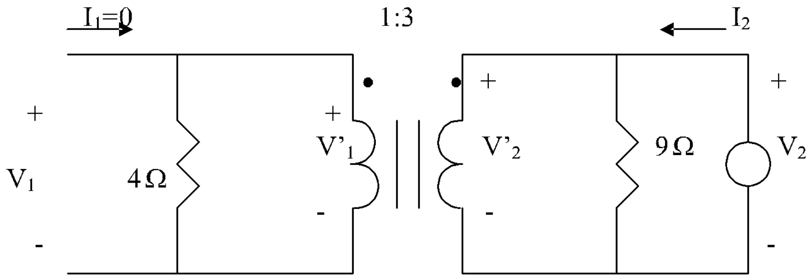


$$Z_R = \frac{9}{n^2} = 1, \qquad n = 3$$

$$V_1 = (4//Z_R)I_1 = \frac{4}{5}I_1 \longrightarrow z_{11} = \frac{V_1}{I_1} = 0.8$$

$$V_2 = V_2' = nV_1' = nV_1 = 3(4/5)I_1 \longrightarrow z_{21} = \frac{V_2}{I_1} = 2.4$$

To get z_{21} and z_{22}, consider the circuit below.



$$Z_R' = n^2(4) = 36, \qquad n = 3$$

$$V_2 = (9//Z_R')I_2 = \frac{9x36}{45}I_2 \longrightarrow z_{22} = \frac{V_2}{I_2} = 7.2$$

$$V_1 = \frac{V_2}{n} = \frac{V_2}{3} = 2.4I_2 \longrightarrow z_{21} = \frac{V_1}{I_2} = 2.4$$

Thus,

$$[z] = \begin{bmatrix} 0.8 & 2.4 \\ 2.4 & 7.2 \end{bmatrix}\Omega$$

Chapter 19, Problem 64.

Determine the y parameters at $\omega = 1{,}000$ rad/s for the op amp circuit in Fig. 19.111. Find the corresponding h parameters.

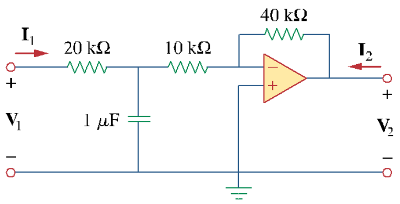


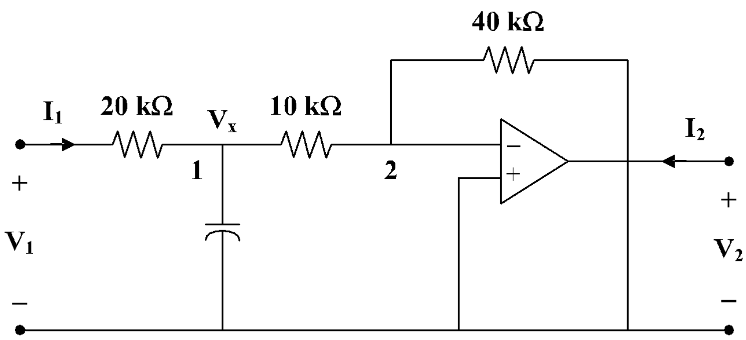
Figure 19.111

For Prob. 19.64.

Chapter 19, Solution 64.

$$1\ \mu F \longrightarrow \frac{1}{j\omega C} = \frac{-j}{(10^3)(10^{-6})} = -j\ k\Omega$$

Consider the op amp circuit below.



At node 1,

$$\frac{V_1 - V_x}{20} = \frac{V_x}{-j} + \frac{V_x - 0}{10}$$

$$V_1 = (3 + j20)V_x \qquad (1)$$

At node 2,

$$\frac{\mathbf{V}_x - 0}{10} = \frac{0 - \mathbf{V}_2}{40} \longrightarrow \mathbf{V}_x = \frac{-1}{4}\mathbf{V}_2 \qquad (2)$$

But $$\mathbf{I}_1 = \frac{\mathbf{V}_1 - \mathbf{V}_x}{20 \times 10^3} \qquad (3)$$

Substituting (2) into (3) gives

$$\mathbf{I}_1 = \frac{\mathbf{V}_1 + 0.25\mathbf{V}_2}{20 \times 10^3} = 50 \times 10^{-6}\,\mathbf{V}_1 + 12.5 \times 10^{-6}\,\mathbf{V}_2 \qquad (4)$$

Substituting (2) into (1) yields

$$\mathbf{V}_1 = \frac{-1}{4}(3 + j20)\mathbf{V}_2$$

or $$0 = \mathbf{V}_1 + (0.75 + j5)\mathbf{V}_2 \qquad (5)$$

Comparing (4) and (5) with the following equations

$$\mathbf{I}_1 = \mathbf{y}_{11}\,\mathbf{V}_1 + \mathbf{y}_{12}\,\mathbf{V}_2$$
$$\mathbf{I}_2 = \mathbf{y}_{21}\,\mathbf{V}_1 + \mathbf{y}_{22}\,\mathbf{V}_2$$

indicates that $\mathbf{I}_2 = 0$ and that

$$[\mathbf{y}] = \begin{bmatrix} \mathbf{50 \times 10^{-6}} & \mathbf{12.5 \times 10^{-6}} \\ \mathbf{1} & \mathbf{0.75 + j5} \end{bmatrix} \mathbf{S}$$

$$\Delta_y = (77.5 + j25. - 12.5) \times 10^{-6} = (65 + j250) \times 10^{-6}$$

$$[\mathbf{h}] = \begin{bmatrix} \dfrac{1}{\mathbf{y}_{11}} & \dfrac{-\mathbf{y}_{12}}{\mathbf{y}_{11}} \\ \dfrac{\mathbf{y}_{21}}{\mathbf{y}_{11}} & \dfrac{\Delta_y}{\mathbf{y}_{11}} \end{bmatrix} = \begin{bmatrix} \mathbf{2 \times 10^4\ \Omega} & \mathbf{-0.25} \\ \mathbf{2 \times 10^4} & \mathbf{1.3 + j5\ S} \end{bmatrix}$$

Chapter 19, Problem 65.

What is the *y* parameter presentation of the circuit in Fig. 19.112?

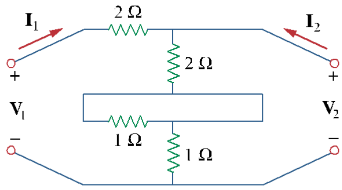


Figure 19.112

For Prob. 19.65.

Chapter 19, Solution 65.

The network consists of two two-ports in series. It is better to work with z parameters and then convert to y parameters.

For N_a, $[\mathbf{z}_a] = \begin{bmatrix} 4 & 2 \\ 2 & 2 \end{bmatrix}$

For N_b, $[\mathbf{z}_b] = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

$$[\mathbf{z}] = [\mathbf{z}_a] + [\mathbf{z}_b] = \begin{bmatrix} 6 & 3 \\ 3 & 3 \end{bmatrix}$$

$$\Delta_z = 18 - 9 = 9$$

$$[\mathbf{y}] = \begin{bmatrix} \frac{\mathbf{z}_{22}}{\Delta_z} & \frac{-\mathbf{z}_{12}}{\Delta_z} \\ \frac{-\mathbf{z}_{21}}{\Delta_z} & \frac{\mathbf{z}_{11}}{\Delta_z} \end{bmatrix} = \underline{\begin{bmatrix} \frac{1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{2}{3} \end{bmatrix} \mathbf{S}}$$

Chapter 19, Problem 66.

In the two-port of Fig. 19.113, let $\mathbf{y}_{12} = \mathbf{y}_{21} = 0$, $\mathbf{y}_{11} = 2$ mS, and $\mathbf{y}_{22} = 10$ mS. Find $\mathbf{V}_o/\mathbf{V}_s$.

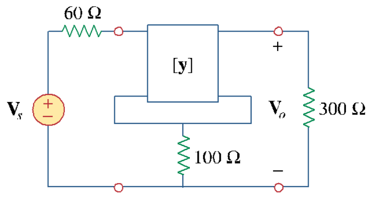


Figure 19.113

For Prob. 19.66.

Chapter 19, Solution 66.

Since we have two two-ports in series, it is better to convert the given y parameters to z parameters.

$$\Delta_y = \mathbf{y}_{11}\mathbf{y}_{22} - \mathbf{y}_{12}\mathbf{y}_{21} = (2\times10^{-3})(10\times10^{-3}) - 0 = 20\times10^{-6}$$

$$[\mathbf{z}_a] = \begin{bmatrix} \frac{\mathbf{y}_{22}}{\Delta_y} & \frac{-\mathbf{y}_{12}}{\Delta_y} \\ \frac{-\mathbf{y}_{21}}{\Delta_y} & \frac{\mathbf{y}_{11}}{\Delta_y} \end{bmatrix} = \begin{bmatrix} 500\,\Omega & 0 \\ 0 & 100\,\Omega \end{bmatrix}$$

$$[\mathbf{z}] = \begin{bmatrix} 500 & 0 \\ 0 & 100 \end{bmatrix} + \begin{bmatrix} 100 & 100 \\ 100 & 100 \end{bmatrix} = \begin{bmatrix} 600 & 100 \\ 100 & 200 \end{bmatrix}$$

i.e.
$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2$$
$$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2$$

or
$$\mathbf{V}_1 = 600\mathbf{I}_1 + 100\mathbf{I}_2 \qquad (1)$$
$$\mathbf{V}_2 = 100\mathbf{I}_1 + 200\mathbf{I}_2 \qquad (2)$$

But, at the input port,
$$\mathbf{V}_s = \mathbf{V}_1 + 60\mathbf{I}_1 \qquad (3)$$

and at the output port,
$$\mathbf{V}_2 = \mathbf{V}_o = -300\mathbf{I}_2 \qquad (4)$$

From (2) and (4),
$$100\mathbf{I}_1 + 200\mathbf{I}_2 = -300\mathbf{I}_2$$
$$\mathbf{I}_1 = -5\mathbf{I}_2 \qquad (5)$$

Substituting (1) and (5) into (3),
$$\mathbf{V}_s = 600\mathbf{I}_1 + 100\mathbf{I}_2 + 60\mathbf{I}_1$$
$$= (660)(-5)\mathbf{I}_2 + 100\mathbf{I}_2$$
$$= -3200\mathbf{I}_2 \qquad (6)$$

From (4) and (6),
$$\frac{\mathbf{V}_o}{\mathbf{V}_2} = \frac{-300\mathbf{I}_2}{-3200\mathbf{I}_2} = \underline{\mathbf{0.09375}}$$

Chapter 19, Problem 67.

If three copies of the circuit in Fig. 19.114 are connected in parallel, find the overall transmission parameters.

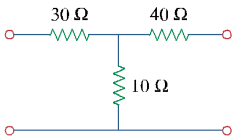
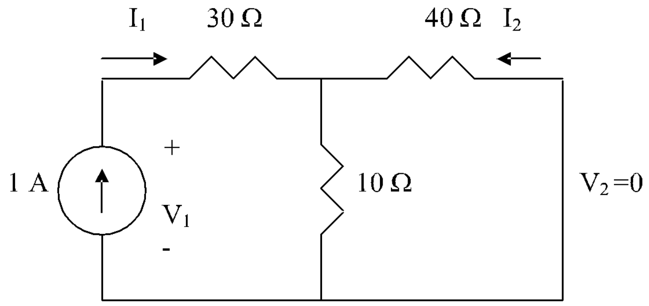


Figure 19.114

For Prob. 19.67.

Chapter 19, Solution 67.

We first the y parameters, to find y_{11} and y_{21} consider the circuit below.

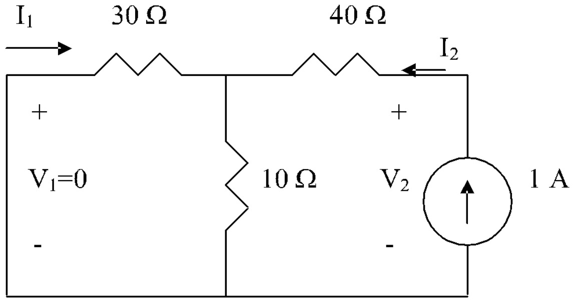


$$V_1 = I_1(30 + 10 // 40) = 38 I_1 \quad \longrightarrow \quad y_{11} = \frac{I_1}{V_1} = \frac{1}{38}$$

By current division,

$$I_2 = \frac{-10}{50} I_1 = -0.2 I_1 \quad \longrightarrow \quad y_{21} = \frac{I_2}{V_1} = \frac{-0.2 I_1}{38 I_1} = \frac{-1}{190}$$

To find y_{22} and y_{12} consider the circuit below.



$$V_2 = (40 + 10 // 30)I_2 = 47.5I_2 \longrightarrow y_{22} = \frac{I_2}{V_2} = \frac{2}{93}$$ $y_{22} = 2/95$

By current division,

$$I_1 = -\frac{10}{30+10}I_2 = -\frac{I_2}{4} \longrightarrow y_{12} = \frac{I_1}{V_2} = \frac{-\frac{1}{4}I_2}{47.5I_2} = -\frac{1}{190}$$

$$[y] = \begin{bmatrix} 1/38 & -1/190 \\ -1/190 & 2/95 \end{bmatrix}$$

For three copies cascaded in parallel, we can use MATLAB.

```
>> Y=[1/38,-1/190;-1/190,2/95]
Y =
   0.0263  -0.0053
  -0.0053   0.0211
>> Y3=3*Y
Y3 =
   0.0789  -0.0158
  -0.0158   0.0632
>> DY=0.0789*0.0632-0.0158*0.158
DY =
   0.0025
>> T=[0.0632/0.0158,1/0.0158;DY/0.0158,0.0789/0.0158]
T =
   4.0000  63.2911
   0.1576   4.9937
```

$$T = \begin{bmatrix} 4 & 63.29 \\ 0.1576 & 4.994 \end{bmatrix}$$

Chapter 19, Problem 68.

Obtain the *h* parameters for the network in Fig. 19.115.

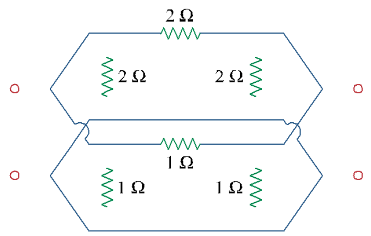


Figure 19.115
For Prob. 19.68.

Chapter 19, Solution 68.

For the upper network N_a, $[\mathbf{y}_a] = \begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix}$

and for the lower network N_b, $[\mathbf{y}_b] = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$

For the overall network,

$$[\mathbf{y}] = [\mathbf{y}_a] + [\mathbf{y}_b] = \begin{bmatrix} 6 & -3 \\ -3 & 6 \end{bmatrix}$$

$$\Delta_y = 36 - 9 = 27$$

$$[\mathbf{h}] = \begin{bmatrix} \frac{1}{\mathbf{y}_{11}} & \frac{-\mathbf{y}_{12}}{\mathbf{y}_{11}} \\ \frac{\mathbf{y}_{21}}{\mathbf{y}_{11}} & \frac{\Delta_y}{\mathbf{y}_{11}} \end{bmatrix} = \underline{\begin{bmatrix} \frac{1}{6}\Omega & \frac{1}{2} \\ \frac{1}{2} & \frac{9}{2}\text{S} \end{bmatrix}}$$

Chapter 19, Problem 69.

* The circuit in Fig. 19.116 may be regarded as two two-ports connected in parallel. Obtain the *y* parameters as functions of *s*.

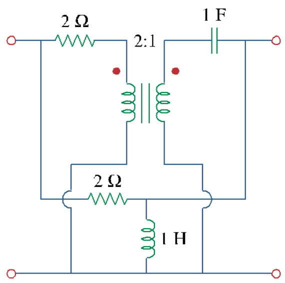


Figure 19.116
For Prob. 19.69.

* An asterisk indicates a challenging problem.

Chapter 19, Solution 69.

We first determine the y parameters for the upper network N_a.
To get $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$, consider the circuit in Fig. (a).

$$n = \frac{1}{2}, \qquad \mathbf{Z}_R = \frac{1/s}{n^2} = \frac{4}{s}$$

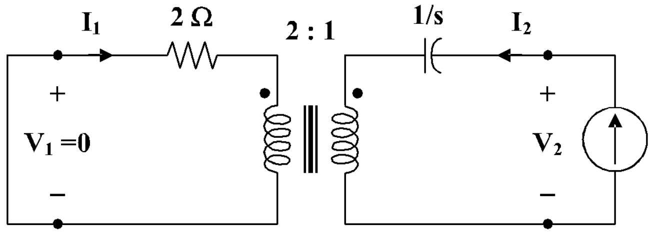
$$\mathbf{V}_1 = (2 + \mathbf{Z}_R)\mathbf{I}_1 = \left(2 + \frac{4}{s}\right)\mathbf{I}_1 = \left(\frac{2s+4}{s}\right)\mathbf{I}_1$$

$$\mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{s}{2(s+2)}$$

$$\mathbf{I}_2 = \frac{-\mathbf{I}_1}{n} = -2\mathbf{I}_1 = \frac{-s\mathbf{V}_1}{s+2}$$

$$\mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \frac{-s}{s+2}$$

To get $\mathbf{y}_{22}$ and $\mathbf{y}_{12}$, consider the circuit in Fig. (b).



(b)

$$\mathbf{Z}_R' = (n^2)(2) = \left(\frac{1}{4}\right)(2) = \frac{1}{2}$$

$$\mathbf{V}_2 = \left(\frac{1}{s} + \mathbf{Z}_R'\right)\mathbf{I}_2 = \left(\frac{1}{s} + \frac{1}{2}\right)\mathbf{I}_2 = \left(\frac{s+2}{2s}\right)\mathbf{I}_2$$

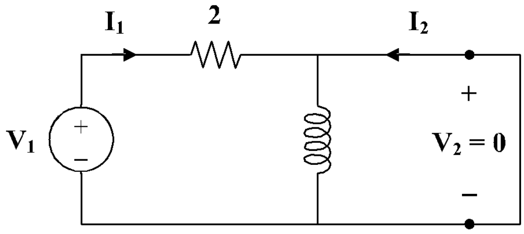
$$\mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{2s}{s+2}$$

$$\mathbf{I}_1 = -n\,\mathbf{I}_2 = \left(\frac{-1}{2}\right)\left(\frac{2s}{s+2}\right)\mathbf{V}_2 = \left(\frac{-s}{s+2}\right)\mathbf{V}_2$$

$$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{-s}{s+2}$$

$$[\mathbf{y}_a] = \begin{bmatrix} \dfrac{s}{2(s+2)} & \dfrac{-s}{s+2} \\ \dfrac{-s}{s+2} & \dfrac{2s}{s+2} \end{bmatrix}$$

For the lower network N_b, we obtain $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$ by referring to the network in Fig. (c).

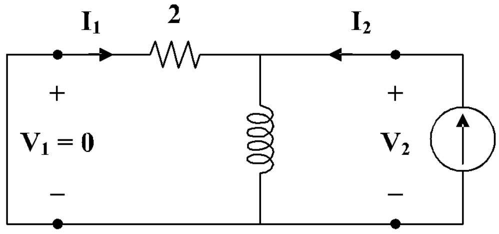


(c)

$$\mathbf{V}_1 = 2\mathbf{I}_1 \longrightarrow \mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{1}{2}$$

$$\mathbf{I}_2 = -\mathbf{I}_1 = \frac{-\mathbf{V}_1}{2} \longrightarrow \mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \frac{-1}{2}$$

To get $\mathbf{y}_{22}$ and $\mathbf{y}_{12}$, refer to the circuit in Fig. (d).



(d)

$$\mathbf{V}_2 = (s \,||\, 2)\mathbf{I}_2 = \frac{2s}{s+2}\mathbf{I}_2 \longrightarrow \mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{s+2}{2s}$$

$$\mathbf{I}_1 = -\mathbf{I}_2 \cdot \frac{-s}{s+2} = \left(\frac{-s}{s+2}\right)\left(\frac{s+2}{2s}\right)\mathbf{V}_2 = \frac{-\mathbf{V}_2}{2}$$

$$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{-1}{2}$$

$$[\mathbf{y}_b] = \begin{bmatrix} 1/2 & -1/2 \\ -1/2 & (s+2)/2s \end{bmatrix}$$

$$[\mathbf{y}] = [\mathbf{y}_a] + [\mathbf{y}_b] = \begin{bmatrix} \dfrac{\mathbf{s+1}}{\mathbf{s+2}} & \dfrac{\mathbf{-(3s+2)}}{\mathbf{2(s+2)}} \\ \dfrac{\mathbf{-(3s+2)}}{\mathbf{2(s+2)}} & \dfrac{\mathbf{5s^2+4s+4}}{\mathbf{2s(s+2)}} \end{bmatrix}$$

Chapter 19, Problem 70.

* For the parallel-series connection of the two two-ports in Fig. 19.117, find the *g* parameters.

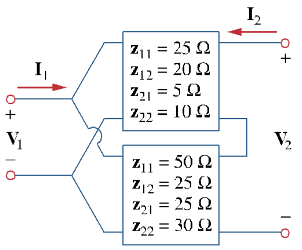


Figure 19.117

For Prob. 19.70.

* An asterisk indicates a challenging problem.

Chapter 19, Solution 70.

We may obtain the g parameters from the given z parameters.

$$[\mathbf{z}_a] = \begin{bmatrix} 25 & 20 \\ 5 & 10 \end{bmatrix}, \qquad \Delta_{z_a} = 250 - 100 = 150$$

$$[\mathbf{z}_b] = \begin{bmatrix} 50 & 25 \\ 25 & 30 \end{bmatrix}, \qquad \Delta_{z_b} = 1500 - 625 = 875$$

$$[\mathbf{g}] = \begin{bmatrix} \dfrac{1}{z_{11}} & \dfrac{-z_{12}}{z_{11}} \\ \dfrac{z_{21}}{z_{11}} & \dfrac{\Delta_z}{z_{11}} \end{bmatrix}$$

$$[\mathbf{g}_a] = \begin{bmatrix} 0.04 & -0.8 \\ 0.2 & 6 \end{bmatrix}, \qquad [\mathbf{g}_b] = \begin{bmatrix} 0.02 & -0.5 \\ 0.5 & 17.5 \end{bmatrix}$$

$$[\mathbf{g}] = [\mathbf{g}_a] + [\mathbf{g}_b] = \underline{\begin{bmatrix} \mathbf{0.06\ S} & \mathbf{-1.3} \\ \mathbf{0.7} & \mathbf{23.5\ \Omega} \end{bmatrix}}$$

Chapter 19, Problem 71.

* Determine the z parameters for the network in Fig. 19.118.

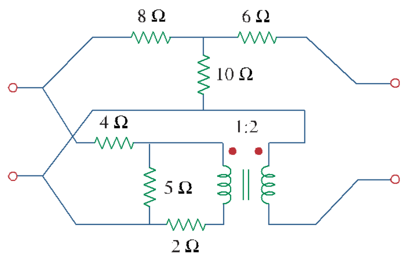


Figure 19.118

For Prob. 19.71.

* An asterisk indicates a challenging problem.

Chapter 19, Solution 71.

This is a parallel-series connection of two two-ports. We need to add their g parameters together and obtain z parameters from there.

For the transformer,

$$V_1 = \frac{1}{2}V_2, \quad I_1 = -2I_2$$

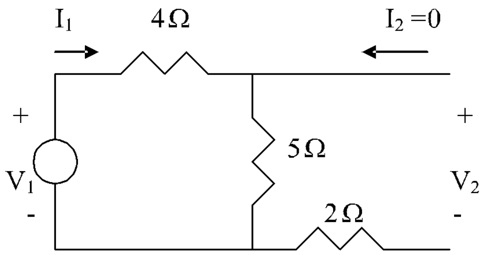
Comparing this with

$$V_1 = AV_2 - BI_2, \qquad I_1 = CV_2 - DI_2$$

shows that

$$[T_{b1}] = \begin{bmatrix} 0.5 & 0 \\ 0 & 2 \end{bmatrix}$$

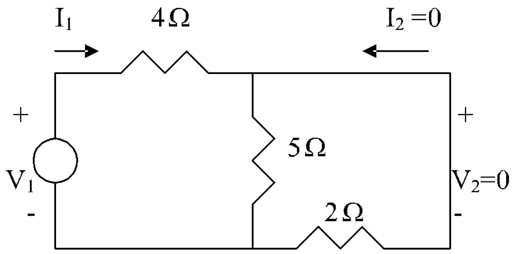
To get A and C for T_{b2} , consider the circuit below.



$$V_1 = 9I_1, \quad V_2 = 5I_1$$

$$A = \frac{V_1}{V_2} = 9/5 = 1.8, \qquad C = \frac{I_1}{V_2} = 1/5 = 0.2$$

We obtain B and D by looking at the circuit below.



$$I_2 = -\frac{5}{7}I_1 \quad\longrightarrow\quad D = -\frac{I_1}{I_2} = 7/5 = 1.4$$

$$V_1 = 4I_1 - 2I_2 = 4(-\frac{7}{5}I_2) - 2I_2 = -\frac{38}{5}I_2 \quad\longrightarrow\quad B = -\frac{V_1}{I_2} = 7.6$$

$$[T_{b2}] = \begin{bmatrix} 1.8 & 7.6 \\ 0.2 & 1.4 \end{bmatrix}$$

$$[T] = [T_{b1}][T_{b2}] = \begin{bmatrix} 0.9 & 3.8 \\ 0.4 & 2.8 \end{bmatrix}, \qquad \Delta_T = 1$$

$$[g_b] = \begin{bmatrix} C/A & -\Delta_T/A \\ 1/A & B/A \end{bmatrix} = \begin{bmatrix} 0.4444 & -1.1111 \\ 1.1111 & 4.2222 \end{bmatrix}$$

From Prob. 19.52,

$$[T_a] = \begin{bmatrix} 1.8 & 18.8 \\ 0.1 & 1.6 \end{bmatrix}$$

$$[g_a] = \begin{bmatrix} C/A & -\Delta_T/A \\ 1/A & B/A \end{bmatrix} = \begin{bmatrix} 0.05555 & -0.5555 \\ 0.5555 & 10.4444 \end{bmatrix}$$

$$[g] = [g_a] + [g_b] = \begin{bmatrix} 0.4999 & -1.6667 \\ 1.6667 & 14.667 \end{bmatrix}$$

Thus,

$$[z] = \begin{bmatrix} 1/g_{11} & -g_{21}/g_{11} \\ g_{21}/g_{11} & \Delta_g/g_{11} \end{bmatrix} = \underline{\begin{bmatrix} 2 & -3.334 \\ 3.334 & 20.22 \end{bmatrix}\Omega}$$

Chapter 19, Problem 72.

* A series-parallel connection of two two-ports is shown in Fig. 19.119. Determine the *z* parameter representation of the network.

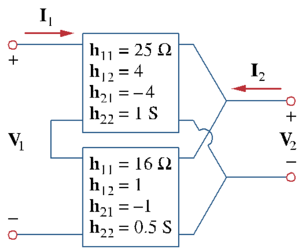


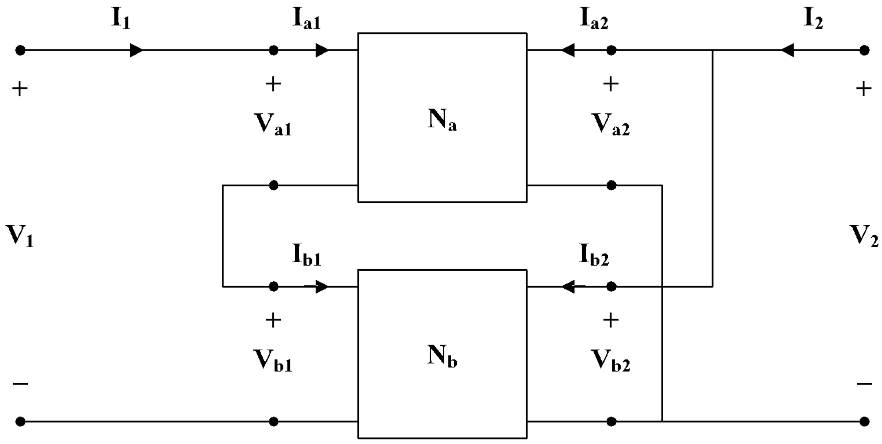
Figure 19.119

For Prob. 19.72.

* An asterisk indicates a challenging problem.

Chapter 19, Solution 72.

Consider the network shown below.



$$\mathbf{V}_{a1} = 25\mathbf{I}_{a1} + 4\mathbf{V}_{a2} \qquad (1)$$
$$\mathbf{I}_{a2} = -4\mathbf{I}_{a1} + \mathbf{V}_{a2} \qquad (2)$$
$$\mathbf{V}_{b1} = 16\mathbf{I}_{b1} + \mathbf{V}_{b2} \qquad (3)$$
$$\mathbf{I}_{b2} = -\mathbf{I}_{b1} + 0.5\mathbf{V}_{b2} \qquad (4)$$

$$\mathbf{V}_1 = \mathbf{V}_{a1} + \mathbf{V}_{b1}$$
$$\mathbf{V}_2 = \mathbf{V}_{a2} = \mathbf{V}_{b2}$$
$$\mathbf{I}_2 = \mathbf{I}_{a2} + \mathbf{I}_{b2}$$
$$\mathbf{I}_1 = \mathbf{I}_{a1}$$

Now, rewrite (1) to (4) in terms of $\mathbf{I}_1$ and $\mathbf{V}_2$

$$\mathbf{V}_{a1} = 25\mathbf{I}_1 + 4\mathbf{V}_2 \qquad (5)$$
$$\mathbf{I}_{a2} = -4\mathbf{I}_1 + \mathbf{V}_2 \qquad (6)$$
$$\mathbf{V}_{b1} = 16\mathbf{I}_{b1} + \mathbf{V}_2 \qquad (7)$$
$$\mathbf{I}_{b2} = -\mathbf{I}_{b1} + 0.5\mathbf{V}_2 \qquad (8)$$

Adding (5) and (7),

$$\mathbf{V}_1 = 25\mathbf{I}_1 + 16\mathbf{I}_{b1} + 5\mathbf{V}_2 \qquad (9)$$

Adding (6) and (8),

$$\mathbf{I}_2 = -4\mathbf{I}_1 - \mathbf{I}_{b1} + 1.5\mathbf{V}_2 \qquad (10)$$
$$\mathbf{I}_{b1} = \mathbf{I}_{a1} = \mathbf{I}_1 \qquad (11)$$

Because the two networks N_a and N_b are independent,

$$\mathbf{I}_2 = -5\mathbf{I}_1 + 1.5\mathbf{V}_2$$

or
$$\mathbf{V}_2 = 3.333\mathbf{I}_1 + 0.6667\mathbf{I}_2 \tag{12}$$

Substituting (11) and (12) into (9),

$$\mathbf{V}_1 = 41\mathbf{I}_1 + \frac{25}{1.5}\mathbf{I}_1 + \frac{5}{1.5}\mathbf{I}_2$$

$$\mathbf{V}_1 = 57.67\mathbf{I}_1 + 3.333\mathbf{I}_2 \tag{13}$$

Comparing (12) and (13) with the following equations

$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2$$

$$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2$$

indicates that

$$[\mathbf{z}] = \underline{\begin{bmatrix} \mathbf{57.67} & \mathbf{3.333} \\ \mathbf{3.333} & \mathbf{0.6667} \end{bmatrix}\Omega}$$

Alternatively,

$$[\mathbf{h}_a] = \begin{bmatrix} 25 & 4 \\ -4 & 1 \end{bmatrix}, \qquad [\mathbf{h}_b] = \begin{bmatrix} 16 & 1 \\ -1 & 0.5 \end{bmatrix}$$

$$[\mathbf{h}] = [\mathbf{h}_a] + [\mathbf{h}_b] = \begin{bmatrix} 41 & 5 \\ -5 & 1.5 \end{bmatrix} \qquad \Delta_h = 61.5 + 25 = 86.5$$

$$[\mathbf{z}] = \begin{bmatrix} \dfrac{\Delta_h}{\mathbf{h}_{22}} & \dfrac{\mathbf{h}_{12}}{\mathbf{h}_{22}} \\ \dfrac{-\mathbf{h}_{21}}{\mathbf{h}_{22}} & \dfrac{1}{\mathbf{h}_{22}} \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{57.67} & \mathbf{3.333} \\ \mathbf{3.333} & \mathbf{0.6667} \end{bmatrix}\Omega}$$

as obtained previously.

Chapter 19, Problem 73.

ML

Three copies of the circuit shown in Fig. 19.70 are connected in cascade. Determine the *z* parameters.

Chapter 19, Solution 73.

From Problem 19.6,

$$[z] = \begin{bmatrix} 25 & 20 \\ 24 & 30 \end{bmatrix}, \qquad \Delta z = 25x30 - 20x24 = 270$$

$$A = \frac{z_{11}}{z_{21}} = \frac{25}{24}, \qquad B = \frac{\Delta z}{z_{21}} = \frac{270}{24}$$

$$C = \frac{1}{z_{21}} = \frac{1}{24}, \; D = \frac{z_{22}}{z_{21}} = \frac{30}{24}$$

The overall ABCD parameters can be found using MATLAB.

```
>> T=[25/24,270/24;1/24,30/24]
T =
   1.0417   11.2500
   0.0417    1.2500
>> T3=T*T*T
T3 =
   2.6928   49.7070
   0.1841    3.6133
>> Z=[2.693/0.1841,(2.693*3.613-0.1841*49.71)/0.1841;1/0.1841,3.613/0.1841]
Z =
  14.6279    3.1407
   5.4318   19.6252
```

$$Z = \underline{\begin{bmatrix} 14.628 & 3.141 \\ 5.432 & 19.625 \end{bmatrix}}$$

Chapter 19, Problem 74.

* Determine the **ABCD** parameters of the circuit in Fig. 19.120 as functions of *s*. (*Hint:* Partition the circuit into subcircuits and cascade them using the results of Prob. 19.43.)

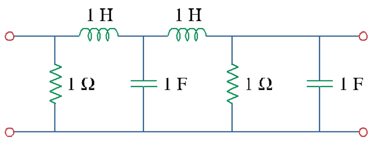


Figure 19.120

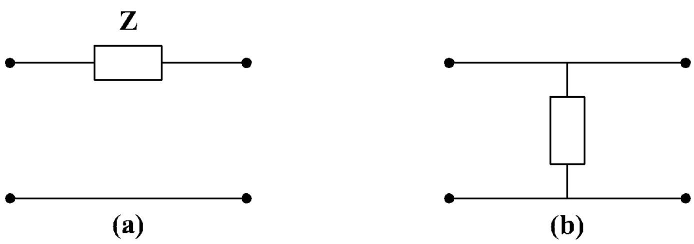
For Prob. 19.74.

* An asterisk indicates a challenging problem.

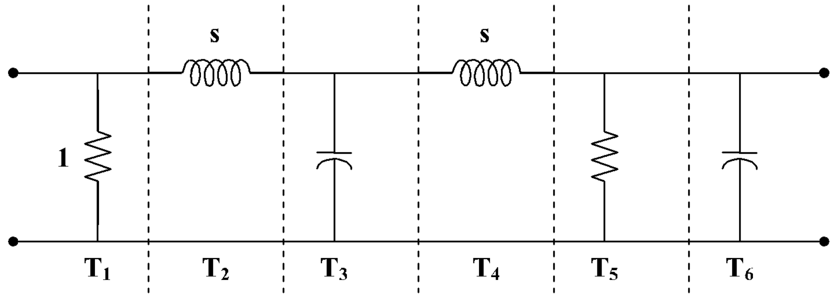
Chapter 19, Solution 74.

From Prob. 18.35, the transmission parameters for the circuit in Figs. (a) and (b) are

$$[\mathbf{T}_a] = \begin{bmatrix} 1 & \mathbf{Z} \\ 0 & 1 \end{bmatrix}, \qquad [\mathbf{T}_b] = \begin{bmatrix} 1 & 0 \\ 1/\mathbf{Z} & 1 \end{bmatrix}$$



We partition the given circuit into six subcircuits similar to those in Figs. (a) and (b) as shown in Fig. (c) and obtain [T] for each.



$$[\mathbf{T}_1]=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix},\qquad [\mathbf{T}_2]=\begin{bmatrix}1 & s\\ 0 & 1\end{bmatrix},\qquad [\mathbf{T}_3]=\begin{bmatrix}1 & 0\\ s & 1\end{bmatrix}$$

$$[\mathbf{T}_4]=[\mathbf{T}_2],\qquad [\mathbf{T}_5]=[\mathbf{T}_1],\qquad [\mathbf{T}_6]=[\mathbf{T}_3]$$

$$[\mathbf{T}]=[\mathbf{T}_1][\mathbf{T}_2][\mathbf{T}_3][\mathbf{T}_4][\mathbf{T}_5][\mathbf{T}_6]=[\mathbf{T}_1][\mathbf{T}_2][\mathbf{T}_3][\mathbf{T}_4]\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ s & 1\end{bmatrix}$$

$$=[\mathbf{T}_1][\mathbf{T}_2][\mathbf{T}_3][\mathbf{T}_4]\begin{bmatrix}1 & 0\\ s+1 & 1\end{bmatrix}=[\mathbf{T}_1][\mathbf{T}_2][\mathbf{T}_3]\begin{bmatrix}1 & s\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ s+1 & 1\end{bmatrix}$$

$$=[\mathbf{T}_1][\mathbf{T}_2]\begin{bmatrix}1 & 0\\ s & 1\end{bmatrix}\begin{bmatrix}s^2+s+1 & s\\ s+1 & 1\end{bmatrix}$$

$$=[\mathbf{T}_1]\begin{bmatrix}1 & s\\ 0 & 1\end{bmatrix}\begin{bmatrix}s^2+s+1 & s\\ s^3+s^2+2s+1 & s^2+1\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}\begin{bmatrix}s^4+s^3+3s^2+2s+1 & s^3+2s\\ s^3+s^2+2s+1 & s^2+1\end{bmatrix}$$

$$[\mathbf{T}]=\begin{bmatrix}\mathbf{s^4+s^3+3s^2+2s+1} & \mathbf{s^3+2s}\\ \mathbf{s^4+2s^3+4s^2+4s+2} & \mathbf{s^3+s^2+2s+1}\end{bmatrix}$$

Note that $\mathbf{AB}-\mathbf{CD}=1$ as expected.

Chapter 19, Problem 75.

* For the individual two-ports shown in Fig. 19.121 where,

$$[\mathbf{z}_a] = \begin{bmatrix} 8 & 6 \\ 4 & 5 \end{bmatrix} \Omega \qquad [\mathbf{y}_b] = \begin{bmatrix} 8 & -4 \\ 2 & 10 \end{bmatrix} \text{S}$$

(a) Determine the y parameters of the overall two-port.
(b) Find the voltage ratio $\mathbf{V}_o / \mathbf{V}_i$ when $\mathbf{Z}_L = 2\ \Omega$.

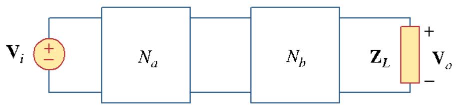


Figure 19.110

For Prob. 19.63.

* An asterisk indicates a challenging problem.

Chapter 19, Solution 75.

(a) We convert $[z_a]$ and $[z_b]$ to T-parameters. For N_a, $\Delta_z = 40 - 24 = 16$.

$$[T_a] = \begin{bmatrix} z_{11}/z_{21} & \Delta_z/z_{21} \\ 1/z_{21} & z_{22}/z_{21} \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 0.25 & 1.25 \end{bmatrix}$$

For N_b, $\Delta_y = 80 + 8 = 88$.

$$[T_b] = \begin{bmatrix} -y_{22}/y_{21} & -1/y_{21} \\ -\Delta_y/y_{21} & -y_{11}/y_{21} \end{bmatrix} = \begin{bmatrix} -5 & -0.5 \\ -44 & -4 \end{bmatrix}$$

$$[T] = [T_a][T_b] = \begin{bmatrix} -186 & -17 \\ -56.25 & -5.125 \end{bmatrix}$$

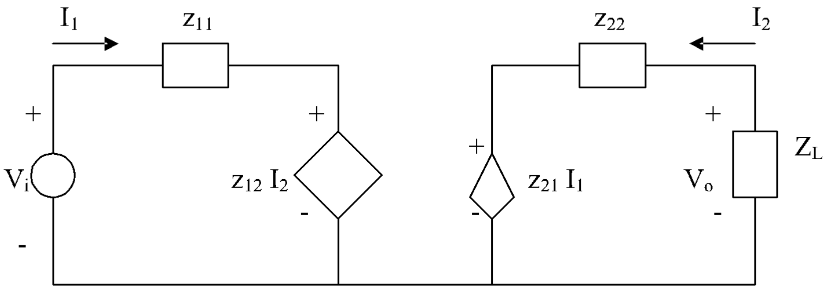
We convert this to y-parameters. $\Delta_T = AD - BC = -3$.

$$[y] = \begin{bmatrix} D/B & -\Delta_T/B \\ -1/B & A/B \end{bmatrix} = \begin{bmatrix} 0.3015 & -0.1765 \\ 0.0588 & 10.94 \end{bmatrix}$$

(b) The equivalent z-parameters are

$$[z] = \begin{bmatrix} A/C & \Delta_T/C \\ 1/C & D/C \end{bmatrix} = \begin{bmatrix} 3.3067 & 0.0533 \\ -0.0178 & 0.0911 \end{bmatrix}$$

Consider the equivalent circuit below.



$$V_i = z_{11}I_1 + z_{12}I_2 \qquad (1)$$

$$V_o = z_{21}I_1 + z_{22}I_2 \qquad (2)$$

But $V_o = -I_2 Z_L \longrightarrow I_2 = -V_o / Z_L$ (3)

From (2) and (3) ,

$$V_o = z_{21}I_1 - z_{22}\frac{V_o}{Z_L} \longrightarrow I_1 = V_o\left(\frac{1}{z_{21}} + \frac{z_{22}}{Z_L z_{21}}\right) \qquad (4)$$

Substituting (3) and (4) into (1) gives

$$\frac{V_i}{V_o} = \left(\frac{z_{11}}{z_{21}} + \frac{z_{11}z_{22}}{z_{21}Z_L}\right) - \frac{z_{12}}{Z_L} = -194.3 \longrightarrow \underline{\frac{V_o}{V_i} = -0.0051}$$

Chapter 19, Problem 76.

Use *PSpice* to obtain the *z* parameters of the network in Fig. 19.122.

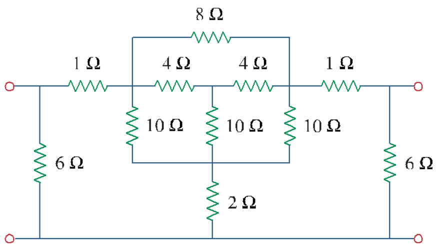


Figure 19.122

For Prob. 19.76.

Chapter 19, Solution 76.

To get z_{11} and z_{21}, we open circuit the output port and let $I_1 = 1A$ so that

$$z_{11} = \frac{V_1}{I_1} = V_1, \qquad z_{21} = \frac{V_2}{I_1} = V_2$$

The schematic is shown below. After it is saved and run, we obtain

$$z_{11} = V_1 = 3.849, \qquad z_{21} = V_2 = 1.122$$

Similarly, to get z_{22} and z_{12}, we open circuit the input port and let $I_2 = 1A$ so that

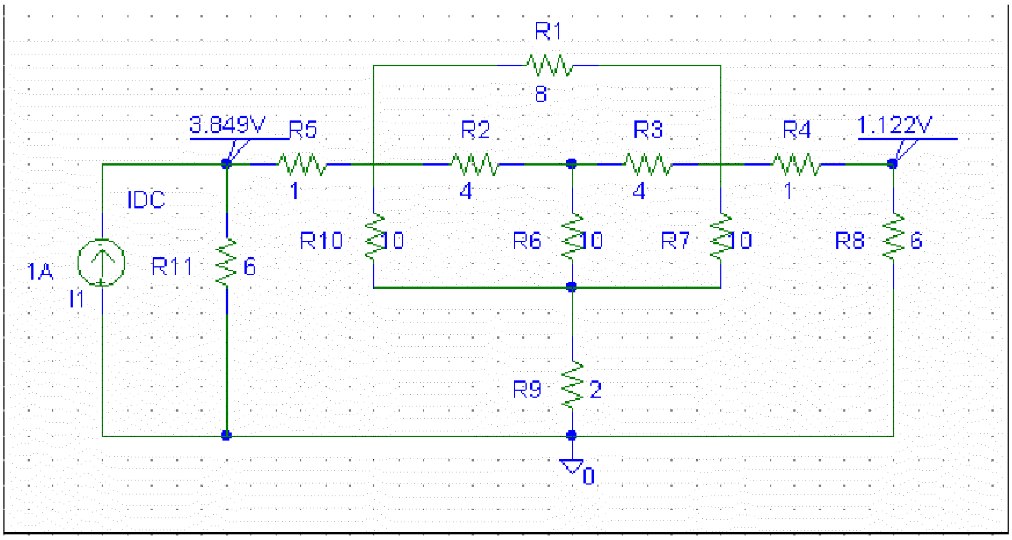
$$z_{12} = \frac{V_1}{I_2} = V_1, \qquad z_{22} = \frac{V_2}{I_2} = V_2$$

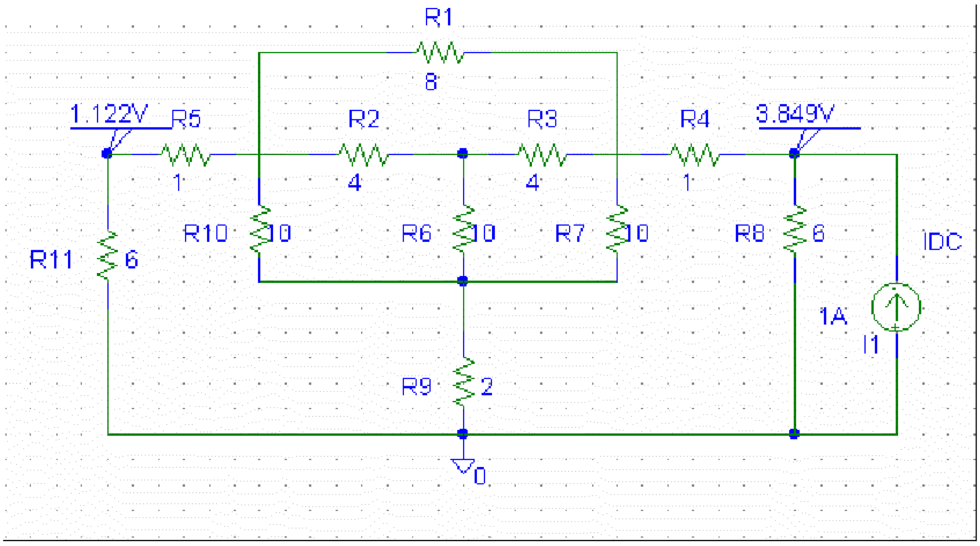
The schematic is shown below. After it is saved and run, we obtain

$$z_{12} = V_1 = 1.122, \qquad z_{22} = V_2 = 3.849$$

Thus,

$$[z] = \begin{bmatrix} 3.949 & 1.122 \\ 1.122 & 3.849 \end{bmatrix} \Omega$$


R1
8
3.849V
R5
R2
R3
R4
1.122V
1
4
4
1
IDC
R10
10
R6
10
R7
10
R8
6
1A
R11
6
I1
R9
2
0


R1
8
1.122V
R5
R2
R3
R4
3.849V
1
4
4
1
R10
10
R6
10
R7
10
R8
6
IDC
R11
6
1A
I1
R9
2
0

Chapter 19, Problem 77.

Using *PSpice*, find the h parameters of the network in Fig. 19.123. Take ω = 1 rad/s

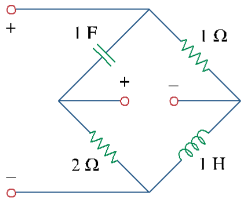


Figure 19.123
For Prob. 19.77.

Chapter 19, Solution 77.

We follow Example 19.15 except that this is an AC circuit.

(a) We set $V_2 = 0$ and $I_1 = 1$ A. The schematic is shown below. In the AC Sweep Box, set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, the output file includes

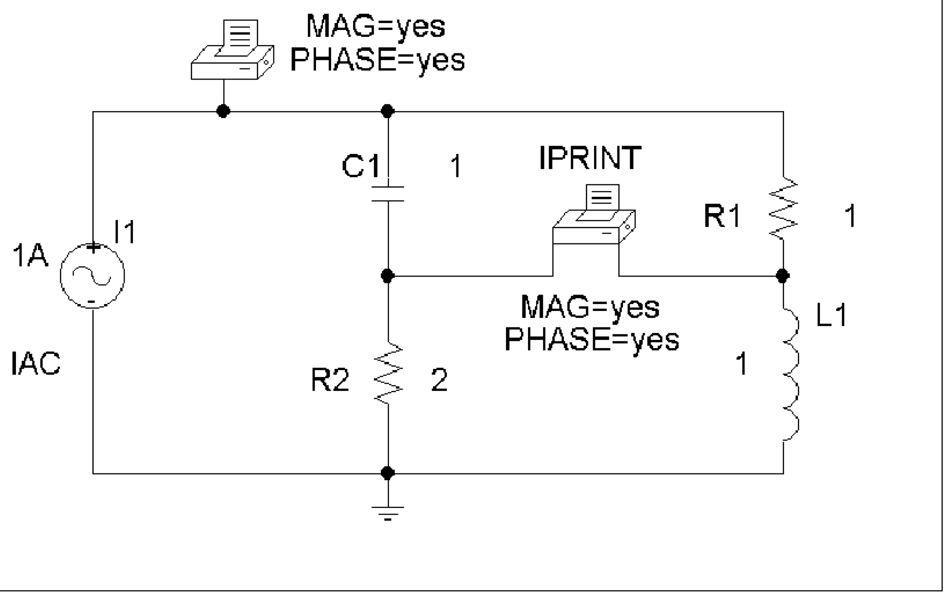
FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E–01	3.163 E–.01	–1.616 E+02

FREQ	VM($N_0001)	VP($N_0001)
1.592 E–01	9.488 E–01	–1.616 E+02

From this we obtain

$$h_{11} = V_1/1 = 0.9488\angle{-161.6°}$$

$$h_{21} = I_2/1 = 0.3163\angle{-161.6°}.$$



(b) In this case, we set $I_1 = 0$ and $V_2 = 1V$. The schematic is shown below. In the AC Sweep box, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, we obtain an output file which includes

FREQ	VM($N_0001)	VP($N_0001)
1.592 E–01	3.163 E–.01	1.842 E+01

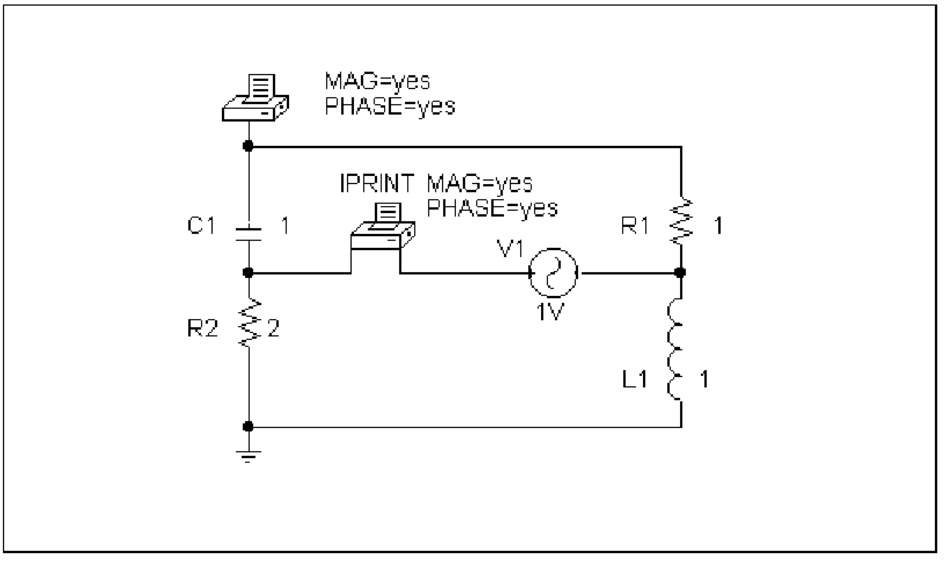
FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E–01	9.488 E–01	–1.616 E+02

From this,

$$h_{12} = V_1/1 = 0.3163\angle 18.42°$$

$$h_{21} = I_2/1 = 0.9488\angle -161.6°.$$

Thus,
$$[h] = \underline{\begin{bmatrix} \mathbf{0.9488\angle -161.6°} & \mathbf{0.3163\angle 18.42°} \\ \mathbf{0.3163\angle -161.6°} & \mathbf{0.9488\angle -161.6°} \end{bmatrix}}$$



Chapter 19, Problem 78.

Obtain the *h* parameters at $\omega = 4$ rad/s for the circuit in Fig. 19.124 using *PSpice*.

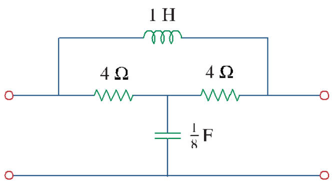


Figure 19.124

For Prob. 19.78.

Chapter 19, Solution 78

For h_{11} and h_{21}, short-circuit the output port and let $I_1 = 1A$. $f = \omega / 2\pi = 0.6366$. The schematic is shown below. When it is saved and run, the output file contains the following:

FREQ IM(V_PRINT1)IP(V_PRINT1)

6.366E-01 1.202E+00 1.463E+02

FREQ VM($N_0003) VP($N_0003)

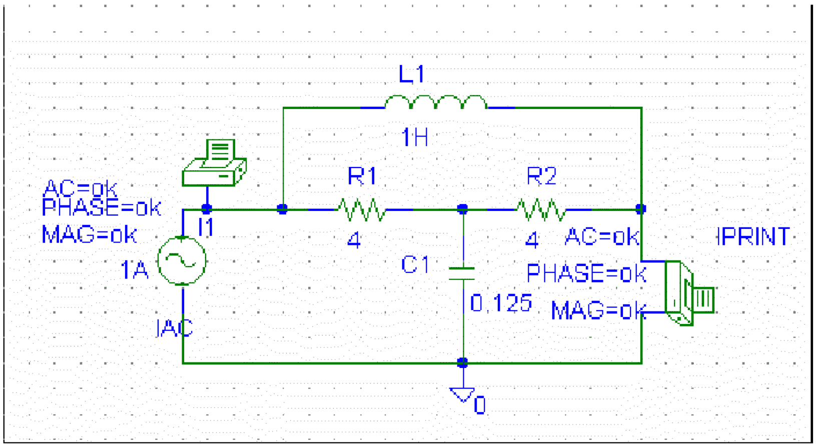
6.366E-01 3.771E+00 -1.350E+02

From the output file, we obtain

$$I_2 = 1.202\angle 146.3^o, \qquad V_1 = 3.771\angle -135^o$$

so that

$$h_{11} = \frac{V_1}{1} = 3.771\angle -135^o, \qquad h_{21} = \frac{I_2}{1} = 1.202\angle 146.3^o$$



For h_{12} and h_{22}, open-circuit the input port and let $V_2 = 1V$. The schematic is shown below. When it is saved and run, the output file includes:

```
FREQ       VM($N_0003) VP($N_0003)

 6.366E-01   1.202E+00  -3.369E+01

FREQ       IM(V_PRINT1)IP(V_PRINT1)

 6.366E-01   3.727E-01  -1.534E+02
```

From the output file, we obtain

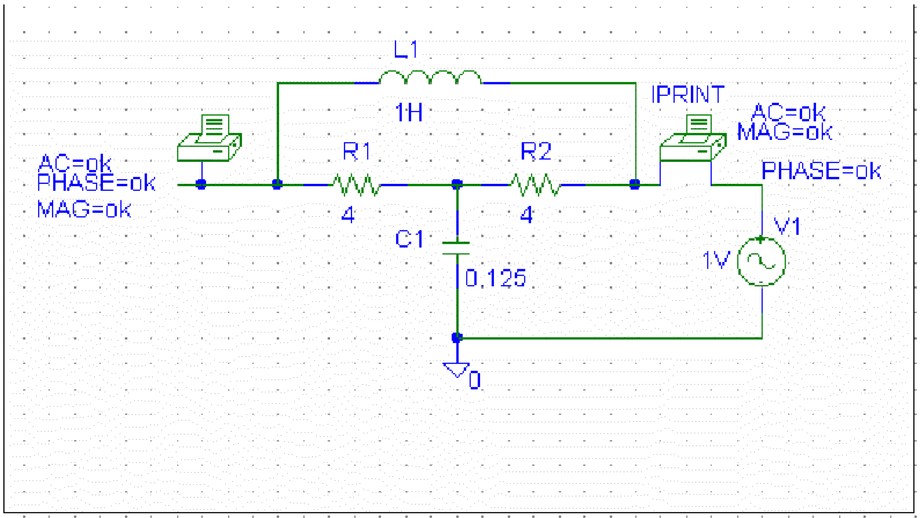
$$I_2 = 0.3727\angle -153.4^o, \qquad V_1 = 1.202\angle -33.69^o$$

so that

$$h_{12} = \frac{V_1}{1} = 1.202\angle -33.69^o, \qquad h_{22} = \frac{I_2}{1} = 0.3727\angle -153.4^o$$

Thus,

$$[h] = \begin{bmatrix} 3.771\angle -135^o & 1.202\angle -33.69^o \\ 1.202\angle 146.3 & 0.3727\angle -153.4^o \end{bmatrix}$$



Chapter 19, Problem 79.

Use *PSpice* to determine the z parameters of the circuit in Fig. 19.125. Take $\omega = 2$ rad/s.

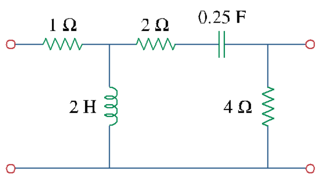


Figure 19.125

For Prob. 19.79.

Chapter 19, Solution 79

We follow Example 19.16.

(a) We set I_1 = 1 A and open-circuit the output-port so that I_2 = 0. The schematic is shown below with two VPRINT1s to measure V_1 and V_2. In the AC Sweep box, we enter Total Pts = 1, Start Freq = 0.3183, and End Freq = 0.3183. After simulation, the output file includes

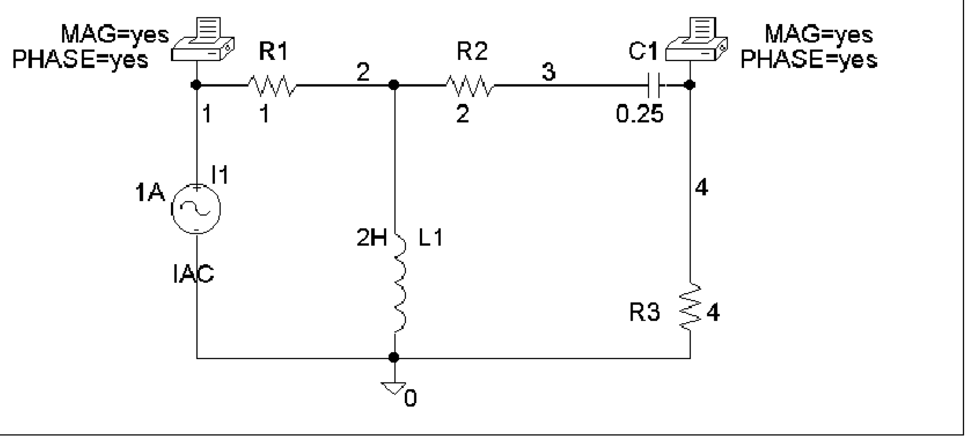
FREQ	VM(1)	VP(1)
3.183 E–01	4.669 E+00	–1.367 E+02

FREQ	VM(4)	VP(4)
3.183 E–01	2.530 E+00	–1.084 E+02

From this,

$$z_{11} = V_1/I_1 = 4.669\angle{-136.7°}/1 = 4.669\angle{-136.7°}$$

$$z_{21} = V_2/I_1 = 2.53\angle{-108.4°}/1 = 2.53\angle{-108.4°}.$$



(b) In this case, we let I_2 = 1 A and open-circuit the input port. The schematic is shown below. In the AC Sweep box, we type Total Pts = 1, Start Freq = 0.3183, and End Freq = 0.3183. After simulation, the output file includes

FREQ	VM(1)	VP(1)
3.183 E–01	2.530 E+00	–1.084 E+02

FREQ	VM(2)	VP(2)
3.183 E–01	1.789 E+00	–1.534 E+02

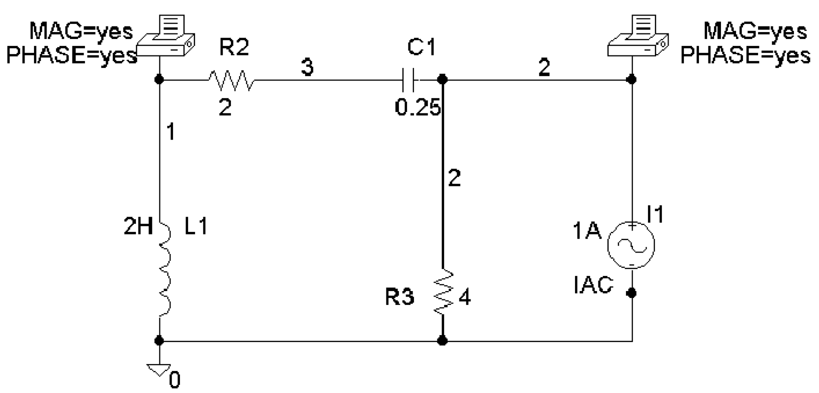
From this,

$$z_{12} = V_1/I_2 = 2.53\angle{-108.4°}/1 = 2.53\angle{-108..4°}$$

$$z_{22} = V_2/I_2 = 1.789\angle{-153.4°}/1 = 1.789\angle{-153.4°}.$$

Thus,

$$[z] = \underline{\begin{bmatrix} \mathbf{4.669\angle -136.7°} & \mathbf{2.53\angle -108.4°} \\ \mathbf{2.53\angle -108.4°} & \mathbf{1.789\angle -153.4°} \end{bmatrix} \boldsymbol{\Omega}}$$



Chapter 19, Problem 80.

Use *PSpice* to find the *z* parameters of the circuit in Fig. 19.71.

Chapter 19, Solution 80

To get z_{11} and z_{21}, we open circuit the output port and let $I_1 = 1A$ so that

$$z_{11} = \frac{V_1}{I_1} = V_1, \quad z_{21} = \frac{V_2}{I_1} = V_2$$

The schematic is shown below. After it is saved and run, we obtain

$$z_{11} = V_1 = 29.88, \quad z_{21} = V_2 = -70.37$$

Similarly, to get z_{22} and z_{12}, we open circuit the input port and let $I_2 = 1A$ so that

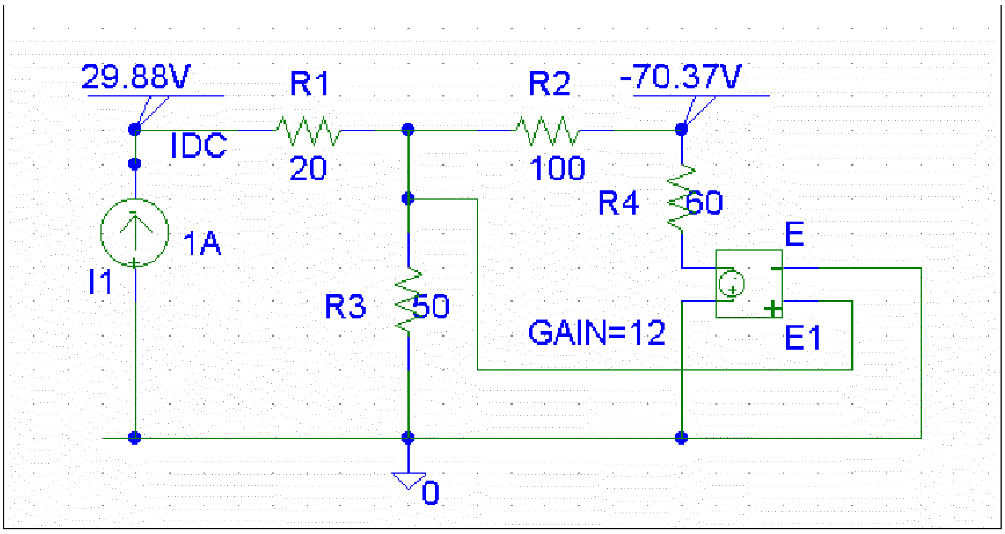
$$z_{12} = \frac{V_1}{I_2} = V_1, \quad z_{22} = \frac{V_2}{I_2} = V_2$$

The schematic is shown below. After it is saved and run, we obtain

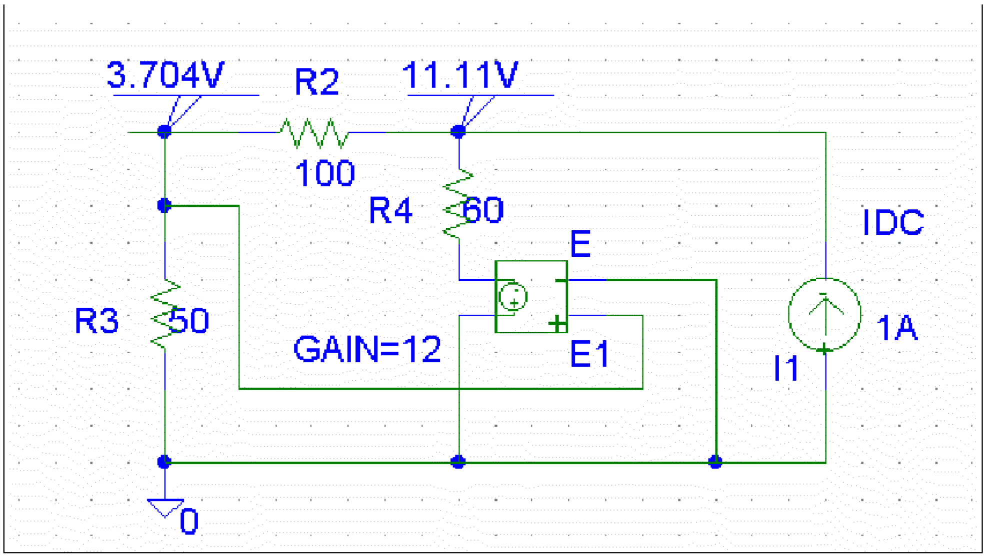
$$z_{12} = V_1 = 3.704, \quad z_{22} = V_2 = 11.11$$

Thus,

$$[z] = \begin{bmatrix} 29.88 & 3.704 \\ -70.37 & 11.11 \end{bmatrix} \Omega$$



29.88V
R1
R2
-70.37V
IDC
20
100
R4
60
1A
E
I1
R3
50
GAIN=12
E1
0



3.704V
R2
11.11V
100
R4
60
IDC
E
R3
50
1A
GAIN=12
E1
I1
0

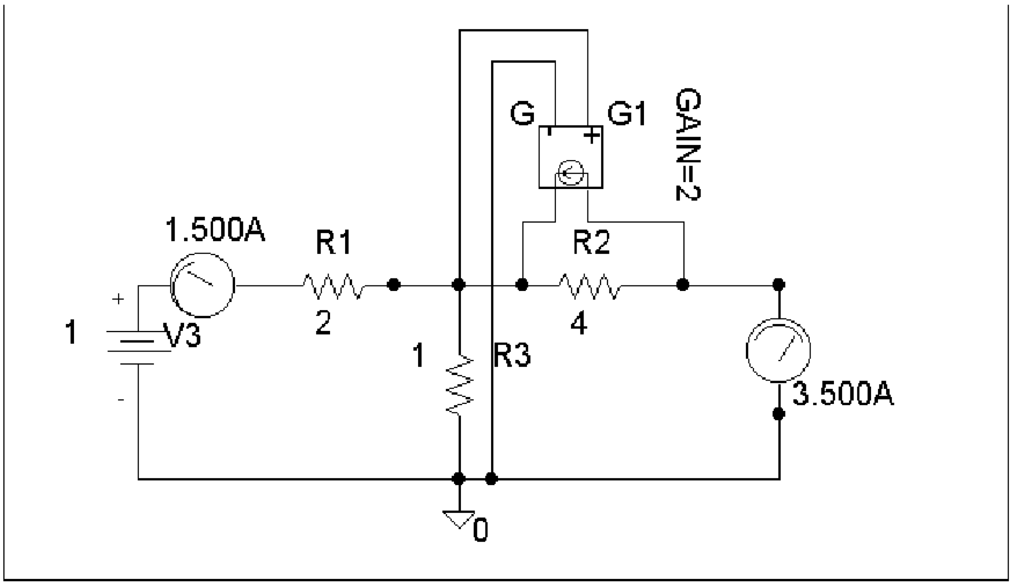
Chapter 19, Problem 81.

Repeat Prob. 19.26 using *PSpice*.

Chapter 19, Solution 81

(a) We set $V_1 = 1$ and short circuit the output port. The schematic is shown below. After simulation we obtain

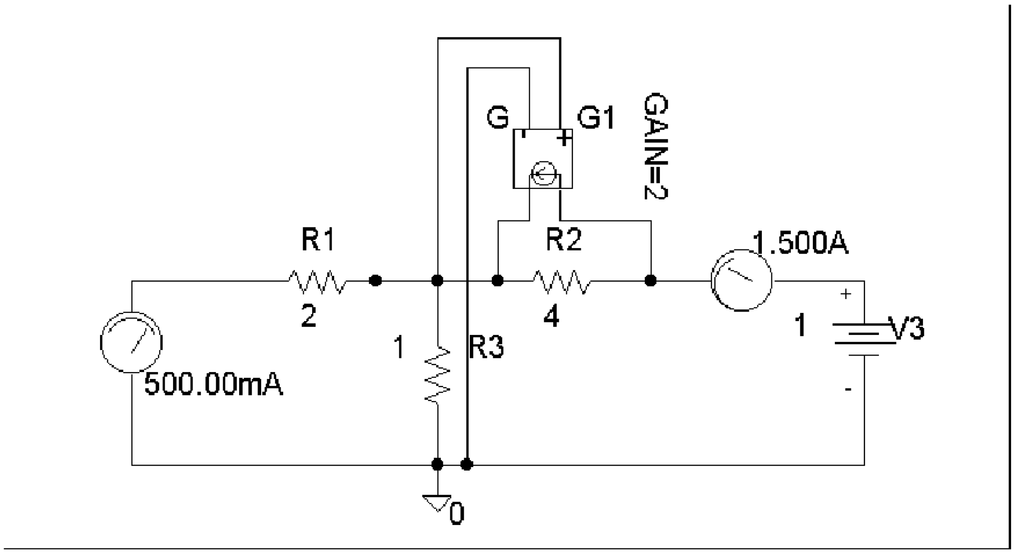
$$y_{11} = I_1 = 1.5, \ y_{21} = I_2 = 3.5$$



(b) We set $V_2 = 1$ and short-circuit the input port. The schematic is shown below. Upon simulating the circuit, we obtain

$$y_{12} = I_1 = -0.5, \quad y_{22} = I_2 = 1.5$$

$$[Y] = \underline{\begin{bmatrix} \mathbf{1.5} & \mathbf{-0.5} \\ \mathbf{3.5} & \mathbf{1.5} \end{bmatrix} \underline{\mathbf{S}}}$$



Chapter 19, Problem 82.

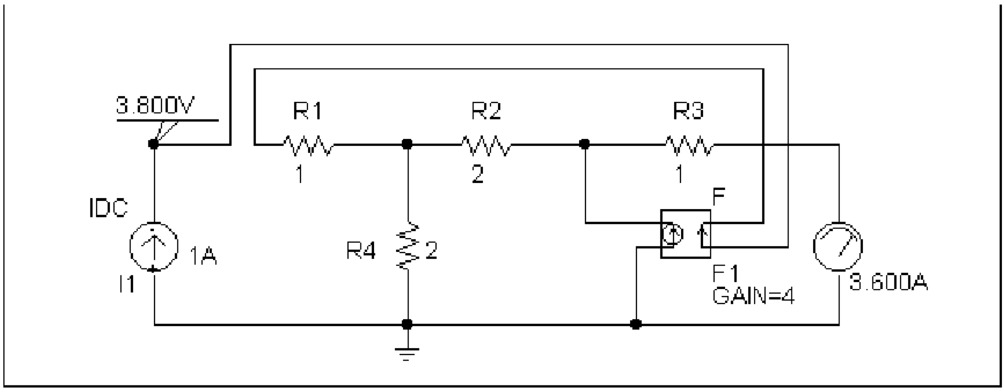
Use *PSpice* to rework Prob. 19.31.

Chapter 19, Solution 82

We follow Example 19.15.

(a) Set $V_2 = 0$ and $I_1 = 1A$. The schematic is shown below. After simulation, we obtain

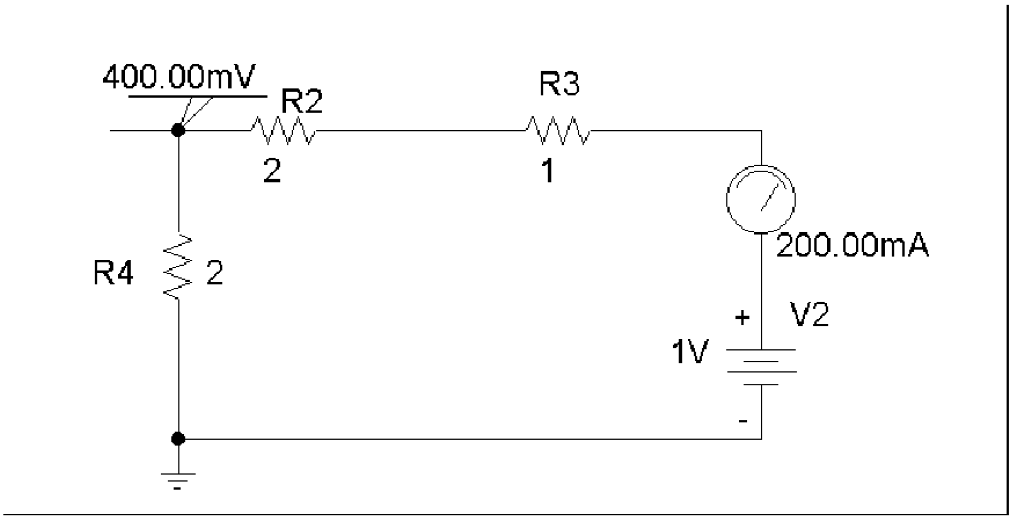
$$h_{11} = V_1/1 = 3.8, \ h_{21} = I_2/1 = 3.6$$



(b) Set $V_1 = 1$ V and $I_1 = 0$. The schematic is shown below. After simulation, we obtain

$$h_{12} = V_1/1 = 0.4, \quad h_{22} = I_2/1 = 0.25$$

Hence, $$[h] = \underline{\begin{bmatrix} \mathbf{3.8} & \mathbf{0.4} \\ \mathbf{3.6} & \mathbf{0.25} \end{bmatrix}}$$



Chapter 19, Problem 83.

Rework Prob. 19.47 using *PSpice*.

Chapter 19, Solution 83

To get A and C, we open-circuit the output and let $I_1 = 1A$. The schematic is shown below. When the circuit is saved and simulated, we obtain $V_1 = 11$ and $V_2 = 34$.

$$A = \frac{V_1}{V_2} = 0.3235, \qquad C = \frac{I_1}{V_2} = \frac{1}{34} = 0.02941$$

Similarly, to get B and D, we open-circuit the output and let $I_1 = 1A$. The schematic is shown below. When the circuit is saved and simulated, we obtain $V_1 = 2.5$ and $I_2 = -2.125$.

$$B = -\frac{V_1}{I_2} = \frac{2.5}{2.125} = 1.1765, \qquad D = -\frac{I_1}{I_2} = \frac{1}{2.125} = 0.4706$$

Thus,

$$[T] = \begin{bmatrix} 0.3235 & 1.1765 \\ 0.02941 & 0.4706 \end{bmatrix}$$

Chapter 19, Problem 84.

Using *PSpice*, find the transmission parameters for the network in Fig. 19.126.

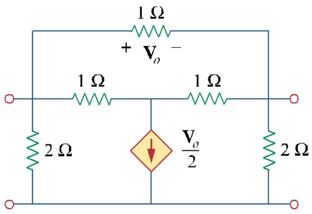


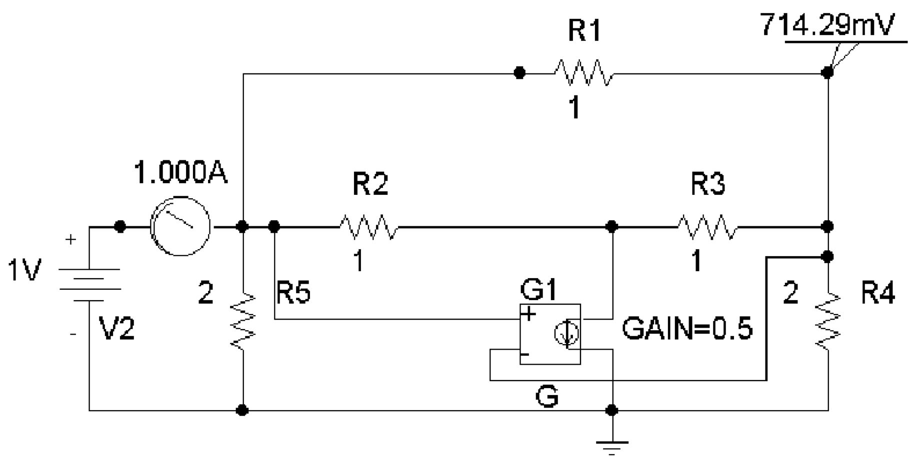
Figure 19.126
For Prob. 19.84.

Chapter 19, Solution 84

(a) Since $A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$ and $C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$, we open-circuit the output port and let V_1 = 1 V. The schematic is as shown below. After simulation, we obtain

$$A = 1/V_2 = 1/0.7143 = 1.4$$

$$C = I_2/V_2 = 1.0/0.7143 = 1.4$$

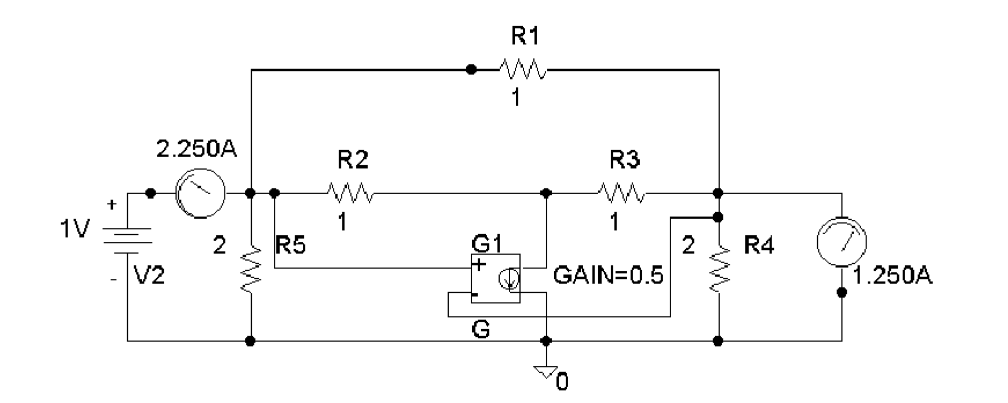


(b) To get B and D, we short-circuit the output port and let $V_1 = 1$. The schematic is shown below. After simulating the circuit, we obtain

$$B = -V_1/I_2 = -1/1.25 = -0.8$$

$$D = -I_1/I_2 = -2.25/1.25 = -1.8$$

Thus $$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{1.4} & \mathbf{-0.8} \\ \mathbf{1.4} & \mathbf{-1.8} \end{bmatrix}}$$



Chapter 19, Problem 85.

At ω = 1 rad/s find the transmission parameters of the network in Fig. 19.127 using *PSpice*.

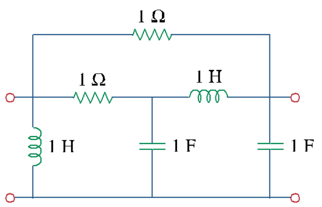


Figure 19.127

For Prob. 19.85.

Chapter 19, Solution 85

(a) Since $A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$ and $C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$, we let V_1 = 1 V and open-circuit the output port. The schematic is shown below. In the AC Sweep box, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, we obtain an output file which includes

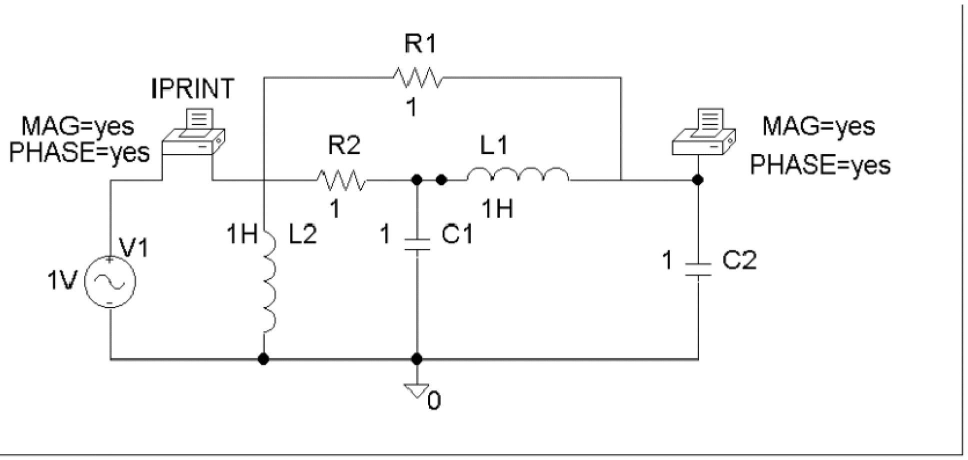
FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592 E–01	6.325 E–01	1.843 E+01

FREQ	VM($N_0002)	VP($N_0002)
1.592 E–01	6.325 E–01	–7.159 E+01

From this, we obtain

$$A = \frac{1}{V_2} = \frac{1}{0.6325\angle -71.59^\circ} = 1.581\angle 71.59^\circ$$

$$C = \frac{I_1}{V_2} = \frac{0.6325\angle 18.43^\circ}{0.6325\angle -71.59^\circ} = 1\angle 90^\circ = j$$



(b) Similarly, since $B = \left.\frac{V_1}{I_2}\right|_{V_2=0}$ and $D = -\left.\frac{I_1}{I_2}\right|_{V_2=0}$, we let $V_1 = 1$ V and short-circuit the output port. The schematic is shown below. Again, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592 in the AC Sweep box. After simulation, we get an output file which includes the following results:

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592 E–01	5.661 E–04	8.997 E+01

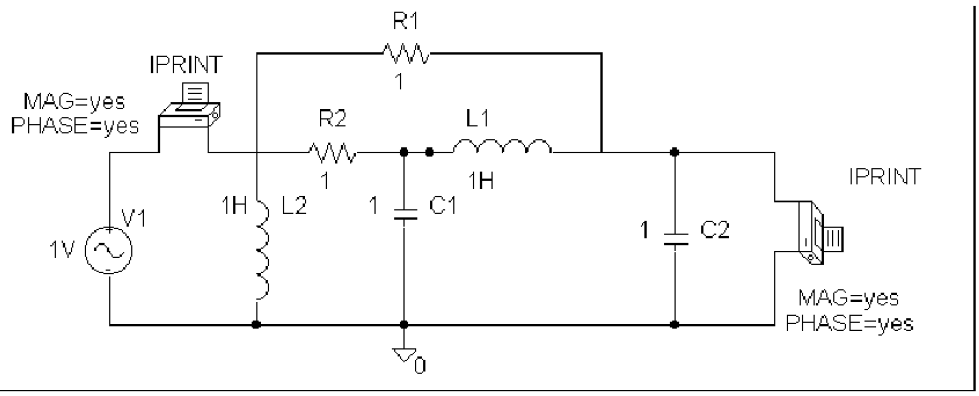
FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.592 E–01	9.997 E–01	–9.003 E+01

From this,

$$B = -\frac{1}{I_2} = -\frac{1}{0.9997\angle -90^\circ} = -1\angle 90^\circ = -j$$

$$D = -\frac{I_1}{I_2} = -\frac{5.661\text{x}10^{-4}\angle 89.97^\circ}{0.9997\angle -90^\circ} = 5.561\text{x}10^{-4}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \underline{\begin{bmatrix} \mathbf{1.581\angle 71.59^\circ} & \mathbf{-j} \\ \mathbf{j} & \mathbf{5.661x10^{-4}} \end{bmatrix}}$$



Chapter 19, Problem 86.

Obtain the *g* parameters for the network in Fig. 19.128 using *PSpice*.

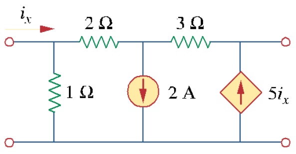


Figure 19.128
For Prob. 19.86.

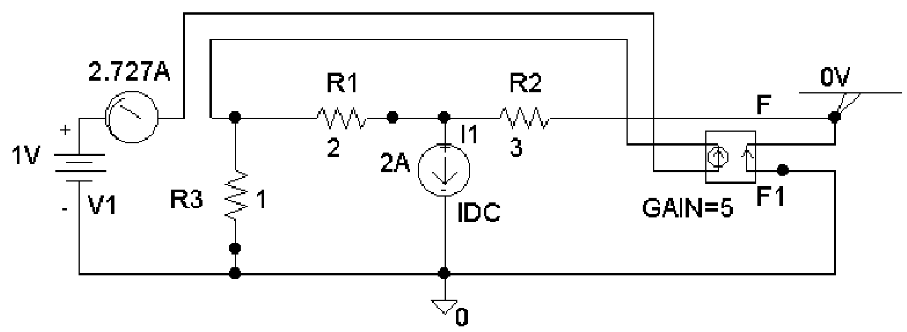
Chapter 19, Solution 86

(a) By definition, $g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0}$, $g_{21} = \left.\frac{V_1}{V_2}\right|_{I_2=0}$.

We let $V_1 = 1$ V and open-circuit the output port. The schematic is shown below. After simulation, we obtain

$$g_{11} = I_1 = 2.7$$

$$g_{21} = V_2 = 0.0$$



(b) Similarly,

$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0}, \quad g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0}$$

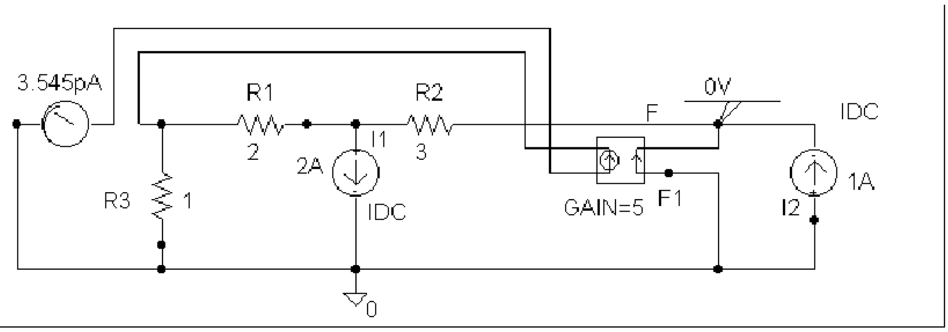
We let $I_2 = 1$ A and short-circuit the input port. The schematic is shown below. After simulation,

$$g_{12} = I_1 = 0$$

$$g_{22} = V_2 = 0$$

Thus

$$[g] = \underline{\begin{bmatrix} \mathbf{2.727S} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}}$$



Chapter 19, Problem 87.

For the circuit shown in Fig. 19.129, use *PSpice* to obtain the *t* parameters. Assume ω = 1 rad/s.

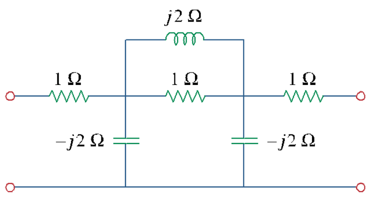


Figure 19.129

For Prob. 19.87.

Chapter 19, Solution 87

(a) Since $\quad a = \left.\dfrac{V_2}{V_1}\right|_{I_1=0} \quad$ and $\quad c = \left.\dfrac{I_2}{V_1}\right|_{I_1=0}$,

we open-circuit the input port and let V_2 = 1 V. The schematic is shown below. In the AC Sweep box, set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, we obtain an output file which includes

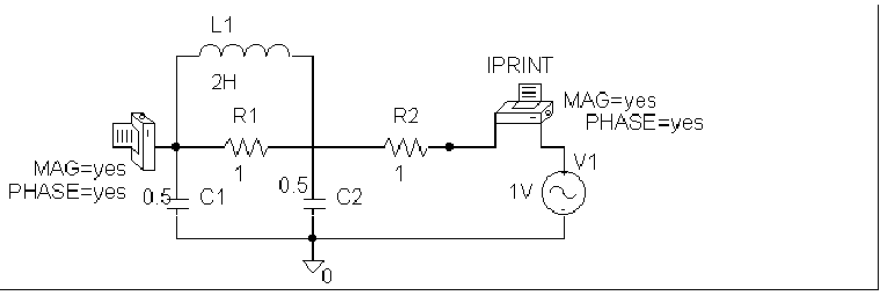
FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E–01	5.000 E–01	1.800 E+02

FREQ	VM($N_0001)	VP($N_0001)
1.592 E–01	5.664 E–04	8.997 E+01

From this,

$$a = \frac{1}{5.664\text{x}10^{-4}\angle 89.97^\circ} = 1765\angle -89.97^\circ$$

$$c = \frac{0.5\angle 180^\circ}{5.664\text{x}10^{-4}\angle 89.97^\circ} = -882.28\angle -89.97^\circ$$



(b) Similarly,

$$b = -\frac{V_2}{I_1}\bigg|_{V_1=0} \quad \text{and} \quad d = -\frac{I_2}{I_1}\bigg|_{V_1=0}$$

We short-circuit the input port and let $V_2 = 1$ V. The schematic is shown below. After simulation, we obtain an output file which includes

FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E–01	5.000 E–01	1.800 E+02

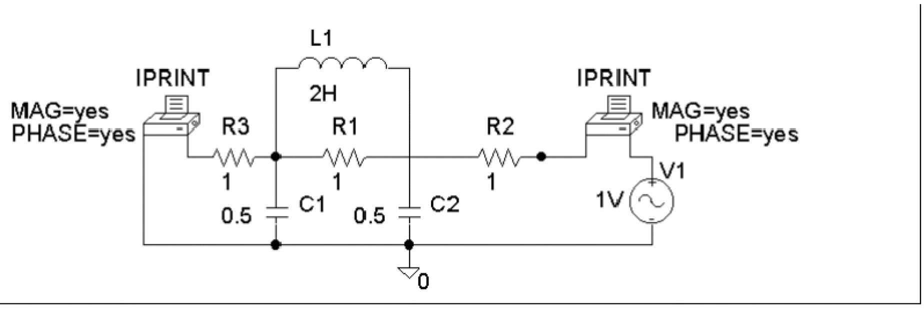
FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.592 E–01	5.664 E–04	–9.010 E+01

From this, we get

$$b = -\frac{1}{5.664\text{x}10^{-4}\angle-90.1^\circ} = -j1765$$

$$d = -\frac{0.5\angle 180^\circ}{5.664\text{x}10^{-4}\angle-90.1^\circ} = j888.28$$

Thus $$[t] = \underline{\begin{bmatrix} \mathbf{-j1765} & \mathbf{-j1765} \\ \mathbf{j888.2} & \mathbf{j888.2} \end{bmatrix}}$$

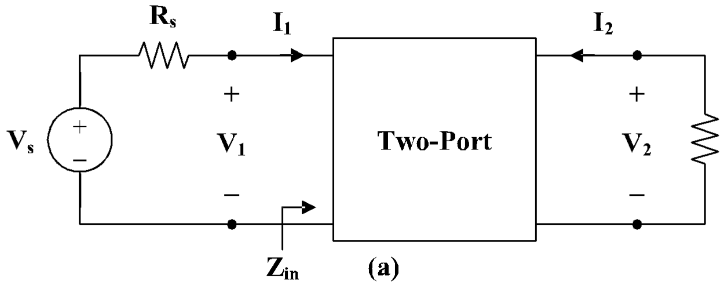


Chapter 19, Problem 88.

Using the y parameters, derive formulas for Z_{in}, Z_{out}, A_i, and A_v for the common-emitter transistor circuit.

Chapter 19, Solution 88

To get Z_{in}, consider the network in Fig. (a).



(a)

$$\mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2 \qquad (1)$$

$$\mathbf{I}_2 = \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2 \qquad (2)$$

But
$$\mathbf{I}_2 = \frac{-\mathbf{V}_2}{R_L} = \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2$$

$$\mathbf{V}_2 = \frac{-\mathbf{y}_{21}\mathbf{V}_1}{\mathbf{y}_{22} + 1/R_L} \qquad (3)$$

Substituting (3) into (1) yields

$$\mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\cdot\left(\frac{-\mathbf{y}_{21}\mathbf{V}_1}{\mathbf{y}_{22} + 1/R_L}\right), \qquad \mathbf{Y}_L = \frac{1}{R_L}$$

$$\mathbf{I}_1 = \left(\frac{\Delta_y + \mathbf{y}_{11}\mathbf{Y}_L}{\mathbf{y}_{22} + \mathbf{Y}_L}\right)\mathbf{V}_1, \qquad \Delta_y = \mathbf{y}_{11}\mathbf{y}_{22} - \mathbf{y}_{12}\mathbf{y}_{21}$$

or
$$Z_{in} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \underline{\frac{\mathbf{y}_{22} + \mathbf{Y}_L}{\Delta_y + \mathbf{y}_{11}\mathbf{Y}_L}}$$

$$A_i = \frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{\mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2}{\mathbf{I}_1} = \mathbf{y}_{21}Z_{in} + \left(\frac{\mathbf{y}_{22}}{\mathbf{I}_1}\right)\left(\frac{-\mathbf{y}_{21}\mathbf{V}_1}{\mathbf{y}_{22} + \mathbf{Y}_L}\right)$$

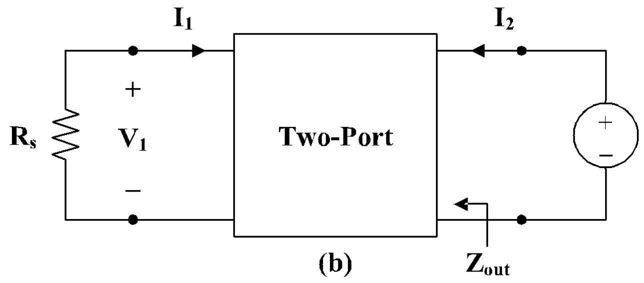
$$= \mathbf{y}_{21}Z_{in} - \frac{\mathbf{y}_{22}\mathbf{y}_{21}Z_{in}}{\mathbf{y}_{22} + \mathbf{Y}_L} = \left(\frac{\mathbf{y}_{22} + \mathbf{Y}_L}{\Delta_y + \mathbf{y}_{11}\mathbf{Y}_L}\right)\left(\mathbf{y}_{21} - \frac{\mathbf{y}_{22}\mathbf{y}_{21}}{\mathbf{y}_{22} + \mathbf{Y}_L}\right)$$

$$A_i = \underline{\frac{\mathbf{y}_{21}\mathbf{Y}_L}{\Delta_y + \mathbf{y}_{11}\mathbf{Y}_L}}$$

From (3),

$$A_v = \frac{\mathbf{V}_2}{\mathbf{V}_1} = \underline{\frac{-\mathbf{y}_{21}}{\mathbf{y}_{22} + \mathbf{Y}_L}}$$

To get Z_{out}, consider the circuit in Fig. (b).



(b)

$$Z_{out} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{\mathbf{V}_2}{\mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2} \qquad (4)$$

But $\qquad \mathbf{V}_1 = -R_s\,\mathbf{I}_1$

Substituting this into (1) yields

$$\mathbf{I}_1 = -\mathbf{y}_{11} R_s \mathbf{I}_1 + \mathbf{y}_{12}\mathbf{V}_2$$

$$(1 + \mathbf{y}_{11} R_s)\mathbf{I}_1 = \mathbf{y}_{12}\mathbf{V}_2$$

$$\mathbf{I}_1 = \frac{\mathbf{y}_{12}\mathbf{V}_2}{1 + \mathbf{y}_{11} R_s} = \frac{-\mathbf{V}_1}{R_s}$$

or

$$\frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{-\mathbf{y}_{12} R_s}{1 + \mathbf{y}_{11} R_s}$$

Substituting this into (4) gives

$$Z_{out} = \frac{1}{\mathbf{y}_{22} - \dfrac{\mathbf{y}_{12}\mathbf{y}_{21} R_s}{1 + \mathbf{y}_{11} R_s}}$$

$$= \frac{1 + \mathbf{y}_{11} R_s}{\mathbf{y}_{22} + \mathbf{y}_{11}\mathbf{y}_{22} R_s - \mathbf{y}_{21}\mathbf{y}_{22} R_s}$$

$$Z_{out} = \underline{\frac{\mathbf{y}_{11} + \mathbf{Y}_s}{\Delta_y + \mathbf{y}_{22}\mathbf{Y}_s}}$$

Chapter 19, Problem 89.

A transistor has the following parameters in a common-emitter circuit:

$h_{ie} = 2{,}640\,\Omega$, $h_{re} = 2.6 \times 10^{-4}$

$h_{fe} = 72$, $h_{oe} = 16\,\mu\text{S}$, $R_L = 100\text{ k}\Omega$

What is the voltage amplification of the transistor? How many decibels gain is this?

Chapter 19, Solution 89

$$A_v = \frac{-h_{fe} R_L}{h_{ie} + (h_{ie} h_{oe} - h_{re} h_{fe}) R_L}$$

$$A_v = \frac{-72 \cdot 10^5}{2640 + (2640 \times 16 \times 10^{-6} - 2.6 \times 10^{-4} \times 72) \cdot 10^5}$$

$$A_v = \frac{-72 \cdot 10^5}{2640 + 1824} = \underline{\mathbf{-1613}}$$

$$\text{dc gain} = 20\log\left|A_v\right| = 20\log(1613) = \underline{\mathbf{64.15}}$$

Chapter 19, Problem 90.

e2d

A transistor with

$h_{fe} = 120,$ $h_{ie} = 2\text{k}\Omega$

$h_{re} = 10^{-4},$ $h_{oe} = 20\,\mu\text{S}$

is used for a CE amplifier to provide an input resistance of $1.5\ \text{k}\Omega$.

(a) Determine the necessary load resistance R_L.

(b) Calculate A_v, A_i, and Z_{out} if the amplifier is driven by a 4-mV source having an internal resistance of $600\,\Omega$.

(c) Find the voltage across the load.

Chapter 19, Solution 90

(a) $$Z_{in} = h_{ie} - \frac{h_{re} h_{fe} R_L}{1 + h_{oe} R_L}$$

$$1500 = 2000 - \frac{10^{-4} \times 120 R_L}{1 + 20 \times 10^{-6} R_L}$$

$$500 = \frac{12 \times 10^{-3}}{1 + 2 \times 10^{-5} R_L}$$

$$500 + 10^{-2} R_L = 12 \times 10^{-3} R_L$$

$$500 \times 10^2 = 0.2 R_L$$

$$R_L = \mathbf{250\ k\Omega}$$

(b) $$A_v = \frac{-h_{fe} R_L}{h_{ie} + (h_{ie} h_{oe} - h_{re} h_{fe}) R_L}$$

$$A_v = \frac{-120 \times 250 \times 10^3}{2000 + (2000 \times 20 \times 10^{-6} - 120 \times 10^{-4}) \times 250 \times 10^3}$$

$$A_v = \frac{-30 \times 10^6}{2 \times 10^3 + 7 \times 10^3} = \underline{-3333}$$

$$A_i = \frac{h_{fe}}{1 + h_{oe} R_L} = \frac{120}{1 + 20 \times 10^{-6} \times 250 \times 10^3} = \mathbf{20}$$

$$Z_{out} = \frac{R_s + h_{ie}}{(R_s + h_{ie}) h_{oe} - h_{re} h_{fe}} = \frac{600 + 2000}{(600 + 2000) \times 20 \times 10^{-6} - 10^{-4} \times 120}$$

$$Z_{out} = \frac{2600}{40}\ k\Omega = \mathbf{65\ k\Omega}$$

(c) $$A_v = \frac{V_c}{V_b} = \frac{V_c}{V_s} \longrightarrow V_c = A_v V_s = -3333 \times 4 \times 10^{-3} = \mathbf{-13.33\ V}$$

Chapter 19, Problem 91.

For the transistor network of Fig. 19.130,

$h_{fe} = 80,$ $h_{ie} = 1.2\text{k}\Omega$

$h_{re} = 1.5 \times 10^{-4},$ $h_{oe} = 20\mu\text{S}$

Determine the following:

(a) voltage gain $A_v = V_o/V_s$,

(b) current gain $A_i = I_0/I_i$,

(c) input impedance Z_{in},

(d) output impedance Z_{out}.

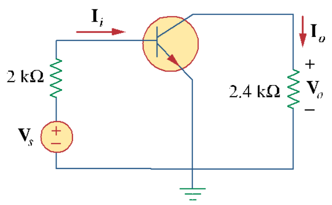


Figure 19.130

For Prob. 19.91.

Chapter 19, Solution 91

$$R_s = 1.2\ \text{k}\Omega, \qquad R_L = 4\ \text{k}\Omega$$

(a) $$A_v = \frac{-h_{fe} R_L}{h_{ie} + (h_{ie} h_{oe} - h_{re} h_{fe}) R_L}$$

$$A_v = \frac{-80 \times 4 \times 10^3}{1200 + (1200 \times 20 \times 10^{-6} - 1.5 \times 10^{-4} \times 80) \times 4 \times 10^3}$$

$A_V = \frac{-32000}{1248} = \underline{\mathbf{-25.64}}$ for the transistor. However, the problem asks for V_o/V_s.

Thus,

$V_b = Vo/ATransV = -V_o/25.64$
$I_b = V_s/(2000 + 1200) = V_s/3200$ (Note, we used Z_{in} from (c) below.)
$V_b = 1200xI_b = (1200/3200)V_s = 0.375V_s = -V_o/25.64$

A_V for the circuit = V_o/V_s = $\underline{\mathbf{-9.615}}$

(b) $$A_i = \frac{h_{fe}}{1 + h_{oe} R_L} = \frac{80}{1 + 20 \times 10^{-6} \times 4 \times 10^3} = \underline{74.07}$$

(c) $$Z_{in} = h_{ie} - h_{re} A_i$$

$$Z_{in} = 1200 - 1.5 \times 10^{-4} \times 74.074 \cong \underline{\mathbf{1.2\ k\Omega}}$$

(d) $$Z_{out} = \frac{R_s + h_{ie}}{(R_s + h_{ie}) h_{oe} - h_{re} h_{fe}}$$

$$Z_{out} = \frac{1200 + 1200}{2400 \times 20 \times 10^{-6} - 1.5 \times 10^{-4} \times 80} = \frac{2400}{0.0468} = \underline{51.28\ \text{k}\Omega}$$

Chapter 19, Problem 92.

* Determine A_v, A_i, Z_{in}, and Z_{out} for the amplifier shown in Fig. 19.131. Assume that

$h_{ie} = 4\text{ k}\Omega$, $h_{re} = 10^{-4}$

$h_{fe} = 100$, $h_{oe} = 30\,\mu\text{S}$

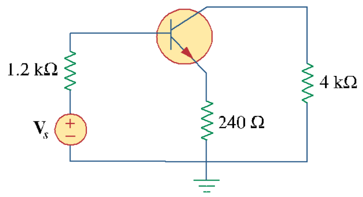
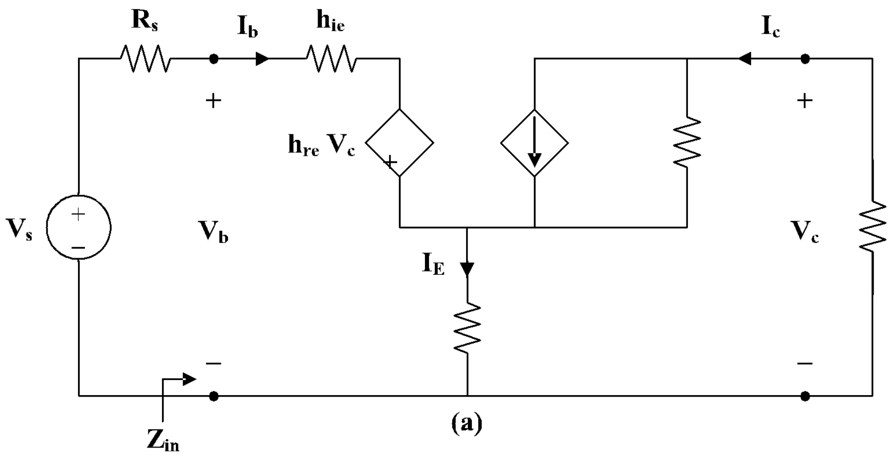


Figure 19.131
For Prob. 19.92.

* An asterisk indicates a challenging problem.

Chapter 19, Solution 92

Due to the resistor $R_E = 240\,\Omega$, we cannot use the formulas in section 18.9.1. We will need to derive our own. Consider the circuit in Fig. (a).



(a)

$$\mathbf{I}_E = \mathbf{I}_b + \mathbf{I}_c \qquad (1)$$

$$\mathbf{V}_b = h_{ie}\,\mathbf{I}_b + h_{re}\,\mathbf{V}_c + (\mathbf{I}_b + \mathbf{I}_c)R_E \qquad (2)$$

$$\mathbf{I}_c = h_{fe}\,\mathbf{I}_b + \frac{\mathbf{V}_c}{R_E + \frac{1}{h_{oe}}} \qquad (3)$$

But $$\mathbf{V}_c = -\mathbf{I}_c R_L \qquad (4)$$

Substituting (4) into (3),

$$\mathbf{I}_c = h_{fe}\,\mathbf{I}_b - \frac{R_L}{R_E + \frac{1}{h_{oe}}}\mathbf{I}_c$$

or $$A_i = \frac{\mathbf{I}_c}{\mathbf{I}_b} = \frac{h_{fe}(1 + R_E h_{oe})}{1 + h_{oe}(R_L} \qquad (5)$$

$$A_i = \frac{100(1 + 240x30x10^{-6})}{1 + 30 \times 10^{-6}(4{,}000 + 240)}$$

$$A_i = \underline{\mathbf{79.18}}$$

From (3) and (5),

$$\mathbf{I}_c = \frac{h_{fe}(1 + R_E)h_{oe}}{1 + h_{oe}(R_L + R_E)}\mathbf{I}_b = h_{fe}\,\mathbf{I}_b + \frac{\mathbf{V}_c}{R_E + \frac{1}{h_{oe}}} \qquad (6)$$

Substituting (4) and (6) into (2),

$$\mathbf{V}_b = (h_{ie} + R_E)\mathbf{I}_b + h_{re}\,\mathbf{V}_c + \mathbf{I}_c R_E$$

$$\mathbf{V}_b = \frac{\mathbf{V}_c(h_{ie} + R_E)}{\left(R_E + \frac{1}{h_{oe}}\right)\left[\frac{h_{fe}(1 + R_E h_{oe})}{1 + h_{oe}(R_L + R_E)} - h_{fe}\right]} + h_{re}\,\mathbf{V}_c - \frac{\mathbf{V}_c}{R_L}R_E$$

$$\frac{1}{A_v} = \frac{\mathbf{V}_b}{\mathbf{V}_c} = \frac{(h_{ie} + R_E)}{\left(R_E + \frac{1}{h_{oe}}\right)\left[\frac{h_{fe}(1 + R_E h_{oe})}{1 + h_{oe}(R_L + R_E)} - h_{fe}\right]} + h_{re} - \frac{R_E}{R_L} \qquad (7)$$

$$\frac{1}{A_v} = \frac{(4000 + 240)}{\left(240 + \frac{1}{30x10^{-6}}\right)\left[\frac{100(1 + 240x30x10^{-6})}{1 + 30 \times 10^{-6} \times 4240} - 100\right]} + 10^{-4} - \frac{240}{4000}$$

$$\frac{1}{A_v} = -6.06x10^{-3} + 10^{-4} - 0.06 = -0.066$$

$$A_v = \underline{\mathbf{-15.15}}$$

From (5),

$$\mathbf{I}_c = \frac{h_{fe}}{1 + h_{oe} R_L} \mathbf{I}_b$$

We substitute this with (4) into (2) to get

$$\mathbf{V}_b = (h_{ie} + R_E)\mathbf{I}_b + (R_E - h_{re} R_L)\mathbf{I}_c$$

$$\mathbf{V}_b = (h_{ie} + R_E)\mathbf{I}_b + (R_E - h_{re} R_L)\left(\frac{h_{fe}(1 + R_E h_{oe})}{1 + h_{oe}(R_L + R_E)}\mathbf{I}_b\right)$$

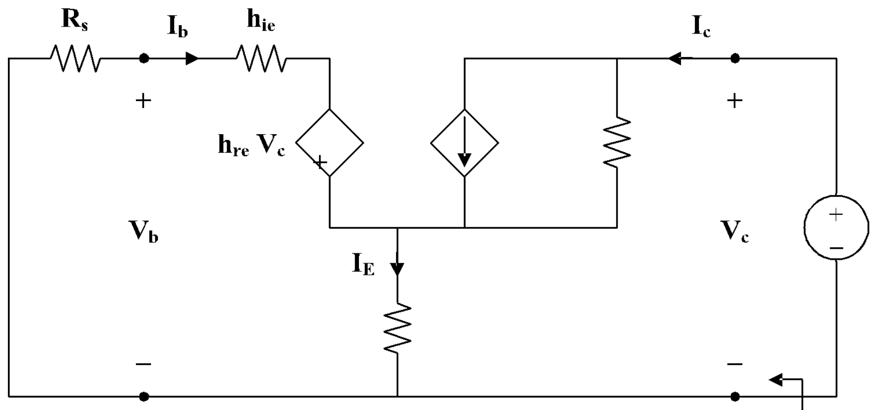
$$Z_{in} = \frac{\mathbf{V}_b}{\mathbf{I}_b} = h_{ie} + R_E + \frac{h_{fe}(R_E - h_{re} R_L)(1 + R_E h_{oe})}{1 + h_{oe}(R_L + R_E)}$$

(8)

$$Z_{in} = 4000 + 240 + \frac{(100)(240 \times 10^{-4} \times 4 \times 10^3)(1 + 240\text{x}30\text{x}10^{-6})}{1 + 30 \times 10^{-6} \times 4240}$$

$$Z_{in} = \underline{\mathbf{12.818\ k\Omega}}$$

To obtain Z_{out}, which is the same as the Thevenin impedance at the output, we introduce a 1-V source as shown in Fig. (b).



(b)

From the input loop,

$$\mathbf{I}_b(R_s + h_{ie}) + h_{re}\mathbf{V}_c + R_E(\mathbf{I}_b + \mathbf{I}_c) = 0$$

But $\quad \mathbf{V}_c = 1$

So,

$$\mathbf{I}_b(R_s + h_{ie} + R_E) + h_{re} + R_E\,\mathbf{I}_c = 0 \qquad (9)$$

From the output loop,

$$\mathbf{I}_c = \frac{\mathbf{V}_c}{R_E + \frac{1}{h_{oe}}} + h_{fe}\mathbf{I}_b = \frac{h_{oe}}{R_E h_{oe} + 1} + h_{fe}\mathbf{I}_b$$

or

$$\mathbf{I}_b = \frac{\mathbf{I}_c}{h_{fe}} - \frac{h_{oe}/h_{fe}}{1 + R_E h_{oe}} \qquad (10)$$

Substituting (10) into (9) gives

$$(R_s + R_E + h_{ie})\left(\frac{\mathbf{I}_c}{h_{fe}}\right) + h_{re} + R_E\,\mathbf{I}_c - \frac{(R_s + R_E + h_{ie})\left(h_{oe}/h_{fe}\right)}{1 + R_E h_{oe}} = 0$$

$$\frac{R_s + R_E + h_{ie}}{h_{fe}}\mathbf{I}_c + R_E\,\mathbf{I}_c = \frac{R_s + R_E + h_{ie}}{1 + R_E h_{oe}}\left(\frac{h_{oe}}{h_{fe}}\right) - h_{re}$$

$$\mathbf{I}_c = \frac{(h_{oe}/h_{fe})\left[\frac{R_s + R_E + h_{ie}}{1 + R_E h_{oe}}\right] - h_{re}}{R_E + (R_s + R_E + h_{ie})/h_{fe}}$$

$$Z_{out} = \frac{1}{\mathbf{I}_c} = \frac{R_E h_{fe} + R_s + R_E + h_{ie}}{\left[\frac{R_s + R_E + h_{ie}}{1 + R_E h_{oe}}\right]h_{oe} - h_{re}h_{fe}}$$

$$Z_{out} = \frac{240 \times 100 + (1200 + 240 + 4000)}{\left[\frac{1200 + 240 + 4000}{1 + 240\text{x}30\text{x}10^{-6}}\right] \times 30 \times 10^{-6} - 10^{-4} \times 100}$$

$$Z_{out} = \frac{24000 + 5440}{0.152} = \underline{\mathbf{193.7\ k\Omega}}$$

***Chapter 19, Problem 93.**

Calculate A_v, A_i, Z_{in}, and Z_{out}, for the transistor network in Fig. 19.132. Assume that

$h_{ie} = 2\ \text{k}\Omega$, $h_{re} = 2.5 \times 10^{-4}$

$h_{fe} = 150$, $h_{oe} = 10\ \mu\text{S}$

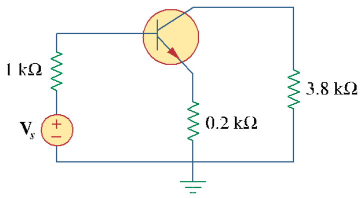


Figure 19.110
For Prob. 19.63.
*An asterisk indicates a challenging problem.

Chapter 19, Solution 93

We apply the same formulas derived in the previous problem.

$$\frac{1}{A_v} = \frac{(h_{ie} + R_E)}{\left(R_E + \frac{1}{h_{oe}}\right)\left[\frac{h_{fe}(1 + R_E h_{oe})}{1 + h_{oe}(R_L + R_E)} - h_{fe}\right]} + h_{re} - \frac{R_E}{R_L}$$

$$\frac{1}{A_v} = \frac{(2000 + 200)}{(200 + 10^5)\left[\frac{150(1 + 0.002)}{1 + 0.04} - 150\right]} + 2.5\times10^{-4} - \frac{200}{3800}$$

$$\frac{1}{A_v} = -0.004 + 2.5\times10^{-4} - 0.05263 = -0.05638$$

$A_v = \underline{\mathbf{-17.74}}$

$$A_i = \frac{h_{fe}(1 + R_E h_{oe})}{1 + h_{oe}(R_L + R_E)} = \frac{150(1 + 200\text{x}10^{-5})}{1 + 10^{-5}\times(200 + 3800)} = \underline{\mathbf{144.5}}$$

$$Z_{in} = h_{ie} + R_E + \frac{h_{fe}(R_E - h_{re}R_L)(1 + R_E h_{oe})}{1 + h_{oe}(R_L + R_E)}$$

$$Z_{in} = 2000 + 200 + \frac{(150)(200 - 2.5\times10^{-4}\times3.8\times10^3)(1.002)}{1.04}$$

$Z_{in} = 2200 + 28966$

$Z_{in} = \underline{\mathbf{31.17\ k\Omega}}$

$$Z_{out} = \frac{R_E h_{fe} + R_s + R_E + h_{ie}}{\left[\frac{R_s + R_E + h_{ie}}{1 + R_E h_{oe}}\right]h_{oe} - h_{re}h_{fe}}$$

$$Z_{out} = \frac{200\times150 + 1000 + 200 + 2000}{\left[\frac{3200\times10^{-5}}{1.002}\right] - 2.5\times10^{-4}\times150} = \frac{33200}{-0.0055}$$

$Z_{out} = \underline{\mathbf{-6.148\ M\Omega}}$

Chapter 19, Problem 94.

e2d

A transistor in its common-emitter mode is specified by

$$[\mathbf{h}] = \begin{bmatrix} 200\Omega & 0 \\ 100 & 10^{-6}\,\text{S} \end{bmatrix}$$

Two such identical transistors are connected in cascade to form a two-stage amplifier used at audio frequencies. If the amplifier is terminated by a 4-kΩ resistor, calculate the overall A_v and Z_{in}.

Chapter 19, Solution 94

We first obtain the **ABCD** parameters.

Given $[\mathbf{h}] = \begin{bmatrix} 200 & 0 \\ 100 & 10^{-6} \end{bmatrix}$, $\Delta_h = \mathbf{h}_{11}\mathbf{h}_{22} - \mathbf{h}_{12}\mathbf{h}_{21} = 2\times 10^{-4}$

$$[\mathbf{T}] = \begin{bmatrix} \dfrac{\Delta_h}{\mathbf{h}_{21}} & \dfrac{-\mathbf{h}_{11}}{\mathbf{h}_{21}} \\ \dfrac{-\mathbf{h}_{22}}{\mathbf{h}_{21}} & \dfrac{-1}{\mathbf{h}_{21}} \end{bmatrix} = \begin{bmatrix} -2\times 10^{-6} & -2 \\ -10^{-8} & -10^{-2} \end{bmatrix}$$

The overall **ABCD** parameters for the amplifier are

$$[\mathbf{T}] = \begin{bmatrix} -2\times 10^{-6} & -2 \\ -10^{-8} & -10^{-2} \end{bmatrix}\begin{bmatrix} -2\times 10^{-6} & -2 \\ -10^{-8} & -10^{-2} \end{bmatrix} \cong \begin{bmatrix} 2\times 10^{-8} & 2\times 10^{-2} \\ 10^{-10} & 10^{-4} \end{bmatrix}$$

$$\Delta_T = 2\times 10^{-12} - 2\times 10^{-12} = 0$$

$$[\mathbf{h}] = \begin{bmatrix} \dfrac{\mathbf{B}}{\mathbf{D}} & \dfrac{\Delta_T}{\mathbf{D}} \\ \dfrac{-1}{\mathbf{D}} & \dfrac{\mathbf{C}}{\mathbf{D}} \end{bmatrix} = \begin{bmatrix} 200 & 0 \\ -10^{4} & 10^{-6} \end{bmatrix}$$

Thus, $h_{ie} = 200$, $h_{re} = 0$, $h_{fe} = -10^4$, $h_{oe} = 10^{-6}$

$$A_v = \frac{(10^4)(4\times 10^3)}{200 + (2\times 10^{-4} - 0)\times 4\times 10^3} = \underline{\mathbf{2\times 10^5}}$$

$$Z_{in} = h_{ie} - \frac{h_{re}h_{fe}R_L}{1 + h_{oe}R_L} = 200 - 0 = \underline{\mathbf{200\,\Omega}}$$

Chapter 19, Problem 95.

Realize an *LC* ladder network such that

$$y_{22} = \frac{s^3 + 5s}{s^4 + 10s^2 + 8}$$

Chapter 19, Solution 95

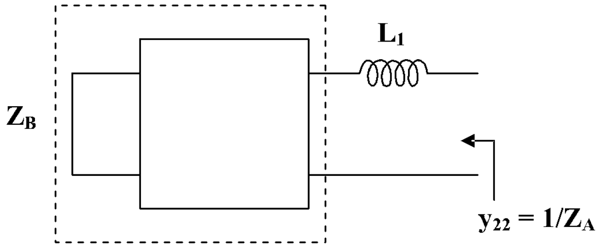
Let $\mathbf{Z}_A = \frac{1}{\mathbf{y}_{22}} = \frac{s^4 + 10s^2 + 8}{s^3 + 5s}$

Using long division,

$$\mathbf{Z}_A = s + \frac{5s^2 + 8}{s^3 + 5s} = sL_1 + \mathbf{Z}_B$$

i.e. $L_1 = 1\text{ H}$ and $\mathbf{Z}_B = \frac{5s^2 + 8}{s^3 + 5s}$

as shown in Fig (a).



(a)

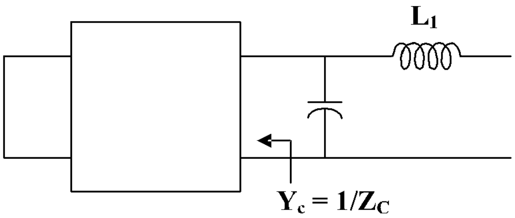
$$\mathbf{Y}_B = \frac{1}{\mathbf{Z}_B} = \frac{s^3 + 5s}{5s^2 + 8}$$

Using long division,

$$\mathbf{Y}_B = 0.2s + \frac{3.4s}{5s^2+8} = sC_2 + \mathbf{Y}_C$$

where $C_2 = 0.2$ F and $\mathbf{Y}_C = \dfrac{3.4s}{5s^2+8}$

as shown in Fig. (b).



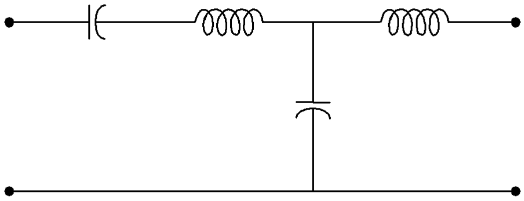
(b)

$$\mathbf{Z}_C = \frac{1}{\mathbf{Y}_C} = \frac{5s^2+8}{3.4s} = \frac{5s}{3.4} + \frac{8}{3.4s} = sL_3 + \frac{1}{sC_4}$$

i.e. an inductor in series with a capacitor

$$L_3 = \frac{5}{3.4} = 1.471 \text{ H} \quad \text{and} \quad C_4 = \frac{3.4}{8} = 0.425 \text{ F}$$

Thus, **<u>the LC network is shown in Fig. (c)</u>**.



(c)

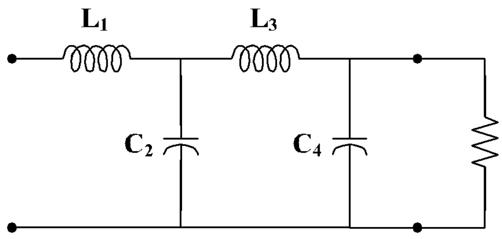
Chapter 19, Problem 96.

Design an *LC* ladder network to realize a lowpass filter with transfer function

$$H(s) = \frac{1}{s^4 + 2.613s^2 + 3.414s^2 + 2.613s + 1}$$

Chapter 19, Solution 96

This is a fourth order network which can be realized with the network shown in Fig. (a).



(a)

$$\Delta(s) = (s^4 + 3.414s^2 + 1) + (2.613s^3 + 2.613s)$$

$$H(s) = \frac{\dfrac{1}{2.613s^3 + 2.613s}}{1 + \dfrac{s^4 + 3.414s^2 + 1}{2.613s^3 + 2.613s}}$$

which indicates that

$$\mathbf{y}_{21} = \frac{-1}{2.613s^3 + 2.613s}$$

$$\mathbf{y}_{22} = \frac{s^4 + 3.414s + 1}{2.613s^3 + 2.613s}$$

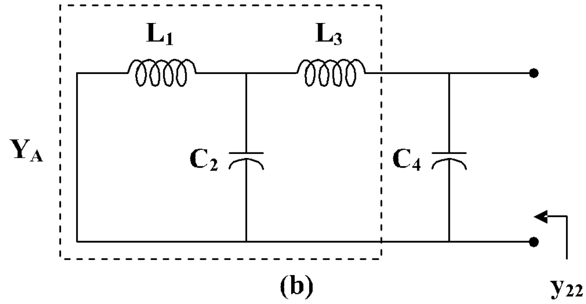
We seek to realize $\mathbf{y}_{22}$.

By long division,

$$\mathbf{y}_{22} = 0.383s + \frac{2.414s^2 + 1}{2.613s^3 + 2.613s} = sC_4 + \mathbf{Y}_A$$

i.e. $C_4 = 0.383$ F and $\mathbf{Y}_A = \dfrac{2.414s^2 + 1}{2.613s^3 + 2.613s}$

as shown in Fig. (b).



(b)

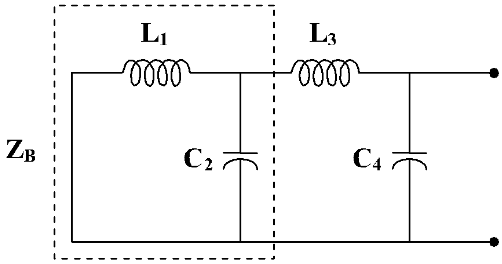
$$\mathbf{Z}_A = \frac{1}{\mathbf{Y}_A} = \frac{2.613s^3 + 2.613s}{2.414s^2 + 1}$$

By long division,

$$\mathbf{Z}_A = 1.082s + \frac{1.531s}{2.414s^2 + 1} = sL_3 + \mathbf{Z}_B$$

i.e. $L_3 = 1.082$ H and $\mathbf{Z}_B = \dfrac{1.531s}{2.414s^2 + 1}$

as shown in Fig.(c).

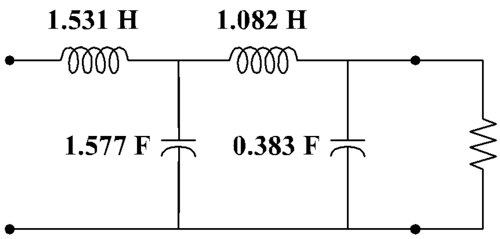


(c)

$$\mathbf{Y}_B = \frac{1}{\mathbf{Z}_B} = 1.577s + \frac{1}{1.531s} = sC_2 + \frac{1}{sL_1}$$

i.e. $C_2 = 1.577\ \text{F}$ and $L_1 = 1.531\ \text{H}$

Thus, **the network is shown in Fig. (d)**.



(d)

Chapter 19, Problem 97.

Synthesize the transfer function

$$H(s) = \frac{V_o}{V_s} = \frac{s^3}{s^3 + 6s + 12s + 24}$$

using the *LC* ladder network in Fig. 19.133.

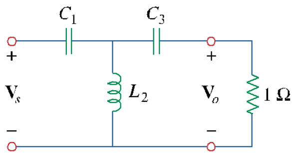


Figure 19.133
For Prob. 19.97.

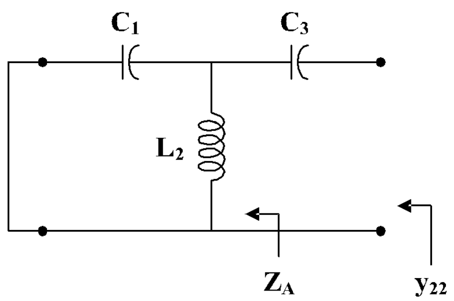
Chapter 19, Solution 97

$$H(s) = \frac{s^3}{(s^3 + 12s) + (6s^2 + 24)} = \frac{\dfrac{s^3}{s^3 + 12s}}{1 + \dfrac{6s^2 + 24}{s^3 + 12s}}$$

Hence,

$$\mathbf{y}_{22} = \frac{6s^2 + 24}{s^3 + 12s} = \frac{1}{sC_3} + \mathbf{Z}_A \qquad (1)$$

where $\mathbf{Z}_A$ is shown in the figure below.



We now obtain C_3 and $\mathbf{Z}_A$ using partial fraction expansion.

Let
$$\frac{6s^2+24}{s(s^2+12)} = \frac{A}{s} + \frac{Bs+C}{s^2+12}$$

$$6s^2+24 = A(s^2+12) + Bs^2 + Cs$$

Equating coefficients :

s^0: $24 = 12A \longrightarrow A = 2$

s^1: $0 = C$

s^2: $6 = A + B \longrightarrow B = 4$

Thus,
$$\frac{6s^2+24}{s(s^2+12)} = \frac{2}{s} + \frac{4s}{s^2+12} \qquad (2)$$

Comparing (1) and (2),
$$C_3 = \frac{1}{A} = \frac{1}{2}\ \text{F}$$

$$\frac{1}{\mathbf{Z}_A} = \frac{s^2+12}{4s} = \frac{1}{4}s + \frac{3}{s} \qquad (3)$$

But
$$\frac{1}{\mathbf{Z}_A} = sC_1 + \frac{1}{sL_2} \qquad (4)$$

Comparing (3) and (4),
$$C_1 = \frac{1}{4}\ \text{F} \qquad \text{and} \qquad L_2 = \frac{1}{3}\ \text{H}$$

Therefore,
$$C_1 = \underline{\mathbf{0.25\ F}}, \qquad L_2 = \underline{\mathbf{0.3333\ H}}, \qquad C_3 = \underline{\mathbf{0.5\ F}}$$

Chapter 19, Problem 98.

A two-stage amplifier in Fig. 19.134 contains two identical stages with

$$[\mathbf{h}] = \begin{bmatrix} 2\,k\Omega & 0.004 \\ 200 & 500\,\mu S \end{bmatrix}$$

If $\mathbf{Z}_L = 20\,k\Omega$, find the required value of $\mathbf{V}_s$ to produce $\mathbf{V}_o = 16$ V.

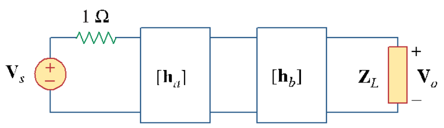


Figure 19.134

For Prob. 19.98.

Chapter 19, Solution 98

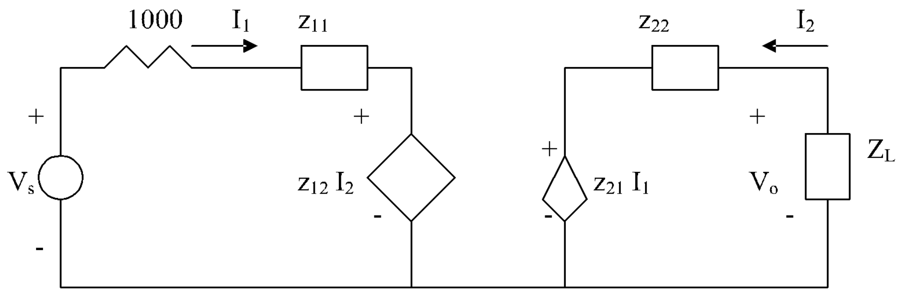
$$\Delta_h = 1 - 0.8 = 0.2$$

$$[T_a] = [T_b] = \begin{bmatrix} -\Delta_h / h_{21} & -h_{11}/h_{21} \\ -h_{22}/h_{21} & -1/h_{21} \end{bmatrix} = \begin{bmatrix} -0.001 & -10 \\ -2.5x10^{-6} & -0.005 \end{bmatrix}$$

$$[T] = [T_a][T_b] = \begin{bmatrix} 2.6x10^{-5} & 0.06 \\ 1.5x10^{-8} & 5x10^{-5} \end{bmatrix}$$

We now convert this to z-parameters

$$[z] = \begin{bmatrix} A/C & \Delta_T / C \\ 1/C & D/C \end{bmatrix} = \begin{bmatrix} 1.733x10^3 & 0.0267 \\ 6.667x10^7 & 3.33x10^3 \end{bmatrix}$$



$$V_s = (1000 + z_{11})I_1 + z_{12}I_2 \qquad (1)$$

$$V_o = z_{22}I_2 + z_{21}I_1 \qquad (2)$$

But $V_o = -I_2 Z_L \longrightarrow I_2 = -V_o / Z_L$ (3)

Substituting (3) into (2) gives

$$I_1 = V_o\left(\frac{1}{z_{21}} + \frac{z_{22}}{z_{21}Z_L}\right) \qquad (4)$$

We substitute (3) and (4) into (1)

$$V_s = (1000 + z_{11})\left(\frac{1}{z_{11}} + \frac{z_{22}}{z_{21}Z_L}\right)V_o - \frac{z_{12}}{Z_L}V_o$$

$$= 7.653\text{x}10^{-4} - 2.136\text{x}10^{-5} = \underline{744\mu V}$$

Chapter 19, Problem 99.

Assume that the two circuits in Fig. 19.135 are equivalent. The parameters of the two circuits must be equal. Using this factor and the z parameters, derive Eqs. (9.67) and (9.68).

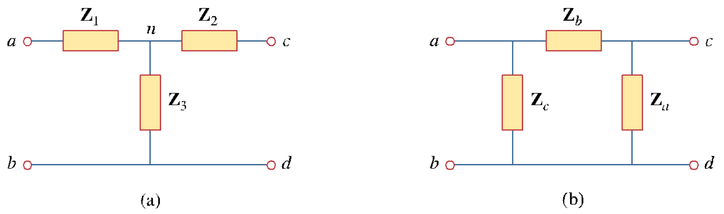


Figure 19.135
For Prob. 19.99.

Chapter 19, Solution 99

$$\mathbf{Z}_{ab} = \mathbf{Z}_1 + \mathbf{Z}_3 = \mathbf{Z}_c \parallel (\mathbf{Z}_b + \mathbf{Z}_a)$$

$$\mathbf{Z}_1 + \mathbf{Z}_3 = \frac{\mathbf{Z}_c(\mathbf{Z}_a + \mathbf{Z}_b)}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c} \qquad (1)$$

$$\mathbf{Z}_{cd} = \mathbf{Z}_2 + \mathbf{Z}_3 = \mathbf{Z}_a \parallel (\mathbf{Z}_b + \mathbf{Z}_c)$$

$$\mathbf{Z}_2 + \mathbf{Z}_3 = \frac{\mathbf{Z}_a(\mathbf{Z}_b + \mathbf{Z}_c)}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c} \qquad (2)$$

$$\mathbf{Z}_{ac} = \mathbf{Z}_1 + \mathbf{Z}_2 = \mathbf{Z}_b \parallel (\mathbf{Z}_a + \mathbf{Z}_c)$$

$$\mathbf{Z}_1 + \mathbf{Z}_2 = \frac{\mathbf{Z}_b(\mathbf{Z}_a + \mathbf{Z}_c)}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c} \qquad (3)$$

Subtracting (2) from (1),

$$\mathbf{Z}_1 - \mathbf{Z}_2 = \frac{\mathbf{Z}_b(\mathbf{Z}_c - \mathbf{Z}_a)}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c} \qquad (4)$$

Adding (3) and (4),

$$\mathbf{Z}_1 = \underline{\frac{\mathbf{Z}_b \mathbf{Z}_c}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c}} \qquad (5)$$

Subtracting (5) from (3),

$$\mathbf{Z}_2 = \frac{\mathbf{Z}_a \mathbf{Z}_b}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c} \qquad (6)$$

Subtracting (5) from (1),

$$\mathbf{Z}_3 = \frac{\mathbf{Z}_c \mathbf{Z}_a}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c} \qquad (7)$$

Using (5) to (7)

$$\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1 = \frac{\mathbf{Z}_a \mathbf{Z}_b \mathbf{Z}_c (\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c)}{(\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c)^2}$$

$$\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1 = \frac{\mathbf{Z}_a \mathbf{Z}_b \mathbf{Z}_c}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c} \qquad (8)$$

Dividing (8) by each of (5), (6), and (7),

$$\mathbf{Z}_a = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_1}$$

$$\mathbf{Z}_b = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_3}$$

$$\mathbf{Z}_c = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_2}$$

as required. Note that the formulas above are not exactly the same as those in Chapter 9 because the locations of $\mathbf{Z}_b$ and $\mathbf{Z}_c$ are interchanged in Fig. 18.122.